Sheet 5 PID

*Notes:

P: (K_P) $\rightarrow$ as $K_P \uparrow\uparrow$, $e_{ss} \downarrow$, $e_{ss} \neq 0$

PI: ($K_P + \frac{K_I}{s}$) $\rightarrow$ Raise Type / order (by 1)
$\rightarrow$ eliminate ess / disturbance (=0)

PD: ($K_P + K_d s$) $\rightarrow$ Transient Requirements
$\zeta, \omega_n, M_P, t_s, t_P, t_r$

PID: ($K_P + K_d s + \frac{K_I}{s}$) $\rightarrow$ Transient & Steady State

*Recall:

* $$\zeta = \frac{-\ln(M_P)}{\sqrt{\pi^2 + (\ln M_P)^2}}$$

* $$t_{rising} = \frac{\pi - \cos^{-1}\zeta}{\omega_n \sqrt{1-\zeta^2}}$$ ψ] ω

* $$t_{settle} = \underbrace{\frac{4}{\zeta \omega_n}}_{2\%}, \underbrace{\frac{3}{\zeta \omega_n}}_{5\%}$$

* $$t_{Peak} = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}}$$] ω

$\boxed{Q.1}$ $MP = 0.1 \rightsquigarrow \zeta = 0.59$
$t_s = 1 \longrightarrow \omega_n = 6.78$ rad/sec } so transient Req so use PD $(K_P + K_d s)$

$c/cs\ Eq \Rightarrow 1 + \text{loop} \rightarrow s^3 + 50s^2 + 10^4(K_P + K_d s)$

$\rightarrow s^3 + 50s^2 + 10^4 K_d s + 10^4 K_P$

$= s^3 + (2\zeta\omega_n + \alpha)s^2 + (2\zeta\omega_n\alpha + \omega_n^2)s + \alpha\omega_n^2$

s^2: $\alpha = 50 - 2\zeta\omega_n = 42 > 5\zeta\omega_n$ ~~is~~
s: $K_d = (2\zeta\omega_n\alpha + \omega_n^2) * 10^{-4} \approx 0.0382$
s^0: $K_P = \alpha\omega_n^2 / 10^4 \approx 0.193$

} $G_c(s) = 0.193 + 0.0382s$

$\boxed{Q.2}$

$$G(s) = \frac{2}{s(s+2)(s+8)}$$ } → Type 1, order 3, $R(s)$ is a unit ramp $\frac{1}{s^2}$

$\boxed{a}$ $K_P = \infty$, $K_a = 0$, $K_v = \lim_{s \to 0} s\,GH(s) = \frac{2}{2*8} = 1/8$

$$E(s) = \frac{R(s)}{1+GH(s)} = \frac{s(s+2)(s+8)}{s^3 + 10s^2 + 16s + 2} * 1/s^2$$ → solve using calc to get $()()() = 0$

$$= \frac{(s+2)(s+8)}{s(s+8.04)(s+0.136)(s+1.823)} = \frac{8.03}{s} + \frac{0.0006}{s+8.04} - \frac{8.088}{s+0.136} + \frac{0.057}{s+1.823}$$ } Partial fractions

$$\frac{a}{s+b} \longleftrightarrow ae^{-b}$$

$$e(t) = u(t) = [8.03 + 0.0006e^{-8.04t} - 8.088e^{-0.136} + 0.057e^{-1.823}]$$

$\boxed{\text{Follow Q:2}}$

(b) we want dominant Poles at $s = -1 \pm j2\sqrt{3}$

but we know that, $s = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$

$\therefore \zeta\omega_n = 1 \rightarrow \omega_n = 1/\zeta$

$\therefore \omega_n\sqrt{1-\zeta^2} = 2\sqrt{3}$

$\frac{\sqrt{1-\zeta^2}}{\zeta} = 2\sqrt{3}$ $\rightarrow \zeta = \sqrt{1/13}$, $\omega_n = \sqrt{13}$

