Chapter 14

# Nested and Split-Plot Designs

**Solutions**

In this chapter we have not shown residual plots and other diagnostics to conserve space. A complete analysis would, of course, include these model adequacy checking procedures.

**14.1S.** A rocket propellant manufacturer is studying the burning rate of propellant from three production processes. Four batches of propellant are randomly selected from the output of each process and three determinations of burning rate are made on each batch. The results follow. Analyze the data and draw conclusions.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | Process 1 | | | |  | Process 2 | | | |  | Process 3 | | | |
| Batch | 1 | 2 | 3 | 4 |  | 1 | 2 | 3 | 4 |  | 1 | 2 | 3 | 4 |
|  | 25 | 19 | 15 | 15 |  | 19 | 23 | 18 | 35 |  | 14 | 35 | 38 | 25 |
|  | 30 | 28 | 17 | 16 |  | 17 | 24 | 21 | 27 |  | 15 | 21 | 54 | 29 |
|  | 26 | 20 | 14 | 13 |  | 14 | 21 | 17 | 25 |  | 20 | 24 | 50 | 33 |

Minitab Output

**ANOVA: Burn Rate versus Process, Batch**

Factor Type Levels Values

Process fixed 3 1 2 3

Batch(Process) random 4 1 2 3 4

Analysis of Variance for Burn Rat

Source DF SS MS F P

Process 2 676.06 338.03 1.46 0.281

Batch(Process) 9 2077.58 230.84 12.20 0.000

Error 24 454.00 18.92

Total 35 3207.64

Source Variance Error Expected Mean Square for Each Term

component term (using restricted model)

1 Process 2 (3) + 3(2) + 12Q[1]

2 Batch(Process) 70.64 3 (3) + 3(2)

3 Error 18.92 (3)

There is no significant effect on mean burning rate among the different processes; however, different batches from the same process have significantly different burning rates.

**14.2S.** A manufacturing engineer is studying the dimensional variability of a particular component that is produced on three machines. Each machine has two spindles, and four components are randomly selected from each spindle. The results follow. Analyze the data, assuming that machines and spindles are fixed factors.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | Machine 1 | |  | Machine 2 | |  | Machine 3 | |
| Spindle | 1 | 2 |  | 1 | 2 |  | 1 | 2 |
|  | 12 | 8 |  | 14 | 12 |  | 14 | 16 |
|  | 9 | 9 |  | 15 | 10 |  | 10 | 15 |
|  | 11 | 10 |  | 13 | 11 |  | 12 | 15 |
|  | 12 | 8 |  | 14 | 13 |  | 11 | 14 |

Minitab Output

**ANOVA: Variability versus Machine, Spindle**

Factor Type Levels Values

Machine fixed 3 1 2 3

Spindle(Machine) fixed 2 1 2

Analysis of Variance for Variabil

Source DF SS MS F P

Machine 2 55.750 27.875 18.93 0.000

Spindle(Machine) 3 43.750 14.583 9.91 0.000

Error 18 26.500 1.472

Total 23 126.000

There is a significant effect on dimensional variability due to the machine and spindle factors.

**14.3.** To simplify production scheduling, an industrial engineer is studying the possibility of assigning one time standard to a particular class of jobs, believing that differences between jobs is negligible. To see if this simplification is possible, six jobs are randomly selected. Each job is given to a different group of three operators. Each operator completes the job twice at different times during the week, and the following results are obtained. What are your conclusions about the use of a common time standard for all jobs in this class? What value would you use for the standard?

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | Operator 1 | |  | Operator 2 | |  | Operator 3 | |
| Job | 1 | 2 |  | 1 | 2 |  | 1 | 2 |
| 1 | 158.3 | 159.4 |  | 159.2 | 159.6 |  | 158.9 | 157.8 |
| 2 | 154.6 | 154.9 |  | 157.7 | 156.8 |  | 154.8 | 156.3 |
| 3 | 162.5 | 162.6 |  | 161.0 | 158.9 |  | 160.5 | 159.5 |
| 4 | 160.0 | 158.7 |  | 157.5 | 158.9 |  | 161.1 | 158.5 |
| 5 | 156.3 | 158.1 |  | 158.3 | 156.9 |  | 157.7 | 156.9 |
| 6 | 163.7 | 161.0 |  | 162.3 | 160.3 |  | 162.6 | 161.8 |

Minitab Output

**ANOVA: Time versus Job, Operator**

Factor Type Levels Values

Job random 6 1 2 3 4 5 6

Operator(Job) random 3 1 2 3

Analysis of Variance for Time

Source DF SS MS F P

Job 5 148.111 29.622 17.21 0.000

Operator(Job) 12 20.653 1.721 1.58 0.186

Error 18 19.665 1.092

Total 35 188.430

Source Variance Error Expected Mean Square for Each Term

component term (using restricted model)

1 Job 4.6502 2 (3) + 2(2) + 6(1)

2 Operator(Job) 0.3143 3 (3) + 2(2)

3 Error 1.0925 (3)

The jobs differ significantly; the use of a common time standard would likely not be a good idea.

**14.4S.** Consider the three-stage nested design shown in Figure 14.5 to investigate alloy hardness. Using the data that follow, analyze the design, assuming that alloy chemistry and heats are fixed factors and ingots are random. Use the restricted form of the mixed model.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | Alloy Chemistry | | | | | | | | | | | | | | | | |
|  | 1 | | | | | | | |  | 2 | | | | | | | |
| Heats | 1 | |  | 2 | |  | 3 | |  | 1 | |  | 2 | |  | 3 | |
| Ingots | 1 | 2 |  | 1 | 2 |  | 1 | 2 |  | 1 | 2 |  | 1 | 2 |  | 1 | 2 |
|  | 40 | 27 |  | 95 | 69 |  | 65 | 78 |  | 22 | 23 |  | 83 | 75 |  | 61 | 35 |
|  | 63 | 30 |  | 67 | 47 |  | 54 | 45 |  | 10 | 39 |  | 62 | 64 |  | 77 | 42 |

