Digital Communications (ELC 325b)

© Samy S. Soliman

Electronics and Electrical Communication Engineering Department
Cairo University, Egypt

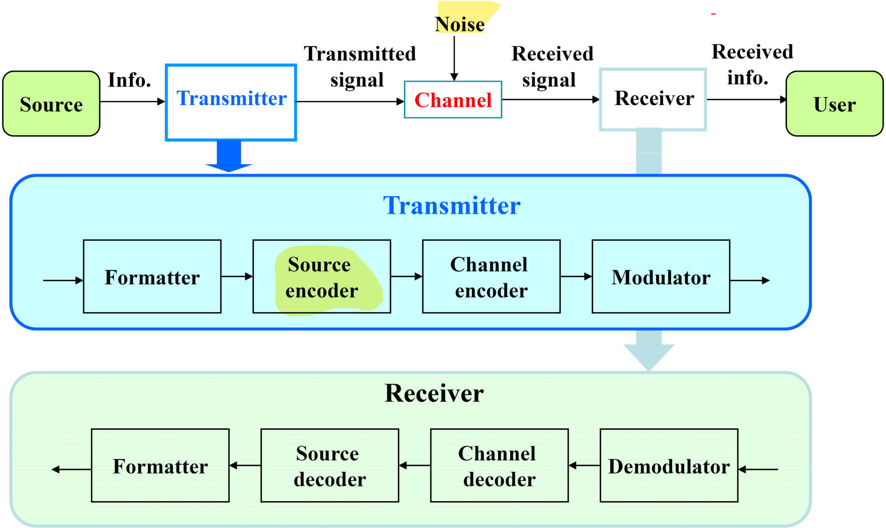
Email: *samy.soliman@cu.edu.eg*
Website: http://scholar.cu.edu.eg/samysoliman

Spring 2016

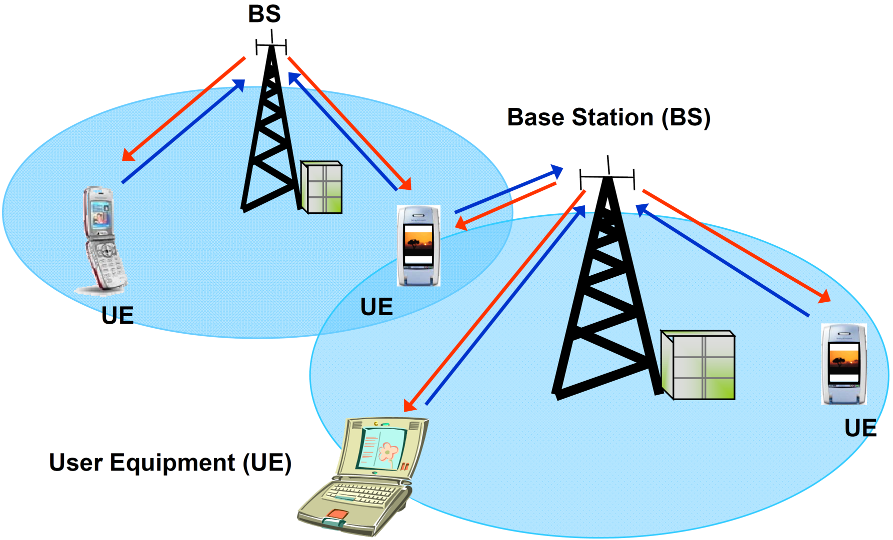
Outline

1. Introduction to Digital Communication Systems
   - Structure of Digital Communication Systems
   - Classification of Signals
   - Review on Sampling, Quantization, PCM
     - Sampling
     - Quantization
     - Encoding
   - Random Processes
   - Noise In Communication Systems
   - Signal Transmission Through Linear Systems
   - Baseband versus Passband - Signal Bandwidth

Structure of Digital Communication Systems



Wireless Cellular System



Why Digital Communication Systems?

