**Chapter 12**

# Robust Parameter Design and Process Robustness Studies

**Solutions**

**12.1.** Reconsider the leaf spring experiment in Table 12.1. Suppose that the objective is to find a set of conditions where the mean free height is as close as possible to 7.6 inches with a variance of free height as small as possible. What conditions would you recommend to achieve these objectives?

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| *A* | *B* | *C* | *D* | *E*(–) | *E*(+) |  | *s*2 |
| – | – | – | – | 7.78,7.78, 7.81 | 7.50, 7,25, 7.12 | 7.54 | 0.090 |
| + | – | – | + | 8.15, 8.18, 7.88 | 7.88, 7.88, 7.44 | 7.90 | 0.071 |
| – | + | – | + | 7.50, 7.56, 7.50 | 7.50, 7.56, 7.50 | 7.52 | 0.001 |
| + | + | – | – | 7.59, 7.56, 7.75 | 7.63, 7.75, 7.56 | 7.64 | 0.008 |
| – | – | + | + | 7.54, 8.00, 7.88 | 7.32, 7.44, 7.44 | 7.60 | 0.074 |
| + | – | + | – | 7.69, 8.09, 8.06 | 7.56, 7.69, 7.62 | 7.79 | 0.053 |
| – | + | + | – | 7.56, 7.52, 7.44 | 7.18, 7.18, 7.25 | 7.36 | 0.030 |
| + | + | + | + | 7.56, 7.81, 7.69 | 7.81, 7.50, 7.59 | 7.66 | 0.017 |

By overlaying the contour plots for Free Height Mean and the Free Height Variance, optimal solutions can be found. To minimize the variance, factor *B* must be at the high level while factors *A* and *D* are adjusted to assure a mean of 7.6. The two overlay plots below set factor *D* at both low and high levels. Therefore, a mean as close as possible to 7.6 with minimum variance of 0.008 can be achieved at   
*A* = 0.78, *B* = +1, and *D* = -1. This can also be achieved with *A* = +0.07, *B* = +1, and *D* = +1.





Factor *D* = -1 Factor *D* = +1

**12.2S.** Consider the bottle filling experiment in Problem 6.20. Suppose that the percentage of carbonation (*A*) is a noise variable (in coded units).

(a) Fit the response model to these data. Is there a robust design problem?

The following is the analysis of variance with all terms in the model followed by a reduced model. Because the noise factor *A* is significant, and the *AB* interaction is moderately significant, there is a robust design problem.

Design Expert Output

**Response:** **Fill Deviation**

**ANOVA for Response Surface Reduced Cubic Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Cor Total 300.05 3

Model 73.00 7 10.43 16.69 0.0003 significant

*A 36.00 1 36.00 57.60 < 0.0001*

*B 20.25 1 20.25 32.40 0.0005*

*C 12.25 1 12.25 19.60 0.0022*

*AB 2.25 1 2.25 3.60 0.0943*

*AC 0.25 1 0.25 0.40 0.5447*

*BC 1.00 1 1.00 1.60 0.2415*

*ABC 1.00 1 1.00 1.60 0.2415*

Pure Error 5.00 8 0.63

Cor Total 78.00 15

Based on the above analysis, the *AC*, *BC*, and *ABC* interactions are removed from the model and used as error.

Design Expert Output

**Response:** **Fill Deviation**

**ANOVA for Response Surface Reduced Cubic Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Model 70.75 4 17.69 26.84 < 0.0001 significant

*A 36.00 1 36.00 54.62 < 0.0001*

*B 20.25 1 20.25 30.72 0.0002*

*C 12.25 1 12.25 18.59 0.0012*

*AB 2.25 1 2.25 3.41 0.0917*

Residual 7.25 11 0.66

Lack of Fit 2.25 3 0.75 1.20 0.3700 not significant

Pure Error 5.00 8 0.63

Cor Total 78.00 15

The Model F-value of 26.84 implies there is a 0.01% chance that a "Model F-Value"

this large could occur due to noise.

Std. Dev. 0.81 R-Squared 0.9071

Mean 1.00 Adj R-Squared 0.8733

C.V. 81.18 Pred R-Squared 0.8033

PRESS 15.34 Adeq Precision 15.424

**Final Equation in Terms of Coded Factors:**

Fill Deviation =

+1.00

+1.50 \* A

+1.13 \* B

+0.88 \* C

+0.38 \* A \* B

(b) Find the mean model and either the variance model or the POE.

From the final equation shown in the above analysis, the mean model and corresponding contour plot is shown below.





Contour and 3-D plots of the POE are shown below.



(c) Find a set of conditions that result in mean fill deviation as close to zero as possible with minimum transmitted variance.

The overlay plot below identifies a an operating region for pressure and speed that in a mean fill deviation as close to zero as possible with minimum transmitted variance.



**12.3.** Reconsider the leaf spring experiment from Table 12.1. Suppose that factors *A*, *B* and *C* are controllable variables, and that factors *D* and *E* are noise factors. Set up a crossed array design to investigate this problem, assuming that all of the two-factor interactions involving the controllable variables are thought to be important. What type of design have you obtained?

The following experimental design has a 23 inner array for the controllable variables and a 22 outer array for the noise factors. A total of 32 runs are required.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  | Outer Array | | | | |
| Inner Array | | | *D* | -1 | 1 | -1 | 1 |
| *A* | *B* | *C* | *E* | -1 | -1 | 1 | 1 |
| -1 | -1 | -1 |  |  |  |  |  |
| 1 | -1 | -1 |  |  |  |  |  |
| -1 | 1 | -1 |  |  |  |  |  |
| 1 | 1 | -1 |  |  |  |  |  |
| -1 | -1 | 1 |  |  |  |  |  |
| 1 | -1 | 1 |  |  |  |  |  |
| -1 | 1 | 1 |  |  |  |  |  |
| 1 | 1 | 1 |  |  |  |  |  |

**12.4. Continuation of Problem 12.3.** Reconsider the leaf spring experiment from Table 12.1. Suppose that *A*, *B* and *C* are controllable factors and that factors *D* and *E* are noise factors. Show how a combined array design can be employed to investigate this problem that allows all two-factor interactions to be estimated and only requires 16 runs. Compare this with the crossed array design from Problem 12.3. Can you see how in general combined array designs have fewer runs than crossed array designs?

The following experiment is a 25-1 fractional factorial experiment where the controllable factors are *A*, *B*, and *C* and the noise factors are *D* and *E*. Only 16 runs are required versus the 32 runs required for the crossed array design in Problem 12.3.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| *A* | *B* | *C* | *D* | *E* | Free Height |
| – | – | – | – | + |  |
| + | – | – | – | – |  |
| – | + | – | – | – |  |
| + | + | – | – | + |  |
| – | – | + | – | – |  |
| + | – | + | – | + |  |
| – | + | + | – | + |  |
| + | + | + | – | – |  |
| – | – | – | + | – |  |
| + | – | – | + | + |  |
| – | + | – | + | + |  |
| + | + | – | + | – |  |
| – | – | + | + | + |  |
| + | – | + | + | – |  |
| – | + | + | + | – |  |
| + | + | + | + | + |  |

**12.5.** Consider the connector pull-off force experiment shown in Table 12.2. Show how an experiment can be designed for this problem that will allow a full quadratic model to be fit in the controllable variables along all main effects of the noise variables and their interactions with the controllable variables. How many runs will be required in this design? How does this compare with the design in Table 12.2?