Minitab Output

**ANOVA: Hardness versus Alloy, Heat, Ingot**

Factor Type Levels Values

Alloy fixed 2 1 2

Heat(Alloy) fixed 3 1 2 3

Ingot(Alloy Heat) random 2 1 2

Analysis of Variance for Hardness

Source DF SS MS F P

Alloy 1 315.4 315.4 0.85 0.392

Heat(Alloy) 4 6453.8 1613.5 4.35 0.055

Ingot(Alloy Heat) 6 2226.3 371.0 2.08 0.132

Error 12 2141.5 178.5

Total 23 11137.0

Source Variance Error Expected Mean Square for Each Term

component term (using restricted model)

1 Alloy 3 (4) + 2(3) + 12Q[1]

2 Heat(Alloy) 3 (4) + 2(3) + 4Q[2]

3 Ingot(Alloy Heat) 96.29 4 (4) + 2(3)

4 Error 178.46 (4)

Alloy hardness differs significantly due to the different heats within each alloy.

**14.5.** Reanalyze the experiment in Problem 14.4 using the unrestricted form of the mixed model. Comment on any differences you observe between the restricted and unrestricted model results. You may use a computer software package.

Minitab Output

**ANOVA: Hardness versus Alloy, Heat, Ingot**

Factor Type Levels Values

Alloy fixed 2 1 2

Heat(Alloy) fixed 3 1 2 3

Ingot(Alloy Heat) random 2 1 2

Analysis of Variance for Hardness

Source DF SS MS F P

Alloy 1 315.4 315.4 0.85 0.392

Heat(Alloy) 4 6453.8 1613.5 4.35 0.055

Ingot(Alloy Heat) 6 2226.3 371.0 2.08 0.132

Error 12 2141.5 178.5

Total 23 11137.0

Source Variance Error Expected Mean Square for Each Term

component term (using unrestricted model)

1 Alloy 3 (4) + 2(3) + Q[1,2]

2 Heat(Alloy) 3 (4) + 2(3) + Q[2]

3 Ingot(Alloy Heat) 96.29 4 (4) + 2(3)

4 Error 178.46 (4)

**14.7.** Derive the expected means squares for a balanced three-stage nested design, assuming that *A* is fixed and that *B* and *C* are random. Obtain formulas for estimating the variance components.

The expected mean squares can be generated in Minitab as follows:

Minitab Output

**ANOVA: y versus A, B, C**

Factor Type Levels Values

A fixed 2 -1 1

B(A) random 2 -1 1

C(A B) random 2 -1 1

Analysis of Variance for y

Source DF SS MS F P

A 1 0.250 0.250 0.06 0.831

B(A) 2 8.500 4.250 0.35 0.726

C(A B) 4 49.000 12.250 2.13 0.168

Error 8 46.000 5.750

Total 15 103.750

Source Variance Error Expected Mean Square for Each Term

component term (using restricted model)

1 A 2 (4) + 2(3) + 4(2) + 8Q[1]

2 B(A) -2.000 3 (4) + 2(3) + 4(2)

3 C(A B) 3.250 4 (4) + 2(3)

4 Error 5.750 (4)

**14.7.** Derive the expected means squares for a balanced three-stage nested design if all three factors are random. Obtain formulas for estimating the variance components. Assume the restricted form of the mixed model.

The expected mean squares can be generated in Minitab as follows:

Minitab Output

**ANOVA: y versus A, B, C**

Factor Type Levels Values

A random 2 -1 1

B(A) random 2 -1 1

C(A B) random 2 -1 1

Analysis of Variance for y

Source DF SS MS F P

A 1 0.250 0.250 0.06 0.831

B(A) 2 8.500 4.250 0.35 0.726

C(A B) 4 49.000 12.250 2.13 0.168

Error 8 46.000 5.750

Total 15 103.750

Source Variance Error Expected Mean Square for Each Term

component term (using unrestricted model)

1 A -0.5000 2 (4) + 2(3) + 4(2) + 8(1)

2 B(A) -2.0000 3 (4) + 2(3) + 4(2)

3 C(A B) 3.2500 4 (4) + 2(3)

4 Error 5.7500 (4)

**14.8.** Verify the expected mean squares given in Table 14.1.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | F | F | R |  |
|  | *a* | *b* | *n* |  |
| Factor | *i* | *j* | *l* | E(*MS*) |
|  | 0 | *b* | *n* |  |
|  | 1 | 0 | *n* |  |
|  | 1 | 1 | 1 |  |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | R | R | R |  |
|  | *a* | *b* | *n* |  |
| Factor | *i* | *j* | *l* | E(*MS*) |
|  | 1 | *b* | *n* |  |
|  | 1 | 1 | *n* |  |
|  | 1 | 1 | 1 |  |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | F | R | R |  |
|  | *a* | *b* | *n* |  |
| Factor | *i* | *j* | *l* | E(*MS*) |
|  | 0 | *b* | *n* |  |
|  | 1 | 1 | *n* |  |
|  | 1 | 1 | 1 |  |

**14.9.** ***Unbalanced nested designs.*** Consider an unbalanced two-stage nested design with *bj* levels of *B* under the *i*th level of *A* and *nij* replicates in the *ij*th cell.

(a) Write down the least squares normal equations for this situation. Solve the normal equations.

The least squares normal equations are:







There are 1+*a*+*b* equations in 1+*a*+*b* unknowns. However, there are *a*+1 linear dependencies in these equations, and consequently, *a*+1 side conditions are needed to solve them. Any convenient set of *a*+1 linearly independent equations can be used. The easiest set is , , for *i*=1,2,…,*a*. Using these conditions we get

, ,

as the solution to the normal equations. See Searle (1971) for a full discussion.

(b) Construct the analysis of variance table for the unbalanced two-stage nested design.

The analysis of variance table is

|  |  |  |
| --- | --- | --- |
| Source | SS | DF |
| *A* |  | *a-*1 |
| *B* |  | *b.-a* |
| Error |  | *n..-b* |
| Total |  | *n..-*1 |

(c) Analyze the following data, using the results in part (b).

