Cairo University
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ELC325B
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Sheet 2

Problem#1

6.2-9 A signal band-limited to 1 MHz is sampled at a rate 50% higher than the Nyquist rate and quantized into 256 levels using a μ-law quantizer with $\mu = 255$.

(a) Determine the signal-to-quantization-noise ratio.

(b) The SNR (the received signal quality) found in part **(a)** was unsatisfactory. It must be increased at least by 10 dB. Would you be able to obtain the desired SNR without increasing the transmission bandwidth if it was found that a sampling rate 20% above the Nyquist rate is adequate? If so, explain how. What is the maximum SNR that can be realized in this way?

Problem#2

Consider a sinusoidal signal with random phase, defined by

$$X(t) = A\cos(2\pi f_c t + \Theta) \qquad (1.15)$$

where A and f_c are constants and Θ is a random variable that is *uniformly distributed* over the interval $[-\pi, \pi]$, that is,

$$f_\Theta(\theta) = \begin{cases} \dfrac{1}{2\pi}, & -\pi \le \theta \le \pi \\ 0, & \text{elsewhere} \end{cases} \qquad (1.16)$$

Find the auto-correlation function and the power spectral density of $X(t)$

Problem#3

Figure 1.6 shows the sample function $x(t)$ of a process $X(t)$ consisting of a random sequence of *binary symbols* 1 and 0. The following assumptions are made:

1. The symbols 1 and 0 are represented by pulses of amplitude $+A$ and $-A$ volts, respectively, and duration T seconds.
2. The pulses are not synchronized, so the starting time t_d of the first complete pulse for positive time is equally likely to lie anywhere between zero and T seconds. That is, t_d is the sample value of a uniformly distributed random variable T_d, with its probability density function defined by

$$f_{T_d}(t_d) = \begin{cases} \dfrac{1}{T}, & 0 \le t_d \le T \\ 0, & \text{elsewhere} \end{cases}$$

3. During any time interval $(n-1)T < t - t_d < nT$, where n is an integer, the presence of a 1 or a 0 is determined by tossing a fair coin; specifically, if the outcome is heads, we have a 1 and if the outcome is tails, we have a 0. These two symbols are thus equally likely, and the presence of a 1 or 0 in any one interval is independent of all other intervals.

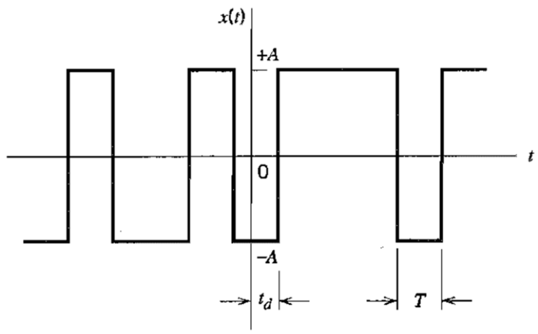

FIGURE 1.6 Sample function of random binary wave.

Find the auto-correlation function and the power spectral density of $x(t)$

Problem#4

1.8 A random process $Y(t)$ consists of a DC component of $\sqrt{3/2}$ volts, a periodic component $g(t)$, and a random component $X(t)$. The autocorrelation function of $Y(t)$ is shown in Figure P1.8.

(a) What is the average power of the periodic component $g(t)$?

(b) What is the average power of the random component $X(t)$?

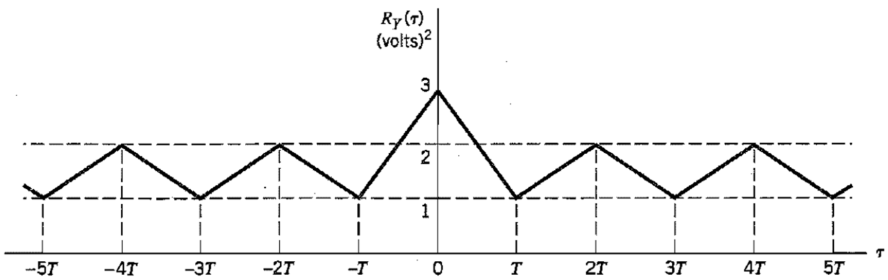

FIGURE P1.8

Problem#5

1.12 The power spectral density of a random process $X(t)$ is shown in Figure P1.12. It consists of a delta function at $f = 0$ and a triangular component.

(a) Determine and sketch the autocorrelation function $R_X(\tau)$ of $X(t)$.

(b) What is the DC power contained in $X(t)$?

(c) What is the AC power contained in $X(t)$?

(d) What sampling rates will give uncorrelated samples of $X(t)$?

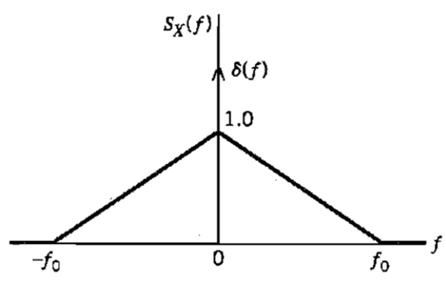

FIGURE P1.12