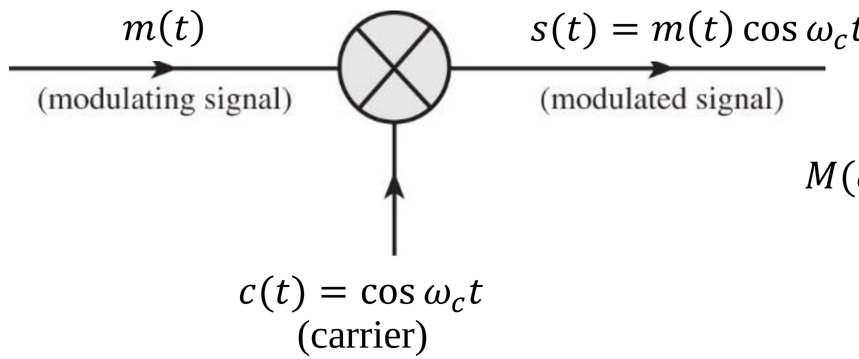
# Lecture 3

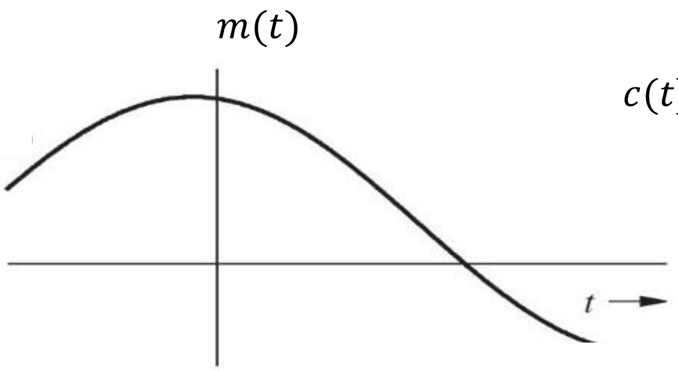
## Amplitude Modulation

# **Review:** 1. Double sideband suppressed carrier (DSB-SC)

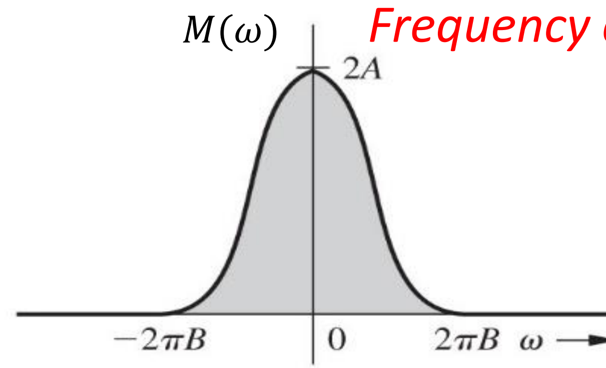
## Modulator



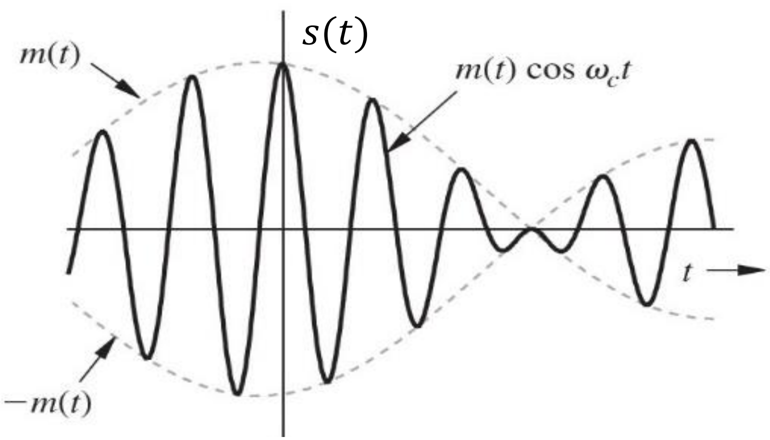

*Time domain*

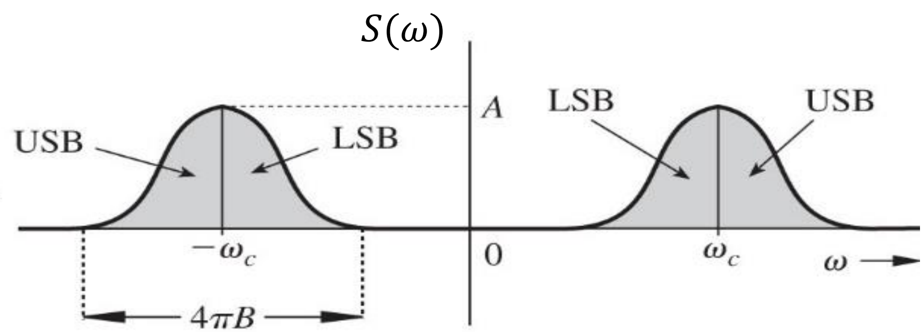


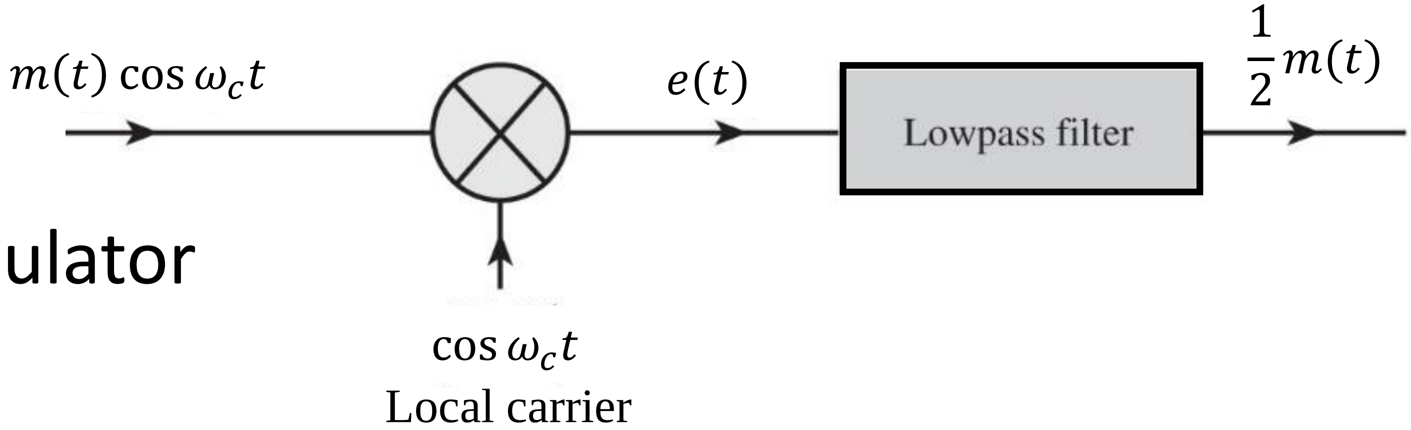

*Frequency domain*



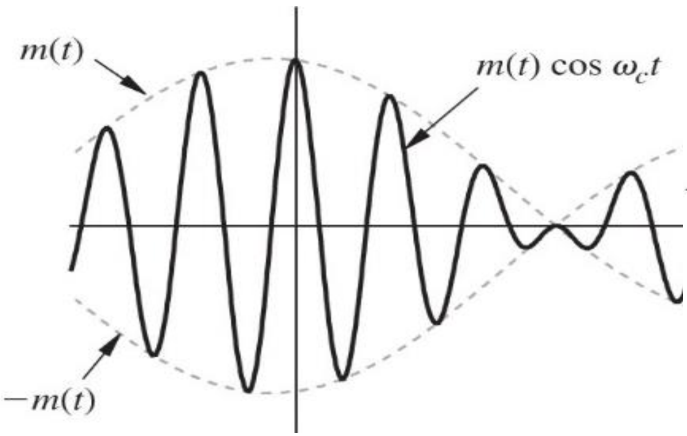

$$m(t)\cos\omega_c t \Leftrightarrow \frac{1}{2}[M(\omega+\omega_c)+M(\omega-\omega_c)]$$

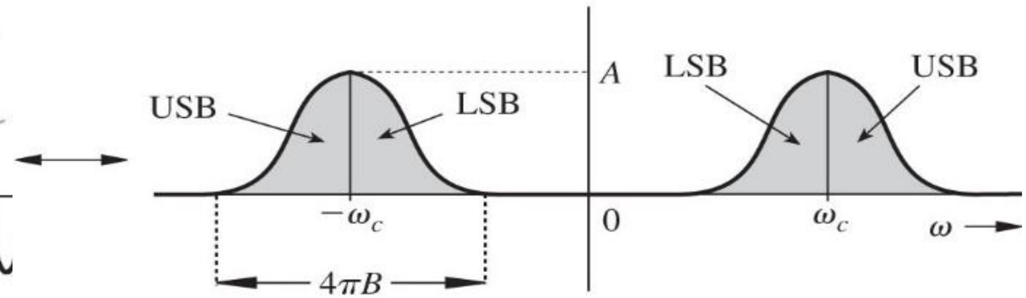


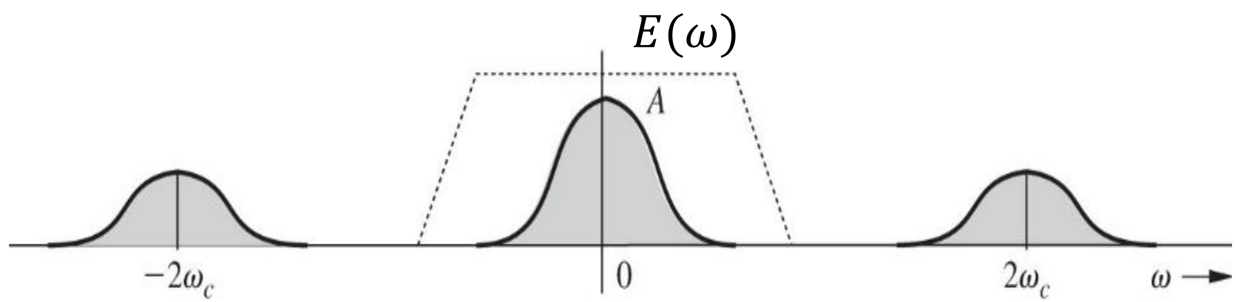






# Demodulator




$$e(t) = m(t)\cos^2\omega_c t = \frac{1}{2}[m(t) + \cancel{m(t)\cos 2\omega_c t}]$$

$$E(\omega) = \frac{1}{2}M(\omega) + \cancel{\frac{1}{4}[M(\omega + 2\omega_c) + M(\omega - 2\omega_c)]}$$

Eliminated by the LPF

# Notes on DSB-SC

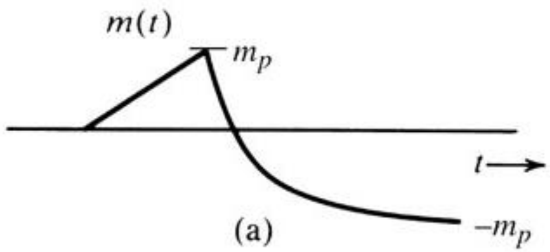
- Simple modulation