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Factor *A* | 1 | |  | 2 | | |
| Factor *B* | 1 | 2 |  | 1 | 2 | 3 |
|  | 6 | 3 |  | 5 | 2 | 1 |
|  | 4 | 1 |  | 7 | 4 | 0 |
|  | 8 |  |  | 9 | 3 | -3 |
|  |  |  |  | 6 |  |  |

Note that *a*=2, *b*1=2, *b*2=3, *b*.=*b*1+*b*2=5, *n*11=3, *n*12=2, *n*21=4, *n*22=3 and *n*23=3

|  |  |  |  |
| --- | --- | --- | --- |
| Source | SS | DF | MS |
| *A* | 0.13 | 1 | 0.13 |
| *B* | 153.78 | 3 | 51.26 |
| Error | 35.42 | 10 | 3.54 |
| Total | 189.33 | 14 |  |

The analysis can also be performed in Minitab as follows. The adjusted sum of squares is utilized by Minitab’s general linear model routine.

Minitab Output

**General Linear Model: y versus A, B**

Factor Type Levels Values

A fixed 2 1 2

B(A) fixed 5 1 2 1 2 3

Analysis of Variance for y, using Adjusted SS for Tests

Source DF Seq SS Adj SS Adj MS F P

A 1 0.133 0.898 0.898 0.25 0.625

B(A) 3 153.783 153.783 51.261 14.47 0.001

Error 10 35.417 35.417 3.542

Total 14 189.333

**14.10.** A process engineer is testing the yield of a product manufactured on three machines. Each machine can be operated at two power settings. Furthermore, a machine has three stations on which the product is formed. An experiment is conducted in which each machine is tested at both power settings, and three observations on yield are taken from each station. The runs are made in random order, and the results are shown in Table P14.1. Analyze this experiment, assuming all three factors are fixed.

**Table P14.1**

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | Machine 1 | | |  | Machine 2 | | |  | Machine 3 | | |
| Station | 1 | 2 | 3 |  | 1 | 2 | 3 |  | 1 | 2 | 3 |
| Power Setting 1 | 34.1 | 33.7 | 36.2 |  | 31.1 | 33.1 | 32.8 |  | 32.9 | 33.8 | 33.6 |
| 30.3 | 34.9 | 36.8 |  | 33.5 | 34.7 | 35.1 |  | 33.0 | 33.4 | 32.8 |
| 31.6 | 35.0 | 37.1 |  | 34.0 | 33.9 | 34.3 |  | 33.1 | 32.8 | 31.7 |
| Power Setting 2 | 24.3 | 28.1 | 25.7 |  | 24.1 | 24.1 | 26.0 |  | 24.2 | 23.2 | 24.7 |
| 26.3 | 29.3 | 26.1 |  | 25.0 | 25.1 | 27.1 |  | 26.1 | 27.4 | 22.0 |
| 27.1 | 28.6 | 24.9 |  | 26.3 | 27.9 | 23.9 |  | 25.3 | 28.0 | 24.8 |

The linear model is 

Minitab Output

**ANOVA: Yield versus Machine, Power, Station**

Factor Type Levels Values

Machine fixed 3 1 2 3

Power fixed 2 1 2

Station(Machine) fixed 3 1 2 3

Analysis of Variance for Yield

Source DF SS MS F P

Machine 2 21.436 10.718 6.25 0.005

Power 1 845.698 845.698 492.96 0.000

Station(Machine) 6 33.583 5.597 3.26 0.012

Machine\*Power 2 0.383 0.191 0.11 0.895

Power\*Station(Machine) 6 29.208 4.868 2.84 0.023

Error 36 61.760 1.716

Total 53 992.068

Source Variance Error Expected Mean Square for Each Term

component term (using restricted model)

1 Machine 6 (6) + 18Q[1]

2 Power 6 (6) + 27Q[2]

3 Station(Machine) 6 (6) + 6Q[3]

4 Machine\*Power 6 (6) + 9Q[4]

5 Power\*Station(Machine) 6 (6) + 3Q[5]

6 Error 1.716 (6)

**14.11.** Suppose that in Problem 14.10 a large number of power settings could have been used and that the two selected for the experiment were chosen randomly. Obtain the expected mean squares for this situation assuming the restricted form of the mixed model and modify the previous analysis appropriately.

The analysis of variance and the expected mean squares can be obtained from Minitab as follows:

Minitab Output

**ANOVA: Yield versus Machine, Power, Station**

Factor Type Levels Values

Machine fixed 3 1 2 3

Power random 2 1 2

Station(Machine) fixed 3 1 2 3

Analysis of Variance for Yield

Source DF SS MS F P

Machine 2 21.436 10.718 56.03 0.018

Power 1 845.698 845.698 492.96 0.000

Station(Machine) 6 33.583 5.597 1.15 0.435

Machine\*Power 2 0.383 0.191 0.11 0.895

Power\*Station(Machine) 6 29.208 4.868 2.84 0.023

Error 36 61.760 1.716

Total 53 992.068

Source Variance Error Expected Mean Square for Each Term

component term (using restricted model)

1 Machine 4 (6) + 9(4) + 18Q[1]

2 Power 31.2586 6 (6) + 27(2)

3 Station(Machine) 5 (6) + 3(5) + 6Q[3]

4 Machine\*Power -0.1694 6 (6) + 9(4)

5 Power\*Station(Machine) 1.0508 6 (6) + 3(5)

6 Error 1.7156 (6)

**14.12.** Reanalyze the experiment in Problem 14.11 assuming the unrestricted form of the mixed model. You may use a computer software program to do this. Comment on any differences between the restricted and unrestricted model analysis and conclusions.

