Sheet 3
Bayes Classification

1 We classify based on $P(x|c_i)\,P(c_i)$

Decision boundry

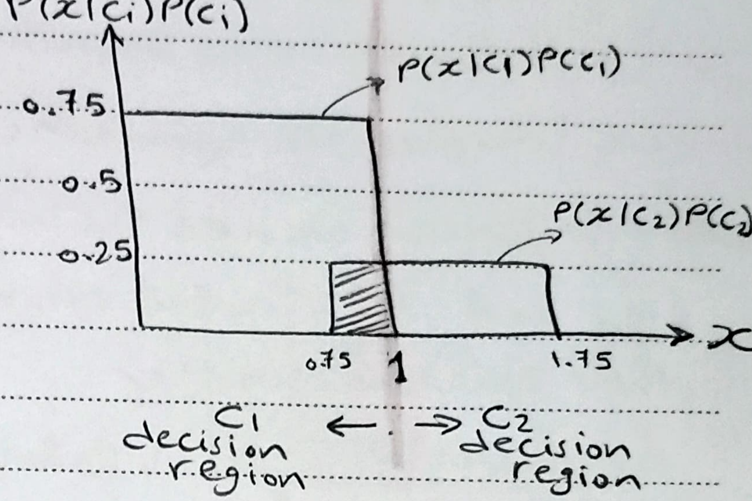
$\boxed{x = 1}$

$P(\text{error}) = \text{Area}$

$= 0.25 * 0.25 = \frac{1}{16}$

$P(\text{correct}) = 1 - P(\text{error})$

$= \frac{15}{16}$



General rule for probability of error:

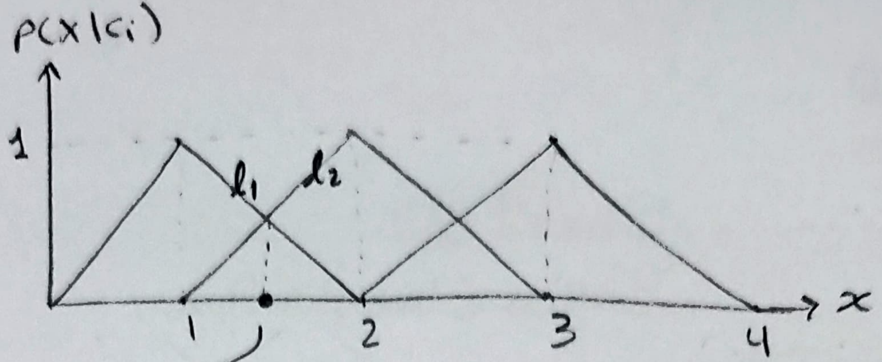
$$P(\text{error}) = \int \min_{k} P(\underline{x}|C_k)\,P(C_k)\,dx$$

$$= \int_{-\infty}^{1} P(x|C_2)\,P(C_2)\,dx + \int_{1}^{\infty} P(x|C_1)\,P(C_1)\,dx$$

$$= \int_{0.75}^{1} 0.25\,dx + \cancel{\int_{1}^{1.75} 0\,dx}$$

$$= \Big[0.25\,x\Big]_{0.75}^{1} = (0.25)(1) - (0.25)(0.75) = \frac{1}{16}$$ #

2



we need to get the intersection point x where we will put the decision boundry.

Here triangles are symmetric and we can predict $\boxed{x = 1.5}$

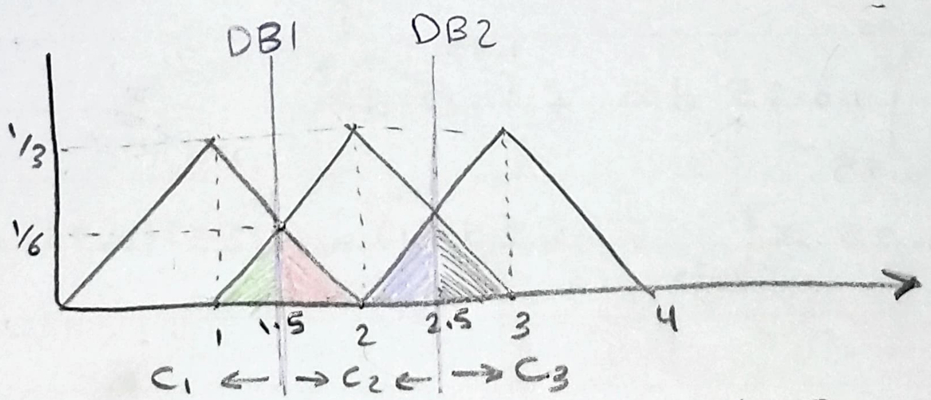
But in general we need the intersection point between l_1 and l_2. So, get eqn of the two lines and solve their equations together