- Complex and expensive demodulation (requires synchronization)
- Waste of bandwidth ($2B$ Hz)

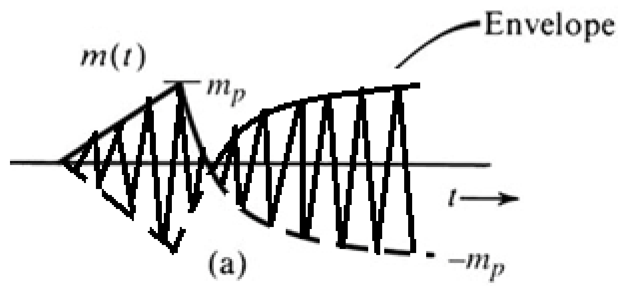
# Lecture outline

- DSB-LC: Another type of AM with **less complex demodulation** (not requiring generation of a local carrier)

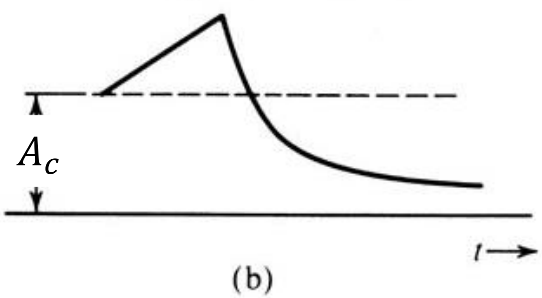
# 2. Double sideband large carrier (DSB-LC) Conventional AM (or simply AM)

$\pm m_p$: maximum and minimum values of $m(t)$

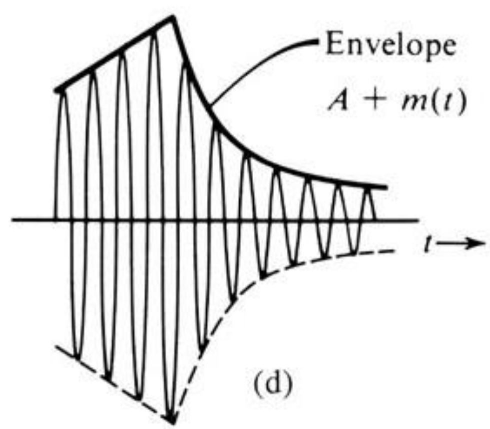



$m(t)\cos\omega_c t$




$A_c + m(t)$




$\left(A_c + m(t)\right)\cos\omega_c t$

# 2. Double sideband large carrier (DSB-LC) Conventional AM (or simply AM)

- **Target:** **Simpler and cheaper demodulators** (without generation of a local carrier)
- Unmodulated carrier:

$$c(t) = A_c \cos \omega_c t, \; A_c\text{: carrier amplitude}$$

$$f_c\text{: carrier frequency} \;\; (\omega_c = 2\pi f_c)$$

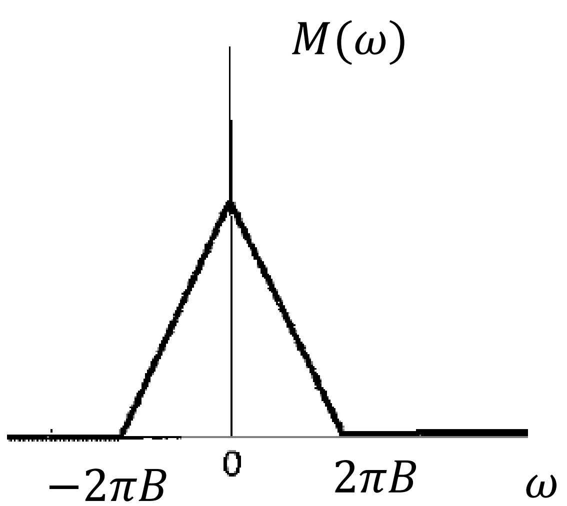
- Modulated signal:

