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Computer Graphics (CMP 205)
Midterm Exam – Fall 2015
(90 minutes) -*Total Marks: 20*

Question 1 (5)	Question 2 (5)	Question 3 (5)	Question 4 (5)	Total (20)
2	2	4.5	5	13.5

$$R_x(\theta)=\begin{pmatrix}1 & 0 & 0\\ 0 & \cos\theta & \sin\theta\\ 0 & -\sin\theta & \cos\theta\end{pmatrix},\quad R_y(\theta)=\begin{pmatrix}\cos\theta & 0 & -\sin\theta\\ 0 & 1 & 0\\ \sin\theta & 0 & \cos\theta\end{pmatrix},\quad R_z(\theta)=\begin{pmatrix}\cos\theta & \sin\theta & 0\\ -\sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{pmatrix}$$

Slope of AB = $\frac{2}{3}$ | Slope of DC = $\frac{1}{2}$

" " DE = -1

" " AE = $\frac{4}{3}$

" " BC = vertical (// y-axis)

Question 1: [5 points] 2

For the polygon shown in Figure 1. [A (2,2) , B (8,6) , C (8,12) , D (2, 9) , E (5, 6)]

A. Perform a proper edge shortening
B. Compute the corresponding edge table including edge structure
C. List the active edge table at Y = 8 and Y = 4

