Ch4 Transient Response [3]

1) 1^{st} order:

$$c(s) = \underbrace{1/s}_{r(s)} \; \underbrace{\frac{K}{(s+P)}}_{TF}$$

2] 2^{nd} order

$$\Rightarrow s^2 + 2\zeta\omega_n s + \omega_n^2$$

3] 

* كل ما ال pole يبعد عن ال origin يبقى
هـ settle أسرع

4] $$c(t) = M\Big[1 - \underbrace{\frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin(\omega t + \psi)}\Big]$$

to settle, this term = 0
at $c(t) = \square\% M$

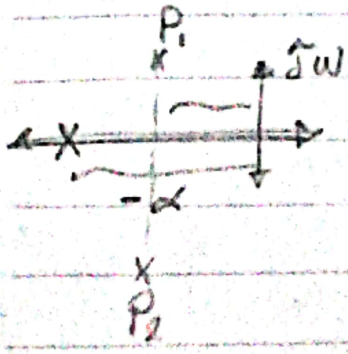
$$* \; -\zeta\omega_n t = \ln(\square\%)$$

$$t_s = \frac{-\ln(\square\%)}{\zeta\omega_n}$$

at 5% $\rightarrow t_s = \frac{3}{\zeta\omega_n}$, at 2% $\rightarrow t_s = \frac{4}{\zeta\omega_n}$

4] 3^{rd} order $\rightarrow \alpha > 5\zeta\omega_n$

$$\frac{\quad}{(s+\alpha)(s+P_1)(s+P_2)} \Rightarrow \frac{\square}{s+\alpha} + \frac{\square}{s+P_1} + \frac{\square}{s+P_2}$$



$$e^{-\alpha t} + \quad\sim\sim\sim\quad \sin(\;)$$

5] $\zeta = \frac{-\ln(Mp)}{\sqrt{\pi^2 + (\ln Mp)^2}}$

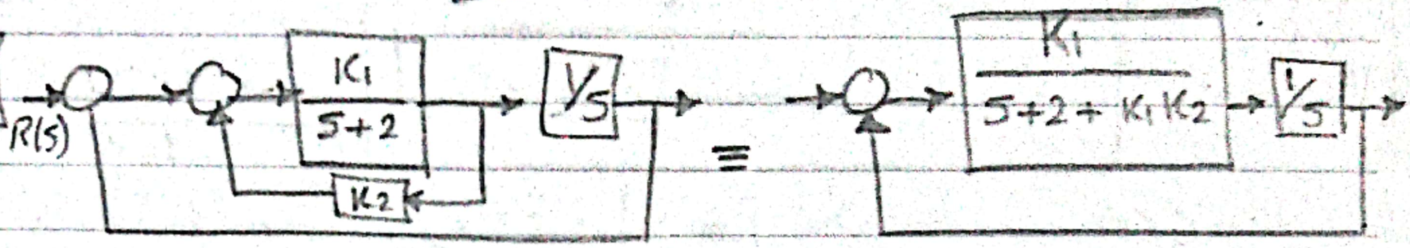
6] trising $\rightarrow \frac{\pi - \cos^{-1}\zeta}{\omega n \sqrt{1-\zeta^2}} \begin{matrix} \psi \\ \omega \end{matrix}$

7] $t_{settle} = \underset{2\%}{\frac{4}{\zeta \omega n}}, \underset{5\%}{\frac{3}{\zeta \omega n}}$

8] $t_{Peak} = \frac{\pi}{\omega n \sqrt{1-\zeta 2}}$

Sh3

Q.1



$$TF = \frac{G(s)}{1+GH(s)} = \frac{K_1/s+2}{1+\frac{K_1K_2}{s+2}} = \frac{K_1}{s+2+K_1K_2}$$

$\because Mp = 0.25$

$Mp = f(\zeta)$

$$Mp = \exp\left[\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}\right]$$