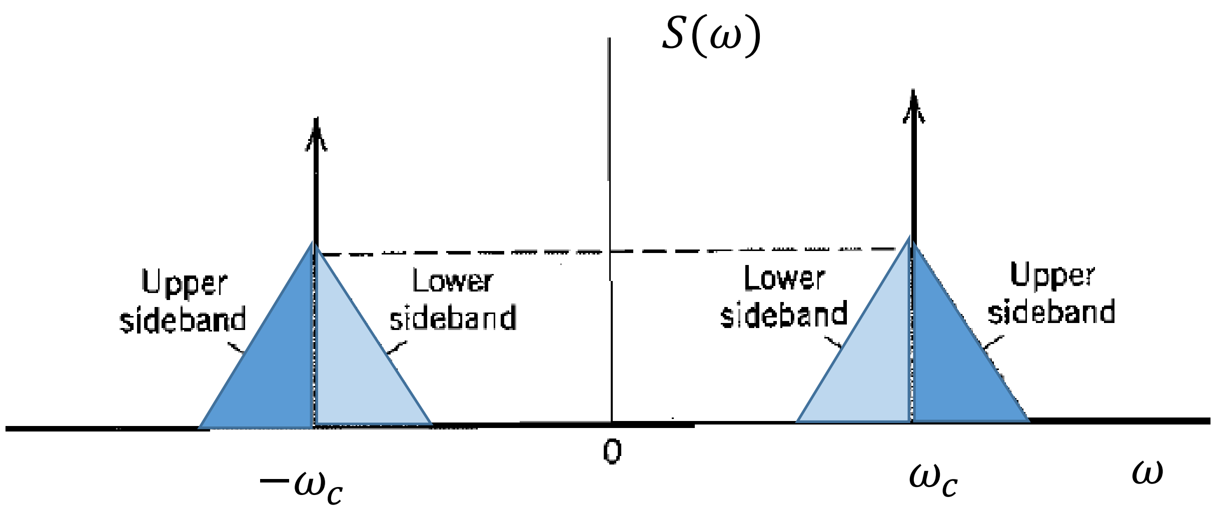
$$\begin{aligned} s(t) &= \left(A_c + m(t)\right) \cos \omega_c t \\ &= A_c \cos \omega_c t + m(t) \cos \omega_c t \end{aligned}$$

$$\begin{aligned} S(\omega) = &\ \pi A_c [\delta(\omega + \omega_c) + \delta(\omega - \omega_c)] \\ &+ \frac{1}{2} [M(\omega + \omega_c) + M(\omega - \omega_c)] \end{aligned}$$

# DSB-LC

- $s(t)$ = un-modulated carrier
  + upper sideband (USB)
  + lower sideband (LSB)




BW $= B$ Hz

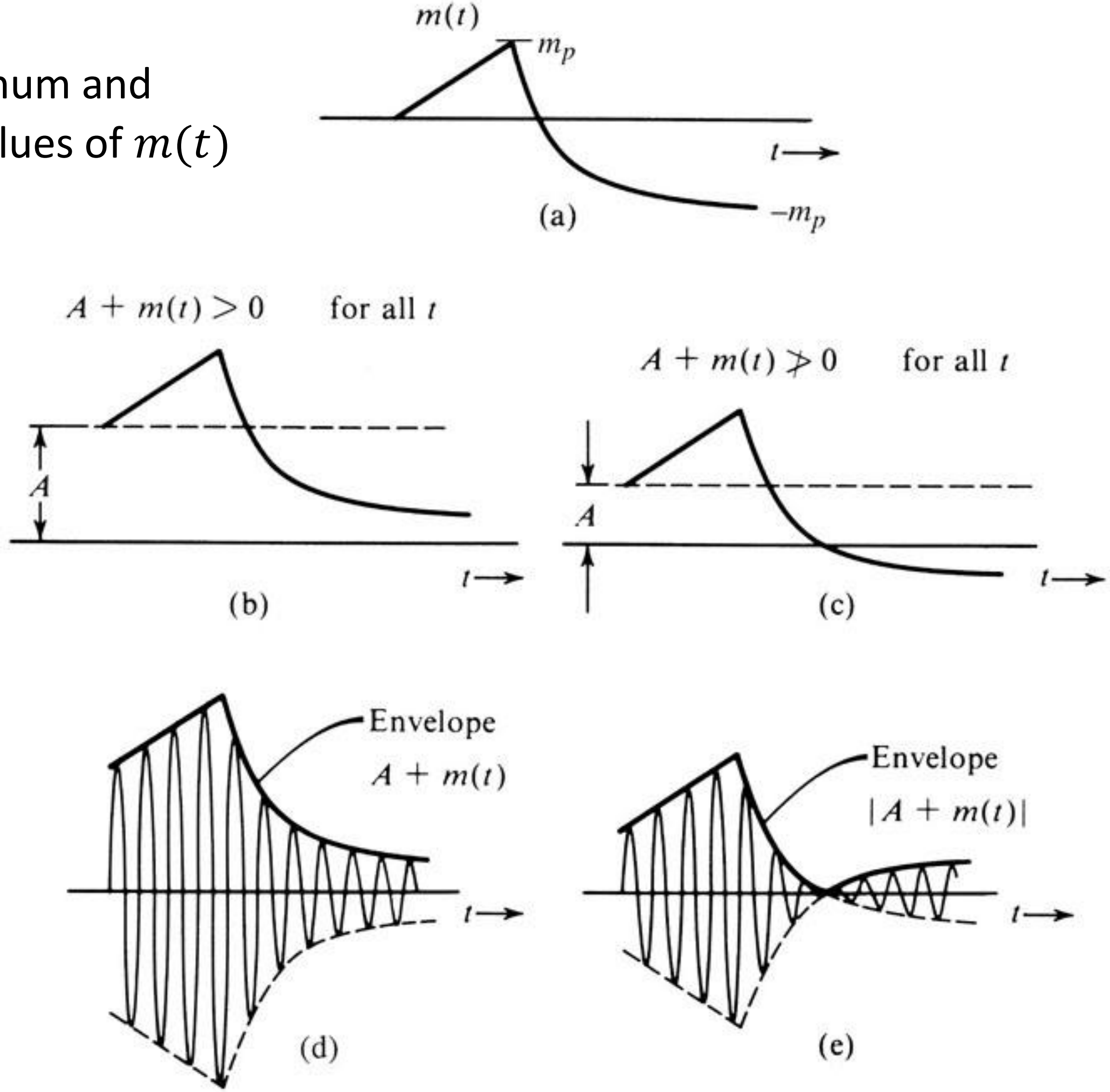
(Baseband)

