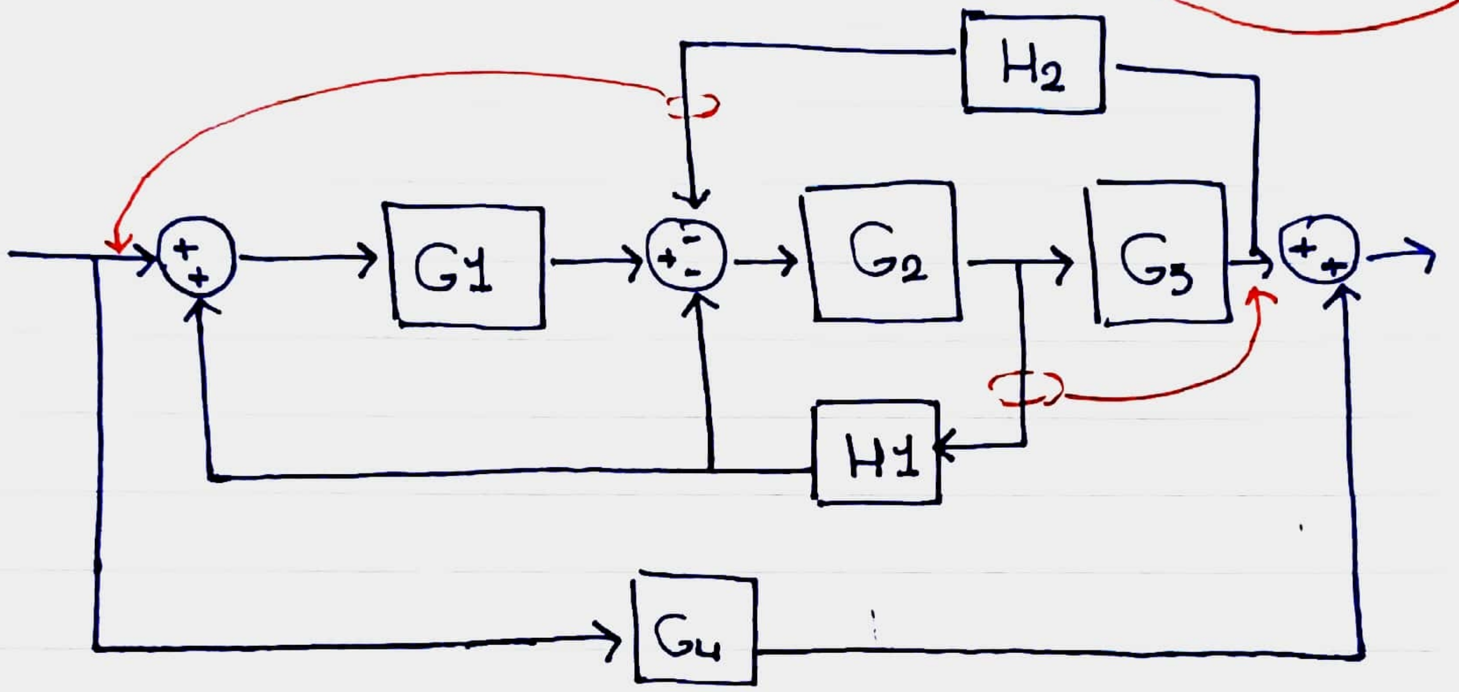
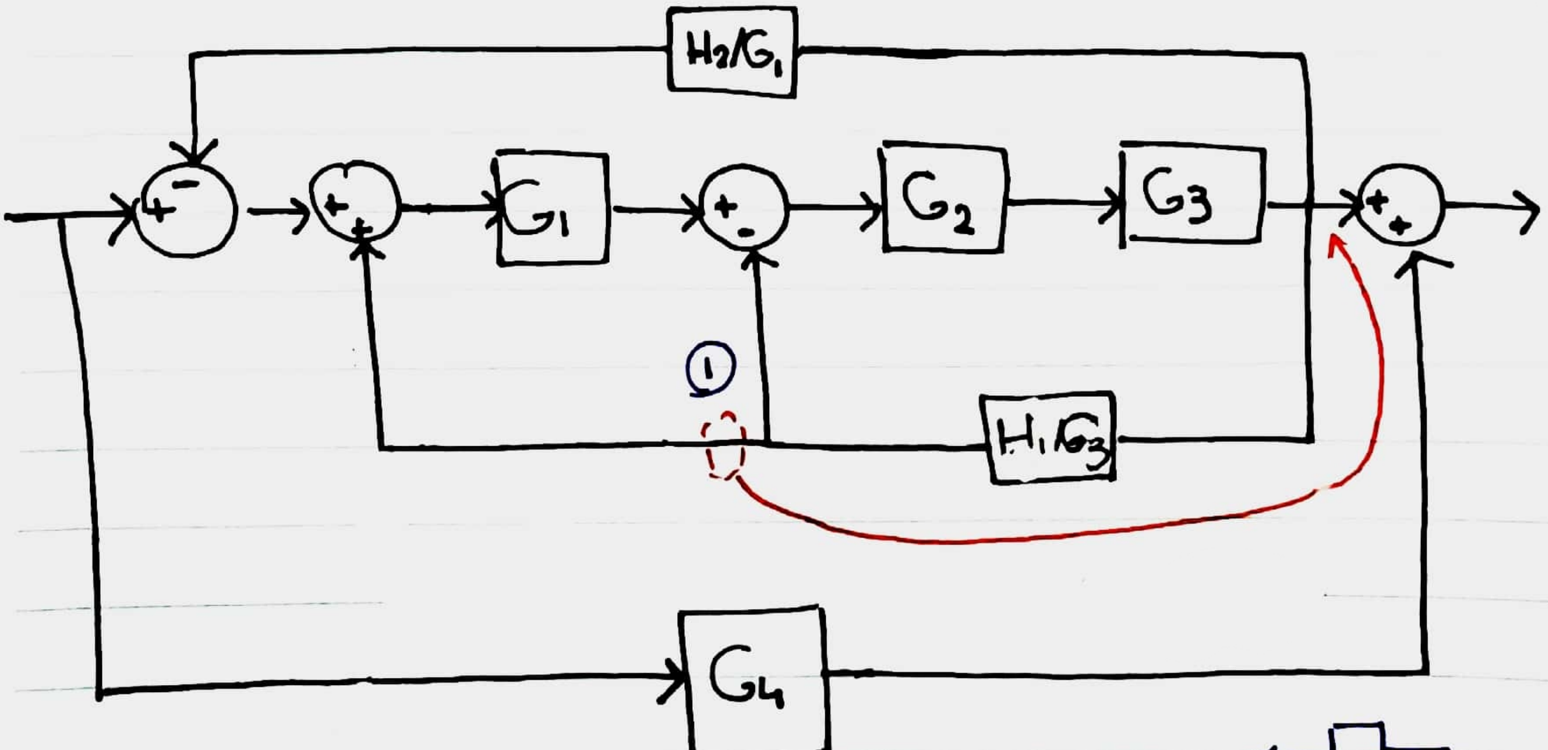
CE Sheet 6 Sol.

