① $\gcd(a,b)$

$a = p_1, \underbrace{p_2, p_3}, p_4$

$b = q_1, q_2, \underbrace{p_2, p_3}, q_4$

$\gcd(a,b) = p_2 p_3.$

$\gcd(a,b) = \gcd(a-b, b)$

$= \gcd(a-2b, b)$

$= \cdots \gcd(a \bmod b, b)$

$\boxed{1}$ show that

a) $\gcd(2a+1, 9a+4) = 1$, $a \in \mathbb{Z}$

$= \gcd(7a+3, 2a+1)$

$= \gcd(5a+2, 2a+1)$

$= \gcd(3a+1, 2a+1)$

$= \gcd(a, 2a+1)$

$= \gcd(a, a+1)$

$= \gcd(a, 1) = \underline{1}$

c) if a is odd, then $\gcd(3a, 3a+2) = 1$

Sol

$\gcd(3a, 3a+2)$

$= \gcd(2, 3a)$

since a is odd, $3a$ is also odd

$= \underline{1}$

$\boxed{2}$ $a \& b \in \mathbb{Z} - \{0\}$

prove that $\gcd(2a-3b, 4a-5b)$ divides b hence $\gcd(2a+3, 4a+5) = \underline{1}$

Sol

$\gcd(2a-3b, 4a-5b)$

$= \gcd(2a-2b, 2a-3b)$

$= \gcd(b, 2a-3b)$

$\therefore \gcd(2a-3b, 4a-5b) \mid b$

Consequently

$\gcd(2a+3, 4a+5) = 1$

$\boxed{3}$ $\gcd(143, 227) = 1$

$143 = 11 \cdot 13$

$227 = 227$

$\boxed{\text{or}}$

$= \gcd(143, 84) = \gcd(84, 59)$
$= \gcd(59, 25) = \gcd(25, 9)$
$= \gcd(9, 7) = 1$

$\gcd(272, 1479) = 17$

$1479 = 3 \cdot 17 \cdot 29$

$272 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 17$

$\boxed{\text{or}}$

$\gcd(272, 119)$
$= \gcd(119, 34)$
$= \gcd(34, 17) = 17$

$\boxed{4}$ find $x \& y \in \mathbb{Z}$ such that

$$\gcd(56,72) = 56x + 72y$$

$$\gcd(56,72)$$
$$= \gcd(56,16)$$
$$= \gcd(16,8)$$
$$= 8$$

$$72 = (1)(56) + 16$$
$$56 = (3)(16) + 8$$
$$56 = 3[72 - 56] + 8$$
$$56 \cdot 4 = 3 \cdot 72 + 8$$
$$\boxed{8 = 56 \cdot 4 - 3 \cdot 72}$$

$$\gcd(24,138) = 24x + 138y$$

$$\gcd(24,138)$$
$$= \gcd(24,18)$$
$$= \gcd(18,6)$$
$$= 6$$

$$138 = (5)(24) + 18$$
$$24 = 18 + 6$$
$$24 = 138 - 5 \cdot 24 + 6$$
$$6 = 6 \cdot 24 - 138$$

$\#$

$\mathrm{lcm}(a,b)$

$\{a, 2a, 3a, 4a, \dots\} = S_1$

$\{b, 2b, 3b, 4b, \dots\} = S_2$

$\mathrm{lcm}(a,b) = \text{minimal } S_1 \cap S_2$

$$\mathrm{lcm}(a,b) = \frac{ab}{\gcd(a,b)}$$

$\mathrm{lcm}(143, 227)$

$143 = 11 \cdot 13$

$227 = 227$

$\mathrm{lcm}(143, 227) = 11 \cdot 13 \cdot 227$
$= ab$

$\mathrm{lcm}(272, 1479) = 3 \cdot 17 \cdot 29$

$1479 = 3 \cdot 17 \cdot 29$

$272 = 2^4 \cdot 17$

$\cdot 2^4$