CMP362/CMPN446: Image Processing and Computer Vision

Morphological Image Processing

Mayada Hadhoud
Computer Engineering Department
Cairo University

Agenda

Mathematical Morphology

Basic Set Theory

Structuring Elements

Basic morphological operations

- Erosion
- Dilation
- Opening
- Closing

Basic Morphological Algorithms

- Boundary Extraction
- Region Filling

Gray-Scale Morphological Image Processing

Reference

(Digital Image Processing – Gonzalez/Woods- Chapter 9)

Introduction

- Morphological operations are used as a step in image processing and analysis.
- It is used to **modify the shape of objects in an image**, by using local operations.
- It can be **used to remove unwanted effects** in segmentation post-processing
  - Remove small objects (that is assumed to be noise)
  - Smooth the edges of larger objects
  - Fill holes in objects
  - Link objects together
- It can be **used as a part of object description and analysis**
  - Locate the boundary of objects
  - Thin objects
  - Locating objects with a certain structure
  - Locating patterns in an image
- The operations are small, and often very fast.

Mathematical Morphology

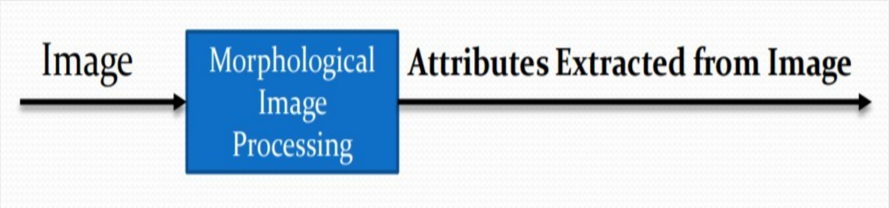
- Used to **extract image components** that are useful in the **representation and description of region shape**
- The applications of Morphological Image Processing are:
  - boundaries extraction
  - Region filling
  - skeletons
  - morphological filtering
  - thinning

Mathematical Morphology

- The language of Mathematical Morphology is **<u>Set Theory</u>**
- **Sets** in Mathematical Morphology represent **objects** in images:
  - **Binary Image**

    the element of the set is the coordinates (x,y) of pixel belong to the object Z_2
  - **Gray-scale Image**

    the element of the set is the coordinates (x,y) of pixel belong to the object and the gray levels Z_3



Basic Set Theory

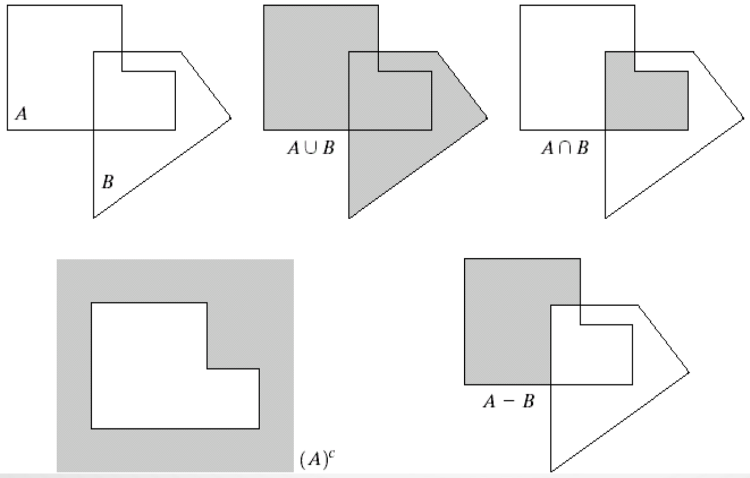


FIGURE 9.1 (a) Two sets A and B. (b) The union of A and B. (c) The intersection of A and B. (d) The complement of A. (e) The difference between A and B.

$$A^c \equiv \{ w \mid w \notin A \}$$

$$A - B \equiv \{ w \mid w \in A, w \notin B \} = A \cap B^c$$

Structuring element (SE)

Structuring Elements(SE): small sets or subimages used to probe an image under study for properties of interest.

- used to **determine the acting range of the operations.**
- The matrix dimensions specify the **size of the structuring element.**
- The pattern of ones and zeros specifies the s**hape of the structuring element.**
- for each SE, define origin(**black dot**). When SE is symmetric and no dot is show the origin is at the center of symmetry.