l_1 : Passes by Points $(1,1)$ & $(2,0)$ $\Rightarrow$ $\boxed{y = 2 - x}$

l_2 : Passes by Points $(1,0)$ & $(2,1)$ $\Rightarrow$ $y = \boxed{x - 1}$

l_1 & l_2 intersects @ $x = 1.5$, $y = 0.5$

So, decision boundry 1 will be @ $x = 1.5$



Probability of error = + + +

$= 4 * (1/2 * 1/2 * \frac{1}{6}) = \frac{1}{6}$

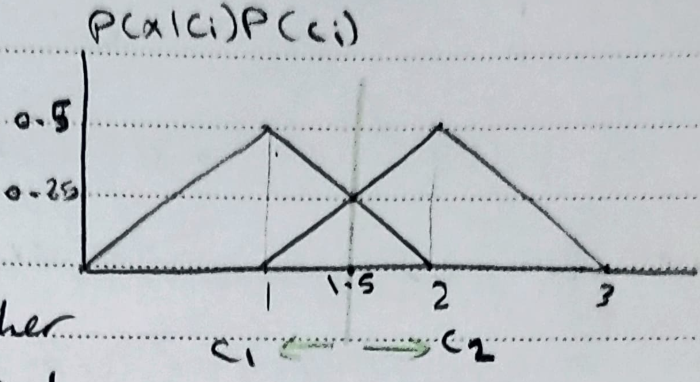
$P(\text{correct}) = 1 - P(\text{error}) = \frac{5}{6}$

$\boxed{3}$ DB @ $x = 1.5$

p(error) =

$= 2 * (\frac{1}{2} * \frac{1}{2} * \frac{1}{4})$

$= \frac{1}{8}$



Req.: Prove that any other DB results in $P(error) > \frac{1}{8}$

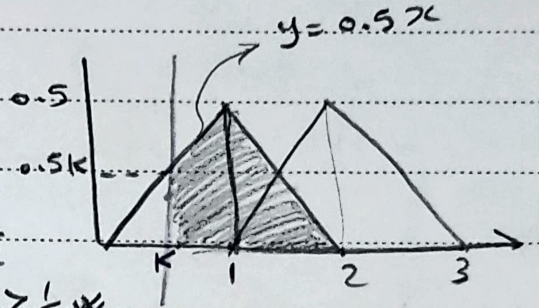
→ For any DB $x = k$

Case $k \leq 1$

P(error) =  + 

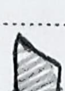
$= \frac{1}{2} * (0.5k + 0.5) * (1 - k) + \frac{1}{2} * 1 * \frac{1}{2}$

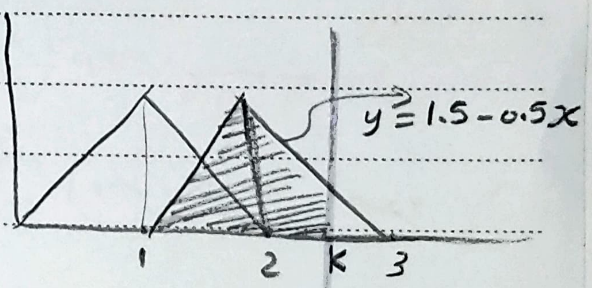
example @ $k = 0.5$ $P(error) = \frac{7}{16} > \frac{1}{8}$ #