$$\zeta = \frac{-\ln(Mp)}{\sqrt{\pi^2+(\ln Mp)^2}} = 0.403$$

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$$

$$\omega_n = \frac{\pi}{t_p\sqrt{1-\zeta^2}} = 1.716\ r/s$$

$$G(s) = \frac{K_1}{s[s+(2+K_1K_2)]}, \quad H(s)=1 \implies c/cs: 1+GH(s)$$

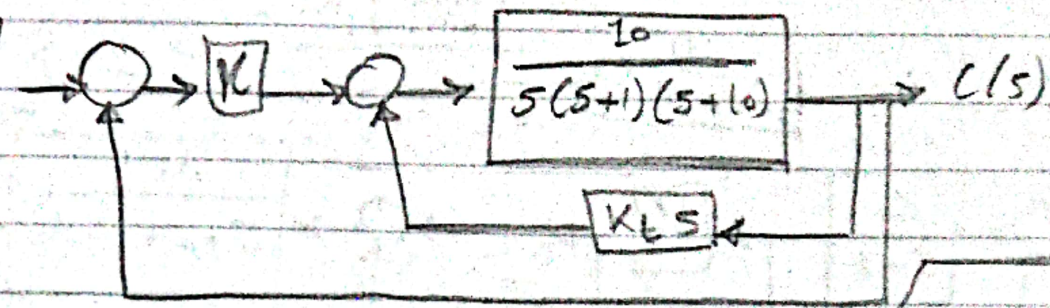
$$\left.\begin{aligned} c/cs: s^2+s(2+K_1K_2)+K_1 \\ s^2+s(2\zeta\omega_n)+\omega_n^2 \end{aligned}\right\} \begin{aligned} K_1 &= \omega_n^2 \\ 2+K_1K_2 &= 2\zeta\omega_n \end{aligned}$$

$\therefore K_1 = 2.945, \quad K_2 = -0.2095$

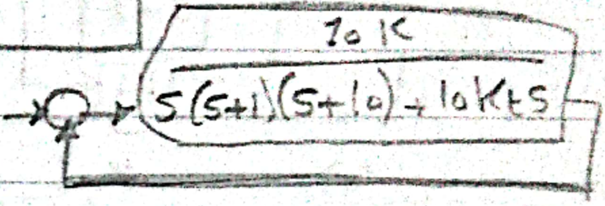
* معنى إنا بنعمل ديزاين لـsys بمواصفات معينة!

يعني الsys ده stable!

Q:2



$$TF = \frac{G(s)}{1+GH(s)} = \frac{Nom}{den + Nom * H(s)} \Rightarrow$$



* Specifications → $\zeta = 0.707$, $K_v = 1$

$$K_v = \lim_{s \to 0} s \underbrace{GH(s)}_{TF} = \frac{10K}{10 + 10K_t} = 1 \Rightarrow K = 1 + K_t$$

$$C/Cs \Rightarrow 1 + GH(s) = s^3 + 11s^2 + (10 + 10K_t)s + 10K$$

$$(s+\alpha)(s^2 + 2\zeta\omega_n s + \omega_n^2) = s^3 + s^2(2\zeta\omega_n + \alpha) + (2\alpha\zeta\omega_n + \omega_n^2)s + \alpha\omega_n^2$$

$\therefore$ $s^2 \rightarrow 2\zeta\omega_n + \alpha = 11$ (1) $\quad$ $s^0 \rightarrow \alpha\omega_n^2 = 10K$ (3)

$s \rightarrow 10(1 + K_t) = 2\alpha\zeta\omega_n + \omega_n^2$ (2) $\quad$ $K_v \rightarrow K = 1 + K_t$ (4)

(4) in (2) → $10K = 2\alpha\zeta\omega_n + \omega_n^2$ (5)

(1) in (5) → $10K = (11 - 2\zeta\omega_n) \cdot 2\zeta\omega_n + \omega_n^2$ (6)

(1) in (3) → $(11 - 2\zeta\omega_n)\omega_n^2 = 10K$ (7)

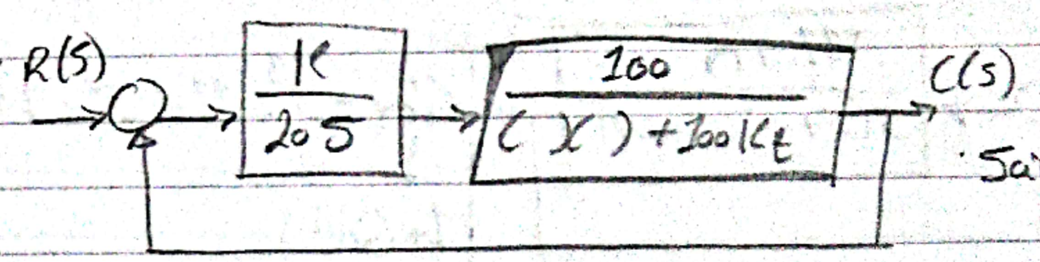
(6) = (7) → $(11 - 2\zeta\omega_n)\omega_n^2 = (11 - 2\zeta\omega_n) \cdot 2\zeta\omega_n + \omega_n^2$, $\zeta = 0.707$

