CE Sheet 3 Sol.

9) a) $S(t) = 10\cos(100\pi t + \frac{\pi}{3})$

- $\theta(t) = 2\pi(50)t + \frac{\pi}{3}$ rad — inst. Phase
  $\phi(t) = \frac{\pi}{3}$ rad — inst. Phase deviation
  $\Delta\theta = \frac{\pi}{3}$ rad — Peak Phase deviation

$\frac{d}{dt}$

- $\omega_i(t) = 2\pi(50)$ rad/s — inst. Frequency
  $\phi'(t) = 0$ rad/s — inst. Freq. deviation
  $\Delta\omega = 0$ rad/s — Peak Freq. deviation

b) $S(t) = 10\cos(200\pi t + \sin\pi t)$

- $\theta(t) = 200\pi t + \sin\pi t$ rad
  $\phi(t) = \sin\pi t$ rad
  $\Delta\theta = 1$ rad

- $\omega_i(t) = 200\pi + \pi\cos\pi t$ rad/s
  $\phi'(t) = \pi\cos\pi t$ rad/s
  $\Delta\omega = \pi$ rad/s

c) $S(t) = e^{j(200\pi t \cdot (1+\sqrt{t}))} \cdot 2$

- $\theta(t) = 200\pi t + 200\pi t^{1.5}$ rad $\quad (t\sqrt{t} = t^{1.5})$
  $\phi(t) = 200\pi t^{1.5}$ rad

- $\omega_i(t) = 200\pi + 300\pi t^{0.5}$ rad/s $\quad (\frac{d}{dt} t^{1.5} = 1.5 t^{0.5})$
  $\phi'(t) = 300\pi t^{0.5}$ rad/s

- $\Delta\theta, \Delta\omega$ diverge

d) $S(t) = \cos(200\pi t)\cos(5\sin(2\pi t))$
$+$
$\sin(200\pi t)\sin(5\sin(2\pi t))$

- $\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$. $\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$

$S(t) = \cos(200\pi t - 5\sin(2\pi t))$

- $\theta(t) = 200\pi t - 5\sin(2\pi t)$ rad
  $\phi(t) = -5\sin(2\pi t)$ rad
  $\Delta\theta = +5$ rad $\quad \max\{\phi(t)\}$

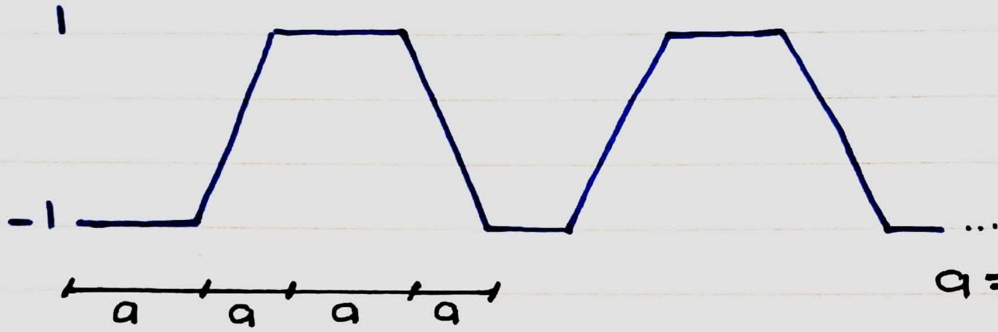
- $\omega_i(t) = 200\pi - 10\pi\cos(2\pi t)$ rad/s
  $\phi'(t) = -10\pi\cos(2\pi t)$ rad/s
  $\Delta\omega = 10\pi$ rad/s

can convert rad/s (angular freq.) to Hz (freq., required) by dividing by 2π

2)

