Pattern Classification

Neural Networks

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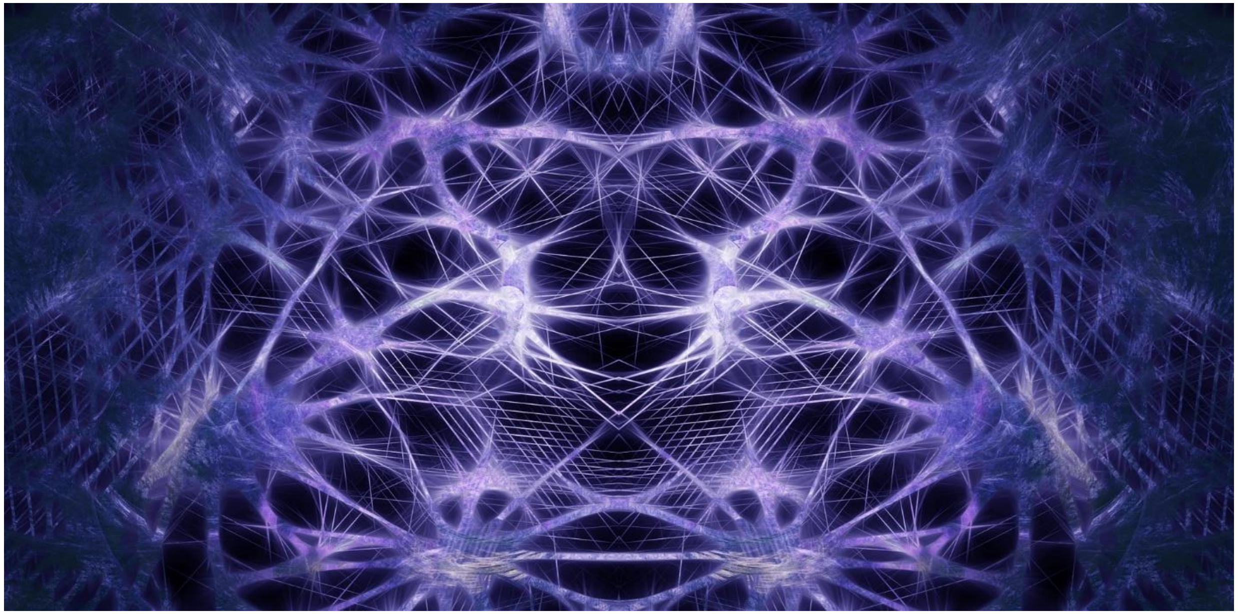
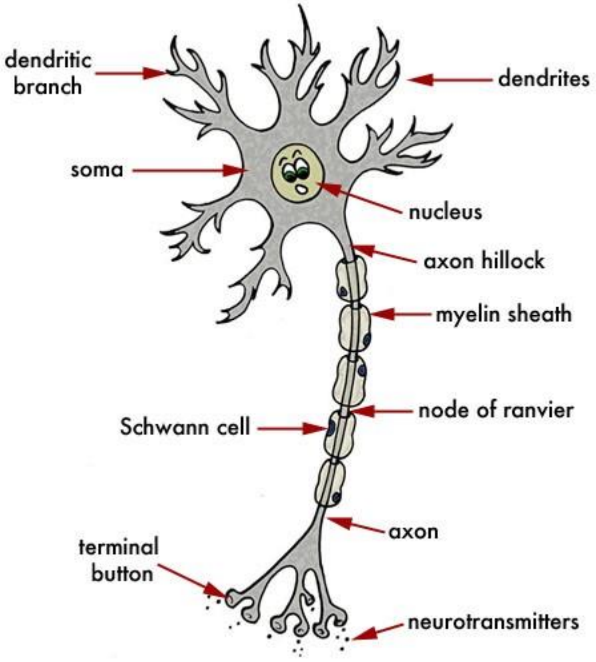
Human Brain

- The brain has 10^{10} – 10^{11} cells that are highly connected
- Every cell (called neuron) is connected to about 10^3 – 10^4 other cells in its neighborhood



