CMP205: Computer Graphics

جامعة القاهرة

Lecture 3: Transformations II

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Fall 2018

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Agenda

- Transformations Vs Coordinate Change
- Arbitrary 3D Rotations
- Transforming Normal Vectors
- Coordinate Transformation
- Windowing Transforms

Acknowledgments: Some slides adapted from Steve Marschner and Fredo Durand.

Transformation Vs Coordinate Change

We can view the same rotation matrix in two ways:

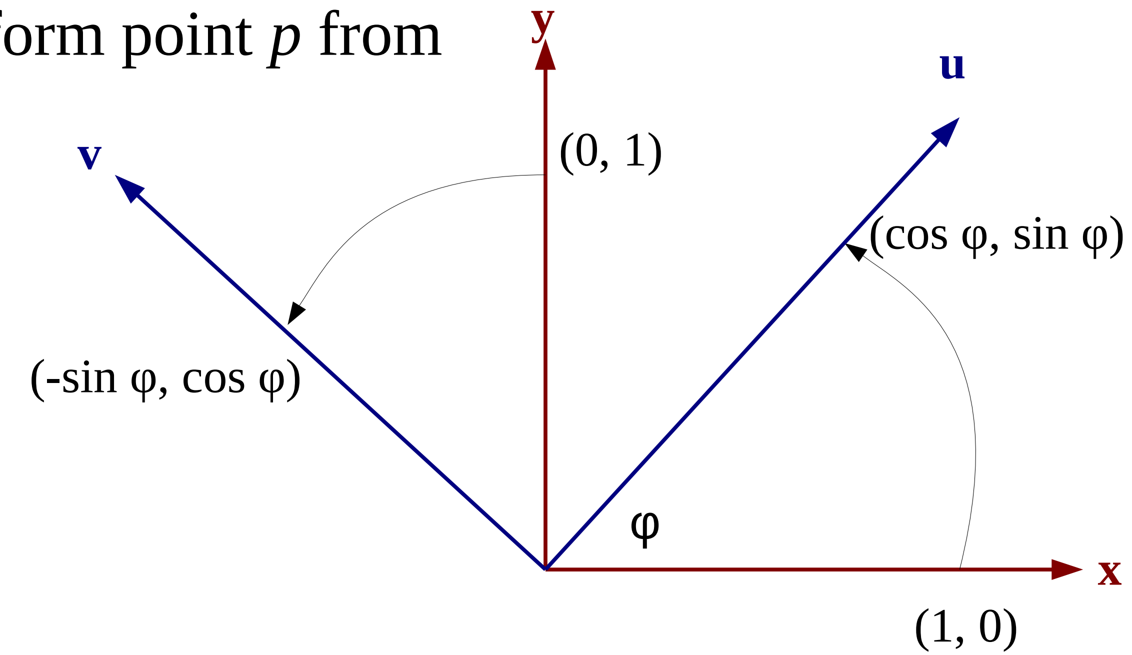
1) As a transformation matrix to transform point p to point p' in the same frame

$$R = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix}$$

2) As a coordinate change to transform point p from frame uv to frame xy

$$\begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos\phi \\ \sin\phi \end{bmatrix}$$

$$\begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -\sin\phi \\ \cos\phi \end{bmatrix}$$