- $\omega_c = 10^8$ rad/s, $K_p = 10^5$, $K_p = 25$

- $m(t)$



$a = \frac{10^{-3}}{4}$

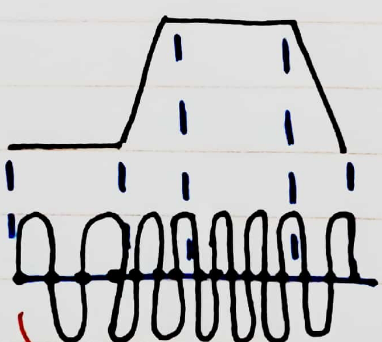
- Sketch $S_{FM}(t)$, $S_{AM}(t)$

FM)

→ $\omega_i = 10^8 + 10^5(m(t))$

→ $m(t)$ is either -1, increasing, 1, decreasing

$w_i = 10^8 - 10^5$ $\quad w_i \nearrow \quad w_i = 10^8 + 10^5 \quad w_i \searrow$



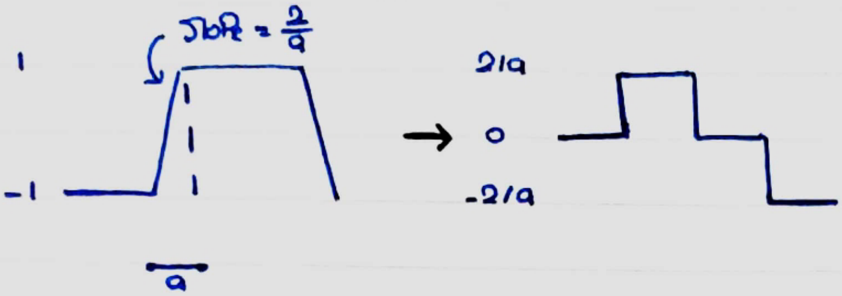
} One Period of

$S_{FM}(t) = \sin(10^8 t + 10^5 \int_{-\infty}^{t} m(\tau)\,d\tau)$

$w_i = 9.99 \times 10^{7}$ $\quad w_i = 1.001 \times 10^8$

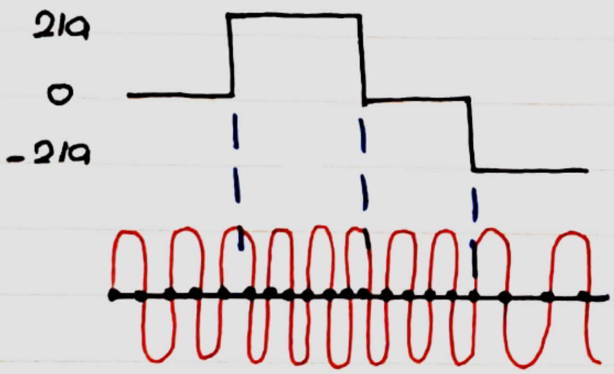
PM)

$$\omega_i = 10^8 + 25\frac{dm(t)}{dt}$$



Hence, $\omega_i = \begin{cases} 10^8 + 25\times 2/a \\ 10^8 \\ 10^8 - 25\times 2/a \end{cases}$

• $a = 10^{-3}/4$



one Period of $S_{PM}(t) = (10^8 t + 25m(t))$

$\omega_i =$ 10^8, 1.002×10^8, 10^8, 9.98×10^7 rad/s

3) $S(t) = 10\cos(13000t)$ $\quad -1 \leq t \leq 1$

• $\theta(t) = 13000t$

$\phi(t) = \theta(t) - \omega_c t = 3000t$

PM

• $\phi(t) = K_p m(t)$

Hence, $m(t) = \dfrac{\phi(t)}{K_p} = \dfrac{3000t}{1000} = 3t$

FM

• $\phi(t) = K_f \cdot a(t)$

Hence, $a(t) = 3t$ and

$m(t) = a'(t) = 3$

4)

