# CMP205: Computer Graphics


## Lecture 1: Line Drawing

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Fall 2018

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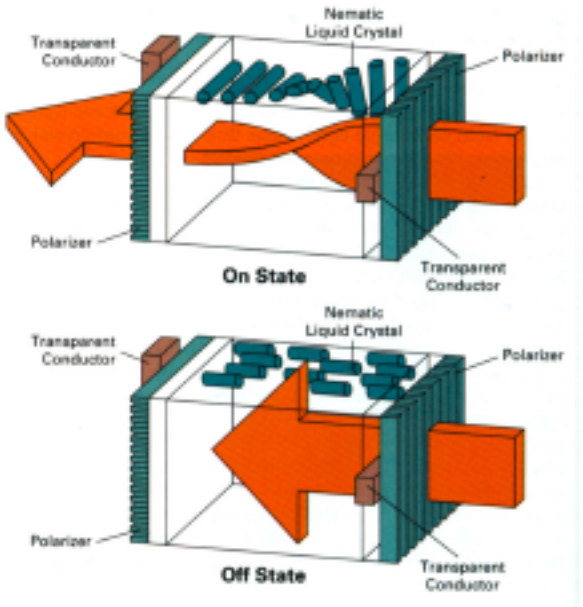
# Agenda

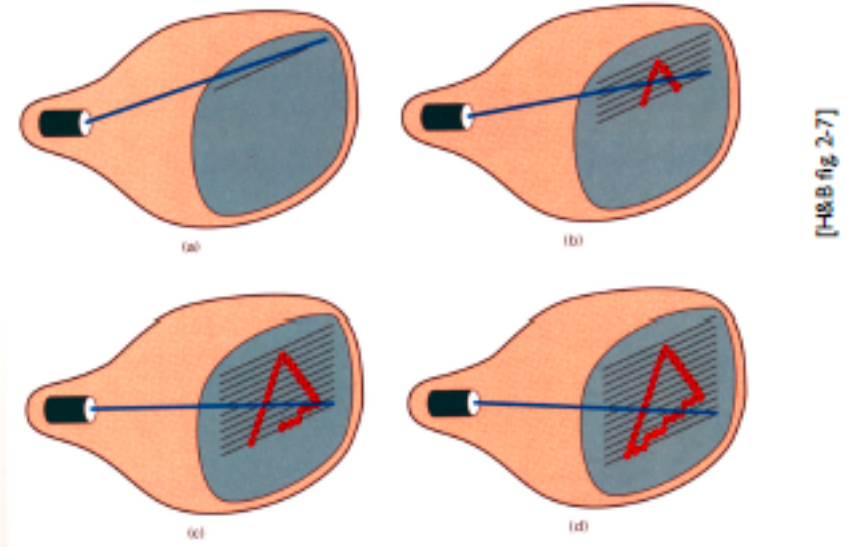
- Raster Displays
- Gamma Correction
- RGB Color & Alpha Channel
- Line Drawing

**Acknowledgments**: Some slides adapted from Steve Marschner and Fredo Durand.

# Raster Displays

- Displays that present *raster* images to the user
- LCD, CRT, Projector, … etc



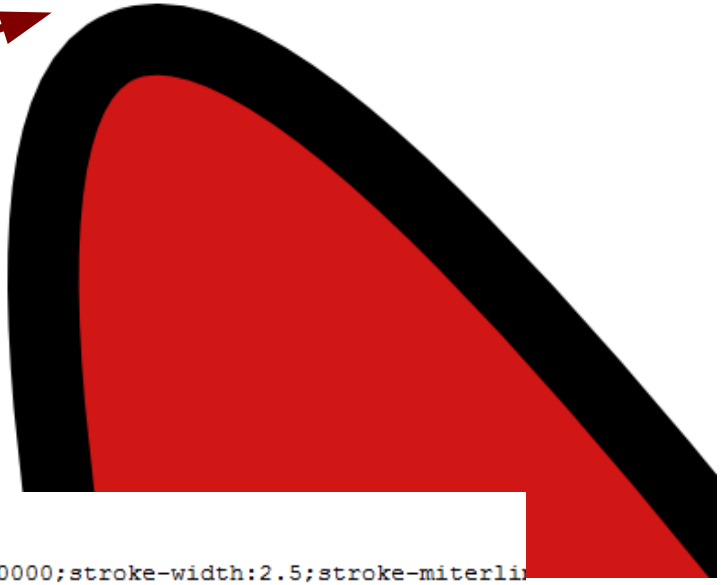



# Raster Images Vs Vector Images

- Raster Images
  - Made up of pixels with a certain resolution
  - "Pixelization" effect on zoom in
  - E.g. JPEG, BMP, PNG, GIF

# Raster Images Vs Vector Images

- Vector Images
  - Made up of “descriptions” of objects
  - Must be “rasterized” for display
  - Resolution independent
  - E.g. SVG



```
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    style="fill:#d11616;fill-opacity:1;stroke:#000000;stroke-width:2.5;stroke-miterli
    id="path3003"
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    sodipodi:cy="448.07648"
    sodipodi:r1="137.35846"
    sodipodi:r2="52.745647"
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    inkscape:randomized="0.005"
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    inkscape:transform-center-x="8.8250402"
    inkscape:transform-center-y="4.4368255" />
```
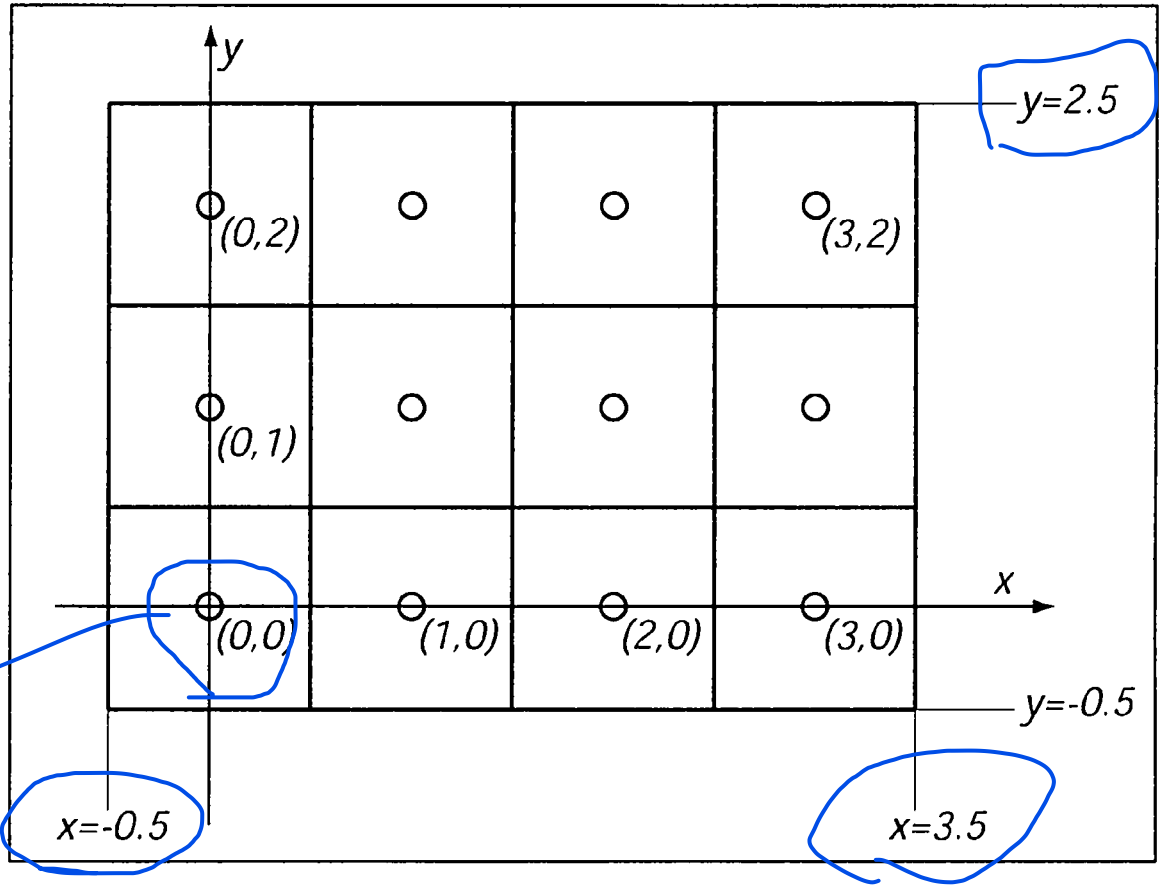

