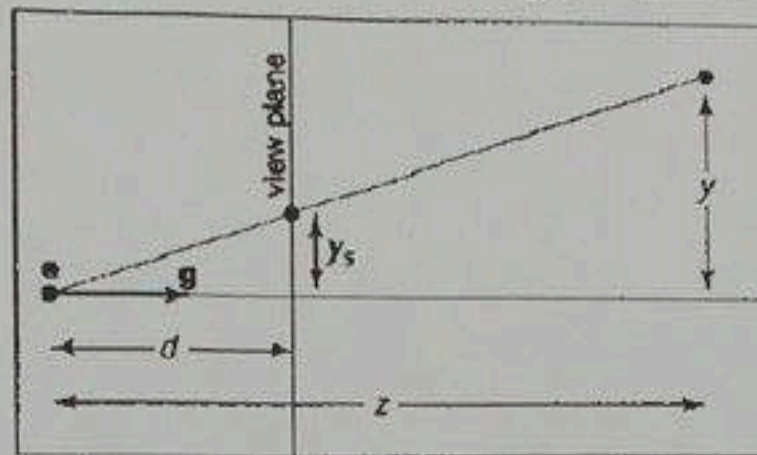


8. The specular component of the shading model depends on the angle between the viewing direction and the \_\_\_\_\_.
9. In the figure below which describes perspective projection,  $y_s =$  \_\_\_\_\_.



10. To represent a material that has perfect red diffuse component (i.e. reflects all incoming red light, and nothing else), we set the diffuse coefficient  $k_d = [ \text{ } , \text{ } , \text{ } ]$ .
11. The three surface rendering methods discussed in the course are flat shading, \_\_\_\_\_, and \_\_\_\_\_.
12. Environment mapping is used to approximate \_\_\_\_\_ in the rasterization pipeline.



13. In Constructive Solid Geometry, the ray intersection with the object above can be found by computing the ray intersection with a \_\_\_\_\_ and a \_\_\_\_\_ and then taking their \_\_\_\_\_.
14. Bounding Volume Hierarchies reduces the time required for ray tracing by \_\_\_\_\_.
15. An ellipse can be modeled by transforming a \_\_\_\_\_, and this is an example of \_\_\_\_\_.

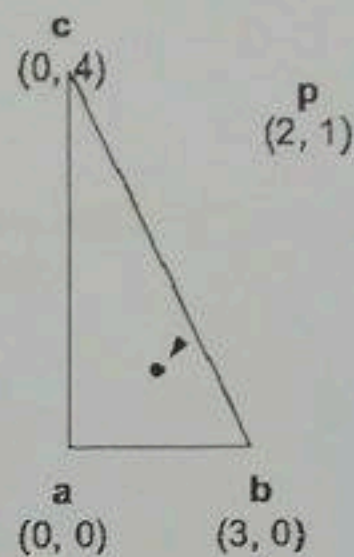
### Question 3 [5 points]

Given a 4x4 transformation matrix  $M$  that is used to transform points, show that to transform *normal vectors* correctly you should use the transformation matrix  $(M^{-1})^T$



#### Question 4 [5 points]

Compute the Barycentric Coordinates  $(\alpha, \beta, \gamma)$  of the point  $p$  on the right such that  $p = \alpha a + \beta b + \gamma c$



## Question 5 [10 points]

Given a ray with starting point  $e$  and direction vector  $d$ :

1. [4 points] Show that the barycentric coordinates of the point of intersection of that ray with a triangle with vertices  $a$ ,  $b$ , and  $c$  can be found by solving the linear system of equation

$$\begin{bmatrix} x_a - x_b & x_a - x_c & x_d \\ y_a - y_b & y_a - y_c & y_d \\ z_a - z_b & z_a - z_c & z_d \end{bmatrix} \begin{bmatrix} \beta \\ \gamma \\ t \end{bmatrix} = \begin{bmatrix} x_a - x_e \\ y_a - y_e \\ z_a - z_e \end{bmatrix}$$

where  $a = [x_a, y_a, z_a]^T$  and similarly for  $e$ ,  $d$ ,  $b$ , and  $c$ .

