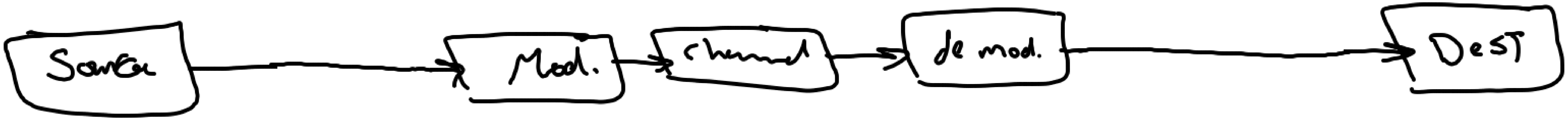
# Information Theory




$I \longrightarrow n$ bits  ex:  states  $\Longrightarrow$ (States) $N =$

if each State has equal Prob $\longrightarrow P(S_i) =$

$\therefore$

if $P = 0.00001$

$N =$

$I =$

# Entropy

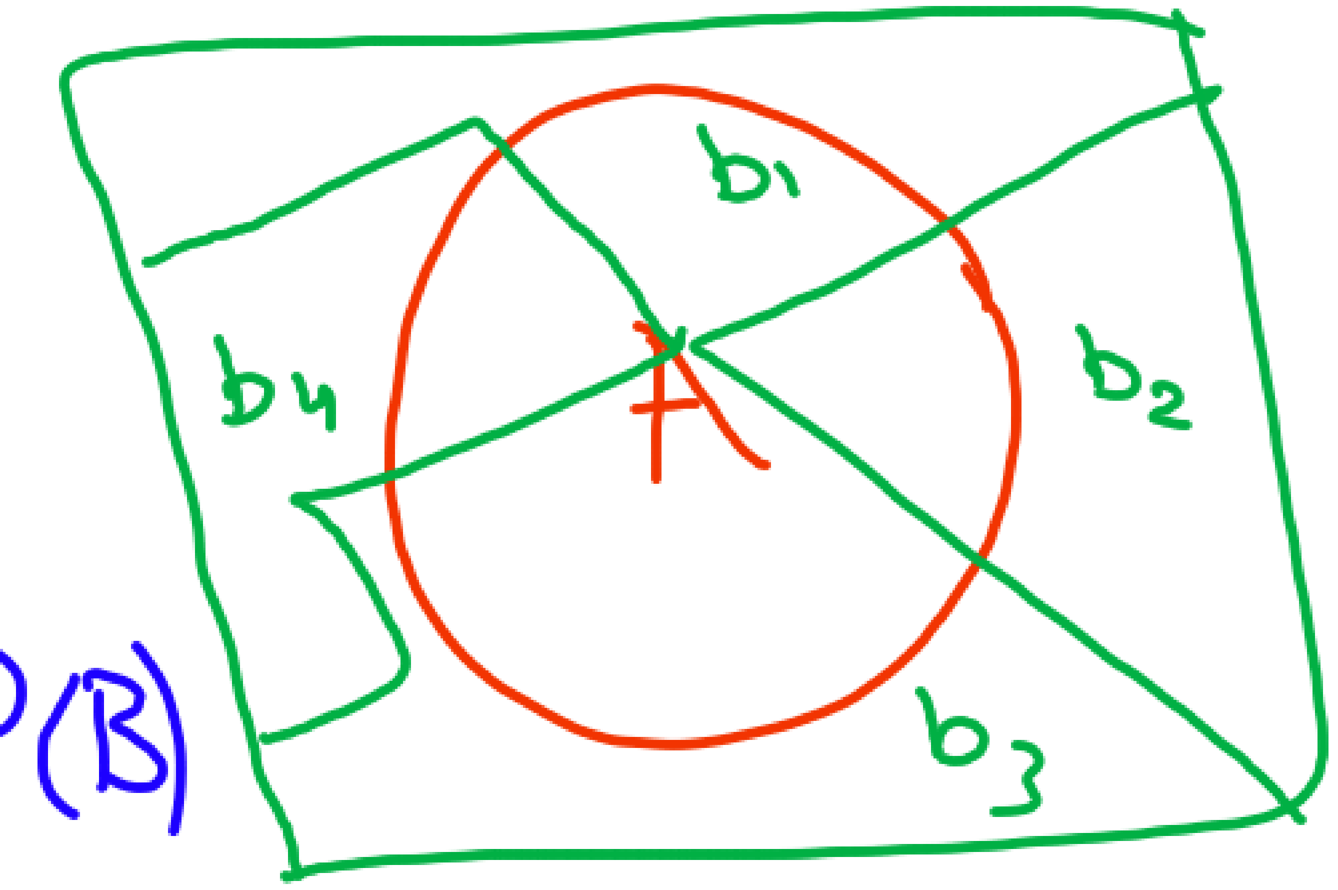
item: $X_i \rightarrow P(x_i)$

$H = \quad x_i$

$$P(A=a, B=b) = P(A,B) = P(A|B)\,P(B) = P(B|A)\,P(A)$$

$$P(A) = \sum_i P(A|b_i)\,P(b_i)$$

$< P <$




if indep.

