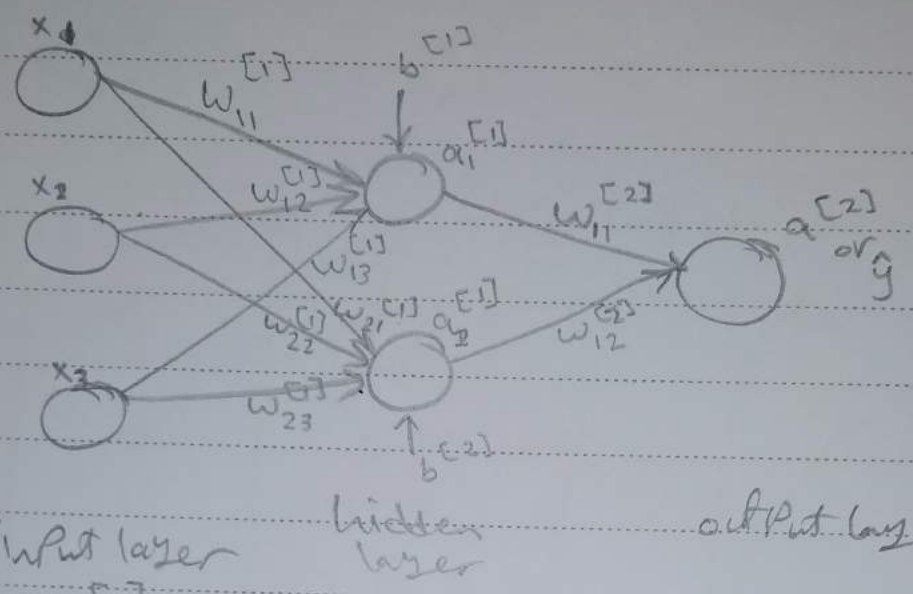
Sheet "5"

Problem ①



$b^{[1]} = 0 \qquad b^{[2]} = 0$

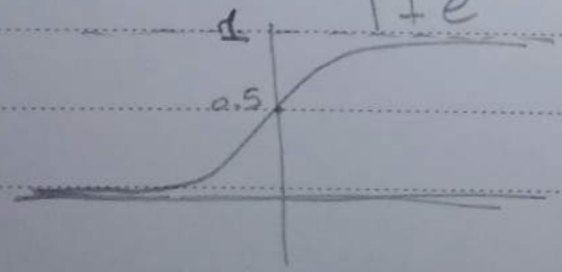
a) $Z^{[1]} = W^{[1]} \underline{X} + \cancel{b^{[1]}}$

$a^{[1]} = \text{sigmoid}(z^{[1]})$

$Z^{[2]} = W^{[2]} a^{[1]} + b^{[2]}$

$a^{[2]} = \text{Sigmoid}(Z^{[2]})$

eqn of Sigmoid $\dfrac{1}{1+e^{-x}} = \dfrac{e^{x}}{e^{x}+1}$



dimension of

$W^{[1]}_{2\times 3}$

$\underline{X} = a^{[0]}_{3\times 1}$

$\underline{Z}^{[1]}$ & $\underline{a}^{[1]}_{2\times 1}$

$W^{[2]}_{1\times 2}$

$a^{[1]}_{2\times 1}$

$Z^{[2]}$ & $a^{[2]} = \hat{y}_{1\times 1}$

b) Loss (cost) function of binary classification

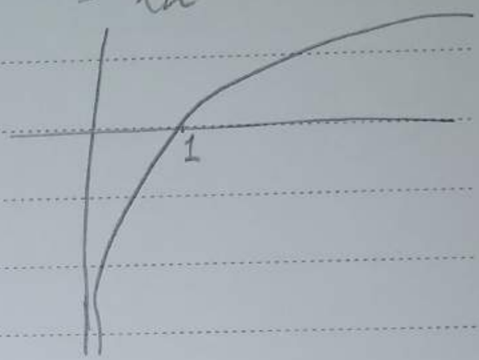
Cross entropy loss function

$y \begin{cases} 0 \\ 1 \end{cases}$ $\hat{y} \begin{cases} 0 \\ 1 \end{cases}$

$$\mathcal{L} = -\left[y \log \hat{y} + (1-y) \log (1-\hat{y}) \right]$$

$\log \rightarrow \ln$

y	$\hat{y}$	$\mathcal{L}$
0	0	0 No loss ✓
0	1	big -ve x
1	0	big -ve x
1	1	0 No loss ✓