Minitab Output

**ANOVA: Yield versus Machine, Power, Station**

Factor Type Levels Values

Machine fixed 3 1 2 3

Power random 2 1 2

Station(Machine) fixed 3 1 2 3

Analysis of Variance for Yield

Source DF SS MS F P

Machine 2 21.436 10.718 56.03 0.018

Power 1 845.698 845.698 4420.88 0.000

Station(Machine) 6 33.583 5.597 1.15 0.435

Machine\*Power 2 0.383 0.191 0.04 0.962

Power\*Station(Machine) 6 29.208 4.868 2.84 0.023

Error 36 61.760 1.716

Total 53 992.068

Source Variance Error Expected Mean Square for Each Term

component term (using unrestricted model)

1 Machine 4 (6) + 3(5) + 9(4) + Q[1,3]

2 Power 31.3151 4 (6) + 3(5) + 9(4) + 27(2)

3 Station(Machine) 5 (6) + 3(5) + Q[3]

4 Machine\*Power -0.5196 5 (6) + 3(5) + 9(4)

5 Power\*Station(Machine) 1.0508 6 (6) + 3(5)

6 Error 1.7156 (6)

There are differences between several of the expected mean squares. However, the conclusions that could be drawn do not differ in any meaningful way from the restricted model analysis.

**14.13.** A structural engineer is studying the strength of aluminum alloy purchased from three vendors. Each vendor submits the alloy in standard-sized bars of 1.0, 1.5, or 2.0 inches. The processing of different sizes of bar stock from a common ingot involves different forging techniques, and so this factor may be important. Furthermore, the bar stock if forged from ingots made in different heats. Each vendor submits two tests specimens of each size bar stock from the three heats. The resulting strength data is shown in Table P14.2. Analyze the data, assuming that vendors and bar size are fixed and heats are random. Use the restricted form of the mixed model.

**Table P14.2**

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | Vendor 1 | | |  | Vendor 2 | | |  | Vendor 3 | | |
|  | Heat | 1 | 2 | 3 |  | 1 | 2 | 3 |  | 1 | 2 | 3 |
| Bar | 1 inch | 1.230 | 1.346 | 1.235 |  | 1.301 | 1.346 | 1.315 |  | 1.247 | 1.275 | 1.324 |
| Size |  | 1.259 | 1.400 | 1.206 |  | 1.263 | 1.392 | 1.320 |  | 1.296 | 1.268 | 1.315 |
|  | 1 ½ | 1.316 | 1.329 | 1.250 |  | 1.274 | 1.384 | 1.346 |  | 1.273 | 1.260 | 1.392 |
|  | inch | 1.300 | 1.362 | 1.239 |  | 1.268 | 1.375 | 1.357 |  | 1.264 | 1.265 | 1.364 |
|  | 2 inch | 1.287 | 1.346 | 1.273 |  | 1.247 | 1.362 | 1.336 |  | 1.301 | 1.280 | 1.319 |
|  |  | 1.292 | 1.382 | 1.215 |  | 1.215 | 1.328 | 1.342 |  | 1.262 | 1.271 | 1.323 |



Minitab Output

**ANOVA: Strength versus Vendor, Bar Size, Heat**

Factor Type Levels Values

Vendor fixed 3 1 2 3

Heat(Vendor) random 3 1 2 3

Bar Size fixed 3 1.0 1.5 2.0

Analysis of Variance for Strength

Source DF SS MS F P

Vendor 2 0.0088486 0.0044243 0.26 0.776

Heat(Vendor) 6 0.1002093 0.0167016 41.32 0.000

Bar Size 2 0.0025263 0.0012631 1.37 0.290

Vendor\*Bar Size 4 0.0023754 0.0005939 0.65 0.640

Bar Size\*Heat(Vendor) 12 0.0110303 0.0009192 2.27 0.037

Error 27 0.0109135 0.0004042

Total 53 0.1359034

Source Variance Error Expected Mean Square for Each Term

component term (using restricted model)

1 Vendor 2 (6) + 6(2) + 18Q[1]

2 Heat(Vendor) 0.00272 6 (6) + 6(2)

3 Bar Size 5 (6) + 2(5) + 18Q[3]

4 Vendor\*Bar Size 5 (6) + 2(5) + 6Q[4]

5 Bar Size\*Heat(Vendor) 0.00026 6 (6) + 2(5)

6 Error 0.00040 (6)

**14.14.** Rework Problem 14.13 using the unrestricted form of the mixed model. You can use a computer software program to do this. Comment on any differences between the restricted and unrestricted model analysis and conclusions.

Minitab Output

**ANOVA: Strength versus Vendor, Bar Size, Heat**

Factor Type Levels Values

Vendor fixed 3 1 2 3

Heat(Vendor) random 3 1 2 3

Bar Size fixed 3 1.0 1.5 2.0

Analysis of Variance for Strength

Source DF SS MS F P

Vendor 2 0.0088486 0.0044243 0.26 0.776

Heat(Vendor) 6 0.1002093 0.0167016 18.17 0.000

Bar Size 2 0.0025263 0.0012631 1.37 0.290

Vendor\*Bar Size 4 0.0023754 0.0005939 0.65 0.640

Bar Size\*Heat(Vendor) 12 0.0110303 0.0009192 2.27 0.037

Error 27 0.0109135 0.0004042

Total 53 0.1359034

Source Variance Error Expected Mean Square for Each Term

component term (using unrestricted model)

1 Vendor 2 (6) + 2(5) + 6(2) + Q[1,4]

2 Heat(Vendor) 0.00263 5 (6) + 2(5) + 6(2)

3 Bar Size 5 (6) + 2(5) + Q[3,4]

4 Vendor\*Bar Size 5 (6) + 2(5) + Q[4]

5 Bar Size\*Heat(Vendor) 0.00026 6 (6) + 2(5)

6 Error 0.00040 (6)

There are some differences in the expected mean squares. However, the conclusions do not differ from those of the restricted model analysis.