Transformation Vs Coordinate Change

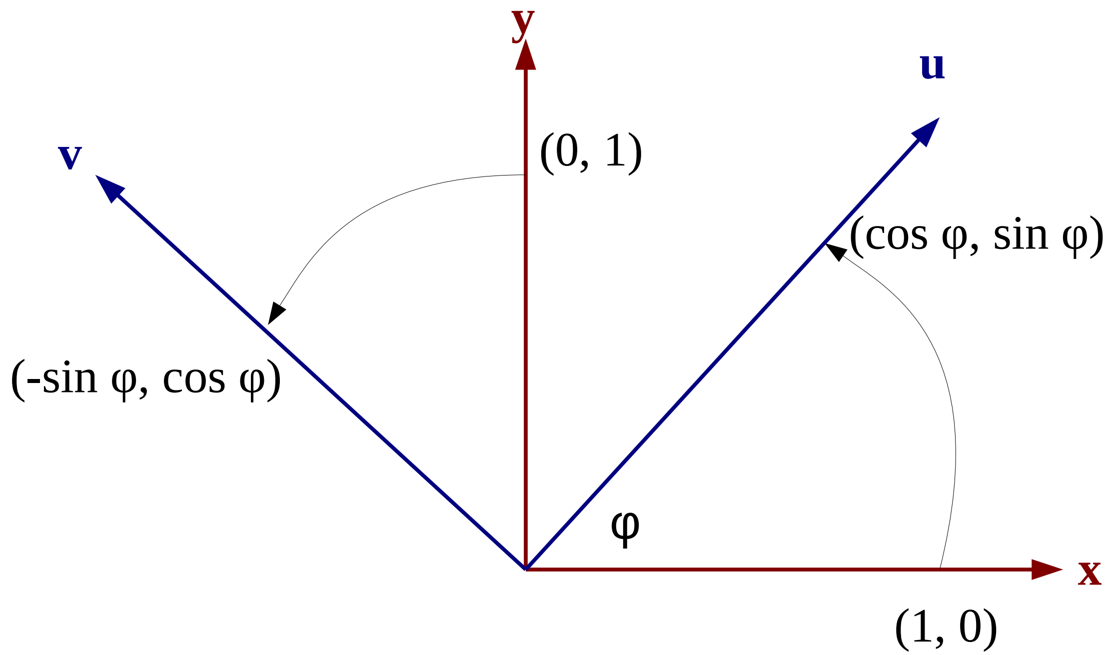
$$R = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix}$$

Transformation:

$$p' = R\,p$$

Coordinate Change:

$${}^{xy}p = R\,{}^{uv}p$$



R transforms points in xy coordinates OR transforms uv coordinates to xy coordinates

What about R^T?

Arbitrary Rotation

A 3x3 unitary matrix can represent arbitrary rotation around any axis

$$R = \begin{bmatrix} x_u & y_u & z_u \\ x_v & y_v & z_v \\ x_w & y_w & z_w \end{bmatrix} = \begin{bmatrix} u^T \\ v^T \\ w^T \end{bmatrix}$$

$$R R^T = I$$

$$Ru = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = x$$

R takes (or rotates) uvw to xyz

$$R^T x = \begin{bmatrix} x_u \\ y_u \\ z_u \end{bmatrix} = u$$

R^T takes (or rotates) xyz to uvw

Arbitrary Rotation

- To rotate about an arbitrary axis *a* that passes through the origin with an angle φ:
  - Create axes *uvw* s.t. *w* coincides with *a*
  - Change *xyz*-frame to *uvw*-frame using *R* (Recall that *R* rotates *uvw* to *xyz*)
  - Perform the rotation in *uvw* around *w*-axis (vector *a*)
  - Change back to *xyz*-frame using R^T

$$\begin{bmatrix} x_u & x_v & x_w \\ y_u & y_v & y_w \\ z_u & z_v & z_w \end{bmatrix} \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_u & y_u & z_u \\ x_v & y_v & z_v \\ x_w & y_w & z_w \end{bmatrix}$$

converting uvw to xyz

rotate around w

converting xyz to uvw

Now, how do we know *uvw*?