Features of Digital Communication Systems

- Transmitter sends a waveform from a finite set of possible waveforms during a limited time
- Channel distorts, attenuates and adds noise to the transmitted signal
- Receiver decides which waveform was transmitted from the noisy received signal
- Probability of erroneous decision is an important measure for the system performance

Advantages of Digital Communication Systems

- The ability to use regenerative repeaters
- Different kinds of digital signals are treated identically
- Immunity to noise

Designing Digital Communication Systems

Necessary Knowledge/Tools for the Design of DCS

1. Classification of signals
2. Random processes
3. Noise in communication systems
4. Signal transmission through linear systems
5. Bandwidth of signal

Classification of Signals

Signal Classifications

- Periodic - Aperiodic
- Continuous - Discrete
- Analog - Digital
- Power - Energy
- Deterministic - Random

Energy Signal - Power Signal

- **Energy Signal:** A signal is an energy signal if, and only if, it has nonzero but finite energy for all time, i.e. $0 < E < \infty$

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \lim_{T \to \infty} \int_{-T/2}^{T/2} |x(t)|^2 dt$$

- **Power Signal:** A signal is a power signal if, and only if, it has finite but nonzero power for all time, i.e. $0 < P < \infty$

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt$$

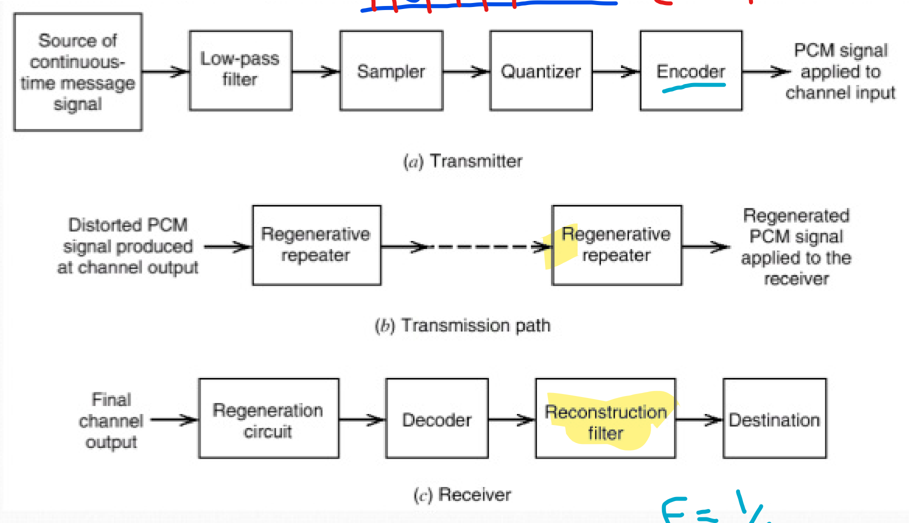
Classification of Signals

Deterministic - Random

- **Deterministic signal:** No uncertainty with respect to the signal value at any time.
- **Random signal:** Some degree of uncertainty in signal values before it actually occurs.
    1. Thermal noise in electronic circuits due to the random movement of electrons.
    2. Reflection of radio waves from different layers of ionosphere.

General rule: Periodic and random signals are power signals. Signals that are both deterministic and non-periodic are energy signals

Pulse Code Modulation: Basic Elements



$F = 1/x$

Instantaneous Sampling

Definition

It is the process of transforming a message signal $m(t)$ into an **analog discrete** signal $m_s(t) = m(nT_s)$ with a sampling frequency f_s which is higher than twice the highest frequency component W of the message signal

$$
\begin{aligned}
m_s(t) &= m(t)\delta_{T_s}(t) \\
M_s(f) &= f_s \sum M(f - nf_s)
\end{aligned}
$$

- Ensure perfect reconstruction at the Receiver
- Narrow rectangular pulses $\Rightarrow$ instantaneous sampling
- Proceeded by an anti-aliasing filter
- Reduces the continuously varying message signal to a limited number of discrete values per second

