CE Sheet 2 Sol.

1.

i) $m(t) = \cos(1000t)$

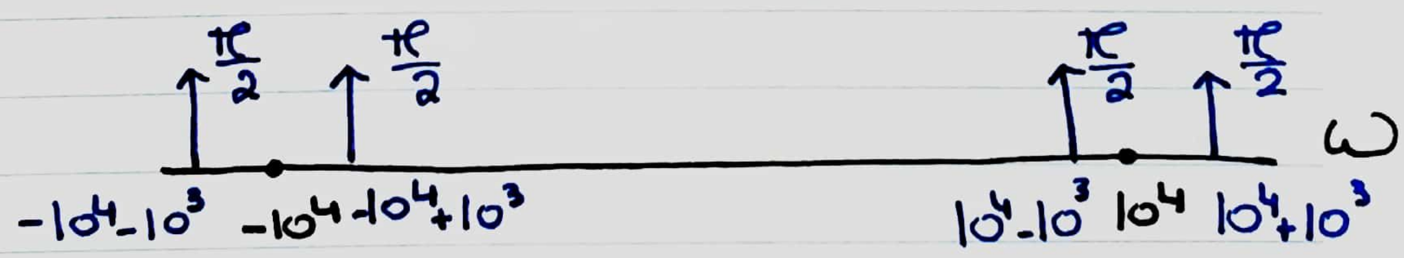
a)

$$M(\omega) = \pi\left(\delta(\omega - 1000) + \delta(\omega + 1000)\right)$$

b)

$$\mathcal{F}_T\{m(t)\cos 10^4 t\} = \frac{\left(M(\omega - 10^4) + M(\omega + 10^4)\right)}{2}$$

Shifts $M(\omega)$ at $\pm 10^4$



c) USB | LSB   LSB | USB

d) Frequencies

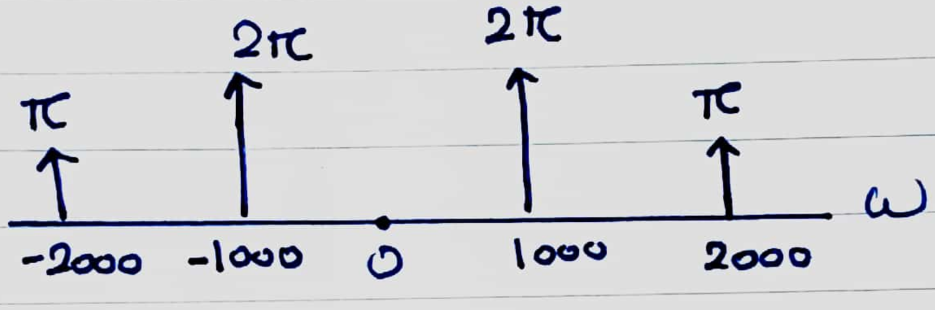
Baseband	Corresponds $\longrightarrow$ add $\pm\omega_c$ then check.	USB	LSB
1000		11000	-9000
-1000		-11000	9000

ii.

$m(t) = 2\cos 1000t + \cos 2000t$

a)

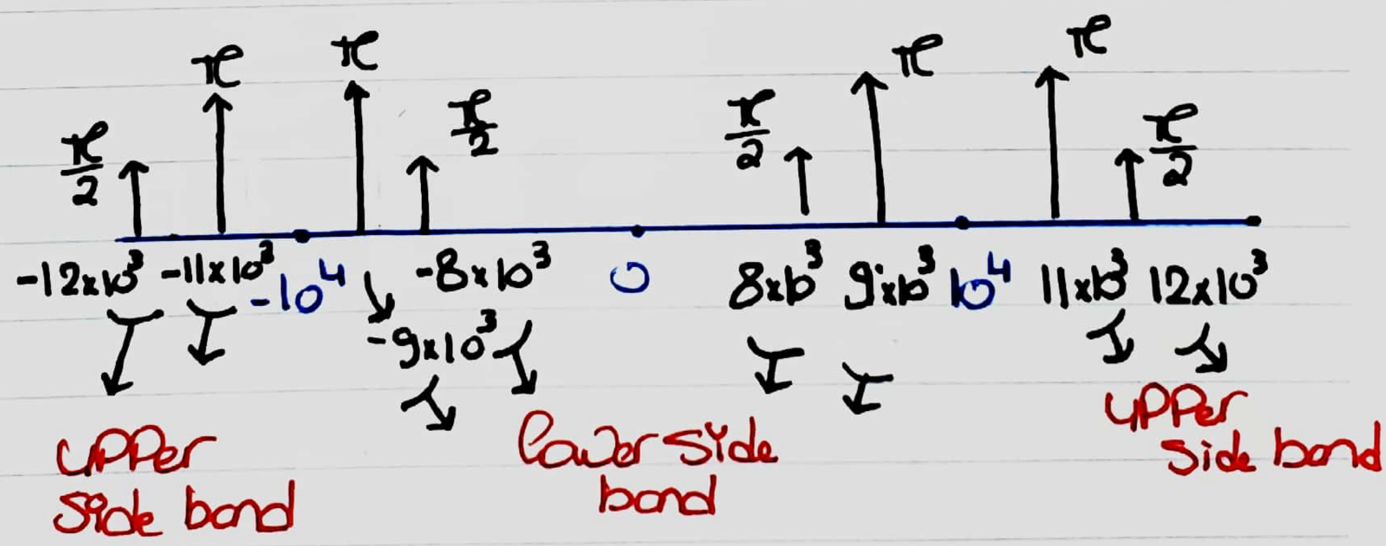
$$M(\omega) = \pi\big(2\delta(\omega-1000) + \delta(\omega-2000) + 2\delta(\omega+1000) + \delta(\omega+2000)\big)$$



b)

$$S(\omega) = \frac{1}{2}\big(M(\omega-10^4) + M(\omega+10^4)\big)$$

c)



d)

Baseband	-2000	-1000	1000	2000
USB	-12000	-11000	11000	12000
LSB	8000	9000	-9000	-8000

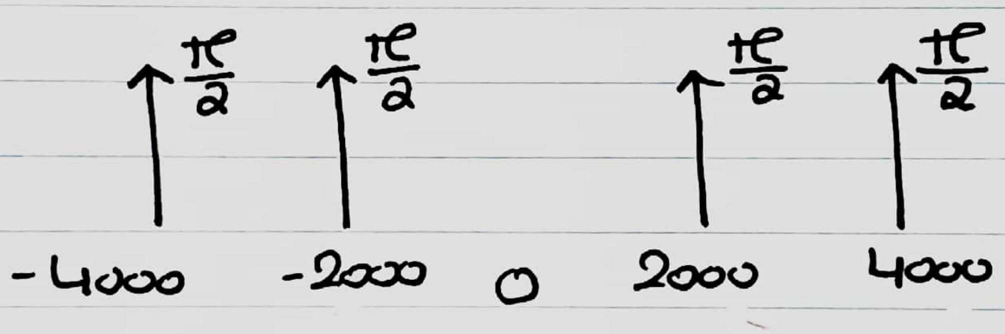
Note that for $m(t)$ that's sinusoid
using $\cos x \cos y = \frac{1}{2}(\cos(x+y) + \cos(x-y))$ ①
yields expressions corresponding to USB and LSB.

iii.