→ $\omega_n = 6.8895$ or $\omega_n = 1.59663$

$5\zeta\omega_n = 24$	$5\zeta\omega_n = 5.644$	To approx 3rd order to 2nd order!
$\alpha = 1.258$	$\alpha = 8.742$	
$\alpha < 5\zeta\omega_n$	$\alpha > 5\zeta\omega_n$	
✗	✓	

$\therefore$ $K = 2.23$, $K_t = 1.23$

Q:3



Same as Sh3 Q:4

Relquirments $\Rightarrow M_p = 0.043$, $t_r = 2$ sec

$$\zeta = \frac{-\ln M_p}{\sqrt{\pi^2 + (\ln M_p)^2}} = 0.707 \quad \Big| \quad t_r = \frac{\pi - \cos^{-1}(\zeta)}{\omega_n \sqrt{1-\zeta^2}} \quad \omega_n = 1.66 \text{ r/sec}$$

C/Cs $\Rightarrow 1 + GH(s) = 0.05s^3 + 0.6s^2 + (1+100K_t)s + 5K = 0 \quad \times 20$

$$s^3 + 12s^2 + (20 + 20000K_t)s + 100K = 0$$

$$(s+\alpha)(s^2 + 2\zeta\omega_n s + \omega_n^2) = s^3 + s^2(2\zeta\omega_n + \alpha) + s(\omega_n^2 + 2\alpha\zeta\omega_n) + \alpha\omega_n$$

$s^2: 2\zeta\omega_n + \alpha = 12$ (1) $\quad s^1: \omega_n^2 + 2\alpha\zeta\omega_n = 20(1+K_t)$ (2)

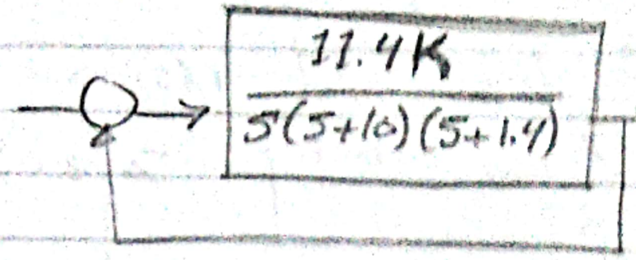
$s^0: 100K = \alpha\omega_n^2$ (3)

$\alpha = 9.639$, $K = 0.268$, $K_t = 0.00277$ $\Big|$ $5\zeta\omega_n = 5.9 < \alpha$

$$K_v = \lim_{s\to 0} s\,GH(s) = \frac{5K}{1 + 100K_t} = 1.05$$

$e_{ss} = 1/K_v = 0.9523$ $\quad \because$ Input is a unit ramp

Q.4



$C/Cs = s^3 + 11.4s^2 + 14s + 11.4K$

$\downarrow$

$1 + GH(s)$

b] From Routh:

s^3	1	14
s^2	11.4	11.4K
s	$\frac{11.4\times14 - 11.4K}{11.4}$	zero
s^0	11.4K	zero

$11.4K > 0 \rightsquigarrow K > 0$

$14 - K > 0 \rightsquigarrow K < 14$

$\therefore 0 < K < 14 \rightsquigarrow \boxed{K_{max} = 14}$

$e_{ss} = 1/K_v$ "ramp"

$K_v \to \lim sGH(s) = \frac{11.4K}{14}$

$e_{ss})_{min} = \frac{14}{11.4\times14} = \frac{1}{11.4}$

$\boxed{e_{ss} < 0.087719}$

a] $C/Cs = 1 + GH(s) = s^3 + 11.4s^2 + 14s + 11.4K$

$= s^3 + (\alpha + 2\zeta\omega_n)s^2 + (\omega_n^2 + 2\zeta\omega_n\alpha)s + \alpha\omega_n^2$

$s^2: \alpha + 2\zeta\omega_n = 11.4$ (1) , $s^1: \omega_n^2 + 2\zeta\omega_n\alpha = 14$ (2)

$s: \alpha\omega_n^2 = 11.4K$ (3)

$MP = 0.095$

$\zeta = 0.5996$

$* \ \omega_n^2 + 2\zeta\omega_n[11.4 - 2\zeta\omega_n] = 14$

$* \ -0.438\omega_n^2 + 13.67\omega_n - 14 = 0$

$\omega_n = 30.149$

$\alpha = -24.7$ X

$\omega_n = 1.06015$

$\alpha = 10.1286 \to K = \frac{\alpha\omega_n^2}{11.4} = 0.9985$

$> 5\zeta\omega_n$

$t_r = \frac{\pi - \cos^{-1}\zeta}{\omega_n\sqrt{1-\zeta^2}} = 2.6092$ sec

$t_s = \frac{4}{\zeta\omega_n} = 6.2926$ sec

$t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 4.6832$ sec