There are several designs that can be employed to achieve the requirements stated above. Below is a small central composite design with the axial points removed for the noise variables. Five center points are also included which brings the total runs to 35. As shown in the alias analysis, the full quadratic model for the controllable variables is achieved.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| *A* | *B* | *C* | *D* | *E* | *F* | *G* |
| +1 | +1 | +1 | -1 | +1 | +1 | +1 |
| +1 | +1 | -1 | +1 | -1 | +1 | -1 |
| +1 | +1 | -1 | +1 | +1 | -1 | +1 |
| +1 | -1 | +1 | +1 | -1 | +1 | +1 |
| -1 | +1 | +1 | -1 | -1 | +1 | -1 |
| +1 | -1 | -1 | -1 | +1 | -1 | -1 |
| -1 | +1 | -1 | +1 | +1 | -1 | +1 |
| +1 | +1 | +1 | +1 | -1 | +1 | -1 |
| +1 | -1 | +1 | -1 | -1 | -1 | +1 |
| -1 | -1 | -1 | -1 | +1 | +1 | -1 |
| -1 | +1 | -1 | +1 | -1 | -1 | -1 |
| +1 | +1 | +1 | -1 | +1 | -1 | -1 |
| +1 | -1 | -1 | +1 | -1 | -1 | -1 |
| -1 | -1 | +1 | -1 | -1 | -1 | +1 |
| -1 | +1 | -1 | -1 | -1 | +1 | +1 |
| +1 | -1 | -1 | -1 | -1 | +1 | +1 |
| -1 | +1 | -1 | -1 | +1 | +1 | +1 |
| +1 | -1 | -1 | +1 | +1 | +1 | +1 |
| -1 | -1 | +1 | +1 | +1 | -1 | +1 |
| -1 | -1 | +1 | +1 | +1 | +1 | -1 |
| -1 | +1 | +1 | +1 | -1 | -1 | +1 |
| -1 | -1 | -1 | -1 | -1 | -1 | -1 |
| -2.17 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2.17 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | -2.17 | 0 | 0 | 0 | 0 | 0 |
| 0 | 2.17 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | -2.17 | 0 | 0 | 0 | 0 |
| 0 | 0 | 2.17 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | -2.17 | 0 | 0 | 0 |
| 0 | 0 | 0 | 2.17 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Design Expert Output

**Alias Matrix**

**[Est. Terms] Aliased Terms**  
 [Intercept] = Intercept

[A] = A

[B] = B

[C] = C

[D] = D

[E] = E + 0.211 \* EG + 0.789 \* FG

[F] = F - EF - EG

[G] = G - EF - 0.158 \* EG + 0.158 \* FG

[A2] = A2

[B2] = B2

[C2] = C2

[D2] = D2

[E2] = E2 + F2 + G2

[AB] = AB - 0.105 \* EG - 0.895 \* FG

[AC] = AC - 0.158 \* EG + 0.158 \* FG

[AD] = AD + 0.421 \* EG + 0.579 \* FG

[AE] = AE - 0.474 \* EG + 0.474 \* FG

[AF] = AF + EF + 1.05 \* EG - 0.0526 \* FG

[AG] = AG + EF + 1.05 \* EG - 0.0526 \* FG

[BC] = BC - 0.263 \* EG + 0.263 \* FG

[BD] = BD - EF - 0.158 \* EG + 0.158 \* FG

[BE] = BE - 0.368 \* EG + 0.368 \* FG

[BF] = BF + 1.11 \* EG - 0.105 \* FG

[BG] = BG + EF + 0.421 \* EG - 0.421 \* FG

[CD] = CD - 0.421 \* EG + 0.421 \* FG

[CE] = CE - EF + 0.158 \* EG + 0.842 \* FG

[CF] = CF - EF - 0.211 \* EG + 0.211 \* FG

[CG] = CG - 1.21 \* EG + 0.211 \* FG

[DE] = DE - 0.842 \* EG - 0.158 \* FG

[DF] = DF - 0.211 \* EG + 0.211 \* FG

[DG] = DG - EF + 0.263 \* EG - 0.263 \* FG

**12.6.** Consider the experiment in Problem 11.8. Suppose that pressure is a noise variable (in coded units). Fit the response model for the viscosity response. Find a set of conditions that result in viscosity as close as possible to 600 and that minimize the variability transmitted from the noise variable pressure.

Design Expert Output

**Response:** **Viscosity**

**ANOVA for Response Surface Quadratic Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Model 85467.33 6 14244.56 12.12 0.0012 significant

*A 703.12 1 703.12 0.60 0.4615*

*B 6105.12 1 6105.12 5.19 0.0522*

*C 5408.00 1 5408.00 4.60 0.0643*

*A2 21736.93 1 21736.93 18.49 0.0026*

*C2 5153.80 1 5153.80 4.38 0.0696*

*AC 47742.25 1 47742.25 40.61 0.0002*

Residual 9404.00 8 1175.50

*Lack of Fit 7922.00 6 1320.33 1.78 0.4022 not significant*

*Pure Error 1482.00 2 741.00*

Cor Total 94871.33 14

The Model F-value of 12.12 implies the model is significant. There is only

a 0.12% chance that a "Model F-Value" this large could occur due to noise.

Std. Dev. 34.29 R-Squared 0.9009

Mean 575.33 Adj R-Squared 0.8265

C.V. 5.96 Pred R-Squared 0.6279

PRESS 35301.77 Adeq Precision 11.731

**Final Equation in Terms of Coded Factors:**

Viscosity =

+636.00

+9.37 \* A

+27.62 \* B

-26.00 \* C

-76.50 \* A2

-37.25 \* C2

+109.25 \* A \* C

From the final equation shown in the above analysis, the mean model is shown below.



The corresponding contour and 3-D plots for this model are shown below followed by the POE contour and 3-D plots. Finally, the stacked contour plot is presented identifying a region with viscosity between 590 and 610 while minimizing the variability transmitted from the noise variable pressure. These conditions are in the region of factor *A* = 0.5 and factor *B* = -1.







**12.7. A variation of Example 12.1.** In example 12.1 (which utilized data from Example 6.2) we found that one of the process variables (*B* = pressure) was not important. Dropping this variable produced two replicates of a 23 design. The data are shown below.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| *C* | *D* | *A*(-) | *A*(+) |  | *s*2 |
| – | – | 45, 48 | 71, 65 | 57.75 | 121.19 |
| + | – | 68, 80 | 60, 65 | 68.25 | 72.25 |
| – | + | 43, 45 | 100, 104 | 73.00 | 1124.67 |
| + | + | 75, 70 | 86, 96 | 81.75 | 134.92 |

Assume that *C* and *D* are controllable factors and that *A* is a noise factor.

(a) Fit a model to the mean response.