Arbitrary Rotation

$$w = \frac{a}{\|a\|}$$

get unit vector in the direction we want

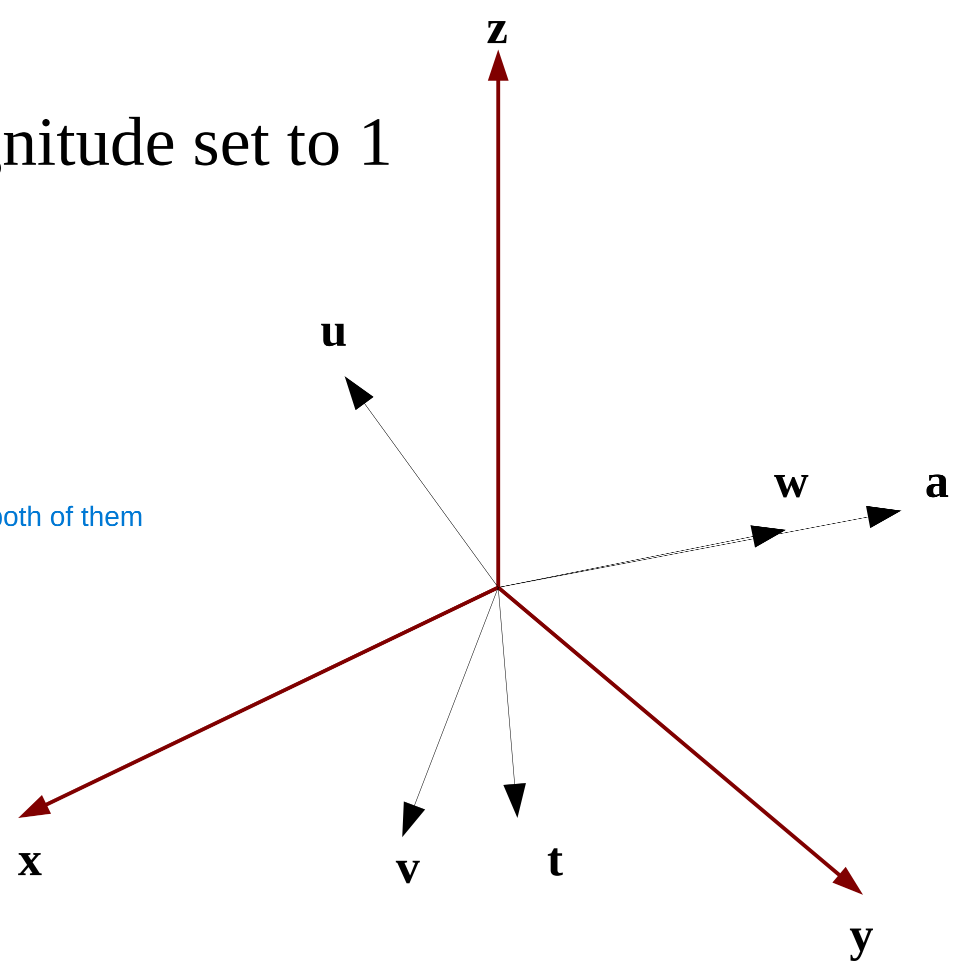
$t = w'$ i.e. w with lowest magnitude set to 1

$$u = \frac{t \times w}{\|t \times w\|}$$

get a perpendicular unit vector on it

$$v = w \times u$$

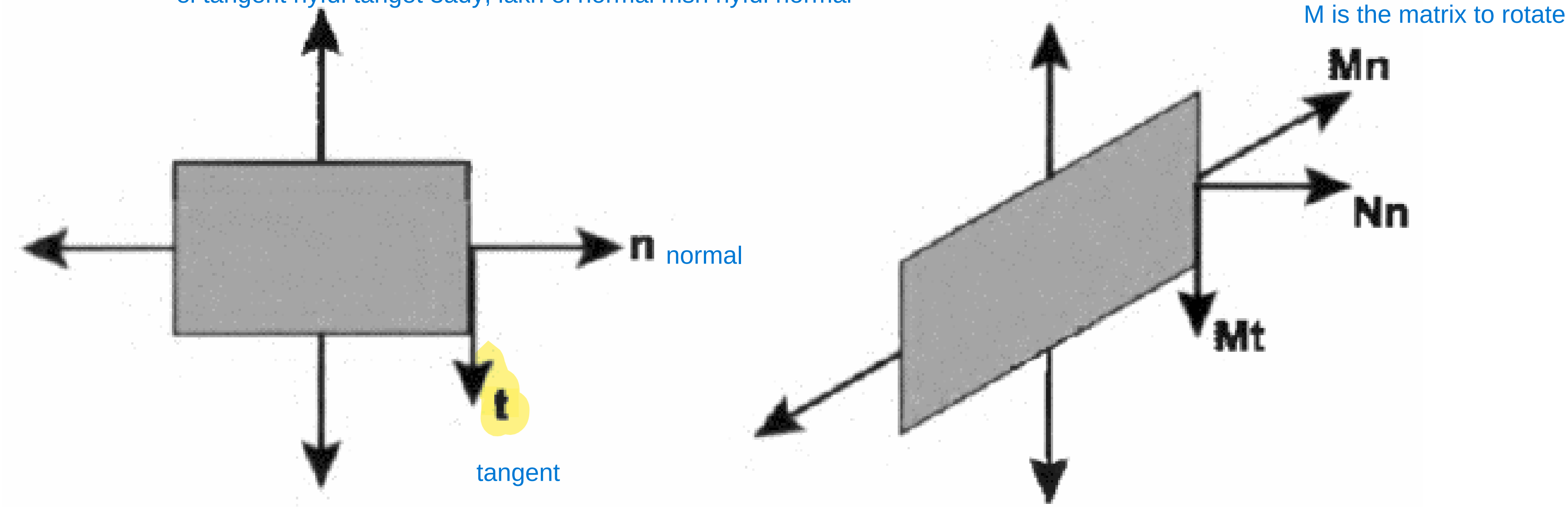
get a third vector perpendicular on both of them



