4.5

Question 1: [5 points]

For the polygon shown in Figure 1. [A (2,2) , B (8,6) , C (8,12) , D (2, 9) , E (5, 6)]

AB: (2, 2) (3.5, 3) (5, 4) (6.5, 5) (8, 6)

AE: (2, 2) (2.75, 3) (3.5, 4) (4.25, 5) (5, 6)

ED: (5, 6) (4, 7) (3, 8) (2, 9)

DC: (2, 9) (4, 10) (6, 11) (8, 12)

$y_{min} < y$

$y_{max} < y$

A. Perform a proper edge shortening

B. Compute the corresponding edge table including edge structure

C. List the active edge table at Y = 8 and Y = 4

AB $\frac{dx}{dy} = \frac{6}{4} = 1.5$

AB → $\frac{dx}{dy} = 1.5$

AE → $\frac{dx}{dy} = \frac{3}{4}$

ED → $\frac{dx}{dy} = \frac{2-5}{9-6} = \frac{-3}{3} = -1$

DC → $\frac{dx}{dy} = \frac{8-2}{12-9} = \frac{6}{3} = 2$

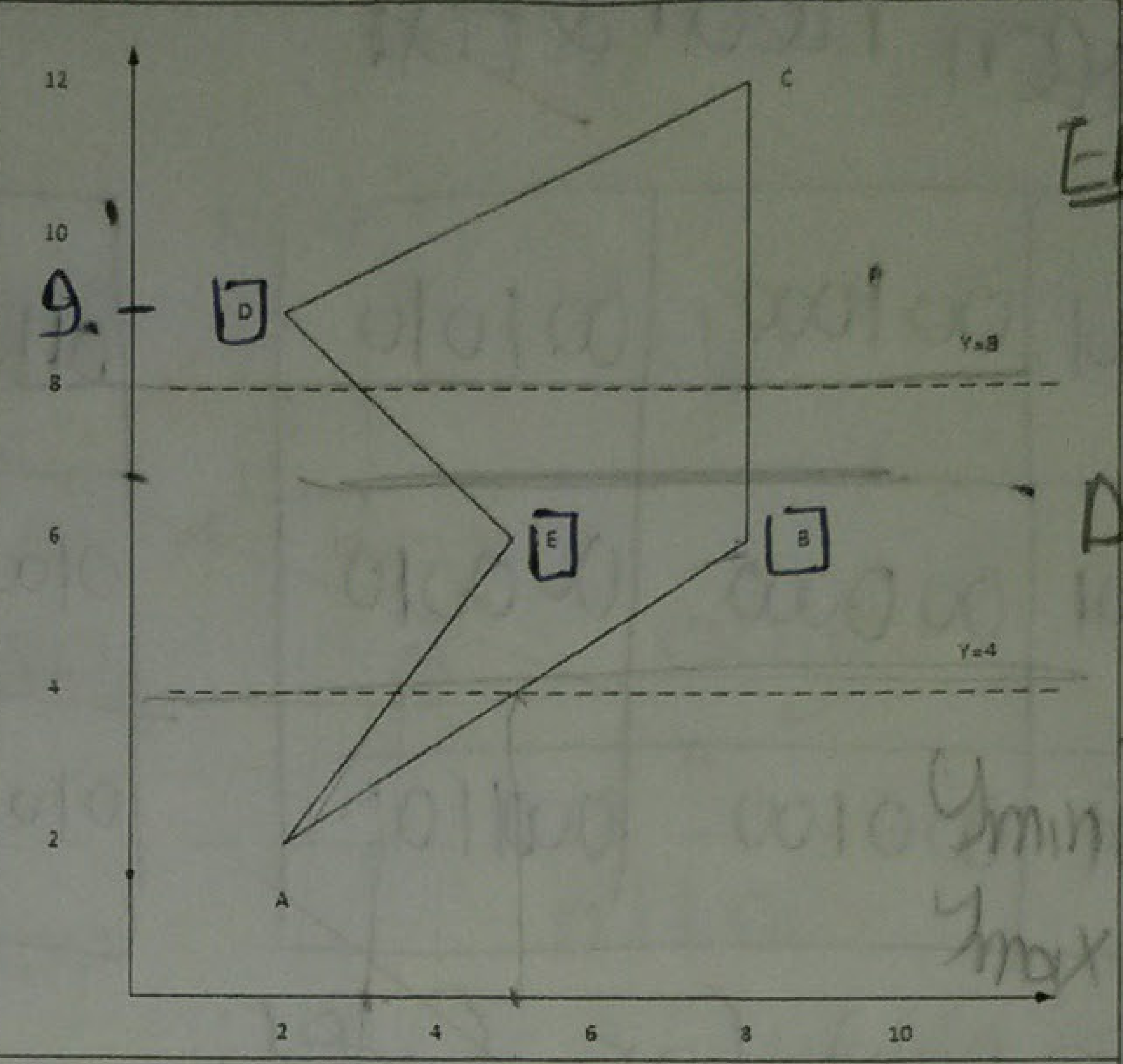


