**Chapter 10**

## Fitting Regression Models

**Solutions**

**10.1S.** The tensile strength of a paper product is related to the amount of hardwood in the pulp. Ten samples are produced in the pilot plant, and the data obtained are shown in the following table.

|  |  |  |  |
| --- | --- | --- | --- |
| Strength | Percent Hardwood | Strength | Percent Hardwood |
| 160 | 10 | 181 | 20 |
| 171 | 15 | 188 | 25 |
| 175 | 15 | 193 | 25 |
| 182 | 20 | 195 | 28 |
| 184 | 20 | 200 | 30 |

(a) Fit a linear regression model relating strength to percent hardwood.

Minitab Output

**Regression Analysis: Strength versus Hardwood**

The regression equation is

Strength = 144 + 1.88 Hardwood

Predictor Coef SE Coef T P

Constant 143.824 2.522 57.04 0.000

Hardwood 1.8786 0.1165 16.12 0.000

S = 2.203 R-Sq = 97.0% R-Sq(adj) = 96.6%

PRESS = 66.2665 R-Sq(pred) = 94.91%



(b) Test the model in part (a) for significance of regression.

Minitab Output

Analysis of Variance

Source DF SS MS F P

Regression 1 1262.1 1262.1 260.00 0.000

Residual Error 8 38.8 4.9

Lack of Fit 4 13.7 3.4 0.54 0.716

Pure Error 4 25.2 6.3

Total 9 1300.9

3 rows with no replicates

No evidence of lack of fit (P > 0.1)

(c) Find a 95 percent confidence interval on the parameter *β*1.

The 95 percent confidence interval is:







**10.2.** Plot the residuals from Problem 10.1 and comment on model adequacy.



There is nothing unusual about the residual plots. The underlying assumptions have been met.

**10.3.** Using the results of Problem 10.1, test the regression model for lack of fit.

The Minitab output below identifies no evidence of lack of fit.

Minitab Output

Analysis of Variance

Source DF SS MS F P

Regression 1 1262.1 1262.1 260.00 0.000

Residual Error 8 38.8 4.9

Lack of Fit 4 13.7 3.4 0.54 0.716

Pure Error 4 25.2 6.3

Total 9 1300.9

3 rows with no replicates

No evidence of lack of fit (P > 0.1)

**10.4.** A study was performed on wear of a bearing *y* and its relationship to *x*1 = oil viscosity and *x*2 = load. The following data were obtained.

|  |  |  |
| --- | --- | --- |
| *y* | *x*1 | *x*2 |
| 193 | 1.6 | 851 |
| 230 | 15.5 | 816 |
| 172 | 22.0 | 1058 |
| 91 | 43.0 | 1201 |
| 113 | 33.0 | 1357 |
| 125 | 40.0 | 1115 |

(a) Fit a multiple linear regression model to the data.

Minitab Output

**Regression Analysis: Wear versus Viscosity, Load**

The regression equation is

Wear = 351 - 1.27 Viscosity - 0.154 Load

Predictor Coef SE Coef T P VIF

Constant 350.99 74.75 4.70 0.018

Viscosit -1.272 1.169 -1.09 0.356 2.6

Load -0.15390 0.08953 -1.72 0.184 2.6

S = 25.50 R-Sq = 86.2% R-Sq(adj) = 77.0%

PRESS = 12696.7 R-Sq(pred) = 10.03%

(b) Test for significance of regression.

Minitab Output

Analysis of Variance

Source DF SS MS F P

Regression 2 12161.6 6080.8 9.35 0.051

Residual Error 3 1950.4 650.1

Total 5 14112.0

No replicates. Cannot do pure error test.

Source DF Seq SS

Viscosit 1 10240.4

Load 1 1921.2

\* Not enough data for lack of fit test

(c) Compute *t* statistics for each model parameter. What conclusions can you draw?

Minitab Output

**Regression Analysis: Wear versus Viscosity, Load**

The regression equation is

Wear = 351 - 1.27 Viscosity - 0.154 Load

Predictor Coef SE Coef T P VIF

Constant 350.99 74.75 4.70 0.018

Viscosit -1.272 1.169 -1.09 0.356 2.6

Load -0.15390 0.08953 -1.72 0.184 2.6

S = 25.50 R-Sq = 86.2% R-Sq(adj) = 77.0%

PRESS = 12696.7 R-Sq(pred) = 10.03%

The *t*-tests are shown in part (a). Notice that overall regression is significant (part(b)), but neither variable has a large *t*-statistic. This could be an indicator that the regressors are nearly linearly dependent.