D(2,9) → (2,8)
-1

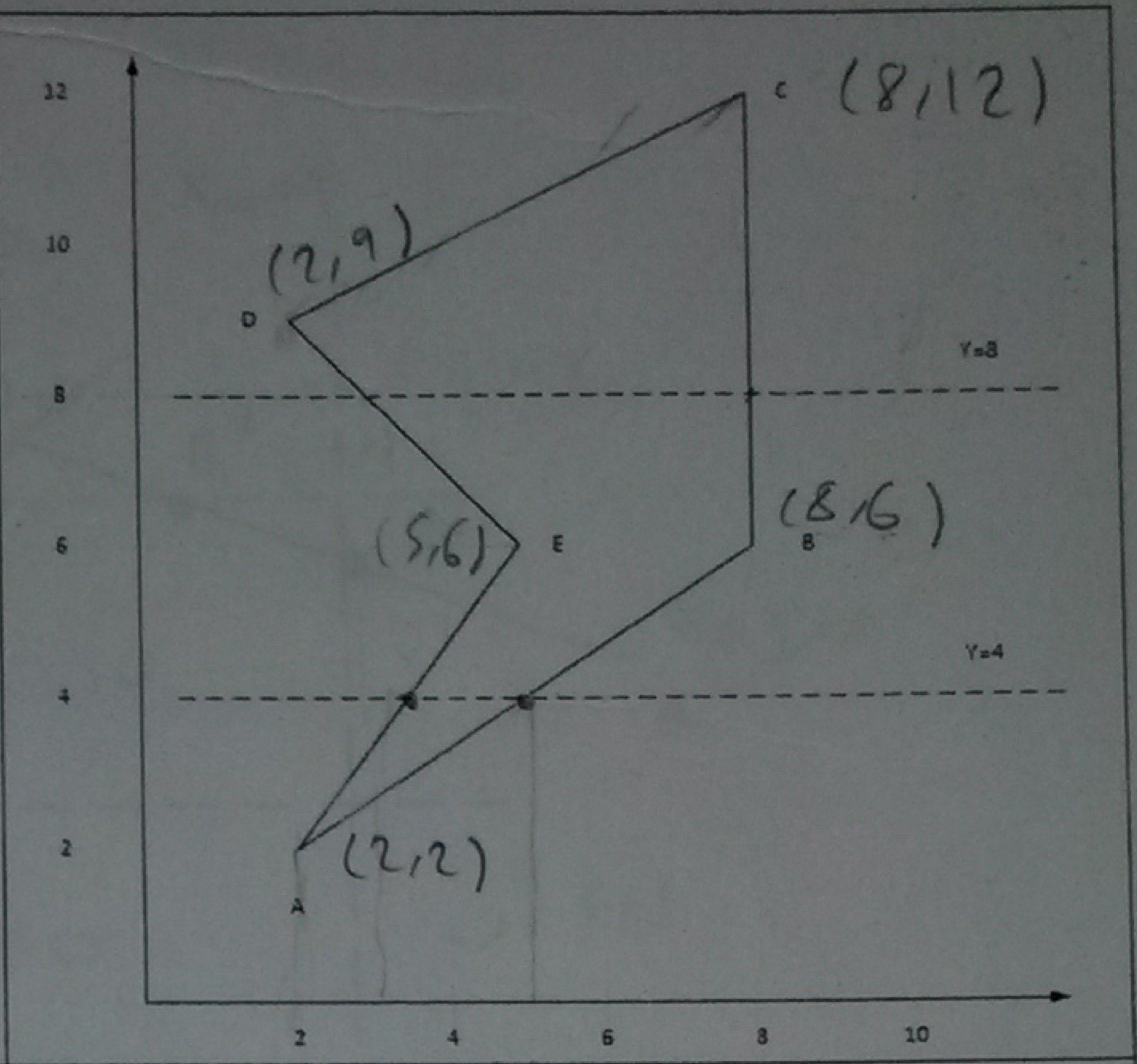


Figure 1

(A) We can shorten edge DC such that D :
-1 (x, y-1) → D(2,8)

(B) ET: -2

12 → | 12 | 2 | 4 | X

11

10

9 → DC | 12 | 2 | 4 |

8

7

6 → ED | 9 | 5 | 4 | → BC | 12 | 8 | 4 |

5

4

3

2 → AE | 6 | 2 | 4 | → AB | 6 | 2 | 4 |

1

0

1/m X

| y_{max} | x_{min} | slope |

of y_{min} — of line

(C) AET: AT y=6)

AE | 6 | 3 | 4 | → AB | 6 | 5 | 4 |

AT y=8)

DE | 9 | 3 | 4 | → CB | 12 | 8 | 4 |

Question 2: [5 points] 2

A. Using Cohen-Sutherland line clipping algorithm, show the steps and the final results of clipping the line shown in Figure 2.