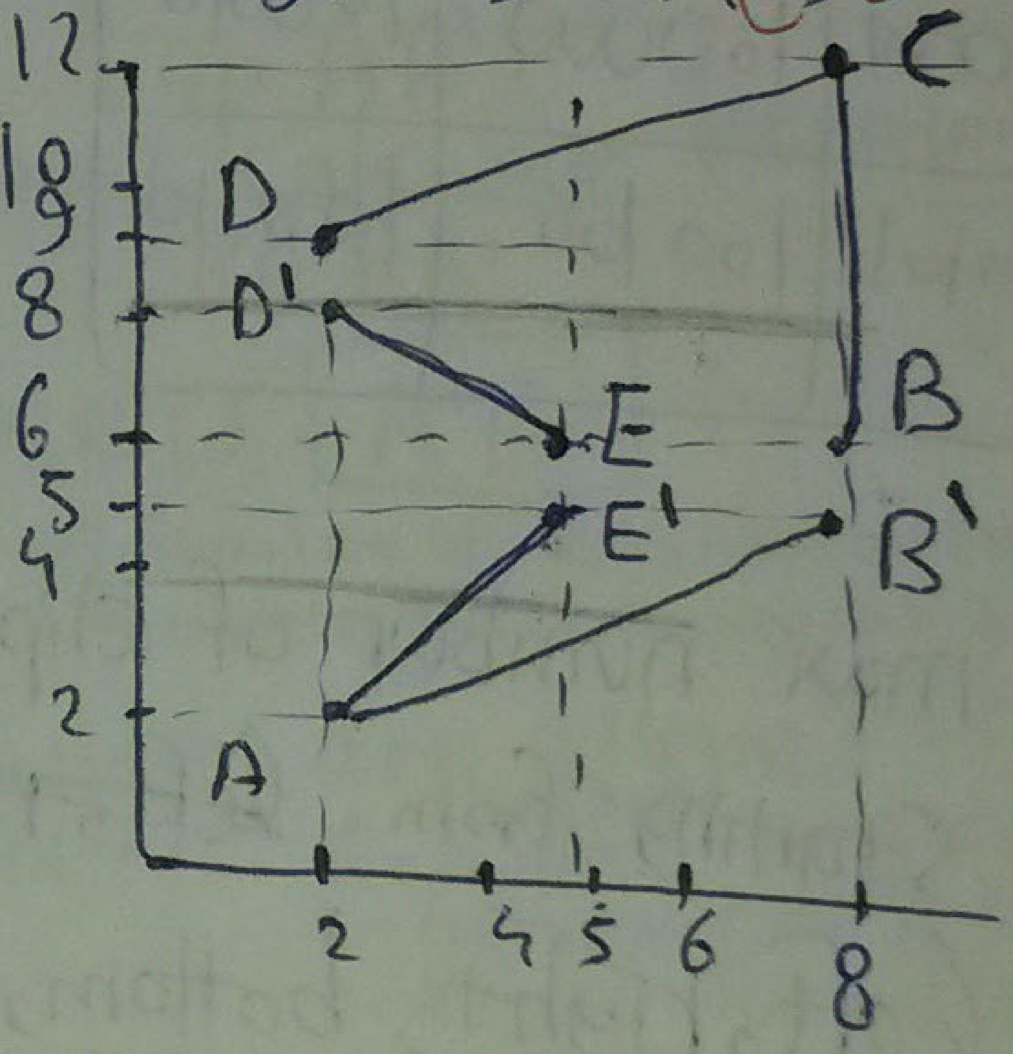
Figure 1

A) Edge Shortening at D: D' for lower Edge at (2, 8) −0.5

E' for lower Edge AE' at (5, 5)

B' for lower Edge AB' at (8, 5)

B) Edge Table:



Ymin	
10	
9	DC: [12 | 2 | 2 | /]
8	
7	
6	ED': [8 | 5 | −1 | •] → BC: [12 | 8 | 0 | /]
5	
4	
3	
2	AE': [5 | 2 | 3/4 | •] → AB': [5 | 2 | 3/2 | /]
1	
0	

Ymax, X at Ymin

⇒ for edges AE', AB' y > ymax so they're dropped

⇒ for edge DC ymin > y so not added yet

AET at y = 8: ED' [8 | 3 | −1 | •] — BC [12 | 8 | 0 | /]

AET at y = 4: AE' [5 | 3½ | 3/4 | •] — AB' [5 | 5 | 3/2 | /]

Question 2: [5 points]

2 A. Using Cohen-Sutherland line clipping algorithm, show the steps and the final results of clipping the line shown in Figure 2.

