Digital Communications (ELC 325b)

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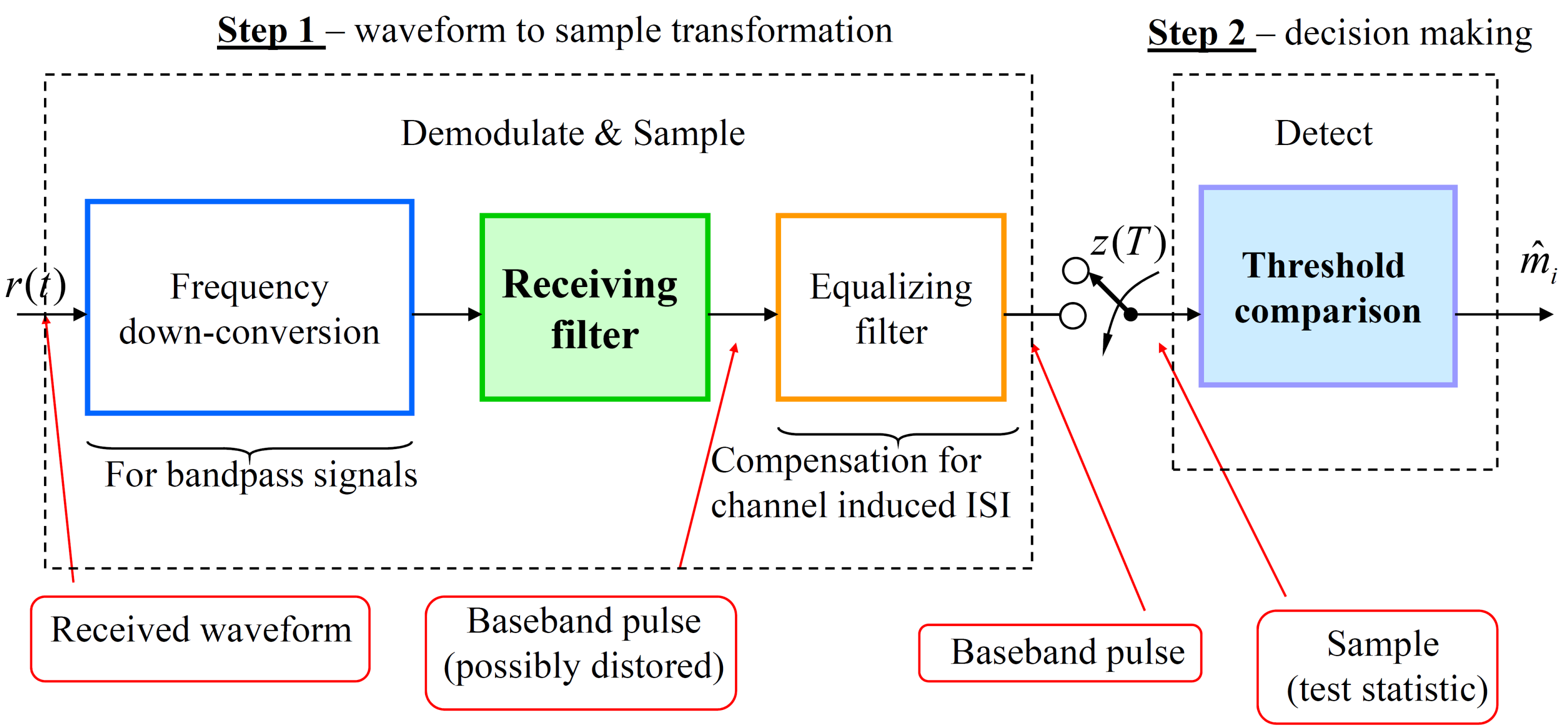
Spring 2016

Outline

1. Base-Band Transmission and Structure of Optimum Receiver
   - Inter-Symbol Interference
   - Design of Optimum ISI-Free Communication System

Inter-Symbol Interference



What is ISI?

It is another source of errors that arises when the **communication channel is dispersive**.

It occurs because dispersed **symbols are expanded beyond the symbol duration** to interfere with adjacent symbols

Inter-Symbol Interference

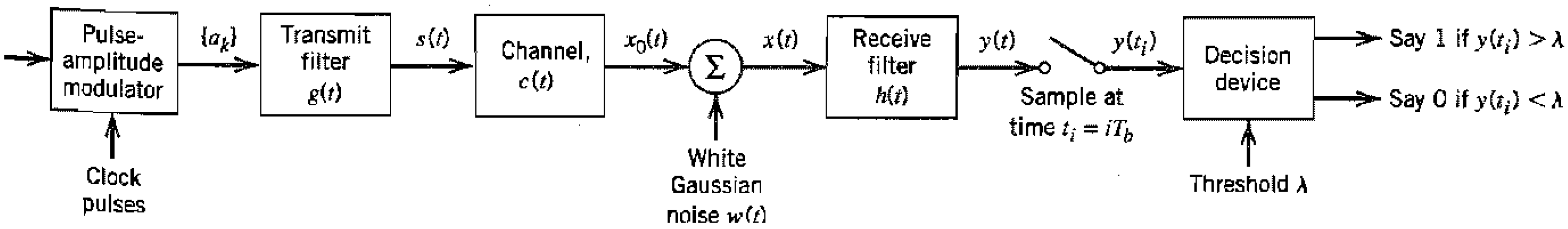
- A rectangular pulse requires an infinite bandwidth on the channel. This is cannot be practically achieved
- A band-limited pulse will be widely spread in time (This is refereed to as **Time Spread**)

Requirements of the pulse shape

- Band-limited
- Does not interfere with adjacent pulses

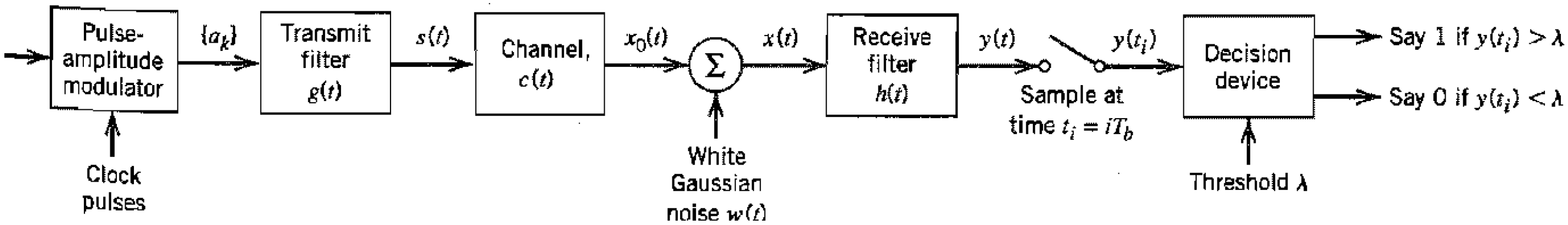
ISI will cause that the amplitudes of the pulses change resulting in less robustness to noise

The analytics of ISI



- The input is binary bits, each of duration T_b
- $\{a_k\}$ is a sequence of amplitude-modulated short pulses. In the case of binary PAM, $a_k = \pm 1$
- $s(t)$ is a sequence of pulse shaped symbols
- $y(t)$ is the output of the receiver filter. It is sampled at $t_i = iT_b$ synchronously with the transmitter
- The decision device finally decides, based on a threshold λ, whether the sample is '1' or '0'