**14.15S.** Steel in normalized by heating above the critical temperature, soaking, and then air cooling. This process increases the strength of the steel, refines the grain, and homogenizes the structure. An experiment is performed to determine the effect of temperature and heat treatment time on the strength of normalized steel. Two temperatures and three times are selected. The experiment is performed by heating the oven to a randomly selected temperature and inserting three specimens. After 10 minutes one specimen is removed, after 20 minutes the second specimen is removed, and after 30 minutes the final specimen is removed. Then the temperature is changed to the other level and the process is repeated. Four shifts are required to collect the data, which are shown below. Analyze the data and draw conclusions, assume both factors are fixed.

|  |  |  |  |
| --- | --- | --- | --- |
|  |  | Temperature (F) | |
| Shift | Time(minutes) | 1500 | 1600 |
| 1 | 10 | 63 | 89 |
|  | 20 | 54 | 91 |
|  | 30 | 61 | 62 |
| 2 | 10 | 50 | 80 |
|  | 20 | 52 | 72 |
|  | 30 | 59 | 69 |
| 3 | 10 | 48 | 73 |
|  | 20 | 74 | 81 |
|  | 30 | 71 | 69 |
| 4 | 10 | 54 | 88 |
|  | 20 | 48 | 92 |
|  | 30 | 59 | 64 |

This is a split-plot design. Shifts correspond to blocks, temperature is the whole plot treatment, and time is the sub-treatments (in the subplot or split-plot part of the design). The expected mean squares and analysis of variance are shown below. The following Minitab Output has been modified to display the results of the split-plot analysis. Minitab will calculate the sums of squares correctly, but the expected mean squares and the statistical tests are not, in general, correct. Notice that the Error term in the analysis of variance is actually the three factor interaction.

Minitab Output

**ANOVA: Strength versus Shift, Temperature, Time**

Factor Type Levels Values

Shift random 4 1 2 3 4

Temperat fixed 2 1500 1600

Time fixed 3 10 20 30

Analysis of Variance for Strength

Standard Split Plot

Source DF SS MS F P F P

Shift 3 145.46 48.49 1.19 0.390

Temperat 1 2340.38 2340.38 29.20 0.012 29.21 0.012

Shift\*Temperat 3 240.46 80.15 1.97 0.220

Time 2 159.25 79.63 1.00 0.422 1.00 0.422

Shift\*Time 6 478.42 79.74 1.96 0.217

Temperat\*Time 2 795.25 397.63 9.76 0.013 9.76 0.013

Error 6 244.42 40.74

Total 23 4403.63

Source Variance Error Expected Mean Square for Each Term

component term (using restricted model)

1 Shift 1.292 7 (7) + 6(1)

2 Temperat 3 (7) + 3(3) + 12Q[2]

3 Shift\*Temperat 13.139 7 (7) + 3(3)

4 Time 5 (7) + 2(5) + 8Q[4]

5 Shift\*Time 19.500 7 (7) + 2(5)

6 Temperat\*Time 7 (7) + 4Q[6]

7 Error 40.736 (7)

**14.16S.** An experiment is designed to study pigment dispersion in paint. Four different mixes of a particular pigment are studied. The procedure consists of preparing a particular mix and then applying that mix to a panel by three application methods (brushing, spraying, and rolling). The response measured is the percentage reflectance of the pigment. Three days are required to run the experiment, and the data obtained follow. Analyze the data and draw conclusions, assuming that mixes and application methods are fixed.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  | Mix | | | |
| Day | App Method | 1 | 2 | 3 | 4 |
| 1 | 1 | 64.5 | 66.3 | 74.1 | 66.5 |
|  | 2 | 68.3 | 69.5 | 73.8 | 70.0 |
|  | 3 | 70.3 | 73.1 | 78.0 | 72.3 |
| 2 | 1 | 65.2 | 65.0 | 73.8 | 64.8 |
|  | 2 | 69.2 | 70.3 | 74.5 | 68.3 |
|  | 3 | 71.2 | 72.8 | 79.1 | 71.5 |
| 3 | 1 | 66.2 | 66.5 | 72.3 | 67.7 |
|  | 2 | 69.0 | 69.0 | 75.4 | 68.6 |
|  | 3 | 70.8 | 74.2 | 80.1 | 72.4 |

This is a split plot design. Days correspond to blocks, mix is the whole plot treatment, and method is the sub-treatment (in the subplot or split plot part of the design). The following Minitab Output has been modified to display the results of the split-plot analysis. Minitab will calculate the sums of squares correctly, but the expected mean squares and the statistical tests are not, in general, correct. Notice that the Error term in the analysis of variance is actually the three factor interaction.

Minitab Output

**ANOVA: Reflectance versus Day, Mix, Method**

Factor Type Levels Values

Day random 3 1 2 3

Mix fixed 4 1 2 3 4

Method fixed 3 1 2 3

Analysis of Variance for Reflecta

Standard Split Plot

Source DF SS MS F P F P

Day 2 2.042 1.021 1.39 0.285

Mix 3 307.479 102.493 135.77 0. 000 135.75 0.000

Day\*Mix 6 4.529 0.755 1.03 0.451

Method 2 222.095 111.047 226.24 0.000 226.16 0.000

Day\*Method 4 1.963 0.491 0.67 0.625

Mix\*Method 6 10.036 1.673 2.28 0.105 2.28 0.105

Error 12 8.786 0.732

Total 35 556.930

Source Variance Error Expected Mean Square for Each Term

component term (using restricted model)

1 Day 0.02406 7 (7) + 12(1)

2 Mix 3 (7) + 3(3) + 9Q[2]

3 Day\*Mix 0.00759 7 (7) + 3(3)

4 Method 5 (7) + 4(5) + 12Q[4]

5 Day\*Method -0.06032 7 (7) + 4(5)

6 Mix\*Method 7 (7) + 3Q[6]

7 Error 0.73213 (7)

**14.17.** Repeat Problem 14.16, assuming that the mixes are random and the application methods are fixed.

The F-tests are the same as those in Problem 14-16. The following Minitab Output has been edited to display the results of the split-plot analysis. Minitab will calculate the sums of squares correctly, but the expected mean squares and the statistical tests are not, in general, correct. Again, the Error term in the analysis of variance is actually the three factor interaction.