Instantaneous Sampling: Sampling Theorem

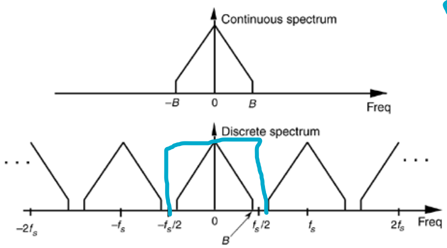
Theorem (Sampling Theorem)

A band-limited signal of finite energy, which has no frequency components higher than B Hz, is completely described by the values of the signal at instants of time separated by $\frac{1}{2B}$ seconds.
The signal may be completely recovered from the knowledge of its samples.

Nyquist rate $= 2B$ Nyquist interval $= \frac{1}{2B}$

f

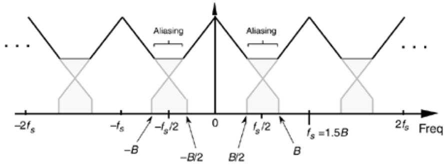
5
10 7



Instantaneous Sampling: Aliasing

Definition of Aliasing

Aliasing is the phenomenon of a high-frequency component in the spectrum of the signal, seemingly taking on the identity of a lower frequency in the spectrum of its sampled version (occurs if $f_s < 2B$)



To combat the effects of aliasing;

1. An anti-aliasing LPF is used prior to sampling to attenuate the non-essential high-frequency components of the signal
2. The filtered signal is sampled at a rate slightly higher than the Nyquist rate

Reconstruction

T_s

In order to reconstruct the signal, a LPF is used such that

$$\begin{aligned} M_{\text{reconstructed}}(f) &= T_s M_s(f), \qquad -f_s/2 < f < f_s/2 \\ &= \underline{M(f)} \end{aligned}$$

Then,

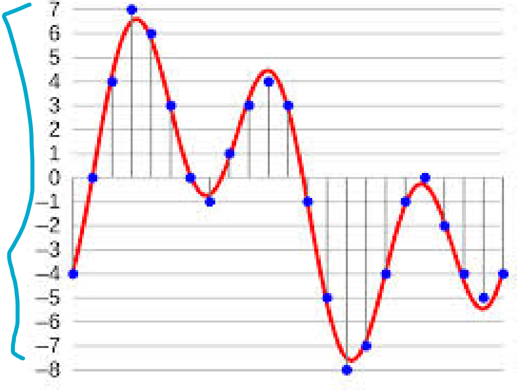
The Reconstructed Signal

$$m_{\text{reconstructed}}(t) = \sum m(nT_s)\, sinc(f_s t - n)$$

Quantization

Definition

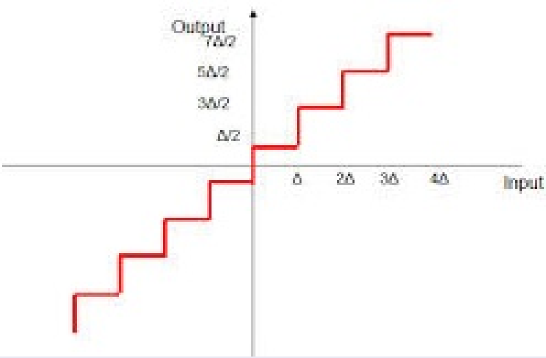
It is the process of transforming the sample amplitude $m(nT_s)$ into a **discrete amplitude** $\nu(nT_s)$ taken from a finite set of possible amplitudes