BW of $s(t)$ = $2B$ Hz = $4\pi B$ rad/s

(Transmission BW)

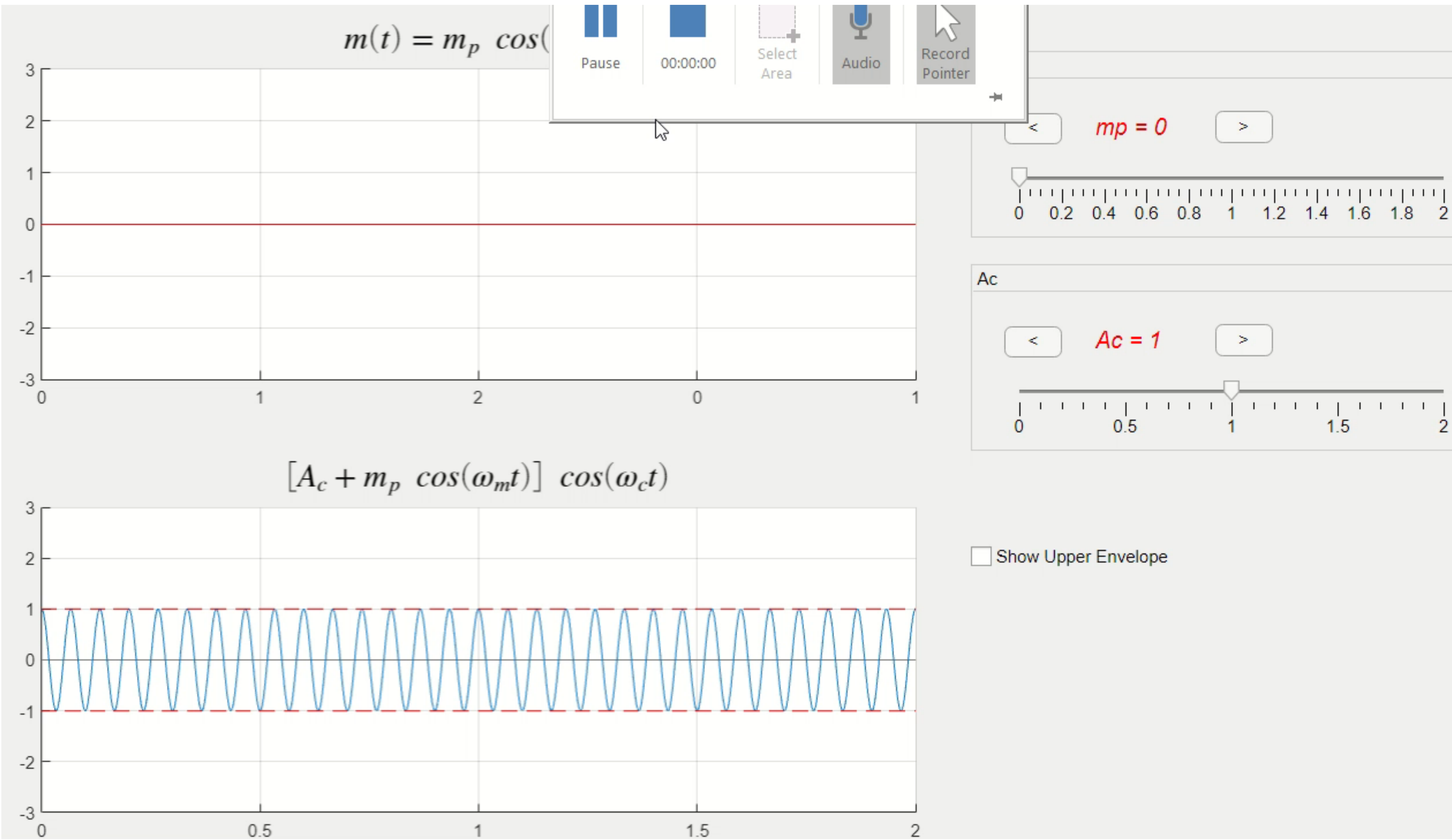
# DSB-LC

$\pm m_p$: maximum and minimum values of $m(t)$



# DSB-LC

- **Condition** for using envelop detection in demodulator:
  $A_c + m(t) > 0$ for all $t$
  $A_c \geq m_p$ ($A_c \geq$ absolute -ve peak amplitude, $|m(t)_{min}|$)
  Thus, **the envelope has the same shape as $\boldsymbol{m(t)}$**
- This makes the demodulator simpler (no need for generation of local carrier)
- **Modulation index:** $\mu = \frac{m_p}{A_c}$ (Generally, $\mu = \frac{|m(t)_{min}|}{A_c}$)
- $0 \leq \mu \leq 1$ since $A_c \geq m_p$ and there is no upper bound on $A_c$



$m(t) = m_p\ cos($
Pause
00:00:00
Select Area
Audio
Record Pointer
mp = 0
Ac
Ac = 1
$[A_c + m_p\ cos(\omega_m t)]\ cos(\omega_c t)$
Show Upper Envelope

# DSB-LC

- $\mu \times 100$: percentage modulation
- $\mu > 1$: over-modulation (envelope detection not viable)
- Coherent detection can be used for both DSB-SC and DSB-LC (for any $\mu$)

$$s(t) = A_c \cos \omega_c t + m(t) \cos \omega_c t$$

# DSB-LC

- We transmit higher power level than DSB-SC (higher cost for transmitter)
- Trade-off: higher power at transmitter, cheaper less complex receiver.
- Suitable for broadcast systems (1 transmitter, many receivers)
  - More economical to have one expensive high power transmitter, and many cheaper receivers.