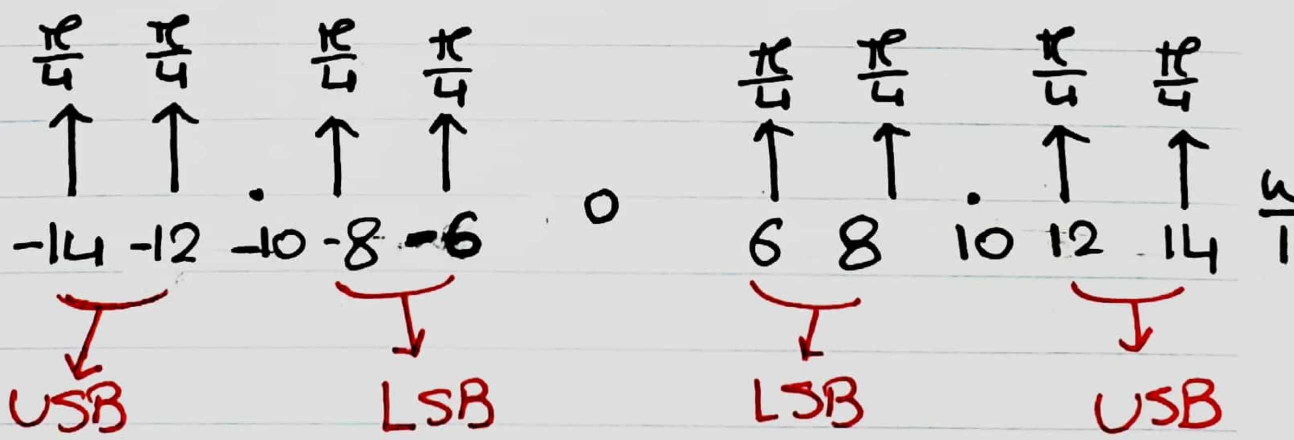
$$m(t) = \cos 1000t \cos 3000t$$
$$= \frac{1}{2}(\cos 4000t + \cos 2000t)$$

a)

$$M(\omega) = \frac{\pi}{2}\big(\delta(\omega+4000) + \delta(\omega+2000) + \delta(\omega-4000) + \delta(\omega-2000)\big)$$



b)



c)

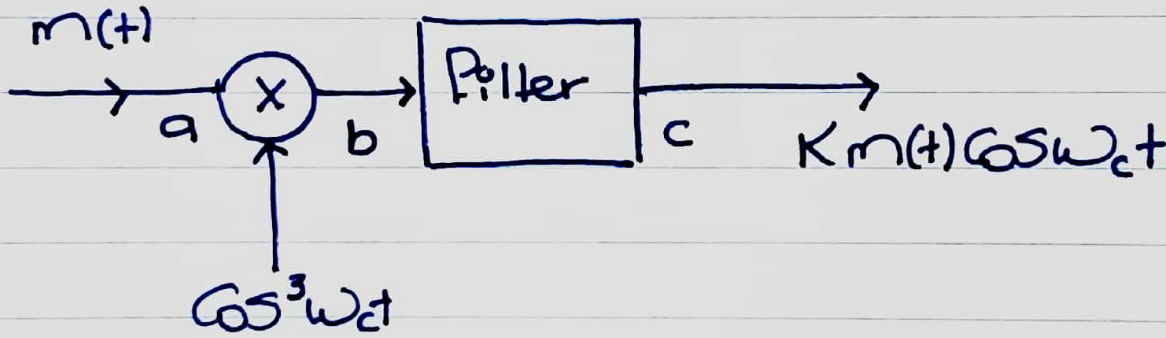
① USB) $\frac{1}{4}(\cos(12000t) + \cos(14000t))$

LSB) $\frac{1}{4}(\cos(8000t) + \cos(6000t))$

Baseband Frequency	USB	10^4 LSB
-4000	-14000	6000
-2000	-12000	8000
2000	12000	-8000
4000	14000	-6000

The nature of frequency shifting in all cases is to shift the message signal to $\pm 10^4$ rad/s (Carrier Frequency)

2.



$\cdot \cos^3\omega_c t = \cos\omega_c t(\cos^2\omega_c t) = \frac{1}{2}\cos\omega_c t(1+\cos 2\omega_c t)$

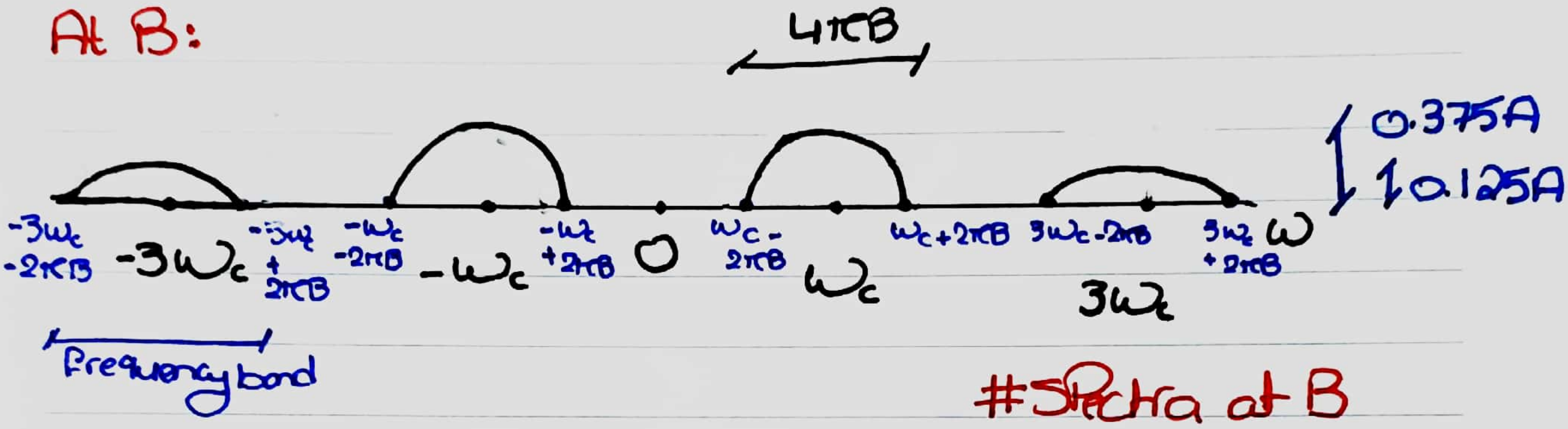
$= \frac{1}{2}(\frac{\cos\omega_c t}{2} + \frac{1}{2}(\cos\omega_c t + \cos 3\omega_c t)$

$= 0.75\cos\omega_c t + 0.25\cos 3\omega_c t$

$$S(\omega) = \mathcal{F}_T\{m(t)\cos^3\omega_c t\}$$

$$= \frac{0.75}{2}\left(M(\omega+\omega_c)+M(\omega-\omega_c)\right) + \frac{0.25}{2}\left(M(\omega+3\omega_c)+M(\omega-3\omega_c)\right)$$

At B:



#Spectra at B

a) Band Pass Filter with bandwith $4\pi B \leq \omega_B < 2\omega_c$* and Central Frequency @ω_c.

(Ideal BPF at $BW = 4\pi B$ works)

*We'll need much larger BW if we use LPF (not efficient)

b) At C:

(or let the filter have gain $\frac{1}{0.375}$)

$$S(\omega)\big|_{BPF} = 0.75\left(\frac{1}{2\pi}\underbrace{\{\cos\omega_c t\}}_{\mathcal{F}_T} * M(\omega)\right)$$

$S(t) = 0.75\cos\omega_c t\, m(t)$ indeed $S(t) = K m(t)\cos\omega_c t$

c)

Should be at least $2\pi B$
(the signals bandwidth)

d) $S(t) = m(t)\cos^2\omega_c t$

$= \frac{1}{2} m(t)(1+\cos 2\omega_c t) = \frac{m(t)}{2} + \frac{1}{2} m(t)\cos 2\omega_c t$

should be t.