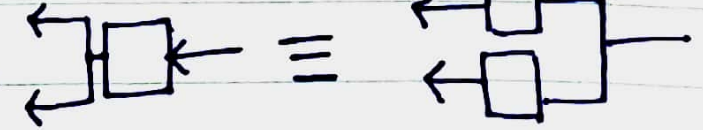
Start With Prob. 2

1)

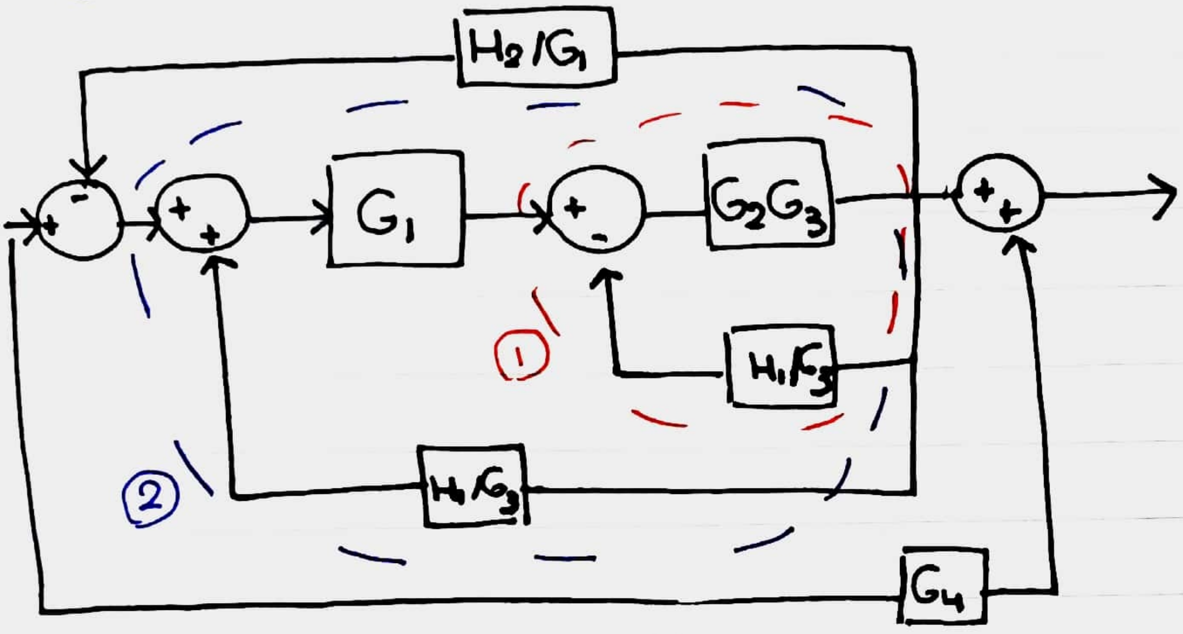


- The question to ask is "which take-off point(s) or summing should be moved one step forward/backward so we're able to apply a closed-loop to open-loop reduction?"



.Can as well think of 1 as 

and we need to do it so we see our usual Standard Form



Reducing ①:

$$F_1 = \frac{G_2G_3^2}{G_2G_3H_1 + G_3} = \frac{G_2G_3}{G_2H_1+1}$$

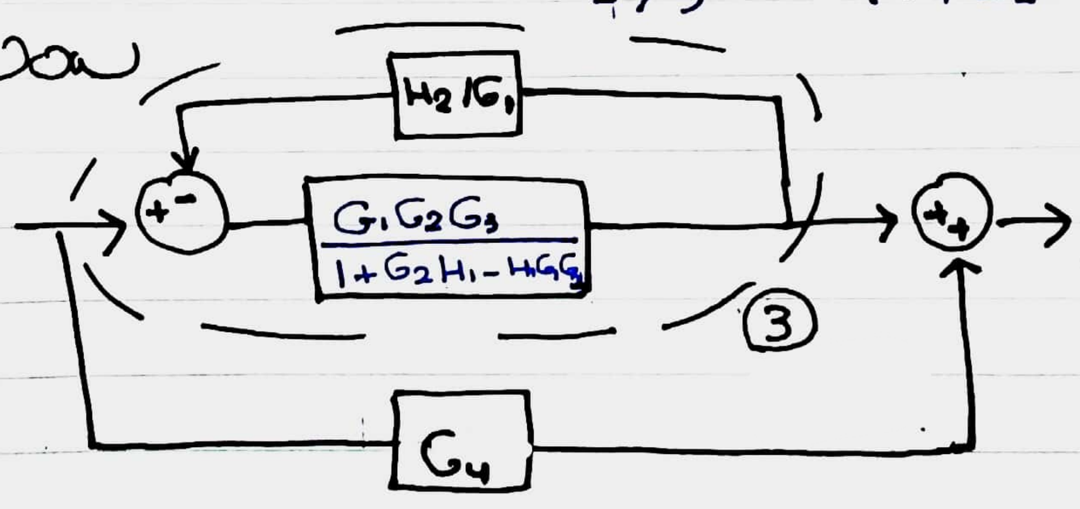
Reducing ②:

$$F_2 = \frac{G_1G_2G_3^2}{-G_1G_2\cancel{G_3}H_1 + (1+G_2H_1)\cancel{G_3}}$$

. Due to (+ +)