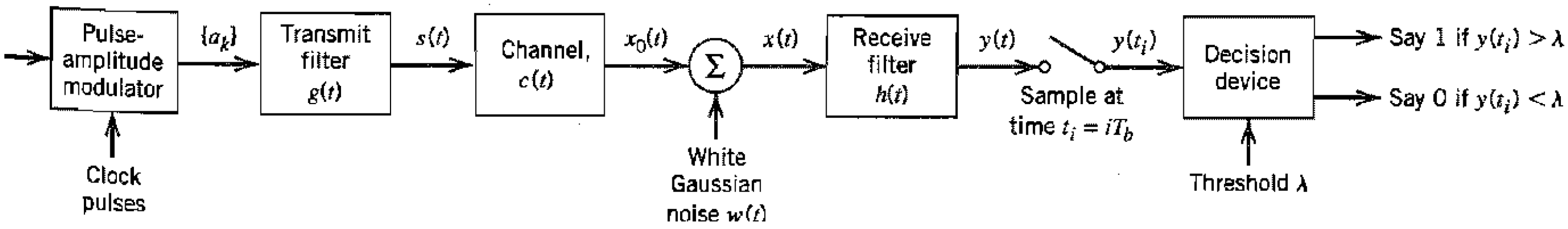
The analytics of ISI



$$
\begin{aligned}
s(t) &= \sum_k a_k g(t - kT_b) \\
x_0(t) &= s(t) * c(t) \\
x(t) &= x_0(t) + w(t) = s(t) * c(t) + w(t) \\
y(t) &= x(t) * h(t) \\
&= s(t) * c(t) * h(t) + w(t) * h(t) \\
&= \mu \sum_k a_k p(t - kT_b) + n(t), \qquad \mu p(t) = g(t) * c(t) * h(t)
\end{aligned}
$$

$$\mu P(f) = G(f)C(f)H(f)$$

The analytics of ISI



$$
\begin{aligned}
y(t_i) &= \mu \sum_k a_k p(t_i - kT_b) + n(t_i) \\
&= \mu a_i + \mu \sum_{k \neq i} a_k p((i-k)T_b) + n(t_i)
\end{aligned}
$$

- **The first term** represents the contribution of the i^{th} bit (This is the bit that needs detection)
- **The second term** represents the effect of all other transmitted bits on the decoding of the i^{th} bit (This is the ISI)
- **The third term** is the noise sample at the sampling time

Dealing with ISI

- The presence of ISI and noise in the DCS is unavoidable. This is introduces errors in the decision of the decision device
- In the design of the transmit and receive filters, $g(t)$ and $h(t)$, the objective of minimizing the effects of ISI as well as noise should be considered

  Note that ideally $y(t_i) = \mu a_i$

Problem Statement

How to design of the optimum $p(t) = g(t) * c(t) * h(t)$, such that effects of ISI and noise are minimized?

Design of ISI-Free Systems

For the system to be ISI-free

$$p(nT_b) = p((i-k)T_b) = \begin{cases} 1, & i = k \\ 0, & i \neq k \end{cases}$$

Note: Such condition results in perfect detection of the transmitted symbols, in the absence of noise.

$p(nT_b)$ are samples of $p(t)$, sampled by a train of impulses, then using the sampling theorem,

$$P_s(f) = \frac{1}{T_b} \sum_{-\infty}^{\infty} P\left(f - n\frac{1}{T_b}\right)$$

where $P_s(f)$ is the Fourier Transform of $p(nT_b)$
Since $p(nT_b) = \delta(n)$, then $P_s(f) = 1$

Nyquist's Criterion

Nyquist's Criterion

The condition of zero ISI is

$$\sum_{-\infty}^{\infty} P\left(f - nR_b\right) = T_b, \qquad \text{where } R_b = \frac{1}{T_b}$$

Nyquist's criterion for distortion-less baseband transmission in the absence of noise states that the frequency function $P(f)$ will eliminate the ISI for samples taken at intervals of T_b provided that the above equation is satisfied

Note: The RHS of the Nyquist's Criterion need not be exactly T_b.
It can be any constant.

Ideal Nyquist Channel

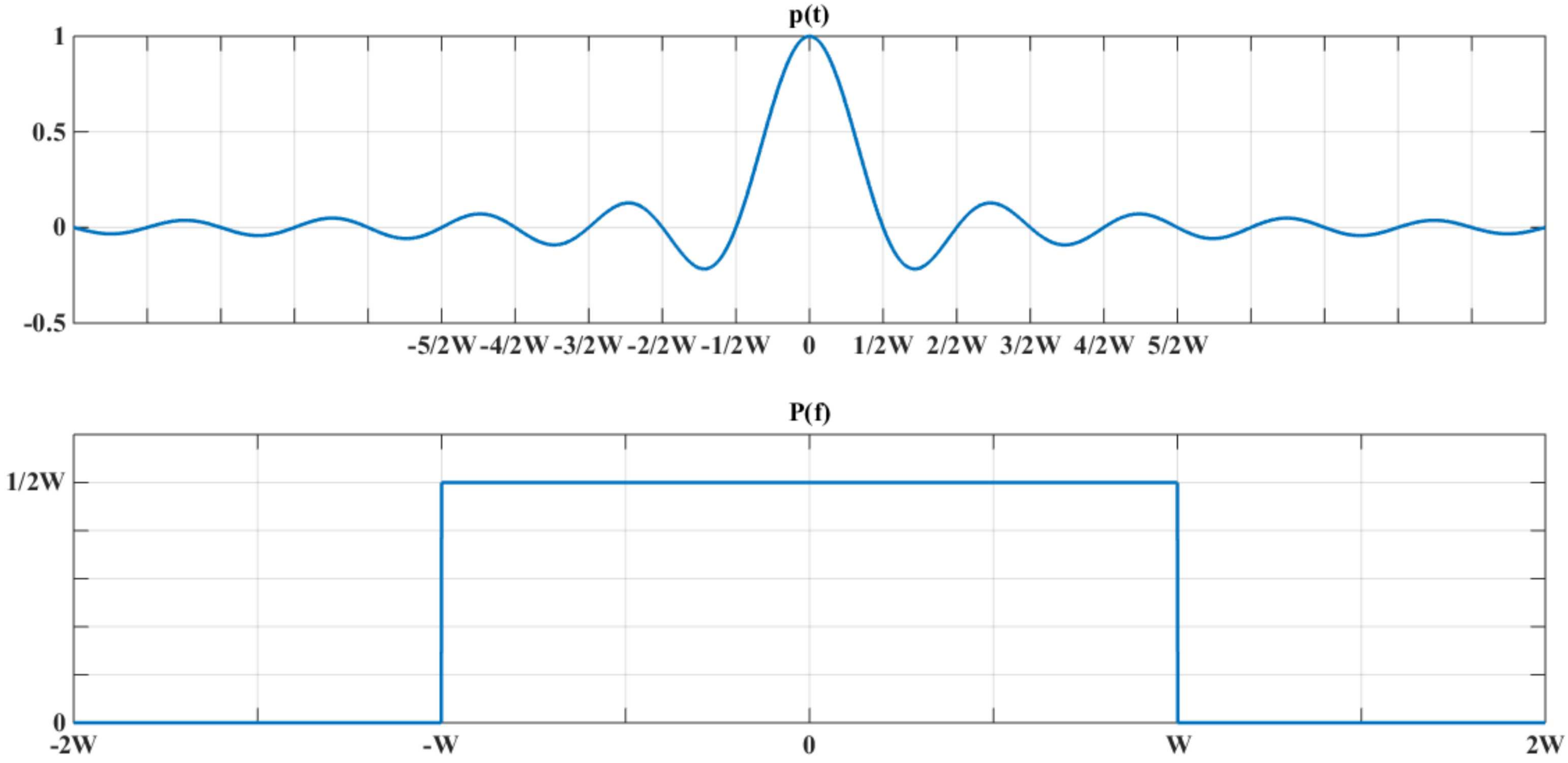
Simplest way to satisfy the Nyquist's Criterion