Source: Luis Bermudez – Machine Vision



Source: Shubham Panchal - Predict

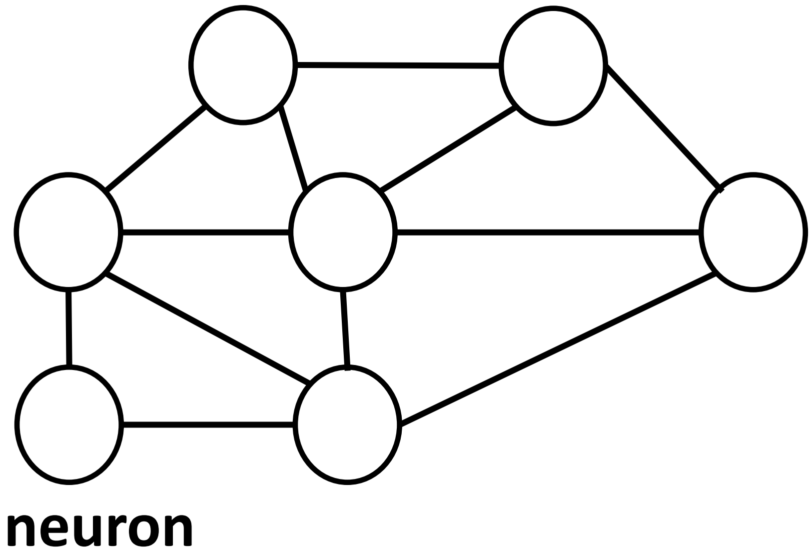
Human Brain

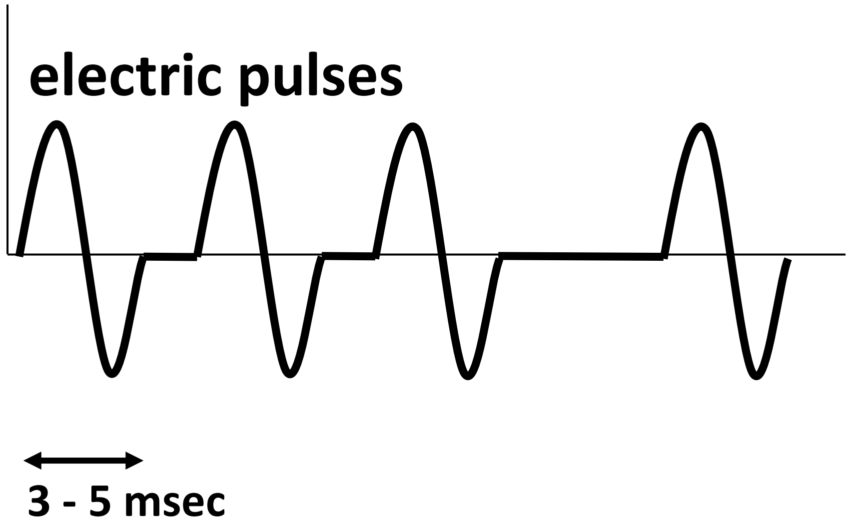
- The cells exchange information in the form of electric signals (pulses)

- That is how the brain processes information and performs intelligent tasks

- Success of the brain is a result of the parallelism of this huge network of cells