**10.5S.** The brake horsepower developed by an automobile engine on a dynamometer is thought to be a function of the engine speed in revolutions per minute (rpm), the road octane number of the fuel, and the engine compression. An experiment is run in the laboratory and the data that follow are collected.

|  |  |  |  |
| --- | --- | --- | --- |
| Brake Horsepower | rpm | Road Octane Number | Compression |
| 225 | 2000 | 90 | 100 |
| 212 | 1800 | 94 | 95 |
| 229 | 2400 | 88 | 110 |
| 222 | 1900 | 91 | 96 |
| 219 | 1600 | 86 | 100 |
| 278 | 2500 | 96 | 110 |
| 246 | 3000 | 94 | 98 |
| 237 | 3200 | 90 | 100 |
| 233 | 2800 | 88 | 105 |
| 224 | 3400 | 86 | 97 |
| 223 | 1800 | 90 | 100 |
| 230 | 2500 | 89 | 104 |

(a) Fit a multiple linear regression model to the data.

Minitab Output

**Regression Analysis: Horsepower versus rpm, Octane, Compression**

The regression equation is

Horsepower = - 266 + 0.0107 rpm + 3.13 Octane + 1.87 Compression

Predictor Coef SE Coef T P VIF

Constant -266.03 92.67 -2.87 0.021

rpm 0.010713 0.004483 2.39 0.044 1.0

Octane 3.1348 0.8444 3.71 0.006 1.0

Compress 1.8674 0.5345 3.49 0.008 1.0

S = 8.812 R-Sq = 80.7% R-Sq(adj) = 73.4%

PRESS = 2494.05 R-Sq(pred) = 22.33%

(b) Test for significance of regression. What conclusions can you draw?

Minitab Output

Analysis of Variance

Source DF SS MS F P

Regression 3 2589.73 863.24 11.12 0.003

Residual Error 8 621.27 77.66

Total 11 3211.00

r No replicates. Cannot do pure error test.

Source DF Seq SS

rpm 1 509.35

Octane 1 1132.56

Compress 1 947.83

Lack of fit test

Possible interactions with variable Octane (P-Value = 0.028)

Possible lack of fit at outer X-values (P-Value = 0.000)

Overall lack of fit test is significant at P = 0.000

(c) Based on *t* tests, do you need all three regressor variables in the model?

Yes, all the regressor variables are important.

**10.6.** Analyze the residuals from the regression model in Problem 10.5. Comment on model adequacy.







The normal probability plot is satisfactory, as is the plot of residuals versus run order (assuming that observation order is run order). The plot of residuals versus predicted exhibits a slight “bow” shape. This could be an indication of lack of fit. It might be useful to add some interaction terms to the model.

**10.7S.** The yield of a chemical process is related to the concentration of the reactant and the operating temperature. An experiment has been conducted with the following results.

|  |  |  |
| --- | --- | --- |
| Yield | Concentration | Temperature |
| 81 | 1.00 | 150 |
| 89 | 1.00 | 180 |
| 83 | 2.00 | 150 |
| 91 | 2.00 | 180 |
| 79 | 1.00 | 150 |
| 87 | 1.00 | 180 |
| 84 | 2.00 | 150 |
| 90 | 2.00 | 180 |

(a) Suppose we wish to fit a main effects model to this data. Set up the **X’X** matrix using the data exactly as it appears in the table.



(b) Is the matrix you obtained in part (a) diagonal? Discuss your response.

The **X’X** is not diagonal, even though an orthogonal design has been used. The reason is that we have worked with the natural factor levels, not the orthogonally coded variables.

(c) Suppose we write our model in terms of the “usual” coded variables

, 

Set up the **X’X** matrix for the model in terms of these coded variables. Is this matrix diagonal? Discuss your response.



The **X’X** matrix is diagonal because we have used the orthogonally coded variables.