Transforming Normal Vectors

el fekra en ay surface bn3rf leha hagten
normal -> vector perpendicular on it
tangent -> vector perpendicular on the normal

lama bn3ml rotation:
el tangent hyfdl tanget 3ady, lakn el normal msh hyfdl normal



Mn is not normal to the surface!

What is N?

fa 34an keda 3auzen ne7sb N elly lama adrbha fe el normal el adem ydeene el normal el gded

Transforming Normal Vectors

Derivation

$n' = N\, n$ and $t' = M\, t$

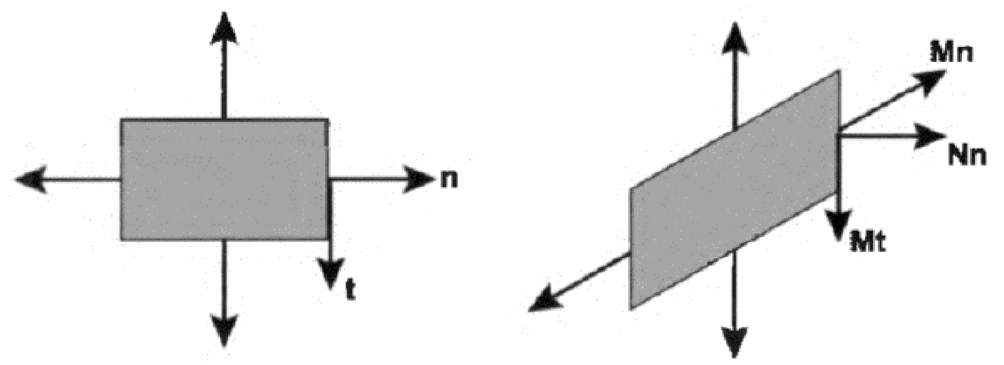
$n^T t = 0$

$n^T M^{-1} M\, t = 0$

$\left(n^T M^{-1}\right)\left(M\, t\right) = 0$

$\left(\left(M^{-1}\right)^T n\right)^T \left(M\, t\right) = 0$