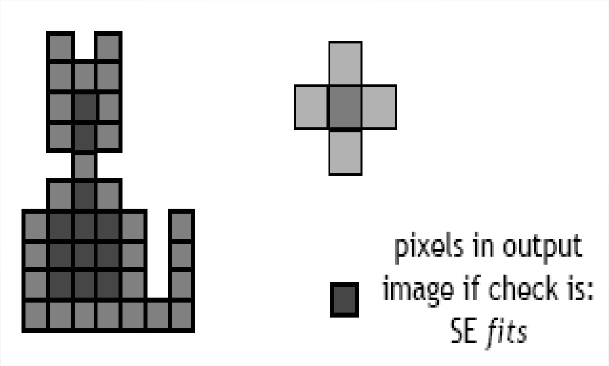
It is typically defined as a binary matrix where pixels valued 0 are not acting, and pixels valued 1 are acting.

When hovering the structuring element over an image, we have three possible scenario for the structuring element (or really the location of the 1’s in the structuring element):

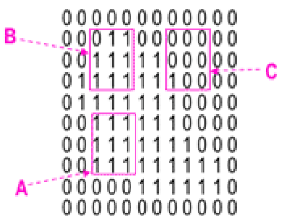
- It is not overlapping the image foreground (a miss).
- It is partly overlapping the image foreground (a hit).
- It is fully overlapping the image foreground (it fits).

Basic idea

- in parallel for each pixel in binary image:
  - check if SE is ”satisfied”
  - output pixel is set to 0 or 1 **depending on used operation**



The structuring element is said to fit the image if, for each of its pixels set to 1, the corresponding image pixel is also 1. Similarly, a structuring element is said to hit, an image if, at least for one of its pixels set to 1 the corresponding image pixel is also 1.



$$s_1 = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \qquad s_2 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

		A	B	C
fit	s_1	yes	no	no
	s_2	yes	yes	no
hit	s_1	yes	yes	yes
	s_2	yes	yes	no

Zero-valued pixels of the structuring element are ignored

Basic morphological operations

- **Erosion** (Erosion: And the image with the SE)

  - removal of structures of certain shape and size, given by SE

- **Dilation** (Dilation = OR the image with the SE)

  - filling of holes of certain shape and size, given by SE

Erosion

- is used for **shrinking** of element A by using element B
- Does the structuring element **fit the set?**
- Erosion for Sets A and B in Z_2 , is defined by the following equation:

$$A \ominus B = \{z \mid (B)_z \subseteq A\}$$

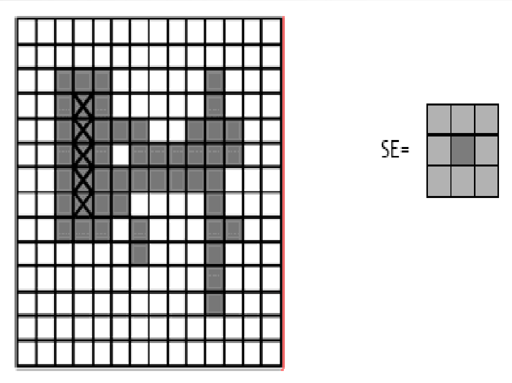
$$A \ominus B = \{z \mid (B)_z \cap A^c = \emptyset\}$$

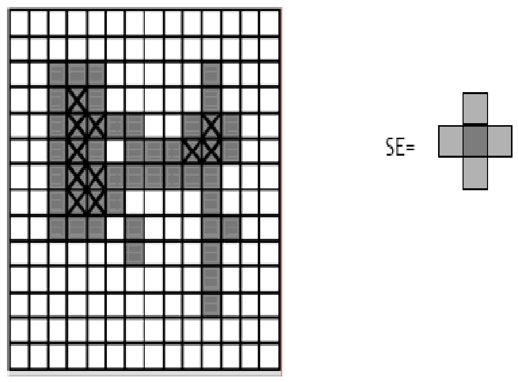
- This equation indicates that the erosion of A by B is the set of all points z such that B, translated by z, is contained in A.

Erosion

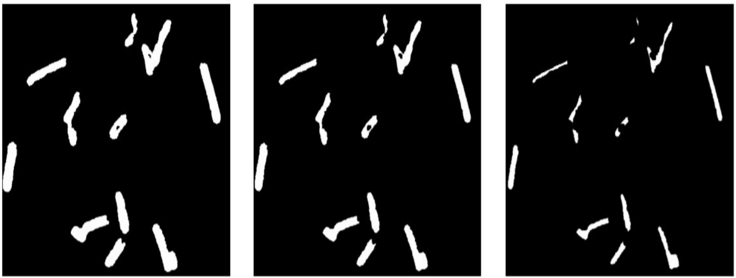
- Erosion can be used to:
  - **Shrinks** or **thins** objects in binary images
  - **Remove image components**
    - Erosion is a **morphological filtering operation** in which image details smaller than the structuring elements are filtered(removed)

