$u(t) \leftrightarrow 1/s$

$t^n u(t) \leftrightarrow \frac{n!}{s^{n+1}}$

$e^{+at}u(t) \leftrightarrow \frac{1}{s-a}$

$\dddot{f}(t) = s^3 F(s) \underbrace{- s^2 f(0) - s\dot{f}(0) - \ddot{f}(0)}_{\text{Initial Condition}}$

Sheet 1

$x(t-t_0) \leftrightarrow X(s)e^{-t_0 s}$

$x(t)e^{+at} \leftrightarrow X(s-a)$

$\boxed{Q.1}$ $\ddot{y}(t) + 3\dot{y}(t) + 2y(t) = u(t)$

$\boxed{a}$ Given $y(0) = -1$, $\dot{y}(0) = 2$

$s^2 Y(s) - s\,y(0) - \dot{y}(0) + 3[s\,Y(s) - y(0)] + 2Y(s) = 1/s$

$Y(s)(s^2+3s+2) + s+1 = 1/s$

Partial Fraction

$Y(s) = \frac{1-s-s^2}{s(s^2+3s+2)} = \frac{1-s-s^2}{s(s+1)(s+2)} = \frac{0.5}{s} + \frac{-1}{s+1} + \frac{-0.5}{s+2}$

$y(t) = [0.5 - e^{-t} - 0.5e^{-2t}]u(t)$

Recall:

Partial fraction $\frac{\square}{(x+\alpha)(x+\beta)} = \frac{A}{x+\alpha} + \frac{B}{x+\beta}$

- عوض في x بـ $(-\alpha)$ وهات A

- حط من فوق وتحت $A = \frac{\square}{x+\beta}$

$\boxed{Q.1}$ $\ddot{y}(t) + 3\dot{y}(t) + 2y(t) = \dot{x}(t) + 3x(t)$ $\rightarrow$ $x(t) = e^{-4t}$

$\boxed{B}$ $y(0) = 1$, $\dot{y}(0) = 0$ $\quad x(0) = 1$

$s^2 Y(s) - s\,y(0) - \dot{y}(0) + 3[sY(s) - y(0)] + 2Y(s) = sX(s) - x(0) + 3X(s)$

$s^2 Y(s) - s + 3[sY(s) - 1] + 2Y(s) = X(s)[s+3] - 1$

$Y(s)[s^2+3s+2] = \frac{s+3}{s+4} - 1 + s + 3 = \frac{s+3}{s+4} + s + 2 \rightarrow \frac{s+3+s^2+6s+8}{s+4}$

$Y(s) = \frac{s^2+7s+11}{(s+4)(s+2)(s+1)} = \frac{-1/6}{s+4} + \frac{-1/2}{s+2} + \frac{5/3}{s+1}$

$y(t) = [-\frac{1}{6}e^{-4t} - \frac{1}{2}e^{-2t} + \frac{5}{3}e^{-t}]u(t)$

Q.2

a) $y(t) = a\,u(t)$, let $u(t) = u_1(t) + u_2(t)$

Hence, $y_1(t) = a\,u_1(t)$

$y_2(t) = a\,u_2(t)$

therfore, $y(t) = a[u_1(t) + u_2(t)]$

$= y_1(t) + y_2(t)$ "Additive"

trivially, $y(t) = a[\alpha u_1(t) + \beta u_2(t)]$

$= \alpha y_1(t) + \beta y_2(t)$ "Homog"

$\therefore$ $y(t)$ is a linear sys ✓

b) $y(t) = u^3(t)$ → trivially, Not additive nor Homog → Not linear

c) $y(t) = e^{u(t)}$ → ——————

d) $\ddot{y}(t) + a\dot{y}(t) + y(t) = u(t)$, Given → $y(0) = \dot{y}(0) = 0$

$u(t)$ → Not a step func!

$s^2 Y(s) + a s Y(s) + Y(s) = U(s)$

$$Y(s) = \frac{U(s)}{s^2 + as + 1} = U(s)\left[\frac{1}{(s+\beta_1)(s+\beta_2)}\right] = U(s)\left[\frac{\alpha_1}{s+\beta_1} + \frac{\alpha_2}{s+\beta_2}\right]$$

let.

$u(t) = a u_1(t) + b u_2(t)$, $y(t) = f(x) + g(x)$

$y(t) = a f_1(t) + b f_2(t)$
$+ \; a g_1(t) + b g_2(t)$

(a لأن مش سيمتري)

$y(t) = a y_1(t) + b(y_2)$ → addi, Homog

$f_1(t) = \int_{-\infty}^{\infty} u_1(t)\, e^{-\beta_1(t-\tau)}\, \alpha_1 \, d\tau$

$f_2(t) = \int_{-\infty}^{\infty} u_2(t)\, e^{-\beta_2(t-\tau)}\, \alpha_2 \, d\tau$

$g_1(t) = \int_{-\infty}^{\infty} u_1(t)\, e^{-\beta_2(t-\tau)}\, \alpha_2 \, d\tau$

$g_2(t) = \int_{-\infty}^{\infty} u_2(t)\, e^{-\beta_2(t-\tau)}\, \alpha_2 \, d\tau$

* كما صح كان ممكن نقف عند متصادل لكن شكله دايما "Linear"
* لو الـ (a) ← 1 الـ B ← C Eng حول لـ sine ~~~~ كده كده!

Q.3 ∴ we want the TF ∴ Assume Zero Init Condi.

a) $\ddot{y}(t) + 3\dot{y}(t) + 2y(t) = \dot{x}(t) + 3x(t)$

$$s^2 Y(s) + 3s Y(s) + 2Y(s) = s X(s) + 3X(s)$$

$$Y(s)\left[s^2 + 3s + 2\right] = X(s)(s+3)$$

$$\therefore T.F. = \frac{Y(s)}{X(s)} = \frac{s+3}{s^2+3s+2} = \frac{s+3}{(s+1)(s+2)}$$

* مش محتاج أعمل
أي Part. Frac عشان
مش عاوز اجيب
ال $y(t)$ زي اللي فاتوا

b) $\dot{y}(t) + y(t) = x(t - \underset{\text{Const}}{T})$

$$s Y(s) + Y(s) = X(s) e^{-Ts}$$

$$Y(s)(s+1) = X(s) e^{-Ts}$$

$$\therefore T.F. = \frac{Y(s)}{X(s)} = \frac{e^{-Ts}}{s+1}$$