Ideal Nyquist Channel: Rectangular Form

$$
\begin{aligned}
P(f) &= \frac{1}{2W}\text{rect}\left(\frac{f}{2W}\right) = \begin{cases} \frac{1}{2W}, & -W < f < W \\ 0, & |f| > W \end{cases} \\
p(t) &= sinc\,(2Wt) = \frac{\sin(2\pi Wt)}{2\pi Wt}
\end{aligned}
$$

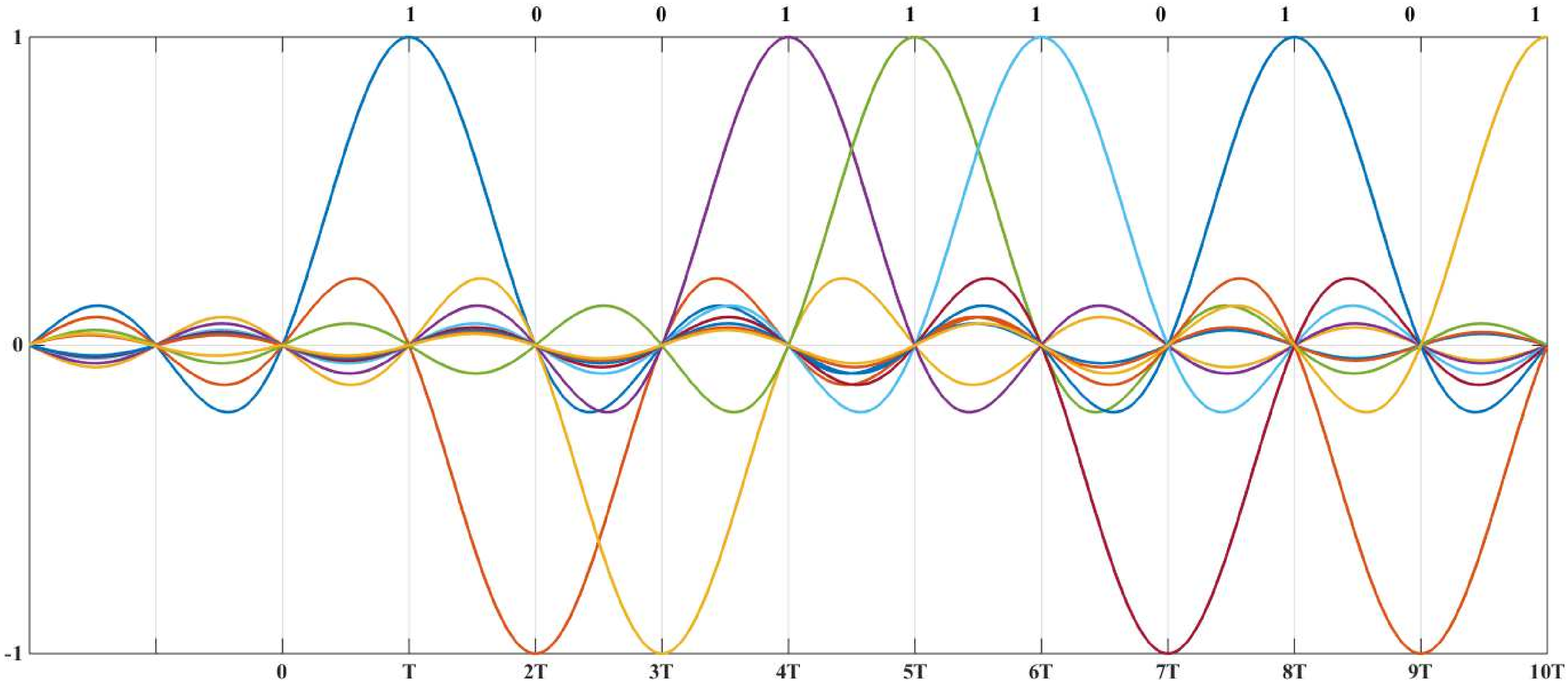
$$
\begin{aligned}
\textbf{Nyquist B.W.} \quad W &= \frac{R_b}{2} = \frac{1}{2T_b} \\
\textbf{Nyquist Rate} \quad R_b &= 2W \\
T_b &= \frac{1}{2W}
\end{aligned}
$$