Uniform Mid-Rise Quantization

Quantizer Characteristic: Mid-Rise Staircase

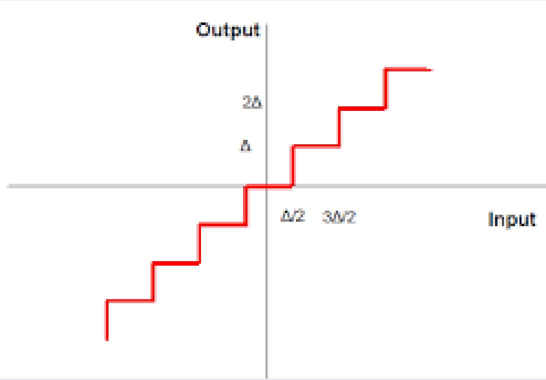
The origin lies in the **middle of a rise**



Uniform Mid-Tread Quantization

Quantizer Characteristic: Mid-Tread Staircase

The origin lies in the **middle of a tread**



Quantization Error

Definition

It is the difference between the input signal, m, and the output signal ν

$$q = m - \nu$$

Notes:

- Maximum error: $q_{max} = \pm\frac{1}{2}$ step size
- Step size: $\Delta = \frac{\max - \min}{L - 1}$
- As the step width $\Downarrow$, the quantization error $\Downarrow$
- It is better to use binary weighted number of levels, *i.e.* $L = 2^R$ bits/sample

Signal-to-Noise Ratio (SNR)

The signal-to-noise ratio (SNR) is one of the performance measures used to describe communication systems.
Quantization error is usually more significant than pulse detection errors.

SNR

It is the ratio of the useful signal power to the noise power.

Assuming a uniform quantizer with $\pm m_p$ peak levels, the average quantization noise level can be evaluated as

$$N_q = \widetilde{q^2} = \frac{\Delta^2}{12} = \frac{m_p^2}{3L^2}$$ Nq -> quantization noise

Quantizer's Output SNR

$$SNR = \frac{\widetilde{m^2}}{N_q} = \frac{3L^2}{m_p^2} P$$

Non-Uniform Quantization

Motivation

- The SNR is a function of the signal average power, it can be different from one user to another. It is needed to have SNR levels close to each other.
- The solution is to use smaller quantization steps for smaller signal amplitudes.
- Achieved through compressing the signal (μ-Law or A-Law), then applying a uniform quantizer. This is equivalent to non-uniform quantization.
- At the reconstruction end, and inverse process is applied using expander.
- The combined system is called **Compander**.

Non-Uniform Quantization

μ-Law Quantizer

$$y = \frac{\ln(1+\mu\hat{m})}{\ln(1+\mu)}$$

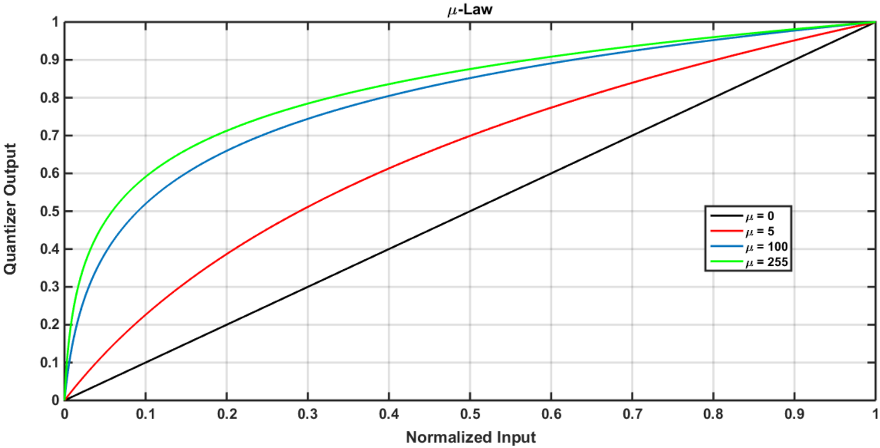
u-law dequantizer:
deq = (sign)((1+u)^abs(y) - 1)
----
u