$\left(n'\right)^T t' = 0$



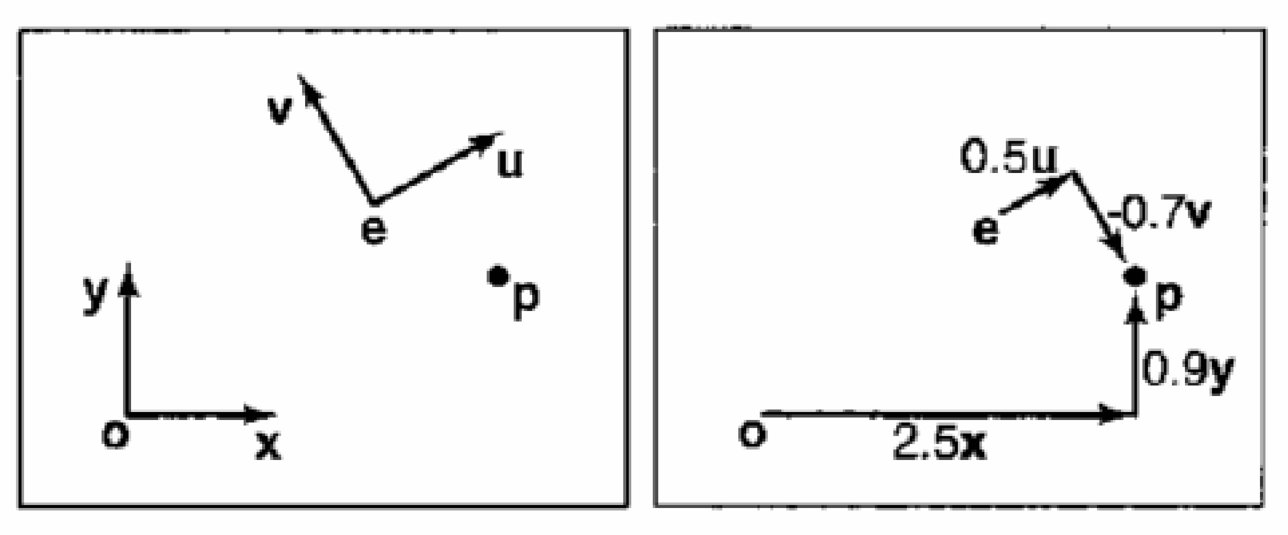
$$N = \left(M^{-1}\right)^T$$

N hya el transpose bta3 inverse el matrix el enta 3mlt beha rotation

Coordinate Transformations

lama kona bn3ml rotation fl awl, kona moftreden en el etnen lehom nfs el origin.
bs da msh lazm. fa 34an keda 3auzen nshof hantsrf ezay baa law el origins mo5tlfa

$$\boldsymbol{p} = \boldsymbol{o} + x_p \boldsymbol{x} + y_p \boldsymbol{y}$$
$$\boldsymbol{p} = \boldsymbol{e} + u_p \boldsymbol{u} + v_p \boldsymbol{v}$$



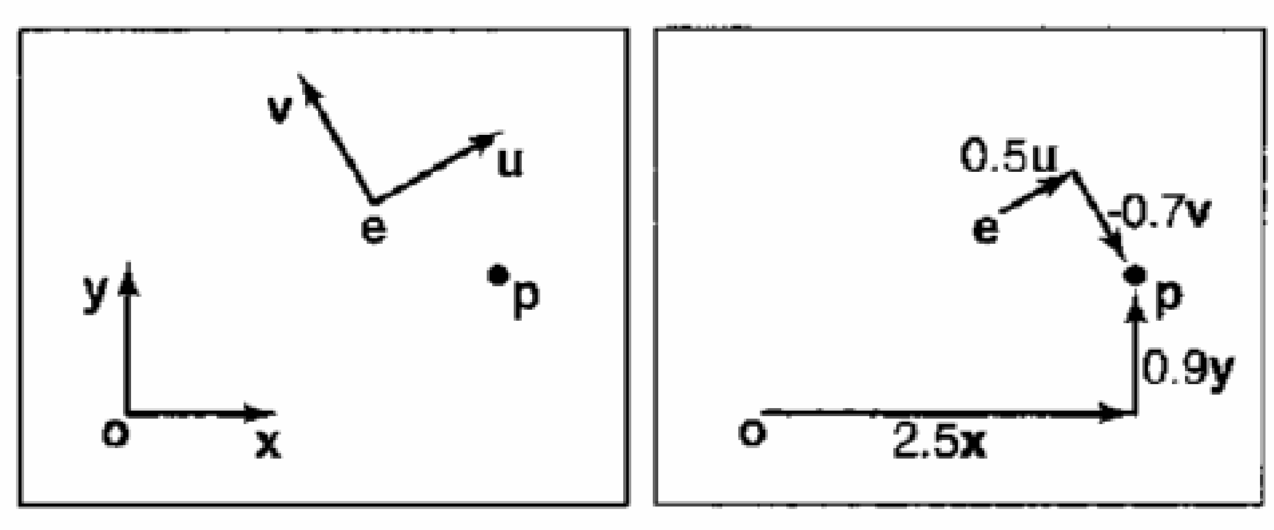
How to find (x_p, y_p) from (u_p, v_p) and vice versa?