Minitab Output

**ANOVA: Reflectance versus Day, Mix, Method**

Factor Type Levels Values

Day random 3 1 2 3

Mix random 4 1 2 3 4

Method fixed 3 1 2 3

Analysis of Variance for Reflecta

Standard Split Plot

Source DF SS MS F P F P

Day 2 2.042 1.021 1.35 0.328

Mix 3 307.479 102.493 135.77 0.000 135.75 0.000

Day\*Mix 6 4.529 0.755 1.03 0.451

Method 2 222.095 111.047 77.58 0.001 x 226.16 0.000

Day\*Method 4 1.963 0.491 0.67 0.625

Mix\*Method 6 10.036 1.673 2.28 0.105 2.28 0.105

Error 12 8.786 0.732

Total 35 556.930

x Not an exact F-test.

Source Variance Error Expected Mean Square for Each Term

component term (using restricted model)

1 Day 0.0222 3 (7) + 3(3) + 12(1)

2 Mix 11.3042 3 (7) + 3(3) + 9(2)

3 Day\*Mix 0.0076 7 (7) + 3(3)

4 Method \* (7) + 3(6) + 4(5) + 12Q[4]

5 Day\*Method -0.0603 7 (7) + 4(5)

6 Mix\*Method 0.3135 7 (7) + 3(6)

7 Error 0.7321 (7)

\* Synthesized Test.

Error Terms for Synthesized Tests

Source Error DF Error MS Synthesis of Error MS

4 Method 3.59 1.431 (5) + (6) - (7)

**14.18.** Consider the split-split-plot design described in Example 14.4. Suppose that this experiment is conducted as described and that the data shown in Table P14.3 are obtained. Analyze the data and draw conclusions.

**Table P14.3**

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | Technician | | | | | | | | | | |
|  |  | 1 | | |  | 2 | | |  | 3 | | |
| Blocks | Dose Strength | 1 | 2 | 3 |  | 1 | 2 | 3 |  | 1 | 2 | 3 |
|  | Wall Thickness |  |  |  |  |  |  |  |  |  |  |  |
| 1 | 1 | 95 | 71 | 108 |  | 96 | 70 | 108 |  | 95 | 70 | 100 |
|  | 2 | 104 | 82 | 115 |  | 99 | 84 | 100 |  | 102 | 81 | 106 |
|  | 3 | 101 | 85 | 117 |  | 95 | 83 | 105 |  | 105 | 84 | 113 |
|  | 4 | 108 | 85 | 116 |  | 97 | 85 | 109 |  | 107 | 87 | 115 |
| 2 | 1 | 95 | 78 | 110 |  | 100 | 72 | 104 |  | 92 | 69 | 101 |
|  | 2 | 106 | 84 | 109 |  | 101 | 79 | 102 |  | 100 | 76 | 104 |
|  | 3 | 103 | 86 | 116 |  | 99 | 80 | 108 |  | 101 | 80 | 109 |
|  | 4 | 109 | 84 | 110 |  | 112 | 86 | 109 |  | 108 | 86 | 113 |
| 3 | 1 | 96 | 70 | 107 |  | 94 | 66 | 100 |  | 90 | 73 | 98 |
|  | 2 | 105 | 81 | 106 |  | 100 | 84 | 101 |  | 97 | 75 | 100 |
|  | 3 | 106 | 88 | 112 |  | 104 | 87 | 109 |  | 100 | 82 | 104 |
|  | 4 | 113 | 90 | 117 |  | 121 | 90 | 117 |  | 110 | 91 | 112 |
| 4 | 1 | 90 | 68 | 109 |  | 98 | 68 | 106 |  | 98 | 72 | 101 |
|  | 2 | 100 | 84 | 112 |  | 102 | 81 | 103 |  | 102 | 78 | 105 |
|  | 3 | 102 | 85 | 115 |  | 100 | 85 | 110 |  | 105 | 80 | 110 |
|  | 4 | 114 | 88 | 118 |  | 118 | 85 | 116 |  | 110 | 95 | 120 |

Using the computer output, the F-ratios were calculated by hand using the expected mean squares found in Table 14.25. The following Minitab Output has been edited to display the results of the split-plot analysis. Minitab will calculate the sums of squares correctly, but the expected mean squares and the statistical tests are not, in general, correct. Notice that the Error term in the analysis of variance is actually the four factor interaction.

Minitab Output

**ANOVA: Time versus Day, Tech, Dose, Thick**

Factor Type Levels Values

Day random 4 1 2 3 4

Tech fixed 3 1 2 3

Dose fixed 3 1 2 3

Thick fixed 4 1 2 3 4

Analysis of Variance for Time

Standard Split Plot

Source DF SS MS F P F P

Day 3 48.41 16.14 3.38 0.029

Tech 2 248.35 124.17 4.62 0.061 4.62 0.061

Day\*Tech 6 161.15 26.86 5.62 0.000

Dose 2 20570.06 10285.03 550.44 0.000 550.30 0.000

Day\*Dose 6 112.11 18.69 3.91 0.004

Tech\*Dose 4 125.94 31.49 3.32 0.048 3.32 0.048

Day\*Tech\*Dose 12 113.89 9.49 1.99 0.056

Thick 3 3806.91 1268.97 36.47 0.000 36.48 0.000

Day\*Thick 9 313.12 34.79 7.28 0.000

Tech\*Thick 6 126.49 21.08 2.26 0.084 2.26 0.084

Day\*Tech\*Thick 18 167.57 9.31 1.95 0.044

Dose\*Thick 6 402.28 67.05 17.13 0.000 17.15 0.000

Day\*Dose\*Thick 18 70.44 3.91 0.82 0.668

Tech\*Dose\*Thick 12 205.89 17.16 3.59 0.001 3.59 0.001

Error 36 172.06 4.78

Total 143 26644.66

Source Variance Error Expected Mean Square for Each Term

component term (using restricted model)