The following is the analysis of variance with all terms in the model:

Design Expert Output

**Response:** **Mean**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Model 300.05 3 100.02

*A* *92.64* *1* *92.64*

*B* *206.64* *1* *206.64*

*AB* *0.77* *1* *0.77*

Pure Error 0.000 0

Cor Total 300.05 3

Based on the above analysis, the *AB* interaction is removed from the model and used as error.

Design Expert Output

**Response:** **Mean**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Model 299.28 2 149.64 195.45 0.0505 not significant

*A* *92.64* *1* *92.64* *121.00* *0.0577*

*B* *206.64* *1* *206.64* *269.90* *0.0387*

Residual 0.77 1 0.77

Cor Total 300.05 3

The Model F-value of 195.45 implies there is a 5.05% chance that a "Model F-Value"

this large could occur due to noise.

Std. Dev. 0.87 R-Squared 0.9974

Mean 70.19 Adj R-Squared 0.9923

C.V. 1.25 Pred R-Squared 0.9592

PRESS 12.25 Adeq Precision 31.672

**Coefficient** **Standard** **95% CI** **95% CI**

**Factor** **Estimate** **DF** **Error** **Low** **High** **VIF**

Intercept 70.19 1 0.44 64.63 75.75

A-Concentration 4.81 1 0.44 -0.75 10.37 1.00

B-Stir Rate 7.19 1 0.44 1.63 12.75 1.00

**Final Equation in Terms of Coded Factors:**

Mean =

+70.19

+4.81 \* A

+7.19 \* B

**Final Equation in Terms of Actual Factors:**

Mean =

+70.18750

+4.81250 \* Concentration

+7.18750 \* Stir Rate

The following is a contour plot of the mean model:



(b) Fit a model to the ln(*s*2) response.

The following is the analysis of variance with all terms in the model:

Design Expert Output

**Response:** **Variance** **Transform:** **Natural log** **Constant:** **0**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Model 4.42 3 1.47

*A* *1.74* *1* *1.74*

*B* *2.03* *1* *2.03*

*AB* *0.64* *1* *0.64*

Pure Error 0.000 0

Cor Total 4.42 3

Based on the above analysis, the *AB* interaction is removed from the model and applied to the residual error.

Design Expert Output

**Response:** **Variance** **Transform:** **Natural log** **Constant:** **0**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Model 3.77 2 1.89 2.94 0.3815 not significant

*A* *1.74* *1* *1.74* *2.71* *0.3477*

*B* *2.03* *1* *2.03* *3.17* *0.3260*

Residual 0.64 1 0.64

Cor Total 4.42 3

The "Model F-value" of 2.94 implies the model is not significant relative to the noise. There is a

38.15 % chance that a "Model F-value" this large could occur due to noise.

Std. Dev. 0.80 R-Squared 0.8545

Mean 5.25 Adj R-Squared 0.5634

C.V. 15.26 Pred R-Squared -1.3284

PRESS 10.28 Adeq Precision 3.954

**Coefficient** **Standard** **95% CI** **95% CI**

**Factor** **Estimate** **DF** **Error** **Low** **High** **VIF**

Intercept 5.25 1 0.40 0.16 10.34

A-Concentration -0.66 1 0.40 -5.75 4.43 1.00

B-Stir Rate 0.71 1 0.40 -4.38 5.81 1.00

**Final Equation in Terms of Coded Factors:**

Ln(Variance) =

+5.25

-0.66 \* A

+0.71 \* B

**Final Equation in Terms of Actual Factors:**

Ln(Variance) =

+5.25185

-0.65945 \* Concentration

+0.71311 \* Stir Rate

The following is a contour plot of the variance model in the untransformed form:



(c) Find operating conditions that result in the mean filtration rate response exceeding 75 with minimum variance.

The overlay plot shown below identifies the region required by the process:



(d) Compare your results with those from Example 12.1 which used the transmission of error approach. How similar are the two answers.

The results are very similar. Both require the Concentration to be held at the high level while the stirring rate is held near the middle.

**12.8S.** In an article (“Let’s All Beware the Latin Square,” *Quality Engineering*, Vol. 1, 1989, pp. 453-465) J.S. Hunter illustrates some of the problems associated with 3*k-p* fractional factorial designs. Factor *A* is the amount of ethanol added to a standard fuel and factor *B* represents the air/fuel ratio. The response variable is carbon monoxide (CO) emission in g/m2. The design is shown below.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Design | | | |  | Observations | |
| *A* | *B* | *x*1 | *x*2 |  | *y* | |
| 0 | 0 | -1 | -1 |  | 66 | 62 |
| 1 | 0 | 0 | -1 |  | 78 | 81 |
| 2 | 0 | 1 | -1 |  | 90 | 94 |
| 0 | 1 | -1 | 0 |  | 72 | 67 |
| 1 | 1 | 0 | 0 |  | 80 | 81 |
| 2 | 1 | 1 | 0 |  | 75 | 78 |
| 0 | 2 | -1 | 1 |  | 68 | 66 |
| 1 | 2 | 0 | 1 |  | 66 | 69 |
| 2 | 2 | 1 | 1 |  | 60 | 58 |

Notice that we have used the notation system of 0, 1, and 2 to represent the low, medium, and high levels for the factors. We have also used a “geometric notation” of -1, 0, and 1. Each run in the design is replicated twice.

(a) Verify that the second-order model



is a reasonable model for this experiment. Sketch the CO concentration contours in the *x*1, *x*2 space.

In the computer output that follows, the “coded factors” model is in the -1, 0, +1 scale.

Design Expert Output

**Response:** **CO Emis**

**ANOVA for Response Surface Quadratic Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Model 1624.00 5 324.80 50.95 < 0.0001 significant

*A* *243.00* *1* *243.00* *38.12* *< 0.0001*

*B* *588.00* *1* *588.00* *92.24* *< 0.0001*

*A2* *81.00* *1* *81.00* *12.71* *0.0039*

*B2* *64.00* *1* *64.00* *10.04* *0.0081*

*AB* *648.00* *1* *648.00* *101.65* *< 0.0001*

Residual 76.50 12 6.37

*Lack of Fit* *30.00* *3* *10.00* *1.94* *0.1944* *not significant*

*Pure Error* *46.50* *9* *5.17*

Cor Total 1700.50 17

The Model F-value of 50.95 implies the model is significant. There is only

a 0.01% chance that a "Model F-Value" this large could occur due to noise.