$S(t) = 10\cos(2\pi 10^6 t + 0.1\sin 2000\pi t)$

a) $P = \frac{A_c^2}{2} = 50$

b) $\phi(t) = 0.1\sin 2000\pi t$ rad

$\phi'(t) = 0.1 \times 2000\pi \times \cos 2000\pi t$ rad/s

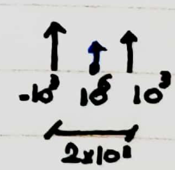
$\Delta\omega = 200\pi$ rad ($\Delta f = 100$ Hz)

c) $\Delta\theta = \max|\phi(t)| = 0.1$ rad

d) we have that $|\phi(t)_{max}| \ll 1$ so narrowband modulation holds

$BW = 2f_m = 2\times10^3$ Hz

↳ single tone @ $\left.\frac{2000\pi}{2\pi}\right\}\omega_m$



Alternatively,

$BW_{KN} = 2(\Delta f + f_m) \simeq 2f_m = 2\times10^3$ Hz

↳ $\Delta f \ll f_m$ ($\beta \ll 1$)

5. $m(t) = \sin(2000\pi t)$, $K_f = 2\times 10^5\pi$, $K_p = 10$

$\searrow f_m = 1000$

a) $\phi'_{FM}(t) = K_f\, m(t) = 2\times 10^5\pi \cdot \sin(2\times 10^3\pi t)$

$$\Delta f = \frac{|\phi'(t)_{max}|}{2\pi} = \frac{2\times 10^5\pi}{2\pi} = 10^5\,Hz > \underset{(f_m)}{10^3}$$

•Not NB

Carson gives

$$BW = 2(\Delta f + f_m) = 202\ KHz$$

$\phi'_{PM}(t) = K_p\, m'(t) = 10 \cdot 2\times 10^3\pi \cos(2\times 10^3\pi t)$

$$\Delta f = \frac{|\phi'(t)_{max}|}{2\pi} = \frac{2\times 10^4\pi}{2\pi} = 10^4\ Hz > \underset{(f_m)}{10^3}$$

•Not NB

Carson gives

$$BW = 2(\Delta f + f_m) = 22\ KHz$$

b) $m(t)$ is doubled $\rightarrow m'(t)$ is doubled

$\rightarrow \phi'(t)$ is doubled •Both FM, PM

$\rightarrow |\phi'(t)|_{max}$ is doubled

$\rightarrow \Delta f$ is doubled

Δf makes most of the BW, so its approximately doubled

$$BW_{FM} = 2(2\times 10^5 + 10^3) = 402\ KHz$$

$$BW_{AM} = 2(2\times 10^4 + 10^3) = 42\ KHz$$

$\rightarrow \Delta f > f_m$

- Clearly, for WB doubling $m(t)$ regardless to whether FM or PM will result in the bandwidth being nearly doubled
- If it was NB instead, then so long as doubling $m(t)$ maintains $\Delta f \ll f_m$ the bandwidth won't be affected (only depends on $m(t)$'s bandwidth)

c) Frequency is doubled

//analysis below is for single tone

Consider $\sin(2\pi f_m t)$

$\phi'_{FM}(t) = k_f \sin(2\pi f_m t)$

$\rightarrow$ if f_m is doubled, $\phi'(t)_{max}$ and hence Δf remain.

$\phi'_{PM}(t) = 2\pi f_m . k_p . \cos(2\pi f_m t)$

$\rightarrow$ $|\phi'(t)_{max}|$ gets doubled as well and hence also Δf

. In both cases BW increases

$$BW_{s(t)} = 2(\Delta f + f_m)$$

$\hookrightarrow$ was doubled

doubled only for PM

$\rightarrow$ Provided $\Delta f \gg f_m$ it will increase somewhat but won't double

$$BW = 2(10^5 + 2\times10^3)$$
$$= 204\ KHz$$

$\rightarrow$ doubles exactly

$$BW = 44\ KHz$$

Clearly, if this was NB ($\Delta f \ll f_m$) then doubling f_m will double the BW for both FM, PM

7. It's a Single-tone Signal

Amplitude Doubled)