Coordinate Transformations

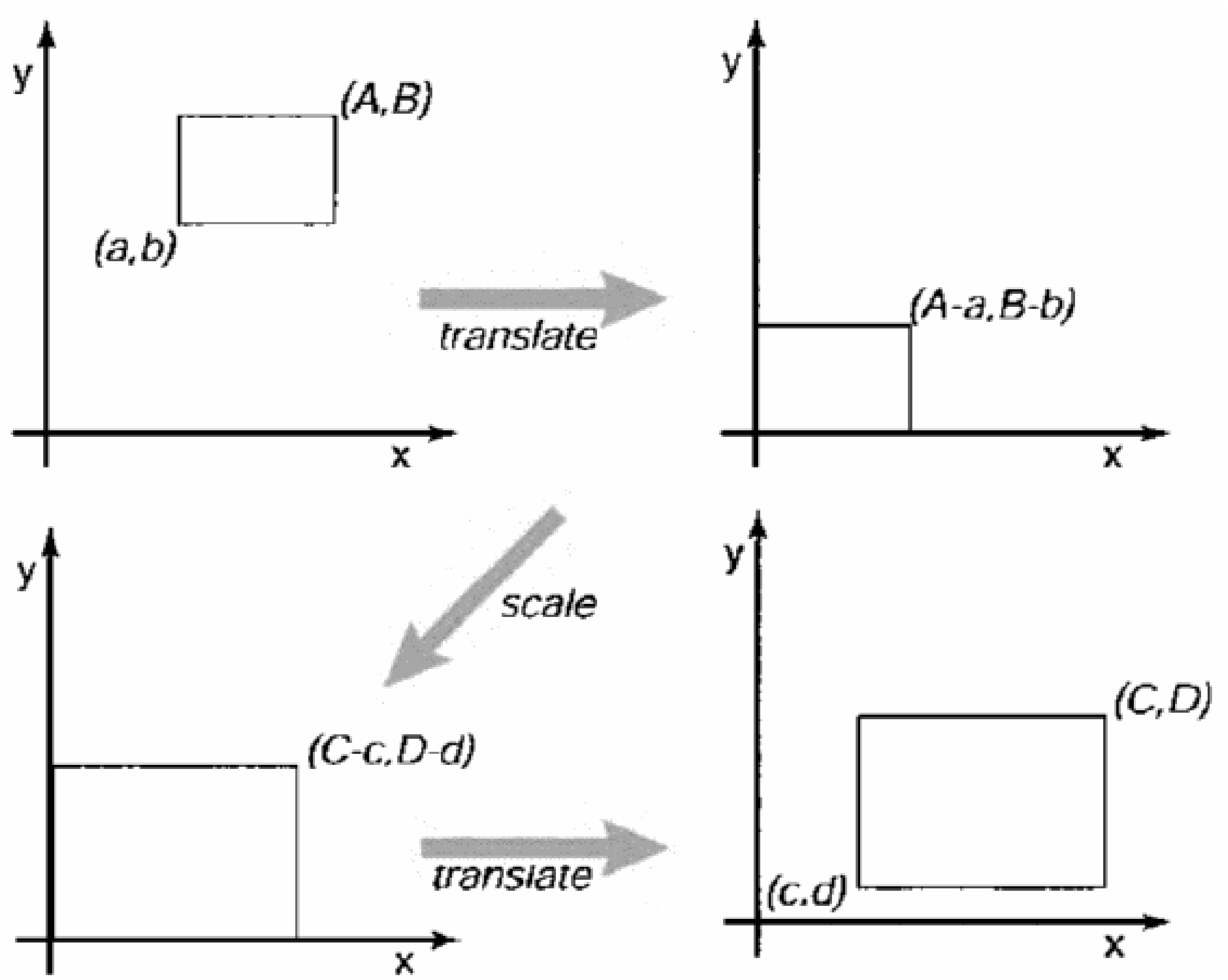
shift the point to the e (ex,ey) rotate it into xy plane multiply by the uvw unit vectors

$$\begin{bmatrix} x_p \\ y_p \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & x_e \\ 0 & 1 & y_e \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_u & x_v & 0 \\ y_u & y_v & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_p \\ v_p \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} u_p \\ v_p \\ 1 \end{bmatrix} = \begin{bmatrix} x_u & y_u & 0 \\ x_v & y_v & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -x_e \\ 0 & 1 & -y_e \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_p \\ y_p \\ 1 \end{bmatrix}$$