2

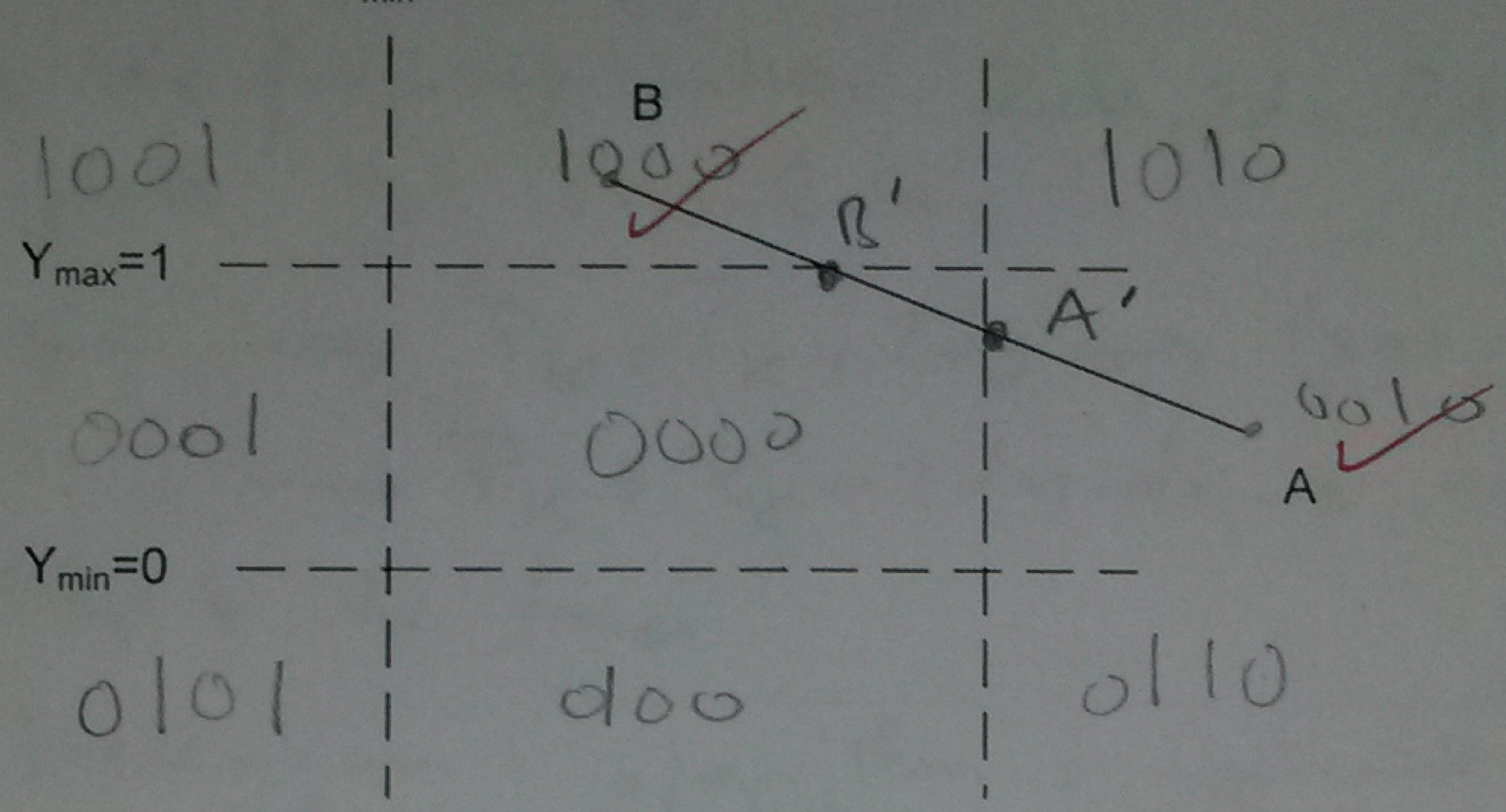


Figure 2

Pick A → clip A to A' (outside Right boundary)

Pick B → clip B to B' (outside Top boundary)

New line clipped → A'B'

0

B. What needs to be done to extend the 2-D Cohen-Sutherland line clipping algorithm to 3-D? What is the maximum number of times that a 3-D line might be clipped by this algorithm? Be sure to briefly justify your answers.

- triangles should be used instead of lines
- We will add three other boundries (far, Near, in between)
- Max. no. is equal to the no. of triangles that the polygon consists of.

Question 3: [5 points]

2 A. Write down the step used in Bresenham's line-drawing algorithm. In your answer, explain under which slope condition your algorithm is applicable.

1 Plot the endpoints of the line in graph

2 Compute the following values: 2dx, 2dy, 2dy-2dx, 2dy-d

3 Starting point $P_0 = 2dy - 2dx$

4 If $P_k < 0 \longrightarrow P_k = P_k + 2dy \longrightarrow (x+1, y)$

else $\longrightarrow P_k = P_k + 2dy - dx \longrightarrow (x+1, y+1)$ ystep

5 Repeat step 4 dx times

Bresenham's algorithm in applicable when slope in either greater than or less than 1

2.5 B. Given a 2D rotation transformation matrix R(θ). Is $R(\theta_1)R(\theta_2)=R(\theta_1+\theta_2)$? Prove your answer.

$$R(\theta_1)R(\theta_2) = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} \cos\theta_1\cos\theta_2 - \sin\theta_2\sin\theta_1 & -\cos\theta_1\sin\theta_2 - \sin\theta_1\cos\theta_2 & 0 \\ \sin\theta_1\cos\theta_2 + \cos\theta_1\sin\theta_2 & -\sin\theta_1\sin\theta_2 + \cos\theta_2\cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R(\theta_1+\theta_2) = \begin{bmatrix} \cos(\theta_1+\theta_2) & -\sin(\theta_1+\theta_2) & 0 \\ \sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore R(\theta_1+\theta_2) = R(\theta_1)R(\theta_2)$$

Question 4: [5 points]

Consider the triangle ABC shown (it lies in the x-y plane). It is required to transform it to the position shown below (to triangle ADC in the x-z plane). The new triangle preserves the same shape and size. The points are

5

A = (0,0,0), B = (1,2,0), C = (1,0,0), D = (0,0,2)

