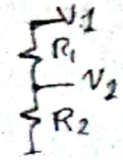
⊣⊢ ⇒ $1/sC$

⏦ ⇒ sL

$V_2 = \frac{R_2}{R_1+R_2} V_1$ "voltage divider"

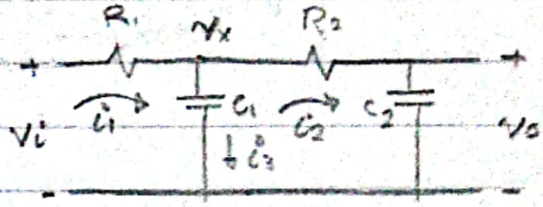


Modelling

Sheet 1, Part 2

4

Q1



$\therefore V_x = V_i - i R_1 = V_i - (i_2 + i_3) R_1$

$\therefore V_x = V_{in} - R_1 \left[\frac{V_x}{1/sC_1} + \frac{V_o}{1/sC_2} \right]$ ①

$\therefore V_o = \frac{1/sC_2}{R_2 + 1/sC_2} V_x$ "voltage divider"

$\therefore V_x = (1 + sC_2R_2) V_o$ ②

$V_o = \frac{1}{sC_2R_2 + 1} V_x$

subst ② in ①

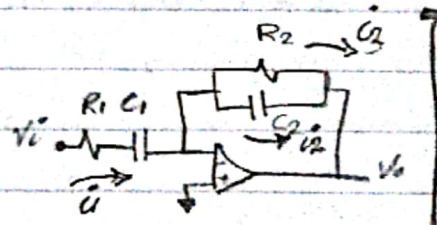
$\rightarrow (1+sC_2R_2) V_o = V_{in} - (sC_1R_1)(1+sC_2R_2) V_o - (sC_2R_1) V_o$

$V_o [sC_2R_1 + (1+sC_1R_1)(1+sC_2R_2)] = V_{in}$

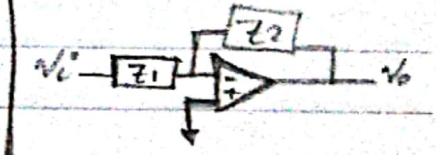
$TF = \frac{V_o}{V_{in}} = \frac{1}{(1+sC_1R_1)(1+sC_2R_2) + sC_2R_1}$

5

Q2



Recall: For ideal OP



* $TF = \frac{-Z_2}{Z_1}$

* $\frac{V_o - 0}{Z_1} = \frac{0 - V_o}{Z_2}$

1st sol:

$i = i_2 + i_3$

$\frac{V_i}{R_1 + 1/sC_1} = \frac{-V_o}{1/sC_2} + \frac{-V_o}{R_2} \Rightarrow \frac{sC_1 V_i}{1+sC_1R_1} = \frac{-V_o}{R_2} \times [1+sC_2R_2]$

2nd sol

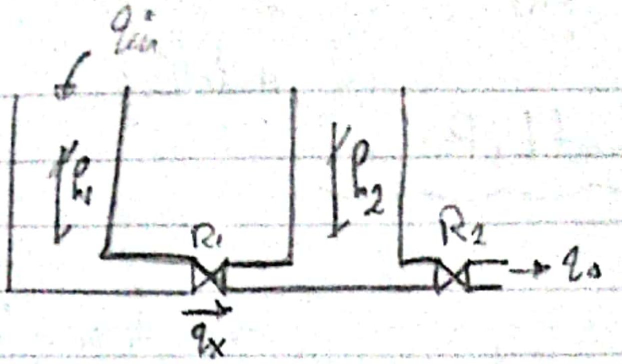
$Z_1 = R_1 + \frac{1}{sC_1} = \frac{1+sC_1R_1}{sC_1}$

$Z_2 = \frac{1}{R_2} + \frac{1}{1/sC_2} = \frac{1+sC_2R_2}{R_2}$

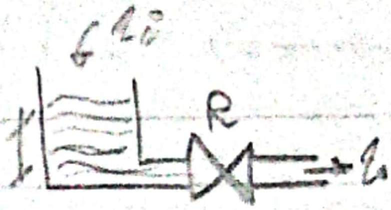
$\Rightarrow TF = \frac{-sC_1R_2}{(1+sC_1R_1)(1+sC_2R_2)}$

6

Q:3



ReCall:



$$q_i - q_o = \frac{dV}{dt} = A\frac{dh}{dt} \rightarrow SAH$$

$$q_o = h/R \rightarrow H/R$$

$$Q_{in} - Q_x = sA_1H_1 \,,\quad Q_x = \frac{H_1 - H_2}{R_1}$$ ← كان بيعتمد عليهم!