$$SNR \simeq \frac{3L^2}{[\ln(1+\mu)]^2}$$

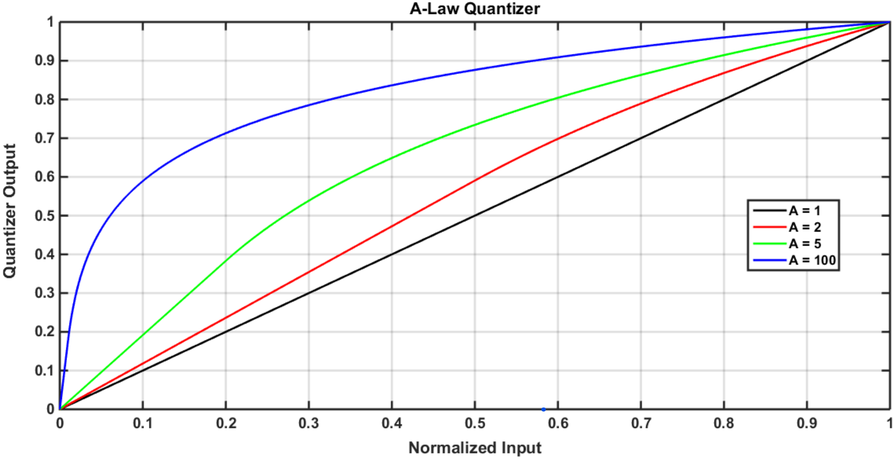
A-Law Quantizer

$$y = \begin{cases} \dfrac{A\hat{m}}{1+\ln(A)}, & 0 \le \hat{m} \le 1/A \\ \dfrac{1+\ln(A\hat{m})}{1+\ln(A)}, & 1/A \le \hat{m} \le 1 \end{cases}$$

Non-Uniform μ-Law Quantization



Non-Uniform A-Law Quantization



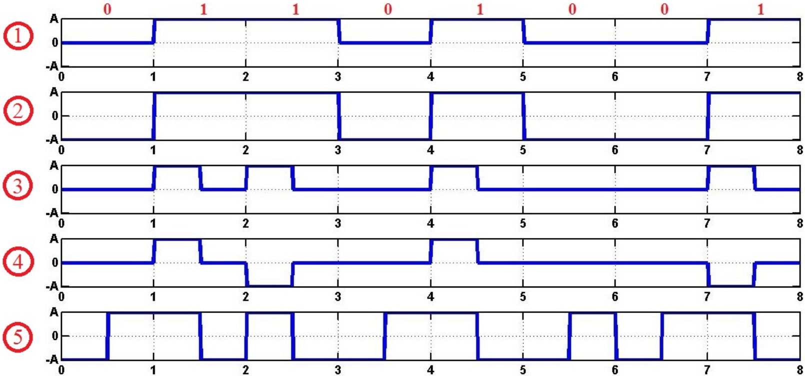
Encoding (Digital Baseband Modulation)

1. Encoding is used to make the transmitted signal more robust to noise, interference and other channel impairments.
2. It translates the discrete set of sample values to a more appropriate form.
3. Binary codes give the maximum advantage over the effects of noise in a transmission medium, because a binary symbol withstands a relatively high level of noise and it is easy to generate.

Line Codes

Line codes are used for the electrical representation of binary data stream.

1. Unipolar NRZ signaling
2. Polar NRZ signaling
3. Unipolar RZ signaling
4. Bipolar BRZ signaling (Alternate Mark Inversion)
5. Split-Phase signaling (Manchester Code)



Line Codes

Line codes usually differ in:

1. **Spectral characteristics** (power spectral density and bandwidth efficiency): BW should be as small as possible + no DC component.
2. **Power Efficiency**: for a given BW and a specified detection error probability, the transmitted power should be as small as possible.
3. **Error detection capability** (Interference and noise immunity): should be possible to detect and preferably correct errors.
4. **Bit synchronization capability**: should be possible to extract timing or clock information from the line code.
5. **Implementation cost and complexity**