$$\frac{G_u H_o}{-G_u H_u + G_o H_o}$$

Now



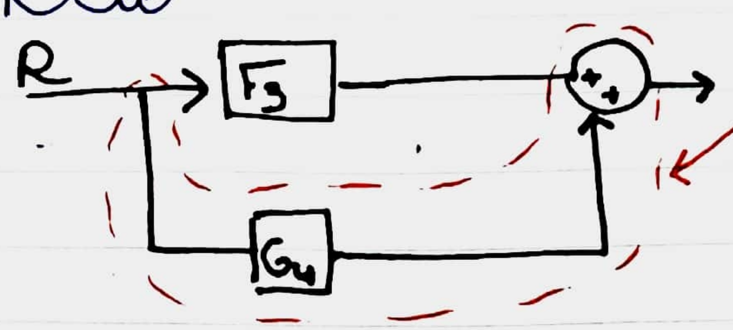
Reducing 3

$$F_3 = \frac{G_1^2 G_2 G_3}{G_1 H_2 G_2 G_3 + G_1 (1 + G_2 H_1 - H_1 G_1 G_2)}$$

$$= \frac{G_1 G_2 G_3}{1 + H_2 G_2 G_3 + G_2 H_1 - H_1 G_1 G_2}$$

$$= \frac{G_1 G_2 G_3}{1 + G_1 G_2 G_3 \left(\frac{H_2}{G_1} + \frac{H_1}{G_1 G_3} - \frac{H_1}{G_3} \right)}$$

- Now

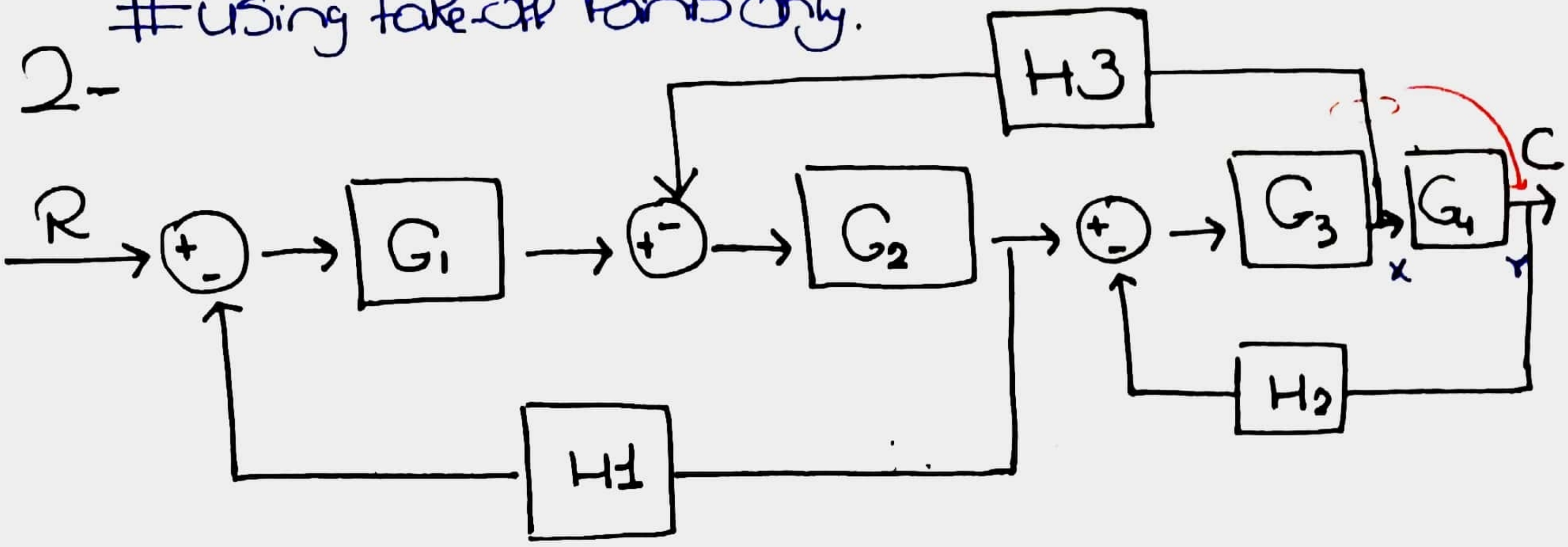


We could've ignored this from the start (just a simple addition)

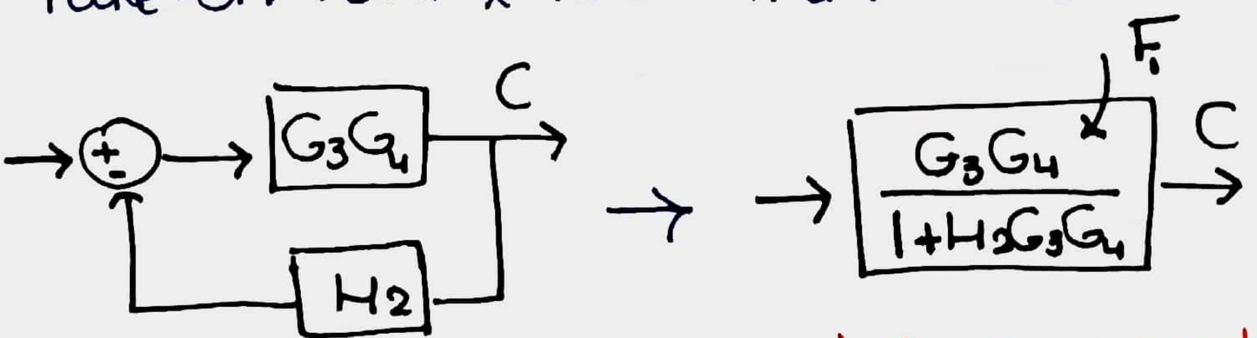
$$\frac{C}{R} = F_3 + G_4$$

- May as well check TA's solution for this one. (Slides)