# Pixels

- Square Grid of “pixels”
- Each pixel stores color values
- Resolution: size of raster




4 x 3 Pixel Grid

Pixels at integer coordinates

ehna bnshtghl en el origin bta3na awl pixel t7t 3la el shmal.
tb3n t3ref el origin haga bt5tlf mn ktab le ktab,

-0.5 de 34an el point btb2a daymaan fe center of the pixels.
fa ba2y el atraf bnzed w n2l nos w hakaza.

# Display Intensity

- How many bits to represent each pixel?
  - Floating point number: 16 or 32 bits
  - Integers: 8, 10, 12, … bits
- Floating points
  - 0 → black (pixel off)
  - 1 → white (pixel fully on)
  - 0.5 → halfway gray
- Integers: quantized values e.g. 8-bits
  - 0 → black
  - 255 → white

# Gamma Correction

- How do displays convert pixel *values* to *intensities*?
  - We would like the screen to:
    - display *white* (intensity 1.0) when we give it 1.0
    - display *black* (intensity 0.0) when we give it 0.0
    - display *grey* (intensity 0.5) when we give it 0.5

- What actually happens?
  - Screens respond non-linearly, e.g. they give intensity
    - 1.0 with input 1.0
    - 0.0 with input 0.0
    - 0.25 with input 0.5

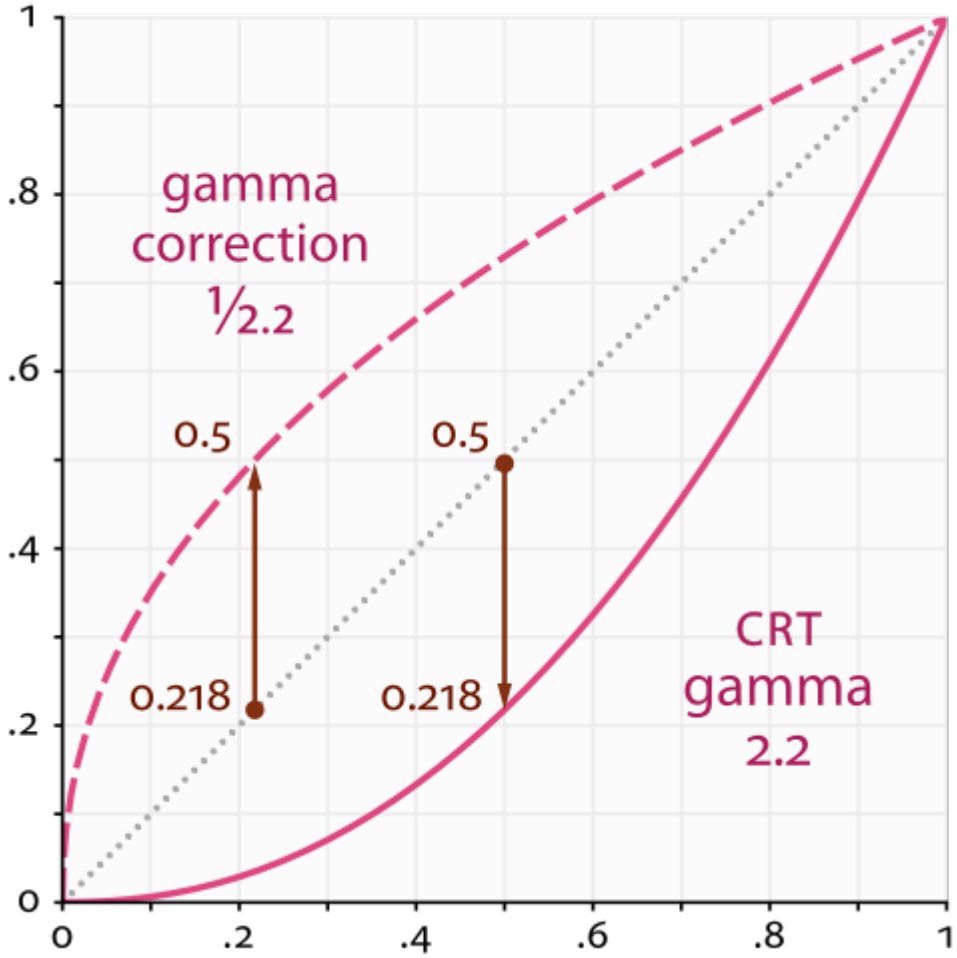
# Gamma Correction

This is modeled as:

$$I_{out} = I_{max} \times I_{\text{in}}^{\gamma}$$

assume Imax = 1

$$\gamma = 2.2$$
$$I_{\text{in}} = 0 \rightarrow I_{out} = 0$$
$$I_{\text{in}} = 0.5 \rightarrow I_{out} = 0.218$$
$$I_{\text{in}} = 1 \rightarrow I_{out} = 1$$