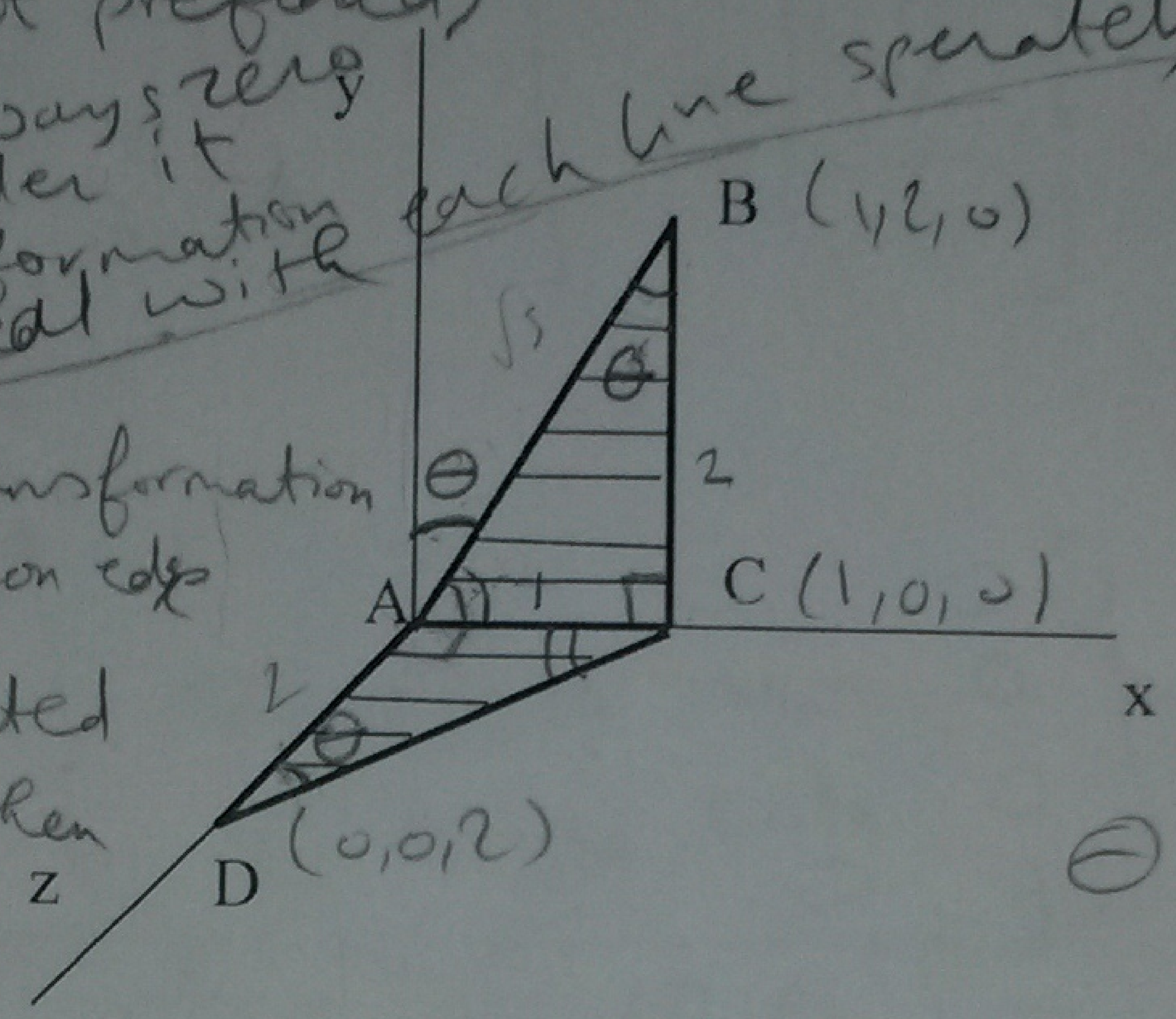
A. Find the steps of the transformations, including the (four-dimensional) transformation matrices. Find the overall transformation matrix that corresponds to all steps combined.