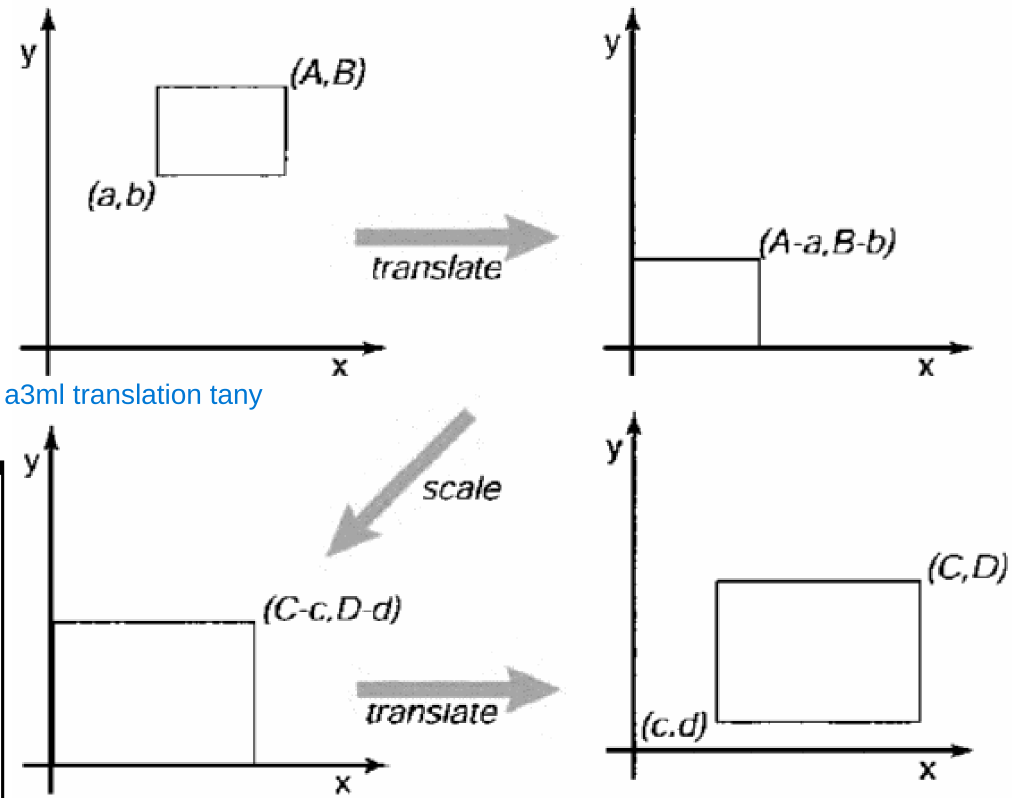


Windowing Transforms



Windowing Transforms

$$\text{translate}(-a,-b)=\begin{bmatrix}1 & 0 & -a\\ 0 & 1 & -b\\ 0 & 0 & 1\end{bmatrix}$$



w b3ml -c 34an lama hagy a3ml translation tany awdeha 3nd el C bzbt.

34an a5ly el corner mazbot C-c

$$\text{scale}\left(\frac{C-c}{A-a},\frac{D-d}{B-b}\right)=\begin{bmatrix}\frac{C-c}{A-a} & 0 & 0\\ 0 & \frac{D-d}{B-b} & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$\text{translate}(c,d)=\begin{bmatrix}1 & 0 & c\\ 0 & 1 & d\\ 0 & 0 & 1\end{bmatrix}$$

$$\begin{bmatrix}\frac{C-c}{A-a} & 0 & \frac{cA-Ca}{A-a}\\ 0 & \frac{D-d}{B-b} & \frac{dB-Db}{B-b}\\ 0 & 0 & 1\end{bmatrix}$$

Recap

- Transformations Vs Coordinate Change
- Arbitrary 3D Rotations
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