Digital Communications (ELC 325b)

© Samy S. Soliman

Electronics and Electrical Communication Engineering Department
Cairo University, Egypt

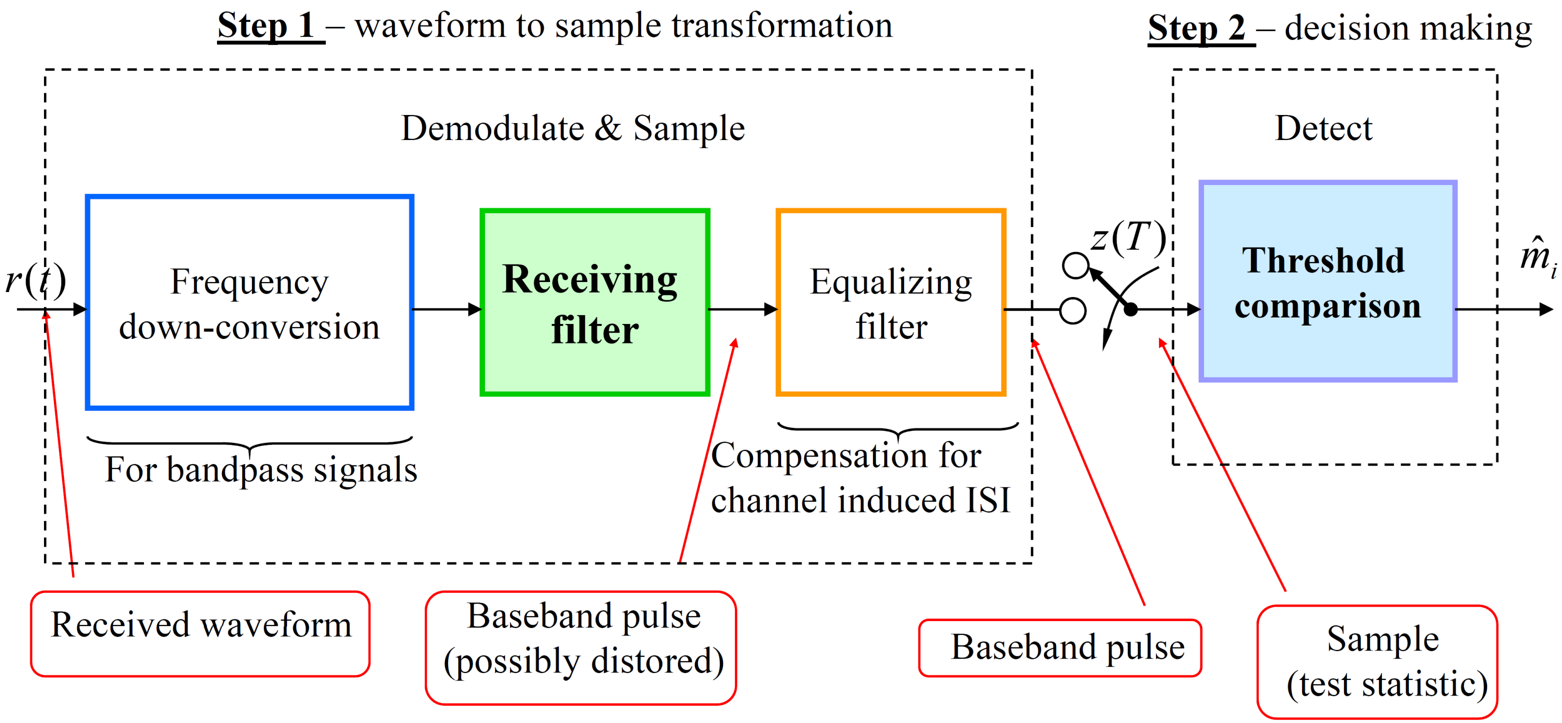
intro to ISI
ISI -> inter symbol inteference

Spring 2016

Outline

1. Base-Band Transmission and Structure of Optimum Receiver
   - Inter-Symbol Interference ISI
   - Design of Optimum ISI-Free Communication System

Inter-Symbol Interference



What is ISI?

It is another source of errors that arises when the **communication channel is dispersive**. symbol here is a combination of bits.

It occurs because dispersed **symbols are expanded beyond the symbol duration** to interfere with adjacent symbols

Inter-Symbol Interference

w ehna practicaly m3ndnash infinite band

- A rectangular pulse requires an infinite bandwidth on the channel. This is cannot be practically achieved
- A band-limited pulse will be widely spread in time (This is refereed to as **Time Spread**)

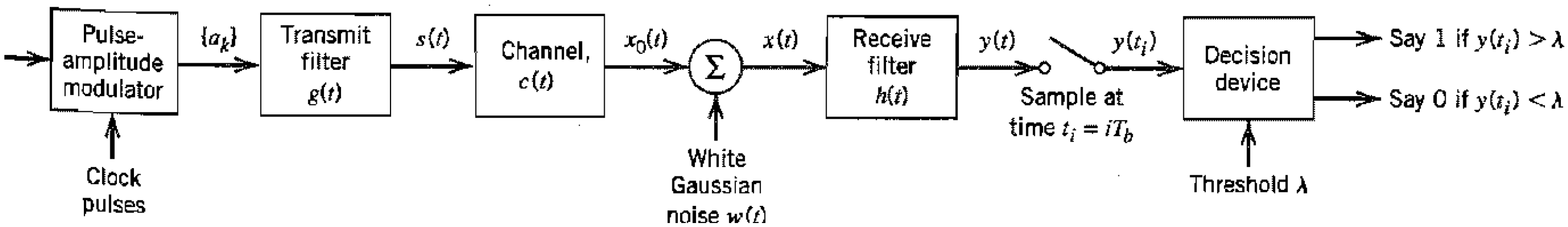
Requirements of the pulse shape

- Band-limited
- Does not interfere with adjacent pulses

by7sl inteference fl time, lakn msh fl amakn el enta bt3ml feha sampling, fa da 34an my7slsh 3ndk aliasing fl frequence.

ISI will cause that the amplitudes of the pulses change resulting in less robustness to noise

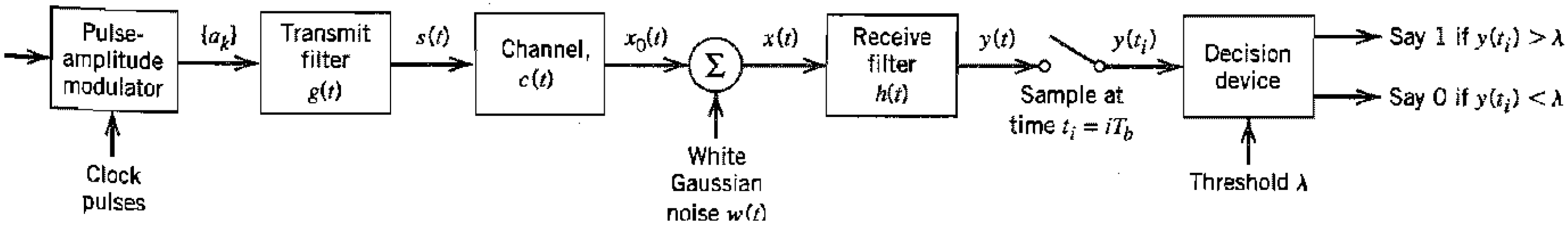
The analytics of ISI



this noise is due to the channel

- The input is binary bits, each of duration T_b
- $\{a_k\}$ is a sequence of amplitude-modulated short pulses. In the case of binary PAM, $a_k = \pm 1$
- $s(t)$ is a sequence of pulse shaped symbols
- $y(t)$ is the output of the receiver filter. It is sampled at $t_i = iT_b$ synchronously with the transmitter
- The decision device finally decides, based on a threshold λ, whether the sample is '1' or '0'