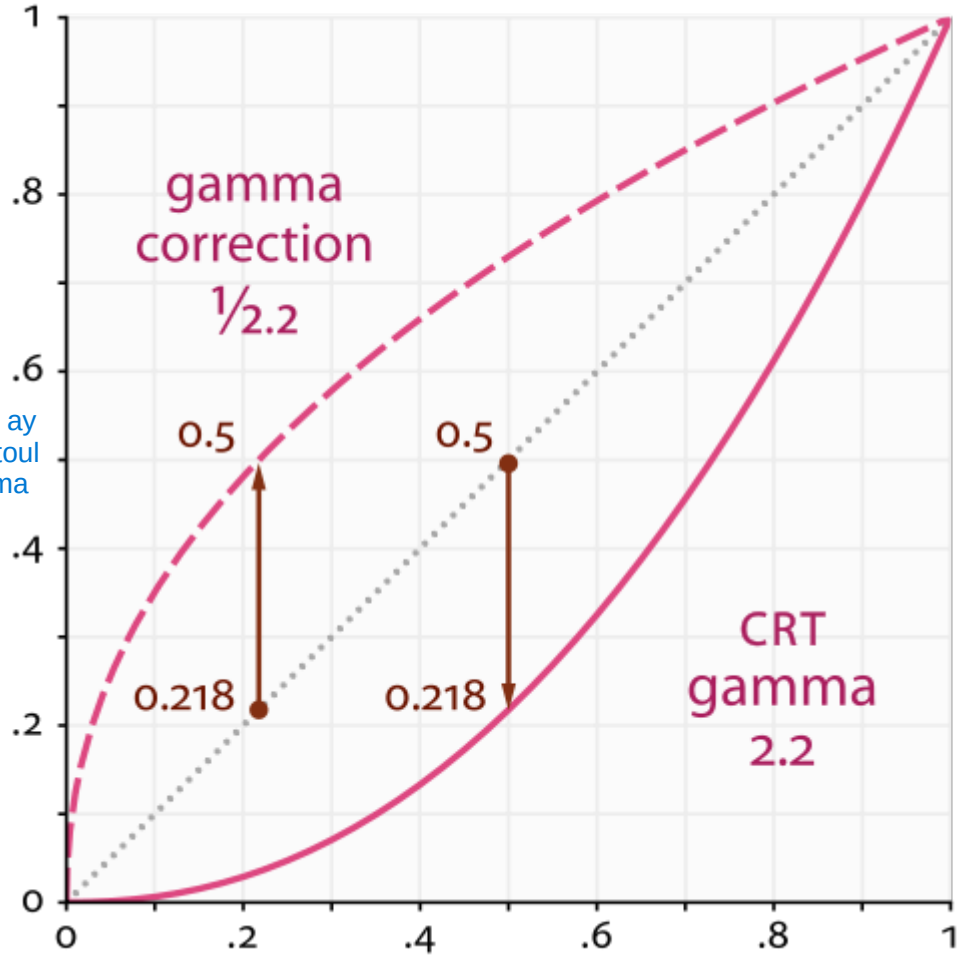
# Gamma Correction

To get the correct intensity from the screen, we modify the input:

Correction

$$I_{out} = I_{max} \times \left( I_{\text{in}}^{1/\gamma} \right)^{\gamma}$$
$$= I_{max} \times I_{\text{in}}$$

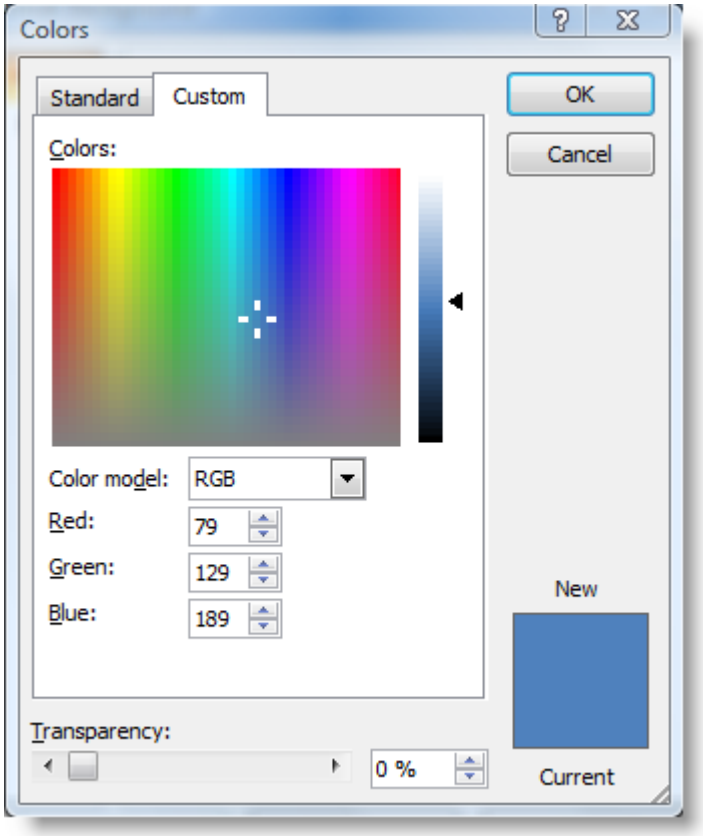
fa 34an t7l el moshkela, bkol bsata enta bttb2a 3aref el gamma correction bta3 ay shasha enta btgbha, fa kol el bt3mlo enk badal ma btdkhl el input intensity 3la toul laa enta bt3mlha power 1/gama 34an el power yro7 m3 el power w ytl3lk el kema el asasya.

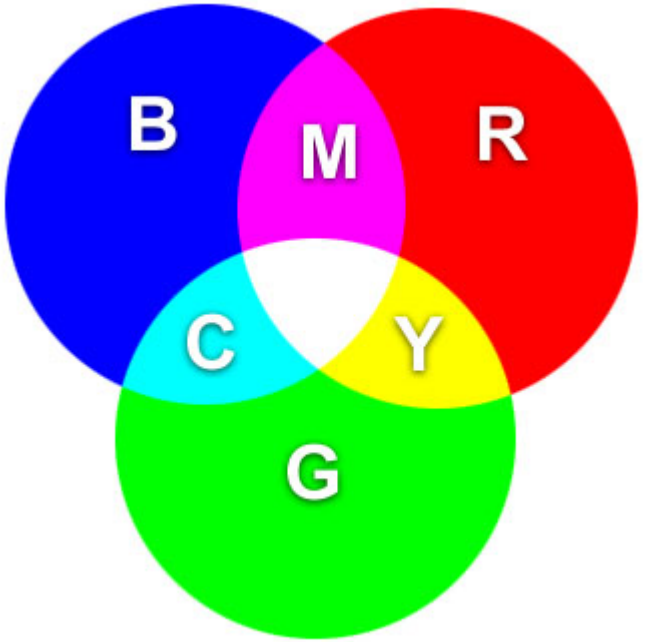


# RGB Color

- For color images, each pixel has three values: Red, Green, and Blue Color
- Additive Property