$$P(A|B) = P(A), \quad P(A,B) = P(A)P(B)$$

$$I = \log_2 \frac{1}{P}$$

$< I <$

$$H(x) = \sum P(x_i) \log \frac{1}{P(x_i)}$$

$< H <$

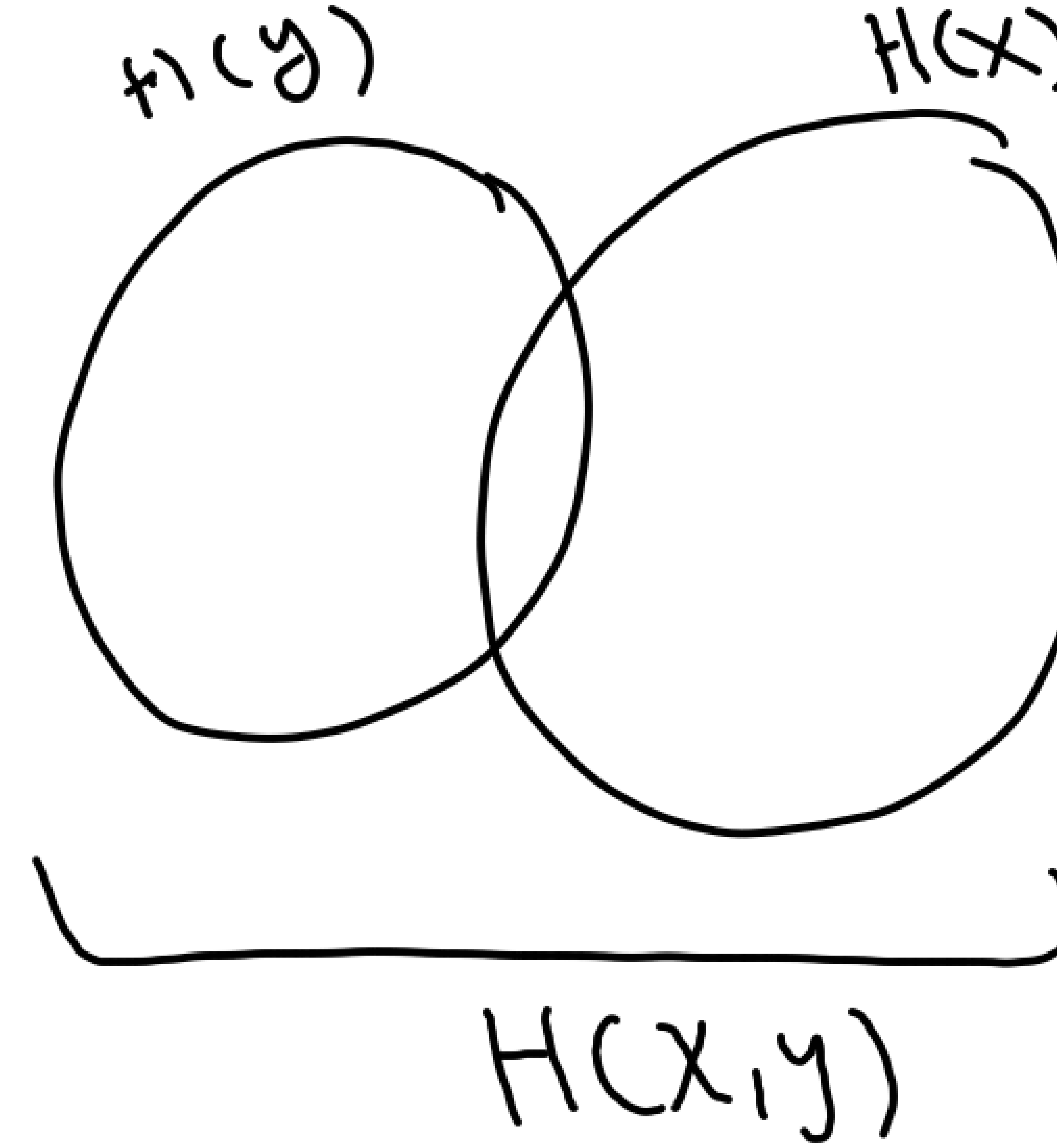
$$H(x,y) = \sum_{x_i}\sum_{y_i} P(x_i,y_i)\log_2\frac{1}{P(x,y)}$$

$$H(y|x) = \sum_{x_i}\sum_{y_j} P(x_i,y_j)\log_2\frac{1}{P(y_j|x_i)}$$

$$= P(x_0)\sum_{y_j} P(y_j|x_0)\log_2\frac{}{P(y_j|x_0)}$$

$$+ P(x_1)\sum_{y_i} P(y_j|x_1)\log_2\frac{1}{P(y_j|x_1)}$$

$$+ \ldots\ldots$$




$$H(x,y) = H(x)+H(y|x) = H(y)+H(x|y)$$

$$< H(y|x) <$$

$$I(x,y) = H(y)-H(y|x) = H(x)-H(x|y)$$

$$= H(x)+H(y)-H(x,y)$$

$$I(x,y) = \sum_{x_i, y_j} P(x_i, y_j) \log_2 \frac{P(x_i, y_j)}{P(x_i)P(y_j)} \longrightarrow$$

$$= \sum_{x_i, y_j} P(y_j|x_i) P(x_i) \log_2 \frac{P(y_j|x_i)}{P(y_j)} \longrightarrow \sum_{x_i} P(y_j|x_i) P(x_i)$$