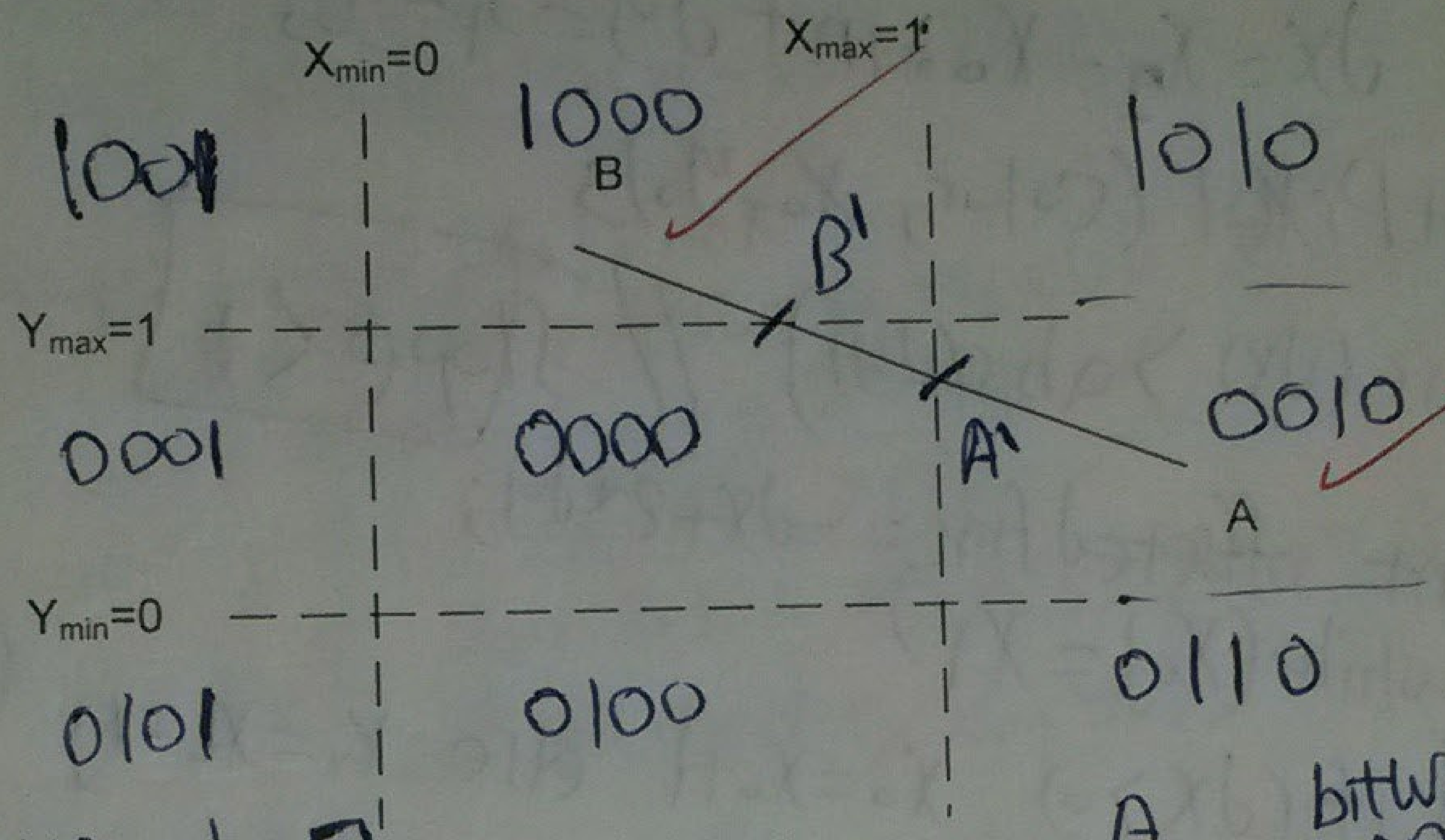


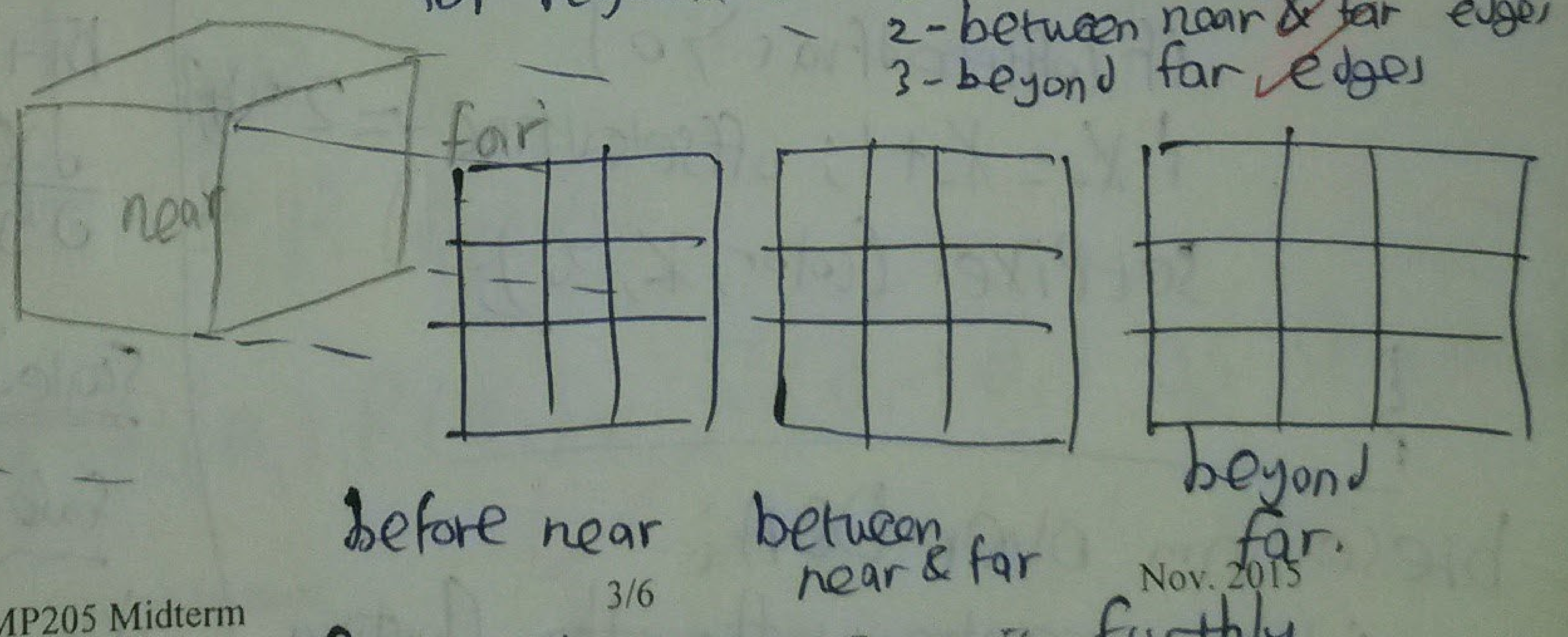
Figure 2

Each region assigned a code
#define left-edge 0x1
#define right-edge 0x2
#define bottom-edge 0x4
#define top-edge 0x8
if(X<Xmin) code|= left-edge
if(X>Xmax) code|= right-edge
if(y<ymin) code|= bottom-edge
if(y>ymax) code|= top-edge