A	Same BW	given	Narrowband
B	Double BW		Wideband
C	Double BW		Wideband

Recall,

NBFM	Same BW
NBPM	Same BW
WBFM	Double BW
WBPM	Double BW

Frequency Doubled

A	Double BW	NBFM or NBPM
B	Double BW	WBPM
C	Same BW	WBFM

the given facts won't help us decide further

Recall,

NBFM	Double BW
NBPM	Double BW
WBFM	Same BW (somewhat)
WBPM	Double BW

8) $m(t) = a\cos(2\pi f_m t)$ $\qquad$ $\int a(t) = \frac{a}{2\pi f_m}\sin(2\pi f_m t)$

$\left.\begin{array}{l} a = 1 \\ f_m = 1\,KHz \end{array}\right\} \to S(t)_{FM} = 100\cos(2\pi 10^7 t + 4\sin(2000\pi t))$

indeed,

$S(t)_{FM} = 100\cos(2\pi 10^7 t + K_f\, a(t))$

for

$K_f = \frac{4}{a/(2\pi f_m)} = 8\pi$ Krad/V

a) $\Phi(t) = 4\sin(2\times10^3\pi t)$

$\Phi'(t) = 8\times10^3\pi\cos(2\times10^3\pi t)$

Hence, $\Delta\omega = 8\times10^3\pi$ rad/s $\quad (\Delta f = 4\,KHz)$

b) $P = \frac{100^2}{2}\cdot\frac{1}{50} = 100\,W$ $\qquad$ $P = \frac{A_c^2}{2}\cdot\frac{1}{R}$ • like in sheet 2

c) Not involved.

d) Carson's Rule

$BW_{S(t)} = 2(\Delta f + B_{m(t)})$

↓ in case m(t) is a sinusoid.

$= 2(\Delta f + f_{max}) = 2(4K + 1K) = 10\,KHz$

e) $S(t) = 100\cos(2\pi 10^7 t + \underbrace{8\pi\times10^3 \frac{75}{2\pi\times2\times10^3}}^{150}\sin(2\pi.2.10^3 t))$

a) $\Phi(t) = 150\sin(4\pi\times10^3 t)$

$\Phi'(t) = 150\times4\pi\times10^3\cos(4\pi\times10^3 t) \to \Delta\omega = 600\pi$ Krad/s

$\Delta f = 300$ KHz

e) b) Power remains the same as $s(t)$'s amplitude is intact.

d) $BW_{s(t)} = 2(\Delta f + f_{max}) = 2(300K + 2K) = 604\ KHz$

g) Need an FM Carrier with

$f_c = 98.1\ MHz$

$\Delta f = 75\ KHz$

NBFM is available where

$f_{c_0} = 100K\ Hz$

$\Delta f = 10\ Hz$

Can use

- Oscillator(s) with frequency ranging from 10 to 100 MHz
- Frequency doublers, triplers, quintuplers
  $2x$ $3x$ $5x$

$\rightarrow n = \dfrac{\Delta f\,|_{final}}{\Delta f\,|_{initial}} = \dfrac{75K}{10} = 7500$

Objective)

if Δf is multiplied by n, so happens to this. (K_f)

$\cos(2\pi(100K)t + K_f a(t)) \rightarrow \cos(2\pi(98.1M)t + nK_f a(t))$

- This cannot be achieved by frequency multipliers only as $\dfrac{f_c}{f_{c_0}} \neq n$

• Consider multiplying then shifting

$$7500 = 75 \times 100 = (5^2 \times 3)(5^2 \times 2^2) = \underset{\text{Two doublers}}{2^2} \times \underset{\text{1 Tripler}}{3} \times \underset{\text{Four Quintuplers}}{5^4}$$

Now we have $K_{f_0} \longrightarrow 7500 K_{f_0}$ ✓

$f_{c0} \longrightarrow \underbrace{7500 f_{c0}}$

→ 750 MHz but we need 98.1 MHz

So in this case we need

$f_{LO} = 750\text{ MHz} - 98.1\text{ MHz}$

$f_{LO} = 651.9\text{ MHz}$ which is beyond the available HW.