# Sideband and carrier power

- $s(t) = A_c \cos \omega_c t + m(t) \cos \omega_c t$

  carrier sidebands (signal)
- Carrier power doesn't carry any information and is a waste of power
- Carrier power $P_c = A_c^2/2$
- Sideband power $P_s = \lim_{T\to\infty} \frac{1}{T} \int_T [m(t) cos\omega_c]^2 dt$

$$= \frac{1}{2} \lim_{T\to\infty} \frac{1}{T} \int_T m^2(t) dt = \frac{1}{2} P_m$$

(1/2 baseband signal power)

# Sideband and carrier power

- Total transmitted power = $P_c + P_s$
- Useful power (containing information) = $P_s$
- Efficiency $\eta = \frac{\text{useful power}}{\text{total power}} = \frac{P_s}{P_c+P_s} = \frac{P_m}{A_c^2+P_m}$

# Sideband and carrier power

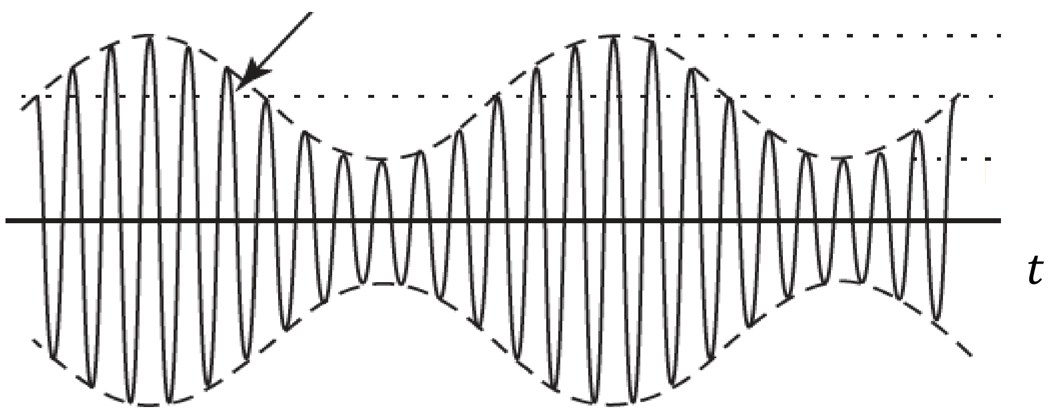
**Example** (tone modulation):

Let $m(t) = A_m \cos \omega_m t$ (single tone signal)

Then $s(t) = (A_c + A_m \cos \omega_m t) \cos \omega_c t$

Modulation index: $\mu = A_m / A_c$

$s(t) = A_c \cos \omega_c t + \mu A_c \cos \omega_m t \, \cos \omega_c t$



# Sideband and carrier power

- $m(t) = A_m \cos \omega_m t = \mu A_c \cos \omega_m t$
- $\overline{m^2(t)} = \frac{(\mu A_c)^2}{2}$
- Efficiency $\eta = \frac{\text{useful power}}{\text{total power}} = \frac{P_s}{P_c + P_s} = \frac{P_m}{A_c^2 + P_m}$

$$= \frac{\mu^2}{2 + \mu^2} 100\%$$

Only for tone modulation

- For $0 \leq \mu \leq 1$, $0\% \leq \eta \leq 33.3\%$

# Sideband and carrier power

- For tone modulation, under the best conditions ($\mu = 1$), only one-third of the transmitted power is carrying messages. (useful information)
- For practical signals, the efficiency is even worse, on the order of 25% or lower.
- Smaller values of $\mu$, degrade efficiency further.

# DSB-LC Example: Tone modulation

1. Sketch the modulated signal for $\mu = 0.5 \; and \; 1$ in time domain. Assume $A_m = 1$. Find the efficiency in each case.
2. Sketch the spectrum of the modulated signal. Find the bandwidth
3. Find the carrier power, USB and LSB power and total sideband power.

# DSB-LC Example: Tone modulation

$$\mu = \frac{A_m}{A_c}$$

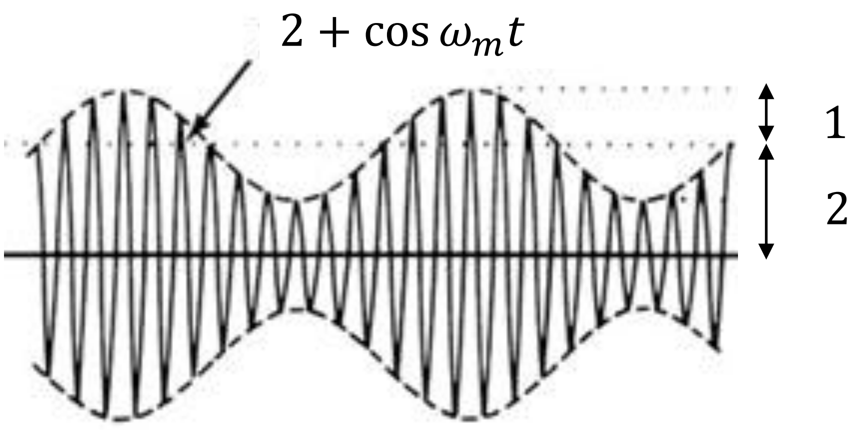
1. $s(t) = (A_c + A_m \cos\omega_m t)\cos\omega_c t$