[1] trivial acceptance A 0010 bitwise OR B 1000 ≠ 0000 (NO)
[2] trivial reject 0010 bitwise AND 1000 = 0000
[if A & B ≠ 0000 then reject but ⇒ No]
[3] Pick A [0010] Calculate Intersection with right which A' clip AA'.
[4] Pick B [1000] Calculate Intersection with top which B' clip BB'
Final line is $\boxed{A'B'}$

3 B. What needs to be done to extend the 2-D Cohen-Sutherland line clipping algorithm to 3-D? What is the maximum number of times that a 3-D line might be clipped by this algorithm? Be sure to briefly justify your answers.

Diff three codes are assigned (2 bits) for regoins at
1- in front (before) to near edges
2- between near & far edges
3- beyond far edges



تكملة السؤال

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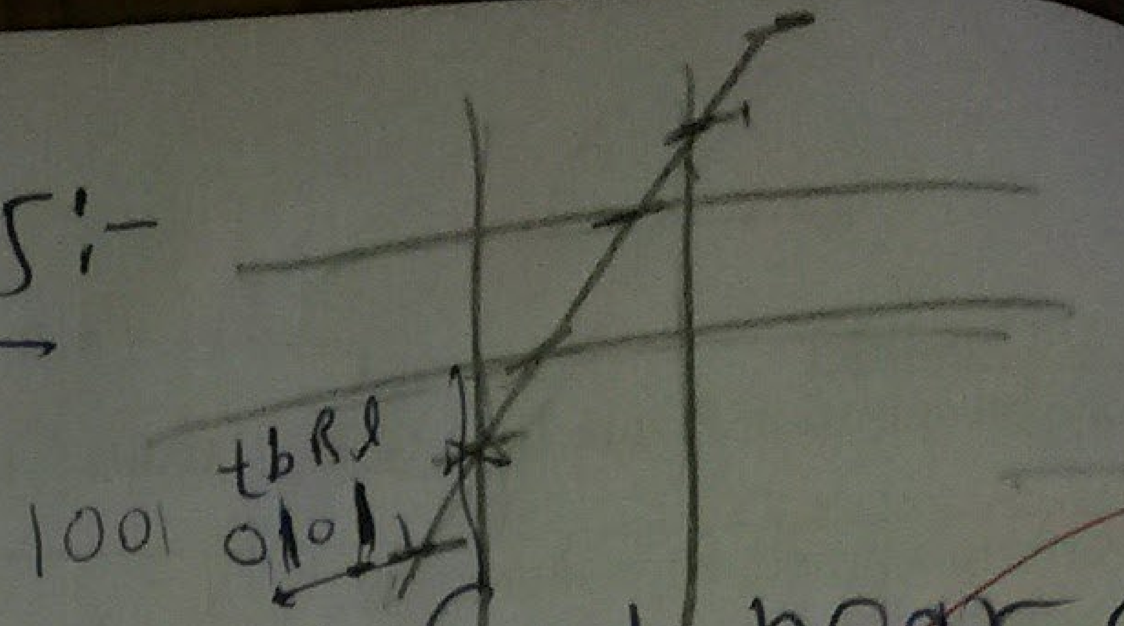
for each region space is furthly subdivided into left-right-bottom-top & combinations of them (8 codes + 0000 for the interior)