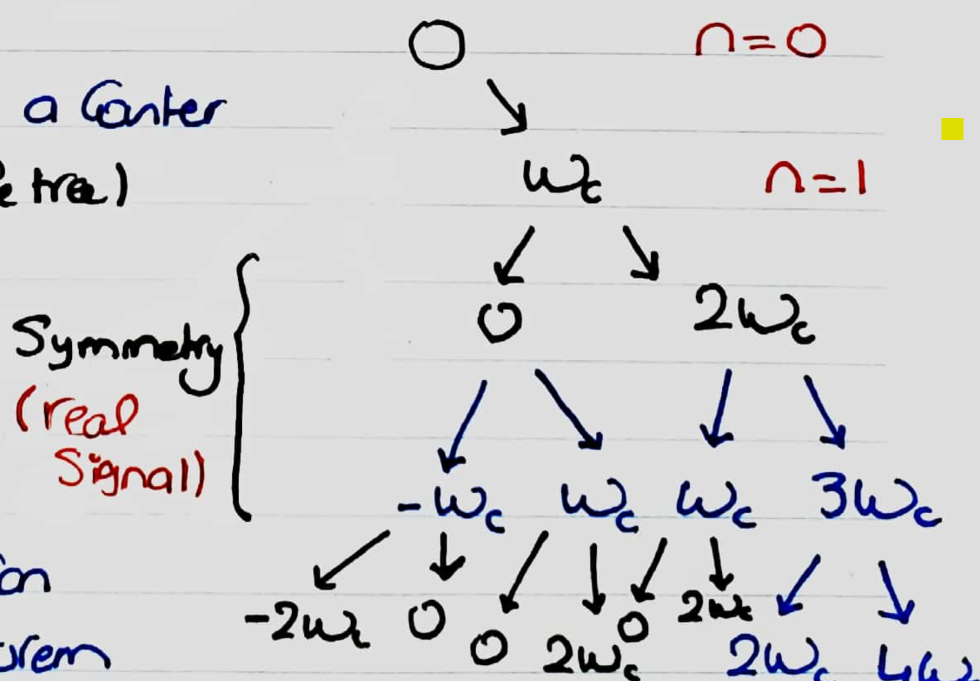
$\xrightarrow{\text{skip}}$ $S(\omega) = \frac{1}{2} M(\omega) + (M(\omega+2\omega_c) + M(\omega-2\omega_c))\frac{1}{4}$

No, we can only get $K m(t)\cos 2\omega_c t$

e)

No, it !

No, Take $n=4$ as a Counter example (leaves of the tree)



* Mathematically, one can use the binomial theorem on $\frac{(e^{jx}+e^{-jx})^n}{2^n}$ to conclude that it only works for odd n

//or using recurrence + tree

3-

- Unmodulated Power Output "Carrier's Power"

↳ unmodulated Signal. ($A_c \cos w_c t$)

$$P_c = \frac{A_c^2}{2} \quad (\text{Volts}^2)$$

// Physical Power should account for the driven load (if represents voltage, divide the energy/Power by the load impedance) ← generally.

• Check wiki

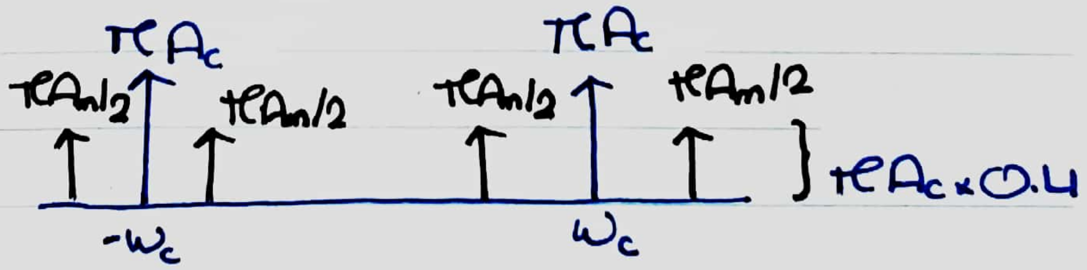
$$P_{c_{act.}} = \frac{A_c^2}{2} \cdot \frac{1}{R} \quad (V^2/\Omega) \quad (\text{Watt})$$

- Sinusoidal test tone ↦ $m(t)$ (applied to input of modulator)

$$m(t) = A_m \cos W_m t$$

↳ 5 (Peak 5 volts)

- In the magnitude Spectrum the Spectral lines (deltas) are 40% of the Carrier line.



Side bands

$$m(t)\cos w_c t = A_m \cos W_m t \cos W_c t$$

$\uparrow \pi A_m$ at $-W_m$, $\uparrow \pi A_m$ at W_m ↲ ; shift at $\pm W_c$, $\times \frac{1}{2}$

$\pi A_c \times 0.4 = \pi A_m \times 0.5$

$A_m = 0.8 A_c$

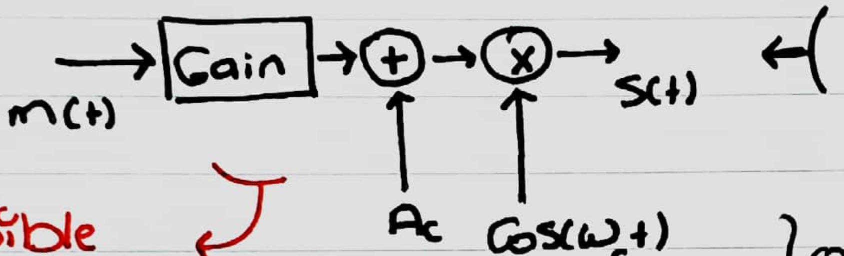
a)

$$P_{c_{\text{Actual}}} = \frac{A_c^2}{2} \cdot \frac{1}{R} = 1000 = \frac{A_c^2}{2} \cdot \frac{1}{50}$$

$A_c = 316.2\text{ V}$

From the output frequency spectrum:

$A_m = 0.8\, A_c$, hence $A_m = 252.96\text{ V}$ which is different from the amplitude at the input.



Possible justification

one might rather think that the gain stage was just before the output but then it doesn't make sense to use such A_c for the carrier.

- thus, $\mu = 0.8$

b) Peak amplitude of lower side band

$$\frac{A_m}{2} \cdot \pi\left(\delta(\omega-(\omega_c-\omega_m)) + \delta(\omega+(\omega_c-\omega_m))\right)$$

$$\longleftrightarrow$$

$$\frac{A_m}{2} \cos(\omega_c t - \omega_m)t$$

↳ Peak amplitude at output is 126.5V

- Can also use $\cos x \cos y = \frac{1}{2}(\cos(x+y) + \cos(x-y))$

c)

$$\frac{P_s}{P_c} = \frac{(A_m^2/4)}{(A_c^2/2)} = \frac{\mu^2}{2} = 0.32$$

d)

$$P_{T_{Actual}} = \frac{1}{R}\left(\frac{A_c^2}{2} + \frac{A_m^2}{4}\right) = 1.312 \quad KW$$