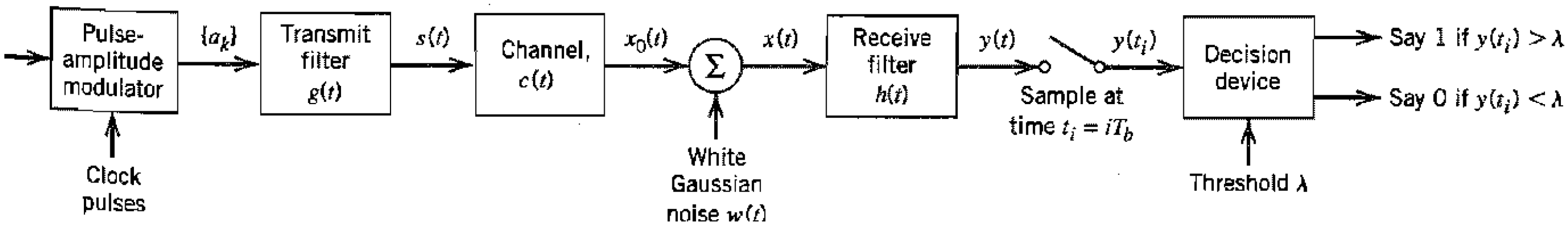
The analytics of ISI



$$
\begin{aligned}
s(t) &= \sum_k a_k g(t - kT_b) \\
x_0(t) &= s(t) * c(t) \\
x(t) &= x_0(t) + w(t) = s(t) * c(t) + w(t) \\
y(t) &= x(t) * h(t) \\
&= s(t) * c(t) * h(t) + w(t) * h(t) \\
&= \mu \sum_k a_k p(t - kT_b) + n(t), \qquad \mu p(t) = g(t) * c(t) * h(t)
\end{aligned}
$$

$$\mu P(f) = G(f)C(f)H(f)$$

The analytics of ISI



$$
\begin{aligned}
y(t_i) &= \mu \sum_k a_k p(t_i - kT_b) + n(t_i) \\
&= \mu a_i + \mu \sum_{k \neq i} a_k p((i-k)T_b) + n(t_i)
\end{aligned}
$$

- **The first term** represents the contribution of the i^{th} bit (This is the bit that needs detection)
- **The second term** represents the effect of all other transmitted bits on the decoding of the i^{th} bit (This is the ISI)
- **The third term** is the noise sample at the sampling time

Dealing with ISI

- The presence of ISI and noise in the DCS is unavoidable. This is introduces errors in the decision of the decision device
- In the design of the transmit and receive filters, $g(t)$ and $h(t)$, the objective of minimizing the effects of ISI as well as noise should be considered
  Note that ideally $y(t_i) = \mu a_i$

Problem Statement

How to design of the optimum $p(t) = g(t) * c(t) * h(t)$, such that effects of ISI and noise are minimized?

Design of ISI-Free Systems

For the system to be ISI-free

$$p(nT_b) = p((i-k)T_b) = \begin{cases} 1, & i = k \\ 0, & i \neq k \end{cases}$$

Note: Such condition results in perfect detection of the transmitted symbols, in the absence of noise.

$p(nT_b)$ are samples of $p(t)$, sampled by a train of impulses, then using the sampling theorem,

$$P_s(f) = \frac{1}{T_b} \sum_{-\infty}^{\infty} P\left(f - n\frac{1}{T_b}\right)$$

where $P_s(f)$ is the Fourier Transform of $p(nT_b)$
Since $p(nT_b) = \delta(n)$, then $P_s(f) = 1$

Nyquist's Criterion

The condition of zero ISI is

$$\sum_{-\infty}^{\infty} P\left(f - nR_b\right) = T_b, \qquad \text{where } R_b = \frac{1}{T_b}$$

Nyquist's criterion for distortion-less baseband transmission in the absence of noise states that the frequency function $P(f)$ will eliminate the ISI for samples taken at intervals of T_b provided that the above equation is satisfied

Note: The RHS of the Nyquist's Criterion need not be exactly T_b.
It can be any constant.

Ideal Nyquist Channel

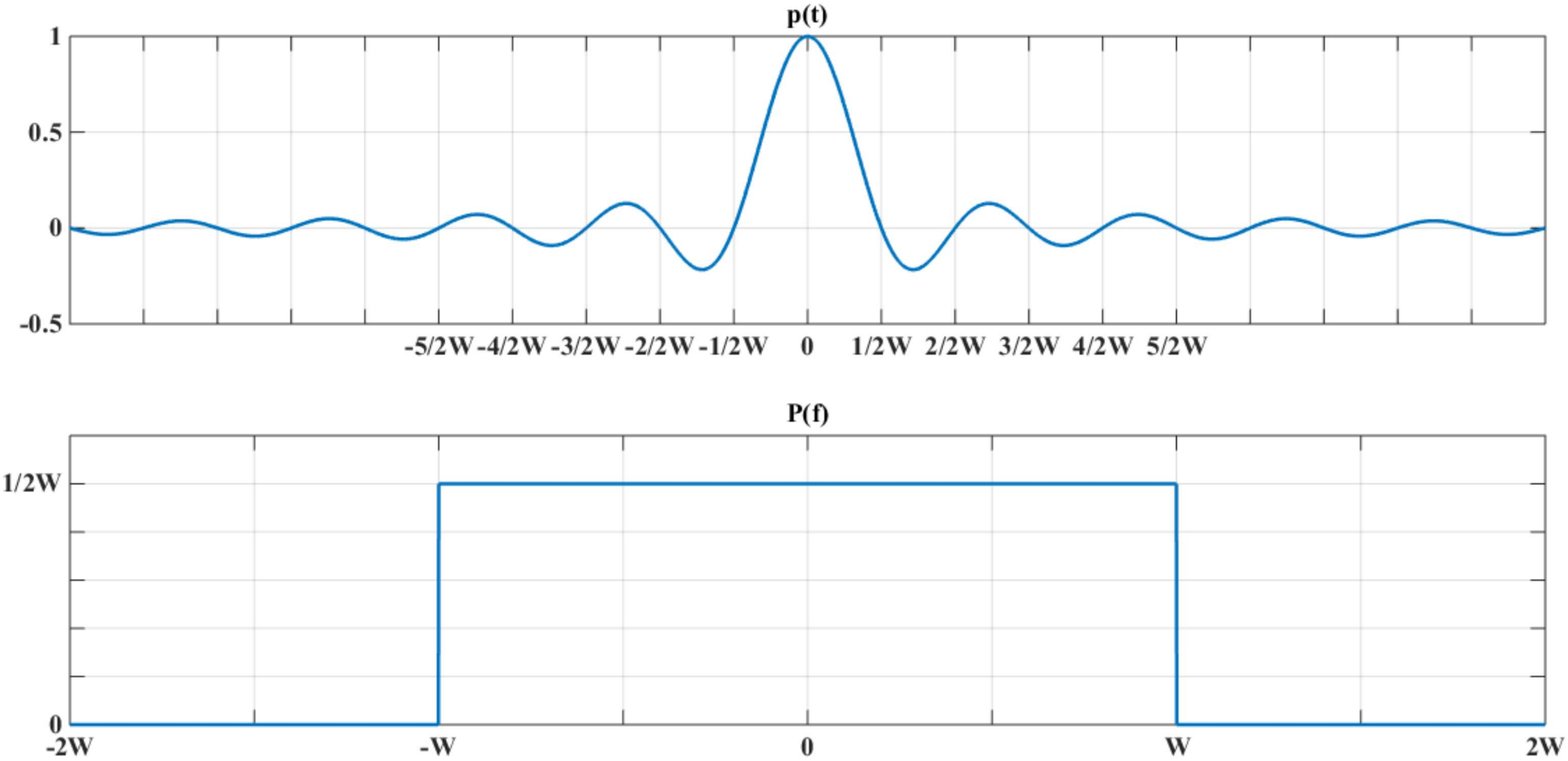
Simplest way to satisfy the Nyquist's Criterion

