CE Sheet 3

1) $G(S) = \frac{K_1}{S+2}$ $\qquad H(S) = \frac{1}{S(S+K_2)}$

Hence,

$$TF(S) = \frac{K_1 S(S+K_2)}{K_1 + S(S+K_2)(S+2)}$$

$$D(S) = S(S+K_2)(S+2)+K_1$$

$$= S(S^2+2S+K_2S+2K_2)+K_1$$

$$= S^3+2S^2+K_2S^2+2SK_2+K_1$$

$$= S^3+S^2(2+K_2)+2SK_2+K_1$$

• For a 3rd order system, need aS^3+bS^2+cS+d where $a,b,c,d > 0$ and $bc-ad>0$

Hence,

$(2+K_2)>0$
$2K_2>0$ $\mapsto$ $K_2>0$ $\qquad$ # Conditions
$K_1>0$ $\mapsto$ $K_1>0$
$(2+K_2)(2K_2)>K_1 \mapsto K_1<2K_2^2+4K_2$

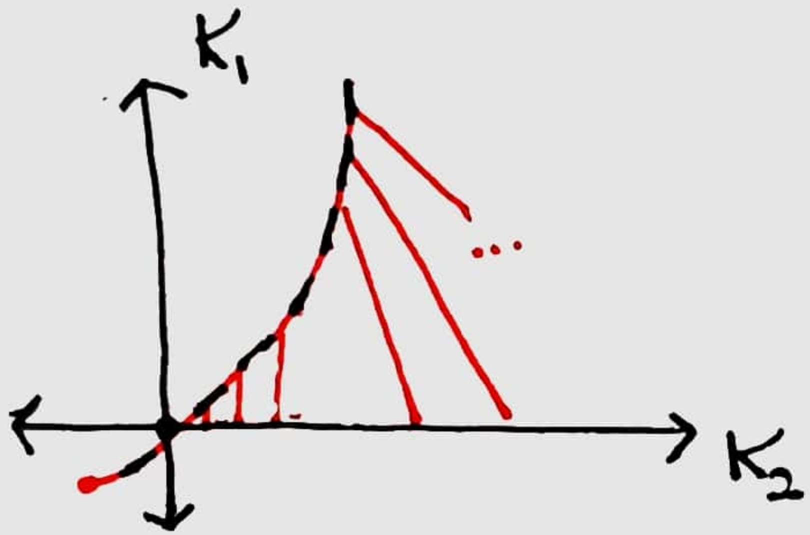
System of inequalities

$K_1 > 0 \quad K_2 > 0 \qquad K_1 < 2K_2^2 + 4K_2$

$$< \underbrace{2(K_2^2 + 2K_2)}_{2((K_2+1)^2-1)}$$

complete the square

← can plot this now



- Any value of K_1, K_2 below the curve but inside the 1st quad works. (System is stable)

2. $G(S) = \dfrac{12(S+4)}{S(S+1)(S+3)(S^2+2S+10)}$ $\qquad H(S) = 1$ (unity feedback)

$// GH(S) = G(S)$

→ Type 1 System

2. $G(S) = \dfrac{12(S+4)}{S(S+1)(S+3)(S^2+2S+10)}$, $H(S) = 1$

- Thus, $GH(S) = G(S)$ loop T.F.
- Clearly a type 1 System

If will ever be needed

$$TF(S) = \frac{12(S+4)}{12(S+4) + S(S+3)(S^2+2S+10)(S+1)}$$

a) n=0)

$$K_p = \lim_{S\to 0} S^0 GH(S) = \infty$$

n=1)

$$K_v = \lim_{S\to 0} S.GH(S) = \frac{12 \times 4}{1 \times 3 \times 10} = 1.6$$

- Will always hit one (N=n)
- $n<N$ (∞) $n>N$ (0)

here N=1

n=2)

$$K_a = \lim_{S\to 0} S^2.GH(S) = 0$$

b) Find e_{ss} and c_{ss}

- $r(t) = 16 + 2t$

$\rightarrow$ By linearity, $e_{ss} = e_{ss_{16}} + e_{ss_{2t}} = \dfrac{\overset{rM}{16}}{1+\underset{\leftarrow K_p}{\infty}} + \dfrac{\overset{rM}{2}}{\underset{\leftarrow K_v}{1.6}} = 1.25$

$\rightarrow$ Since $E(s) = R(s) - C(s).\underset{=1}{H(s)}$ then

$c_{ss} = r_{ss} - e_{ss}$, in other words

$$c_{ss} = \lim_{t\to\infty} c(t) = \underbrace{\lim_{t\to\infty} r(t)}_{\infty} - \underbrace{\lim_{t\to\infty} e(t)}_{1.25} = \infty$$

- Doing it this way (sidestepping FVT) was possible as $e_{ss} < \infty$ and $r_{ss} < \infty$ else $\infty - \infty$?