Q2 (B) more details:-



between near & ~~far~~ Far

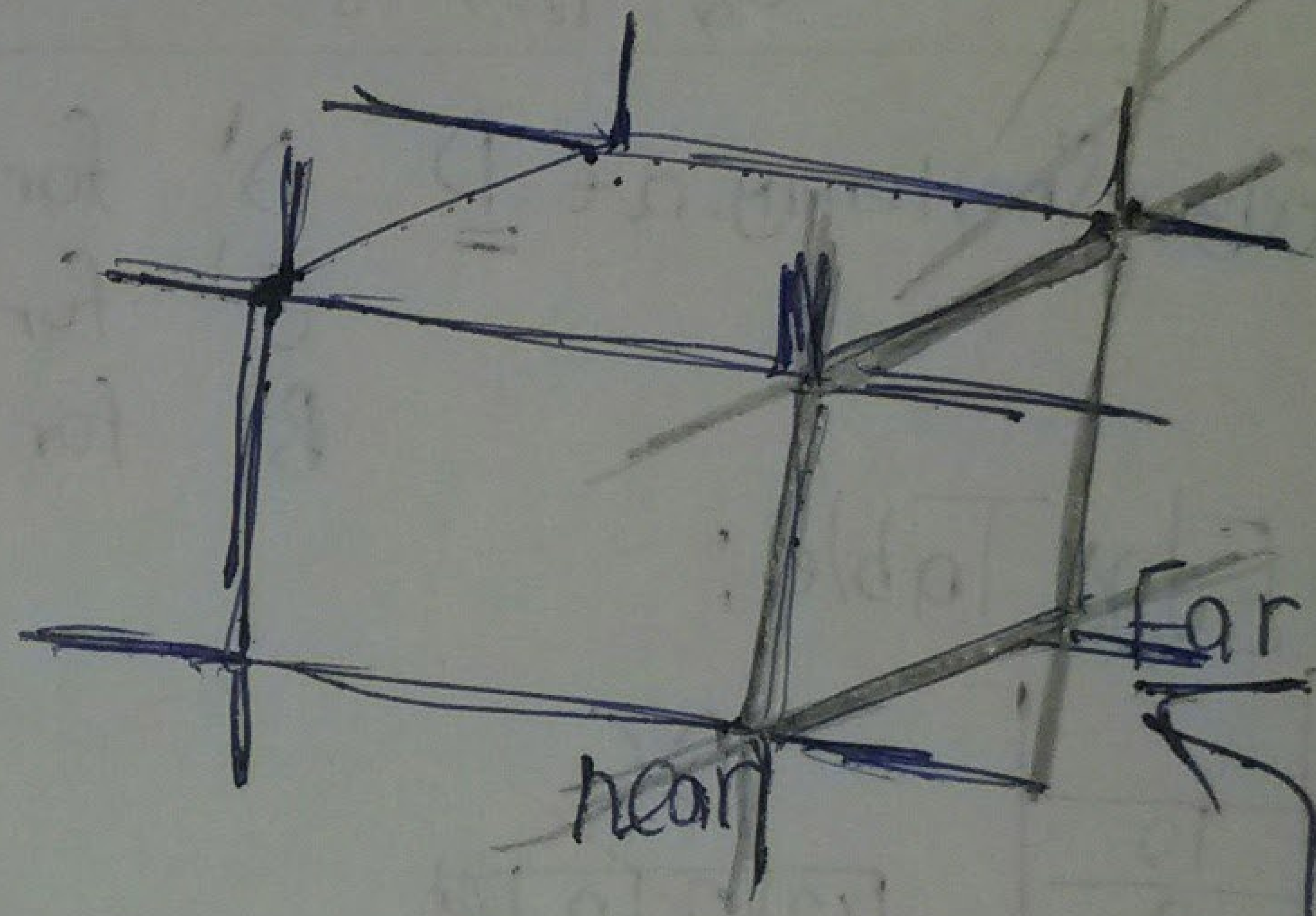
001001	001000	001010
000001	000000	000010
000101	000100	000110

infront near codes:

011001	011000	011010
010001	01000	010010
010101	010100	010110

beyond far Codes:

101001	101000	101010
100001	100000	100010
100101	100100	100110



max number of clipping = 6

Starting from ~~left, right, top, bottom,~~

left, right, bottom, top, infront near, beyond far

for endpoints P_1 P_2

P_1 at max may clipped by

: left, ~~bottom~~ bottom, infront near

P_2 at max may clipped by: right, top, beyond far

Note: P_1 more than this (trivial rejection)

Code P_1 bitwide AND Code P_2 = 000000

here we are between far & near.

Code: 00TBRL

(for $|m|<1$) **Proof:-** $\text{frac} = \frac{1}{2} + m = \frac{1}{2} + \frac{dy}{dx} = \frac{dx + 2dy}{2dx}$

From DDA $y_0 = \text{int}(y_0 + m + \frac{1}{2})$

scaled $= dx + 2dy$

frac ≥ 1
scaled $\geq 2dx$
scaled $-= 2dx$

offseted $= dx + 2dy - 2dx = -dx + 2dy$

Question 3: [5 points]

2.5 A. Write down the step used in Bresenham's line-drawing algorithm. In your answer, explain under which slope condition your algorithm is applicable.

for $|m|<1$

Outside loop: offseted frac = -dX + 2*dy;

Inside loop:
```
while (x0 != x1)
{  if (dX > 0) x0 = x0 + 1; else x0 = x0 - 1;
   offseted frac += 2*dy;
   if (offseted frac > 0)
   {  y0 = y0 + 1; offseted frac -= 2*dX; }
   setPixel (color, x0, y0);
}
```

more details back of page 3

B. Given a 2D rotation transformation matrix R(θ). Is $R(\theta_1)R(\theta_2)=R(\theta_1+\theta_2)$? Prove your answer.

2.5

$$R(\theta_1) = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad R(\theta_2) = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R(\theta_1)R(\theta_2) = \begin{bmatrix} \cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2 & -\sin\theta_2\cos\theta_1 - \sin\theta_1\cos\theta_2 & 0 \\ \sin\theta_1\cos\theta_2 + \cos\theta_1\sin\theta_2 & -\sin\theta_2\sin\theta_1 + \cos\theta_2\cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\theta_1+\theta_2) & -\sin(\theta_1+\theta_2) & 0 \\ \sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= R(\theta_1+\theta_2)$$

Yes $R(\theta_1)R(\theta_2) = R(\theta_1+\theta_2)$ as shown above.