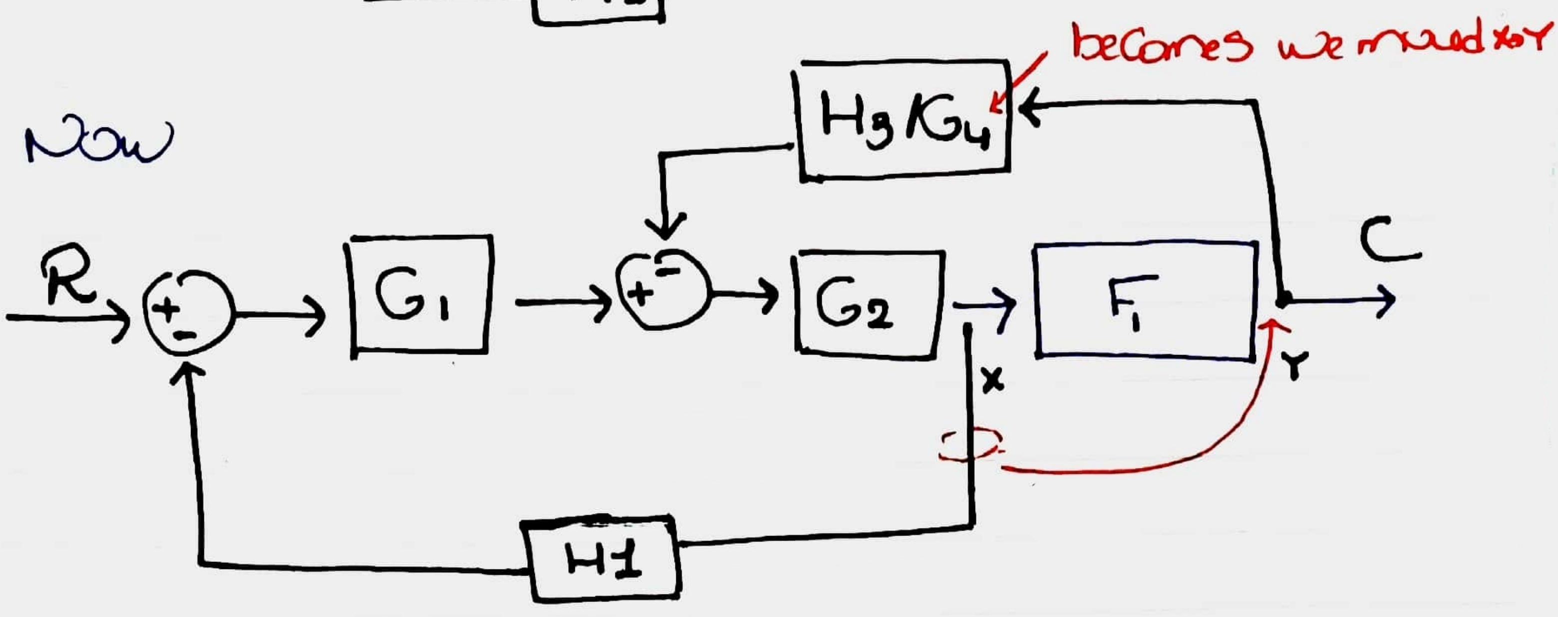
2- # using take-off points only.



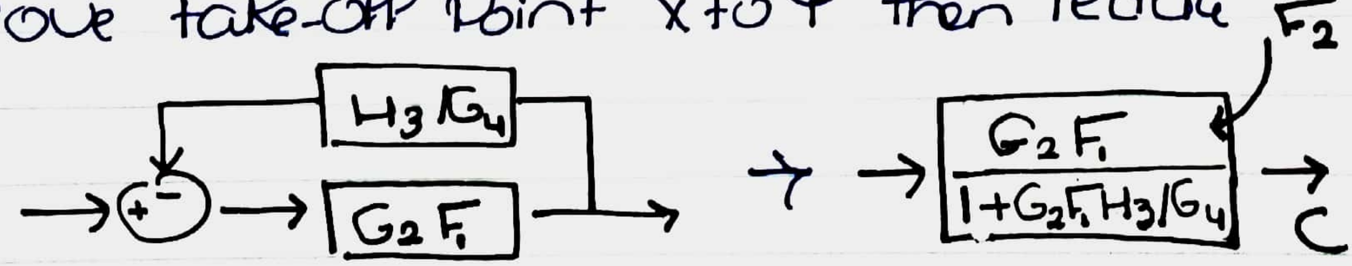
• Move take-off point X to Y then reduce

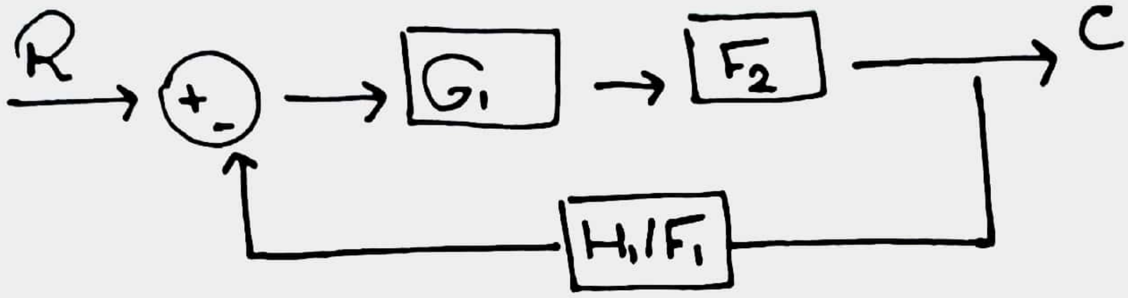


Now



• Move take-off point X to Y then reduce





Thus,

$$\bullet\; G_1F_2 = \frac{G_1G_2F_1}{1+G_2F_1H_3/G_4} = \frac{G_1G_2G_4F_1}{G_4+F_1G_2H_3}$$

Aug F_1

$$= \frac{G_1G_2G_4G_3G_4}{G_4(1+H_2G_3G_4)+G_3G_4G_2H_3}$$

$\div G_4$

$$= \frac{G_1G_2G_3G_4}{1+G_3(H_2G_4+H_3G_2)}$$

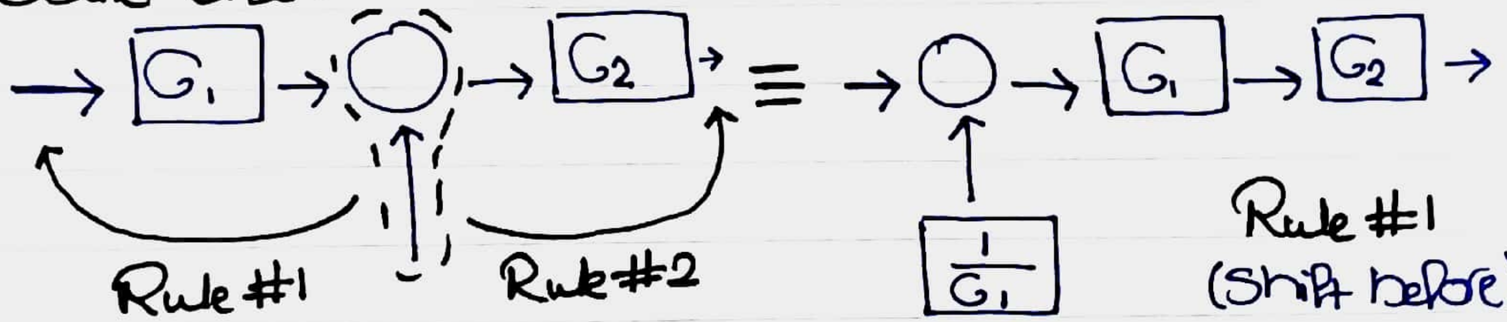
$$\bullet\; \frac{H_1}{F_1} = \frac{H_1(1+H_2G_3G_4)}{G_3G_4}$$