Human Brain





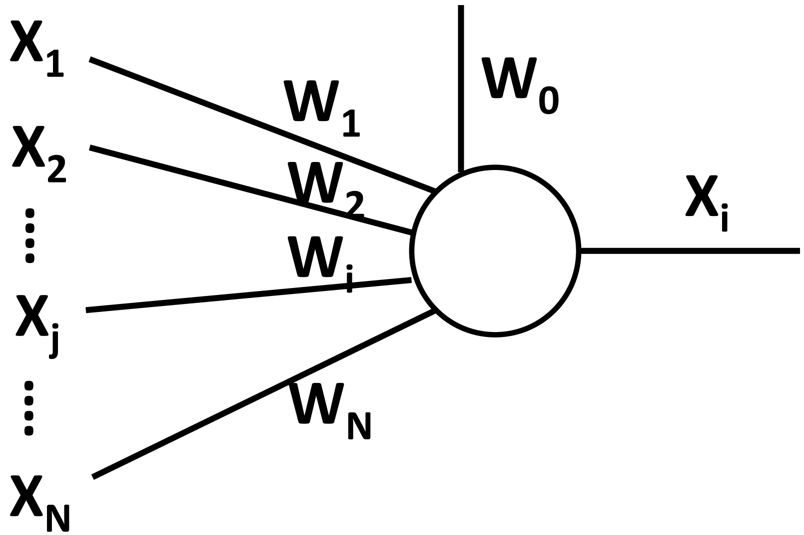
- Time constant ≈ 3 – 5 msec
- Traditional computer's time constant ~ 10^{-9} or nano sec

Human Brain

- The “time constant” of the brain is significantly larger than that of the traditional computers
- Brain is superior because of the massive parallelism

Neural Networks

- Mimic how the brain works

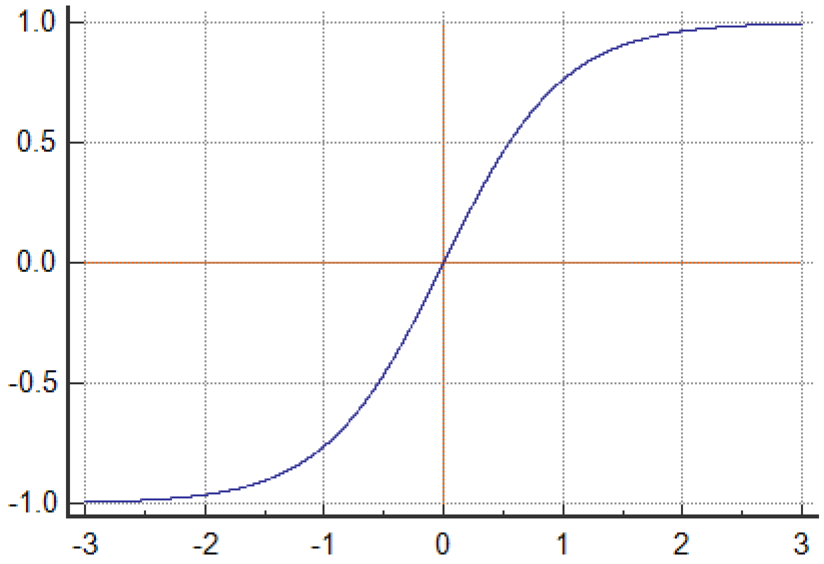


$$X_i = f\left(\sum_{j=1}^{N} W_j X_j + W_0\right)$$

- W_j ≡ weight → gives the degree of how input cell j affects the receiving cell I
- W_0 ≡ some constant called bias or threshold
- f(x) ≡ activation function

Neural Networks

$$X_i = f\left(\sum_{j=1}^{N} W_j X_j + W_0\right)$$



tanh

$$f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

Neural Networks

- Every neuron produces output Xj
- If neuron j is connected to neuron i then Xj will have an affect on Xi
- Each neuron can have a different effect, hence, we multiply by weight wj
- Weights are the parameters that encode the information in the brain

Neural Networks

- Weights act like the “storage” in computers
- When a human encounters a new experience the weights of his brain gets adjusted
- This adjustment results in his learning of the new experience
- Memories are encoded in the weights

Neural Networks

- We try in the field of neural networks (NNs) to exploit the learning feature observed in the brain

- The main concept of NNs is that we have a model given in terms of parameters, i.e. weights.