$$Q_x - Q_o = sA_2H_2 \,,\quad Q_o = \frac{H_2}{R_2}$$

* منعوض ال H لتطلع ال TF

$$H_2 = Q_oR_2 \,,\quad H_1 = Q_xR_1 + Q_oR_2$$

$$Q_x = Q_o + sA_2H_2 = Q_o + sA_2Q_oR_2 = Q_o(1 + sA_2R_2)$$

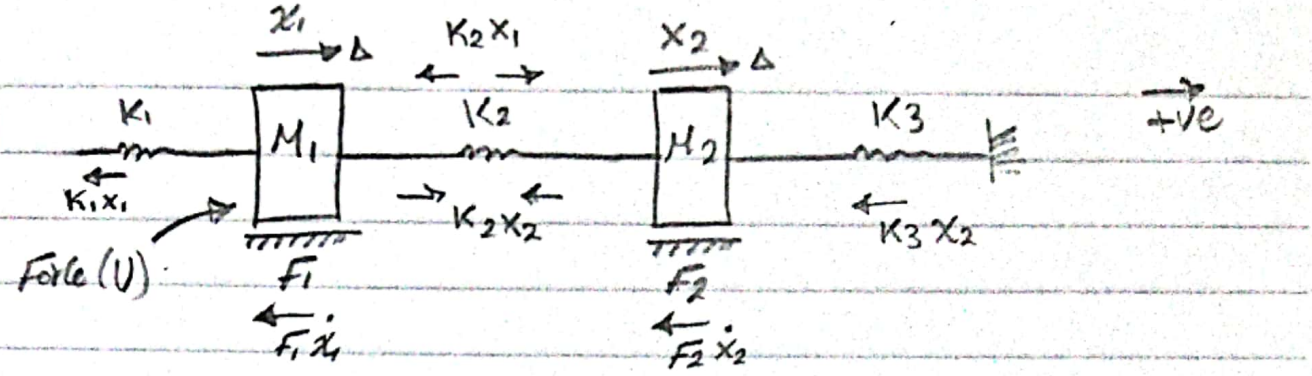
$$Q_{in} = Q_x + sA_1H_1 = Q_o(1 + sA_2R_2) + sA_1[Q_oR_1(1 + sA_2R_2) + Q_oR_2]$$

$$Q_{in} = Q_o[(1 + sA_2R_2) + sA_1R_1 + s^2A_1A_2R_1R_2 + sA_1R_2]$$

$$TF = \frac{Q_o}{Q_{in}} = \frac{1}{s^2R_1R_2A_1A_2 + s(A_1R_1 + A_1R_2 + A_2R_2) + 1}$$

7

Q:4



$$M_1\ddot{x}_1 = \Sigma F)_{m_1} = U - K_1x_1 - F_1\dot{x}_1 - K_2x_1 + K_2x_2$$

$$M_2\ddot{x}_2 = \Sigma F)_{m_2} = K_2x_1 - K_2x_2 - F_2\dot{x}_2 - K_3x_2$$

S-Domain

$$M_1s^2x_1 = U - K_1x_1 - F_1sx_1 - K_2x_1 + K_2x_2 \quad (1)$$

Assume Zero-initial Condition

$$M_2s^2x_2 = K_2x_1 - K_2x_2 - F_2sx_2 - K_3x_2 \quad (2)$$

$$x_2 = \frac{K_2x_1}{M_2s^2 + F_2s + K_2 + K_3} \quad (3) \qquad \text{From (2)}$$

$$\frac{x_1}{U} = \frac{M_2s^2 + F_2s + K_2 + K_3}{(M_1s^2 + K_1 + K_2 + F_1s)(M_2s^2 + F_2s + K_2 + K_3) - K_2^2}$$

$$\frac{x_2}{U} = \frac{x_1}{U} * \frac{x_2}{x_1} = [\quad] * \frac{K_2}{\square} = \frac{K_2}{\square}$$

If $u(t) = 1 \rightarrow U(s) = 1/s$

Get the final value at $t = \infty$, $s \Rightarrow 0$

$$x_1 = \lim_{s\to 0} sx_1 = \lim_{s\to 0} s\,U(s)\,TF = \frac{K_2 + K_3}{(K_1+K_2)(K_2+K_3) - K_2^2}$$

$$x_2 = \lim_{s\to 0} s * 1/s * TF = \frac{K_2}{(K_1+K_2)(K_2+K_3) - K_2^2}$$