Ideal Nyquist Channel: Rectangular Form

$$
\begin{aligned}
P(f) &= \frac{1}{2W}\mathrm{rect}\left(\frac{f}{2W}\right) = \begin{cases} \frac{1}{2W}, & -W < f < W \\ 0, & |f| > W \end{cases} \\
p(t) &= sinc\,(2Wt) = \frac{\sin(2\pi Wt)}{2\pi Wt}
\end{aligned}
$$

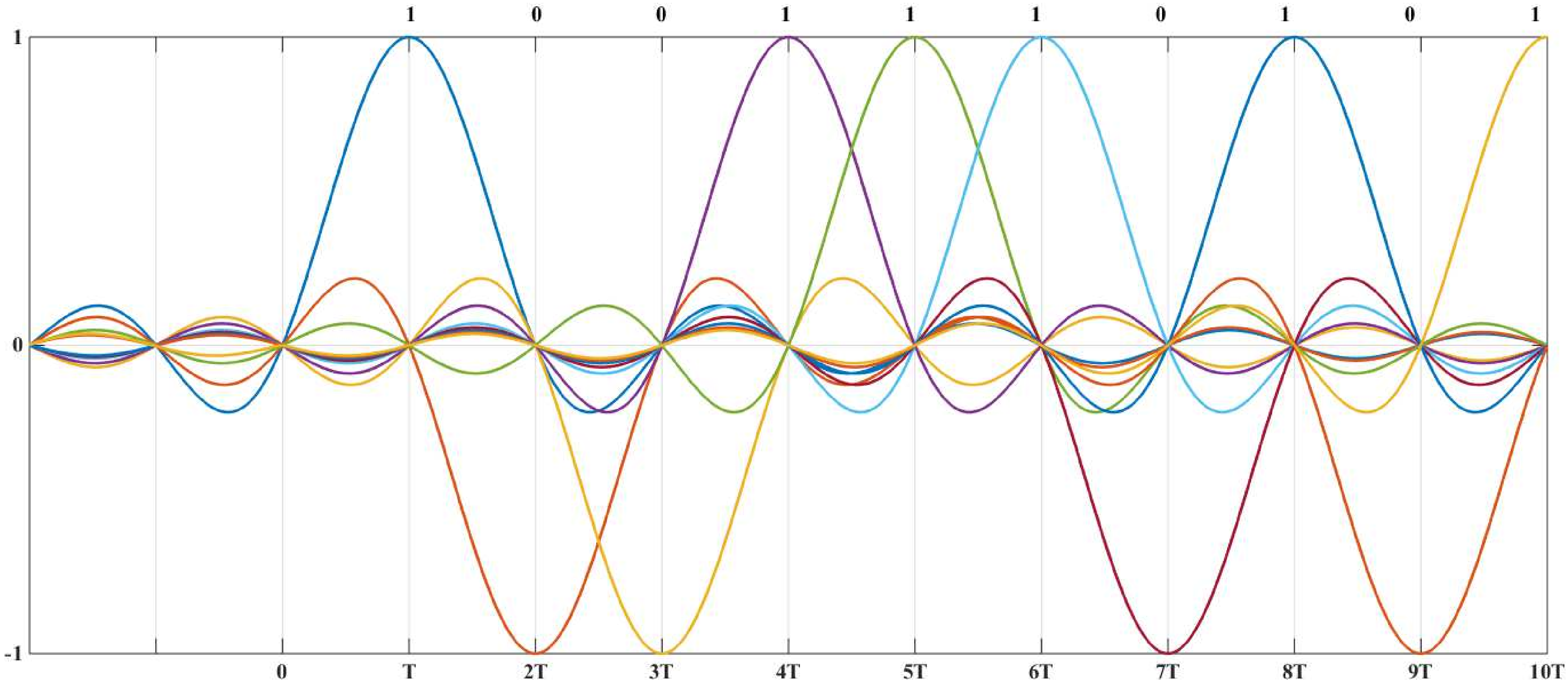
$$
\begin{aligned}
\textbf{Nyquist B.W.} \quad W &= \frac{R_b}{2} = \frac{1}{2T_b} \\
\textbf{Nyquist Rate} \quad R_b &= 2W \\
T_b &= \frac{1}{2W}
\end{aligned}
$$

