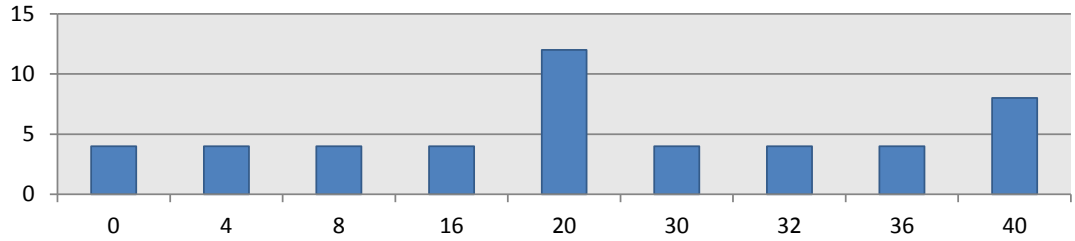
1.



We need to choose 3 random centroids (K = 3). Let's choose 8, 20 and 36.
Next we need to cluster the gray levels around our chosen centroids.

Gray Level	Distance to 8	Distance to 20	Distance to 36
0	**8**	20	36
4	**4**	16	32
16	8	**4**	20
30	22	10	**6**
32	24	12	**8**
40	32	20	**4**

So we have three clusters:

- 0, 4, 8
- 16, 20
- 30, 32, 36, 40

For every cluster, we calculate a new centroid.

$$C_1 = \frac{(0 \times 4) + (4 \times 4) + (8 \times 4)}{4 + 4 + 4} = 4$$
$$C_2 = \frac{(16 \times 4) + (20 \times 12)}{4 + 12} = 19$$
$$C_3 = \frac{(30 \times 4) + (32 \times 4) + (36 \times 4) + (40 \times 8)}{4 + 4 + 4 + 8} = 35.6$$

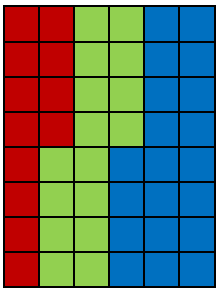
For the new centroids, we repeat the steps above.

Gray Level	Distance to 4	Distance to 19	Distance to 35.6
0	**4**	19	35.6
8	**4**	11	27.6
16	12	**3**	19.6
20	16	**1**	15.6
30	26	11	**5.6**
32	28	13	**3.6**
36	32	17	**0.4**
40	36	21	**4.4**

For every cluster, we calculate a new centroid.

$$C_1 = \frac{(0 \times 4) + (4 \times 4) + (8 \times 4)}{4 + 4 + 4} = 4$$
$$C_2 = \frac{(16 \times 4) + (20 \times 12)}{4 + 12} = 19$$
$$C_3 = \frac{(30 \times 4) + (32 \times 4) + (36 \times 4) + (40 \times 8)}{4 + 4 + 4 + 8} = 35.6$$

We notice that the centroids are unchanged. This means that we have reached the end of our segmentation process. Our segmented image will look like this:



2. We number the pixels along the x and y axes as shown:

	0	**1**	**2**	**3**	**4**	**5**
0	0	8	16	20	32	40
1	0	8	16	20	32	40
2	0	8	16	20	32	40
3	0	8	16	20	32	40
4	4	20	20	30	36	40
5	4	20	20	30	36	40
6	4	20	20	30	36	40
7	4	20	20	30	36	40

For each pixel, calculate its distance from the centroids, where the distance is:

$$\sqrt{(Gray\ level_{pixel} - Gray\ level_{centroid})^2 + (X_{pixel} - X_{centroid})^2 + (Y_{pixel} - Y_{centroid})^2}$$

Note: The value of the gray levels is very high compared to the values of *x* and *y*. This means that the coordinates of a pixel will not affect our calculations, so the result of this segmentation technique will yield the same result as Question 2. To overcome this we have to either:

- Divide the gray levels by an appropriate value to make them of the same weights of the coordinates. (In this example divide by 5 so that the gray levels range from 0 to 8 and the x coordinates range from 0 to 5 and the y coordinates range from 0 to 7).
- Normalize all dimensions by dividing them by their maximum value, so that all dimensions range from 0 to 1 and have equal weights. (In this example divide the gray levels by 40, the x coordinates by 5 and the y coordinates by 7).

Repeat the steps above for getting the centroids.

3. As a measurement of performance, can measure the standard deviation of the colors of each object. A good segmentation will yield objects with the least standard deviation in colors (gray levels in this example). We can also measure the distance between the pixels forming each object, because an object is most likely to be continuous with no gaps in between its pixels.