Erosion





Example



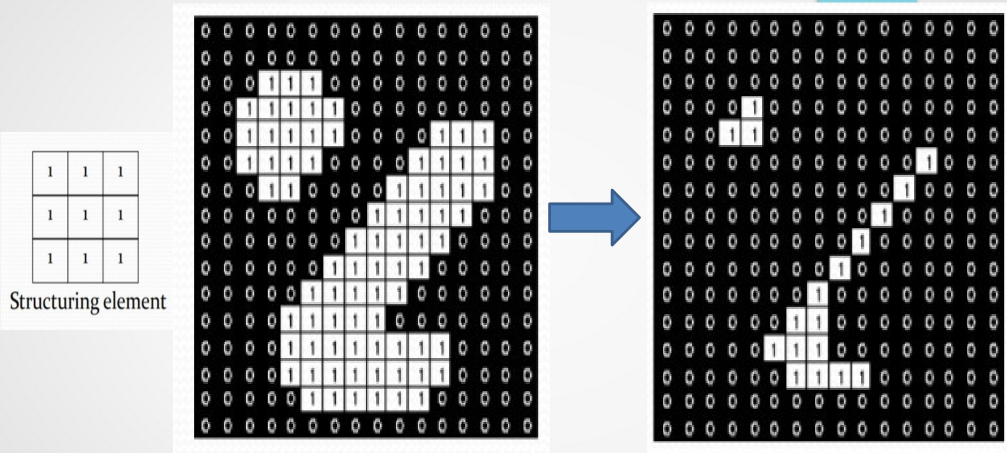
Original (701x781)

erosion with
3x3 structuring element

erosion with
7x7 structuring element

- Shrinks the size of 1-valued objects
- Smoothes object boundaries
- Removes peninsulas, fingers, and small objects

Erosion: Example



Structuring element

The structuring element is now superimposed over each foreground pixel (input pixel) in the image. **If all the pixels below the structuring element are foreground pixels then the input pixel retains it's value.** But if any of the pixels is a background pixel then the input pixel gets the background pixel value.

Dilation

- is used for **expanding** an element A by using structuring element B
- Dilation of A by B and is defined by the following equation:

$$A \oplus B = \left\{ z \,\middle|\, (\hat{B})_z \cap A \neq \emptyset \right\}$$

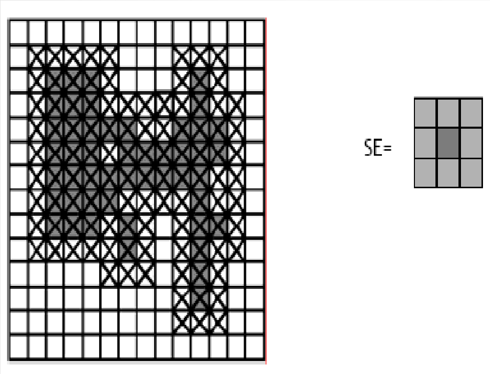
- **Does the structuring element hit the set?**
- The dilation of A by B is the set of all displacements z, such that and A overlap by at least one element.

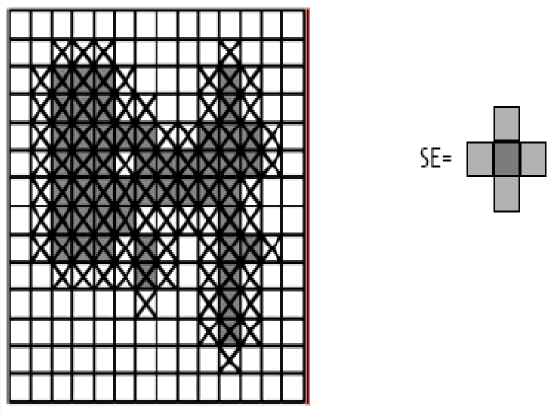
This equation is based 0n obtaining the reflection of B about its origin and shifting this reflection by z.

Dilation

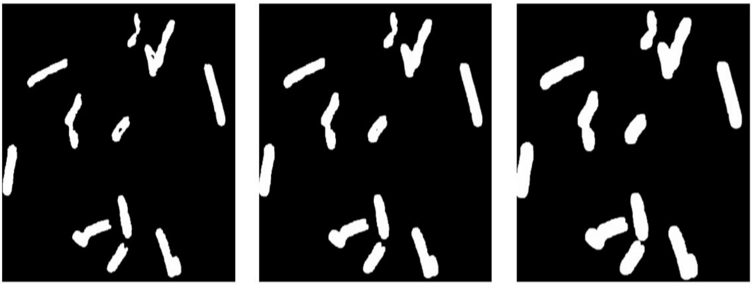
- Dilation can be used to:
  - **Grows** or **thickens** object in a binary image
  - Bridging gaps
  - Fill small holes of sufficiently small size