Std. Dev. 2.52 R-Squared 0.9550

Mean 72.83 Adj R-Squared 0.9363

C.V. 3.47 Pred R-Squared 0.9002

PRESS 169.71 Adeq Precision 21.952

**Coefficient** **Standard** **95% CI** **95% CI**

**Factor** **Estimate** **DF** **Error** **Low** **High** **VIF**

Intercept 78.50 1 1.33 75.60 81.40

A-Ethanol 4.50 1 0.73 2.91 6.09 1.00

B-Air/Fuel Ratio -7.00 1 0.73 -8.59 -5.41 1.00

A2 -4.50 1 1.26 -7.25 -1.75 1.00

B2 -4.00 1 1.26 -6.75 -1.25 1.00

AB -9.00 1 0.89 -10.94 -7.06 1.00

**Final Equation in Terms of Coded Factors:**

CO Emis =

+78.50

+4.50 \* A

-7.00 \* B

-4.50 \* A2

-4.00 \* B2

-9.00 \* A \* B



(b) Now suppose that instead of only two factors, we had used *four* factors in a 34-2 fractional factorial design and obtained *exactly* the same data in part (a). The design would be as follows:

Design Observations

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *A* | *B* | *C* | *D* | *x*1 | *x*2 | *x*3 | *x*4 | *y* | *y* |
| 0 | 0 | 0 | 0 | -1 | -1 | -1 | -1 | 66 | 62 |
| 1 | 0 | 1 | 1 | 0 | -1 | 0 | 0 | 78 | 81 |
| 2 | 0 | 2 | 2 | +1 | -1 | +1 | +1 | 90 | 94 |
| 0 | 1 | 2 | 1 | -1 | 0 | +1 | 0 | 72 | 67 |
| 1 | 1 | 0 | 2 | 0 | 0 | -1 | +1 | 80 | 81 |
| 2 | 1 | 1 | 0 | +1 | 0 | 0 | -1 | 75 | 78 |
| 0 | 2 | 1 | 2 | -1 | +1 | 0 | +1 | 68 | 66 |
| 1 | 2 | 2 | 0 | 0 | +1 | +1 | -1 | 66 | 69 |
| 2 | 2 | 0 | 1 | +1 | +1 | -1 | 0 | 60 | 58 |

Calculate the marginal averages of the CO response at each level of four factors *A*, *B*, *C*, and *D*. Construct plots of these marginal averages and interpret the results. Do factors *C* and *D* appear to have strong effects? Do these factors *really* have any effect on CO emission? Why is their appearance effect strong?

The marginal averages are shown below. Both Factors *C* and *D* appear to have an effect on CO emission. This is probably because both *C* and *D* are aliased with components of the interaction involving *A* and *B*, and there is a strong *AB* interaction.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Level | *A* | *B* | *C* | *D* |
| 0 | 66.83 | 78.50 | 67.83 | 69.33 |
| 1 | 75.83 | 75.50 | 74.33 | 69.33 |
| 2 | 75.83 | 64.50 | 76.33 | 79.83 |





(c) The design in part (b) allows the model



to be fitted. Suppose that the *true* model is



Show that if represents the least squares estimates of the coefficients in the fitted model, then



Does this help explain the strong effects for factors *C* and *D* observed graphically in part (b)?

 





**12.9.** An experiment has been run in a process that applies a coating material to a wafer. Each run in the experiment produced a wafer, and the coating thickness was measured several times at different locations on the wafer. Then the mean *y*1, and standard deviation *y*2 of the thickness measurement was obtained. The data [adapted from Box and Draper (1987)] are shown in the table below.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Run | Speed | Pressure | Distance | Mean (*y*1) | Std Dev (*y*2) |
| 1 | -1 | -1 | -1 | 24.0 | 12.5 |
| 2 | 0 | -1 | -1 | 120.3 | 8.4 |
| 3 | 1 | -1 | -1 | 213.7 | 42.8 |
| 4 | -1 | 0 | -1 | 86.0 | 3.5 |
| 5 | 0 | 0 | -1 | 136.6 | 80.4 |
| 6 | 1 | 0 | -1 | 340.7 | 16.2 |
| 7 | -1 | 1 | -1 | 112.3 | 27.6 |
| 8 | 0 | 1 | -1 | 256.3 | 4.6 |
| 9 | 1 | 1 | -1 | 271.7 | 23.6 |
| 10 | -1 | -1 | 0 | 81.0 | 0.0 |
| 11 | 0 | -1 | 0 | 101.7 | 17.7 |
| 12 | 1 | -1 | 0 | 357.0 | 32.9 |
| 13 | -1 | 0 | 0 | 171.3 | 15.0 |
| 14 | 0 | 0 | 0 | 372.0 | 0.0 |
| 15 | 1 | 0 | 0 | 501.7 | 92.5 |
| 16 | -1 | 1 | 0 | 264.0 | 63.5 |
| 17 | 0 | 1 | 0 | 427.0 | 88.6 |
| 18 | 1 | 1 | 0 | 730.7 | 21.1 |
| 19 | -1 | -1 | 1 | 220.7 | 133.8 |
| 20 | 0 | -1 | 1 | 239.7 | 23.5 |
| 21 | 1 | -1 | 1 | 422.0 | 18.5 |
| 22 | -1 | 0 | 1 | 199.0 | 29.4 |
| 23 | 0 | 0 | 1 | 485.3 | 44.7 |
| 24 | 1 | 0 | 1 | 673.7 | 158.2 |
| 25 | -1 | 1 | 1 | 176.7 | 55.5 |
| 26 | 0 | 1 | 1 | 501.0 | 138.9 |
| 27 | 1 | 1 | 1 | 1010.0 | 142.4 |

(a) What type of design did the experimenters use? Is this a good choice of design for fitting a quadratic model?

The design is a 33. A better choice would be a 23 central composite design. The CCD gives more information over the design region with fewer points.

(b) Build models of both responses.

The model for the mean is developed as follows:

Design Expert Output

**Response:** **Mean**

**ANOVA for Response Surface Reduced Cubic Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Model 1.289E+006 7 1.841E+005 60.45 < 0.0001 significant

*A* *5.640E+005* *1* *5.640E+005* *185.16* *< 0.0001*

*B* *2.155E+005* *1* *2.155E+005* *70.75* *< 0.0001*

*C* *3.111E+005* *1* *3.111E+005* *102.14* *< 0.0001*

*AB* *52324.81* *1* *52324.81* *17.18* *0.0006*

*AC* *68327.52* *1* *68327.52* *22.43* *0.0001*

*BC* *22794.08* *1* *22794.08* *7.48* *0.0131*

*ABC* *54830.16* *1* *54830.16* *18.00* *0.0004*

Residual 57874.57 19 3046.03

Cor Total 1.347E+006 26

The Model F-value of 60.45 implies the model is significant. There is only

a 0.01% chance that a "Model F-Value" this large could occur due to noise.

Std. Dev. 55.19 R-Squared 0.9570

Mean 314.67 Adj R-Squared 0.9412

C.V. 17.54 Pred R-Squared 0.9056

PRESS 1.271E+005 Adeq Precision 33.333

**Coefficient** **Standard** **95% CI** **95% CI**

**Factor** **Estimate** **DF** **Error** **Low** **High** **VIF**

Intercept 314.67 1 10.62 292.44 336.90

A-Speed 177.01 1 13.01 149.78 204.24 1.00

B-Pressure 109.42 1 13.01 82.19 136.65 1.00

C-Distance 131.47 1 13.01 104.24 158.70 1.00

AB 66.03 1 15.93 32.69 99.38 1.00

AC 75.46 1 15.93 42.11 108.80 1.00

BC 43.58 1 15.93 10.24 76.93 1.00

ABC 82.79 1 19.51 41.95 123.63 1.00

**Final Equation in Terms of Coded Factors:**

Mean =

+314.67

+177.01 \* A

+109.42 \* B

+131.47 \* C

+66.03 \* A \* B

+75.46 \* A \* C

+43.58 \* B \* C

+82.79 \* A \* B \* C

**Final Equation in Terms of Actual Factors:**

Mean =

+314.67037

+177.01111 \* Speed

+109.42222 \* Pressure

+131.47222 \* Distance

+66.03333 \* Speed \* Pressure

+75.45833 \* Speed \* Distance

+43.58333 \* Pressure \* Distance

+82.78750 \* Speed \* Pressure \* Distance

The model for the Std. Dev. response is as follows. A square root transformation was applied to correct problems with the normality assumption.

