APPENDIX MATHEMATICAL TABLES

TABLE A.1 Summary of properties of the Fourier transform

Property	Mathematical Description
1. Linearity	$ag_1(t) + bg_2(t) \rightleftharpoons aG_1(f) + bG_2(f)$ where a and b are constants
2. Time scaling	$g(at) \rightleftharpoons \frac{1}{\lvert a\rvert} G\left(\frac{f}{a}\right)$ where a is a constant
3. Duality	If $g(t) \rightleftharpoons G(f)$, then $G(t) \rightleftharpoons g(-f)$
4. Time shifting	$g(t - t_0) \rightleftharpoons G(f)\exp(-j2\pi f t_0)$
5. Frequency shifting	$\exp(j2\pi f_c t)g(t) \rightleftharpoons G(f - f_c)$
6. Area under *g(t)*	$\int_{-\infty}^{\infty} g(t)\,dt = G(0)$
7. Area under *G(f)*	$g(0) = \int_{-\infty}^{\infty} G(f)\,df$
8. Differentiation in the time domain	$\frac{d}{dt}g(t) \rightleftharpoons j2\pi f G(f)$
9. Integration in the time domain	$\int_{-\infty}^{t} g(\tau)d\tau \rightleftharpoons \frac{1}{j2\pi f} G(f) + \frac{G(0)}{2}\delta(f)$
10. Conjugate functions	If $g(t) \rightleftharpoons G(f)$, then $g^*(t) \rightleftharpoons G^*(-f)$
11. Multiplication in the time domain	$g_1(t)g_2(t) \rightleftharpoons \int_{-\infty}^{\infty} G_1(\lambda)G_2(f-\lambda)\,d\lambda$
12. Convolution in the time domain	$\int_{-\infty}^{\infty} g_1(\tau)g_2(t-\tau)d\tau \rightleftharpoons G_1(f)G_2(f)$
13. Rayleigh's energy theorem	$\int_{-\infty}^{\infty} \lvert g(t)\rvert^2 dt = \int_{-\infty}^{\infty} \lvert G(f)\rvert^2 df$

TABLE A.2 Fourier-transform pairs

Time Function	Fourier Transform
$\text{rect}\left(\frac{t}{T}\right)$	$T\,\text{sinc}(fT)$
$\text{sinc}(2Wt)$	$\frac{1}{2W}\text{rect}\left(\frac{f}{2W}\right)$
$\exp(-at)u(t), \quad a > 0$	$\frac{1}{a + j2\pi f}$
$\exp(-a\|t\|), \quad a > 0$	$\frac{2a}{a^2 + (2\pi f)^2}$
$\exp(-\pi t^2)$	$\exp(-\pi f^2)$
$\begin{cases} 1 - \frac{\|t\|}{T}, & \|t\| < T \\ 0, & \|t\| \geq T \end{cases}$	$T\,\text{sinc}^2(fT)$
$\delta(t)$	1
1	$\delta(f)$
$\delta(t - t_0)$	$\exp(-j2\pi f t_0)$
$\exp(j2\pi f_c t)$	$\delta(f - f_c)$
$\cos(2\pi f_c t)$	$\frac{1}{2}[\delta(f - f_c) + \delta(f + f_c)]$
$\sin(2\pi f_c t)$	$\frac{1}{2j}[\delta(f - f_c) - \delta(f + f_c)]$
$\text{sgn}(t)$	$\frac{1}{j\pi f}$
$\frac{1}{\pi t}$	$-j\,\text{sgn}(f)$
$u(t)$	$\frac{1}{2}\delta(f) + \frac{1}{j2\pi f}$
$\sum_{i=-\infty}^{\infty} \delta(t - iT_0)$	$\frac{1}{T_0}\sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T_0}\right)$

Notes: $u(t)$ = unit step function
$\delta(t)$ = Dirac delta function
$\text{rect}(t)$ = rectangular function
$\text{sgn}(t)$ = signum function
$\text{sinc}(t)$ = sinc function