e) Instead of

$5V \longrightarrow 252.96\,V$

we have

$4V \longrightarrow \left.202.37V\right\} A_m$

$$P_{T_{Actual}} = 1.204 \quad KW$$

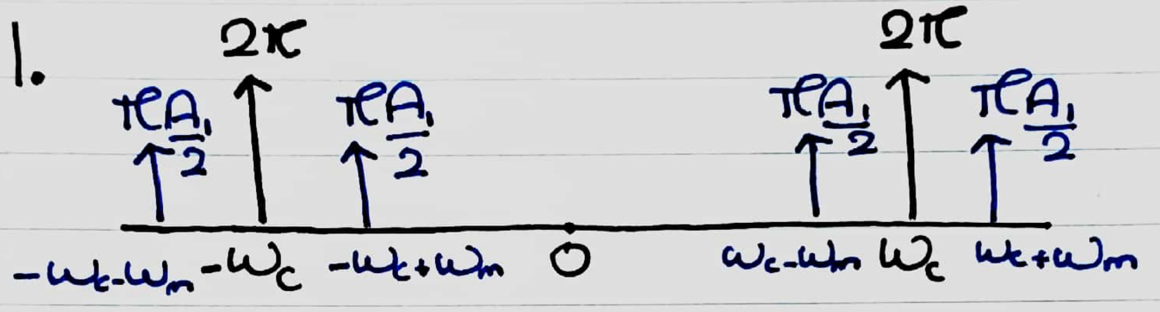
4-

$\Phi \to S, \; E \to A$

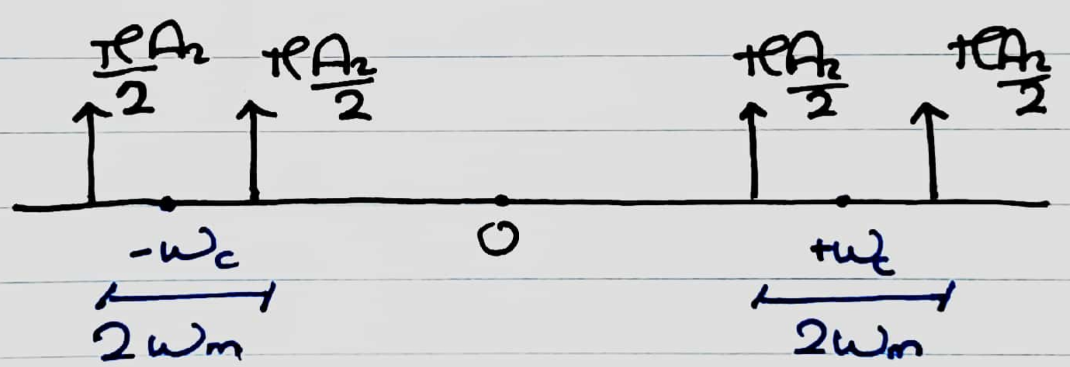
1. $S_1(t) = (2 + A_1 \cos W_m t) \cos W_c t$

2. $S_2(t) = A_2 \cos W_m t \cos W_c t$

a) 1.



2.



b) 1.

$A_c = 2$

For $M = \frac{A_1}{A_c} = 1$

need $A_1 = 2$

Satisfies 100% modulation in the large carrier signal

Same average Power in both:

$$P_{s_1} = \frac{A_c^2}{2} + \frac{A_m^2}{4} = \frac{2^2}{2} + \frac{2}{4} = 2.$$

A_1

$$P_{S_2} = \frac{A_2^2}{4}$$

For $P_{S_1} = P_{S_2}$ also need

C) ratio of output signals when applied to Synchronous detector:

↳ Multiplier + LPF

//Google

$$S_1(t) \cos \omega_c t = (2 + A_1 \cos \omega_m t) \frac{1}{2} (1 + \cos 2\omega_c t)$$

↳ Cleared by LPF

$$S_2(t) \cos \omega_c t = A_2 \frac{1}{2} (1 + \cos 2\omega_c t)$$

$\cos \omega_m t$

↳ Cleared by LPF

the ratio is thus $\dfrac{2 + A_1 \cos \omega_m t}{A_2 \cos \omega_m t}$ *

5-

"Tone Modulation"
"Sine wave Modulation"

- $m(t) = A_m \sin \omega_m t$

//Shifting Cosine doesn't change its Power.

- $\mu = 0.707$
- $P_T = 50$ KW

b) $$\frac{P_s}{P_T} = \frac{\mu^2}{2+\mu^2} = 0.2 = \frac{P_T - P_c}{P_T}$$

a) thus, $P_{c_{Act.}} = 40KW$

c) $$P_{c_{Actual}} = \frac{A_c^2}{2} \cdot \frac{1}{R} = 40 \times 10^3 = \frac{A_c^2}{2} \cdot \frac{1}{50}$$

$A_c = 2000$ Volt

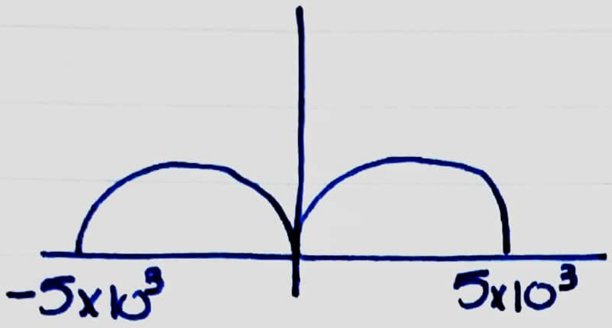
d) $$P_T = 40 \times 10^3 + \frac{A_m^2}{4} \cdot \frac{1}{R} = 45 \times 10^3$$

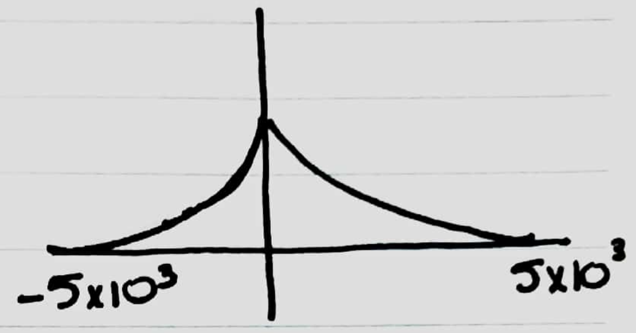
$A_m = 10^3$ V

$\rightarrow \mu = 10^3 / (2 \times 10^3) = 0.5$

$\rightarrow \eta = 0.5^2 / (2 + 0.5^2) = 11.11\%$

6.

$M_1(\omega)$ 

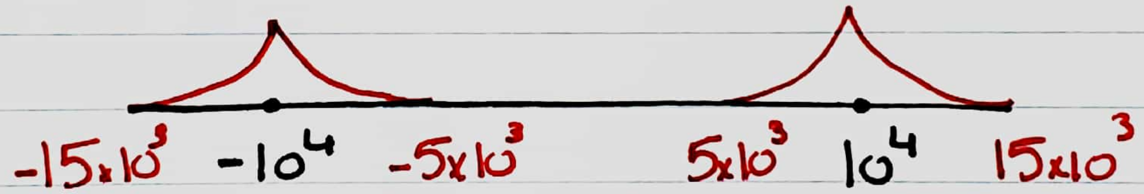
$M_2(\omega)$ 

a)

at a:

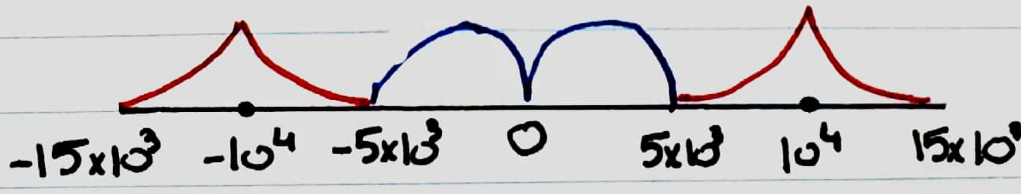
$S_2(t) = m_2(t)\, 2\cos 10^4 t$

$S_2(\omega) = M_2(\omega + 10^4) + M_2(\omega - 10^4)$

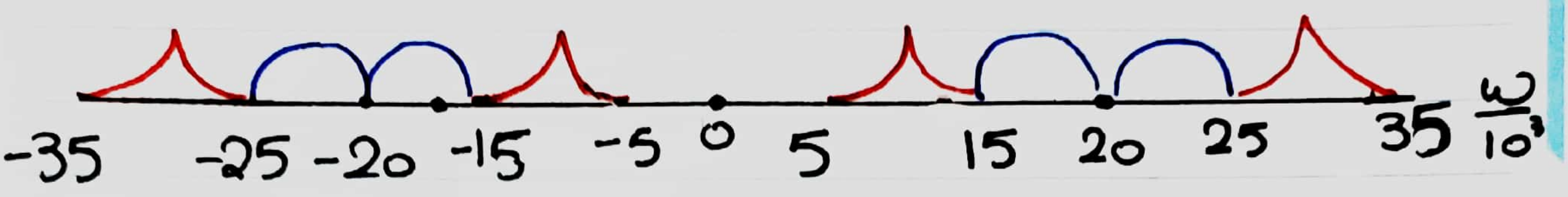


at b:

$S_b(t) = M_1(\omega) + S_2(\omega)$



At C



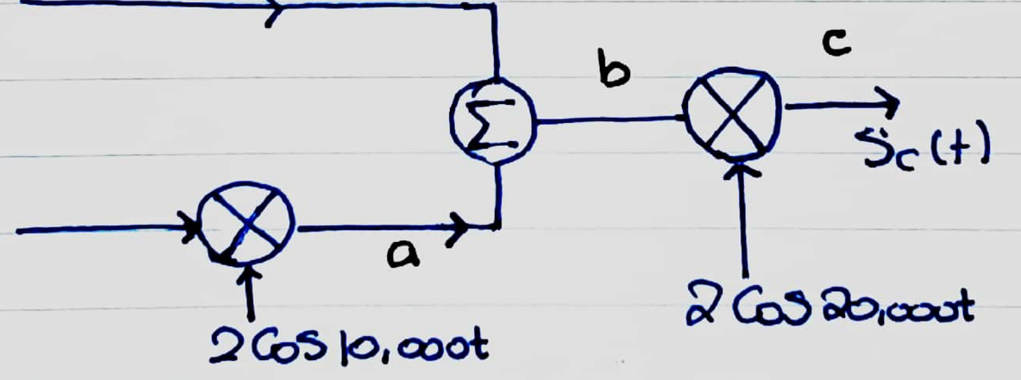
b) the bandwidth of the channel must be

$(35-5)\times10^3 = 30\times10^3$ rad/s or more.

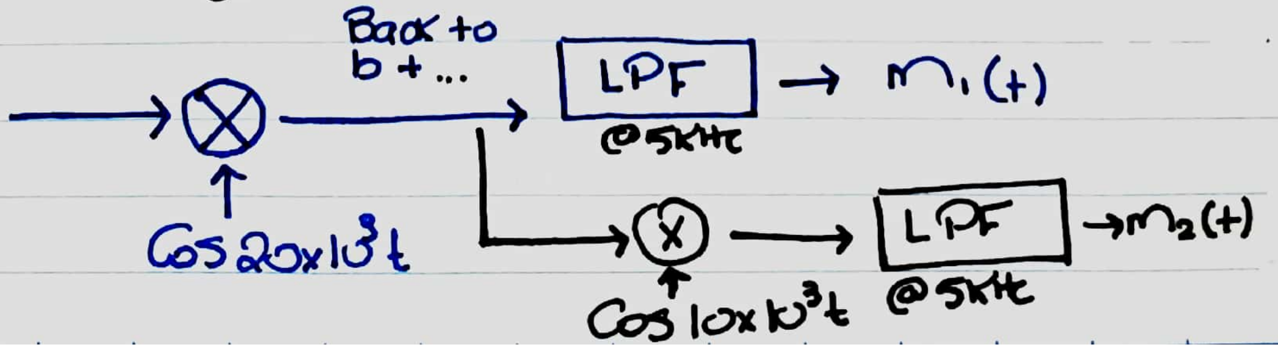
c)

Modulator

* if the signal we want is at w_c → convolve with w_c so its back to 0.
→ make sure no other signal come along the way. *



• According to how DSB-LC works



Bonus Check

to Confirm no images taking into account BW=15

20

0 40

25→55

* Conv.

$\cos 20\times10^3 t$

-10 10 30 50

15→45 35→65

* Conv.

$\cos 10\times10^3 t$

doesn't touch

0→5

-20

0 -40

... Symmetry with ←

$\frac{1}{4}\times2 + \frac{1}{4}\times2$

with each level A=A/2

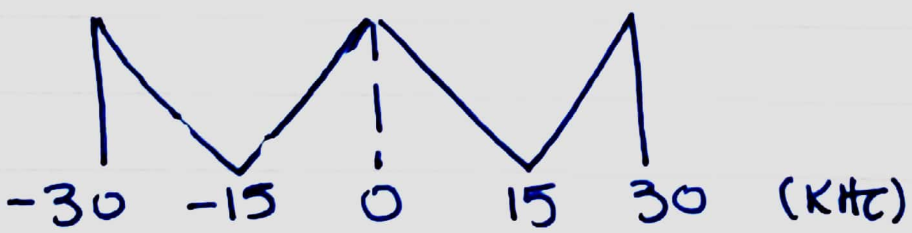
in the other tree the same will happen (10, -10)

7.

M(f) =

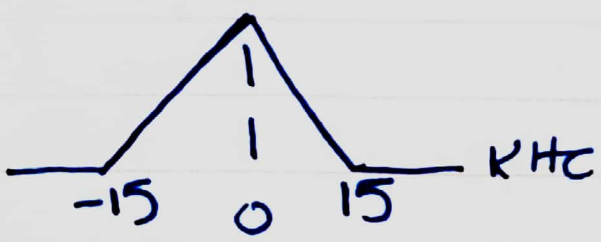
$* \pi(\delta(\omega-3\times10^4)+\delta(\omega+3\times10^4)).2$

$\cdot \frac{1}{2\pi}$



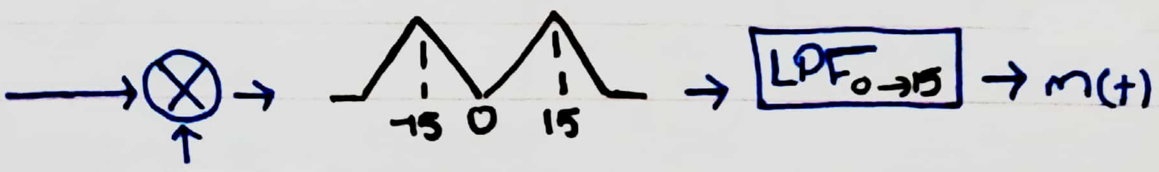
LPF (real filter)
0→15

Y(f)

