Cairo University Faculty of Engineering		3rd Year Comp. MTH3251- Fall 2022 Number theory - Sheet 1

(1) Prove or disprove: If a | (b + c), then either a | b or a | c.
(2) Prove that for any integer a, one of the integers a, a+ 2, a + 4 is divisible by 3.
(3) Prove that if a and bare both odd integers, then if $16|a^4 + b^4 - 2$.
(4) Prove or disprove that if $a|bc$, where a, b, and c are positive integers and a ≠ 0, then $a|b$ or $a|c$.
(5) Show that if n is an integer then $n^2 \equiv 0 \; or \; 1 \; (mod \; 4)$
(6) Prove that if n is an odd positive integer, then $n^2 \equiv 1 \pmod 8$.
(7) Show that if n | m, where n and m are integers greater than 1, and if a ≡ b (mod m), where a and b are integers, then a ≡ b (mod n).
(8) Determine whether each of these integers is prime.
a) 19 b) 27
c) 93 d) 101
(9) Determine whether the integers in each of these sets are pairwise relatively prime.
a) 21, 34, 55 b) 14, 17, 85
(10) How many zeros are there at the end of 100! ?
(11) Prove that the product of any three consecutive integers is divisible by 6.
(12) We call a positive integer perfect if it equals the sum of its positive divisors other than itself.
a) Show that 6 and 28 are perfect.
b) Show that $2^{p-1}(2^p - 1)$ is a perfect number when 2^{p-1} is prime.