1 Day 0.3155 15 (15) + 36(1)

2 Tech 3 (15) + 12(3) + 48Q[2]

3 Day\*Tech 1.8400 15 (15) + 12(3)

4 Dose 5 (15) + 12(5) + 48Q[4]

5 Day\*Dose 1.1588 15 (15) + 12(5)

6 Tech\*Dose 7 (15) + 4(7) + 16Q[6]

7 Day\*Tech\*Dose 1.1779 15 (15) + 4(7)

8 Thick 9 (15) + 9(9) + 36Q[8]

9 Day\*Thick 3.3346 15 (15) + 9(9)

10 Tech\*Thick 11 (15) + 3(11) + 12Q[10]

11 Day\*Tech\*Thick 1.5100 15 (15) + 3(11)

12 Dose\*Thick 13 (15) + 3(13) + 12Q[12]

13 Day\*Dose\*Thick -0.2886 15 (15) + 3(13)

14 Tech\*Dose\*Thick 15 (15) + 4Q[14]

15 Error 4.7793 (15)

**14.19.** Rework Problem 14.18, assuming that the dosage strengths are chosen at random. Use the restricted form of the mixed model.

The following Minitab Output has been edited to display the results of the split-plot analysis. Minitab will calculate the sums of squares correctly, but the expected mean squares and the statistical tests are not, in general, correct. Again, the Error term in the analysis of variance is actually the four factor interaction.

Minitab Output

**ANOVA: Time versus Day, Tech, Dose, Thick**

Factor Type Levels Values

Day random 4 1 2 3 4

Tech fixed 3 1 2 3

Dose random 3 1 2 3

Thick fixed 4 1 2 3 4

Analysis of Variance for Time

Standard Split Plot

Source DF SS MS F P F P

Day 3 48.41 16.14 0.86 0.509

Tech 2 248.35 124.17 2.54 0.155 4.62 0.061

Day\*Tech 6 161.15 26.86 2.83 0.059

Dose 2 20570.06 10285.03 550.44 0.000 550.30 0.000

Day\*Dose 6 112.11 18.69 3.91 0.004

Tech\*Dose 4 125.94 31.49 3.32 0.048 3.32 0.048

Day\*Tech\*Dose 12 113.89 9.49 1.99 0.056

Thick 3 3806.91 1268.97 12.96 0.001 x 36.48 0.000

Day\*Thick 9 313.12 34.79 8.89 0.000

Tech\*Thick 6 126.49 21.08 0.97 0.475 x 2.26 0.084

Day\*Tech\*Thick 18 167.57 9.31 1.95 0.044

Dose\*Thick 6 402.28 67.05 17.13 0.000 17.15 0.000

Day\*Dose\*Thick 18 70.44 3.91 0.82 0.668

Tech\*Dose\*Thick 12 205.89 17.16 3.59 0.001 3.59 0.001

Error 36 172.06 4.78

Total 143 26644.66

x Not an exact F-test.

Source Variance Error Expected Mean Square for Each Term

component term (using restricted model)

1 Day -0.071 5 (15) + 12(5) + 36(1)

2 Tech \* (15) + 4(7) + 16(6) + 12(3) + 48Q[2]

3 Day\*Tech 1.447 7 (15) + 4(7) + 12(3)

4 Dose 213.882 5 (15) + 12(5) + 48(4)

5 Day\*Dose 1.159 15 (15) + 12(5)

6 Tech\*Dose 1.375 7 (15) + 4(7) + 16(6)

7 Day\*Tech\*Dose 1.178 15 (15) + 4(7)

8 Thick \* (15) + 3(13) + 12(12) + 9(9) + 36Q[8]

9 Day\*Thick 3.431 13 (15) + 3(13) + 9(9)

10 Tech\*Thick \* (15) + 4(14) + 3(11) + 12Q[10]

11 Day\*Tech\*Thick 1.510 15 (15) + 3(11)

12 Dose\*Thick 5.261 13 (15) + 3(13) + 12(12)

13 Day\*Dose\*Thick -0.289 15 (15) + 3(13)

14 Tech\*Dose\*Thick 3.095 15 (15) + 4(14)

15 Error 4.779 (15)

\* Synthesized Test.

Error Terms for Synthesized Tests

Source Error DF Error MS Synthesis of Error MS

2 Tech 6.35 48.85 (3) + (6) - (7)

8 Thick 10.84 97.92 (9) + (12) - (13)

10 Tech\*Thick 15.69 21.69 (11) + (14) - (15)

There are no exact tests on technicians , dosage strengths , wall thickness , or the technician x wall thickness interaction . The approximate *F*-tests are as follows:





































**14.20.** Suppose that in Problem 14.18 four technicians had been used. Assuming that all the factors are fixed, how many blocks should be run to obtain an adequate number of degrees of freedom on the test for differences among technicians?

The number of degrees of freedom for the test is (*a*-1)(4-1)=3(*a*-1), where *a* is the number of blocks used.

|  |  |
| --- | --- |
| Number of Blocks (*a*) | DF for test |
| 2 | 3 |
| 3 | 6 |
| 4 | 9 |
| 5 | 12 |

At least three blocks should be run, but four would give a better test.

**14.21.** Consider the experiment described in Example 14.4. Demonstrate how the order in which the treatments combinations are run would be determined if this experiment were run as

(a) a split-split-plot

Randomization for the split-split plot design is described in Example 14.4.

(b) a split-plot

In the split-plot, within a block, the technicians would be the main treatment and within a block-technician plot, the 12 combinations of dosage strength and wall thickness would be run in random order. The design would be a two-factor factorial in a split-plot.

(c) a factorial design in a randomized block

To run the design in a randomized block, the 36 combinations of technician, dosage strength, and wall thickness would be run in random order within each block. The design would be a three factor factorial in a randomized block.

(d) a completely randomized factorial design.

The blocks would be considered as replicates, and all 144 observations would be 4 replicates of a three factor factorial.