Complete bresenham:-

bresenham (int X_0, int y_0, int X_1, int y_1)

```
int dX = X1 - X0; int dy = y1 - y0;
 setPixel (color, X0, y0);
if( abs (dX) > abs(dy)) // Slope<1
{   int offseted frac = -dX + 2*dy;
    while(X0 != X1)
    { if (dX > 0)  X0 = X0+1 else X0 = X0-1;
      if (offseted frac > 0)
      { y0 = y0+1; offseted frac -= 2*dX; }
      setPixel ( color, X0, y0) }
}
elseif (abs (dX) != 0)        // Slope > 1
{   int offseted frac = -dy + 2*dX;
    while( y0 != y1)
    { if (dy > 0)  y0 = y0+1 else y0 = y0-1;
      if (offseted frac > 0)
      { X0 = X0+1; offseted frac -= 2*dy; }
      setPixel (color, X0, y0);
}
```

bresenham over DDA:

- integer operations rather than floating point.
- comparison with zero

Proof for $|m| > 1$

$X = \bar{m} y + b$

$\bar{m} = \frac{dX}{dy}$

from DDA:

$X_0 = int(X_0 + \bar{m} + \frac{1}{2})$

$\bar{m} + \frac{1}{2}$

$\frac{dX}{dy} + \frac{1}{2}$

scaled frac = $dy + 2*dX$

scaled frac $\geq$ $2*dy$

$dy - 2*dX - 2*dy$

Question 4: [5 points]

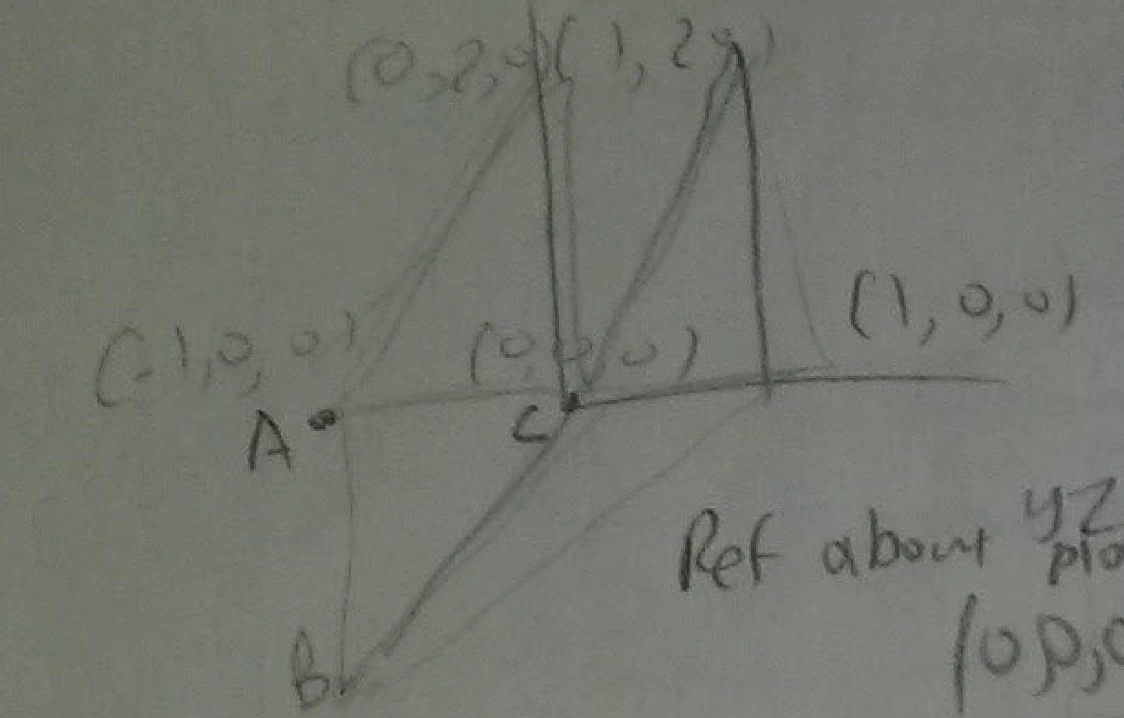
5

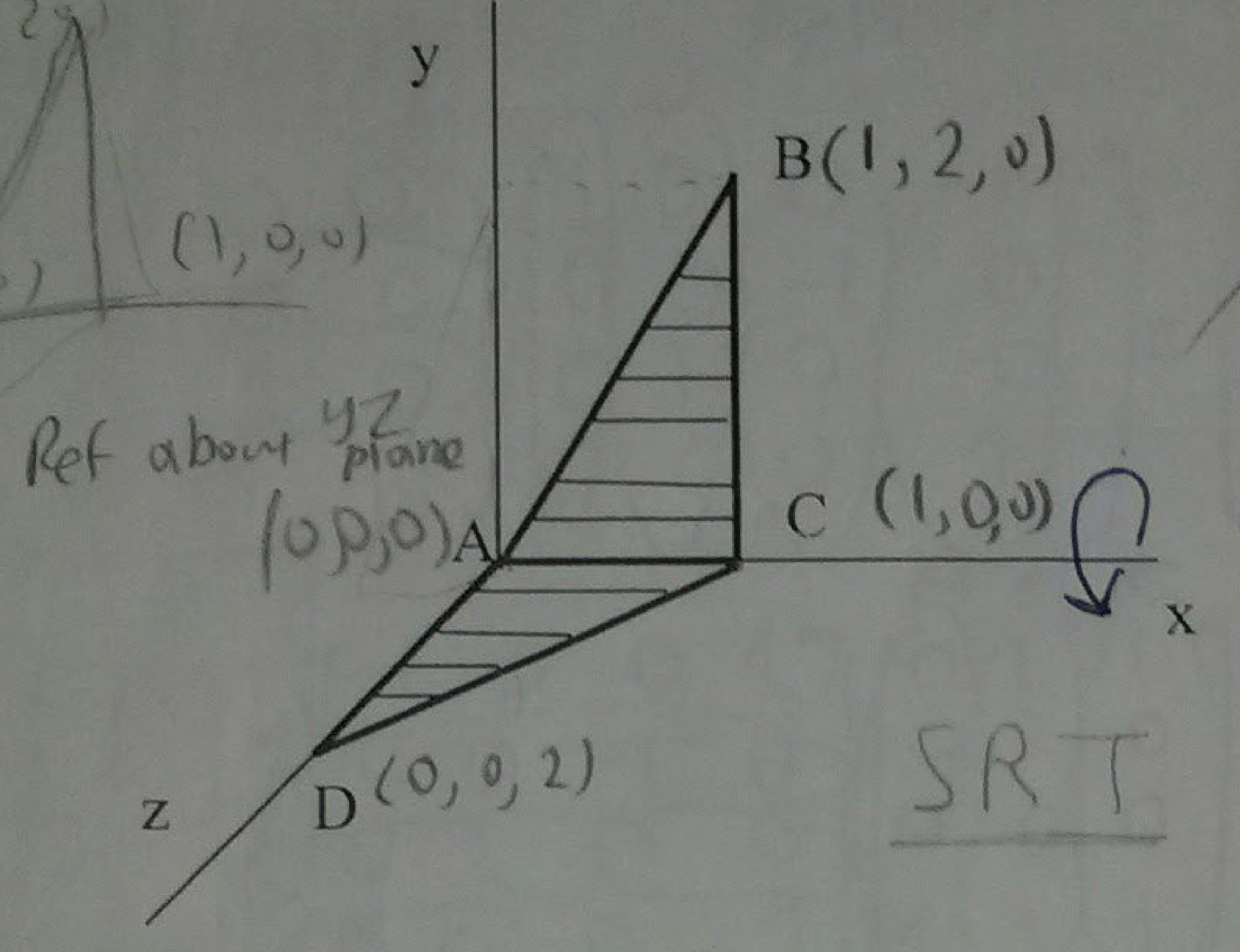
Consider the triangle ABC shown (it lies in the x-y plane). It is required to transform it to the position shown below (to triangle ADC in the x-z plane). The new triangle preserves the same shape and size. The points are

