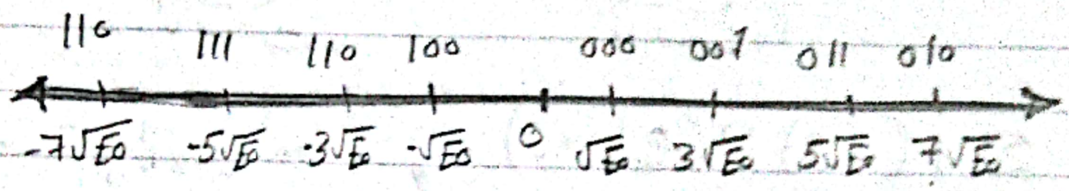
Sheet 7 Pass-band Transmission

Given

↳ $s_i(t) = a_i\sqrt{E_0} * \sqrt{\frac{2}{T}}\cos(2\pi f_c t)$, $0 \leq t \leq T$

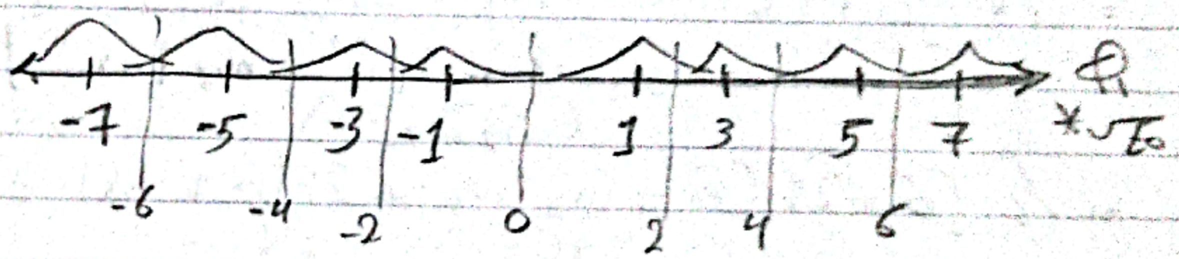
, $i = 1, 2, \dots M$, $a_i = \pm1, \pm3, \dots$

8-ASK

↳


---

①



* $P(m_i)\,P(\hat{m} = m_i | m_i)$ ↑↑

↳ estimated ↳ actual

④ Both ① & ② follow the Gray encoding criteria

الفرق بين كل تنين والل جنبه ب bit واحدة بس !

③ $R_b = 1$ Mbps, $f_c = 5$ MHz | Can we use $M = 8$?

$BW_{channel} = 0.5$ MHz, | What is $M)_{min}$?

* $BW|_{M\text{-ary}} = 2R_s = \frac{2}{\log_2 M} R_b$, but $BW)_{M\text{-ary}} \leq BW)_{channel}$

$\frac{2}{\log_2 M} R_b \leq 0.5\text{MHz} \Rightarrow \log_2 M \geq 4 \Rightarrow \boxed{M \geq 16}$

* For $M = 8 \Rightarrow BW|_{M\text{-ary}} = \frac{2}{\log_2 M} R_b = \frac{2}{3} \times R_b = \frac{2}{3}$ MHz

$> BW)_{channel}$

② Derive the theortical Bit Error Rate!

* Start with [Symbol Error Rate] then get BER

* نحسب لـ $2 \le i \le 7$ لوحدهم و $i=1,8$ لوحدهم، ليه؟

ناحية واحدة

ناحيتين

* $P(e') = \sum_{i=1}^{8} P(m_i)\,P(e|m_i)$, $P(m_i) = 1/8$ كله زي بعض!

$= 1/8 \sum^{8} P(e|m_i)$

$= 1/8 \left[6P(\hat{m} \neq m_3 | m_3) + 2P(\hat{m} \neq m_8 | m_8) \right]$ (for 2→7 ; or m_1)

$= 1/8 \left[6 * ② P(\hat{m} \neq m_8 | m_8) + 2P(\hat{m} \neq m_8 | m_8) \right]$

$= \frac{14}{8} P(\hat{m} \neq m_8 | m_8)$

$= 14/8\, P(y > -6\sqrt{E_0} | m_8)$

$= 7/4\, P\left(z > \frac{-6\sqrt{E_0} + 7\sqrt{E_0}}{\sqrt{N_0/2}}\right)$ let $z = \frac{y - (-7\sqrt{E_0})}{\sqrt{N_0/2}}$ (mean$_8$; σ_8^2)

$= 7/4\, Q\left(\sqrt{\frac{E_0}{N_0/2}}\right)$ ∵ $Q(x) = P(Z > x)$ (std G R var)

$= \frac{7}{4} * \frac{1}{2}\, \text{Erfc}\left(\sqrt{E_0/N_0}\right)$

$= \frac{7}{8}\, \text{erfc}\left(\sqrt{E_0/N_0}\right)$ ∵ $Q(x) = \frac{1}{2}\text{erfc}\left(\frac{x}{\sqrt{2}}\right)$

② $P(e) = \frac{7}{4} Q\left(\sqrt{\frac{2E_0}{N_0}}\right)$, $E_S = \sum_1^8 P(m_i) E_i$

$E_S = \left(2 * \sqrt{7E_0}^2 + 2*25*E_0 + 2*9*E_0 + 2*1*E_0\right) * \frac{1}{8}$

$= 1/8 * E_0 * 2(49+25+9+1) = 21E_0$

$E_S = 21E_0$) 3 bits per symbol
$= 3E_b$

so $P(\text{error})_{\text{symbol}} = 3P(\text{error})|_{\text{bit}}$

$BER = 1/3 SER$

$= 1/3 * \frac{7}{4} Q\left(\sqrt{\frac{2E_0}{N_0}}\right)$

$= \frac{7}{12} Q\left(\sqrt{\frac{2E_0}{N_0}}\right)$

$= \frac{7}{24} erfc\left(\sqrt{E_0/N_0}\right)$