Case $k \geq 2$

Same as Case 1

P(error) =  + 



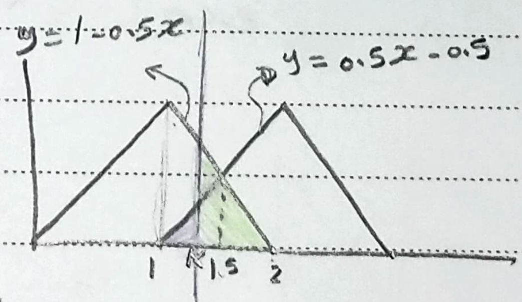
Case $1 < k < 1.5$

p(error) =  + 

$= \frac{1}{2}(k-1)(0.5k - 0.5)$

$+ \frac{1}{2}(2-k)(1 - 0.5k)$

@ $k = 1.25$ $P(error) = \frac{10}{64} > \frac{1}{8}$ ☆

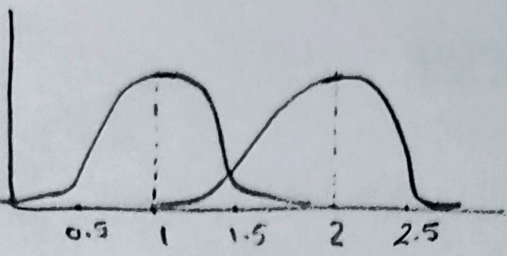


Case $1.5 < k < 2$ Same as Case 3

4

$\mu_1 = 1, \sigma_1 = 0.5$

$\mu_2 = 2, \sigma_2 = 0.5$

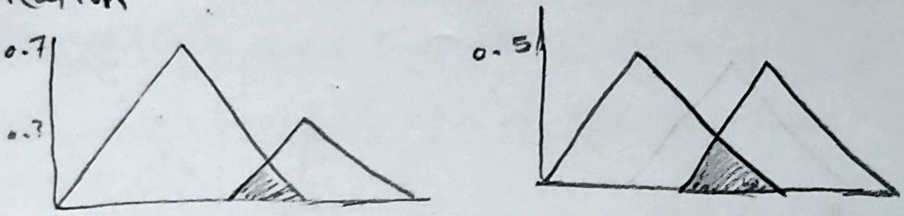


when μ_1 gets farther from μ_2 $\Rightarrow$ P(error) decreases
because the intersection increases

when σ_1 or σ_2 increases $\Rightarrow$ P(error) increases
because the intersection increases

when $P(C_1) = P(C_2) = 0.5$ $\Rightarrow$ P(error) increases
because the intersection increases

clarification



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$$P(x|C_i) = \frac{1}{(2\pi)^{N/2} |\Sigma_i|^{1/2}} \exp\left(\frac{-1}{2} (\underline{x} - \underline{\mu}_i)^T \Sigma_i^{-1} (\underline{x} - \underline{\mu}_i)\right)$$

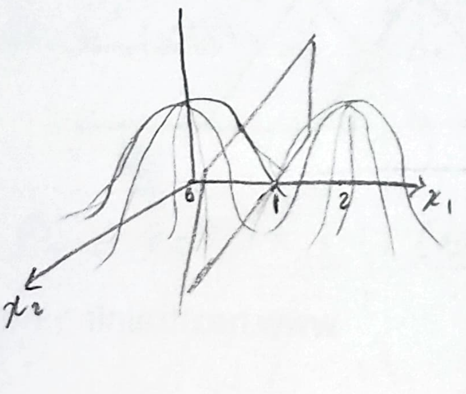
<u>For class 1</u>

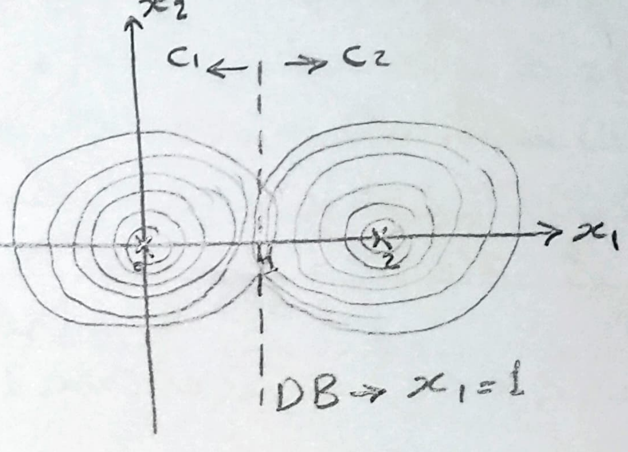
$$P(x|C_1) = \frac{1}{2\pi * (1)^{1/2}} \exp\left(-\frac{1}{2} * [x_1 \ x_2] \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right)$$

$$= \frac{1}{2\pi} \exp\left(-1/2 * [x_1 \ x_2] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \frac{1}{2\pi} \exp\left(-1/2 (x_1^2 + x_2^2)\right)$$

