DC Sheet 5

• For M-ary Transmission (general case)

$$T_s = \log_2 M \, T_b$$ (Symbol duration)

(Pulse duration)

T_b ↑ System's bit rate

$$R_s = \frac{1}{T_s}$$ (Symbol rate)

$$B_T = (1+\alpha)\frac{1}{2T_s}$$ (Channel BW needed)

(Pulse bandwidth)

→ Set $M=2$ and it becomes binary transmission.

→ Further set $\alpha = 0$ and the system becomes an ideal Nyquist channel.

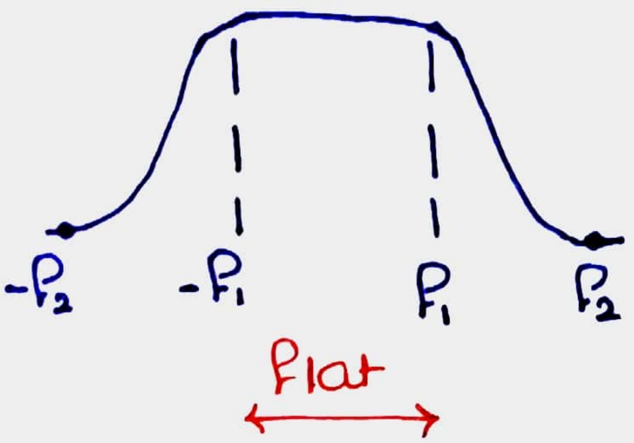
↳ $R_s = R_b = 2B_T$ 

(Nyquist rate)

• Smallest BW to transmit the symbols.

$$B_T = \frac{1}{2T_s} = \frac{1}{2T_b} = \frac{R_b}{2}$$

(Nyquist Bandwidth)



- $f_1 = (1-\alpha) \cdot \dfrac{1}{2T_s}$
- $f_2 = (1+\alpha) \cdot \dfrac{1}{2T_s}$ (Bandwidth)
- $0 \leq \alpha \leq 1$ "Roll off factor"

$0 \to$ ideal Nyquist (least BW)

$1 \to$ largest BW but slowest transition and fastest decay. (least error vs. samp jitter)

Problem 1)

- Computer has $R_b = 56 \times 10^3$ b/s $\left(R_b = \frac{1}{T_b}\right)$
- Uses baseband binary PAM with raised cosine

$$T_s = \log_2 2 \, T_b = T_b$$

$$B_T = (1+\alpha)\frac{1}{2T_b}$$

$$= (1+\alpha)\frac{R_b}{2} = (1+\alpha)\, 28 \times 10^3 \text{ Hz}$$

α	0.25	0.5	0.75	1
B_T	35KHz	42KHz	49KHz	56KHz

Problem 2)

- Binary PAM wave $\rightarrow T_s = T_b$
- $B_T|_{Max} = 75\,KHz$
- $T_b = 10 \times 10^{-6}\,s$

$\rightarrow$ Find Raised-Cosine Spectrum that satisfy the given requirements.

$$B_T = (1+\alpha)\frac{1}{2T_b} = (1+\alpha)\frac{1}{2\times10^{-5}}$$

- $B_T|_{max} = (1+\alpha_{max})\dfrac{1}{2\times10^{-5}} = 75K\,Hz$

$$\alpha_{max} = 0.5$$

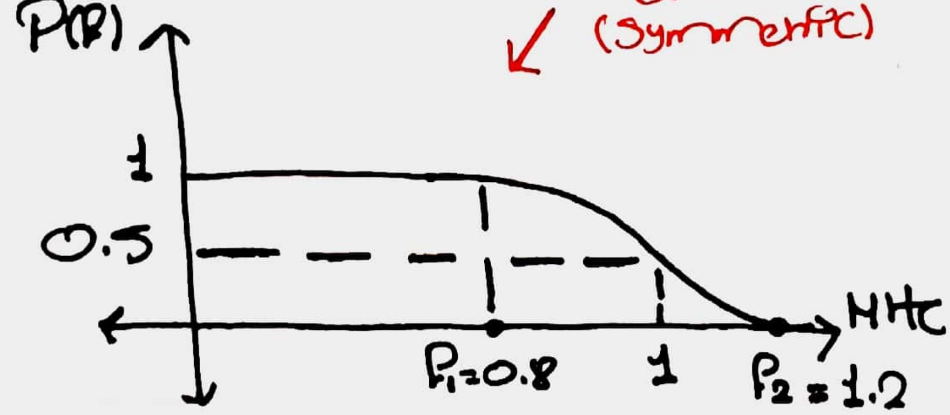
- A Raised-Cosine Spectrum with roll off factor $0 \leq \alpha \leq 0.5$ will satisfy the requirements. (e.g. take $\alpha = 0.5$)

• Recall that for a pulse shape $P(t)$ to satisfy Nyquist criterion for zero ISI then

$$\sum_{K=-\infty}^{\infty} P\left(f - \frac{K}{T_s}\right) = \text{Const.} \quad (1)$$

Problem 3)

Raised Cosine (symmetric)



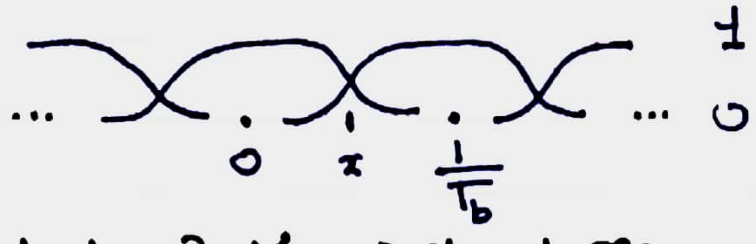
• Transmitting binary data by this pulse ($T_s = T_b$)

→ Find $R_b|_{max}$

→ Find α

• We know that this satisfies (1)

→ will use the fact to find T_b

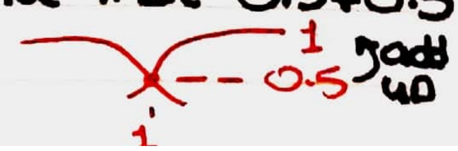


what we know about x:

→ Clearly at $\frac{1}{2T_b}$ (midpoint)

→ whole thing should be constant ($=1$)

• Hence, x must be at 1 since there $0.5 + 0.5 = 1$



• That is, the only value of T_b for which the Nyquist criterion is satisfied is that for which $x = 1$.
(the intersection)

$$1\,MHz = \frac{1}{2T_b}$$

$$B_T = 1.2 \times 10^6 = (1+\alpha) \cdot \overbrace{\frac{1}{2T_b}}^{10^6}$$

Thus, $\alpha = 0.2$

• $R_b = \frac{1}{T_b} = 2 \times 10^6$ bit/s $= 2$ Mbps

Problem 4)

→ $T_s = T_b = \frac{1}{10^6}$ $\quad (R_s = R_b = 10^6$ b/s$)$

→ Will use P(f) from last problem

→ $B_{channel} = 700\,KHz$ $(B_T|_{max})$

• Need α, f_1, f_2

• $700K = (1+\alpha_{max}) \cdot \frac{1}{2T_b}$ ← 10^{-6} → $\alpha_{max} = 0.4$

$$f_1 = (1-\alpha) \cdot \frac{1}{2T_b} = 0.3\,MHz$$

$$f_2 = (1+\alpha) \cdot \frac{1}{2T_b} = 0.7\,MHz$$

($f_2 \in [0.5, 0.7]$, $f_1 \in [0.5, 0.3]$)
• for $0 \le \alpha \le 0.4$