(d) Define a new set of coded variables

, 

Set up the **X’X** matrix for the model in terms of this set of coded variables. Is this matrix diagonal? Discuss your response.



The **X’X** is not diagonal, even though an orthogonal design has been used. The reason is that we have not used orthogonally coded variables.

(e) Summarize what you have learned from this problem about coding the variables.

If the design is orthogonal, use the orthogonal coding. This not only makes the analysis somewhat easier, but it also results in model coefficients that are easier to interpret because they are both dimensionless and uncorrelated.

**10.8.** Consider the 24 factorial experiment in Example 6.2. Suppose that the last two observations are missing. Reanalyze the data and draw conclusions. How do these conclusions compare with those from the original example?

The regression analysis with two data points missing indicates that the same effects are important.

Minitab Output

**Regression Analysis: Rate versus A, B, C, D, AB, AC, AD, BC, BD, CD**

The regression equation is

Rate = 71.4 + 10.1 A + 2.87 B + 6.25 C + 8.62 D - 0.66 AB - 9.78 AC + 7.59 AD

+ 2.50 BC + 1.12 BD + 0.75 CD

Predictor Coef SE Coef T P VIF

Constant 71.375 1.673 42.66 0.000

A 10.094 1.323 7.63 0.005 1.1

B 2.875 1.673 1.72 0.184 1.7

C 6.250 1.673 3.74 0.033 1.7

D 8.625 1.673 5.15 0.014 1.7

AB -0.656 1.323 -0.50 0.654 1.1

AC -9.781 1.323 -7.39 0.005 1.1

AD 7.594 1.323 5.74 0.010 1.1

BC 2.500 1.673 1.49 0.232 1.7

BD 1.125 1.673 0.67 0.549 1.7

CD 0.750 1.673 0.45 0.684 1.7

S = 4.732 R-Sq = 98.7% R-Sq(adj) = 94.2%

PRESS = 1493.06 R-Sq(pred) = 70.20%

Analysis of Variance

Source DF SS MS F P

Regression 10 4943.17 494.32 22.07 0.014

Residual Error 3 67.19 22.40

Total 13 5010.36

No replicates. Cannot do pure error test.

Source DF Seq SS

A 1 1543.50

B 1 1.52

C 1 177.63

D 1 726.01

AB 1 1.17

AC 1 1702.53

AD 1 738.11

BC 1 42.19

BD 1 6.00

CD 1 4.50







The residual plots are acceptable; therefore, the underlying assumptions are valid.

**10.9.** Given the following data, fit the second-order polynomial regression model



|  |  |  |
| --- | --- | --- |
| *y* | *x*1 | *x*2 |
| 26 | 1.0 | 1.0 |
| 24 | 1.0 | 1.0 |
| 175 | 1.5 | 4.0 |
| 160 | 1.5 | 4.0 |
| 163 | 1.5 | 4.0 |
| 55 | 0.5 | 2.0 |
| 62 | 1.5 | 2.0 |
| 100 | 0.5 | 3.0 |
| 26 | 1.0 | 1.5 |
| 30 | 0.5 | 1.5 |
| 70 | 1.0 | 2.5 |
| 71 | 0.5 | 2.5 |

After you have fit the model, test for significance of regression.

Minitab Output

**Regression Analysis: y versus x1, x2, x1^2, x2^2, x1x2**

The regression equation is

y = 24.4 - 38.0 x1 + 0.7 x2 + 35.0 x1^2 + 11.1 x2^2 - 9.99 x1x2

Predictor Coef SE Coef T P VIF

Constant 24.41 26.59 0.92 0.394

x1 -38.03 40.45 -0.94 0.383 89.6

x2 0.72 11.69 0.06 0.953 52.1

x1^2 34.98 21.56 1.62 0.156 103.9

x2^2 11.066 3.158 3.50 0.013 104.7

x1x2 -9.986 8.742 -1.14 0.297 105.1

S = 6.042 R-Sq = 99.4% R-Sq(adj) = 98.9%

PRESS = 1327.71 R-Sq(pred) = 96.24%

r Analysis of Variance

Source DF SS MS F P

Regression 5 35092.6 7018.5 192.23 0.000

Residual Error 6 219.1 36.5

Lack of Fit 3 91.1 30.4 0.71 0.607

Pure Error 3 128.0 42.7

Total 11 35311.7

7 rows with no replicates

Source DF Seq SS

x1 1 11552.0

x2 1 22950.3

x1^2 1 21.9

x2^2 1 520.8

x1x2 1 47.6







**10.10.**

(a) Consider the quadratic regression model from Problem 10.9. Compute *t* statistics for each model parameter and comment on the conclusions that follow from the quantities.