<u>For class 2</u>

$$p(x|C_2) = \frac{1}{2\pi} \exp\left(-1/2 * ((x_1 - 2)^2 + x_2^2)\right)$$





$$P(\text{error}) = \int_{x_2=-\infty}^{\infty}\int_{x_1=-\infty}^{1} P(x|C_2)P(C_2)\,dx_1\,dx_2$$

$$+ \int_{x_2=-\infty}^{\infty}\int_{x_1=1}^{\infty} P(x|C_1)\,P(C_1)\,dx_1 + dx_2$$

$$= I_1 + I_2$$

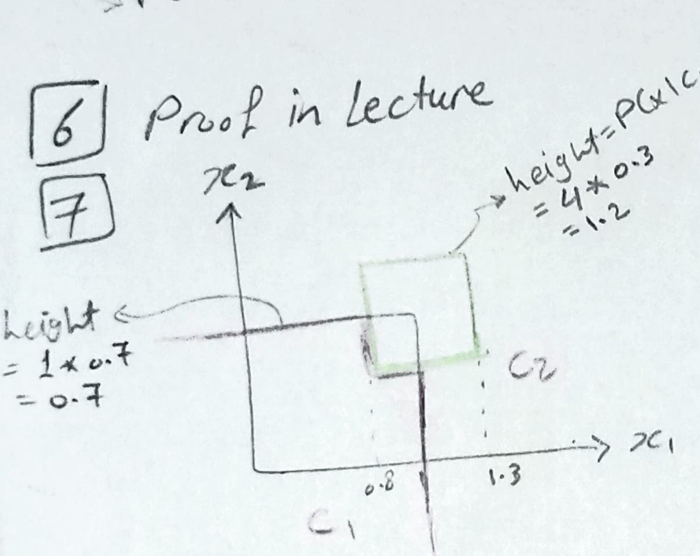
$$I_1 = \frac{1}{2}\int_{x_2=-\infty}^{\infty}\int_{x_1=-\infty}^{1} \frac{1}{2\pi} e^{-\frac{1}{2}\left[(x_1-2)^2 + x_2^2\right]}\,dx_1\,dx_2$$

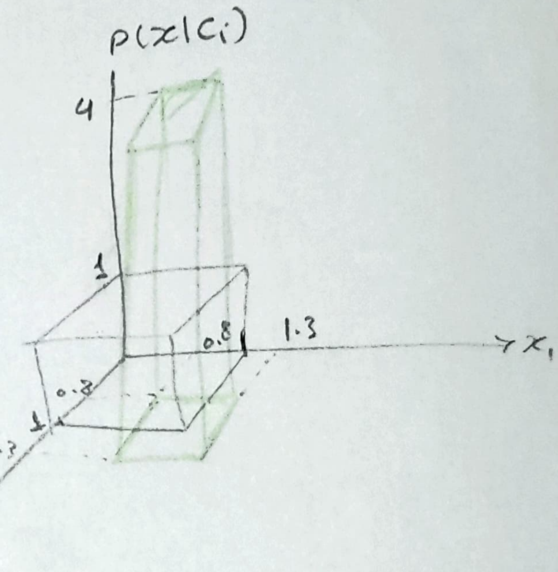
$$I_2 = \frac{1}{2}\int_{x_2=-\infty}^{\infty}\int_{x_1=1}^{\infty} \frac{1}{2\pi} e^{-\frac{1}{2}(x_1^2 + x_2^2)}\,dx_1\,dx_2$$

$\frac{1}{2} \to P(C_1)$

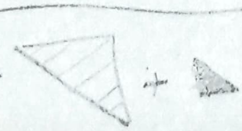
$\boxed{6}$ Proof in Lecture

$\boxed{7}$





$$P(\text{error}) = 0.2 \times 0.2 \times 0.7$$
$$= 0.028$$

P(error) = 

$$= \frac{1}{2} * 0.3 * 0.3 * 0.7 + \frac{1}{2} * 0.1 * 0.1 * 1.2$$
$$= 0.0375$$