- The weights determine the functionality of the model

Neural Networks

- We apply some learning algorithm that adjusts the weights in small steps

- The goal is to lead the NN to learn to implement the given problem

Recall: Feature Vector

- Let $\underline{\boldsymbol{X}}(\boldsymbol{m}) = \begin{bmatrix} \boldsymbol{X_1}(\boldsymbol{m}) \\ \boldsymbol{X_2}(\boldsymbol{m}) \\ \vdots \\ \boldsymbol{X_N}(\boldsymbol{m}) \end{bmatrix}$ be the feature vector of the mth training pattern

- N is the number of features, i.e., the dimension of $\underline{X}(m)$

- M is the number of the training patterns

Example

- Given a training set $\underline{X}(1), \underline{X}(2) \cdots \underline{X}(M)$, design a linear classifier
  - Determine the parameters $W_0, W_1 \cdots W_N$

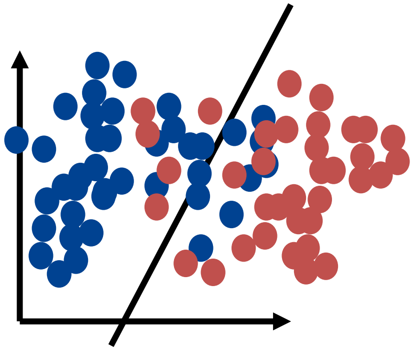
$$y(m) = f\left(W_0 + \underline{w}^T \underline{X}(m)\right) = \begin{cases} 1, & if\, \underline{X}(m) \epsilon\ C_1 \\ 0, & if\, \underline{X}(m) \epsilon\ C_2 \end{cases}$$

$$\underline{w} = \begin{bmatrix} W_1 \\ W_2 \\ \vdots \\ W_N \end{bmatrix} \equiv \text{weight vector}$$

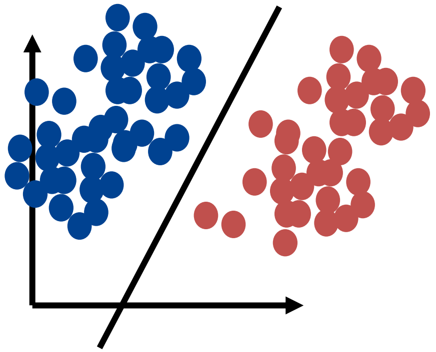
Recall: Types of Problems

- A problem is said to be ***linearly separable*** if there is a hyperplane that can separate the training data points of class C_1 from those of C_2