Minitab Output

Predictor Coef SE Coef T P VIF

Constant 24.41 26.59 0.92 0.394

x1 -38.03 40.45 -0.94 0.383 89.6

x2 0.72 11.69 0.06 0.953 52.1

x1^2 34.98 21.56 1.62 0.156 103.9

x2^2 11.066 3.158 3.50 0.013 104.7

x1x2 -9.986 8.742 -1.14 0.297 105.1

 is the only model parameter that is statistically significant with a *t*-value of 3.50. A logical model might also include *x*2 to preserve model hierarchy.

(b) Use the extra sum of squares method to evaluate the value of the quadratic terms, and  to the model.

The extra sum of squares due to  is



 sum of squares of regression for the model in Problem 10.12 = 35092.6







Since, then the addition of the quadratic terms to the model is significant. The *p*-values indicate that it’s probably the term  that is responsible for this.

**10.11.** ***Relationship between analysis of variance and regression.*** Any analysis of variance model can be expressed in terms of the general linear model **y** = **Xβ** + **ε** , where the **X** matrix consists of zeros and ones. Show that the single-factor model , *i*=1,2,3, *j*=1,2,3,4 can be written in general linear model form. Then

(a) Write the normal equations and compare them with the normal equations found for the model in Chapter 3.

The normal equations are 

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which are in agreement with the results of Chapter 3.

(b) Find the rank of . Can  be obtained?

is a 4 x 4 matrix of rank 3, because the last three columns add to the first column. Thusdoes not exist.

(c) Suppose the first normal equation is deleted and the restriction  is added. Can the resulting system of equations be solved? If so, find the solution. Find the regression sum of squares , and compare it to the treatment sum of squares in the single-factor model.

Imposing  yields the normal equations

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The solution to this set of equations is





This solution was found by solving the last three equations for , yielding , and then substituting in the first equation to find 

The regression sum of squares is

****

with *a* degrees of freedom. This is the same result found in Chapter 3. For more discussion of the relationship between analysis of variance and regression, see Montgomery and Peck (1992).

**10.12.** Suppose that we are fitting a straight line and we desire to make the variance of  as small as possible. Restricting ourselves to an even number of experimental points, where should we place these points so as to minimize ? (Note: Use the design called for in this exercise with great caution because, even though it minimizes , it has some undesirable properties; for example, see Myers and Montgomery (1995). Only if you are *very sure* the true functional relationship is linear should you consider using this design.

Since , we may minimize by making *Sxx* as large as possible. *Sxx* is maximized by spreading out the *xj*’s as much as possible. The experimenter usually has a “region of interest” for *x*. If *n* is even, *n*/2 of the observations should be run at each end of the “region of interest”. If *n* is odd, then run one of the observations in the center of the region and the remaining (*n*-1)/2 at either end.

**10.13.** ***Weighted least squares.*** Suppose that we are fitting the straight line , but the variance of the *y*’s now depends on the level of *x*; that is,



where the *w*i are known constants, often called weights. Show that if we choose estimates of the regression coefficients to minimize the weighted sum of squared errors given by , the resulting least squares normal equations are





The least squares normal equations are found:



which simplify to



**10.14.** Consider the . Suppose after running the experiment, the largest observed effects are *A + BD*, *B + AD*, and *D + AB*. You wish to augment the original design with a group of four runs to de-alias these effects.

(a) Which four runs would you make?

Take the first four runs of the original experiment and change the sign on *A*.