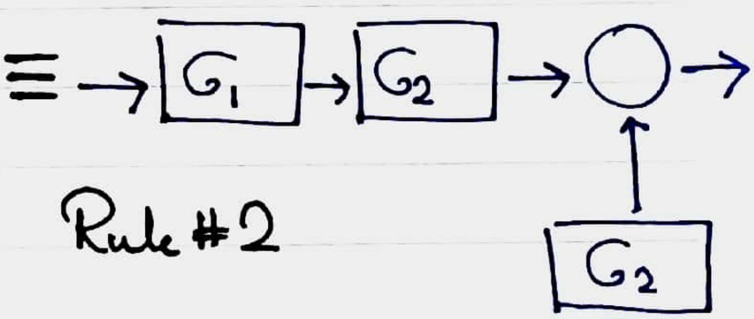
• By lec.3, 14

$$\frac{C(S)}{R(S)} = \frac{G_1G_2G_3G_4(G_3G_4)}{G_1G_2G_3G_4H_1(1+H_2G_3G_4)+(1+G_3(H_2G_4+G_2H_3))G_3G_4}$$

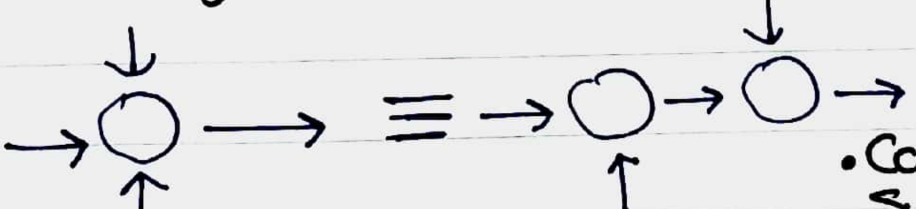
$$\frac{C(S)}{R(S)} = \frac{G_1 G_2 G_3 G_4}{G_1 G_2 H_1 (1 + H_2 G_3 G_4) + 1 + G_3 (H_2 G_4 + G_2 H_3)}$$

- A more efficient Solution (by moving summing Points) would use:



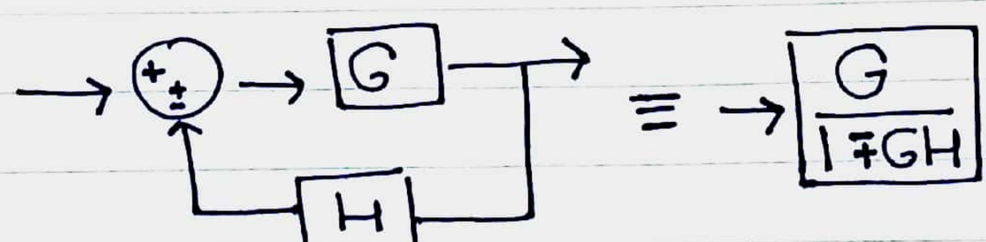


- what if there's a Sum already at the desired Position → Can merge both as

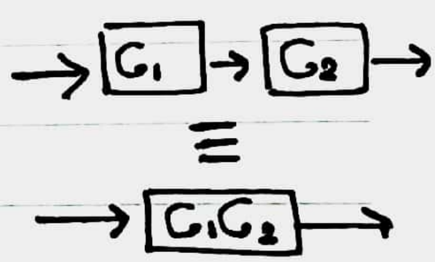


- Can swap sums as well

Recall as well,



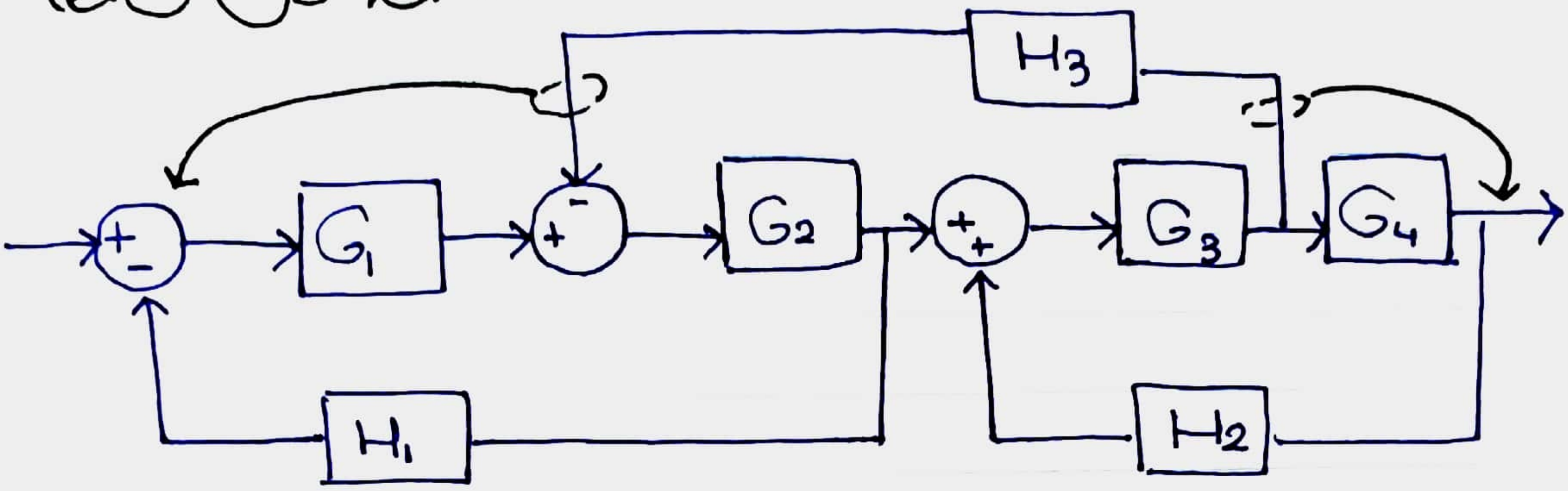
,



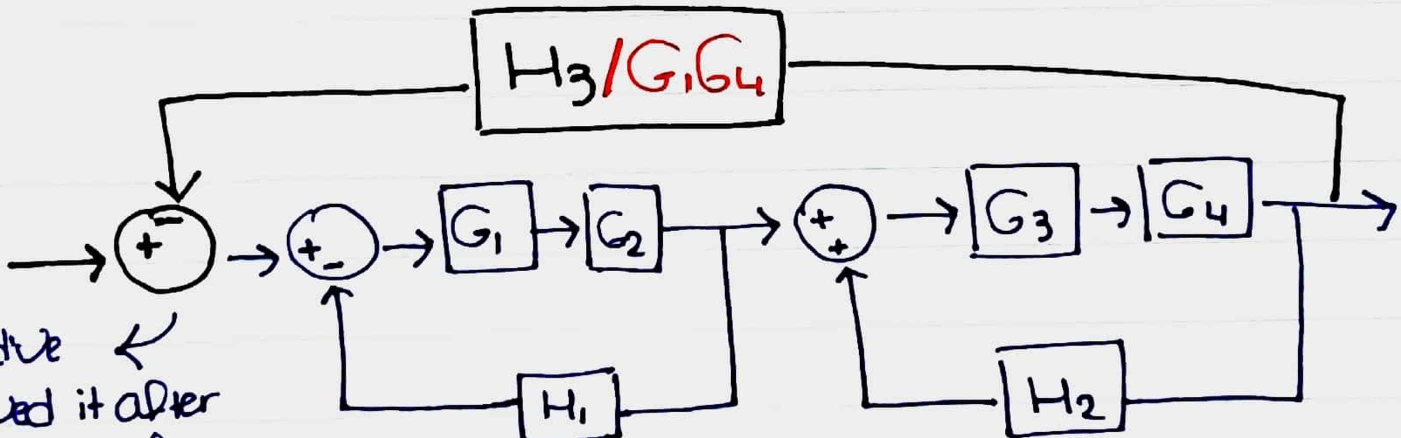
and

- Check lecture if interested in Proofs.

Let's go for it:

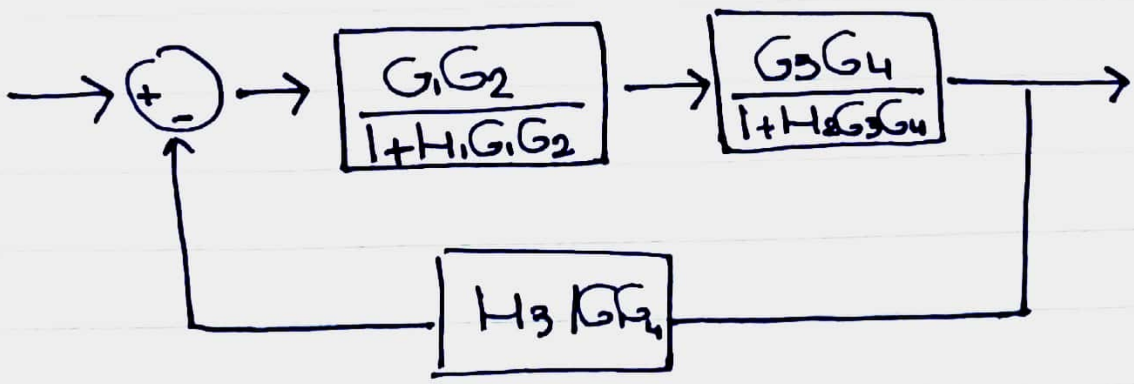


$$\equiv$$



Could've moved it after or at ($\pm$) the other sum as well.

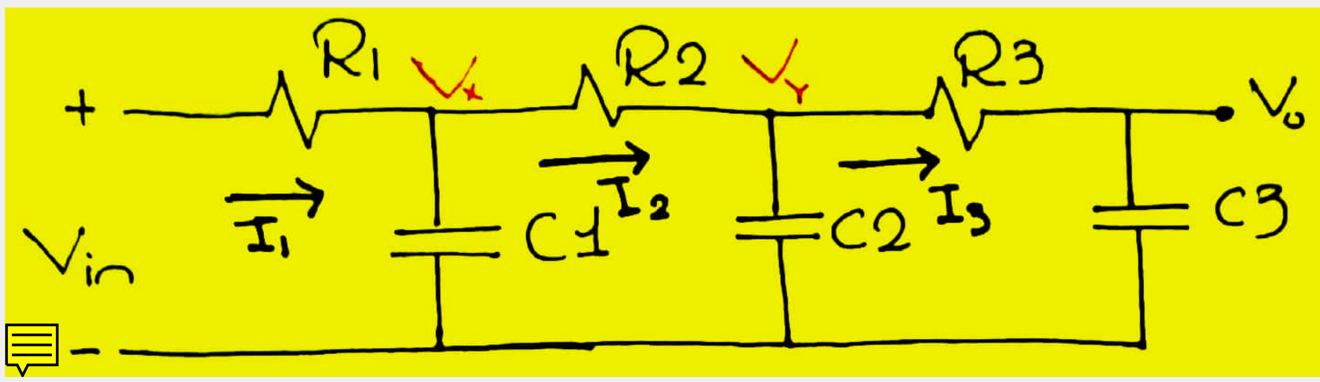
$$\equiv$$



by Lec. 3,14

$$\frac{C(S)}{R(S)} = \frac{G_1G_2\cancel{G_3G_4}(G_3G_4)}{G_1G_2\cancel{G_3}G_4H_3 + \cancel{G_3}G_4(1+H_1G_1G_2)(1+H_2G_3G_4)}$$

Q3)