- Otherwise it is said to be ***not linearly separable***



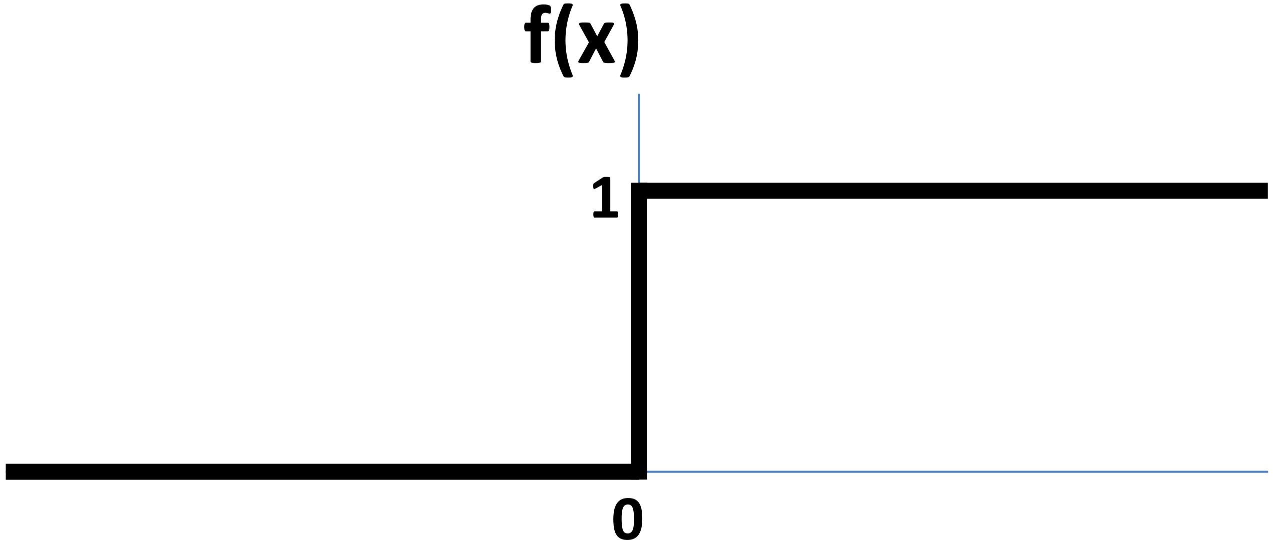
Not linearly separable



Linearly separable

Example – Contd.

- Define the activation function f(x) as a unit step function



unit step fn.
<u>or</u> hard limiting fn.
<u>or</u> hard threshold fn.

Example – Contd.

- $y(m) = f\left(W_0 + \underline{w}^T \underline{X}(m)\right) = \begin{cases} 1, & if\, \underline{X}(m) \epsilon\, C_1 \\ 0, & if\, \underline{X}(m) \epsilon\, C_2 \end{cases}$

- The neuron can implement a linear classifier

Example – Contd.

- Define augmented vectors:

$$\underline{W} = \begin{bmatrix} W_0 \\ \vdots \\ W_N \end{bmatrix} = \begin{bmatrix} W_0 \\ \underline{w} \end{bmatrix} \quad \text{where } \underline{w} = \begin{bmatrix} W_1 \\ \vdots \\ W_N \end{bmatrix}$$

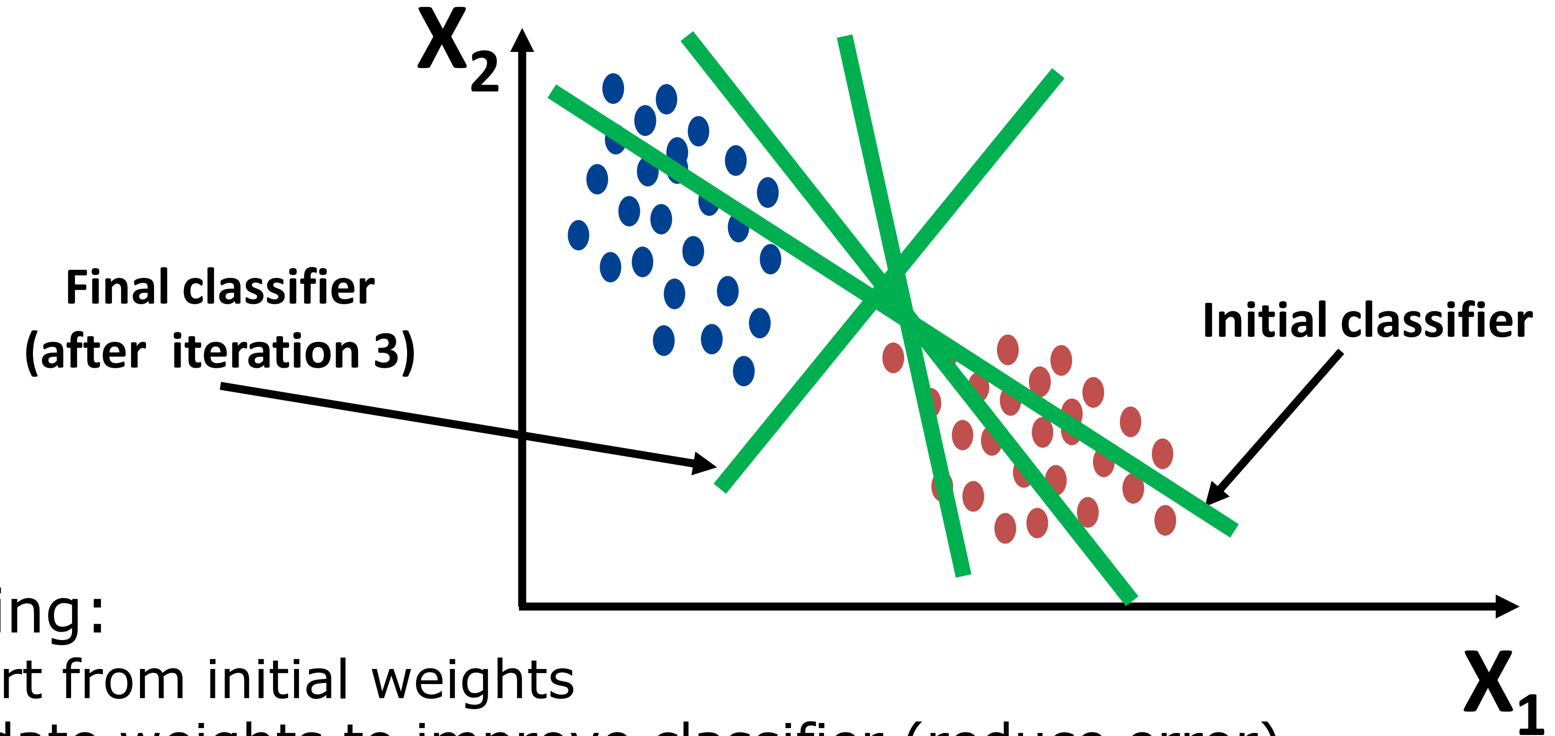
$$\underline{u}(m) = \begin{bmatrix} 1 \\ \underline{X}(m) \end{bmatrix} = \begin{bmatrix} 1 \\ X_1(m) \\ \vdots \\ X_N(m) \end{bmatrix}$$