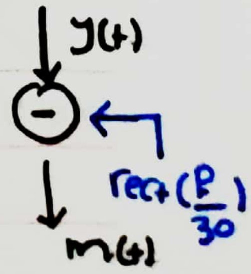


How about

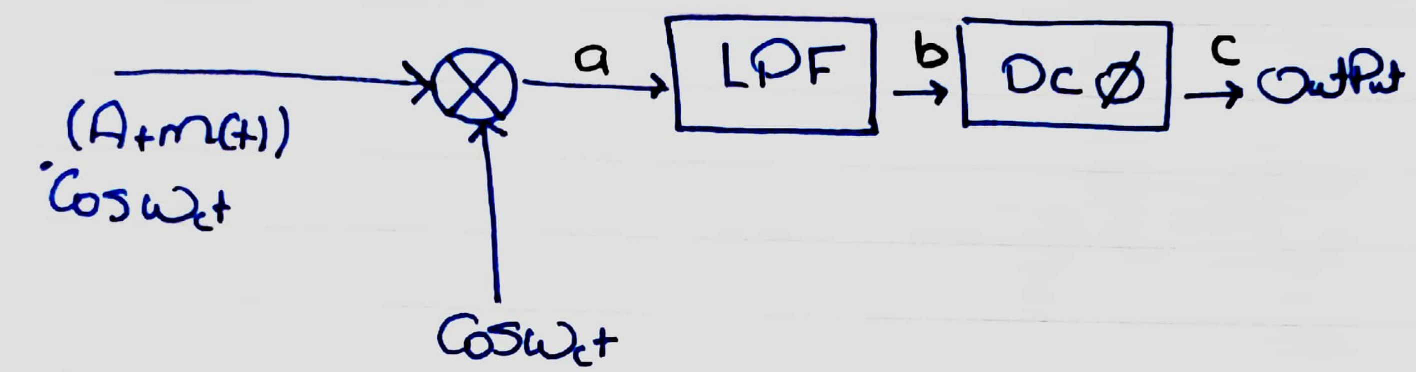
· Demodulation (Descrambling)



$\cos 15\times10^3 t$



8.

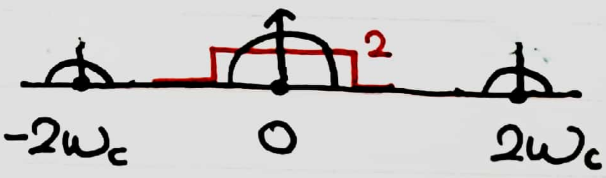


At a:

$$S(t) = (A+m(t))\cos\omega_c t \cos\omega_c t$$

$$= (A+m(t))\cdot\frac{1}{2}(\cos 2\omega_c t + 1)$$

Filtered by LPF



$$= (A+m(t))$$

//let the filter have gain 2

$$= m(t)$$

the dc blocker will block A

Indeed, if we use DSB-SC's expensive demodulator for DSB-LC modulated signals we can get $m(t)$ back regardless of A (no need for $A \geq |m(t)|_{min}$)

9-

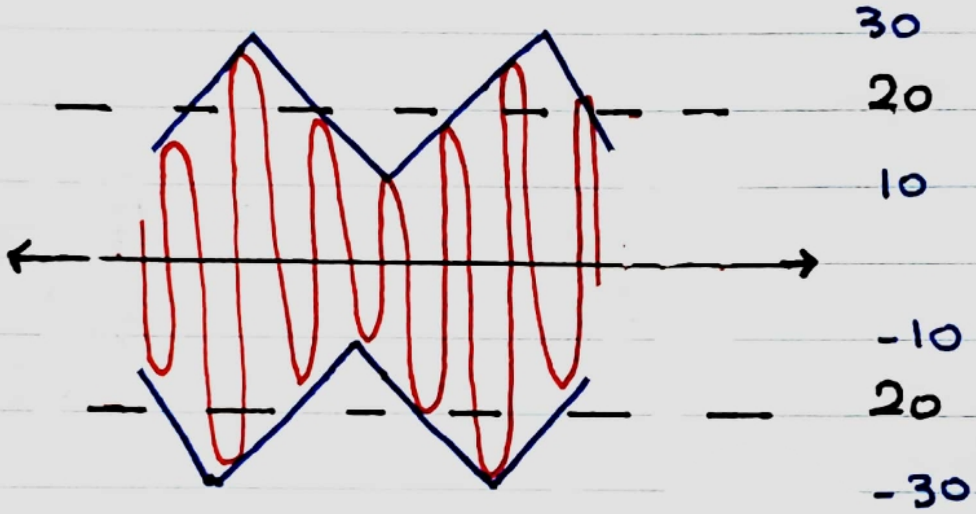
a) $\mu = 0.5$

#check Lec 3

$$\mu = \frac{|m(t)_{min}|}{A_c}$$

$$A_c = \frac{10}{\mu}$$

$A_c = 20$

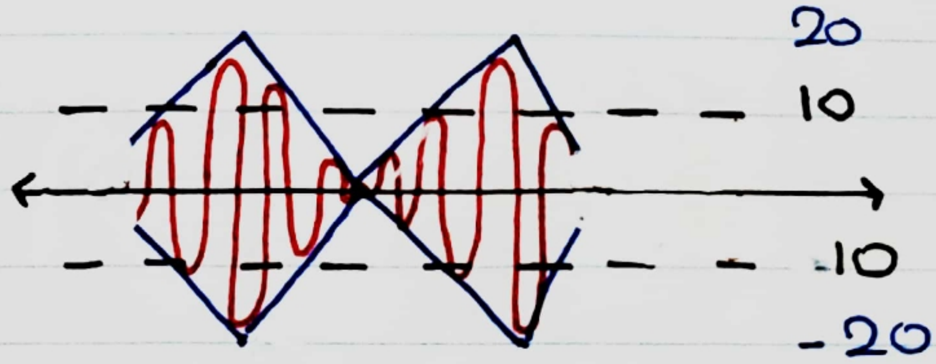


(red)

• The AM signal is what's in red.

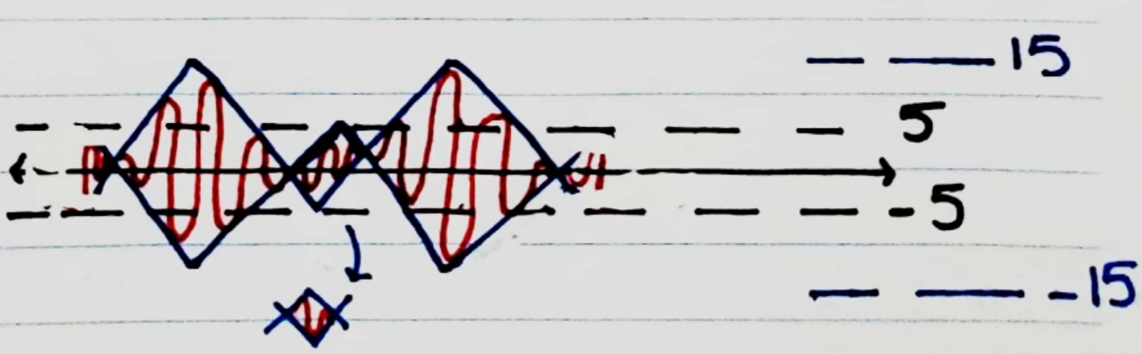
b) $\mu = 1$

$A_c = 10$



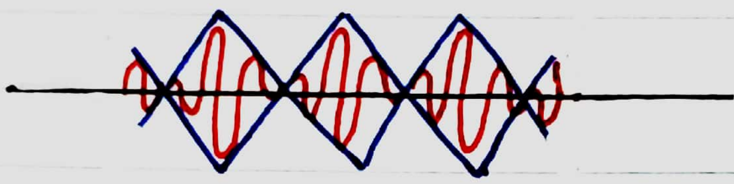
c) $\mu = 2$

$A_c = 5$



d) $\mu = \infty$

$A_c = 0$



$$S(t) = (m(t) + A_c)\cos w_c t = m(t)\cos w_c t$$

this is DSB-SC modulation

For $\mu = 0.8$ $\quad A_c = 12.5$ $\quad$ ($A_c = 10/\mu$ is the carrier's amplitude)