Design Expert Output

**Response:** **Std. Dev.** **Transform:** **Square root** **Constant:** **0**

**ANOVA for Response Surface Linear Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Model 116.75 3 38.92 4.34 0.0145 significant

*A* *16.52* *1* *16.52* *1.84* *0.1878*

*B* *26.32* *1* *26.32* *2.94* *0.1001*

*C* *73.92* *1* *73.92* *8.25* *0.0086*

Residual 206.17 23 8.96

Cor Total 322.92 26

The Model F-value of 4.34 implies the model is significant. There is only

a 1.45% chance that a "Model F-Value" this large could occur due to noise.

Std. Dev. 2.99 R-Squared 0.3616

Mean 6.00 Adj R-Squared 0.2783

C.V. 49.88 Pred R-Squared 0.1359

PRESS 279.05 Adeq Precision 7.278

**Coefficient** **Standard** **95% CI** **95% CI**

**Factor** **Estimate** **DF** **Error** **Low** **High** **VIF**

Intercept 6.00 1 0.58 4.81 7.19

A-Speed 0.96 1 0.71 -0.50 2.42 1.00

B-Pressure 1.21 1 0.71 -0.25 2.67 1.00

C-Distance 2.03 1 0.71 0.57 3.49 1.00

**Final Equation in Terms of Coded Factors:**

Sqrt(Std. Dev.) =

+6.00

+0.96 \* A

+1.21 \* B

+2.03 \* C

**Final Equation in Terms of Actual Factors:**

Sqrt(Std. Dev.) =

+6.00273

+0.95796 \* Speed

+1.20916 \* Pressure

+2.02643 \* Distance

Because Factor *A* is insignificant, it is removed from the model. The reduced linear model analysis is shown below:

Design Expert Output

**Response:** **Std. Dev.** **Transform:** **Square root** **Constant:** **0**

**ANOVA for Response Surface Reduced Linear Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Model 100.23 2 50.12 5.40 0.0116 significant

*B 26.32 1 26.32 2.84 0.1051*

*C 73.92 1 73.92 7.97 0.0094*

Residual 222.68 24 9.28

Cor Total 322.92 26

The Model F-value of 5.40 implies the model is significant. There is only

a 1.16% chance that a "Model F-Value" this large could occur due to noise.

Std. Dev. 3.05 R-Squared 0.3104

Mean 6.00 Adj R-Squared 0.2529

C.V. 50.74 Pred R-Squared 0.1476

PRESS 275.24 Adeq Precision 6.373

**Coefficient** **Standard** **95% CI** **95% CI**

**Factor** **Estimate** **DF** **Error** **Low** **High** **VIF**

Intercept 6.00 1 0.59 4.79 7.21

B-Pressure 1.21 1 0.72 -0.27 2.69 1.00

C-Distance 2.03 1 0.72 0.54 3.51 1.00

**Final Equation in Terms of Coded Factors:**

Sqrt(Std. Dev.) =

+6.00

+1.21 \* B

+2.03 \* C

**Final Equation in Terms of Actual Factors:**

Sqrt(Std. Dev.) =

+6.00273

+1.20916 \* Pressure

+2.02643 \* Distance

The following contour plots graphically represent the two models.



(c) Find a set of optimum conditions that result in the mean as large as possible with the standard deviation less than 60.

The overlay plot identifies a region that meets the criteria of the mean as large as possible with the standard deviation less than 60. The optimum conditions in coded terms are approximately Speed = 1.0, Pressure = 0.75 and Distance = 0.25.



**12.10S.** Suppose that there are four controllable variables and two noise variables. It is necessary to estimate the main effects and two-factor interactions of all of the controllable variables, the main effects of the noise variables, and the two-factor interactions between all controllable and noise factors. If all factors are at two levels, what is the minimum number of runs that can be used to estimate all of the model parameters using a combined array design? Use a *D*-optimal algorithm to find a design.

Twenty-one runs are required for the model, with five additional runs for lack of fit, and five as replicates for a total of 31 runs as follows. It should be noted that *Design* *Expert’s* *D*-optimal algorithm might not create the same design if repeated.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Std | *A* | *B* | *C* | *D* | *E* | *F* |
| 1 | +1 | +1 | -1 | +1 | +1 | +1 |
| 2 | -1 | +1 | -1 | +1 | -1 | -1 |
| 3 | +1 | -1 | -1 | +1 | -1 | -1 |
| 4 | +1 | +1 | -1 | -1 | -1 | +1 |
| 5 | -1 | +1 | -1 | -1 | +1 | +1 |
| 6 | -1 | +1 | +1 | +1 | +1 | +1 |
| 7 | +1 | +1 | -1 | -1 | +1 | -1 |
| 8 | -1 | -1 | +1 | +1 | -1 | -1 |
| 9 | -1 | +1 | +1 | -1 | +1 | -1 |
| 10 | -1 | +1 | +1 | -1 | -1 | +1 |
| 11 | +1 | -1 | +1 | +1 | +1 | +1 |
| 12 | +1 | +1 | +1 | +1 | -1 | +1 |
| 13 | +1 | -1 | -1 | -1 | +1 | +1 |
| 14 | +1 | +1 | +1 | -1 | +1 | +1 |
| 15 | -1 | -1 | -1 | -1 | -1 | -1 |
| 16 | +1 | +1 | +1 | +1 | +1 | -1 |
| 17 | -1 | -1 | -1 | +1 | -1 | +1 |
| 18 | -1 | -1 | -1 | +1 | +1 | -1 |
| 19 | -1 | -1 | +1 | -1 | +1 | +1 |
| 20 | +1 | -1 | +1 | -1 | +1 | -1 |
| 21 | +1 | -1 | +1 | -1 | -1 | +1 |
| 22 | +1 | +1 | +1 | -1 | -1 | -1 |
| 23 | +1 | -1 | -1 | -1 | -1 | -1 |
| 24 | -1 | +1 | -1 | -1 | -1 | -1 |
| 25 | +1 | +1 | -1 | -1 | -1 | -1 |
| 26 | -1 | -1 | +1 | -1 | -1 | -1 |
| 27 | +1 | +1 | +1 | +1 | -1 | +1 |
| 28 | -1 | -1 | -1 | +1 | -1 | +1 |
| 29 | +1 | +1 | +1 | +1 | +1 | -1 |
| 30 | -1 | -1 | -1 | +1 | +1 | -1 |
| 31 | -1 | +1 | -1 | -1 | +1 | +1 |

**12.11S.** Suppose that there are four controllable variables and two noise variables. It is necessary to fit a complete quadratic model in the controllable variables, the main effects of the noise variables, and the two-factor interactions between all controllable and noise factors. Set up a combined array design for this by modifying a central composite design.