Dilation





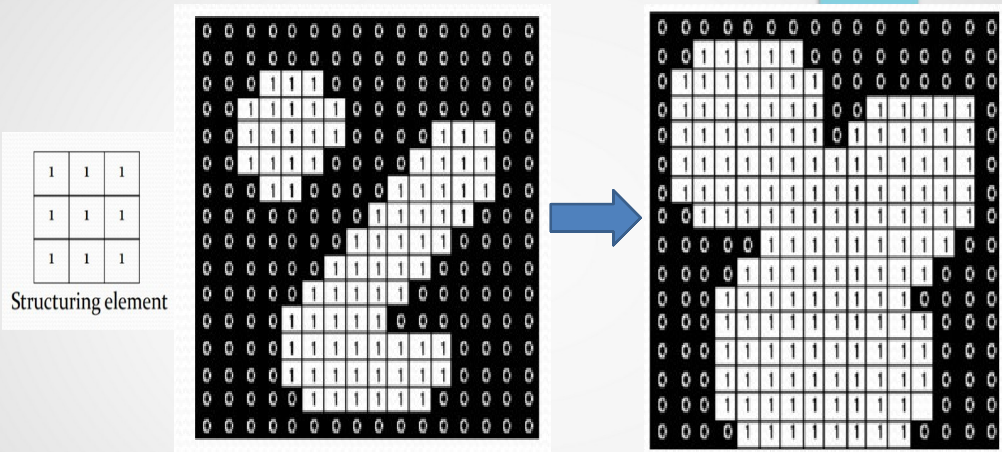
Example



Original (701x781) | dilation with 3x3 structuring element | dilation with 7x7 structuring element

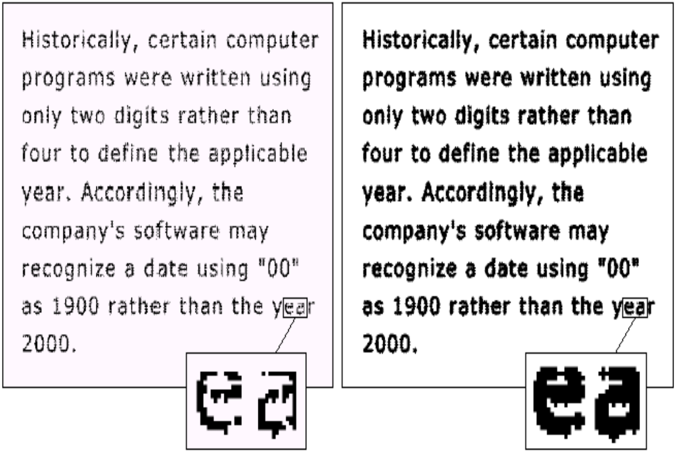
- Expands the size of 1-valued objects
- Smoothes object boundaries
- Closes holes and gaps

Dilation: Example



To compute the dilation of a binary input image by this structuring element, we superimpose the structuring element on top of the input image so that the origin of the structuring element coincides with the input pixel position.

Dilation : Bridging gaps

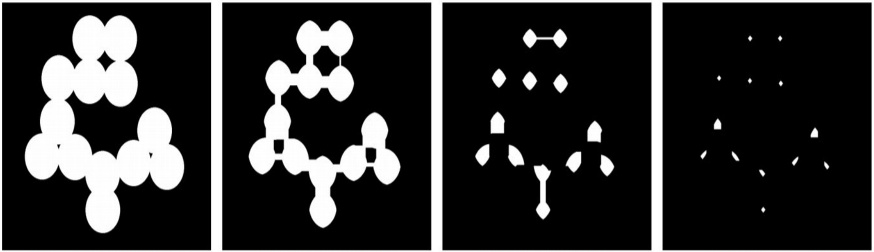


0	1	0
1	1	1
0	1	0

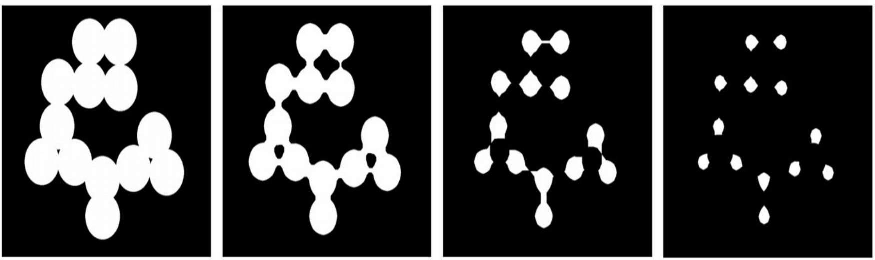
a c
b

FIGURE 9.5
(a) Sample text of poor resolution with broken characters (magnified view).
(b) Structuring element.
(c) Dilation of (a) by (b). Broken segments were joined.