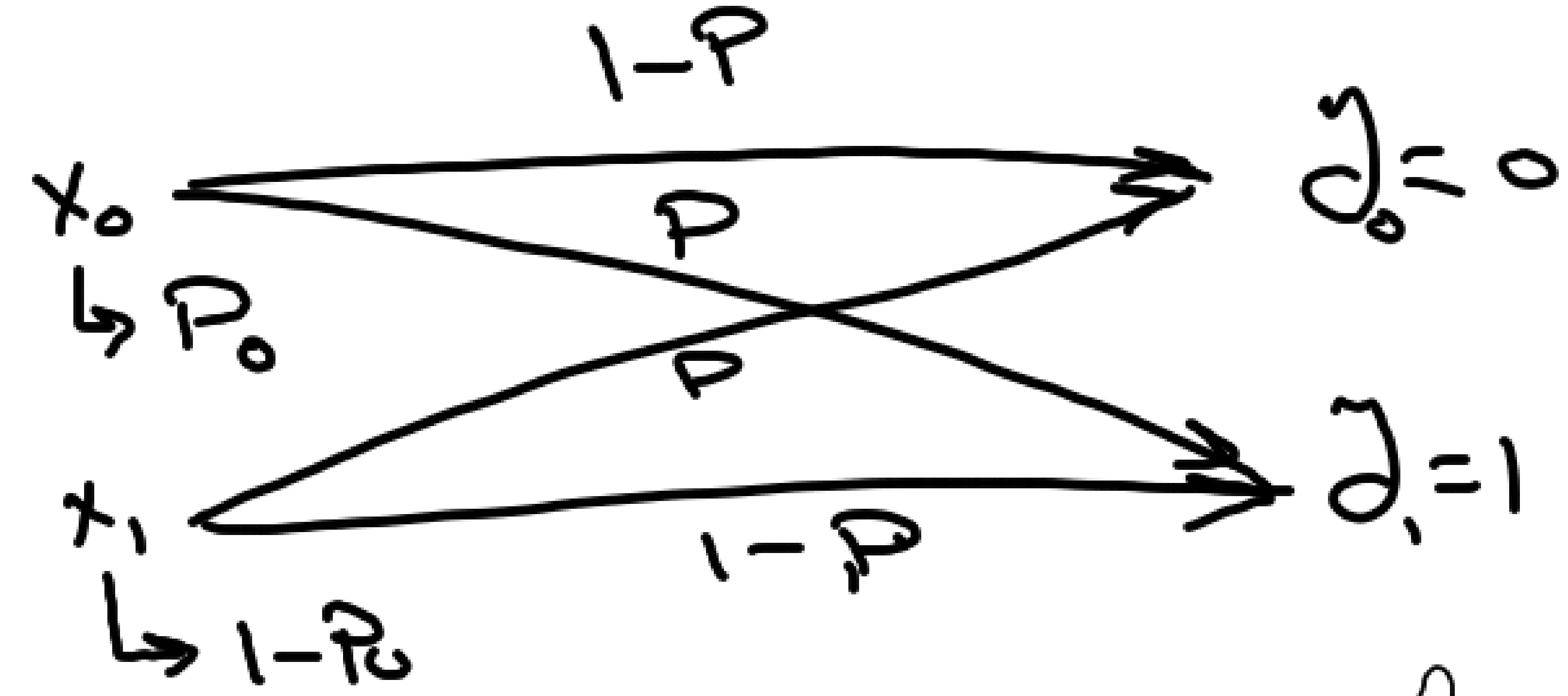
$x_0$ ———————— $y_0$

$x_1$ ———————— $y_1$

$$P(Y|X) = \begin{array}{c|cc|} & y & y \\ x & P(y|x) & P(y|x) \\ x & P(y|x) & P(y|x) \end{array}$$

$$C = \max_{P(x)} \left[ I(x,y) \right] \Rightarrow @$$

1]




$$\mathcal{H}(Z) = Z \log_2 \frac{1}{Z} + (1-Z)\log_2 \frac{1}{1-Z}$$

$\hookrightarrow Z = P_0 P + (1-P_0)(1-P$

a) Show that $I(X;Y) = \mathcal{H}(Z) - \mathcal{H}(P)$

b) $P_0$ Maximize $I(X;Y)$ is $\frac{1}{2}$ Show that $C = 1 - \mathcal{H}(P)$

2\] Binary Channel Matrix $\begin{vmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{10} & \frac{9}{10} \end{vmatrix}$, $P(X) = \begin{bmatrix} 1/3 \\ 2/3 \end{bmatrix}$

find $H(X)$, $H(y)$, $H(X|y)$, $H(y|x)$, $I(x,y)$