Bit Rate - Transmission Bandwidth - Output SNR

A baseband signal with maximum power **P Watts** and bandwidth **B Hz**, sampled at the Nyquist rate, **2B Hz**, and quantized into $\mathbf{L = 2^R}$ **PCM levels**, using a uniform quantizer with $\pm\mathbf{m_p}$ **peak levels**, to be transmitted over a channel of efficiency η **bits/sec/Hz**

Bit Rate

$$R_b = 2BR$$

Transmission Bandwidth

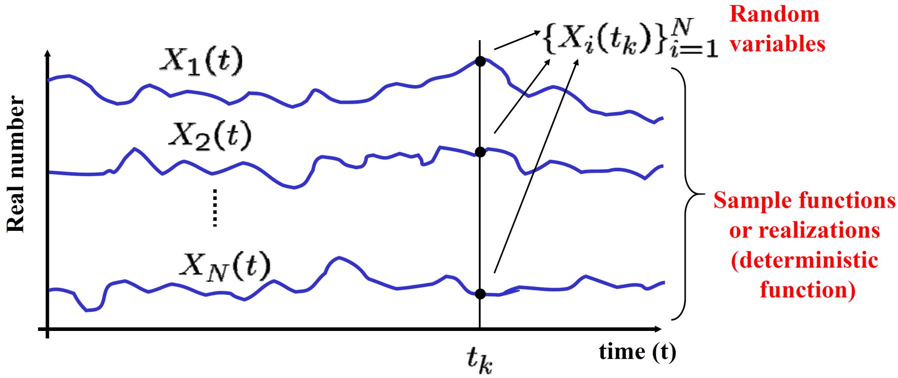
$$B_T = \frac{R_b}{\eta}$$

Output SNR

$$SNR = \frac{3P}{m_p^2} 2^{2R}$$

What is a Random Process?

A random process is a collection of time functions, or signals, corresponding to various outcomes of a random experiment. For each outcome, there exists a deterministic function, which is called a sample function or a realization.



Autocorrelation Function

ACF of a Random Process

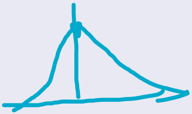
$$R_x(t_i, t_j) = \mathcal{E}\{X(t_i)X^*(t_j)\}$$

ACF of a WSS Process

$$R_x(\tau) = \mathcal{E}\{X(t)X^*(t - \tau)\}$$

Properties of ACF

The ACF of a real WSS process is characterized by:



1. Autocorrelation is symmetric around zero.
2. Its maximum value occurs at the origin.
3. Its value at the origin is equal to the average power or energy.
4. The Fourier Transform of the ACF is called the **Spectral Density**

Spectral Density

t

Power SD of a WSS Random Process

$$G_X(f) = \mathcal{F}\{R_x(\tau)\}$$

Energy SD of an Energy Signal

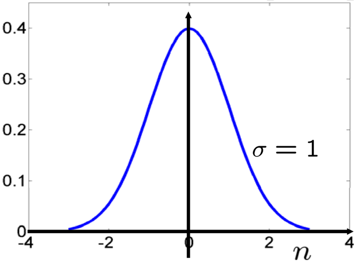
$$\Psi_x(f) = |\mathcal{F}\{x(t)\}|^2 = |X(f)|^2$$

Noise In Communication Systems

There are many types of noise (unwanted signals) in communications systems. The most common type of noise is the **White Gaussian Noise**.