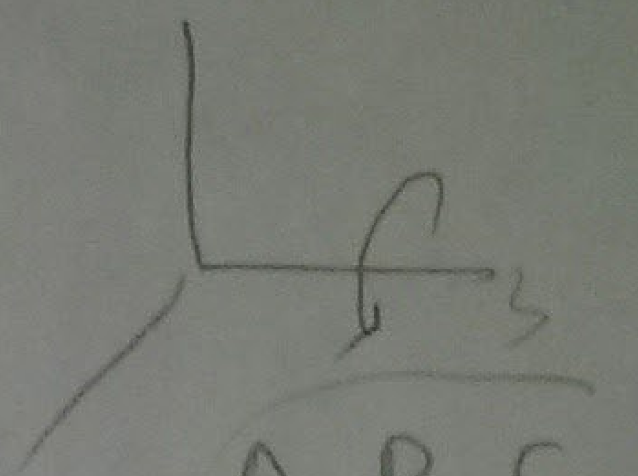
A = (0,0,0), B = (1,2,0), C = (1,0,0), D = (0,0,2)

A. Find the steps of the transformations, including the (four-dimensional) transformation matrices. Find the overall transformation matrix that corresponds to all steps combined.

B. Find the overall inverse transformation.







$$\begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{pmatrix} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{pmatrix} \end{array} \qquad \begin{pmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

$T(-1,0,0)$

$R_x(90^\circ)$

$S(-1,1,1)$

SRT

1^{st} Translate $T(-1,0,0)$ [make C @ origin]

2^{nd} - Rotate by angle $+90^\circ$ around X-axis.

3^{rd} - Scale by $S(-1,1,1)$ [Reflect about YZ plane]

$$T(-1,0,0) = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \quad R_x(90^\circ) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\cos 90^\circ = 0$; $\sin 90^\circ = 1$

$$S(-1,1,1) = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

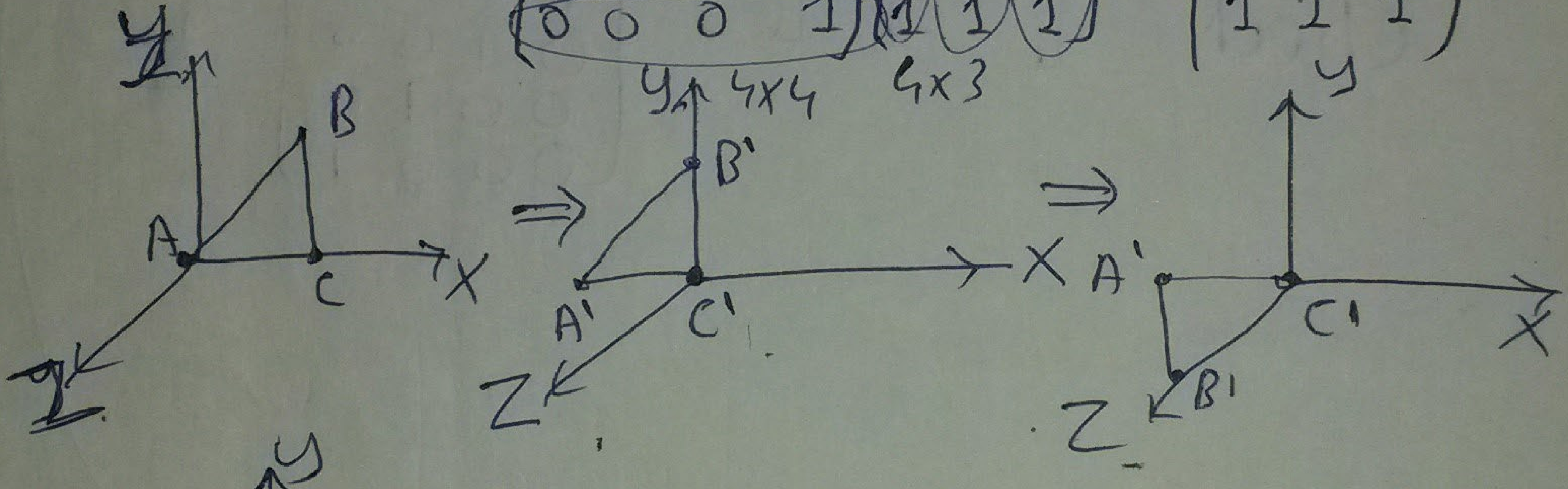
Combined Transformation $= S(-1,1,1)\, R_x(90^\circ)\, T(-1,0,0)$