$A_m = 1$

- $\mu = 0.5$

$Ac = 2$

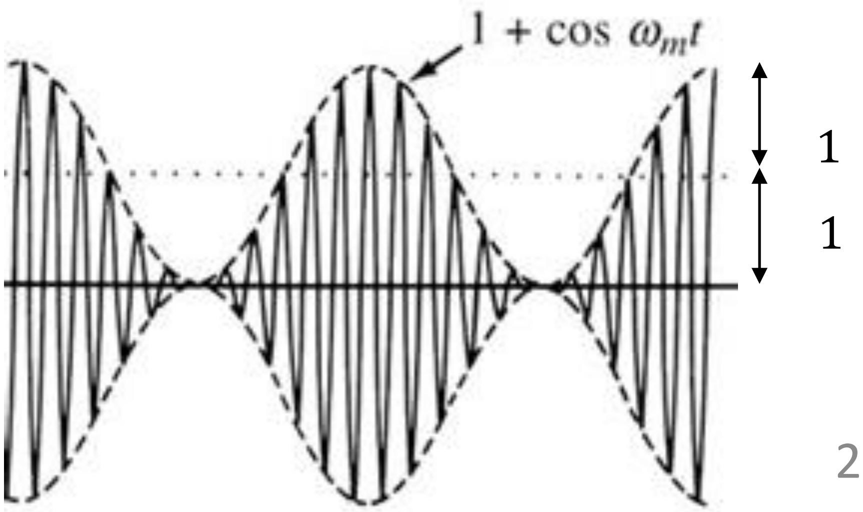
$\eta = 11.1\%$



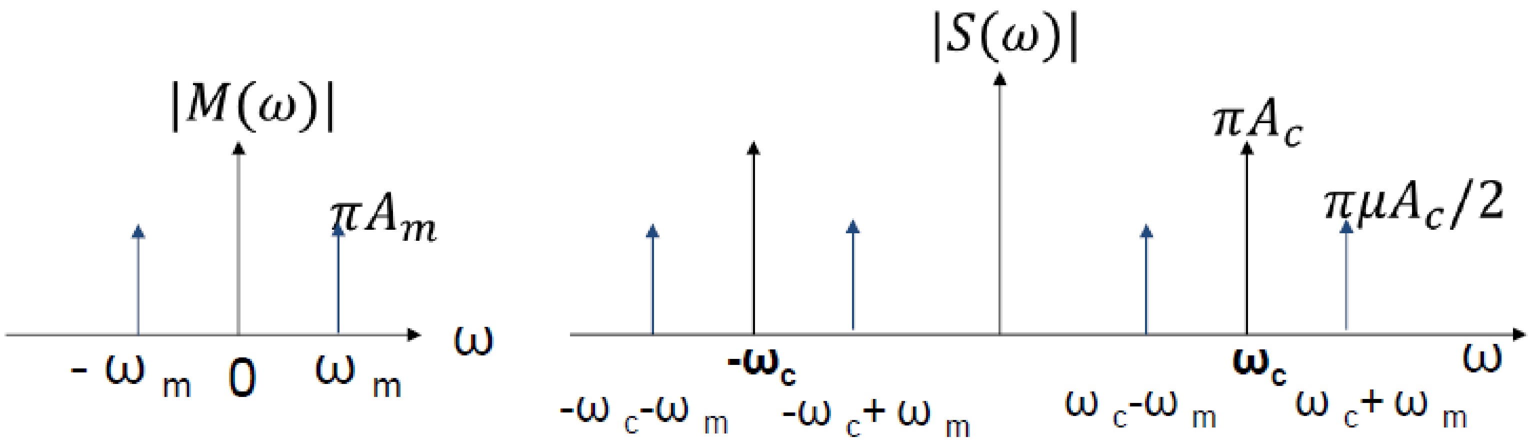

- $\mu = 1$

$Ac = 1$

$\eta = 33.3\%$



# DSB-LC Example: Tone modulation




- $s(t) = A_c \cos \omega_c t + (A_m \cos \omega_m t\ ) \cos \omega_c t$
- $BW\ = 2f_m \text{ Hz} = 2\omega_m \text{ rad/s}$

# DSB-LC Example: Tone modulation

$$S(\omega) = \pi A_c[\delta(\omega - \omega_c) + \delta(\omega + \omega_c)] + \frac{1}{2}[M(\omega - \omega_c) + M(\omega + \omega_c)]$$

$$= \pi A_c [\delta(\omega - \omega_c) + \delta(\omega + \omega_c)]$$
$$+ (\pi\mu A_c/2) [\delta(\omega + (\omega_c + \omega_m))]$$
$$+ (\pi\mu A_c/2) [\delta(\omega - (\omega_c + \omega_m))]$$
$$+ (\pi\mu A_c/2) [\delta(\omega - (\omega_c - \omega_m))]$$
$$+ (\pi\mu A_c/2) [\delta(\omega + (\omega_c - \omega_m))]$$

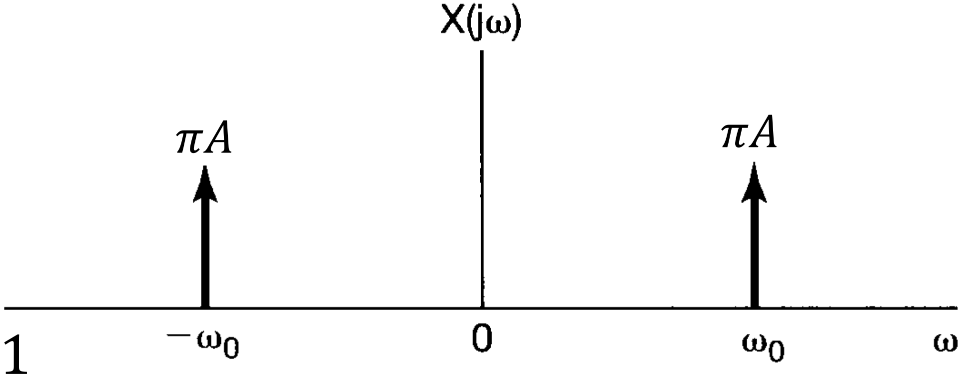
# Review
# Fourier Transform for Periodic Signals