The following design is a half fraction central composite design with the axial points removed from the noise factors. The total number of runs is forty-eight which includes eight center points.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Std | *A* | *B* | *C* | *D* | *E* | *F* |
| 1 | -1 | -1 | -1 | -1 | -1 | -1 |
| 2 | +1 | -1 | -1 | -1 | -1 | +1 |
| 3 | -1 | +1 | -1 | -1 | -1 | +1 |
| 4 | +1 | +1 | -1 | -1 | -1 | -1 |
| 5 | -1 | -1 | +1 | -1 | -1 | +1 |
| 6 | +1 | -1 | +1 | -1 | -1 | -1 |
| 7 | -1 | +1 | +1 | -1 | -1 | -1 |
| 8 | +1 | +1 | +1 | -1 | -1 | +1 |
| 9 | -1 | -1 | -1 | +1 | -1 | +1 |
| 10 | +1 | -1 | -1 | +1 | -1 | -1 |
| 11 | -1 | +1 | -1 | +1 | -1 | -1 |
| 12 | +1 | +1 | -1 | +1 | -1 | +1 |
| 13 | -1 | -1 | +1 | +1 | -1 | -1 |
| 14 | +1 | -1 | +1 | +1 | -1 | +1 |
| 15 | -1 | +1 | +1 | +1 | -1 | +1 |
| 16 | +1 | +1 | +1 | +1 | -1 | -1 |
| 17 | -1 | -1 | -1 | -1 | +1 | +1 |
| 18 | +1 | -1 | -1 | -1 | +1 | -1 |
| 19 | -1 | +1 | -1 | -1 | +1 | -1 |
| 20 | +1 | +1 | -1 | -1 | +1 | +1 |
| 21 | -1 | -1 | +1 | -1 | +1 | -1 |
| 22 | +1 | -1 | +1 | -1 | +1 | +1 |
| 23 | -1 | +1 | +1 | -1 | +1 | +1 |
| 24 | +1 | +1 | +1 | -1 | +1 | -1 |
| 25 | -1 | -1 | -1 | +1 | +1 | -1 |
| 26 | +1 | -1 | -1 | +1 | +1 | +1 |
| 27 | -1 | +1 | -1 | +1 | +1 | +1 |
| 28 | +1 | +1 | -1 | +1 | +1 | -1 |
| 29 | -1 | -1 | +1 | +1 | +1 | +1 |
| 30 | +1 | -1 | +1 | +1 | +1 | -1 |
| 31 | -1 | +1 | +1 | +1 | +1 | -1 |
| 32 | +1 | +1 | +1 | +1 | +1 | +1 |
| 33 | -2.378 | 0 | 0 | 0 | 0 | 0 |
| 34 | +2.378 | 0 | 0 | 0 | 0 | 0 |
| 35 | 0 | -2.378 | 0 | 0 | 0 | 0 |
| 36 | 0 | +2.378 | 0 | 0 | 0 | 0 |
| 37 | 0 | 0 | -2.378 | 0 | 0 | 0 |
| 38 | 0 | 0 | +2.378 | 0 | 0 | 0 |
| 39 | 0 | 0 | 0 | -2.378 | 0 | 0 |
| 40 | 0 | 0 | 0 | +2.378 | 0 | 0 |
| 41 | 0 | 0 | 0 | 0 | 0 | 0 |
| 42 | 0 | 0 | 0 | 0 | 0 | 0 |
| 43 | 0 | 0 | 0 | 0 | 0 | 0 |
| 44 | 0 | 0 | 0 | 0 | 0 | 0 |
| 45 | 0 | 0 | 0 | 0 | 0 | 0 |
| 46 | 0 | 0 | 0 | 0 | 0 | 0 |
| 47 | 0 | 0 | 0 | 0 | 0 | 0 |
| 48 | 0 | 0 | 0 | 0 | 0 | 0 |

**12.12.** Reconsider the situation in Problem 12.11. What is the minimum number of runs that can be used to estimate all of the model parameters using a combined array design? Use a *D*-optimal algorithm to find a reasonable design for this problem.

A minimum of 25 runs is required. The following design is a 36 run D-optimal with six additional runs included for lack of fit and five as replicates. It should be noted that Design Expert’s *D*-optimal algorithm might not create the same design if repeated.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Std | *A* | *B* | *C* | *D* | *E* | *F* |
| 1 | +1 | +1 | +1 | -1 | -1 | -1 |
| 2 | -1 | +1 | -1 | -1 | +1 | +1 |
| 3 | -1 | +1 | +1 | +1 | -1 | +1 |
| 4 | +1 | +1 | -1 | +1 | -1 | -1 |
| 5 | -1 | -1 | +1 | +1 | -1 | -1 |
| 6 | -1 | +1 | -1 | -1 | -1 | -1 |
| 7 | +1 | -1 | -1 | +1 | +1 | -1 |
| 8 | +1 | -1 | +1 | -1 | +1 | -1 |
| 9 | +1 | +1 | -1 | +1 | +1 | +1 |
| 10 | +1 | -1 | -1 | -1 | -1 | -1 |
| 11 | +1 | -1 | +1 | +1 | -1 | +1 |
| 12 | -1 | +1 | -1 | +1 | +1 | -1 |
| 13 | +1 | +1 | +1 | -1 | +1 | +1 |
| 14 | +1 | +1 | -1 | -1 | -1 | +1 |
| 15 | +1 | +1 | +1 | +1 | +1 | -1 |
| 16 | -1 | -1 | -1 | +1 | -1 | +1 |
| 17 | 0 | -1 | -1 | -1 | +1 | -1 |
| 18 | 0 | -1 | +1 | -1 | -1 | +1 |
| 19 | 0 | +1 | 0 | 0 | 0 | 0 |
| 20 | 0 | 0 | 0 | -1 | 0 | 0 |
| 21 | 0 | 0 | +1 | 0 | 0 | 0 |
| 22 | -1 | +1 | +1 | -1 | +1 | -1 |
| 23 | -1 | -1 | +1 | 0 | +1 | +1 |
| 24 | +1 | +1 | -1 | -1 | +1 | -1 |
| 25 | 0 | -1 | +1 | +1 | +1 | +1 |
| 26 | +1 | -1 | -1 | -1 | +1 | +1 |
| 27 | -1 | -1 | -1 | 0 | -1 | -1 |
| 28 | +1 | -1 | 0 | +1 | -1 | -1 |
| 29 | -1 | -1 | 0 | +1 | +1 | -1 |
| 30 | +1 | -1 | -1 | 0 | -1 | +1 |
| 31 | -1 | 0 | -1 | +1 | +1 | +1 |
| 32 | +1 | +1 | +1 | +1 | +1 | -1 |
| 33 | +1 | -1 | +1 | -1 | +1 | -1 |
| 34 | -1 | +1 | +1 | +1 | -1 | +1 |
| 35 | +1 | +1 | +1 | -1 | -1 | -1 |
| 36 | +1 | +1 | -1 | +1 | +1 | +1 |

**12.13.** Rework Problem 12.12 using the *I*-criterion to construct the design. Compare this design to the *D*-optimal design in Problem 12.12. Which design would you prefer?