Ideal Nyquist Channel



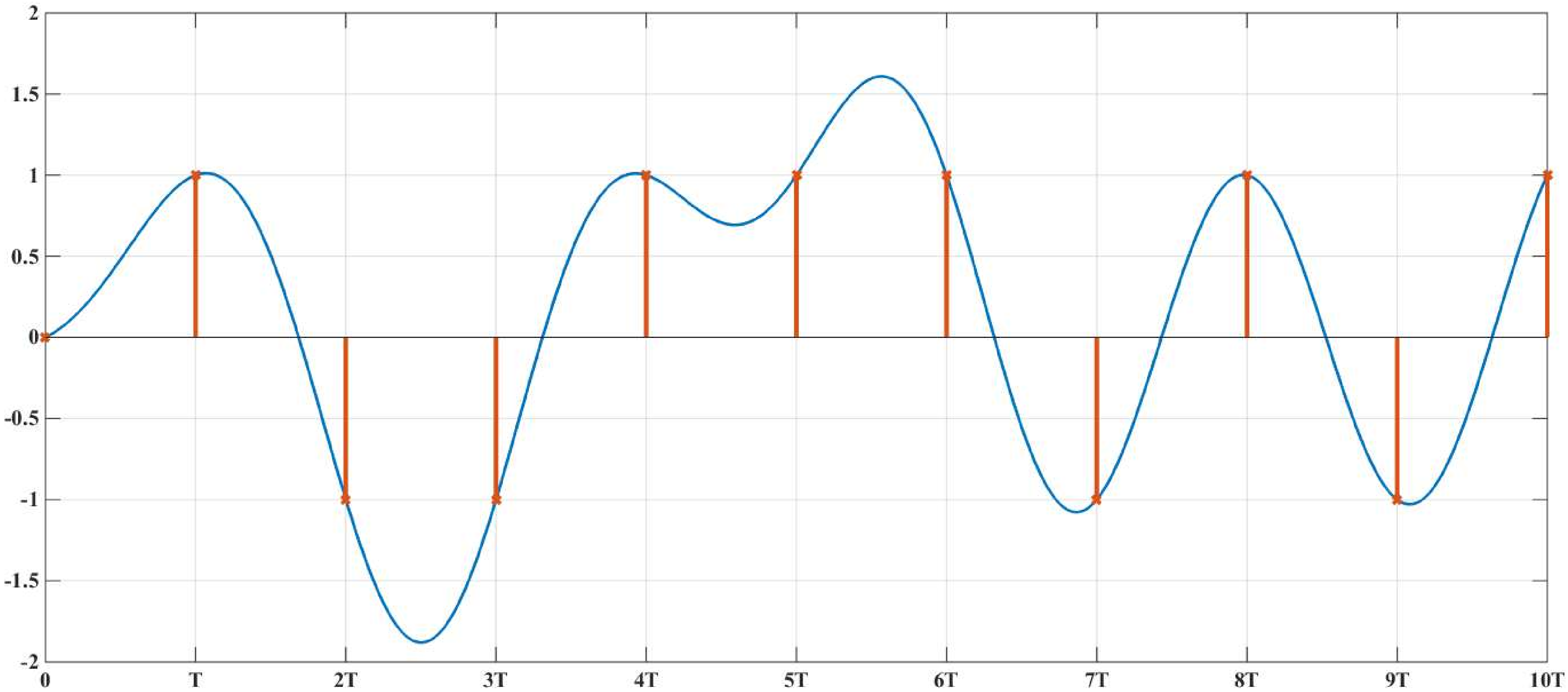
Ideal Nyquist Channel

Transmitted Stream = 1 0 0 1 1 1 0 1 0 1



Ideal Nyquist Channel

Transmitted Stream = 1 0 0 1 1 1 0 1 0 1



Sampling at $t_i = iT$

Detected Stream = 1 0 0 1 1 1 0 1 0 1

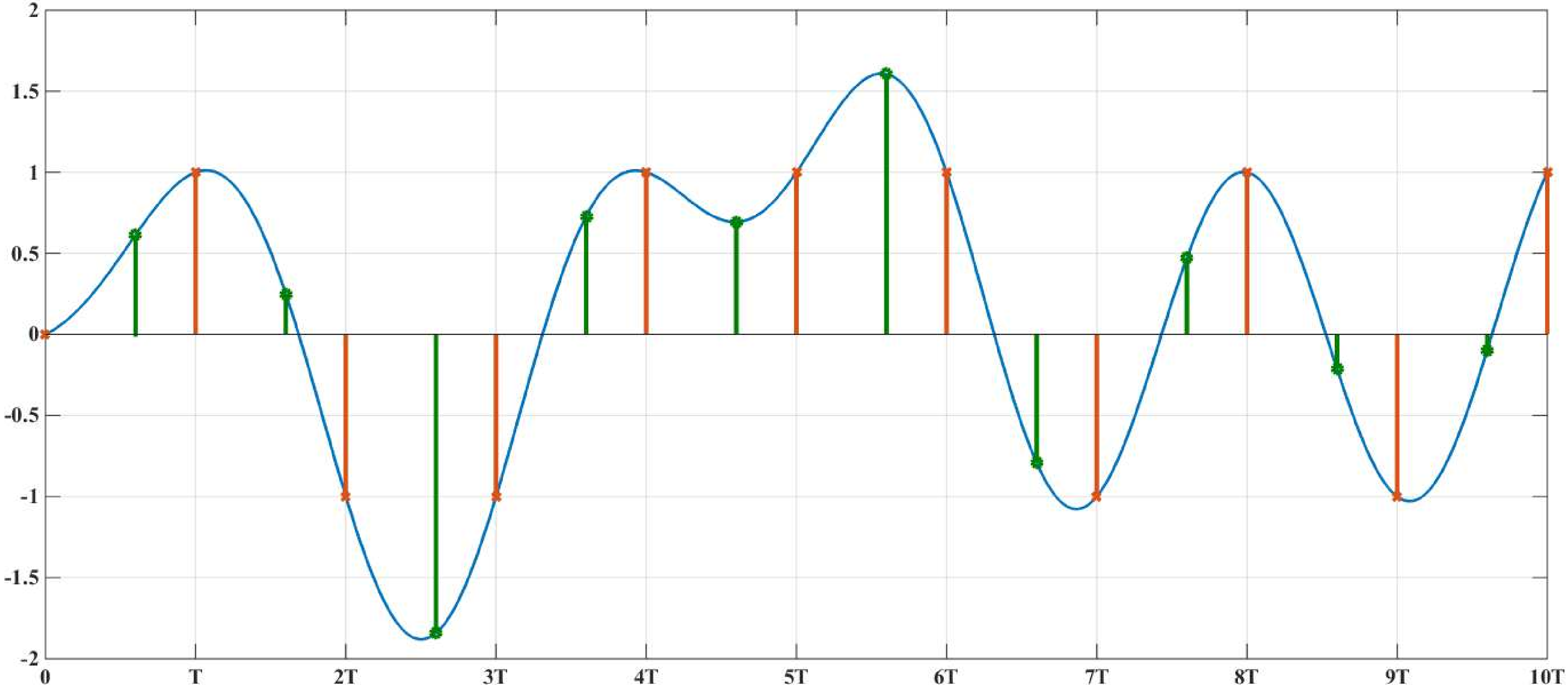
Ideal Nyquist Channel

Advantages and Disadvantages

1. Economic bandwidth usage: Solves the problem of ISI using the minimum possible bandwidth
2. There are practical difficulties:
    - The sudden transition at $f = \pm W$ is physically unrealizable
    - $p(t)$ decreases as $\dfrac{1}{|t|}$ resulting in a slow rate of decay. So, if the sampling times are slightly shifted, large ISI (because of many previous and following pulse signals) will be caused.