$(s+(1-j2\sqrt{3}))(s+(1+j2\sqrt{3}))$

$= s^2 + 2s + 13$

$\equiv s^2 + 2\zeta\omega_n + \omega_n^2$

* using PD

* C/CS EQ $\Rightarrow s^3 + 10s^2 + (16 + 2K_d)s + 2K_P$

$\equiv$

$s^3 + (2\zeta\omega_n + \alpha)s^2 + (\omega_n^2 + 2\alpha\zeta\omega_n)s + \alpha\omega_n^2$

s^2: $2 + \alpha = 10 \rightarrow \alpha = 8 > 5\zeta\omega_n$ ✓✓

s^1: $13 + 2\alpha = 16 + 2K_d \rightarrow K_d = 6.5$

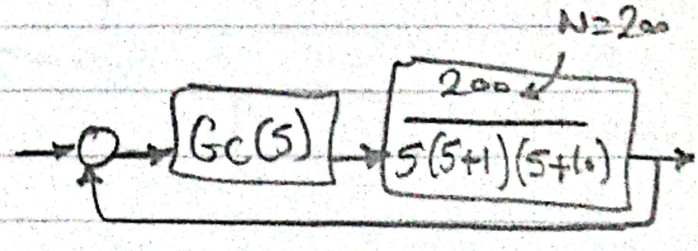
s^0: $2K_P = \alpha\omega_n^2 \rightarrow K_P = 52$

$G_C(s) = 52 + 6.5s$

$\boxed{Q:3}$ $M_p = 0 \xrightarrow{\text{critically Damped}} \zeta = 1$

$t_s = 2.5\ \text{Sec}$ $\quad \omega_n = 1.6$

$\frac{4}{\zeta\omega_n} \longrightarrow 2\%$



* Since, only transient requirments $\longrightarrow$ use PD Controller

$$G_c(S) = K_P + K_d S$$

* C/CS Eq.

$\hookrightarrow 1 + \text{Loop} \Rightarrow S(S+1)(S+10) + 200(K_P + K_d S) = 0$

$$\Rightarrow S^3 + 11S^2 + S(10 + 200K_d) + 200K_P = 0$$
$$= S^3 + (2\zeta\omega_n + \alpha)S^2 + (\omega_n^2 + 2\zeta\omega_n\alpha)S + \alpha\omega_n^2 = 0$$

* S^2: $\alpha = 11 - 2\zeta\omega_n = 7.8 \approx 8 \longrightarrow 5\zeta\omega_n$
* S^1: $K_d = (1.6^2 + 2(1.6)(7.8) - 10)/200 = 0.0876$
* S^0: $K_P = (1.6^2 * 7.8)/200 \approx 0.1$

$$G_c(S) = 0.1 + 0.0876S$$

$\because$ order $= 3$, type $= 1$ $\quad \therefore e_{ss} = 1/K_v$

* Before PD $\Rightarrow K_v = \lim_{S\to 0} S\,GH(S) = \frac{200}{1*10} = 20$, $e_{ss} = \frac{1}{20} = 0.05$
* After PD $\Rightarrow K_v = \frac{200 * 0.1}{1*10} = 2$, $e_{ss} = 1/2 = 0.5$

$\Rightarrow$ The steady state error increased by a factor of $\boxed{10}$

$\boxed{Q{:}4}$ $M_p = 0.1 \rightarrow \zeta = 0.5916$
$t_s = 2\ sec \rightarrow \omega_n = 3.38$ $\Big\}$ $\therefore$ use PD Controller

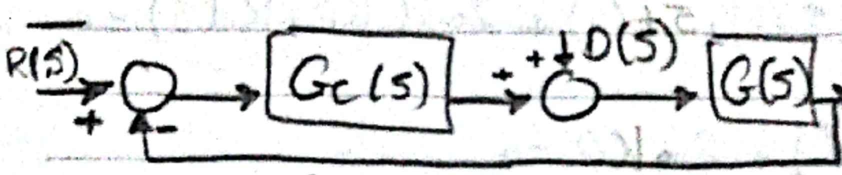
$c/cs\ Eq \Rightarrow 1 + loop \rightarrow s(s+5)(s+10) + 10(K_P + K_d s)$