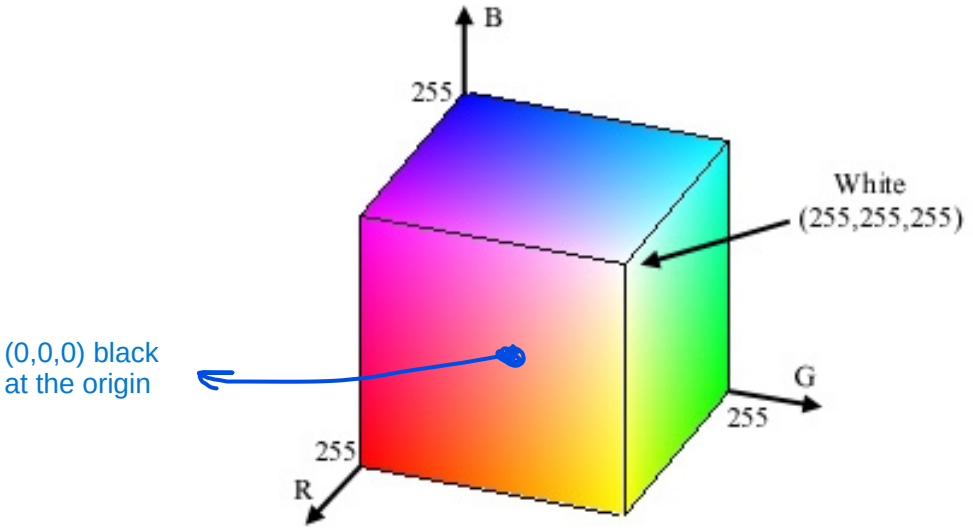
kol pixel mokawana mn 3 channels, red, green w blue 34an behom te2dr ttl3 kol el alwan el momkena



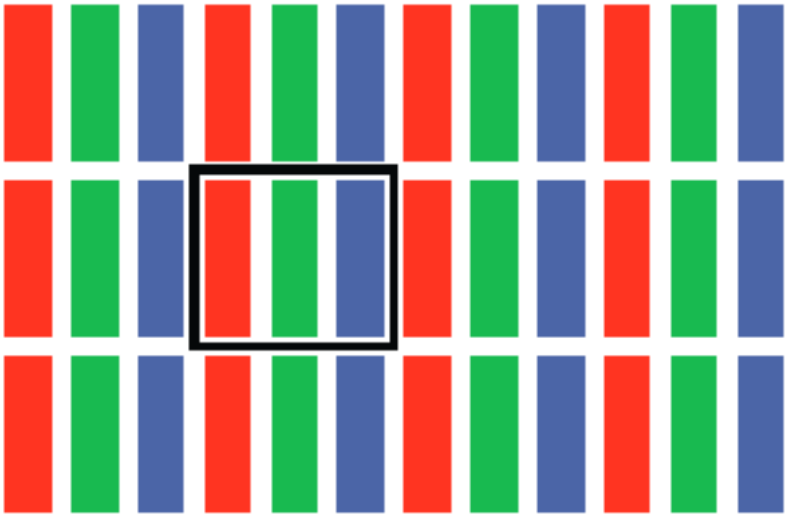



# RGB Colors

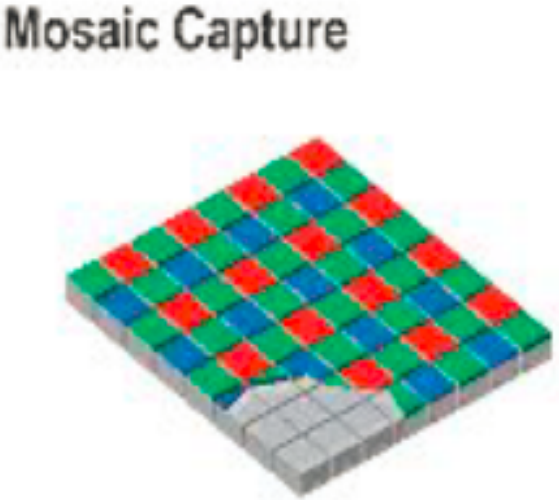
- Represent colors as 3 dimensional vectors (r, g, b)
  - Black = (0, 0, 0)
  - Red = (1, 0, 0)
  - …
  - White = (1, 1, 1)