- $r(t) = 5t^2 = 10\dfrac{t^2}{2!}$

//always put in this form (now $M = 10$)

$\rightarrow e_{ss} = \dfrac{\overset{rM}{10}}{\underset{\leftarrow K_a}{0}}$ (write $\lim_{K_a\to 0^+} \dfrac{10}{K_a} = \infty$)

$\dfrac{10}{s^3} \leftrightarrow \dfrac{10t^2}{2!}$

$\rightarrow c_{ss} = \lim_{s\to 0} s.C(s) = \lim_{s\to 0} s.TF(s).R(s)$

$= \lim_{s\to 0} \underbrace{TF(s)}_{\frac{12\times4}{12\times4}}.\underbrace{sR(s)}_{\infty}$

Could've also wrote the rational function and used the opposite fact. $= \infty$

Recall that limits as $x\to\infty$:
- $\rightarrow 0$: Denominator of higher degree
- $\rightarrow$ Const.: Both of = degree
- $\rightarrow \infty$: Numerator of higher degree

c) $D(s) = 12(s+4) + s\underbrace{(s+3)(s+1)}_{s^2+4s+3}(s^2+2s+10)$

	s^2	$4s$	3
s^2	1	4	3
$2s$	2	8	6
10	10	40	30

$s^4 + 6s^3 + 21s^2 + 46s + 30$

2. Each diagonal //// is a Power of the Poly. (Sum over it)

1. Multiply Coefficients

4	3	2
3	2	1
2	1	0

$D(s) = s^5 + 6s^4 + 21s^3 + 46s^2 + 42s + 48$

s^5	1	21	42
s^4	6	46	48
s^3	40/3	34	0
s^2	30.7	48	0
s^1	$\frac{4038}{307}$	0	0
s^0	48	0	0

Could've divided by 2

• Zero Sign Changes and thus no Poles in the right half Plane
→ System is Stable

Love automation?
→ Write $\frac{6x - 1y}{6}$ on Calc.
→ For each take next Col up as x, y

Q3) $GH(S) = \dfrac{K_o}{S(4S+1)(S+1)}$

4min.

- K_v renamed to avoid confusion.
- $H(S) = 1$

- Type 1 System

a) $r(t) = 1 + t$ Hence, $e_{ss} = \dfrac{1}{1+K_p} + \dfrac{1}{K_v}$

$n=0)$
$K_p = \lim_{S\to 0} S^0 GH(S) = \infty$

$n=1)$
$K_v = \lim_{S\to 0} S^1 . GH(S) = \dfrac{K_o}{1\times 1} = K_o$

Hence, $e_{ss} = \dfrac{1}{K_o}$

else any -ve K_o works

- For $e_{ss} \leq 0.1 \rightarrow \dfrac{1}{K_o} \leq 0.1 \xrightarrow{\text{let } K_o > 0} K_o \geq 10$

- $K_{o_{min}} = 10$

b) $D(S) = K_o + S\underbrace{(4S+1)(S+1)}_{4S^2+5S+1} = 4S^3 + 5S^2 + S + K_o$

need:

- 3rd order system) $* \; 4>0, \; 5>0, \; 1>0, \; K_o > 0$ ✓
$* \; 5\times 1 > 4K_o$ X $\Leftarrow$ unstable!

$\Rightarrow$ Condition can never be satisfied, need $0 < K_o < \frac{5}{4}$ else unstable

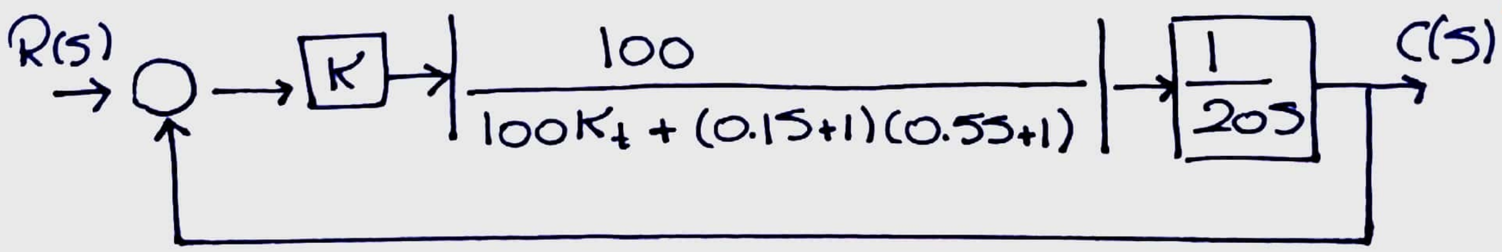
4)