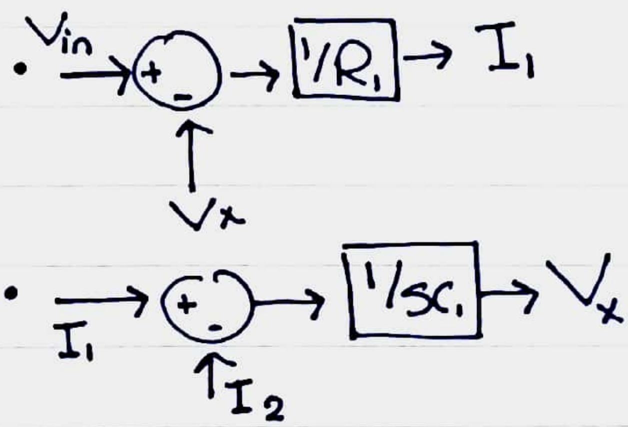


• give nodes names so we can apply nodal analysis.
→ introduce I_1, I_2, I_3

• $$\frac{V_{in} - V_x}{R_1} = I_1$$

$$I_1 = I_2 + \frac{V_x - 0}{1/SC_1}$$

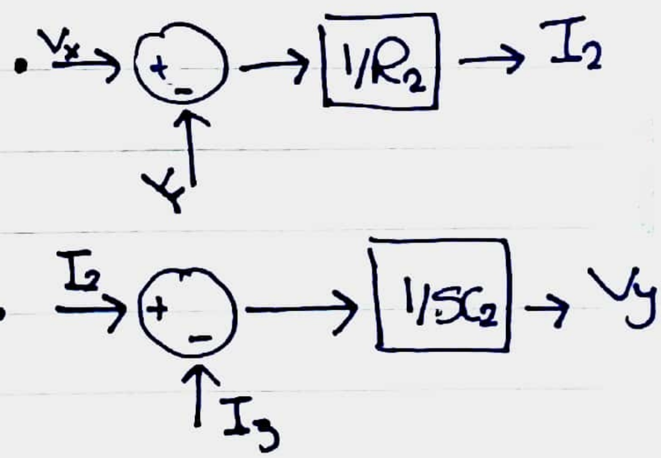
1st node



• $$\frac{V_x - V_Y}{R_2} = I_2$$

$$I_2 = I_3 + \frac{V_y}{1/SC_2}$$

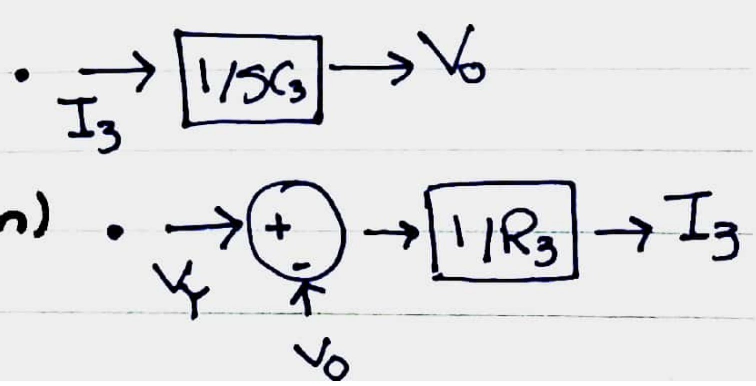
2nd node



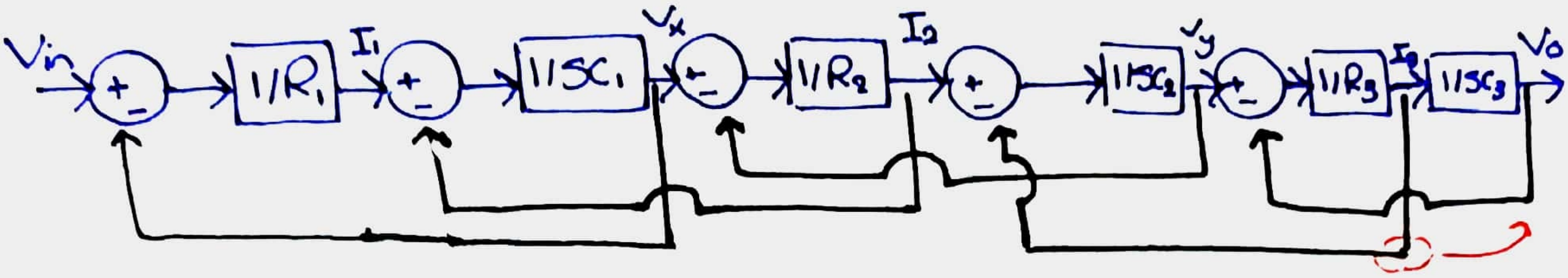
• $$I_3 = \frac{V_o}{1/SC_3}$$

$$\frac{V_Y - V_o}{R_3} = I_3$$

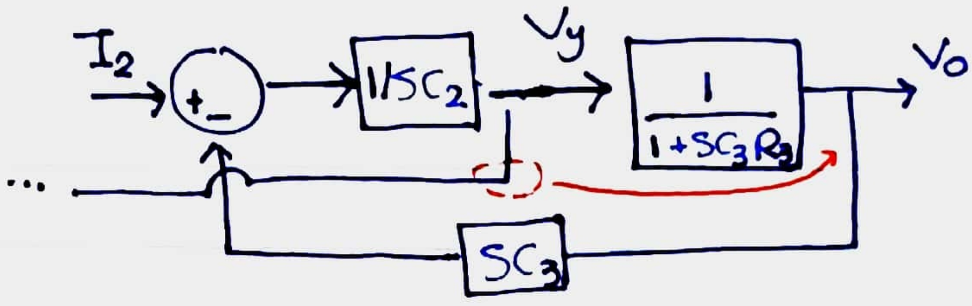
3rd node (unknown)