So:

$$\boldsymbol{y}(\boldsymbol{m}) = \boldsymbol{f}\left(\underline{\boldsymbol{W}}^{\boldsymbol{T}}\underline{\boldsymbol{u}}(\boldsymbol{m})\right)$$

Example – Contd.

- $y(m) = f(\underline{W}^T \underline{u}(m)) = \begin{cases} 1, & if\, \underline{u}(m) \epsilon\ C_1 \\ 0, & if\, \underline{u}(m) \epsilon\ C_2 \end{cases}$



- Learning:
  – Start from initial weights
  – Update weights to improve classifier (reduce error)
  – Repeat until arriving to the best possible classifier

Linear Perception

- The linear neuron (linear perception):
  The linear perceptron can classify correctly only linearly separable problems

- It can be used for non-linearly separable problems as long as it produces a low classification error rate

Linear Perceptron Algorithm

1. Initialize the weights and threshold (bias) randomly

2. Present the augmented input (or feature) vector of the mth training $\underline{u}(m)$ and its corresponding desired output $\mathrm{d}(m)$

$$\boldsymbol{d(m)} = \begin{cases} \mathbf{1}, & \boldsymbol{if\ \underline{u}(m) \in C_1} \\ \mathbf{0}, & \boldsymbol{if\ \underline{u}(m) \in C_2} \end{cases}$$

3. Calculate the actual output for pattern m:

$$\boldsymbol{y(m) = f\left(\underline{W}^T \underline{u}(m)\right)}$$

4. Adapt the weights according to the following rule (called Widrow-Hoff rule):

$$\boldsymbol{\underline{W}(new) = \underline{W}(old) + \eta[d(m) - y(m)]\underline{u}(m)}$$

where η is a constant called the learning rate

5. Go to step 2 until all patterns are classified correctly, i.e., d(m)=y(m) for m=1, … , M

Note: the algorithm is sequential w.r.t. the patterns $\underline{u}(m)$

Understanding Widrow-Hoff Update

- If d(m)=y(m) then no change is needed in the weights, i.e., $\underline{W}(new) = \underline{W}(old)$, because d(m)-y(m)=0

- If d(m)≠y(m) then weights get updated

$$\boldsymbol{\underline{W}(new) = \underline{W}(old) + \eta[d(m) - y(m)]\underline{u}(m)}$$

Acknowledgment

- These slides have been created relying on lecture notes of Prof. Dr. Amir Atiya