Examples : Blob Detection/ Separation(Erosion)



Original binary image *Circles* (792x892) | Erosion by 30x30 structuring element | Erosion by 70x70 structuring element | Erosion by 96x96 structuring element



Original binary image *Circles* (792x892) | Erosion by disk-shaped structuring element Diameter=15 | Erosion by disk-shaped structuring element Diameter=35 | Erosion by disk-shaped structuring element Diameter=48

Combining Erosion and Dilation

- **WANTED:**
  - remove structures / fill holes
  - without affecting remaining parts
- **SOLUTION:**
  - combine erosion and dilation (using same SE)

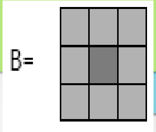
Opening

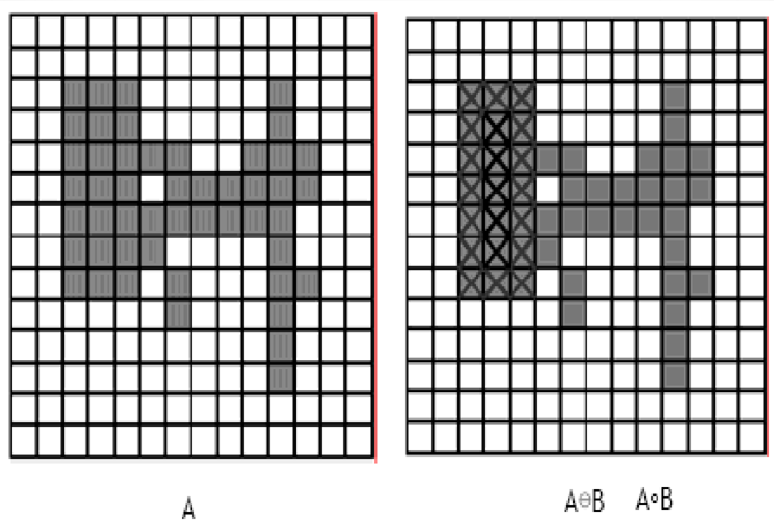
Erosion followed by **Dilation** with the same SE,

- Eliminates protrusions
- Smooths contour

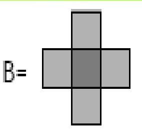
$$A \circ B = (A \ominus B) \oplus B$$

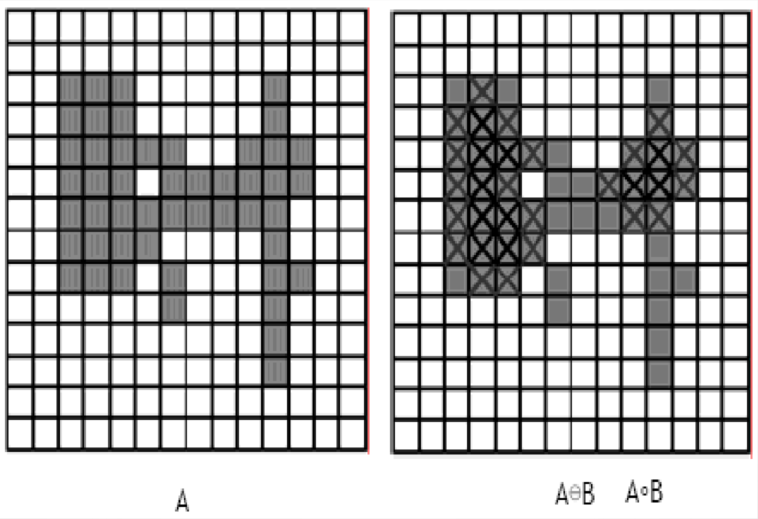
Opening





Opening





Opening : eliminating irrelevant detail

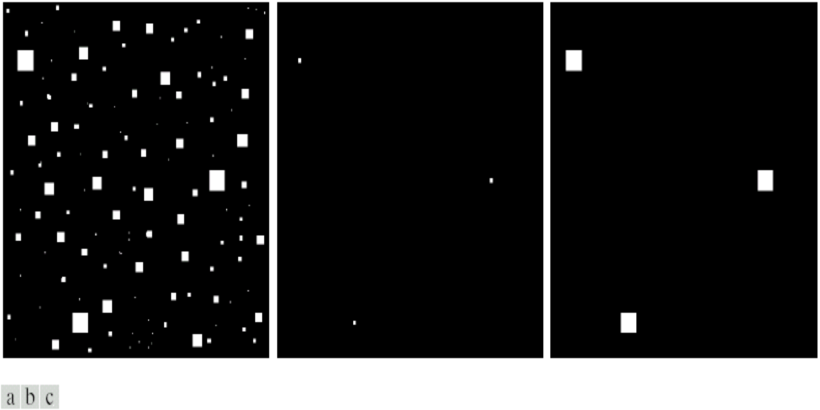


FIGURE 9.7 (a) Image of squares of size 1, 3, 5, 7, 9, and 15 pixels on the side. (b) Erosion of (a) with a square structuring element of 1's, 13 pixels on the side. (c) Dilation of (b) with the same structuring element.