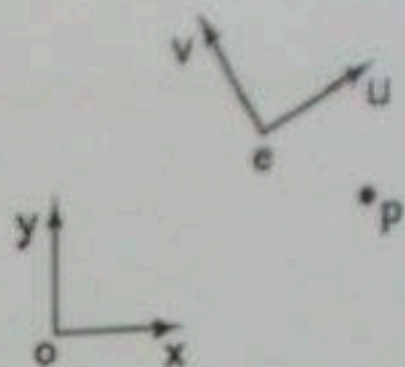
2. [3 points] Show also how to solve this system of equations and find the coordinates of the intersection point.
3. [3 points] Find the point of intersection of the ray  $e = [1, 2, 1]^T$  and  $d = [1, 2, 1]^T / \sqrt{6}$  with the triangle  $a = [6, 0, 0]^T$ ,  $b = [0, 6, 0]^T$  and  $c = [0, 0, 6]^T$ .

[Hint: find the parametric equation of the line and the barycentric representation of the triangle.]



### Question 6 [10 points]

You are given two 2D coordinate frames  $xy$  (with origin at  $o$ ) and  $uv$  (with origin at  $e$ ) such that the frame  $uv$  is obtained from the frame  $xy$  by applying a  $2 \times 2$  rotation matrix  $R$  followed by a translation by a 2D vector  $e$ .



- [3 points] In terms of  $R$  and  $e$ , what is the  $3 \times 3$  transformation matrix that can be used to convert coordinates of a point  $p$  from the  $uv$  frame to the  $xy$  frame i.e.  $p_{xy} = T_1 p_{uv}$

$$T_1 = \begin{bmatrix} \boxed{\phantom{00}} & \boxed{\phantom{00}} & \boxed{\phantom{00}} \\ \boxed{\phantom{00}} & \boxed{\phantom{00}} & \boxed{\phantom{00}} \end{bmatrix}$$

- [3 points] In terms of  $R$  and  $e$ , what is the  $3 \times 3$  transformation matrix that can be used to convert coordinates of a point  $p$  from the  $xy$  frame to the  $uv$  frame i.e.  $p_{uv} = T_2 p_{xy}$

$$T_2 = \begin{bmatrix} \phantom{00} & \phantom{00} & \phantom{00} \\ \phantom{00} & \phantom{00} & \phantom{00} \end{bmatrix}$$

- [4 points] Let  $R = \begin{bmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{bmatrix}$ ,  $e = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ , and  $p_{xy} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ , find  $p_{uv}$  i.e. the point  $p$  in the  $uv$  frame.

## Question 7 [10 points]

1. [5 points] Describe briefly the different transformations in the pipeline to transform points from the 3D model and project them onto the screen:

2. [5 points] Briefly describe one advantage and one disadvantage for the rasterization pipeline and ray tracing.

Rasterization Pipeline	Ray Tracing



### Question 8 [5 points]

Assume a material with ambient coefficient  $k_a=[0.5,0.5,0.5]$ , diffuse coefficient  $k_d=[1.0,1.0,0.5]$ , specular coefficient  $k_s=[1.0,0,0]$ , and specular exponent  $p=0.25$ .

Assume the ambient light is  $I_a=[1.0,1.0,1.0]$  and there are two light sources  $I_1=[1.0,0.0,0.0]$  at position  $[2,3,5]$  and  $I_2=[0.0,1.0,0.0]$  at position  $[-2,3,5]$ .

Compute the lighting at the point  $[0,0,0]$  with surface normal  $n=[0,0,1]^T$  as seen by a camera at position  $[0,0,10]$ .

[Recall: the lighting at a point is computed as  $R=k_a I_a + \sum_i [k_d I_i \max(0, l_i \cdot n) + k_s I_i \max(0, e \cdot r_i)^p]$  where  $l_i$  is the direction of light  $i$ ,  $e$  is the direction of the eye, and  $r_i = -l_i + 2(l_i \cdot n)n$  is the reflection direction.]