Ideal Nyquist Channel



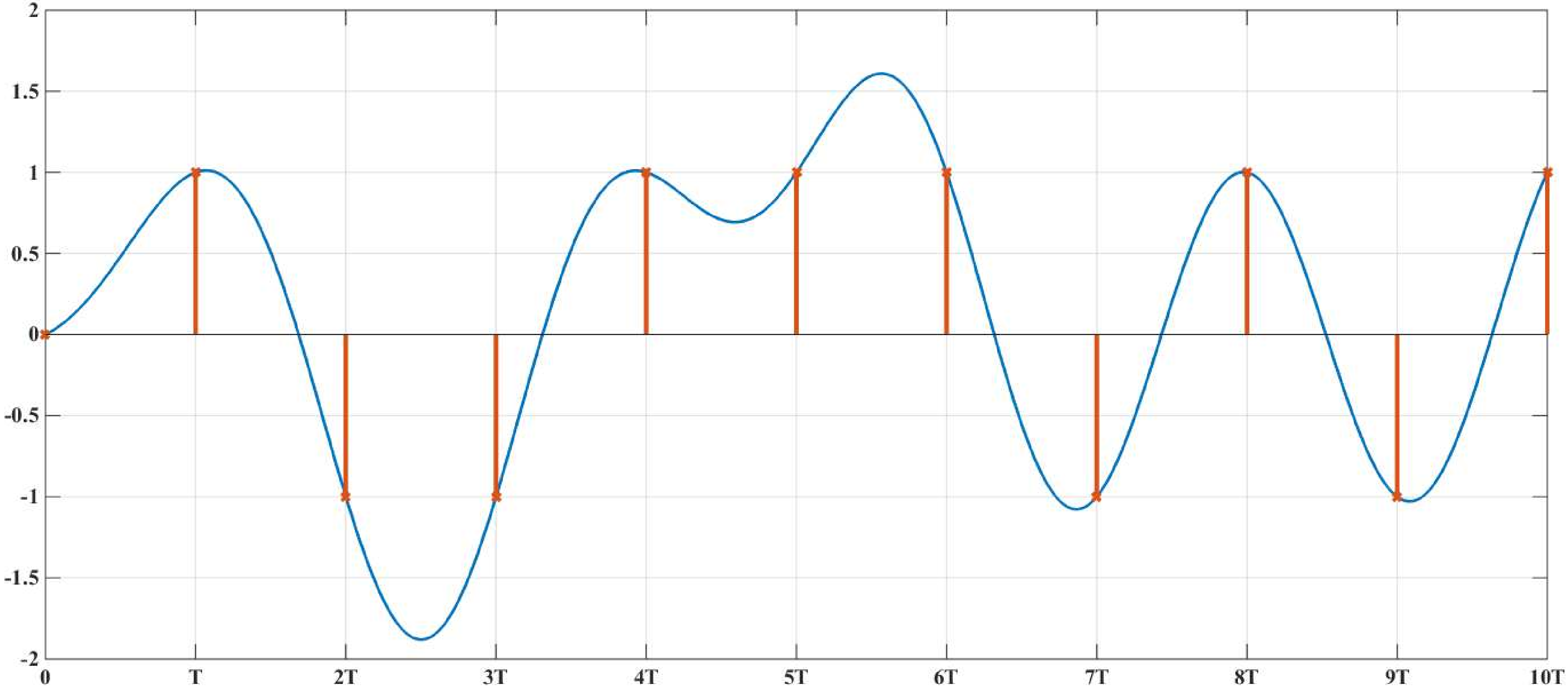
Ideal Nyquist Channel

Transmitted Stream = 1 0 0 1 1 1 0 1 0 1



Ideal Nyquist Channel

Transmitted Stream = 1 0 0 1 1 1 0 1 0 1



Sampling at $t_i = iT$

Detected Stream = 1 0 0 1 1 1 0 1 0 1

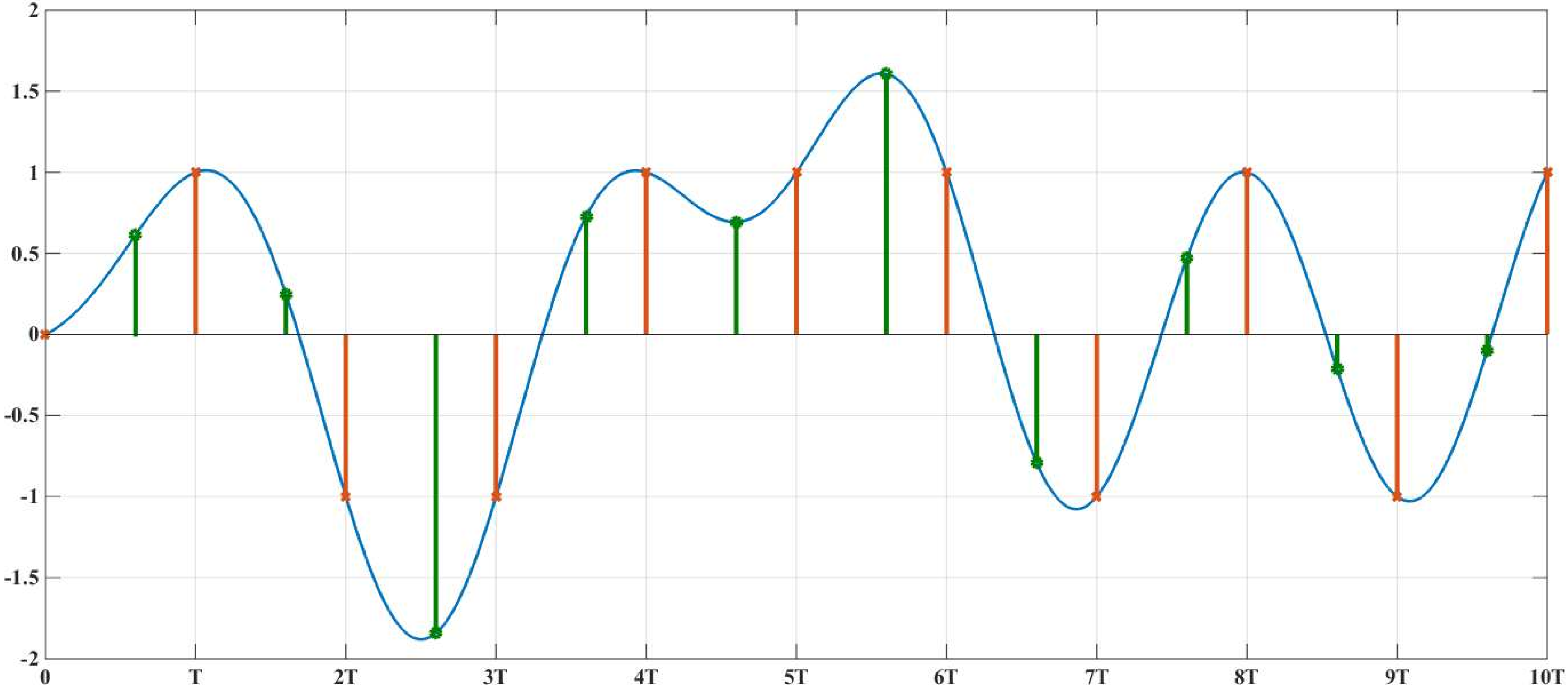
Ideal Nyquist Channel

Advantages and Disadvantages

1. Economic bandwidth usage: Solves the problem of ISI using the minimum possible bandwidth
2. There are practical difficulties:
   - The sudden transition at $f = \pm W$ is physically unrealizable
   - $p(t)$ decreases as $\dfrac{1}{|t|}$ resulting in a slow rate of decay. So, if the sampling times are slightly shifted, large ISI (because of many previous and following pulse signals) will be caused.