Design Expert Output

Factor 1 Factor 2 Factor 3 Factor 4 Factor 5 Factor 6 Factor 7

Std Run Block A:x1 B:x2 C:x3 D:x4 E:x5 F:x6 G:x7

1 1 Block 1 -1 -1 -1 1 1 1 -1

2 2 Block 1 1 -1 -1 -1 -1 1 1

3 3 Block 1 -1 1 -1 -1 1 -1 1

4 4 Block 1 1 1 -1 1 -1 -1 -1

5 5 Block 1 -1 -1 1 1 -1 -1 1

6 6 Block 1 1 -1 1 -1 1 -1 -1

7 7 Block 1 -1 1 1 -1 -1 1 -1

8 8 Block 1 1 1 1 1 1 1 1

9 9 Block 2 1 -1 -1 1 1 1 -1

10 10 Block 2 -1 -1 -1 -1 -1 1 1

11 11 Block 2 1 1 -1 -1 1 -1 1

12 12 Block 2 -1 1 -1 1 -1 -1 -1

Main effects and interactions of interest are:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| x1 | x2 | x4 | x1x2 | x1x4 | x2x4 |
| -1 | -1 | 1 | 1 | -1 | -1 |
| 1 | -1 | -1 | -1 | -1 | 1 |
| -1 | 1 | -1 | -1 | 1 | -1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| -1 | -1 | 1 | 1 | -1 | -1 |
| 1 | -1 | -1 | -1 | -1 | 1 |
| -1 | 1 | -1 | -1 | 1 | -1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| **1** | **-1** | **1** | **-1** | **1** | **-1** |
| **-1** | **-1** | **-1** | **1** | **1** | **1** |
| **1** | **1** | **-1** | **1** | **-1** | **-1** |
| **-1** | **1** | **1** | **-1** | **-1** | **1** |

(b) Find the variances and covariances of the regression coefficients in the model







(c) Is it possible to de-alias these effects with fewer than four additional runs?

It is possible to de-alias these effects in only two runs. By utilizing *Design Expert’s* design augmentation *D*-optimal factorial function, the runs 9 and 10 (Block 2) were generated as follows:

Design Expert Output

Factor 1 Factor 2 Factor 3 Factor 4 Factor 5 Factor 6 Factor 7

Std Run Block A:x1 B:x2 C:x3 D:x4 E:x5 F:x6 G:x7

1 1 Block 1 -1 -1 -1 1 1 1 -1

2 2 Block 1 1 -1 -1 -1 -1 1 1

3 3 Block 1 -1 1 -1 -1 1 -1 1

4 4 Block 1 1 1 -1 1 -1 -1 -1

5 5 Block 1 -1 -1 1 1 -1 -1 1

6 6 Block 1 1 -1 1 -1 1 -1 -1

7 7 Block 1 -1 1 1 -1 -1 1 -1

8 8 Block 1 1 1 1 1 1 1 1

9 9 Block 2 1 -1 -1 1 -1 -1 -1

10 10 Block 2 -1 -1 -1 -1 -1 -1 -1

**10.15.** Consider the computer output from a regression model shown below.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Source | Sum of Squares | Degrees of Freedom | Mean Square | *F* |
| Model | 532,567.14 | 2 | ? | ? |
| Error | ? | ? | ? |  |
| Total | 539,534.33 | 14 |  |  |

(a) Fill in the blanks in the table.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Source | Sum of Squares | Degrees of Freedom | Mean Square | *F* |
| Model | 532,567.14 | 2 | 266,283.57 | 458.64 |
| Error | 6,967.12 | 12 | 580.60 |  |
| Total | 539,534.33 | 14 |  |  |

(b) How many regressor or predictor variables are in this model?

2 regressors.

(c) Find bounds on the *P*-value for the test on significance of regression.

0.0000

(d) Suppose that the *t*-statistic for one of the predictor variables in this model is 4.22. Find bounds on the *P*-value.

Minitab Output

**Probability Density Function**

Student’s t distribution with 12 DF

x f( x )

4.22 0.0010552

**10.16.** Consider the computer output from a regression model shown below.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Source | Sum of Squares | Degrees of Freedom | Mean Square | *F* |
| Model | 534.66 | ? | 133.67 | ? |
| Error | ? | ? | ? |  |
| Total | 590.25 | 19 |  |  |

(a) Fill in the blanks in the table.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Source | Sum of Squares | Degrees of Freedom | Mean Square | *F* |
| Model | 534.66 | 4 | 133.67 | 36.07 |
| Error | 55.59 | 15 | 3.71 |  |
| Total | 590.25 | 19 |  |  |

(b) How many regressor or predictor variables are in this model?