For a periodic $x(t)$ with FS coefficients $\{a_k\}$

$$X(\omega) = \sum_{k=-\infty}^{\infty} 2\pi a_k \, \delta(\omega - k\omega_0)$$

- **Example**

$x(t) = A\cos(\omega_0 t)$




$a_1 = a_{-1} = \frac{A}{2}, a_k = 0, k \neq \pm 1$

$$X(\omega) = \pi A\delta(\omega - \omega_0) + \pi A\delta(\omega + \omega_0)$$

# Review
# Power of sinusoidal signals

- **Parseval's theorem**

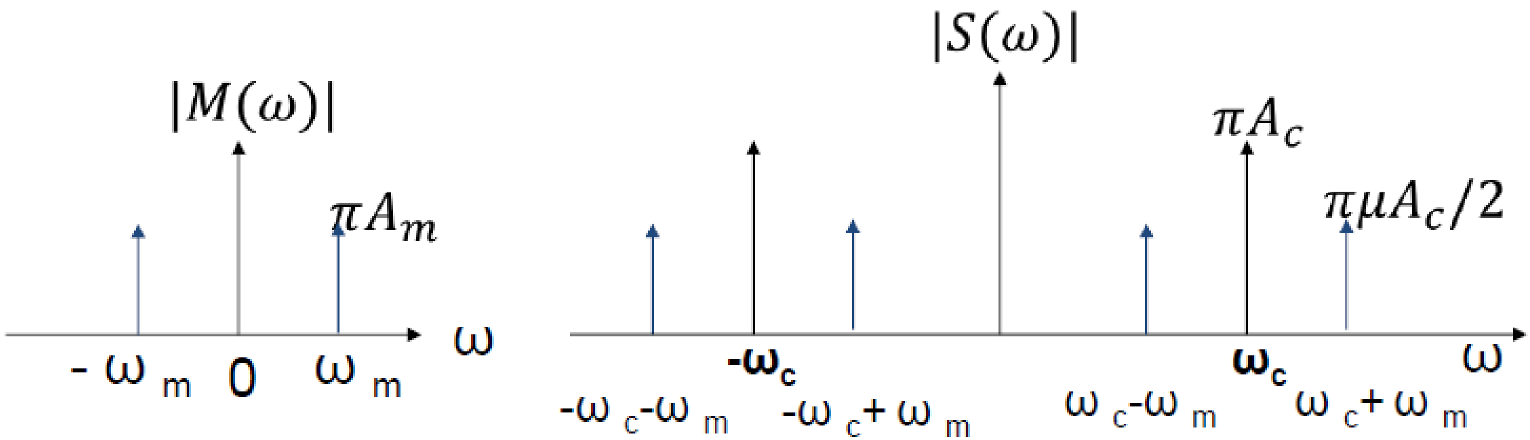
Periodic

$$P = \frac{1}{T}\int_T |x(t)|^2 dt = \sum_{k=-\infty}^{\infty} |a_k|^2$$

- For $x(t) = A\cos\omega t$

$$P = \frac{A^2}{2}$$

# DSB-LC Example: Tone modulation



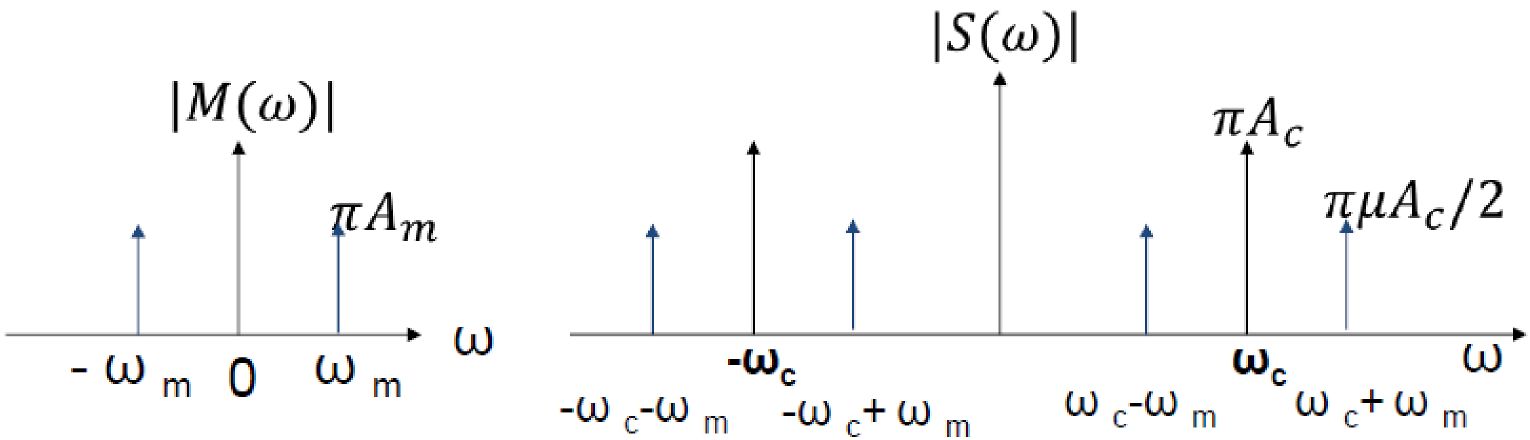

- $s(t) = A_c \cos \omega_c t + (A_m \cos \omega_m t\ ) \cos \omega_c t$

$$= A_c \cos \omega_c t + \frac{A_m}{2} \cos(\omega_c + \omega_m)t$$

$$+ \frac{A_m}{2} \cos(\omega_c - \omega_m)t$$

$$X(\omega) = \sum_{k=-\infty}^{\infty} 2\pi a_k \, \delta(\omega - k\omega_0)$$

# DSB-LC Example: Tone modulation




- $s(t) = A_c \cos \omega_c t + (A_m \cos \omega_m t\ ) \cos \omega_c t$
- To get the power: divide S(ω) by 2π (to get F.S. coeff), square the magnitude and add all terms
  $(P = \sum_{k=-\infty}^{\infty} |a_k|^2)$

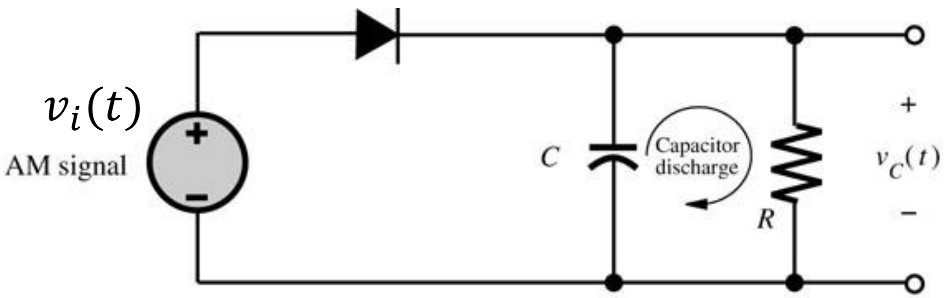
# DSB-LC Example: Tone modulation

- Carrier power $= Ac^2 / 2$
- U.S.B power = L.S.B. power = $\mu^2 Ac^2 / 8$
- Total sidebands power = $\mu^2 Ac^2 / 4$
- $P_t = Pc + Ps = (Ac^2 / 2) + (\mu^2 Ac^2 / 4)$

$$= Pc\,[\,1 + (\mu^2 / 2)\,]$$

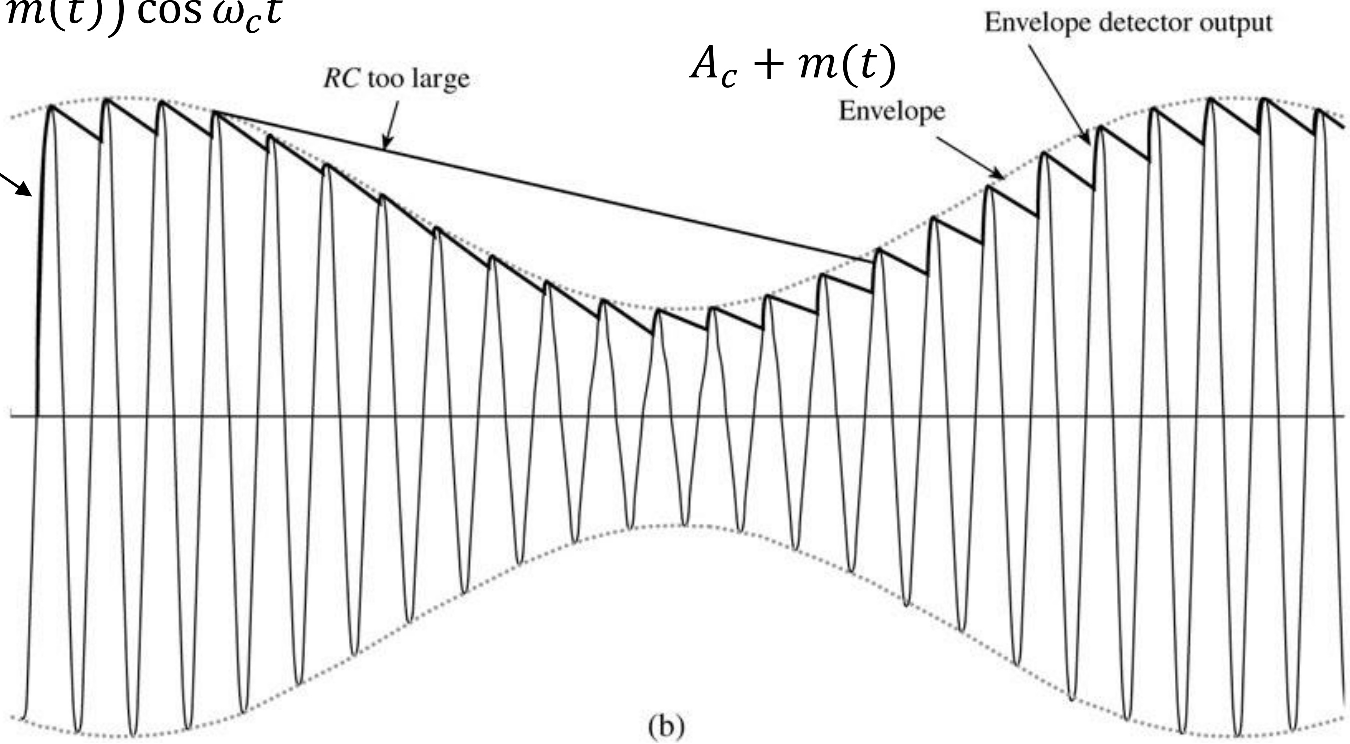
Efficiency $\eta = \dfrac{\text{useful power}}{\text{total power}} = \dfrac{\mu^2}{2+\mu^2}$ (only for tone modulation)

