Dc Sheet 7

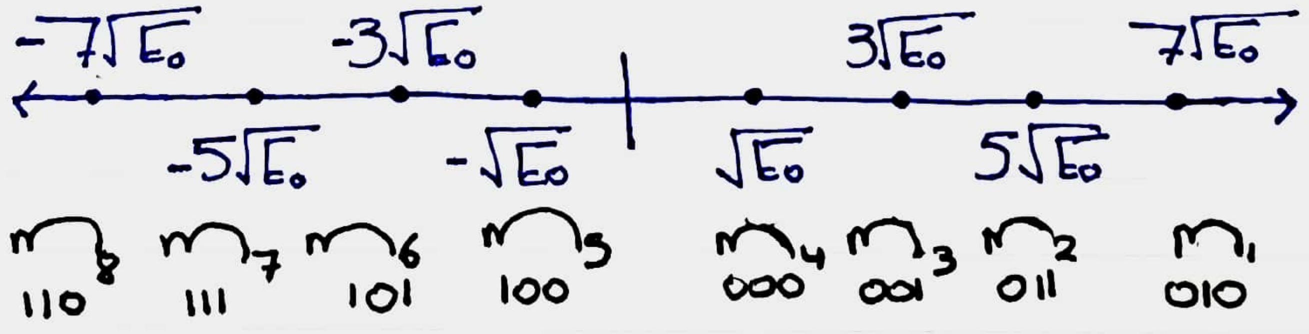
• Consider M-ary ASK where the input symbol modulates the amplitude of the carrier such that

$$g_i(t) = a_i\sqrt{E_0} \times \sqrt{\frac{2}{T}}\cos(2\pi f_c t),\quad 0 \leq t \leq T$$

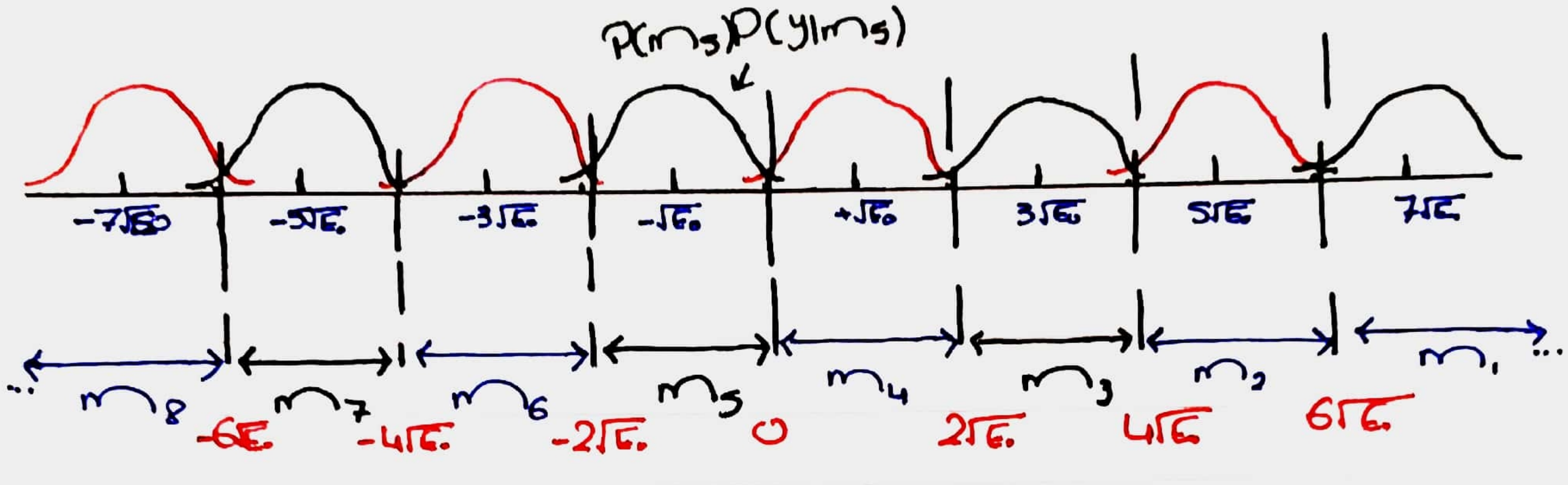
• For $i = 1, 2, \dots, M$

• $a_i = \pm 1, \pm 3, \pm 5, \dots$

which for 8-ASK implies the constellation



1- Mark the decision regions of each symbol then sketch the constellation



- We decide for a certain symbol m_i everywhere where $P(m_i)P(\hat{m}=m_i|m_i)$ is the largest (Bayes optimum rule)