**14.22.** An article in *Quality Engineering* (“Quality Quandaries: Two-Level Factorials Run as Split-Plot Experiments”, Bisgaard, et al, Vol. 8, No. 4, pp. 705-708, 1996) describes a 25 factorial experiment on a plasma process focused on making paper more susceptible to ink. Four of the factors (*A-D*) are difficult to change from run-to-run, so the experimenters set up the reactor at the eight sets of conditions specified by the low and high levels of these factors, and then processed the two paper types (factor *E*) together. The placement of the paper specimens in the reactors (right versus left) was randomized. This produces a split-plot design with *A-D* as the whole-plot factors and factor *E* as the subplot factor. The data from this experiment are shown in Table P14.4. Analyze the data from this experiment and draw conclusions.

**Table P14.4**

| Standard Order | Run Number | A = Pressure | B = Power | C =  Gas Flow | D =  Gas Type | E =  Paper Type | y Contact Angle |
| --- | --- | --- | --- | --- | --- | --- | --- |
| 1 | 23 | -1 | -1 | -1 | Oxygen | E1 | 48.6 |
| 2 | 3 | +1 | -1 | -1 | Oxygen | E1 | 41.2 |
| 3 | 11 | -1 | +1 | -1 | Oxygen | E1 | 55.8 |
| 4 | 29 | +1 | +1 | -1 | Oxygen | E1 | 53.5 |
| 5 | 1 | -1 | -1 | +1 | Oxygen | E1 | 37.6 |
| 6 | 15 | +1 | -1 | +1 | Oxygen | E1 | 47.2 |
| 7 | 27 | -1 | +1 | +1 | Oxygen | E1 | 47.2 |
| 8 | 25 | +1 | +1 | +1 | Oxygen | E1 | 48.7 |
| 9 | 19 | -1 | -1 | -1 | SiCl4 | E1 | 5.0 |
| 10 | 5 | +1 | -1 | -1 | SiCl4 | E1 | 56.8 |
| 11 | 9 | -1 | +1 | -1 | SiCl4 | E1 | 25.6 |
| 12 | 31 | +1 | +1 | -1 | SiCl4 | E1 | 41.8 |
| 13 | 13 | -1 | -1 | +1 | SiCl4 | E1 | 13.3 |
| 14 | 7 | +1 | -1 | +1 | SiCl4 | E1 | 47.5 |
| 15 | 21 | -1 | +1 | +1 | SiCl4 | E1 | 11.3 |
| 16 | 17 | +1 | +1 | +1 | SiCl4 | E1 | 49.5 |
| 17 | 24 | -1 | -1 | -1 | Oxygen | E2 | 57.0 |
| 18 | 4 | +1 | -1 | -1 | Oxygen | E2 | 38.2 |
| 19 | 12 | -1 | +1 | -1 | Oxygen | E2 | 62.9 |
| 20 | 30 | +1 | +1 | -1 | Oxygen | E2 | 51.3 |
| 21 | 2 | -1 | -1 | +1 | Oxygen | E2 | 43.5 |
| 22 | 16 | +1 | -1 | +1 | Oxygen | E2 | 44.8 |
| 23 | 28 | -1 | +1 | +1 | Oxygen | E2 | 54.6 |
| 24 | 26 | +1 | +1 | +1 | Oxygen | E2 | 44.4 |
| 25 | 20 | -1 | -1 | -1 | SiCl4 | E2 | 18.1 |
| 26 | 6 | +1 | -1 | -1 | SiCl4 | E2 | 56.2 |
| 27 | 10 | -1 | +1 | -1 | SiCl4 | E2 | 33.0 |
| 28 | 32 | +1 | +1 | -1 | SiCl4 | E2 | 37.8 |
| 29 | 14 | -1 | -1 | +1 | SiCl4 | E2 | 23.7 |
| 30 | 8 | +1 | -1 | +1 | SiCl4 | E2 | 43.2 |
| 31 | 22 | -1 | +1 | +1 | SiCl4 | E2 | 23.9 |
| 32 | 18 | +1 | +1 | +1 | SiCl4 | E2 | 48.2 |

Half normal probability plots of the effects for both the whole plot with factors *A*, *B*, *C*, *D*, and their corresponding interactions, as well as the sub-plot with factor *E* and all interactions involving *E*, are shown below. The analysis of variance is not shown because of the known errors in the calculations; however, the models are also shown below.



Design Expert Output

**Response:** **Contact Angle**

**Final Equation in Terms of Coded Factors:**

Contact Angle =

+40.98

+5.91 \* A

-7.55 \* D

+1.57 \* E

+8.28 \* A \* D

-2.95 \* A \* E

**Final Equation in Terms of Actual Factors:**

Gas Type Oxygen

Paper Type E1

Contact Angle =

+46.96250

+0.58125 \* Pressure

Gas Type SiCl4

Paper Type E1

Contact Angle =

+31.86250

+17.14375 \* Pressure

Gas Type Oxygen

Paper Type E2

Contact Angle =

+50.10000

-5.31875 \* Pressure

Gas Type SiCl4

Paper Type E2

Contact Angle =

+35.00000

+11.24375 \* Pressure

**14.23.** There is always an interaction term in a nested design.

True **False**

**14.24.** The nested factor is always random.

True **False**

**14.25.** Both the ANOVA and the REML can be applied to the nested design.

**True** False

**14.26.** If there are 3 levels of the main factor and 5 levels of the nested factor there will be 4 degrees of freedom for the nested factor.

True **False**

**14.28.** A good reason to consider a split-plot design is a situation where some experimental units are larger than others.

**True** False

**14.29.** In a split-plot design the subplot error is usually smaller than the whole plot error.

**True** False

**14.30.** If an experiment is conducted as a split-plot but analyzed as a factorial, the resulting error estimate will usually be too large for properly testing the subplot factor and could lead to misleading conclusions.

**True** False

**14.31.** It is usually cheaper to change the levels of the whole plot factor in a split-plot design.

True **False**

**14.32.** In and experiment that has at least one nested factor

(a) The nested factor will always be a random effect.

(b) The primary factor will always be a fixed effect.

**(c) One may identify the key factor by arbitrarily renumbering the levels.**

(d) None of the above (a-c) are true.

(e) All of above (a-c) are true.