

Common subroutines

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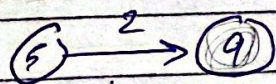
Common subroutines: algorithms

Initializing - single source (G, s)

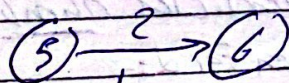
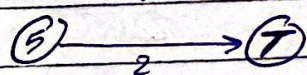
1. For each vertex $v \in G \setminus V$ $G \rightarrow$ graph.
2. $v.d = \infty$ // distance from $s = \infty$ $s \rightarrow$ node to be source
3. $v.\pi = NIL$ // no parents
4. $s.d = 0$ // distance of source = 0.

π // path

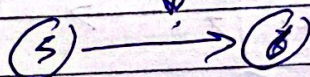
Relaxing an edge



after relaxation



After relaxation



$$(5+2) < 6$$

no
do nothing

$$5+2 < 9$$

yes then put 7.

make 5 the parent of 7

relaxation // if u is source and v is destination
if $u.d + w(u, v) < v.d$ then
make u the parent of v and update $v.d$

if $u.d + w(u, v) > v.d$ then do nothing

Relax (u, v, w)

$u \rightarrow$ source $w \rightarrow$ edge weight
 $v \rightarrow$ destination

1. if $v.d > u.d + w(u, v)$

2. $v.d = u.d + w(u, v)$

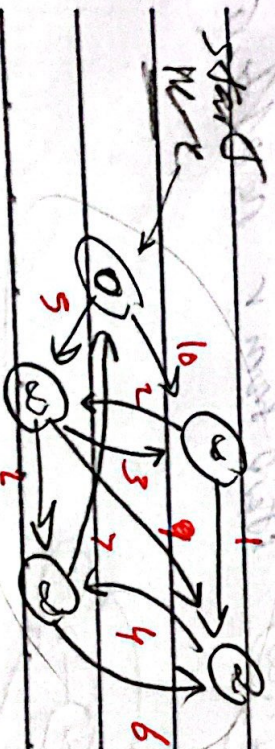
3. $v.\pi = u$

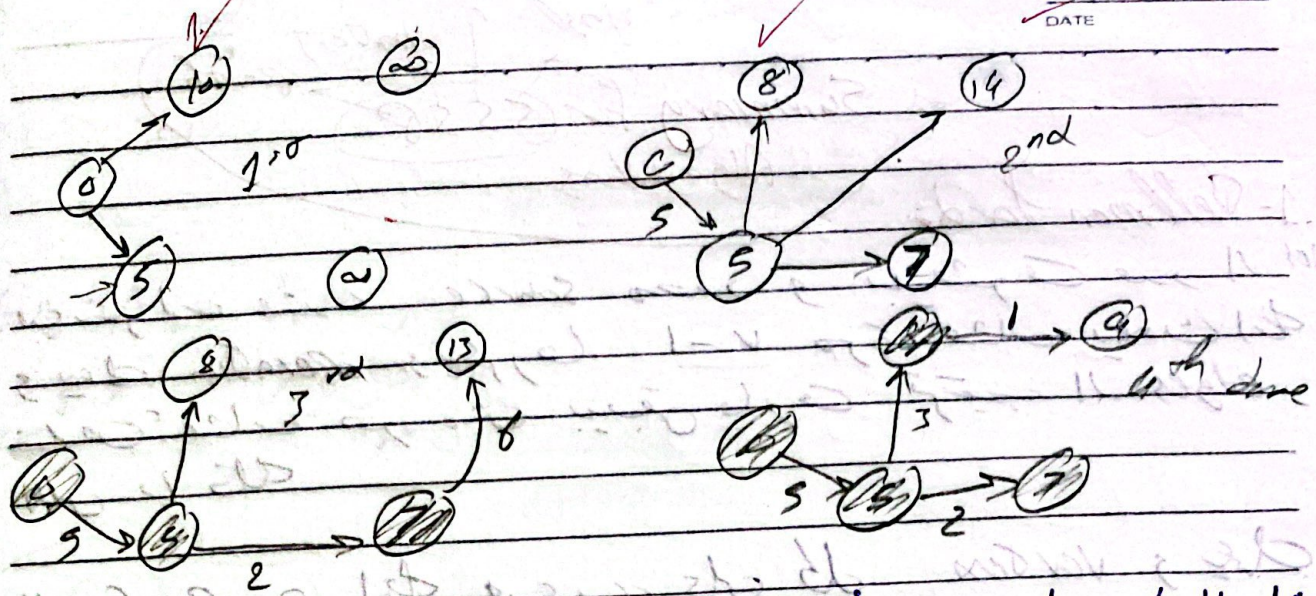
Done

3. Dijkstra's algorithm

- works only with positive edges.
- Greedily takes the nodes with smallest distance from source & relax its edges.
- Similar to BFS but in the non-decreasing order of node distance from root.
- BFS can be seen as special case of Dijkstra when all edges have the same weight (1).

Example:- on the following graph apply Dijkstra.





بشکوف فیت اصغر Node و اصل relax Δ edges بتایو
 وی حالتنا دس فیت Δ [5] بعد کدرا فلا فی ان اصغر Node
 عینن ص Δ [7] اعمالی relax و اشتروقت فیت اصغر
 فلا فی انی Δ [6] بعد Δ [8] اصل relax و فیت خاصه

1. Initialize single source (G, s)
2. $S = \emptyset$ Empty set \rightarrow contain finished nodes
3. $Q = \{s, v\}$ place all vertices in Queue. \rightarrow Build heap
4. while Q .size \rightarrow To get minimum of them
5. $u = \text{Extract_min}(Q)$
6. $S = S \cup \{u\}$ & mark it as source of finished nodes
7. for each vertex $v \in G, G.\text{Adj}[u]$
8. $\text{Relax}(u, v, w) \rightarrow$ Decrease Key

| | Array | Min heap | Bibonacci heap |
|--------------|-----------------|---------------------------|---------------------------|
| Insert (n/t) | $O(1) / O(V)$ | $- / O(V)$ | $- / O(V)$ |
| Extract min | $O(V) / O(V^2)$ | $O(\log V) / O(V \log V)$ | $O(\log V) / O(V \log V)$ |
| Decrease key | $O(1) / O(E)$ | $O(\log V) / O(E \log V)$ | $O(1) / O(E)$ |
| Total | $O(V^2)$ | $O(E \log V)$ | $O(V \log V + E)$ |