• Consider multiply then shift then multiply

$$K_f = n_1 n_2 K_{f_0} \quad \rightarrow \quad n_1 n_2 = 7500$$

$$f_c = n_2(n_1 f_{c0} - f_{LO}) \quad \rightarrow \quad f_{LO} = n_1 \underset{0.1\text{MHz}}{f_{c0}} - \frac{f_c \rightarrow 98.1\text{MHz}}{n_2}$$

Due to the constraints, need n_1, n_2 such that

• $n_1 n_2 = 7500$ • $10 < 0.1 n_1 - \frac{98.1}{n_2} < 100$

We can try out n_1, n_2 or solve this formally.

$$10 < 0.1\left(\frac{7500}{n_2}\right) - \frac{98.1}{n_2} < 100$$

$$10 < \frac{1}{n_2}(651.9) < 100$$

$$\frac{1}{100} < \frac{n_2}{651.9} < \frac{1}{10} \rightarrow 6.519 < n_2 < 65.19 \quad (1)$$

- Since $7500 = 2^2 \times 3 \times 5^4$ and we have hardware for all of these and they're Primes
  - → There's no Constrain on n_2 (any divisor of n must include $2^{k_1}, 3^{k_2}, 5^{k_3}$)
    - if one of the Primes wasn't supported and rather some other divisor did → Constraint.

* Any n_2 that satisfies (1) and is a divisor of 7500 works.

let $n_2 = 60$ then $n_1 = 125$ (also $f_{LO} = 10.865$ MHz)

$n_2 = 60 \rightarrow 2^2 \times 3 \times 5$, $\quad n_1 = 125 \rightarrow 5^3$

Design)

$m(t) \rightarrow$ [NBFM Mod.] $\xrightarrow{A}$ [3x Quintupler] $\xrightarrow{B}$ [Mixer] $\xrightarrow{C}$ [BPF] $\xrightarrow{D}$ [2x Double then triple then Quintuple] $\xrightarrow{E}$ $s(t)$

Mixer input: 10.865 MHz

A) $\cos(2\pi(100K)t + K_f a(t))$

B) $\cos(2\pi(100K \times 5^3)t + 5^3 K_f a(t))$

C) $\cos(2\pi(100K \times 5^3 - 10.865K \times 10^3)t + 5^3 K_f a(t)) + \underline{\cos(2\pi(100K \times 5^3 + 10.8\ldots}$

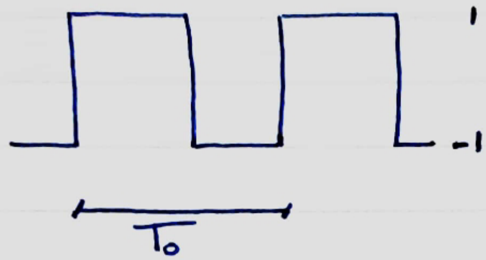
will be cleared by BPF

D) $\cos(2\pi(1635K)t + 125 K_f a(t))$

E) $\cos(2\pi\underbrace{(1635K \times 2^2 \times 3 \times 5)}_{98.1M}t + \underbrace{125 \times 2^2 \times 3 \times 5}_{7500} K_f a(t))$

• If the constraints make it impossible to fulfill the required specs (e.g. no quintuplers) then try to at least fulfill it approximately (e.g. $75 = 5 \times 5 \times 3 \rightarrow \underbrace{4 \times 6 \times 3}_{72}$) • 4, 6 using doublers

10) $m(t) =$

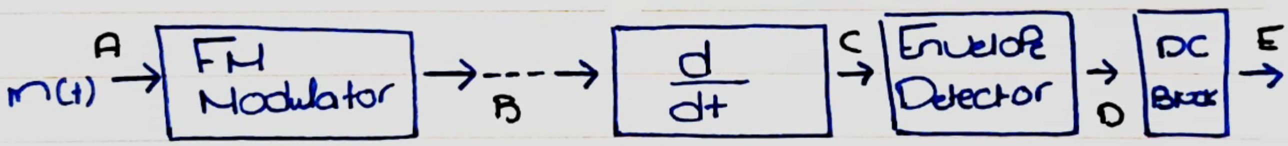


← @A

• $f_c = 10KHz$, $\Delta f = 1KHz$ ← max freq. deviation. Thus, $f_i = \begin{cases} 11KHz & m(t) = 1 \\ 9KHz & m(t) = -1 \end{cases}$