The JMP Output shown below identifies an *I*-optimal design with 25 runs. It should be noted that JMP’s *I*-optimal design algorithm might not create the same design if repeated. We would prefer the *I*-optimal over the *D*-optimal because a response surface model is of interest in this problem.

JMP Output

**Custom Design**

**Factors**

**Add N Factors**

1

X1 Continuous

X2 Continuous

X3 Continuous

X4 Continuous

Noise1 Continuous

Noise2 Continuous

**Design**

Run X1 X2 X3 X4 Noise1 Noise2

1 1 -1 0 -1 -1 1

2 -1 1 -1 1 0 1

3 0 0 1 1 1 -1

4 1 -1 -1 -1 -1 -1

5 1 1 1 -1 -1 -1

6 1 1 -1 0 -1 1

7 -1 1 1 1 -1 -1

8 1 -1 1 1 0 1

9 -1 1 0 -1 1 1

10 -1 -1 -1 0 -1 1

11 -1 -1 0 -1 0 -1

12 1 0 0 0 1 -1

13 0 0 0 1 -1 1

14 -1 -1 0 1 1 1

15 0 -1 1 0 -1 -1

16 -1 1 1 -1 1 -1

17 1 1 -1 1 1 -1

18 -1 0 1 -1 -1 0

19 0 -1 1 -1 1 1

20 0 1 0 -1 -1 -1

21 1 -1 -1 1 1 1

22 1 1 -1 -1 1 1

23 0 0 -1 0 1 -1

24 -1 -1 -1 1 -1 -1

25 0 1 1 0 1 1

**12.14.** Rework Problem 12.10S using the *I*-criterion. Compare this to the *D*-optimal design in Problem 12.12. Which design would you prefer?

The JMP Output shown below identifies an *I*-optimal design with 21 runs. It should be noted that JMP’s *I*-optimal design algorithm might not create the same design if repeated. Because the problem requires the experiment be run in at only two levels for each variable, and *I*-optimal algorithm in JMP generates center points for continuous variables, categorical variables were used. We would prefer the *D*-optimal over the *I*-optimal assuming that the interest is to identify the important factors more so than to fit a response surface model. Also, the *D*-optimal algorithm in Design Expert will generate a design for this model with continuous variables and still maintain the requirement for running the experiment at only two levels for each variable.

JMP Output

**Custom Design**

**Factors**

**Add N Factors**

1

X1 Continuous

X2 Continuous

X3 Continuous

X4 Continuous

Noise1 Continuous

Noise2 Continuous

**Design**

Run X1 X2 X3 X4 Noise1 Noise2

1 1 1 1 1 -1 -1

2 -1 1 -1 -1 1 -1

3 -1 1 -1 1 1 -1

4 -1 1 -1 -1 -1 -1

5 -1 -1 1 1 -1 -1

6 1 -1 -1 1 -1 -1

7 1 1 1 -1 1 -1

8 -1 1 1 1 -1 -1

9 -1 -1 1 -1 -1 1

10 -1 -1 -1 1 -1 -1

11 1 -1 -1 1 -1 -1

12 1 -1 1 -1 -1 -1

13 1 1 -1 -1 1 1

14 1 1 1 1 -1 1

15 -1 1 1 -1 1 1

16 -1 -1 1 -1 -1 1

17 -1 -1 1 1 -1 1

18 -1 1 1 1 1 -1

19 -1 -1 -1 -1 1 1

20 1 1 -1 -1 -1 1

21 -1 -1 -1 1 -1 1

22 1 1 -1 -1 1 1

**12.15.** An experiment was run in a wave soldering process. There are five controllable variables and three noise variables. The response variable is the number of solder defects per million opportunities. The experimental design employed was the crossed array shown below.

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  | Outer Array | | | | |
|  |  |  |  |  | *F* | -1 | 1 | 1 | -1 |
| Inner Array | | | | | *G* | -1 | 1 | -1 | 1 |
| *A* | *B* | *C* | *D* | *E* | *H* | -1 | -1 | 1 | 1 |
| 1 | 1 | 1 | -1 | -1 |  | 194 | 197 | 193 | 275 |
| 1 | 1 | -1 | 1 | 1 |  | 136 | 136 | 132 | 136 |
| 1 | -1 | 1 | -1 | 1 |  | 185 | 261 | 264 | 264 |
| 1 | -1 | -1 | 1 | -1 |  | 47 | 125 | 127 | 42 |
| -1 | 1 | 1 | 1 | -1 |  | 295 | 216 | 204 | 293 |
| -1 | 1 | -1 | -1 | 1 |  | 234 | 159 | 231 | 157 |
| -1 | -1 | 1 | 1 | 1 |  | 328 | 326 | 247 | 322 |
| -1 | -1 | -1 | -1 | -1 |  | 186 | 187 | 105 | 104 |

(a) What types of designs were used for the inner and outer arrays?

The inner array is a 25-2 fractional factorial design with a defining relation of *I = -ACD = -BCE = ABDE*. The outer array is a 23-1 fractional factorial design with a defining relation of *I = -FGH*.

(b) Develop models for the mean and variance of solder defects. What set of operating conditions would you recommend?

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| *A* | *B* | *C* | *D* | *E* |  | *s*2 |
| 1 | 1 | 1 | -1 | -1 | 214.75 | 1616.25 |
| 1 | 1 | -1 | 1 | 1 | 135.00 | 4.00 |
| 1 | -1 | 1 | -1 | 1 | 243.50 | 1523.00 |
| 1 | -1 | -1 | 1 | -1 | 85.25 | 2218.92 |
| -1 | 1 | 1 | 1 | -1 | 252.00 | 2376.67 |
| -1 | 1 | -1 | -1 | 1 | 195.25 | 1852.25 |
| -1 | -1 | 1 | 1 | 1 | 305.75 | 1540.25 |
| -1 | -1 | -1 | -1 | -1 | 145.50 | 2241.67 |

The following analysis identifies factors A, C, and E as being significant for the solder defects mean model.



Design Expert Output

**Response:** **Solder Defects Mean**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Model 36068.63 3 12022.88 194.31 < 0.0001 significant

*A 6050.00 1 6050.00 97.78 0.0006*

*C 25878.13 1 25878.13 418.23 < 0.0001*

*E 4140.50 1 4140.50 66.92 0.0012*

Residual 247.50 4 61.88

Cor Total 36316.13 7

The "Model F-value" of 194.31implies the model is not significant relative to the noise. There is a

0.01 % chance that a "Model F-value" this large could occur due to noise.

Std. Dev. 7.87 R-Squared 0.9932

Mean 197.13 Adj R-Squared 0.9881

C.V. 3.99 Pred R-Squared 0.9727

PRESS 990.00 Adeq Precision 38.519

**Final Equation in Terms of Coded Factors:**

Solder Defects Mean =

+197.13

-27.50 \* A

+56.88 \* C

+22.75 \* E

Although the natural log transformation is often utilized for variance response, a power transformation actually performed better for this problem per the Box-Cox plot below. The analysis for the solder defect variance follows.