structuring element B = 13x13 pixels of gray level 1

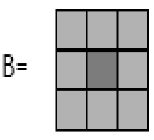
Closing

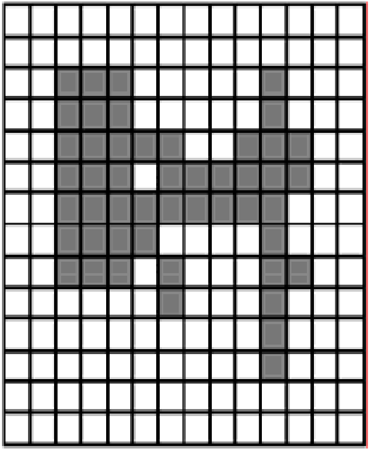
Dilation followed by **Erosion**

- smooth contour
- fuse narrow breaks and long thin gulfs
- eliminate small holes
- fill gaps in the contour

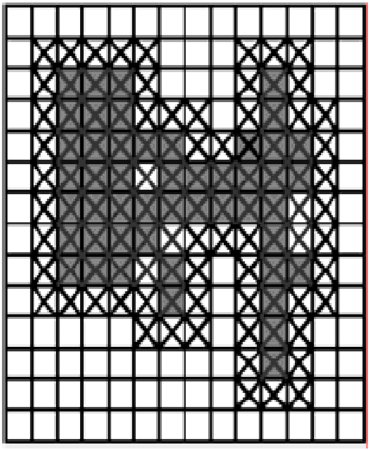
$$A \cdot B = (A \oplus B) \ominus B$$

Closing



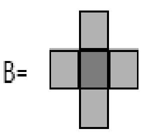


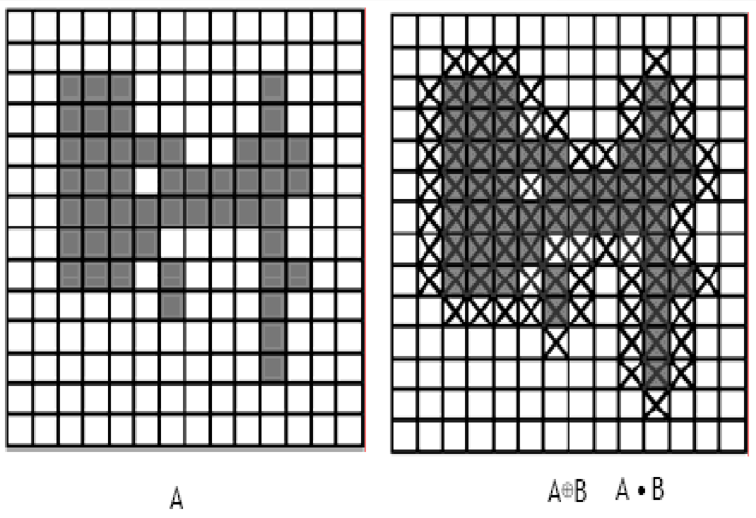
A



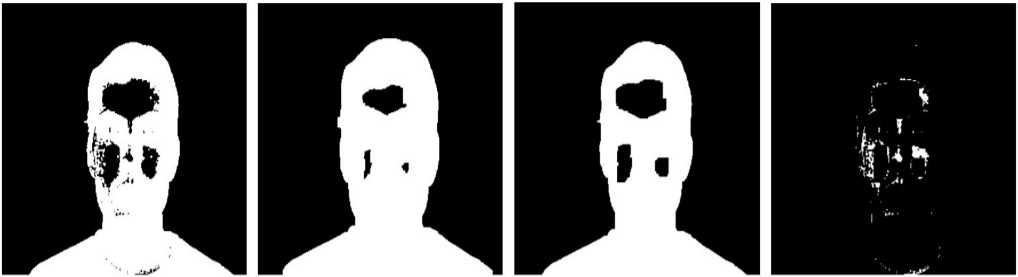
$A \oplus B$ $A \bullet B$

Closing



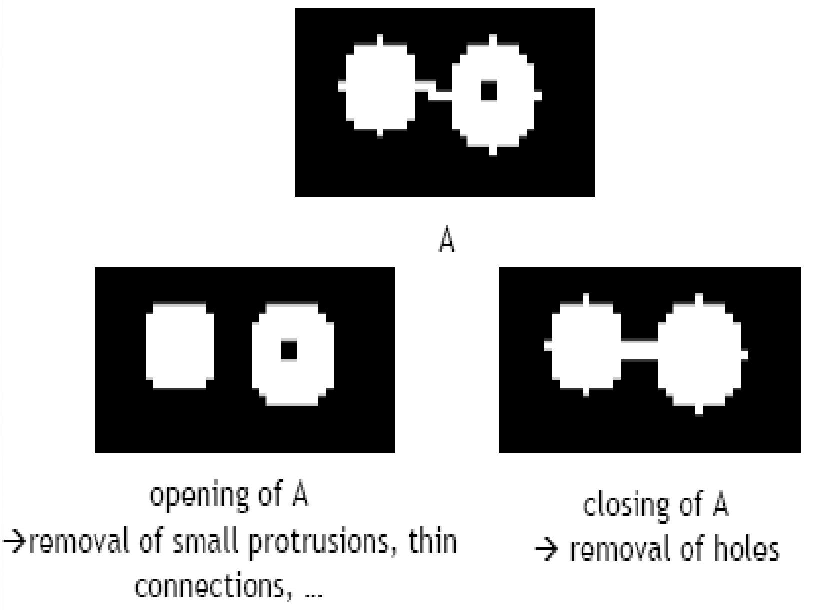


Example: small hole removal using Closing



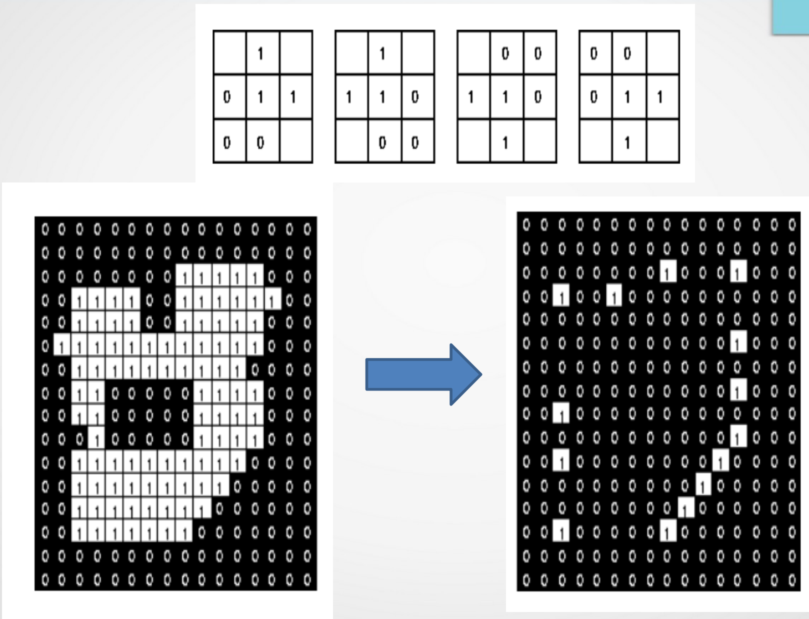
Original binary mask | Dilation 10x10 | Closing 10x10 | Difference to original mask

Opening & Closing



Application: Morphological Filtering

Application: Corner Detection



The hit-and-miss transform is used to look for occurrences of particular binary patterns in fixed orientations. It can be used to look for several patterns (or alternatively, for the same pattern in several orientations as above) simply by running successive transforms using different structuring elements, and then ORing the results together.

Basic Morphological Algorithms

- Boundary Extraction
- Region Filling
- **Thinning** (Assignment)
- **Skeletons** (Assignment)

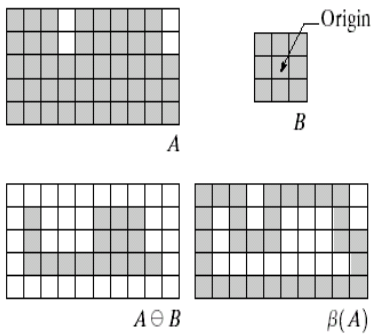
Boundary Extraction

- First **erode A by B**, then **make set difference between A and the erosion**
- The thickness of the contour depends on the size of constructing object – B

$$\beta(A) = A - (A \ominus B)$$

a b
c d

FIGURE 9.13 (a) Set A. (b) Structuring element B. (c) A eroded by B. (d) Boundary, given by the set difference between A and its erosion.