Ideal Nyquist Channel

Transmitted Stream = 1 0 0 1 1 1 0 1 0 1



Sampling at $t_i = iT - \epsilon$

Detected Stream = 1 1 0 1 1 1 0 1 0 0

Effect of Sync = ✓X ✓✓✓✓✓✓X

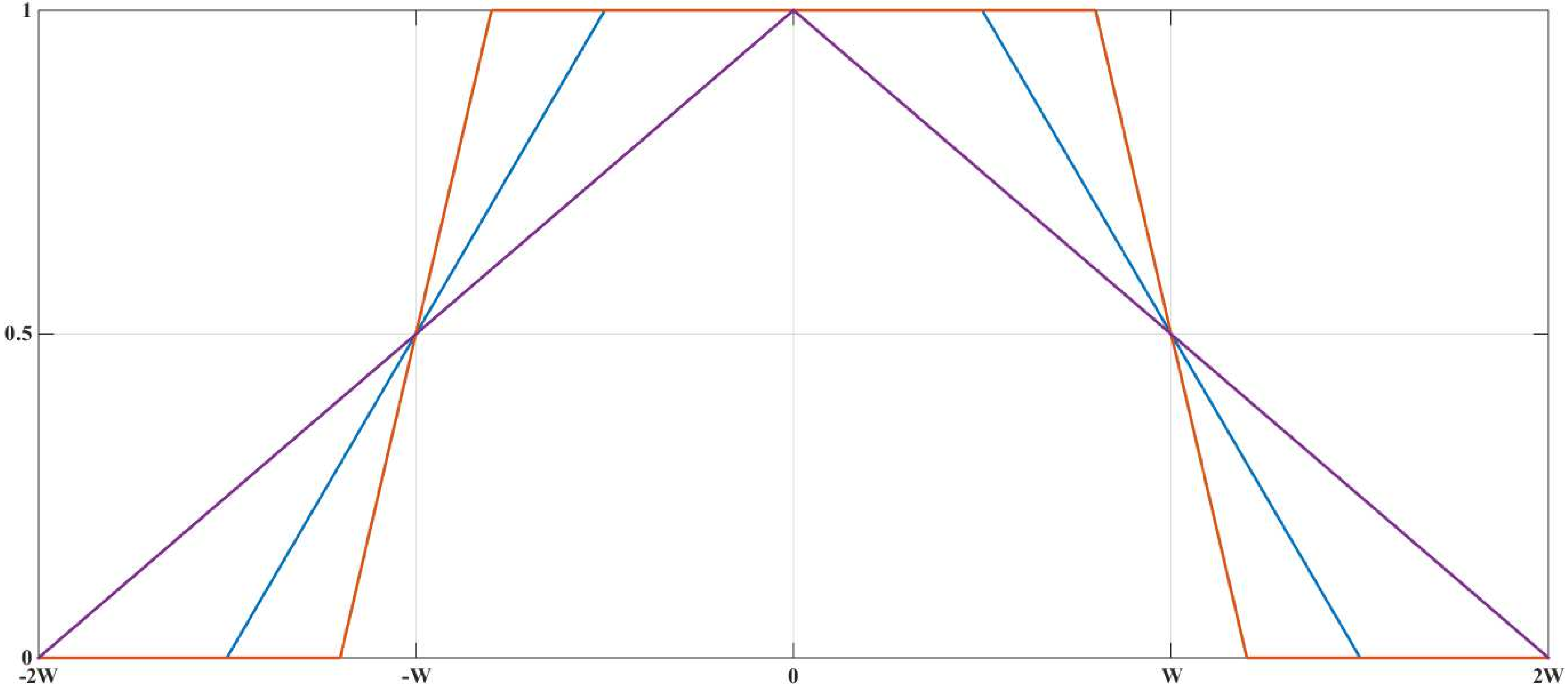
Reduced Nyquist's Criterion

In order to overcome the practical difficulties arising from using the ideal Nyquist channel, we can extend the bandwidth from its minimum value, $W = \frac{R_b}{2}$ to an adjustable value between W and $2W$, such that

$$P(f) + P(f - 2W) + P(f + 2W) = \frac{1}{2W}, \quad -W \leq f \leq W$$

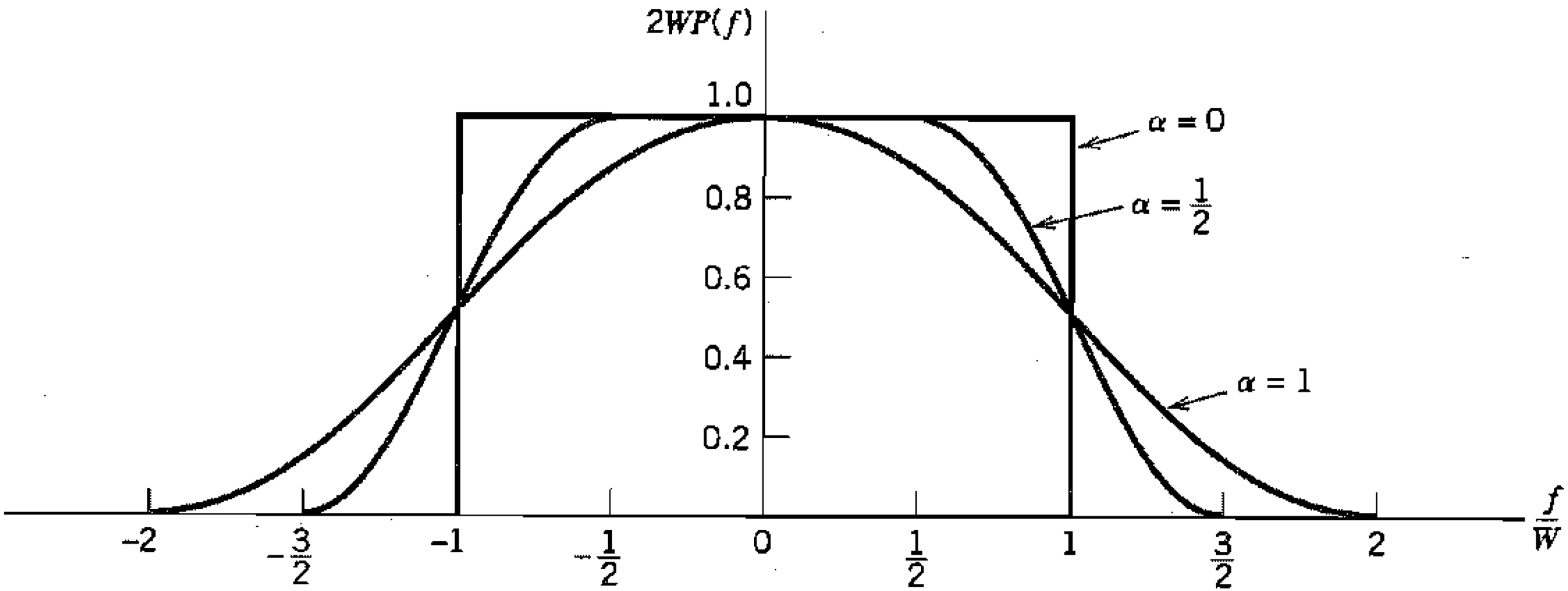
Many band-limited functions that satisfy the above equation can be found.

Reduced Nyquist's Criterion



Raised Cosine Spectrum

One of the most common forms of $P(f)$ is the **Raised Cosine Spectrum**, which consists of a **flat portion** and a **roll-off portion** of a sinusoidal form.



Raised Cosine Spectrum

$$P(f) = \begin{cases} \frac{1}{2W}, & 0 \leq |f| \leq (1-\alpha)W \\ \frac{1}{4W}\left[1 - \sin\left(\frac{\pi(|f|-W)}{2\alpha W}\right)\right], & (1-\alpha)W \leq |f| \leq (1+\alpha)W \\ 0, & (1+\alpha)W \leq |f| \end{cases}$$