$\rightarrow s^3 + 15s^2 + s(50 + 10K_d) + 10K_P$

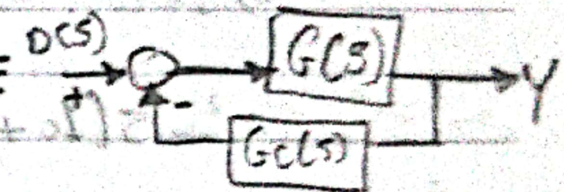
$\equiv s^3 + (2\zeta\omega_n + \alpha)s^2 + (\omega_n^2 + 2\alpha\zeta\omega_n)s + \alpha\omega_n^2$

$s^2: \alpha = 15 - 2\zeta\omega_n \simeq 11 > 5\zeta\omega_n$
$s: K_d = (\omega_n^2 + 2\alpha\zeta\omega_n - 50)/10 = 0.542$
$s^0: K_P = \alpha\omega_n^2/10 = 12.56$

$G_c(s) = 12.56 + 0.542\,s$



$\equiv$



* Calc the effect of D(s) at steady state, $R(s) = 0,\ E(s) \equiv -C(s)$

$$TF = \frac{Y(s)}{D(s)} = \frac{G(s)}{1 + GG_c(s)} = \frac{10}{s(s+5)(s+10) + 10(K_P + K_d s)}$$

$$\therefore Y(s) = D(s)\,TF = \frac{Q}{s} * \square$$

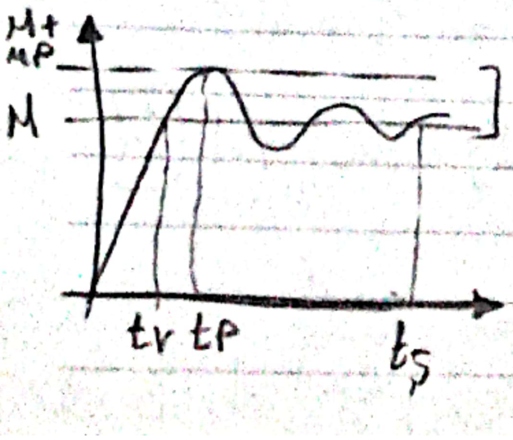
$$y_{ss}|_D = \lim_{s\to 0} sY(s) = \lim_{s\to 0} s * \frac{Q}{s} * \frac{10}{10K_P} = \frac{Q}{K_P} \simeq 0.079\,Q$$

Let $R(s) = \frac{M}{s}$] step $\rightarrow y_{ss}|_R = M$

$$\boxed{y_{ss}|_{total} = M + 0.079\,Q}$$

* the resulting Response (M_p, t_p, t_s, t_r)

$M_p = 0.1M,\ t_s = 2\ sec$
$t_p = 1.15,\ t_r = 0.806$



$\boxed{Q:5}$ $MP = 0.05 \rightarrow \zeta = 0.69$

$t_s = 0.2 \rightarrow \omega_n = 28.98 \approx 29 \text{ rad/sec}$

$ess = 0$

} use PID Controller

$$G(s) = \frac{K_P + K_d s + \frac{K_I}{s}}{0.03 s^2 + 0.4 s + 1}$$

$$C/Cs\ Eq \Rightarrow \cancel{0.03} s^{\cancel{2}\,3} + \frac{(0.4 + K_d)}{0.03} s^{\cancel{1}\,2} + \frac{(1 + K_P)}{0.03} s^{\cancel{0}\,1} + \frac{K_I}{0.03}$$

$$\equiv s^3 + (\alpha + 2\zeta\omega_n) s^2 + (\omega_n^2 + 2\alpha\zeta\omega_n) s + \alpha\omega_n^2$$

∴ Needs an Extra Eq $\rightarrow \alpha > 5\zeta\omega_n \rightarrow \alpha \approx 100$

∴ s^2: $K_d = (100 + 2(0.69)(29)) * 0.03 - 0.4$, $K_d \approx 3.8$

∴ s^1: $K_P = (\omega_n^2 + 2\alpha\omega_n\zeta) * 0.03 - 1$, $K_P = 144.29$

∴ s^0: $K_I = 0.03 * 100 * (29)^2$, $K_I \approx 2520$

$$G_c(s) = 144.3 + 3.8 s + \frac{2520}{s}$$