Example

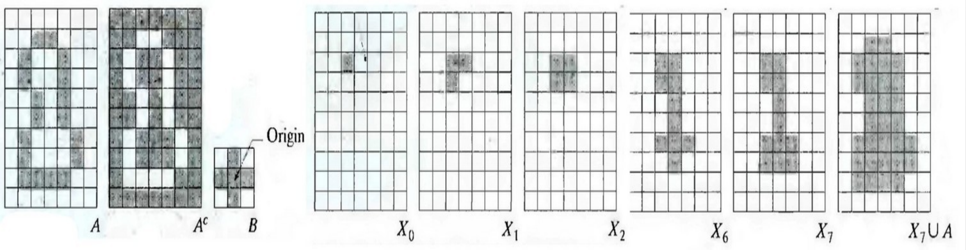
FIGURE 9.14 (a) A simple binary image, with 1's represented in white. (b) Result of using Eq. (9.5-1) with the structuring element in Fig. 9.13(b).

Region Filling

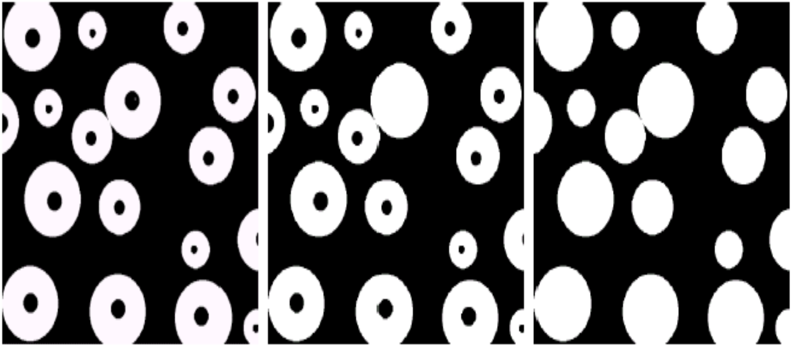
- This algorithm is based on a set of **dilations**, **complementation** and **intersections**
- p is the point inside the boundary, with the value of 1
- $X_{(k)}$ = ($X_{(k-1)}$ dilate B) conjunction with complemented A

$$X_k = (X_{k-1} \oplus B) \cap A^c \qquad k = 1, 2, 3, \ldots$$

- The process stops when $X_{(k)} = X_{(k-1)}$
- The result that given by union of A and $X_{(k)}$, is a set contains the filled set and the boundary



Example



a b c

FIGURE 9.16 (a) Binary image (the white dot inside one of the regions is the starting point for the region-filling algorithm). (b) Result of filling that region (c) Result of filling all regions.

Gray-Scale Morphological Image Processing

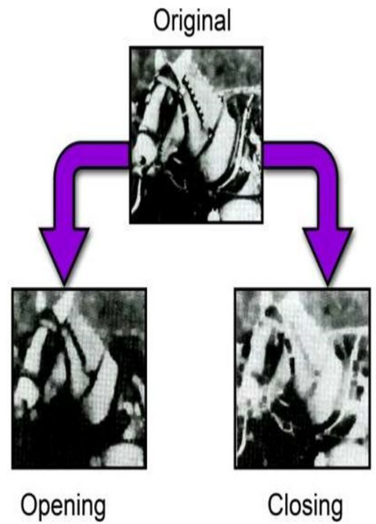
- **Erosion – Gray-Scale**
  - The erosion of f by a flat structuring element b at any location (x,y) is defined as **the minimum value of the image in the region coincides with b when the origin of b is at (x,y).**
- General effect of performing an erosion in grayscale images:
  - The output image tends to be **darker** than the input image.
  - The effect of **bright details** in the input image that are smaller in area than the structuring element is **reduced**, with the degree of reduction being determined by the grayscale values surrounding by the bright detail and by shape and amplitude values of the structuring element itself.

Gray-Scale Morphological Image Processing

- **Dilation– Gray-Scale**
  - The Dilation of f by a flat structuring element b at any location (x,y) is defined as **the maximum value of the image in the region outlined by b when the origin of b is at (x,y).**
- General effect of performing dilation on grayscale images:
  - The output image tends to be **brighter** than the input image.
  - **Dark details** either are **reduced** or **eliminated**, depending on how their values and shape relate to the structuring element used for dilation

Gray-Scale Morphological Image Processing

- **Opening & Closing– Gray-Scale**
  - In the **opening** of a gray-scale image, we **remove small light details**, while relatively undisturbed overall gray levels and larger bright features
  - In the **closing** of a gray-scale image, we **remove small dark details**, while relatively undisturbed overall gray levels and larger dark features



Opening: Decreased size of small bright details. No changes to dark region

Closing: Decreased size of small dark details. No changes to bright region