4 regressors.

(c) Compute the *F*-statistic for significance of regression and find the bounds on the *P*-value. 0.0000

0.0000

(d) Suppose that the *t*-statistic for one of the predictor variables in this model is 6.11. Find bounds on the *P*-value.

Minitab Output

**Probability Density Function**

Student’s t distribution with 15 DF

x f( x )

6.11 0.0000179

(e) What is the value of the *R*2 statistic for this model?



**10.17.** A regression model with 5 regressors has been built to 75 observations. The individual *t*-statistics for each predictor are as follows: 4.21, 2.12, 6.98, 3.55, and 2.45.

(a) Find the bounds on the *P*-values for each predictor.

Minitab Output

**Probability Density Function**

Student’s t distribution with 69 DF

x f( x )

4.21 0.0001331

**Probability Density Function**

Student’s t distribution with 69 DF

x f( x )

2.12 0.0436673

**Probability Density Function**

Student’s t distribution with 69 DF

x f( x )

6.98 0.0000000

**Probability Density Function**

Student’s t distribution with 69 DF

x f( x )

3.55 0.0011206

**Probability Density Function**

Student’s t distribution with 69 DF

x f( x )

2.45 0.0214488

(b) Based on your answers to part (a), are all of the regressor variables currently in the model necessary?

Yes.

**10.18.** A regression model with 3 regressors has been fit to a sample of 45 observations. The total sum of squares for the model is 275.6 and the model sum of squares is 245.86.

(a) Find the value of the mean square error.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Source | Sum of Squares | Degrees of Freedom | Mean Square | *F* |
| Model | 245.86 | 3 | 81.95 | 112.98 |
| Error | 29.74 | 41 | 0.725 |  |
| Total | 275.60 | 45-1=44 |  |  |

(b) Find the model mean square. 81.95

(c) Compute the value of the *F*-statistic for significance of regression and find bounds on the   
*P*-value. What are your conclusions? P=0.0000

(d) What is the value of *R*2 for this model?



(e) What is the value of the adjusted *R*2 statistic for this model?



**10.19.** A regression model with 4 regressors has been fit to a sample of 65 observations. The total sum of squares for the model is 300 and the model sum of squares is 280.

(a) What is the value of *R*2 for this model?



(b) What is the value of the adjusted *R*2 statistic for this model?



(c) Suppose that one of the regressors is removed from the model and the new model sum of squares is 250. What is the value of the adjusted *R*2 for this reduced model? What is the impact on model fit of removing this regressor?





Removing the regressor reduced both *R*2 values. Recommend the model with 3 regressors.

**10.20.** A regression model with 3 regressors has been built to a sample of 30 observations. The individual *t*-statistics for each predictor are as follows: 2.12, 5.45, and 17.50.

(a) Find bounds on the *P*-values for each predictor.

Minitab Output

**Probability Density Function**

Student’s t distribution with 26 DF

x f( x )

2.12 0.0459097

**Probability Density Function**

Student’s t distribution with 26 DF

x f( x )

5.45 0.0000135

**Probability Density Function**

Student’s t distribution with 26 DF

x f( x )

17.5 0.0000000

(b) Based on your answers to part (a), are all of the regressor variables currently in the model necessary?

Yes.

**10.21.** The value of the adjusted *R*2 statistic always increases when a new regressor variable is added to the model.

True **False**

**10.22.** The value of the ordinary *R*2 statistic can decrease when a new regressor variable is added to the model.

True **False**

**10.23.** The ordinary *R*2 statistic is a good indicator of the prediction capability of a regression model.

True **False**

**10.24.** If the P-value for the test for significance of regression is <0.01, this is an indication that all of the regressor variables in the model are necessary.

True **False**

**10.25.** If the *P*-value for the test for significance of regression is <0.01, this is an indication that the regression model will be a good predictor of new observations.

True **False**

**10.26.** A small value of the PRESS statistic is a good indication that the regression model will be a good predictor of new observations.

**True** False

**10.27.** The *t*-test on individual regression coefficients is a test on the utility of the regressor as if it were the last regressor added to a model that already contains the other regressors.

**True** False