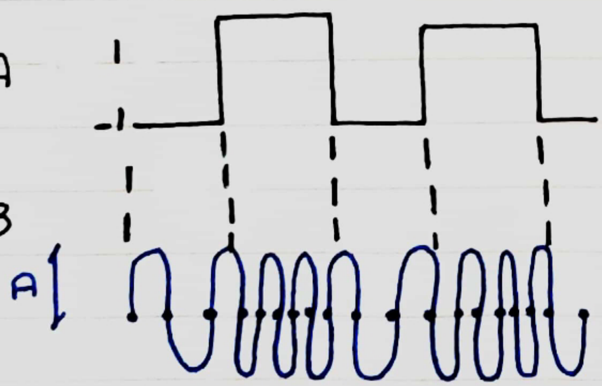
• Carrier amplitude is A

• $k_f = \frac{f_i - f_c}{m(t)} = 1KHz/volt$



@A

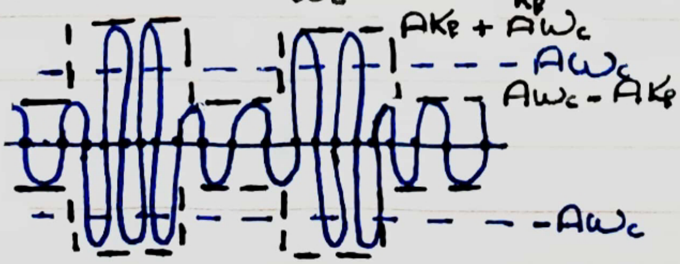
@B



$\leftarrow S(t) = A\sin(2\pi(10K)t + 2\pi 10^3 a(t))$

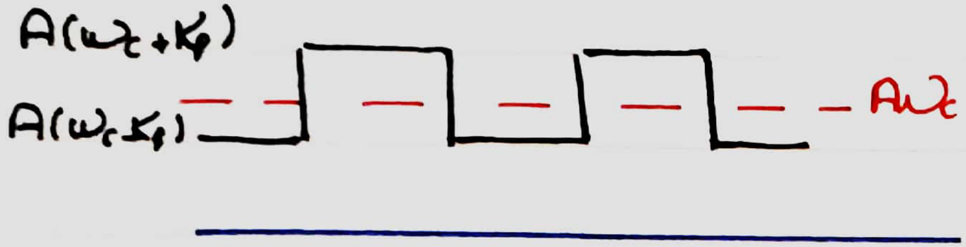
@C

$$\frac{dS(t)}{dt} = A \cdot \overbrace{(\underbrace{2\pi(10K)}_{\omega_c} + \underbrace{2\pi(10^3)}_{k_f} m(t))}^{\text{envelope}} \cdot \cos(2\pi(10K)t + 2\pi(10^3)a(t))$$



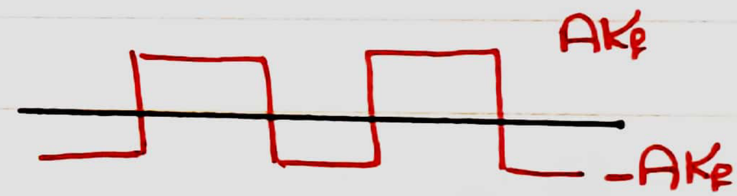
@D

$A\omega_c > AK_f m(t)$ and thus envelope detection to get $A(\omega_c + K_f m(t))$ is Possible



@E

The DC Block stops $A\omega_c$



- Need to divide by AK_f to get back