Ideal Nyquist Channel

Transmitted Stream = 1 0 0 1 1 1 0 1 0 1



Sampling at $t_i = iT - \epsilon$

Detected Stream = 1 1 0 1 1 1 0 1 0 0

Effect of Sync = ✓X ✓✓✓✓✓✓X

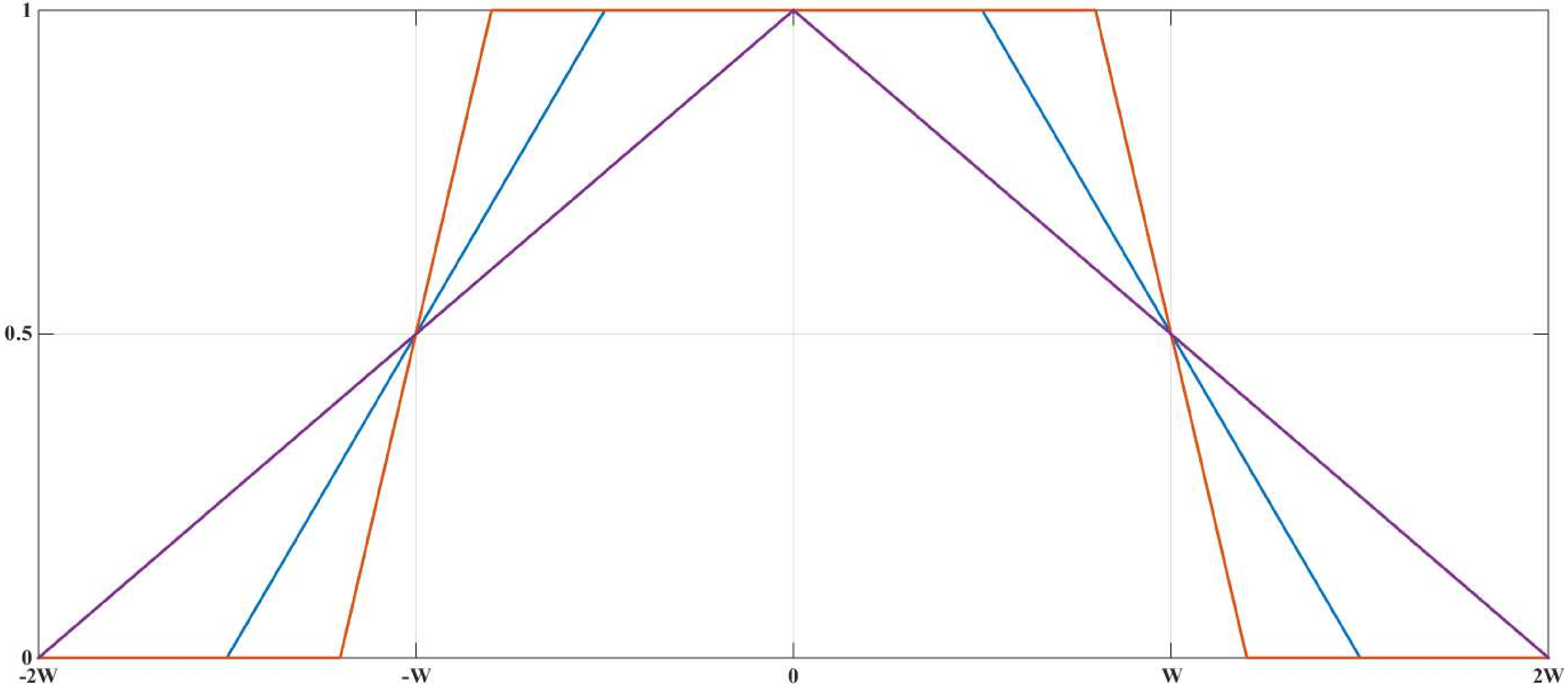
Reduced Nyquist's Criterion

In order to overcome the practical difficulties arising from using the ideal Nyquist channel, we can extend the bandwidth from its minimum value, $W = \frac{R_b}{2}$ to an adjustable value between W and $2W$, such that

$$P(f) + P(f - 2W) + P(f + 2W) = \frac{1}{2W}, \quad -W \leq f \leq W$$

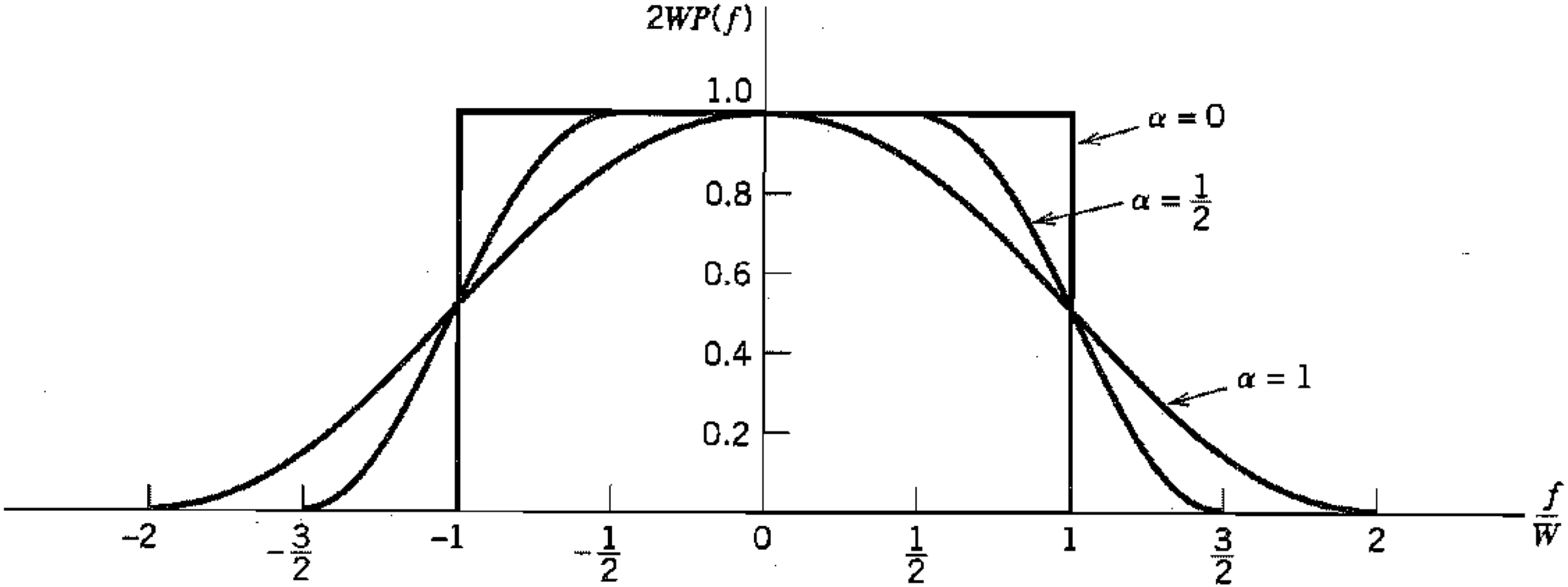
Many band-limited functions that satisfy the above equation can be found.

Reduced Nyquist's Criterion



Raised Cosine Spectrum

One of the most common forms of $P(f)$ is the **Raised Cosine Spectrum**, which consists of a **flat portion** and a **roll-off portion** of a sinusoidal form.