$$\text{Power}(A_c \cos(w_c t + \phi)) = \frac{A_c^2}{2} = 78.125$$

Recall,

$$S(t) = m(t)\cos w_c t + A_c \cos w_c t$$

↳ Side band Power

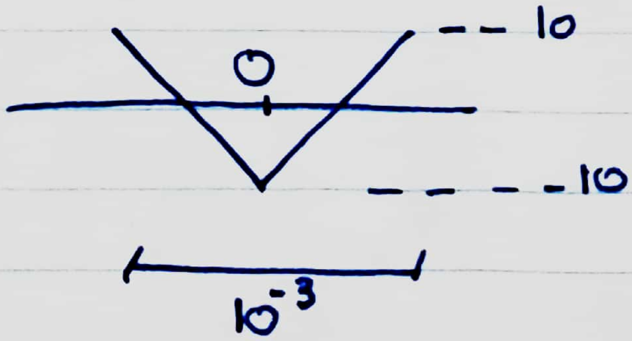
$\approx \frac{1}{2}\,\text{Power}(m(t))$

$= \frac{1}{2}\,\overline{m^2}(t)$

Lec. 3

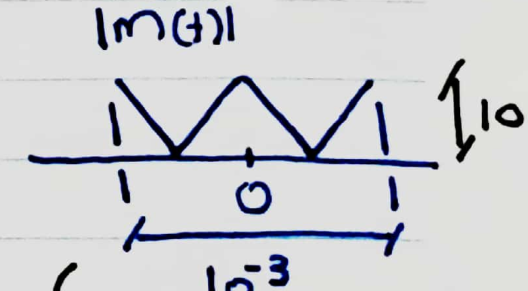
↳ Power of the carrier

$m(t)|_{\text{one Period}}$



→

$|m(t)|$



Can be divided into 4 equal Pieces of area, so is $|m(t)|^2$

$$\text{slope} = \frac{10-0}{0-\frac{10^{-3}}{4}} = -4\times10^{4}, \quad \text{intercept} = 10$$

Area under $|m(t)|^2 =$

$$4 \times \int_0^{10^{-3}/4} (10 - 4\times10^{4} t)^2 \, dt$$

$$= \frac{1}{30}$$

doesn't really matter where you set the origin (think of $\sum |a_k|^2$)

$$\rightarrow \overline{m^2(t)} = \frac{\text{area}}{T} = \frac{1}{30} \cdot \frac{1}{10^{-3}} = \frac{100}{3}$$

$$\text{Power}\Big|_{\substack{\text{side}\\ \text{band}}} = \frac{100}{3} \cdot \frac{1}{2} = \frac{100}{6}$$

$$\eta = \frac{P_s}{P_s + P_c} = \frac{100/6}{100/6 + 78.125} = \boxed{22.5\%}$$

best efficiency at $\mu = 1$.

Note that if $m(t) = A_m \cos \omega_m t$ then

$$P_s = \underbrace{\frac{1}{2}\overline{m^2(t)}}_{\text{Power of } m(t)} = \frac{1}{2}\left(\frac{1}{2}A_m^2\right) = \frac{1}{4}A_m^2 \quad \text{and} \quad \mu = \frac{A_m}{A_c}$$

// Lec. 3, Page 5

$$\eta = \frac{A_m^2/4}{\frac{A_c^2}{2} + \frac{A_m^2}{4}} = \frac{(\mu A_c^2)^2/2}{A_c^2 + \frac{(\mu A_c)^2}{2}} = \frac{\mu^2}{2+\mu^2}$$

10a.

$$\xrightarrow{a} \boxed{\text{Crystal Detector}} \xrightarrow{b} \boxed{\text{LPF}} \xrightarrow{c} \boxed{\text{DC Block}} \xrightarrow{d}$$

(squaring)

a:

$$S(t) = (A + m(t)) \cos \omega_c t$$

// DSB-LC used for radio.

b:

$$S_b(t) = (A + m(t))^2 \cos^2 \omega_c t$$

$$= (A^2 + 2Am(t) + m^2(t)) \frac{1}{2} (1 + \cos 2\omega_c t)$$

C:

$$S_c(t) = \frac{1}{2} (A^2 + 2Am(t) + m^2(t)) (1 + 0)$$

d:

$$S_d(t) = \frac{1}{2} (0 + 2Am(t) + m^2(t))$$

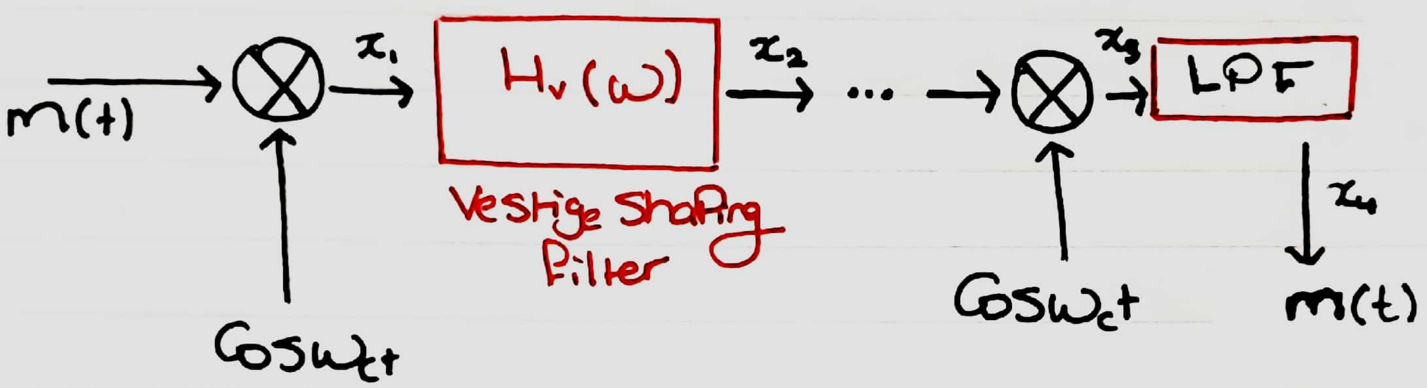
$$= \underbrace{Am(t)}_{\text{what we want}} + \underbrace{m^2(t)/2}_{\text{distortion}}$$

∴ if $A \gg m(t)$ then $Am(t) \gg m^2(t)$ hence, distortion is small relative to the wanted signal.

11.

VSB Modulation

//Recap



at x_1:

$$S_1(t) = m(t)\cos\omega_c t$$

at x_2:

$$S_2(\omega) = \frac{1}{2}\left(M(\omega-\omega_c) + M(\omega+\omega_c)\right) H_v(\omega)$$

at x_3:

$$S_3(\omega) = \frac{1}{2}\left(S_2(\omega-\omega_c) + S_2(\omega+\omega_c)\right)$$

LPF around M(ω)'s bandwidth

$$= \frac{1}{4}\left(M(\underline{\omega-2\omega_c}) + M(\omega)\right) H(\omega-\omega_c)$$

$$+ \frac{1}{4}\left(M(\omega) + M(\underline{\omega+2\omega_c})\right) H(\omega+\omega_c)$$

at x_4:

$$S_4(\omega) = \frac{1}{4}\left(M(\omega)\right)\left(H(\omega+\omega_c) + H(\omega-\omega_c)\right)$$

So either that $H(\omega+\omega_c)+H(\omega-\omega_c)=1$ (Const.)

for $\underbrace{-2\pi B<\omega<2\pi B}_{\text{Bandwidth}}$ or equalizer must be there to
↳ extra stage

multiply the spectrum by $$\left.\frac{1}{H(\omega+\omega_c)+H(\omega-\omega_c)}\right\}$$ the equalizer's transfer function.