Design Expert Output

**Response: Solder Defects Variance Transform: Power Lambda: 2.04 Constant: 0**  **ANOVA for Selected Factorial Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Model 4.542E+013 4 1.136E+013 325.30 0.0003 significant

*A 1.023E+013 1 1.023E+013 293.08 0.0004*

*B 1.979E+012 1 1.979E+012 56.70 0.0049*

*E 2.392E+013 1 2.392E+013 685.33 0.0001*

*AB 9.289E+012 1 9.289E+012 266.11 0.0005*

Residual 1.047E+011 3 3.491E+010

Cor Total 4.553E+013 7

The "Model F-value" of 325.30 implies the model is not significant relative to the noise. There is a

0.03 % chance that a "Model F-value" this large could occur due to noise.

Std. Dev. 1.868E+005 R-Squared 0.9977

Mean 4.461E+006 Adj R-Squared 0.9946

C.V. 4.19 Pred R-Squared 0.9836

PRESS 7.447E+011 Adeq Precision 53.318

**Final Equation in Terms of Coded Factors:**

(Solder Defects Variance)2.04 =

+4.461E+006

-1.131E+006 \* A

-4.974E+005 \* B

-1.729E+006 \* E

-1.078E+006 \* A \* B

The contour plots of the mean and variance models are shown below along with the overlay plot. Assuming that we wish to minimize both solder defects mean and variance, a solution is shown in the overlay plot with factors *A* = +1, *B* = +1, *C* = -1, *D* = 0, and *E* near -1.





**12.16.** Reconsider the wave soldering experiment in Problem 12.15. Find a combined array design for this experiment that requires fewer runs.

The following experiment is a 28-4, resolution IV design with the defining relation *I = BCDE = ACDF = ABCG = ABDH*. Only 16 runs are required.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| *A* | *B* | *C* | *D* | *E* | *F* | *G* | *H* |
| -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 |
| +1 | -1 | -1 | -1 | -1 | +1 | +1 | +1 |
| -1 | +1 | -1 | -1 | +1 | -1 | +1 | +1 |
| +1 | +1 | -1 | -1 | +1 | +1 | -1 | -1 |
| -1 | -1 | +1 | -1 | +1 | +1 | +1 | -1 |
| +1 | -1 | +1 | -1 | +1 | -1 | -1 | +1 |
| -1 | +1 | +1 | -1 | -1 | +1 | -1 | +1 |
| +1 | +1 | +1 | -1 | -1 | -1 | +1 | -1 |
| -1 | -1 | -1 | +1 | +1 | +1 | -1 | +1 |
| +1 | -1 | -1 | +1 | +1 | -1 | +1 | -1 |
| -1 | +1 | -1 | +1 | -1 | +1 | +1 | -1 |
| +1 | +1 | -1 | +1 | -1 | -1 | -1 | +1 |
| -1 | -1 | +1 | +1 | -1 | -1 | +1 | +1 |
| +1 | -1 | +1 | +1 | -1 | +1 | -1 | -1 |
| -1 | +1 | +1 | +1 | +1 | -1 | -1 | -1 |
| +1 | +1 | +1 | +1 | +1 | +1 | +1 | +1 |

**12.17.** Reconsider the wave soldering experiment in Problem 12.15. Suppose that it was necessary to fit a complete quadratic model in the controllable variables, all main effects of the noise variables, and all controllable variable-noise variable interactions. What design would you recommend?

The following experiment is a small central composite design with five center points; the axial points for the noise factors have been removed. A total of 45 runs are required.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| *A* | *B* | *C* | *D* | *E* | *F* | *G* | *H* |
| +1 | +1 | +1 | -1 | -1 | +1 | +1 | +1 |
| -1 | +1 | +1 | -1 | +1 | +1 | +1 | -1 |
| +1 | +1 | -1 | -1 | +1 | +1 | -1 | -1 |
| +1 | -1 | -1 | +1 | +1 | -1 | +1 | -1 |
| -1 | -1 | +1 | +1 | +1 | +1 | -1 | -1 |
| -1 | +1 | +1 | +1 | -1 | +1 | -1 | -1 |
| -1 | +1 | +1 | +1 | +1 | +1 | -1 | -1 |
| +1 | +1 | +1 | +1 | +1 | -1 | +1 | -1 |
| +1 | +1 | -1 | +1 | -1 | +1 | -1 | +1 |
| +1 | -1 | +1 | +1 | -1 | -1 | +1 | -1 |
| +1 | +1 | +1 | -1 | +1 | -1 | -1 | +1 |
| -1 | +1 | +1 | -1 | -1 | -1 | +1 | +1 |
| +1 | -1 | -1 | -1 | -1 | +1 | -1 | -1 |
| +1 | -1 | +1 | -1 | -1 | -1 | -1 | +1 |
| -1 | +1 | -1 | -1 | +1 | +1 | +1 | -1 |
| +1 | -1 | -1 | +1 | +1 | +1 | -1 | +1 |
| -1 | -1 | -1 | -1 | -1 | +1 | +1 | -1 |
| -1 | -1 | +1 | +1 | +1 | -1 | -1 | +1 |
| -1 | +1 | -1 | -1 | +1 | -1 | +1 | +1 |
| -1 | -1 | +1 | +1 | -1 | +1 | +1 | +1 |
| +1 | +1 | -1 | +1 | -1 | -1 | +1 | -1 |
| -1 | -1 | +1 | -1 | +1 | -1 | -1 | -1 |
| +1 | +1 | +1 | -1 | -1 | +1 | -1 | -1 |
| +1 | -1 | -1 | -1 | +1 | +1 | +1 | +1 |
| +1 | -1 | +1 | -1 | -1 | +1 | +1 | +1 |
| -1 | +1 | -1 | +1 | +1 | +1 | +1 | +1 |
| -1 | -1 | -1 | -1 | +1 | -1 | +1 | +1 |
| +1 | -1 | +1 | +1 | +1 | -1 | +1 | -1 |
| -1 | +1 | -1 | +1 | -1 | -1 | -1 | +1 |
| -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 |
| -2.34 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2.34 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | -2.34 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 2.34 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | -2.34 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 2.34 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | -2.34 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 2.34 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | -2.34 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 2.34 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

**12.18.** Signal-to-noise factors generally confound location and dispersion effects.

**True** False

**12.19.** A crossed array design allows estimation of the control-by-noise factor interactions.

**True** False

**12.20.** If a 33-1 design is used as the inner array and a 23 is used as the outer array, all interactions can be estimated.

True **True**

**12.21.** Suppose that the response model is



The value of the controllable factor *x*1 that minimizes the transmitted variance is

**(a)** –**1**

(b) +1

(c) –0.5

(d) None of the above

**12.22.** Consider the response model in Problem 12.25. The value of the controllable factor *x*2 that minimizes the transmitted variance is

**(a)** –**1**

(b) +1

(c) –0.5

(d) None of the above

**12.23.** A log transformation could be useful in directly modeling the variance as a response.

**True** False

**12.24.** Computer-generated optimal designs can be a useful approach for designing combined array experiments with both control and noise variables.

**True** False