$$p(t) = sinc(2Wt)\frac{\cos(2\pi\alpha Wt)}{1 - 16\alpha^2 W^2 t^2}$$

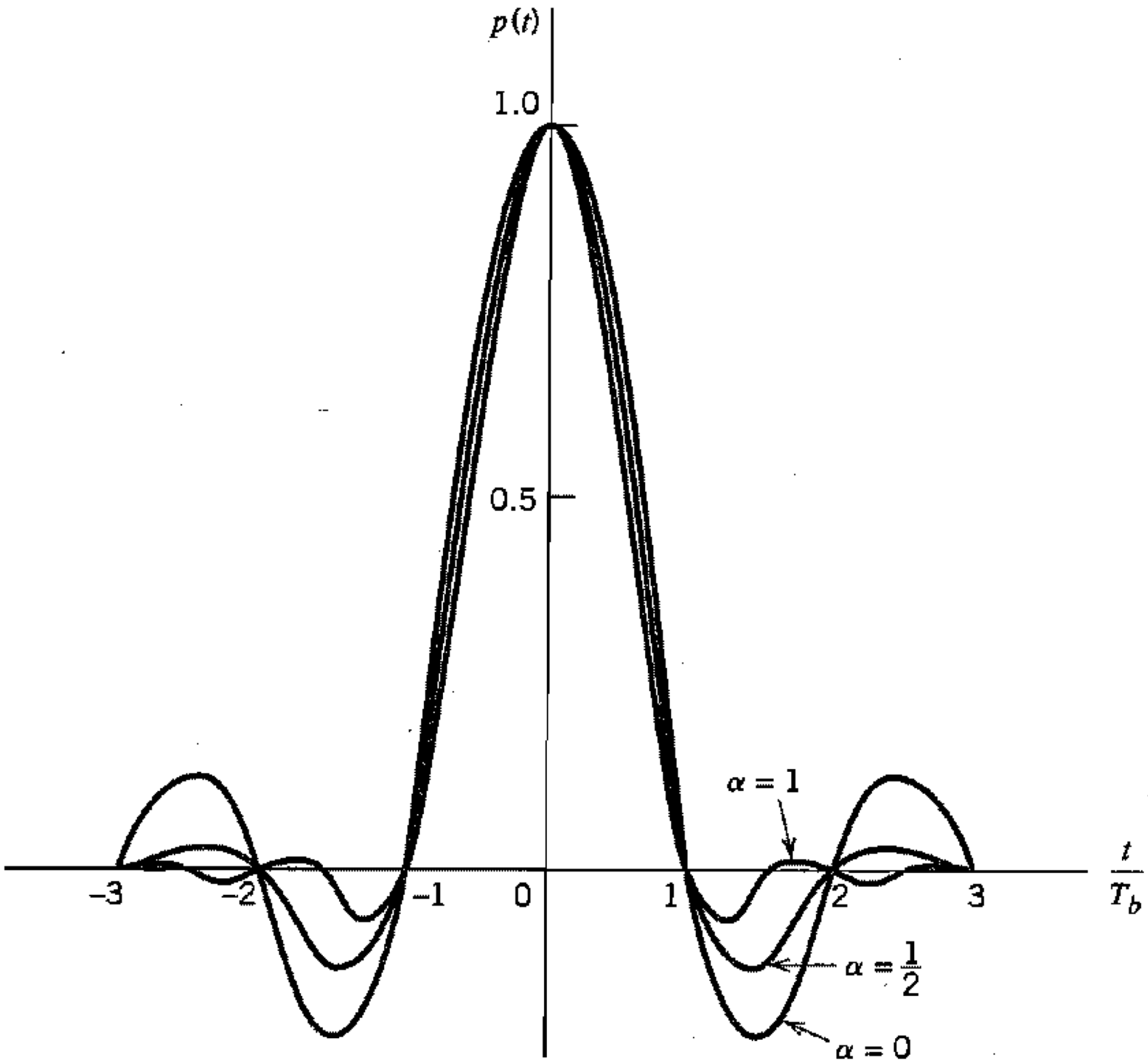
$$f_1 = (1-\alpha)W$$

Roll-off Factor $\alpha = 1 - \dfrac{f_1}{W}$

$$0 \leq \alpha \leq 1$$

Transmission B.W. $B_T = 2W - f_1 = (1+\alpha)W$

Raised Cosine Spectrum

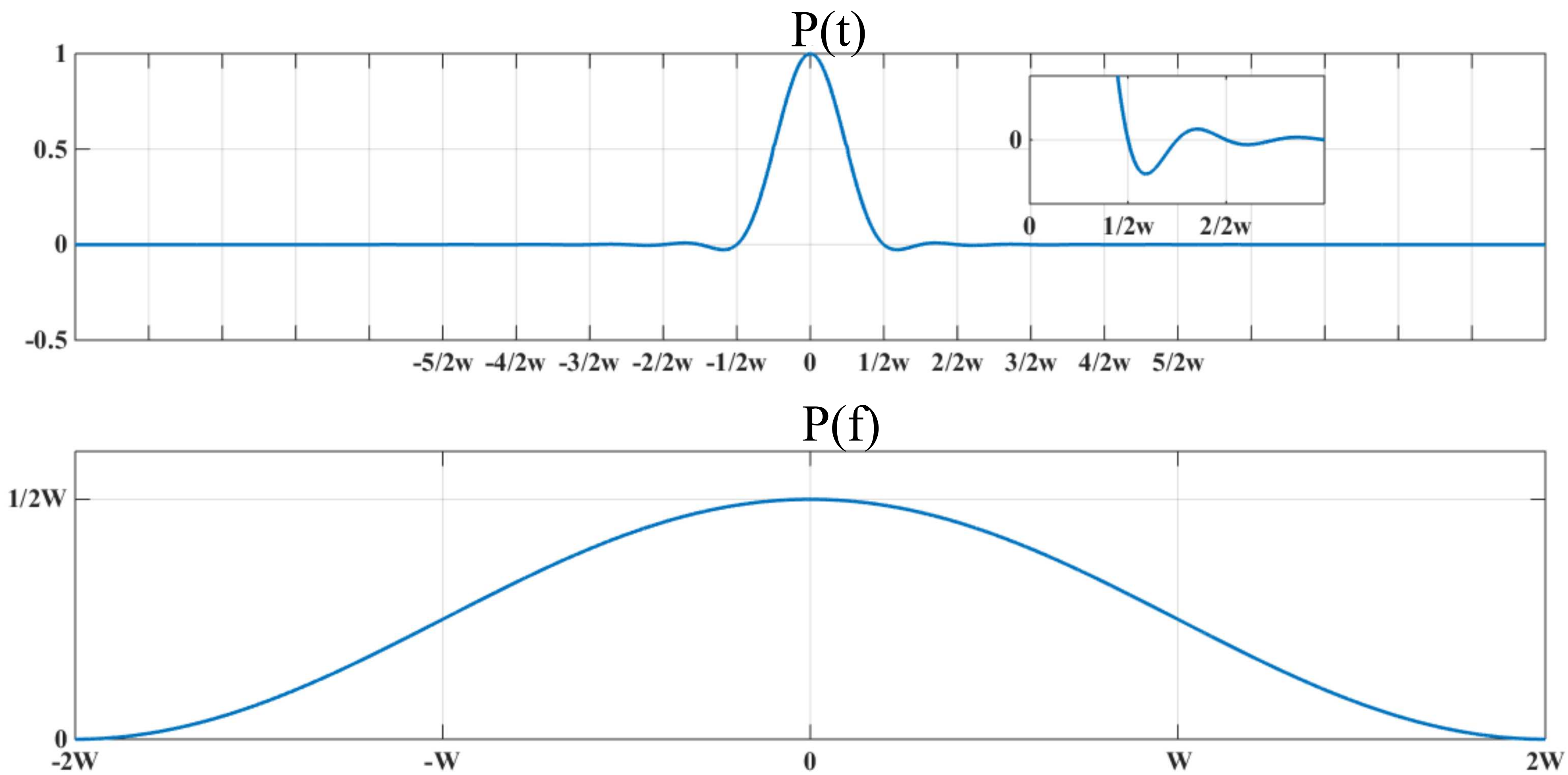


Raised Cosine Spectrum

Features of the Raised Cosine Spectrum