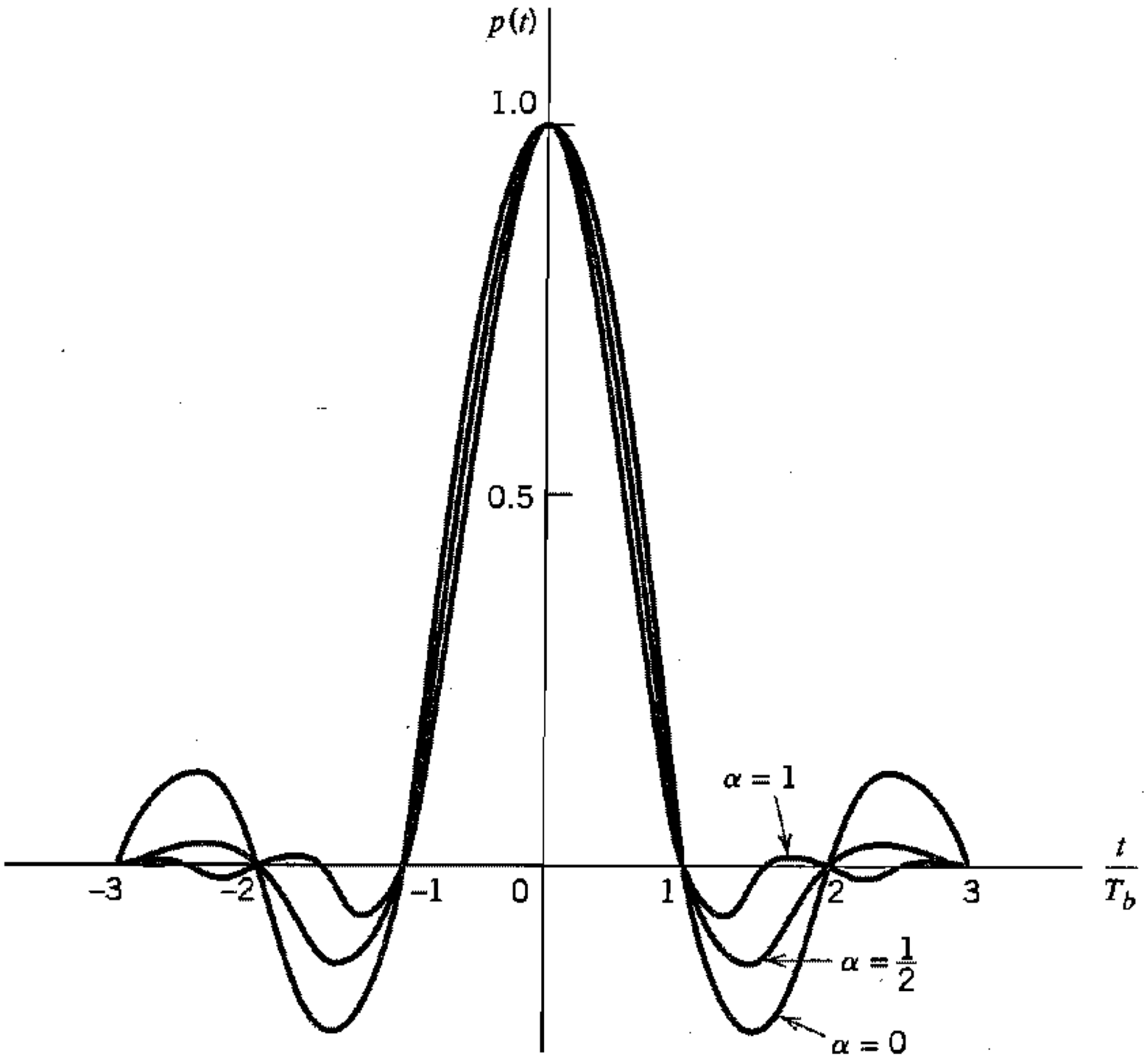
Raised Cosine Spectrum

Raised Cosine Spectrum

$$
\begin{aligned}
P(f) &= \begin{cases} \frac{1}{2W}, & 0 \leq |f| \leq (1-\alpha)W \\ \frac{1}{4W}\left[1 - \sin\left(\frac{\pi(|f|-W)}{2\alpha W}\right)\right], & (1-\alpha)W \leq |f| \leq (1+\alpha)W \\ 0, & (1+\alpha)W \leq |f| \end{cases} \\
p(t) &= sinc(2Wt)\frac{\cos(2\pi\alpha Wt)}{1-16\alpha^2 W^2 t^2}
\end{aligned}
$$

$$
\begin{aligned}
f_1 &= (1-\alpha)W \\
\textbf{Roll-off Factor} \quad \alpha &= 1 - \frac{f_1}{W} \\
0 \leq \alpha &\leq 1 \\
\textbf{Transmission B.W.} \quad B_T &= 2W - f_1 = (1+\alpha)W
\end{aligned}
$$