1. **Gaussian:** because it is a random process that can be described by a zero-mean Gaussian distribution.
2. **White:** because it's PSD is flat.

$$p(n) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{n^2}{2\sigma^2}\right]$$

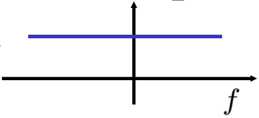


Probability density function

density 34an el power lw7do hwa aslun scalar, lakn enta btshof el 3laka benha w ben el time, fa lw 3auz tgeb el power fe interval mo3yna bt3ml integeration

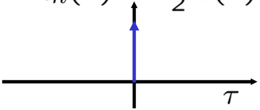
Power spectral density

$$G_n(f) = \frac{N_0}{2} \text{ [w/Hz]}$$

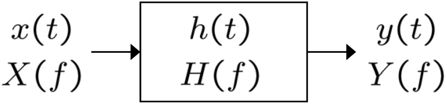


Autocorrelation function

$$R_n(\tau) = \frac{N_0}{2}\delta(\tau)$$



Signal Transmission Through Linear Systems



Input - Output Relationship

Deterministic Signals : $Y(f) = X(f)H(f)$ this is in the frequency domain

Random Signals : $G_Y(f) = G_X(f)|H(f)|^2$

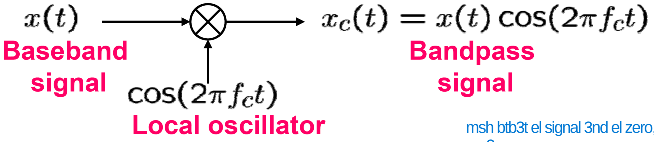
power spectrul density

Ideal Distortionless Transmission

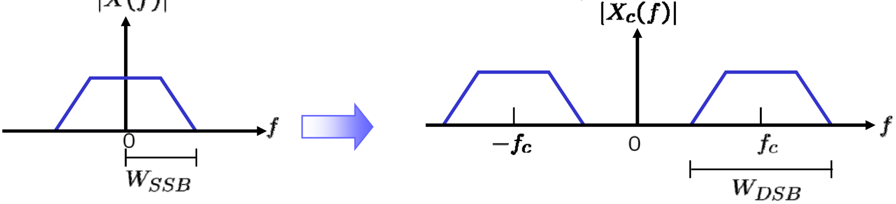
All the frequency components of the signal arrive at the destination with an identical time delay, and they are amplified or attenuated equally.

$$y(t) = K\, x(t - t_o) \quad \Rightarrow \quad Y(f) = K\, X(f) e^{-j2\pi f t_o}$$

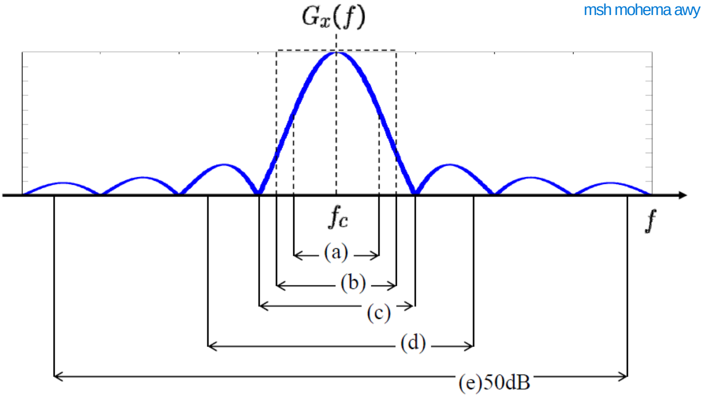
mshakel el channel enha tkon bt3ml attenuation w enha tkon bandlimited fa da bykhly el shape el tal3lk msh mzbot



msh btb3t el signal 3nd el zero, la btb3ha 3nd frequence mo3yna



Definitions of Bandwidth



(a) Half-power bandwidth
(b) Noise equivalent bandwidth
(c) Null-to-null bandwidth
(d) Fractional power containment bandwidth
(e) Bounded power spectral density
(f) Absolute bandwidth

References

Simon Haykin (2001)
Communication Systems, 4th Edition.
John Wiley.

B. P. Lathi (1998)
Modern Digital and Analog Communication Systems, 3rd Edition.
Oxford University Press.

Thank You

Questions ?

samy.soliman@cu.edu.eg

http://scholar.cu.edu.eg/samysoliman