- The inner system is part of the outer one's feedforward, has

$$G(S) = \frac{100}{(0.1S+1)(0.5S+1)} , \quad H(S) = \frac{K_t}{1}$$

$$TF(S) = \frac{100}{100K_t + (0.1S+1)(0.5S+1)}$$

So the entire system becomes



with

$$H(S) = 1$$

$$GH(S) = G(S) = \frac{100K}{(100K_t + (0.1S+1)(0.5S+1)).20S}$$

type 1 system

$$K_p = \lim_{S\to 0} S^0 . GH(S) = \infty \qquad (n=0)$$

$$K_v = \lim_{S\to 0} S^1 . GH(S) = \frac{100K}{(100K_t+1)20} = \frac{5K}{1+100K_t} \qquad (n=1)$$

$$K_a = \lim_{S\to 0} S^2 . GH(S) = 0 \qquad (n=2)$$

a) $r(t) = 6 + 8t$

$$e_{ss} = e_{ss_6} + e_{ss_{8t}} = \frac{6}{1+\underset{\nwarrow K_p}{\infty}} + \frac{8}{\underset{\nwarrow K_v}{\left(\frac{5K}{1+100K_t}\right)}}$$

$$= \frac{1.6}{K}(1+100K_t)$$

b) $r(t) = 2t + 7t^2 = 2t + \frac{14}{2!}t^2$

$$e_{ss} = e_{ss_{2t}} + e_{ss_{7t^2}} = \frac{2}{\left(\frac{5K}{1+100K_t}\right)} + \frac{14}{0} = \infty$$

• Constraints on K, K_t for the System to be Stable (answers are Valid).

$$D(S) = 5K + S(100K_t + \underbrace{(0.1S+1)(0.5S+1)}_{0.05S^2+0.6S+1})$$

$$= 0.05S^3 + 0.6S^2 + S(1+100K_t) + 5K$$

• Need $0.05 > 0$, $0.6 > 0$, $(1+100K_t) > 0$, $5K > 0$

$\checkmark \quad \checkmark \quad K_t > \frac{-1}{100} \quad K > 0$

and

$0.6(1+100K_t) > 5K \cdot 0.05$

$\rightarrow K < (1+100K_t)2.4$ (with $K_t > \frac{-1}{100}$ and $K > 0$)

. unit ramp $r(t) = t$ has

$$e_{ss} = \frac{1}{K_v} = \frac{1}{\frac{5K}{1+100K_t}} = \frac{1+100K_t}{5K}$$

needs to be minimized under constraints

$0 < K < 2.4(1+100K_t)$, $K_t > \frac{-1}{100}$ redundant (covered by other eqn.)

Flip all (all +ve)

$\infty > \frac{1}{K} > \frac{1}{2.4(1+100K_t)}$

• always true

$\frac{1+100K_t}{K} > \frac{1}{2.4} \longrightarrow e_{ss} > \frac{1}{2.4 \times 5} = \frac{1}{12}$

• To maintain a stable system the smallest possible steady state error is beyond $\frac{1}{12}$.

$$\rightarrow G(S) = \frac{K}{S^2(S+25)} \quad , \quad H(S) = K_t S$$

Q5)

- Can reduced the embedded closed loop system into an openloop system of transfer function

$$TF(S) = \frac{K}{KK_tS + S^2(S+25)}$$

- The entire system now has

$$G(S) = \frac{(1+0.02S)K}{KK_tS + S^2(S+25)}$$

- Take S as a common factor → Type 1 system

$$H(S) = 1$$

$$K_v = \lim_{S\to 0} S^1 . GH(S) = \frac{K}{KK_t + 0} = \frac{1}{K_t}$$

- Thus, $e_{ss} = \frac{1}{K_v} = K_t$

⇒ Answer Validity ⟷ System Stability

$$D(S) = (1+0.02S)K + KK_tS + S^2(S+25)$$
$$= S^3 + 25S^2 + S(KK_t + 0.02K) + K$$

- Need $K > 0$ and $KK_t > -0.02K$ and $25(KK_t + 0.02K) > K$

$\rightarrow K_t > -0.02$

- Know $K > 0$

$\rightarrow 25(K_t + 0.02) > 1 \rightarrow K_t > 0.02$

* So need $K_t > 0.02$ and $K > 0$