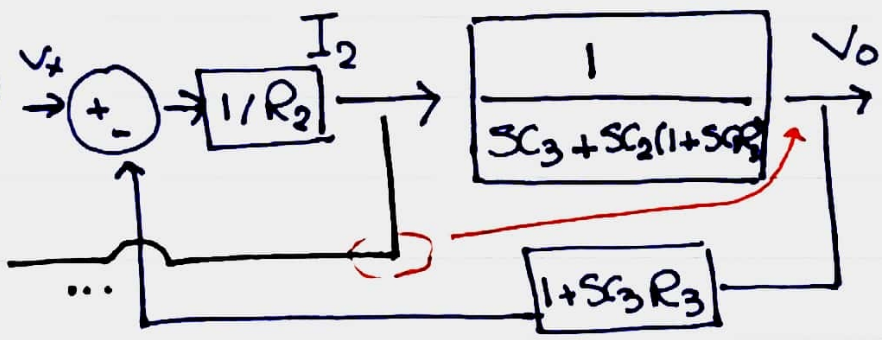
Compiling into one block diagram we get



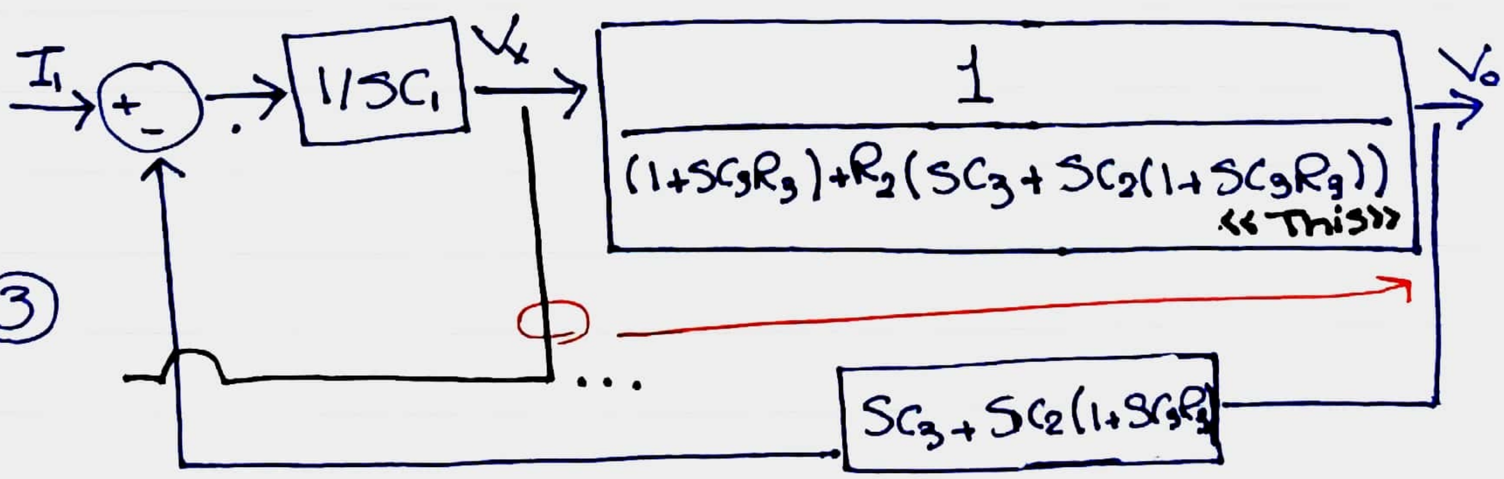
1



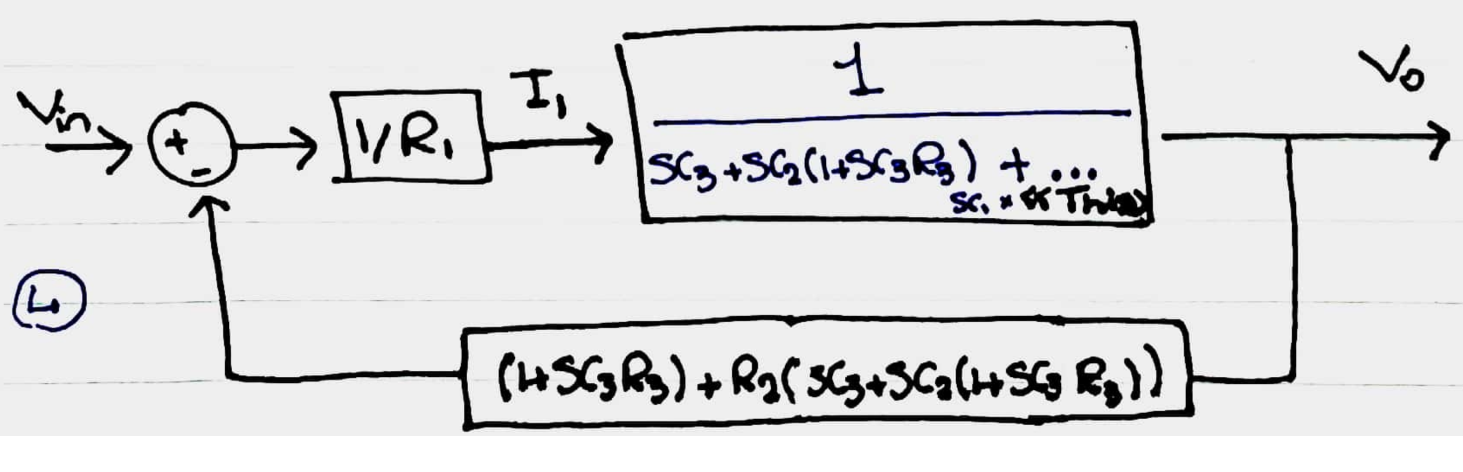
2



3



4



Thus,

$$\frac{V_o}{V_{in}} = \frac{1}{[R_1(SC_3+SC_2(1+SC_3R_3)+(SC_1[(1+SC_3R_3)+R_2 \ldots}$$

$$\ldots (SC_3 + SC_2(1+SC_3R_3))]])) + (1+SC_3R_3) \ldots$$

$$\ldots + R_2(SC_3 + SC_2(1+SC_3R_3)))]$$

which for $C_1 = C_2 = C_3$, $R_1 = R_2 = R_3$ yields

$$\frac{V_o}{V_{in}} = \frac{1}{1 + 6SCR + 5S^2C^2R^2 + S^3C^3R^3}$$

Flipboard :-

Alternatively,

- $$\frac{V_{in} - V_x}{R} = \frac{V_x}{1/SC} + \frac{V_x - V_Y}{R}$$
- $$\frac{V_x - V_Y}{R} = \frac{V_Y}{1/SC} + \frac{V_Y - V_o}{R} \rightarrow \frac{V_x}{R} = V_Y\left(\frac{2}{R} + SC\right) - \frac{V_o}{R}$$
- $$\frac{V_Y - V_o}{R} = \frac{V_o}{1/SC} \rightarrow V_Y = V_o(1+SCR)$$

$$V_x = R V_o (1+SCR)\left(\frac{2}{R} + SC\right) - V_o$$

$$= V_o\left((1+SCR)(2+SCR) - 1\right)$$

1st eqn:

$$\frac{V_{in}}{R} = V_x\left(\frac{2}{R} + SC\right) - \frac{V_o}{R}(1+SCR)$$

$$V_{in} = V_x(2+SCR) - V_o(1+SCR)$$

$$V_{in} = V_o\left(\left(((1+SCR)(2+SCR)-1)(2+SCR)\right) - (1+SCR)\right)$$

$$\frac{V_{in}}{V_o} = S^3C^3R^3 + 5S^2C^2R^2 + 6SCR + 1$$

$$\frac{V_o}{V_{in}} = \frac{1}{S^3C^3R^3 + 5S^2C^2R^2 + 6SCR + 1}$$

I almost cried here.

thank you <3