CAIRO UNIVERSITY
Elected Topics 3
Fourth Year

DEPT COMPUTER ENG
Pattern Classification
November 2017

Time: 1.5 Hours

1) Consider the following problem:

Class 1 patterns: $(2\ \ 2)^T$, $(2\ \ 3)^T$, $(3\ \ 2)^T$, $(3\ \ 3)^T$.
Class 2 patterns: $(0\ \ 0)^T$, $(1\ \ 0)^T$, $(0\ \ 1)^T$, $(1\ \ 1)^T$

a) Assume that we would like to use the nearest neighbor classifier. What would be the classification of the following pattern: $(1.5\ \ 1.4)^T$?

b) Assume that we would like to use the K-nearest neighbor classifier, where $K = 3$. What would be the classification of the following pattern: $(1.1\ \ 1.3)^T$?

2) a) State why the Bayes classifier is theoretically the optimal classifier. Discuss why in realistic situations it cannot be considered optimal.

b) The linear classifier is a classifier that has a decision boundary of the form of a .... (insert missing word).

c) Consider the two-dimensional three-class classification problem, where the class centers are given by the vectors $(0,\ \ 1)^T$, $(2,\ \ 0)^T$, $(2,\ \ 2)^T$. Plot the decision regions and the decision boundaries for the minimum distance classifier. What are the coordinates of the point of equal distance among all class centers.

3) a) Let x be a random variable having a Gaussian density with mean equal 0 and standard deviation equal 2. Write down the expression of the density $p(x)$. Compute $E(e^{-2x^2})$.

b) Consider the joint density

$$p(x, y) = \begin{cases} 2e^{-x}(1-y) & \text{if } 0 \leq x < \infty \text{ and } 0 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Find $p(x)$ (consider $x \geq 0$).

4) Consider a two-class single dimensional problem, where the a priori probabilities are given by $P(C_1) = 0.65$ and $P(C_2) = 0.35$, and the class-conditional densities are given by:

$$p(x|C_1) = \begin{cases} a(1-x) & \text{if } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

$$p(x|C_2) = \begin{cases} 1 & \text{if } 0.6 \leq x < 1.6 \\ 0 & \text{otherwise} \end{cases}$$

a) Find the value of a. b) Plot the densities, and plot the decision regions and the decision boundaries for the Bayes classifier. c) Find the probability error for the Bayes classifier.