Cairo University Faculty of Engineering		3rd Year Comp. MTH3251- Fall 2022 Number theory - Sheet 2

(1) For any integer a, show the following:
   (a) $\gcd(2a+1, 9a+4) = 1$.
   (b) $\gcd(5a+2, 7a+3) = 1$.
   (c) If a is odd, then $\gcd(3a, 3a+2) = 1$.

(2) If a and b are integers, not both of which are zero, prove that $\gcd(2a - 3b, 4a - 5b)$ divides b; hence, $\gcd(2a+3, 4a+5) = 1$.

(3) Find gcd(143, 227), gcd(306, 657), and gcd(272, 1479).

(4) Use the Euclidean Algorithm to obtain integers x and y satisfying the following:
   (a) $\gcd(56, 72) = 56x + 72y$.
   (b) $\gcd(24, 138) = 24x + 138y$.
   (c) $\gcd(119, 272) = 119x + 272y$.

(5) Find lcm(143, 227), lcm(306, 657), and lcm(272, 1479).

(6) Find integers x, y, z satisfying
   $\gcd(198, 288, 512) = 198x + 288y + 512z$
   [Hint: Put $d = \gcd(198, 288)$. Because $\gcd(198, 288, 512) = \gcd(d, 512)$, first find integers u and v for which $\gcd(d, 512) = du + 512v$.]

(7) Which of the following Diophantine equations cannot be solved?
   (a) $6x + 51y = 22$. X
   (b) $33x + 14y = 115$. ✓
   (c) $14x + 35y = 93$. X

(8) Determine all solutions in the integers of the following Diophantine equations:
   (a) $56x + 72y = 40$.
   (b) $24x + 138y = 18$.
   (c) $221x + 35y = 11$.

(9) Determine all solutions in the positive integers of the following Diophantine equations:
   (a) $18x + 5y = 48$.
   (b) $54x + 21y = 906$.
   (c) $123x + 360y = 99$.
   (d) $158x - 57y = 7$.

(10) A man has \$4.55 in change composed entirely of dimes and quarters. What are the maximum and minimum number of coins that he can have? Is it possible for the number of dimes to equal the number of quarters?

(11) The neighborhood theater charges \$1.80 for adult admissions and \$.75 for children. On a particular evening the total receipts were \$90. Assuming that more adults than children were present, how many people attended?

(12) A certain number of sixes and nines is added to give a sum of 126; if the number sixes and nines is interchanged, the new sum is 114. How many of each were there originally?

(13) Using the Extended Euclid’s theorem, find the smallest positive value of x that satisfies:

(a) $5x \equiv 6 \pmod{8}$.

(b) $5x \equiv 4 \pmod{6}$.

(c) $3x - 2 \equiv 0 \pmod{11}$.

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