Raised Cosine Spectrum

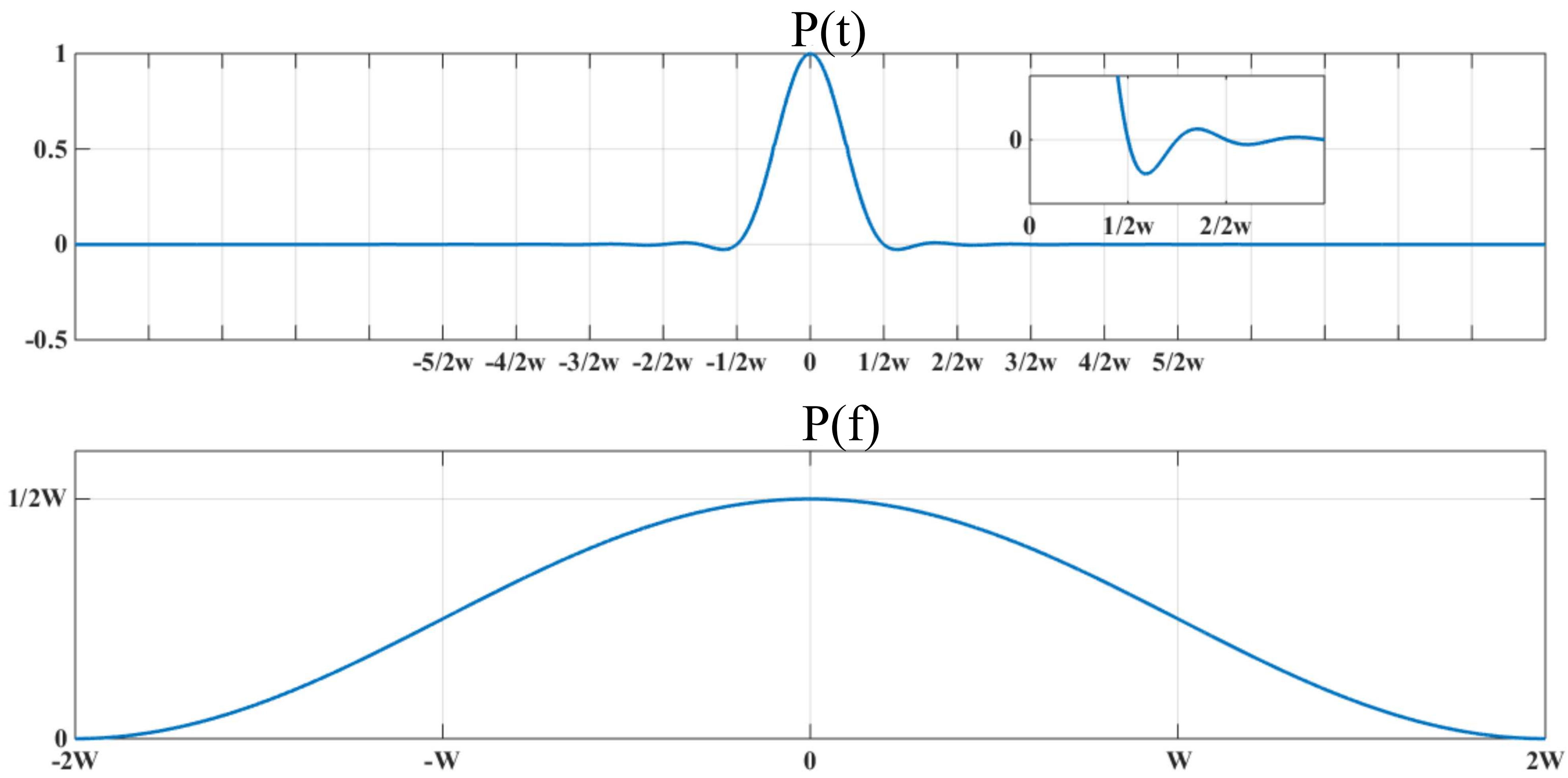


Raised Cosine Spectrum

Features of the Raised Cosine Spectrum

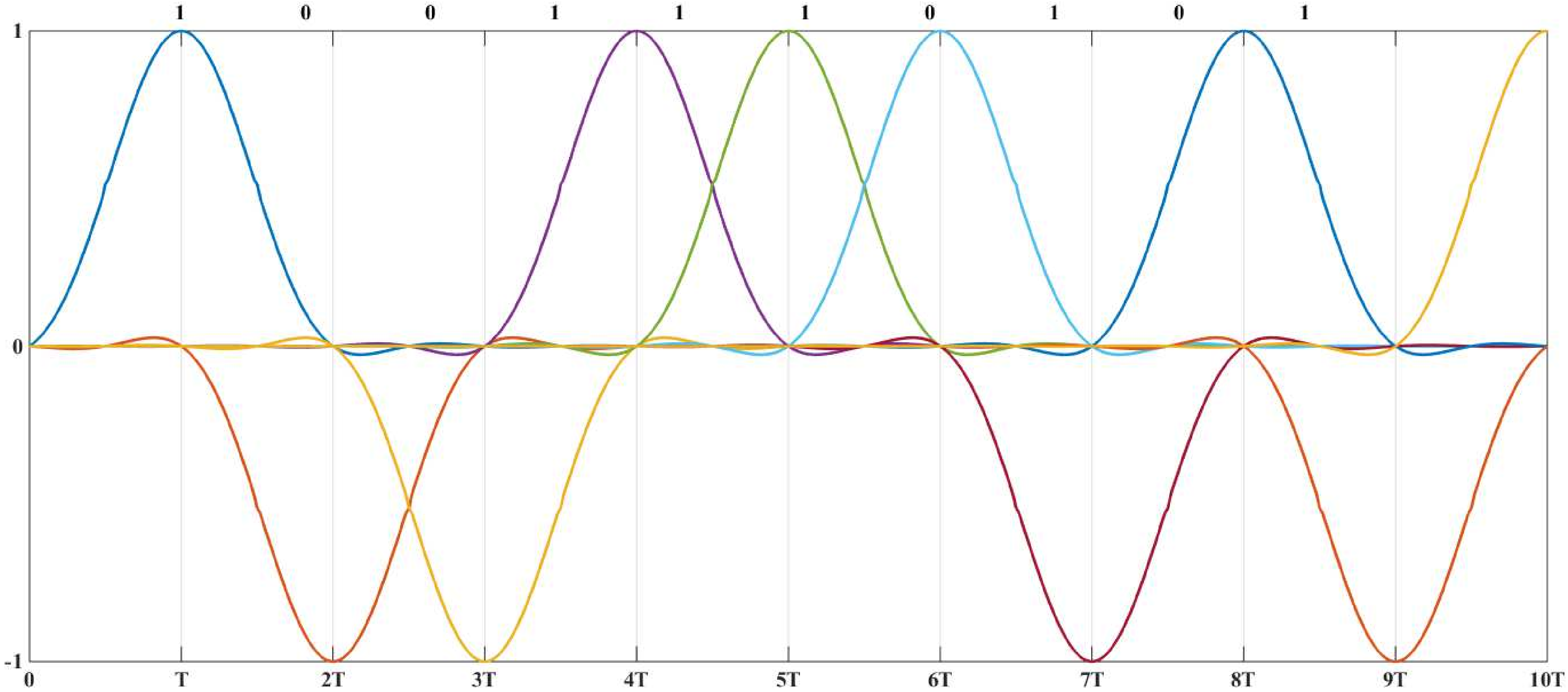
1. $P(f)$: Gradual cut-off
2. $p(t)$: Sinc component $\rightarrow$ zero-crossings at $t_i = iT \rightarrow$ ISI-Free
3. $p(t)$: Fast decay $\dfrac{1}{|t|^2}$
4. For $\alpha = 1$: Full Roll-Off - Larger B.W. - Extra Zero-crossings

Raised Cosine Spectrum: Full Cosine Roll-Off



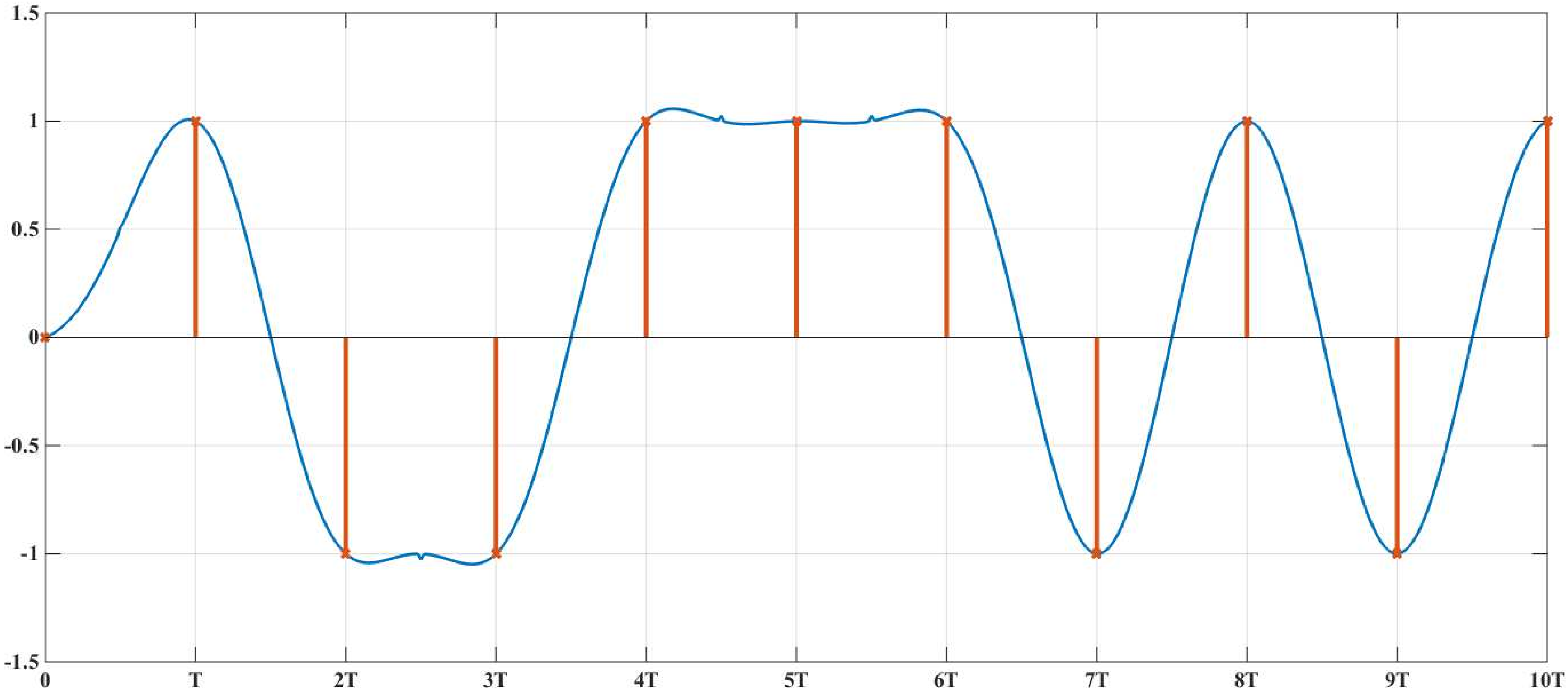
Raised Cosine Spectrum: Full Cosine Roll-Off