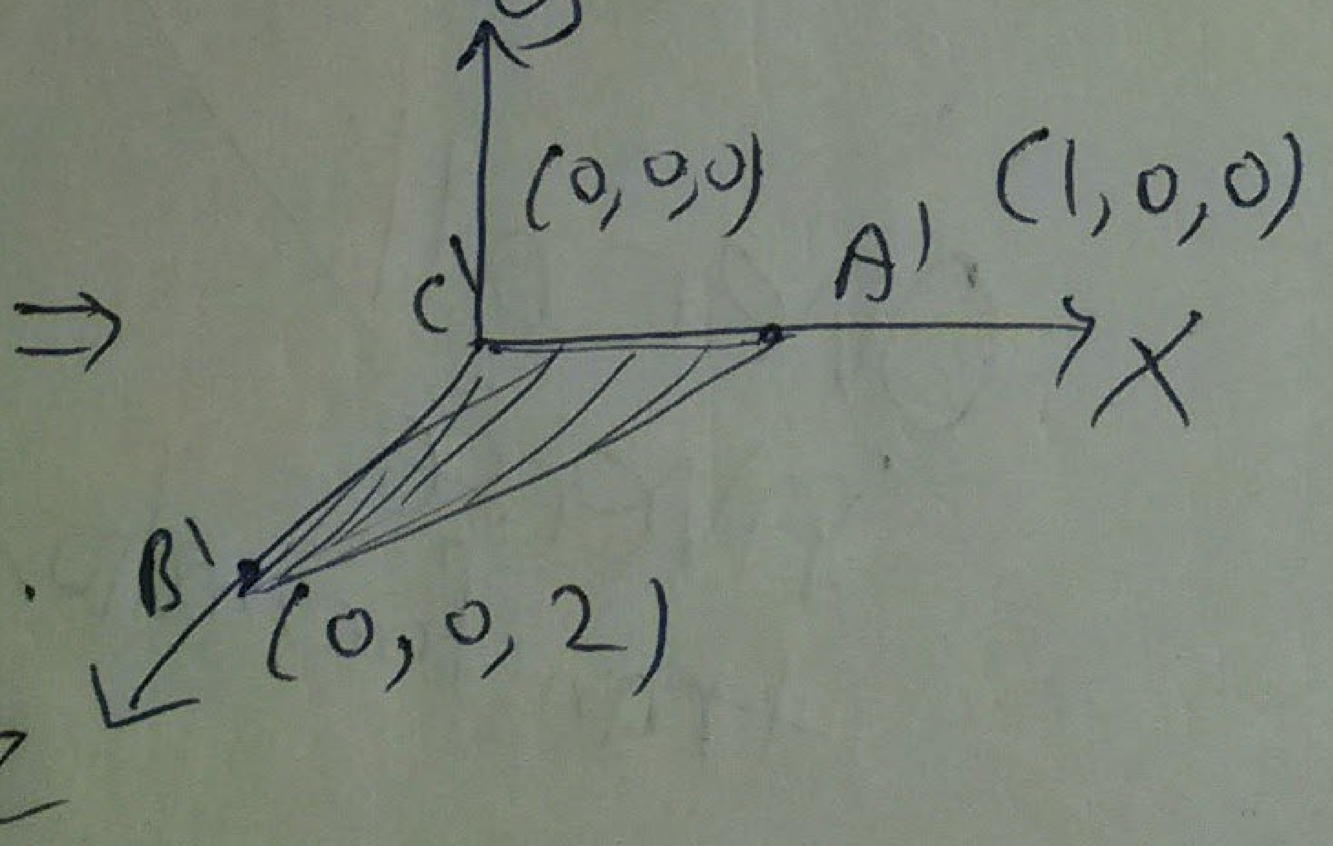
Combined Transformation matrix

$$= \underbrace{\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{S(-1,1,1)} \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{R_x(90^\circ)} \underbrace{\begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{T(-1,0,0)}$$

$$= \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

<u>check</u>: $$T * [P] = \underset{4\times4}{\begin{bmatrix} -1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}} \underset{4\times3}{\begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \end{array}} = \begin{array}{c} \begin{array}{ccc} A' & B' & C' \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 2 & 0 \\ 1 & 1 & 1 \end{bmatrix} \end{array}$$





* Inverse Transformation

$$= \left(S(-1,1,1) \cdot R_x(90^\circ) \cdot T(-1,0,0)\right)^{-1}$$

$$= T^{-1}(-1,0,0)\ R_x^{-1}(90^\circ)\ S^{-1}(-1,1,1)$$

$$= T(1,0,0)\ R_x(-90^\circ)\ S(-1,1,1)$$

$$= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Check $T^{-1} * T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ #

Overall inverse transformation.