2- Derive the theoritical BER

→ We'll start with the probability of error (which is the symbol error rate) and then use it to find the bit error rate

$$P(e) = \sum_{i=1}^{8} P(m_i)P(\hat{m} \neq m_i | m_i)$$

$$= \frac{1}{8} \sum_{i=1}^{8} P(\hat{m} \neq m_i | m_i)$$

$\mu = a_i\sqrt{E_0}$

- Each distribution is a Gaussian where $\sigma^2 = \frac{N_0}{2}$

- Notice
  $P(\hat{m} \neq m_i | m_i)$ is the same for $2 \leq i \leq 7$ (double tail) & for $i = 1, 8$ (single tail)

Hence,

$$P(e) = \frac{1}{8}\left(6P(\hat{m} \neq m_4 | m_4) + 2P(\hat{m} \neq m_8 | m_8)\right)$$

$$= \frac{1}{8}\left(14P(\hat{m} \neq m_8 | m_8)\right)$$

$$= \frac{14}{8} P(y > -6\sqrt{E_0} \mid m$$

$$\cdot\ Z = \frac{y-(-7\sqrt{E_0})}{\sqrt{N_0/2}}$$

$$= \frac{14}{8} P\left(Z > \frac{-6\sqrt{E_0} - (-7\sqrt{E_0})}{\sqrt{N_0/2}} \mid m_1\right)$$

$$= \frac{14}{8} P\left(Z > \frac{\sqrt{E_0}}{\sqrt{N_0/2}} \mid m_1\right)$$

$$= \frac{14}{8} Q\left(\sqrt{\frac{2E_0}{N_0}}\right) = \frac{7}{8} \operatorname{erfc}\left(\sqrt{\frac{E_0}{N_0}}\right)$$

Side note

- Recall that we may also write $P(e)$ in terms of E_b (energy per bit):

$$E_b = \sum_{i=1}^{8} P(m_i) E_i = \frac{1}{8}\left(2(E_0 + 9E_0 + 25E_0 + 49E_0)\right) = 21E_0$$

- Thus, $E_0 = \frac{E_b}{21}$ is the needed sub. in $P(e)$

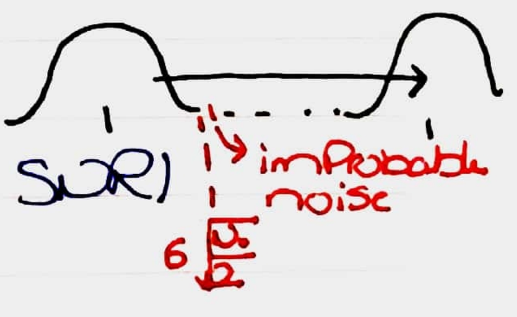
Now need to go from Probability of error (Symbol error rate) to BER (bit error rate)

→ Observe that

$$P(e) = P\left(\bigcup_{i=1}^{\#\text{bits}} \text{error in bit}(i)\right)$$

(Since an error in any of the bits causes an error in the symbols)

- By the encoding scheme in Page 1 (gray encoding), No two codes differ in more than 1 bit and lie in neighboring distributions.
  i.e. the Probabibility of an error in 2 or more bits is very low/negligble (assuming sufficiently large SNR)



→ Hence, errors in the 3 bit positions are approximately disjoint and we can write

$$P(e) = P(e \text{ in } b_2) + P(e \text{ in } b_1) + P(e \text{ in } b_0)$$

which can all be considered to be equal yielding

$$P(e)|_{symbol} = 3\,P(e)|_{bit}$$

#SER #BER

Hence, $BER = \frac{14}{24} Q\left(\sqrt{\frac{2E_0}{N_0}}\right)$

$$= \frac{7}{12} Q\left(\sqrt{\frac{2E_0}{N_0}}\right)$$

$$= \frac{7}{24} \operatorname{erfc}\left(\sqrt{\frac{E_0}{N_0}}\right)$$

where $E_0 = \frac{E_b}{21}$

3- Suppose the required bitrate is 1 Mbps

- The available BW is 0.5 MHz
- with carrier frequency 5 MHz
- BW required by Passband modulation (M-ary ASK) is $2R_s$ (Known fact)

→ Can 8-ary ASK be used? why?

Will answer this 1st { what's the min M that satisfies the (M-ary ASK) requirement?

- $T_b = 10^{-6} s$ $(1/R_b)$

$BW_{channel} = 0.5 MHz$, $f_c = 5 MHz$

$$BW|_{\substack{M\text{-ary}\\ ASK}} = 2R_s = \frac{2}{T_s} = \frac{2}{\log_2 M \, T_b}$$

→ Need $BW_{channel} \geq BW|_{M\text{-ary}}$

$$0.5 \times 10^{6} \geq \frac{2}{10^{-6} \log_2 M}$$

$$\log_2 M \geq 4$$

$$M \geq 16$$

- M needs to be at least 16 (8-ary ASK won't work)

→ in particular, it will take $\frac{2}{10^{-6} \log_2 8} = \frac{2}{3} MHz$ channel BW (less than $\frac{MHz}{2}$)

4- Which bit symbol assignment satisfies gray encoding?

110 111 101 100 000 001 011 010 ✓

\#Both!

000 001 011 010 110 111 101 100 ✓