Transmitted Stream = 1 0 0 1 1 1 0 1 0 1



Raised Cosine Spectrum: Full Cosine Roll-Off

Transmitted Stream = 1 0 0 1 1 1 0 1 0 1

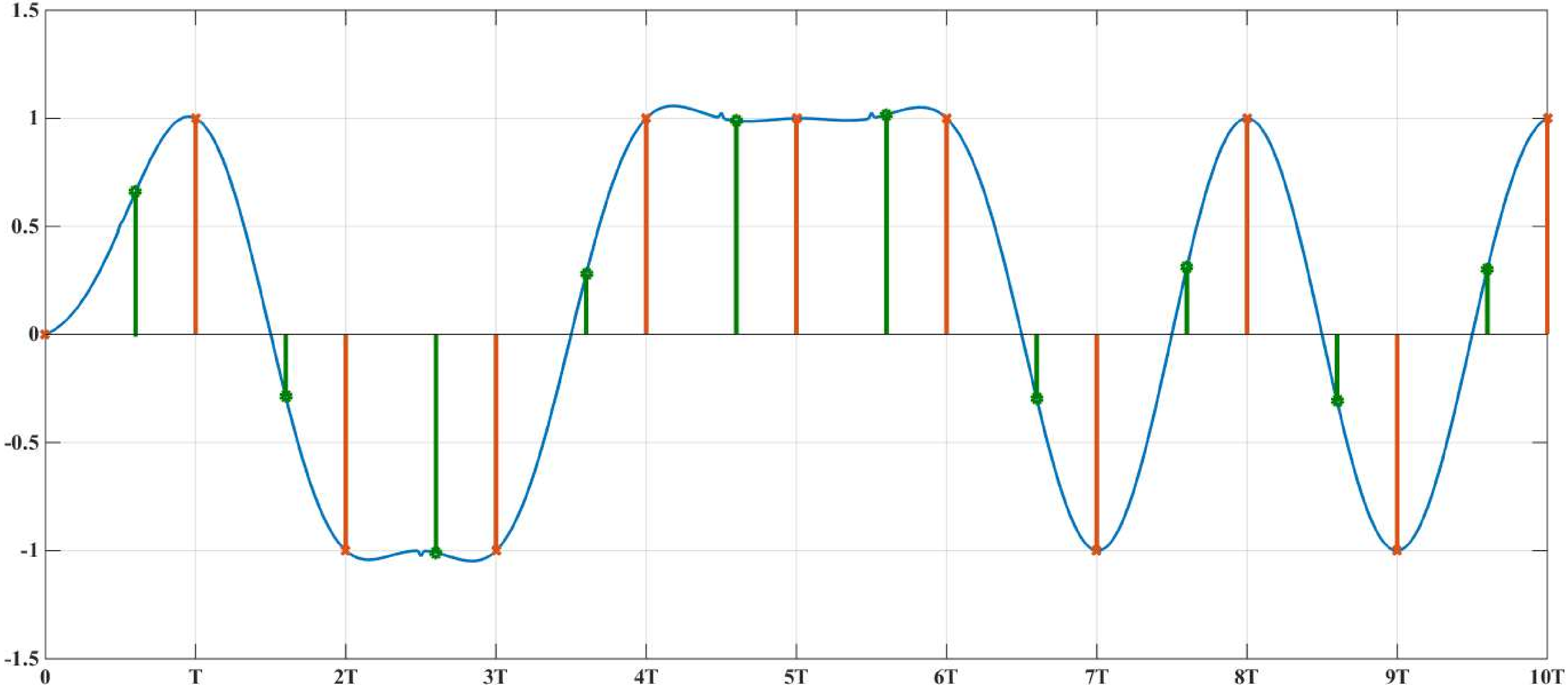


Sampling at $t_i = iT$

Detected Stream = 1 0 0 1 1 1 0 1 0 1

Raised Cosine Spectrum: Full Cosine Roll-Off

Transmitted Stream = 1 0 0 1 1 1 0 1 0 1

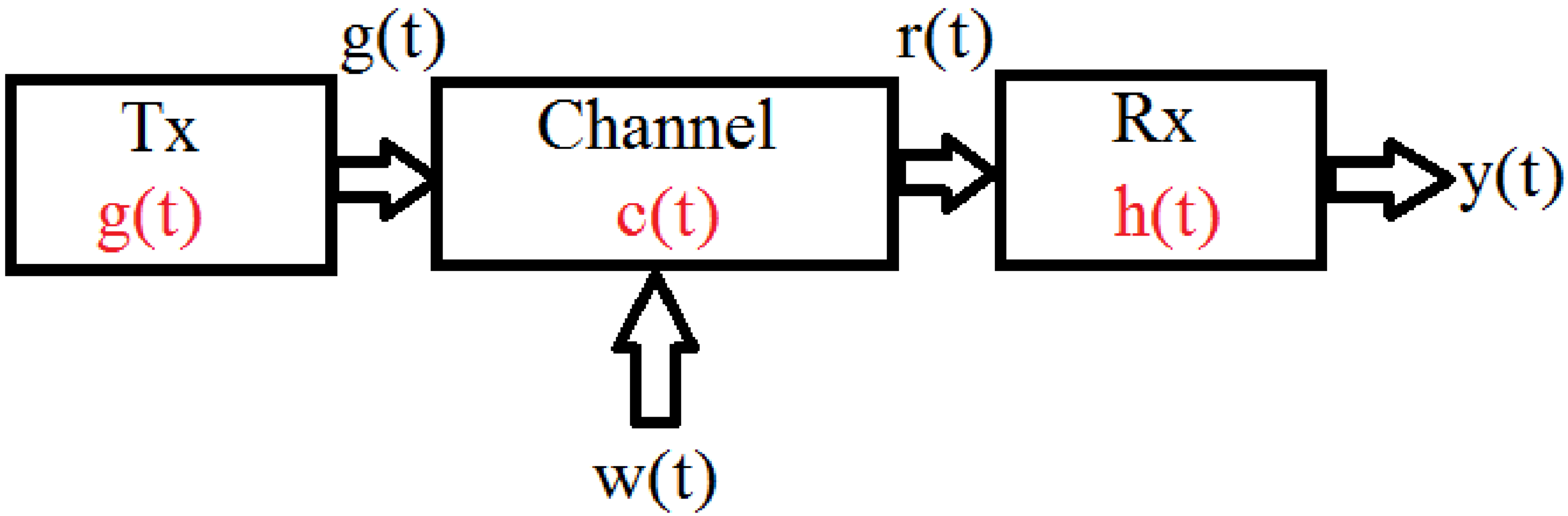


Sampling at $t_i = iT - \epsilon$

Detected Stream = 1 0 0 1 1 1 0 1 0 1

Effect of Sync = ✓✓✓✓✓✓✓✓✓

Design of Optimum ISI-Free Communication System



Recall:

$$
\begin{aligned}
p(t) &= g(t) * c(t) * h(t) \\
P(f) &= G(f)C(f)H(f)
\end{aligned}
$$

Conditions to satisfy:

1. Condition of Matched Filter
2. Condition of ISI-Free

Design of Optimum ISI-Free Communication System

Condition of Matched Filter

$$H(f) = G^*(f)e^{-j2\pi fT}$$

The exponential term represents a phase shift, equivalent to time delay.

Condition of ISI-Free

$$H(f)G(f) = P(f)$$

It is assumed that the channel is flat, $C(f) =$ const., for at least the maximum possible B.W. of the pulse $P(f)$.

$$\begin{aligned} G(f) &= \sqrt{P(f)} \\ H(f) &= \sqrt{P(f)} \end{aligned}$$

Square-Root Raised Cosine Spectrum

Transmission Bandwidth

Assuming a symbol duration of T,

$$\begin{aligned} B_T &= (1+\alpha)W \\ &= (1+\alpha)\frac{R_\mathrm{s}}{2} = (1+\alpha)\frac{1}{2T_\mathrm{s}} \end{aligned}$$

In the case of Binary Transmission,

$$\begin{aligned} T_\mathrm{s} &= T_b \\ B_T &= (1+\alpha)\frac{1}{2T_b} \end{aligned}$$

In the case of M-ary Transmission,

$$T_s = (\log_2 M)\, T_b$$

$$\begin{aligned} B_T &= (1+\alpha)\frac{1}{2T_s} \\ &= (1+\alpha)\frac{1}{\log_2 M}\frac{1}{2T_b} \end{aligned}$$

Transmission Bandwidth

Example

A computer puts out binary data at 56 kbps. The computer output is transmitted using a baseband binary PAM system that has a raised-cosine spectrum. Determine the transmission bandwidth for $\alpha = 0.25, 0.5, 0.75, 1$. Repeat if each of 3 successive binary digits are coded into one of eight PAM levels.

$$B_T = (1+\alpha)W, \qquad W = \frac{R_b}{2} = 28 \text{ kbps}$$

$$B_T = (1+\alpha)W, \qquad W = \frac{R_s}{2} = \frac{\frac{R_b}{\log_2 8}}{2} = \frac{28}{3} \text{ kbps}$$

References

Simon Haykin (2001)
Communication Systems, 4th Edition.
John Wiley.

Thank You

Questions ?