Given:

→ $f_c = 10\,KHz$ BW= 4KHz

. the given band Pass filter clearly exhibits no odd Symmetry with 50% response at ω_c (will need an equalizer)

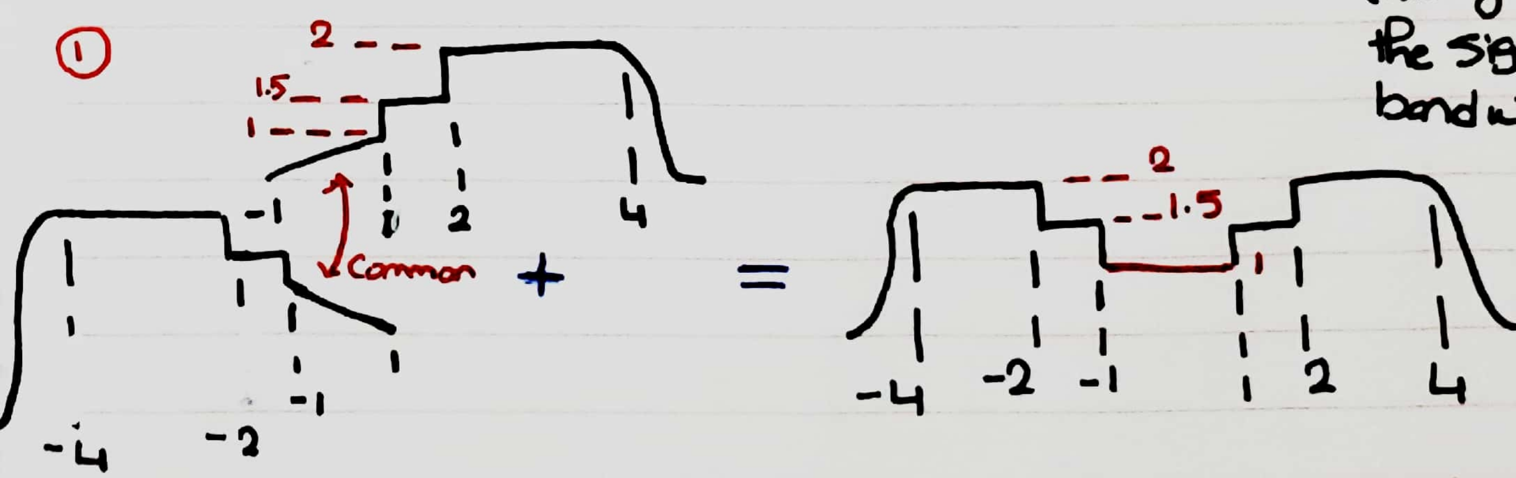
* Need to Compute $H(f+f_c)+H(f-f_c)$

Given is  } H(f)

gets shifted $+f_c$ ② and $-f_c$

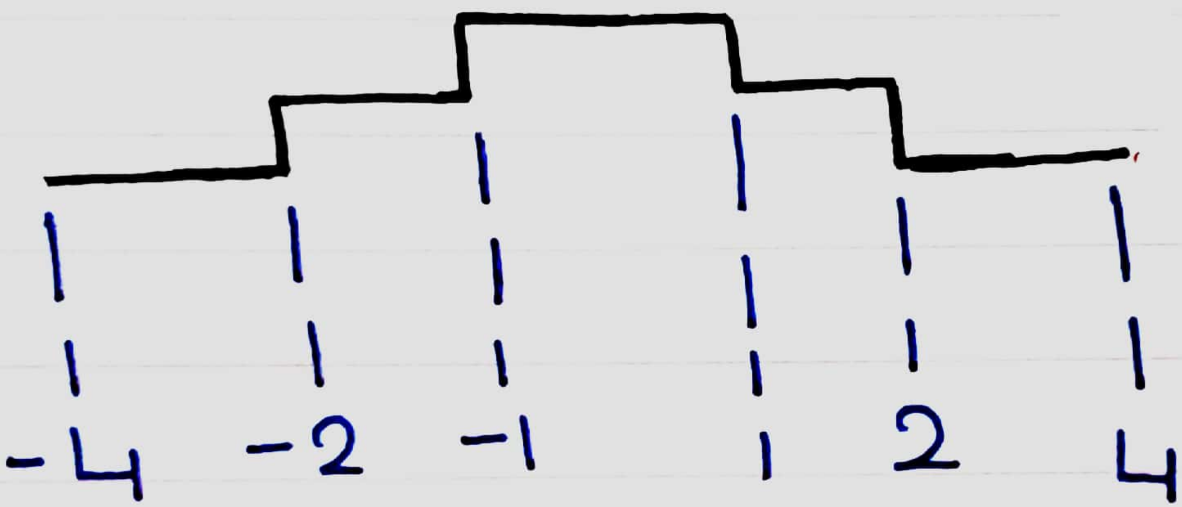
gets shifted by $-f_c$ ① and $+f_c$

« Not important (away from the signals bandwidth)



} the Part Outside -4+4 is outside BW

The transfer function of the equaliser here should be



Rest is outside the signal's bandwidth and should be cleared by LPF regardless.

12.

$f_{carrier} = 1500$ KHz $\quad f_I = 455$ KHz

Inexpensive radio receiver

→ Poor Selectivity @ RF Stage

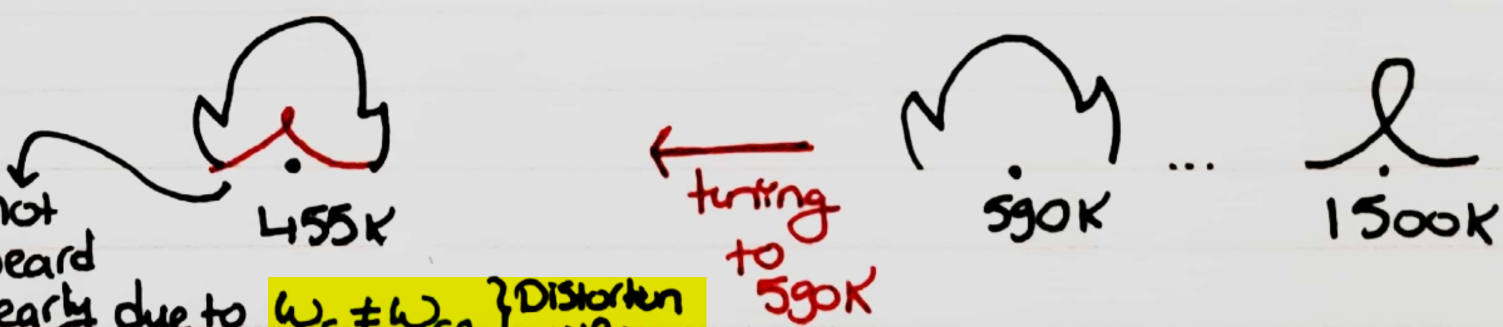
.It's heard loud and clear when the RF is tuned to 1500 KHz

» the signal's central frequency is at 1500 KHz

→ At another dial setting (Common tuning of local oscillator and RF Section to f_{c2}) the signal at $f_c = 1500$ KHz will be heard if

$$f_c = f_{c2} + 2f_I$$

that is $f_{c2} = 590$ KHz and in this case the former signal is an image frequency relative to this one.



not heard clearly due to $\omega_c \neq \omega_{c2}$ } Distortion with time