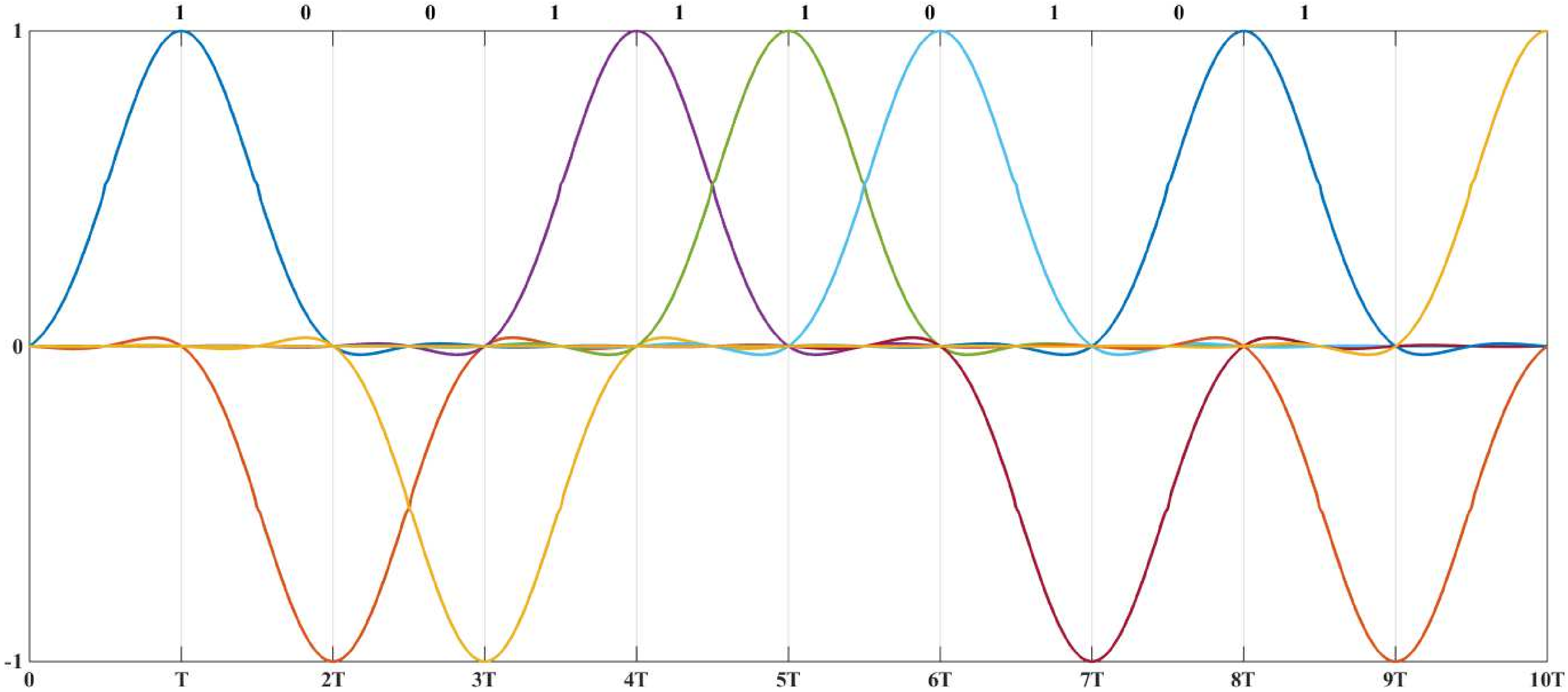
1. $P(f)$: Gradual cut-off
2. $p(t)$: Sinc component $\rightarrow$ zero-crossings at $t_i = iT$ $\rightarrow$ ISI-Free
3. $p(t)$: Fast decay $\dfrac{1}{|t|^2}$
4. For $\alpha = 1$: Full Roll-Off - Larger B.W. - Extra Zero-crossings

Raised Cosine Spectrum: Full Cosine Roll-Off



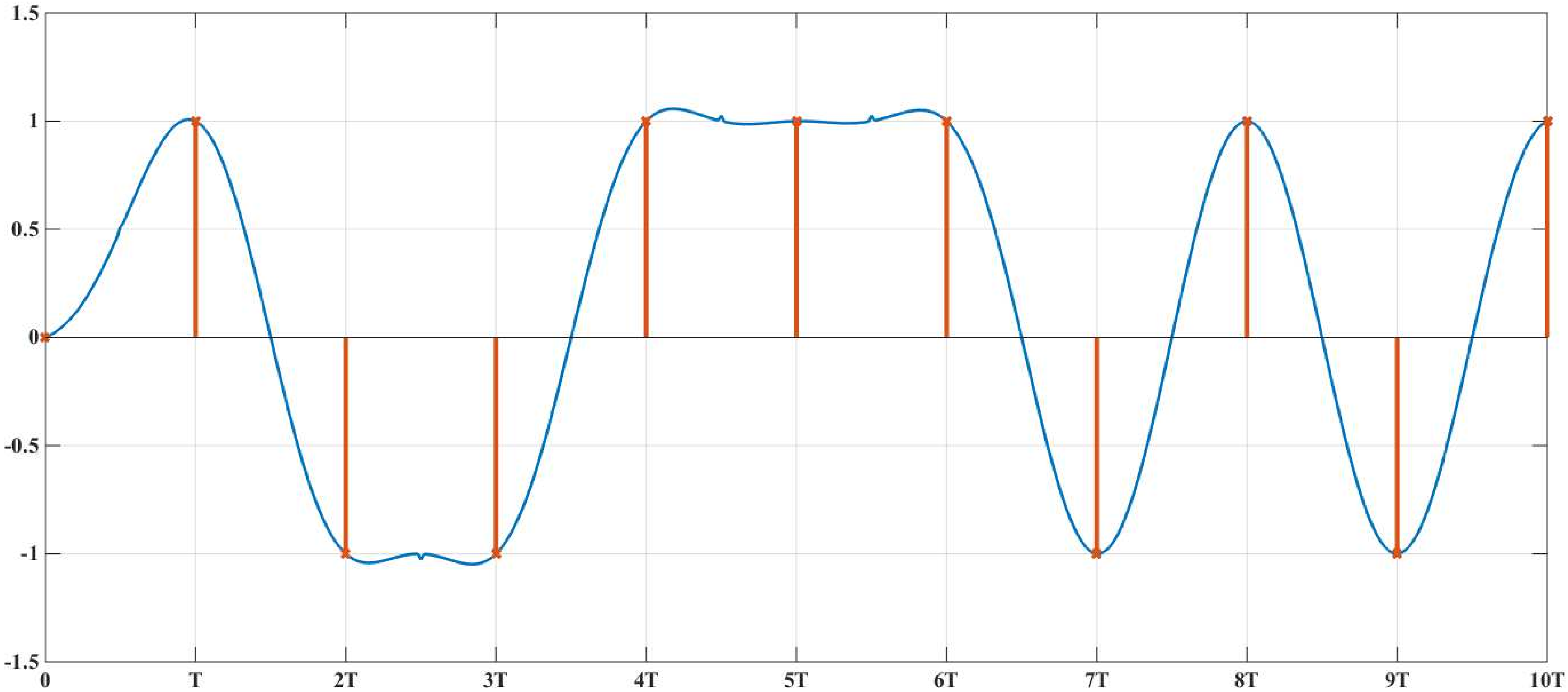
Raised Cosine Spectrum: Full Cosine Roll-Off