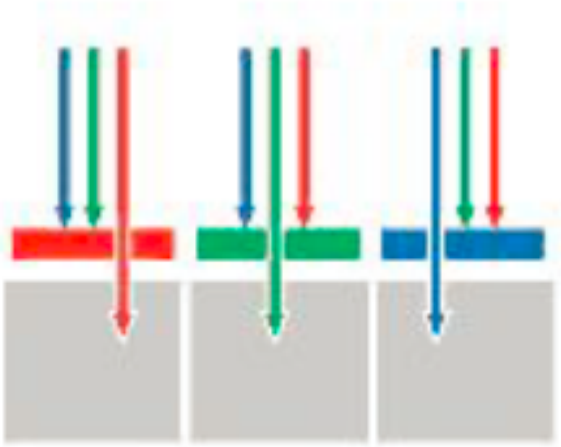


# Displays and Cameras



LCD pixels have 3 sub-pixels



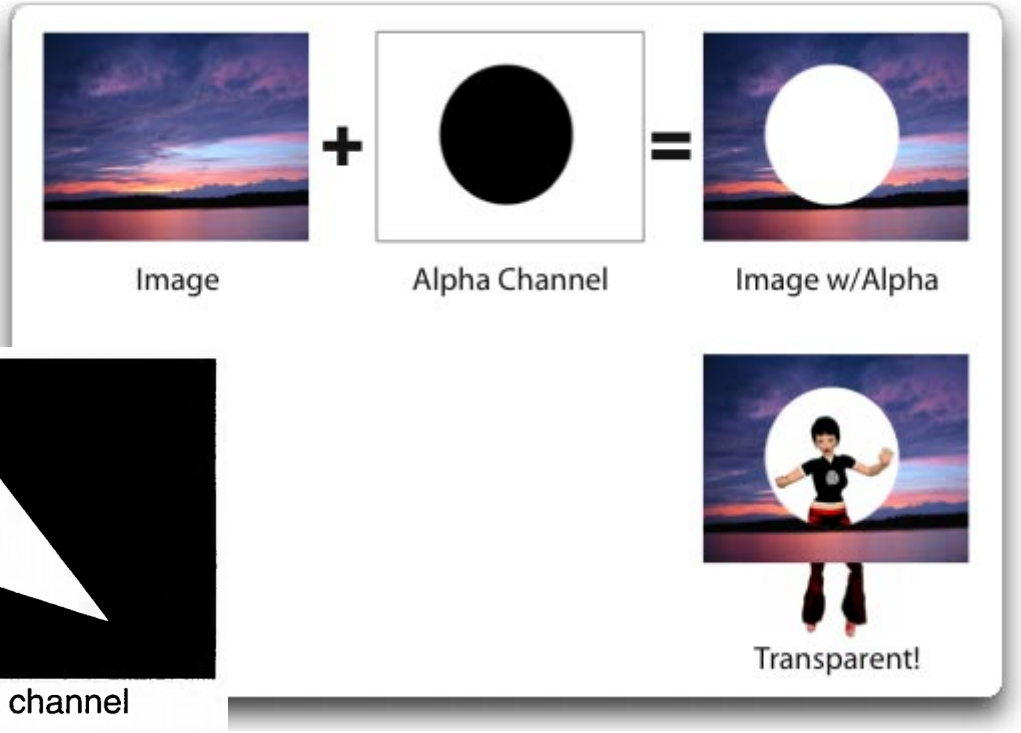



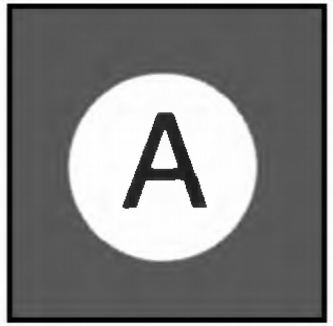
Bayer Mosaic in Digital Cameras

# Alpha Channel

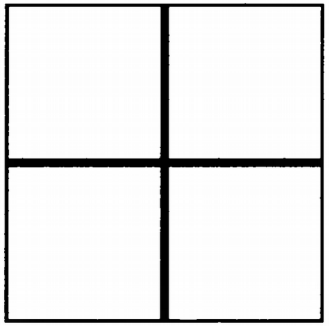
- Handles (partially) transparent objects

$$c = \alpha c_f + (1-\alpha) c_b$$



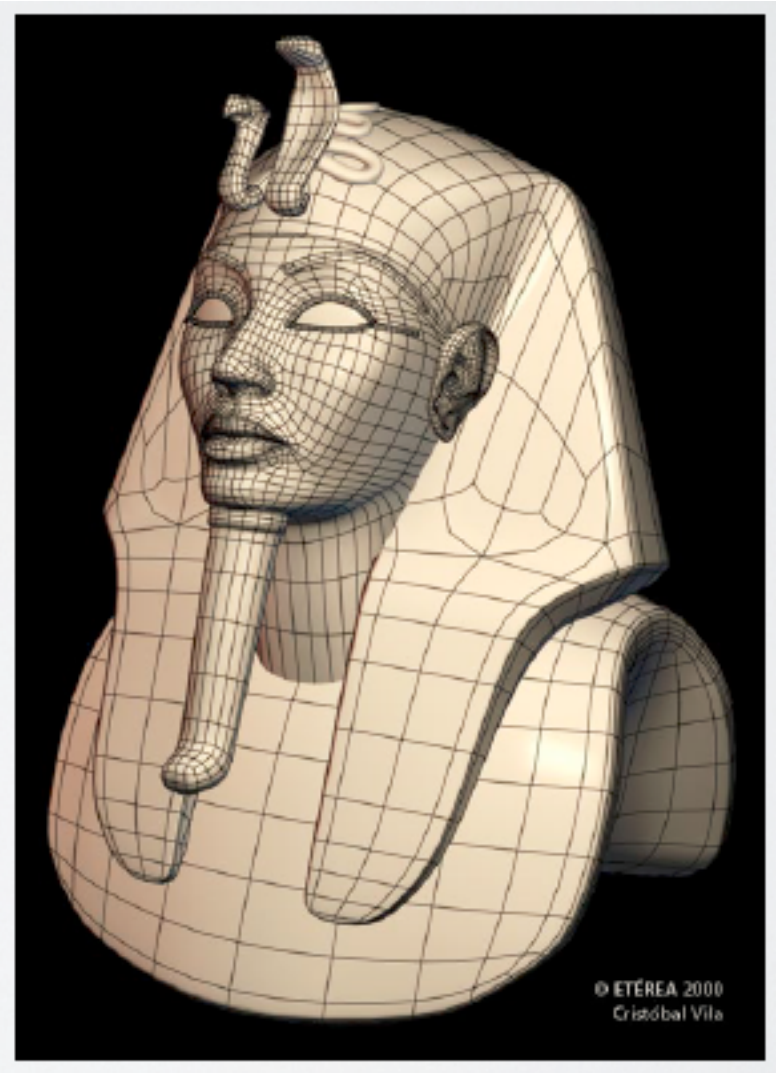
background RGB


foreground RGB


α channel

# Line Drawing

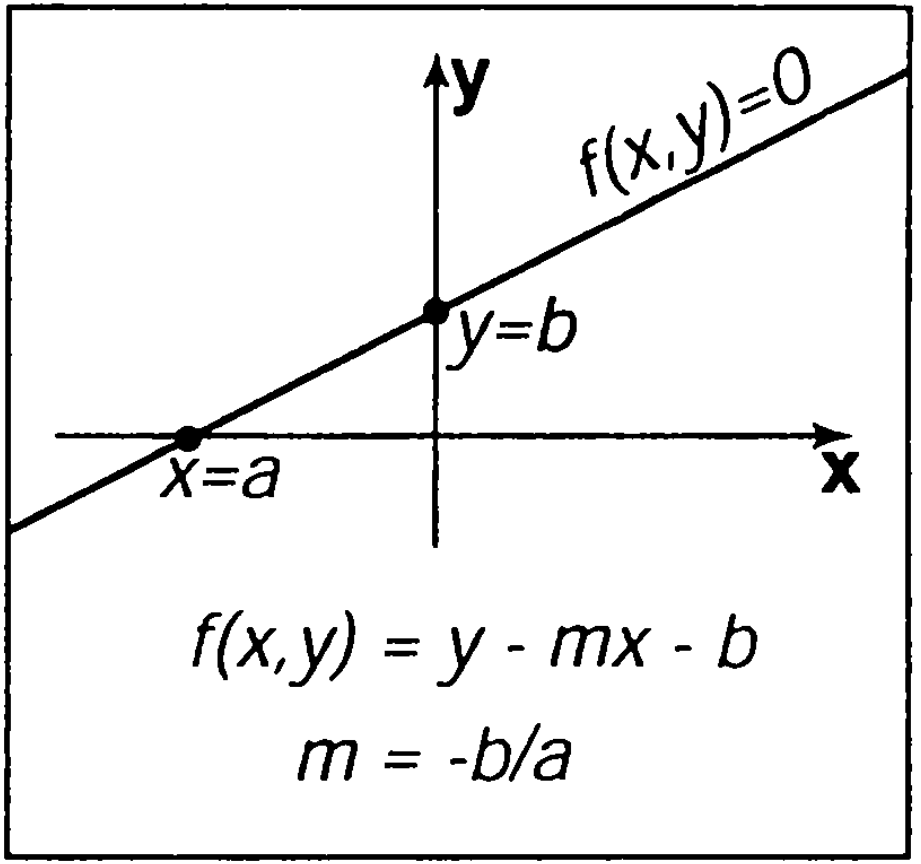
- Basic Operation



# Line Representation

- Slope-Intercept
- Problem?
  - Vertical lines!
  - $m = \infty$
- Solution?

$$y = mx + b$$



# Line Representation

- Implicit Form $f(x,y)=Ax+By+C=0$

Both points are on the line:

$Ax_0+By_0+C=0$

$Ax_1+By_1+C=0$

The normal vector on the line is:

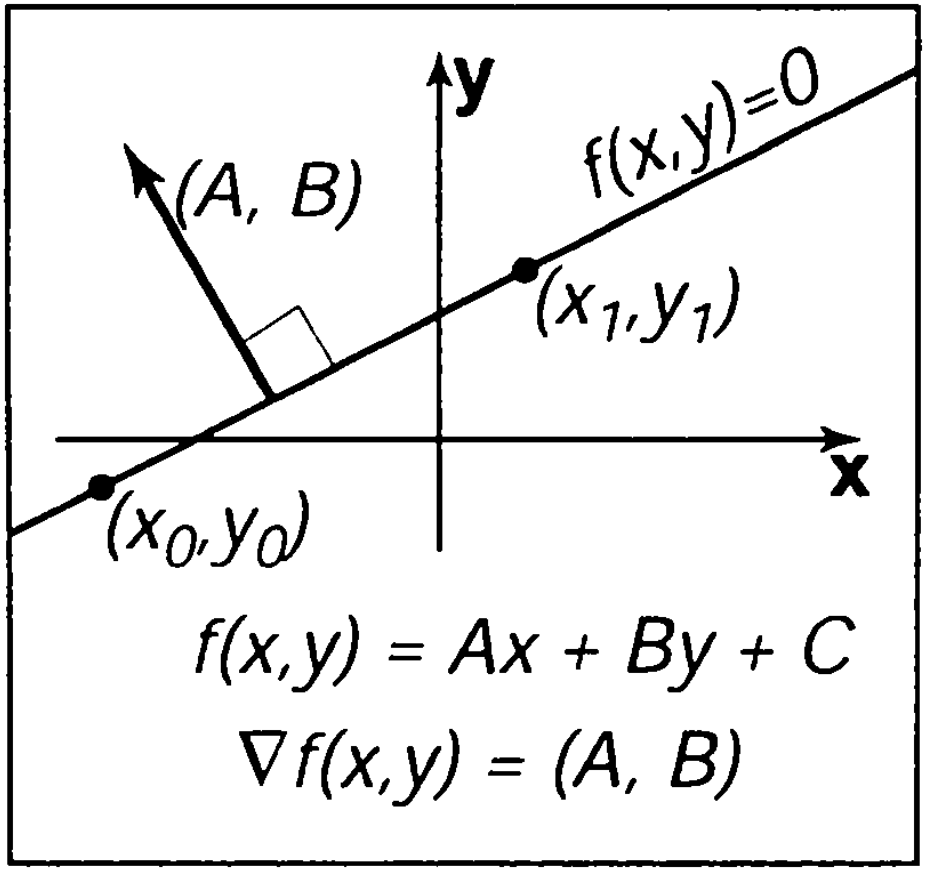
$(A,B)$

$(A,B)^T(x_1-x_0,y_1-y_0)=0$

why?

One possible $(A,B)$ is:

$(A,B)=(y_0-y_1,x_1-x_0)$



# Line Representation

- Implicit Form $f(x, y) = Ax + By + C = 0$

Plug in $(x_{0,} y_0)$ we get:

$(y_0 - y_1)x_0 + (x_1 - x_0)y_0 + C = 0$

after simplification:
-y1x0 + x1y0 + c = 0
c = x0y1 - x1y0

Solve for $C$:

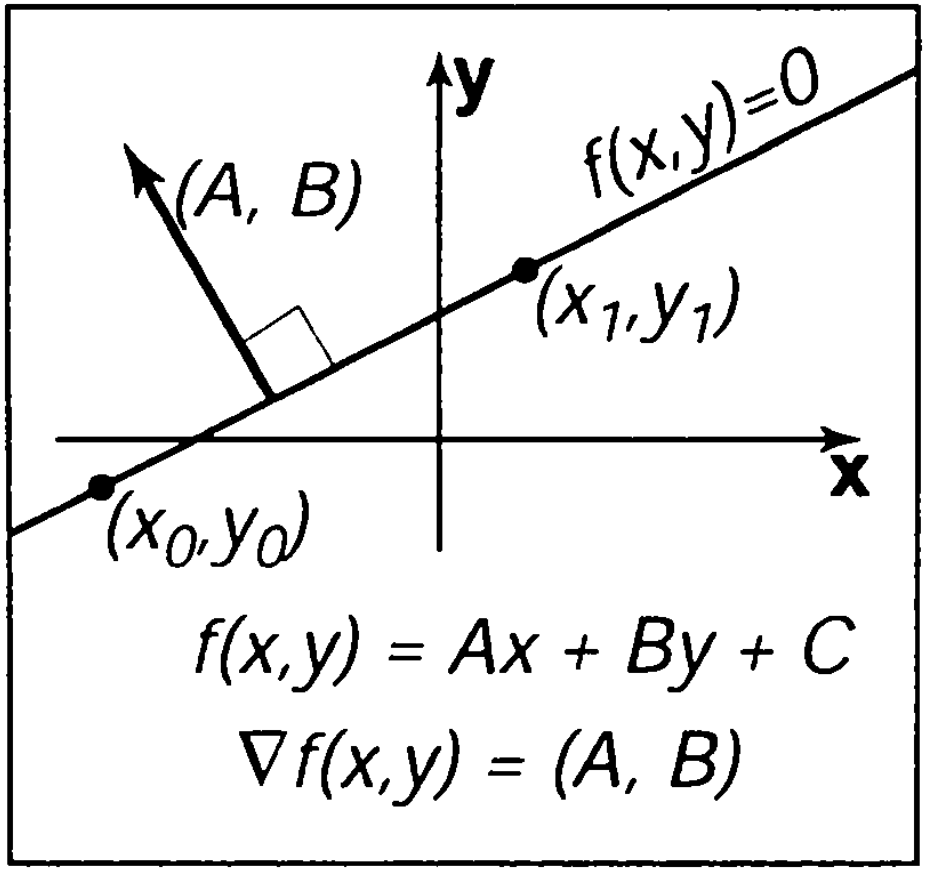
$C = x_0 y_1 - x_1 y_0$

$A = y_0 - y_1$

$B = x_1 - x_0$

$C = x_0 y_1 - x_1 y_0$

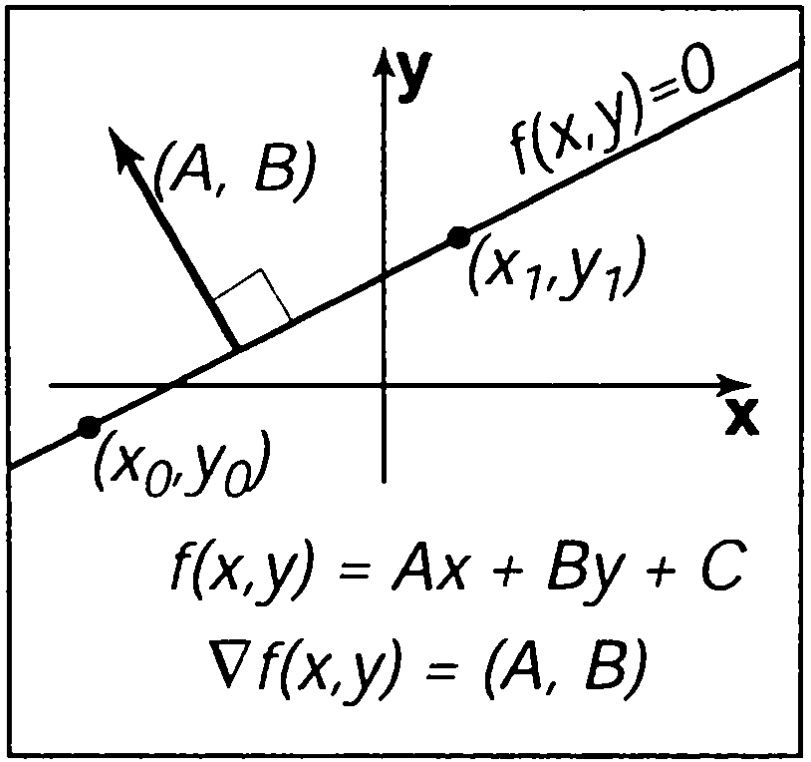
yeb2a keda na edrt eny ageb A,B&C bdalalet el 2 points el kano m3ana.



# Line Representation

$$f(x, y) = Ax + By + C = 0$$

$f(x, y)$ is the signed scaled distance from the point $(x, y)$ to the line




$f(x, y) = 0$ → On the line

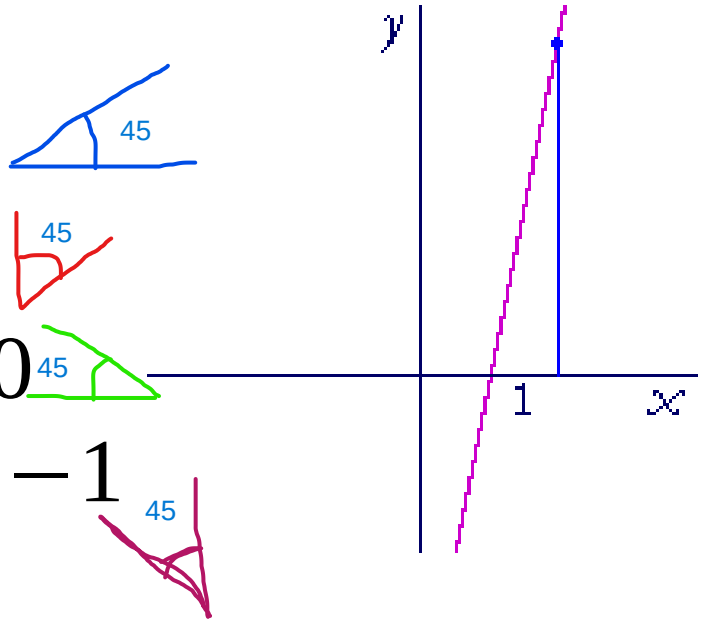
$f(x, y) > 0$ → Above the line

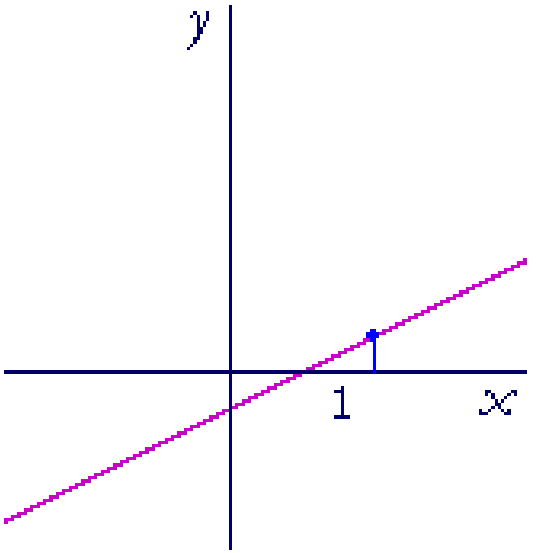
$f(x, y) < 0$ → Below the line

Note that, X must have positive cofficient for these equations to be valid. take care!!!.

# Midpoint Algorithm

- Line Drawing Algorithm
- Uses the Implicit Equation
- Similar to Bresenham's Algorithm
- Integer end-points
- Four Cases:
  - $0 < m \leqslant 1$
  - $1 < m < \infty$
  - $-1 < m \leqslant 0$
  - $-\infty < m \leqslant -1$



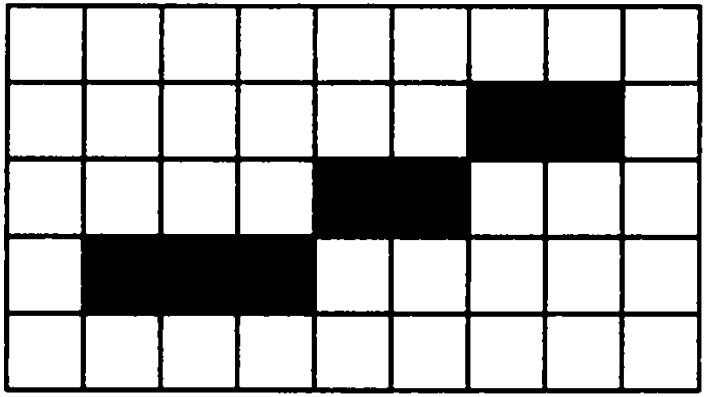

tb ehna leh asmnahom keda aslun?
34an bkol bsata fe awl case msln, enta 3aref en el x btzeed asr3 bkter mn el y

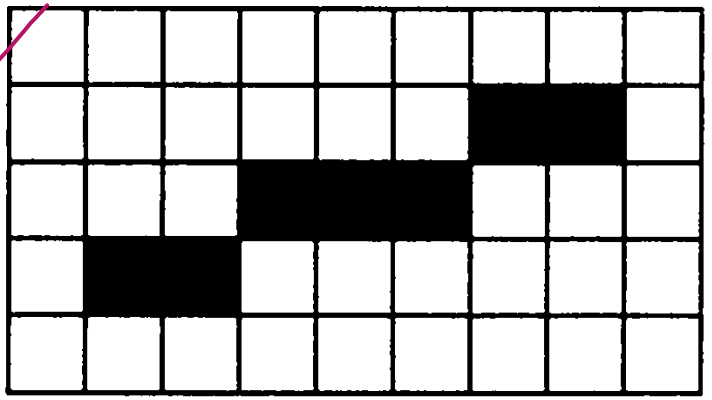
lakn fe el case el tanya laa el y hya el btzed asr3 w hakaza