# DSB-LC Demodulator: Envelope detector (Non-coherent/asynchronous)



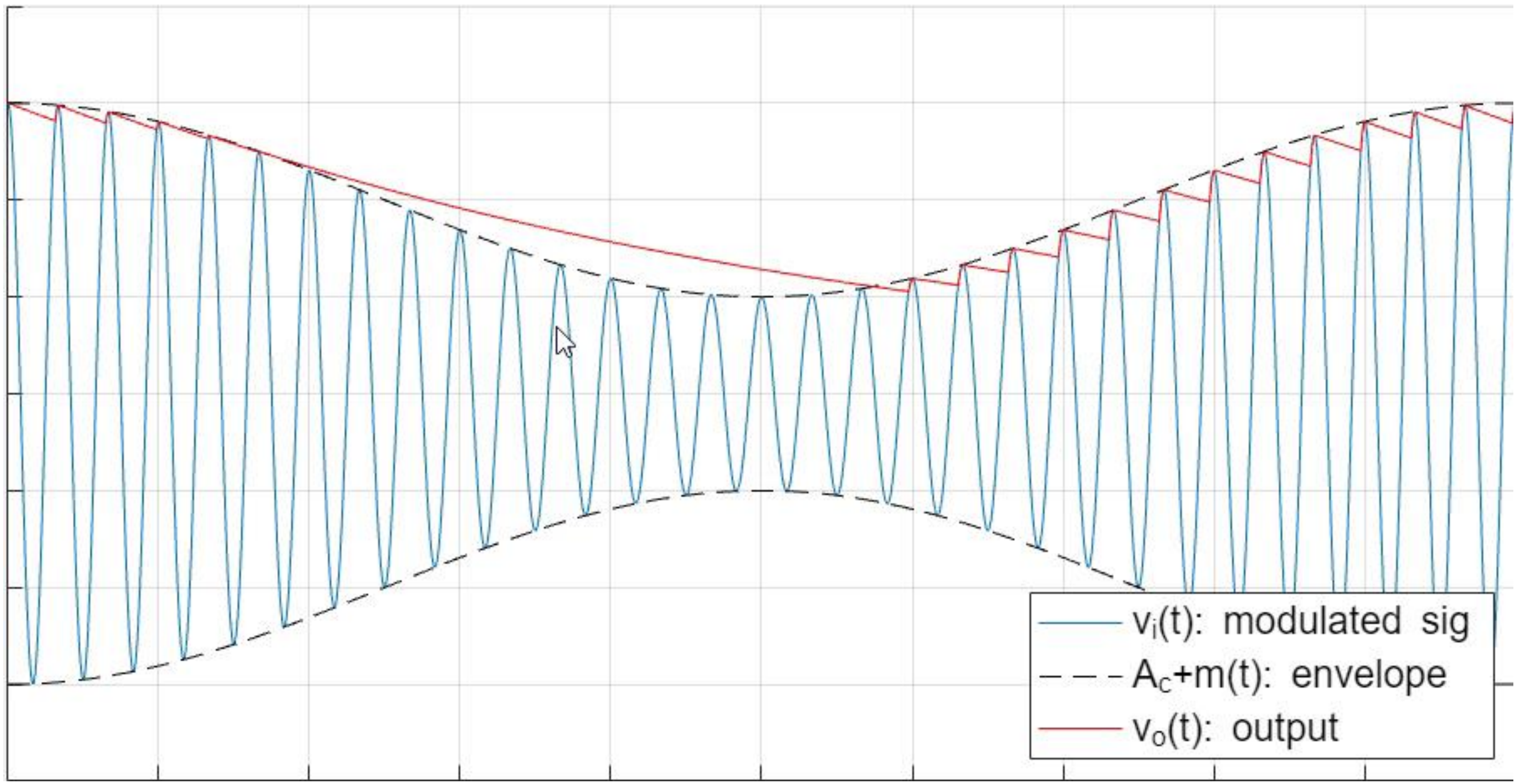

(a)

$$v_i(t) = \left(A_c + m(t)\right) \cos \omega_c t$$




(b)

# DSB-LC Demodulator: Envelope detector (Non-coherent/asynchronous)

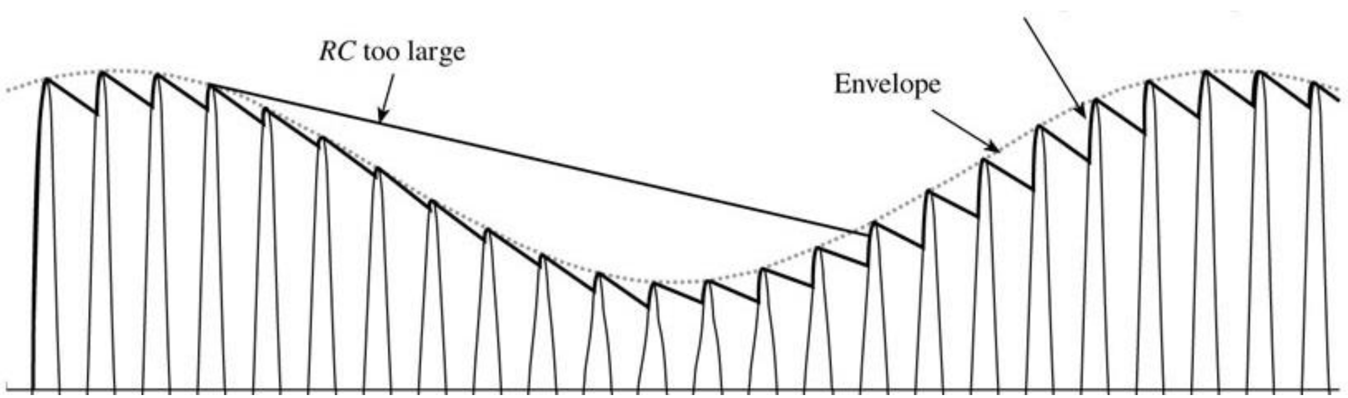


# DSB-LC Demodulator: Envelope detector (Non-coherent/asynchronous)

$$\frac{1}{W} > RC > \left(\frac{1}{f_c} = T_c\right)$$

signal BW carrier freq.

- The dc term $A_c$ can be blocked by a capacitor
- The ripple may be reduced further by a LPF



# Notes on DSB-LC (conventional AM)

- Advantages: Ease of Modulation and demodulation (cheap to build the system)

- Disadvantages

  - Waste of power.

  - Waste of B.W.

# So Far …

- DSC-SC
  - More complicated demodulator (synchronization issues)
  - Excellent power efficiency
  - Need BW = 2W to send a signal of BW=W
- DSB-LC (Conventional AM)
  - Easy to demodulate (envelope detector)
  - Low power efficiency
  - Need BW = 2W to send a signal of BW=W
- Can we improve the usage of the BW?