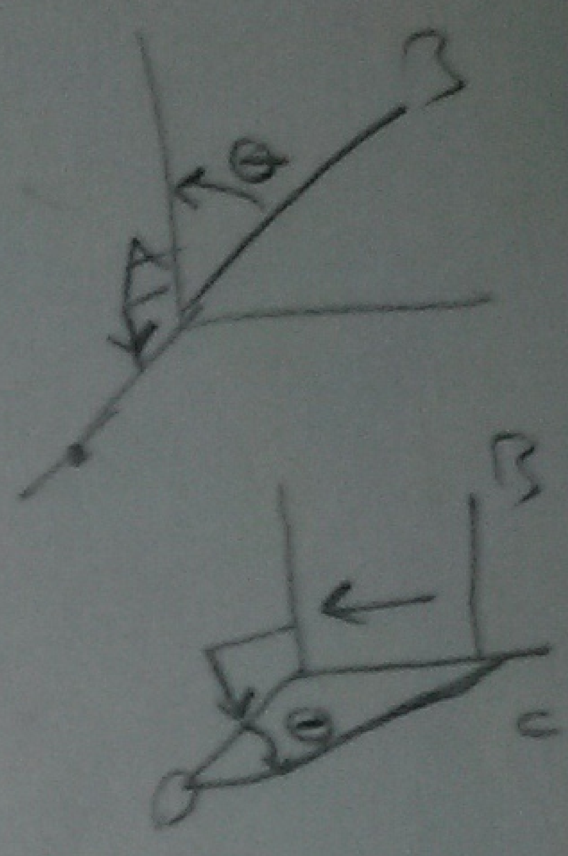
B. Find the overall inverse transformation.

Approach 1; (not prefered)

Since z is always zero

> we can consider it 2D transformation and deal with each line sperately:





1) AC needs no transformation as its a common edge

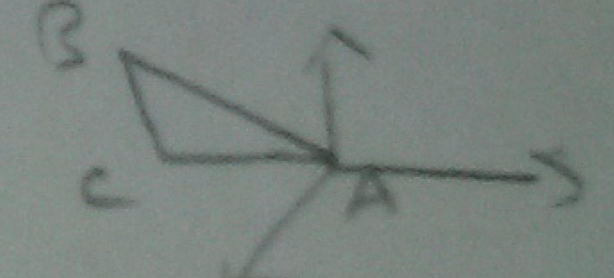
2) AB 'll be rotated around angle θ, then rotated around x-axis

θ = 63.43°

3) BC 'll be translated to the Y-axis, then rotated around x-axis, then rotated again around angle θ

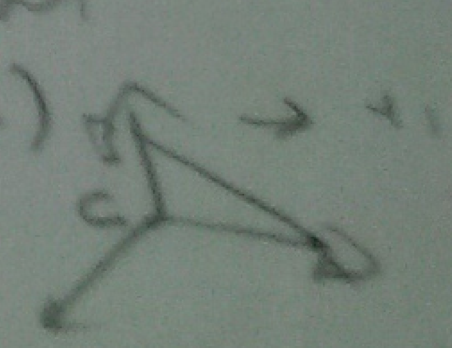
Approach 2; (preferred)

Deal with the whole figure;

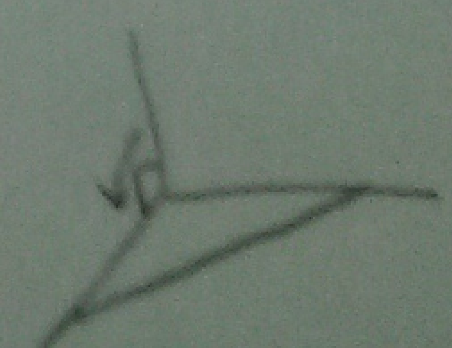


1) Rotate the Δ around Point A 180°
around Y-axis

2) Translate the Δ such that becomes at the origin respectively ($\sqrt{5}$ unit trans.)



3) Rotate the Δ around Point C 90°
around x-axis



Approach 2 matrices:

$$R_y(180) = \begin{bmatrix} \cos 180 & 0 & -\sin 180 & 0 \\ 0 & 1 & 0 & 0 \\ \sin 180 & 0 & \cos 180 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{*} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$T_x = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{*} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$R_x(90) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 90 & \sin 90 & 0 \\ 0 & -\sin 90 & \cos 90 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{*} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

→ Overall trans. matrix $= R_x(90)\, T_x\, R_y(180)$ ✓

Overall inv. trans. matrix $= R_y^{-1}(180)\, T_x^{-1}\, R_x^{-1}(90)$ ✓

Such that:
$$R_y^{-1}(\theta) = R_y(-\theta)$$
$$R_x^{-1}(\theta) = R_x(-\theta)$$
$$T_x^{-1} = -T_x$$

N.B.: I could have neglected the z transformation since it's always zero, but I preferred to include it for generalization