Transmitted Stream = 1 0 0 1 1 1 0 1 0 1



Raised Cosine Spectrum: Full Cosine Roll-Off

Transmitted Stream = 1 0 0 1 1 1 0 1 0 1

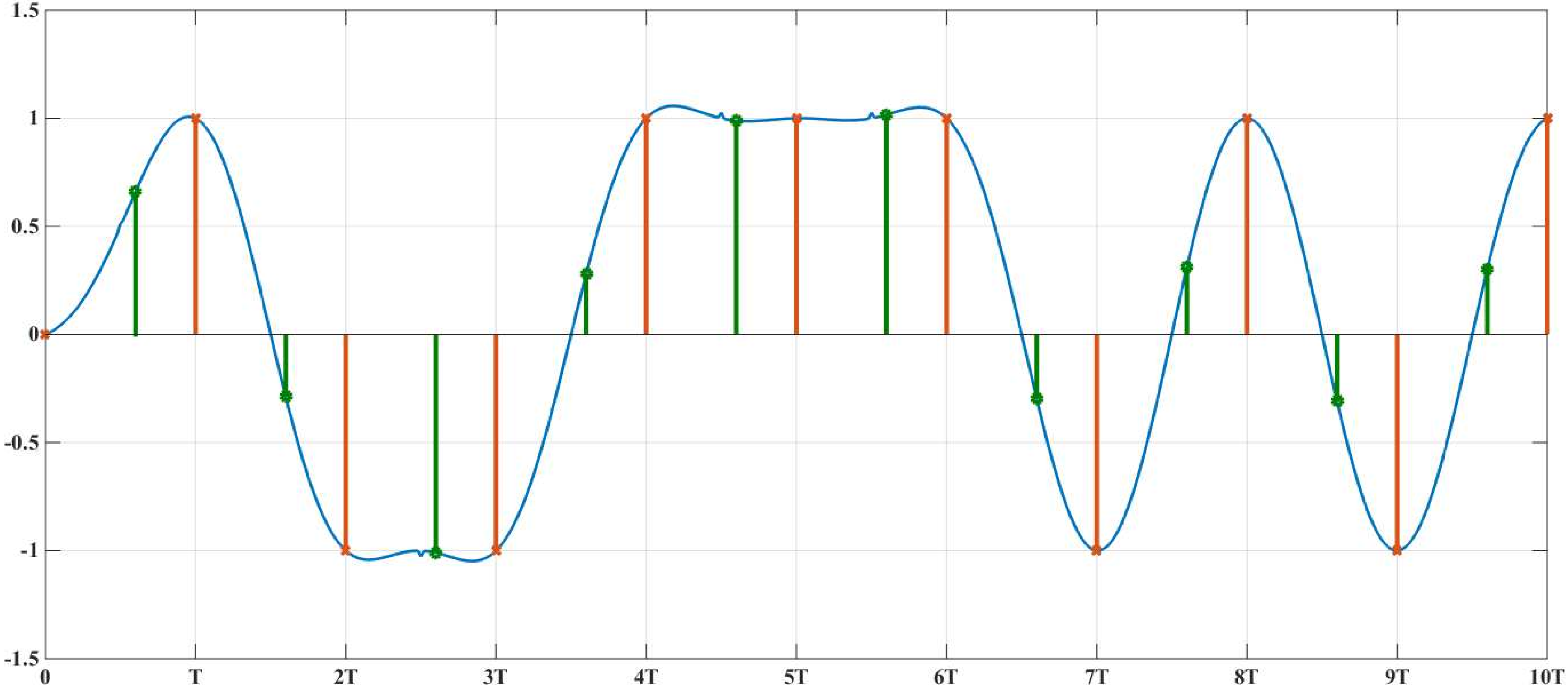


Sampling at $t_i = iT$

Detected Stream = 1 0 0 1 1 1 0 1 0 1

Raised Cosine Spectrum: Full Cosine Roll-Off

Transmitted Stream = 1 0 0 1 1 1 0 1 0 1

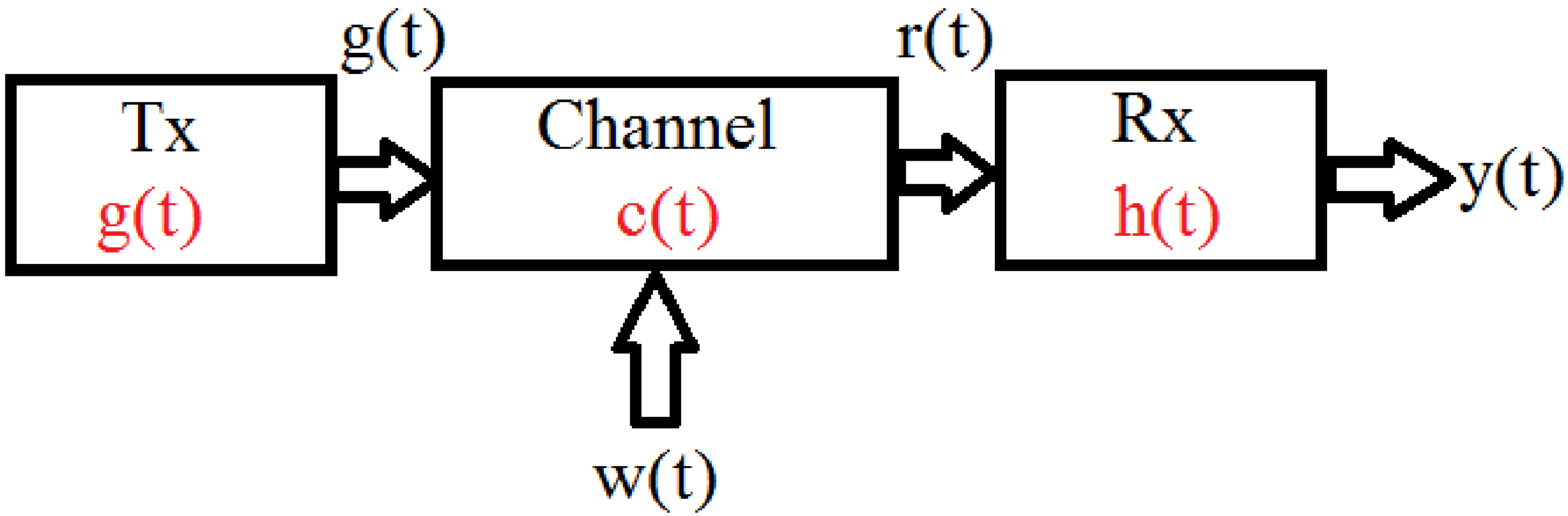


Sampling at $t_i = iT - \epsilon$

Detected Stream = 1 0 0 1 1 1 0 1 0 1

Effect of Sync = ✓✓✓✓✓✓✓✓✓

Design of Optimum ISI-Free Communication System



Recall:

$$
\begin{aligned}
p(t) &= g(t) * c(t) * h(t) \\
P(f) &= G(f)C(f)H(f)
\end{aligned}
$$

Conditions to satisfy:

1. Condition of Matched Filter
2. Condition of ISI-Free

Design of Optimum ISI-Free Communication System

Condition of Matched Filter

$$H(f) = G^*(f)e^{-j2\pi fT}$$

The exponential term represents a phase shift, equivalent to time delay.

Condition of ISI-Free

$$H(f)G(f) = P(f)$$

It is assumed that the channel is flat, $C(f) = \text{const.}$, for at least the maximum possible B.W. of the pulse $P(f)$.

$$\begin{aligned} G(f) &= \sqrt{P(f)} \\ H(f) &= \sqrt{P(f)} \end{aligned}$$

Square-Root Raised Cosine Spectrum

Transmission Bandwidth

Assuming a symbol duration of T,

$$\begin{aligned} B_T &= (1+\alpha)W \\ &= (1+\alpha)\frac{R_{\mathrm{s}}}{2} = (1+\alpha)\frac{1}{2T_{\mathrm{s}}} \end{aligned}$$

In the case of Binary Transmission,

$$\begin{aligned} T_{\mathrm{s}} &= T_b \\ B_T &= (1+\alpha)\frac{1}{2T_b} \end{aligned}$$

In the case of M-ary Transmission,

$$T_s = (\log_2 M)T_b$$

$$\begin{aligned} B_T &= (1+\alpha)\frac{1}{2T_s} \\ &= (1+\alpha)\frac{1}{\log_2 M}\frac{1}{2T_b} \end{aligned}$$

Transmission Bandwidth

Example

A computer puts out binary data at 56 kbps. The computer output is transmitted using a baseband binary PAM system that has a raised-cosine spectrum. Determine the transmission bandwidth for $\alpha = 0.25, 0.5, 0.75, 1$. Repeat if each of 3 successive binary digits are coded into one of eight PAM levels.

$$B_T = (1+\alpha)W, \qquad W = \frac{R_b}{2} = 28 \text{ kbps}$$

$$B_T = (1+\alpha)W, \qquad W = \frac{R_s}{2} = \frac{\frac{R_b}{\log_2 8}}{2} = \frac{28}{3} \text{ kbps}$$

References

Simon Haykin (2001)
Communication Systems, 4th Edition.
John Wiley.

Thank You

Questions ?