NT Sheet 4

6. Verify that $17 | 11^{104} + 1$

7. Let $n \geq 0$ show that $13 | 11^{12n+6} + 1$

8. Let $\gcd(a, 35) = 1$ then establish $a^{12} \equiv 1\ (mod\ 35)$

9. Prove that $a^{21} \equiv a\ (mod\ 15)$

10. Show that $1^{p-1} + 2^{p-1} + \cdots + (p-1)^{p-1} \equiv -1\ (mod\ p)$

11. Let p and q be two primes and $p \neq q$, establish that $p^{q-1} + q^{p-1} \equiv 1\ (mod\ pq)$

12. If $gcd(m, n) = 1$ show that $m^{\varphi(n)} + n^{\varphi(n)} \equiv 1\ (mod\ mn)$

13. Find the least positive residue of 3^{10^5} modulo 35 (compute 3^{10^5} % 35)

14. Solve $5x\ \equiv 3\ (mod\ 14)$ and $4x\ \equiv 7\ (mod\ 15)$ using Euler's theorem.

15. Show that $\sigma(n) = \sigma(n+1)$ for $n = 14,\ 206$

16. Prove that if $\tau(n)$ is odd then n must be a perfect square

17. Prove that $\frac{\sigma(n)}{n} = \sum_{d|n} \frac{1}{d}$

18. Find all integers satisfying $\tau(n) = 10$, what's the smallest of such integers?

19. If $k \geq 2$, establish that

a. If $n = 2^{k-1}$ then $\sigma(n) = 2n - 1$

b. If $2^k - 1$ is a prime, then if $n = 2^{k-1}(2^k - 1)$ then $\sigma(n) = 2n$

20. Compute $\varphi(1001)$ and $\varphi(5040)$

21. Show that $\varphi(2n) = \varphi(n)$ if n is odd and $\varphi(2n) = 2\varphi(n)$ if n is even