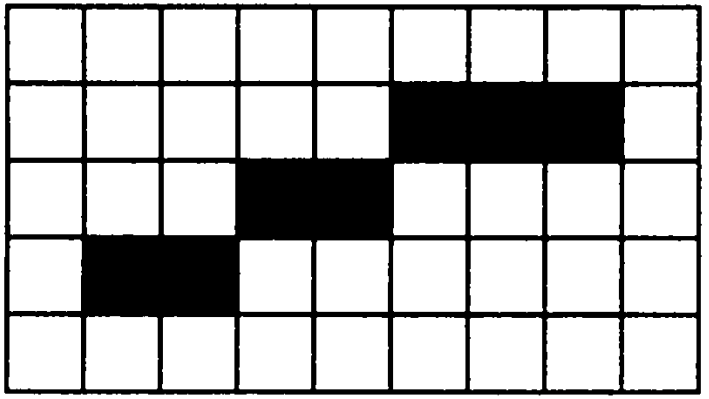
lw s2ltny t2oly tb da kda hwa m8ty el cases en y >=0 bs a2olak laa ya beeh el lines extended 3ady

# Midpoint Algorithm

- First case:
  - more “run” than “rise”
  - Assume moving right







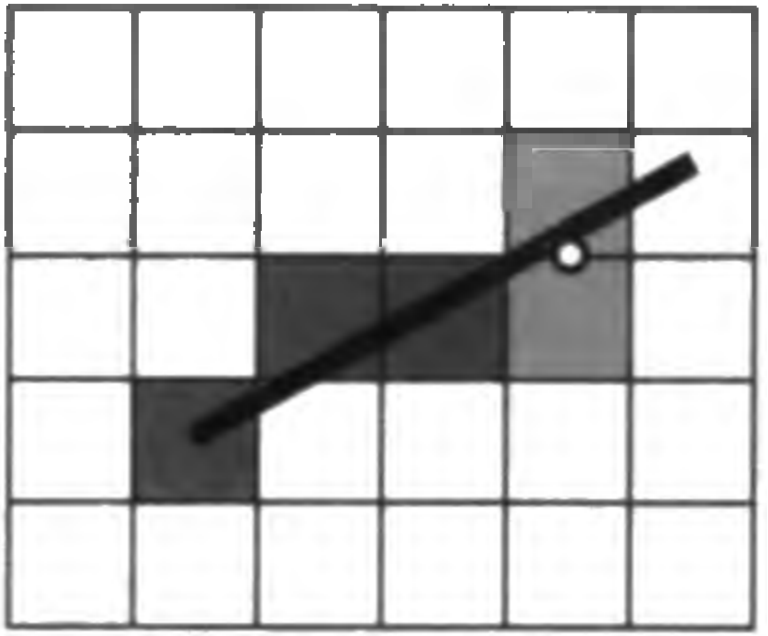
- Which one is better?

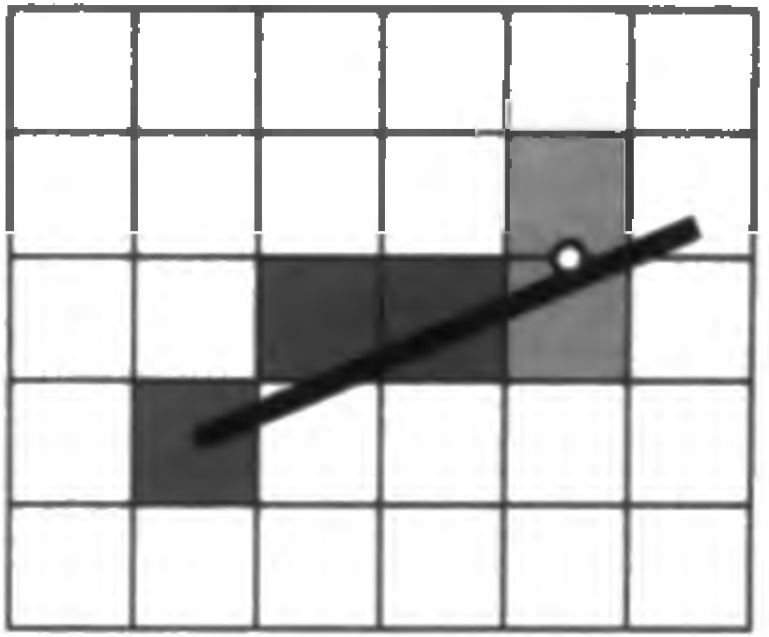
the one in the middle, due to the presence of symmetry.

# Midpoint Algorithm

- First version

$$
\begin{aligned}
&y = y_0 \\
&\text{for } x = x_0 : x_1 \\
&\quad draw(x, y) \\
&\quad \text{if } f(x+1, y+0.5) < 0 \\
&\qquad y = y+1
\end{aligned}
$$





- Optimizations?

# Midpoint Algorithm

- Optimizations
  - Incremental Calculations
  - Integer Operations

$$
\begin{aligned}
&y = y_0 \\
&\text{for } x = x_0 : x_1 \\
&\quad draw(x, y) \\
&\quad \text{if } f(x+1, y+0.5) < 0 \\
&\quad\quad y = y+1
\end{aligned}
$$

# Midpoint Algorithm

- Incremental Calculations
  - Note that:

$$f(x, y) = (y_0 - y_1)x + (x_1 - x_0)y + C = 0$$

  - Which implies that:

$$f(x+1, y) = f(x, y) + (y_0 - y_1)$$
$$f(x+1, y+1) = f(x, y) + (y_0 - y_1) + (x_1 - x_0)$$

# Midpoint Algorithm

- Second Pass

$$
\begin{aligned}
&y = y_0 \\
&d = f\left(x_0 + 1, y_0 + 0.5\right) \\
&\text{for } x = x_0 : x_1 \\
&\quad draw\left(x, y\right) \\
&\quad \text{if } d < 0 \\
&\qquad y = y + 1 \\
&\qquad d = d + \left(x_1 - x_0\right) + \left(y_0 - y_1\right) \\
&\quad \text{else} \\
&\qquad d = d + \left(y_0 - y_1\right)
\end{aligned}
$$

# Midpoint Algorithm

- Integer Operations
  - Note that:

$$f(x,y) = 2\mathrm{f}(x,y) = 0$$

- Which implies:

$$d = 2\mathrm{f}(x_0 + 1, y_0 + 0.5)$$

$$d = 2(y_0 - y_1)(x_0 + 1) + (x_1 - x_0)(2\mathrm{y}_0 + 1) + 2\mathrm{x}_0 y_1 - 2 x_1 y_0$$

we can rephrase this equation to be
d= 2f(x0,y0) + 2(y0-y1) + (x0-x1)

# Midpoint Algorithm

- Third Pass

$$y = y_0$$
$$d = 2(y_0 - y_1)(x_0 + 1) + (x_1 - x_0)(2y_0 + 1) + 2x_0 y_1 - 2x_1 y_0$$
for $x = x_0 : x_1$
  $draw(x, y)$
  if $d < 0$
    $y = y + 1$
    $d = d + 2(x_1 - x_0) + 2(y_0 - y_1)$
  else
    $d = d + 2(y_0 - y_1)$

# Recap

- Raster Displays
- Alpha Channels
- Gamma Correction
- Line Drawing