c) Back propagation

Required: Minimize Loss function w.r.t. W & b

Get:

$$\frac{\partial L}{\partial W^{[2]}} \ \& \ \frac{\partial L}{\partial b^{[2]}} \ \& \ \frac{\partial L}{\partial W^{[1]}} \ \& \ \frac{\partial L}{\partial b^{[1]}}$$

$$\frac{\partial L}{\partial W^{[2]}} = \frac{\partial L}{\partial a^{[2]}} * \frac{\partial a^{[2]}}{\partial Z^{[2]}} * \frac{\partial Z^{[2]}}{\partial W^{[2]}}$$

$\hat{y}^{L}$

$$\boxed{\frac{d}{dx} \ln(x) = \frac{1}{x}}$$

$$\frac{\partial L}{\partial a^{[2]}} = \frac{-y}{a^{[2]}} + \frac{(1-y)}{1-a^{[2]}}$$

$$= \frac{a^{[2]} - y}{a^{[2]}(1-a^{[2]})}$$

$$\frac{\partial L}{\partial z^{[2]}} = \frac{\partial L}{\partial a^{[2]}} * \frac{\partial a^{[2]}}{\partial z^{[2]}}$$

$$= \frac{a^{[2]} - y}{\cancel{a^{[2]}(1-a^{[2]})}} * \cancel{a^{[2]}(1-a^{[2]})}$$

$$= a^{[2]} - y$$

$$y = \text{sigmoid}(x)$$

$$= \frac{1}{1+e^{-x}}$$

$$= (1+e^{-x})^{-1}$$

$$= -(1+e^{-x})^{-2}(-e^{-x})$$

$$= \frac{e^{-x}}{(1+e^{-x})^2}$$

$$= \frac{1}{1+e^{-x}} * \frac{e^{-x}+1-1}{1+e^{-x}}$$

$$= y * \left[\frac{1+e^{-x}}{1+e^{-x}} - \frac{1}{1+e^{-x}}\right]$$

$$= y * [1-y]$$

$$* \frac{\partial L}{\partial W^{[2]}_{1\times 2}} = \frac{\partial L}{a^{[2]}} * \frac{\partial a^{[2]}}{\partial z^{[2]}} * \frac{\partial z^{[2]}}{\partial W^{[2]}}$$

$$= (a^{[2]} - y)_{1\times 1} (a^{[1]})^T$$ should be 1×2

$$* \frac{\partial L}{\partial b^{[2]}} = \frac{\partial L}{\partial a^{[2]}} * \frac{\partial a^{[2]}}{\partial z^{[2]}} * \frac{\partial z^{[2]}}{\partial b^{[2]}}$$

$$= (a^{[2]} - y)$$

$$\frac{\partial L}{\partial a^{[1]}_{2\times 1}} = \frac{\partial L}{\partial a^{[2]}} * \frac{\partial a^{[2]}}{\partial z^{[2]}} * \frac{\partial z^{[2]}}{\partial a^{[1]}}$$

$$= ((a^{[2]} - y)_{1\times 1} \; W^{[2]}_{1\times 2})^T$$ $\rightarrow$ should be 2×1

$$\frac{\partial L}{\partial z^{1}} = \underset{\text{chain of}}{\frac{\partial L}{\partial a^{[1]}}} * \frac{\partial a^{[1]}}{\partial z^{[1]}}$$

$$= (a^{[2]} - y)[W^{[2]}]^T * a^{[1]}(1-a^{[1]})$$

$$* \frac{\partial L}{\partial W^{[1]}_{2\times 3}} = \underset{\text{chain of}}{\frac{\partial L}{\partial z^{[1]}}} * \frac{\partial z^{[1]}}{\partial W^{[1]}}$$

$$= \frac{\partial L}{\partial z^{[1]}_{2\times 1}} * (a^{[0]})^T_{1\times 3}$$

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$$* \frac{\partial L}{\partial b^{[1]}} = \frac{\partial L}{\partial z^{[1]}} * \frac{\partial z^{[1]}}{\partial b^{[1]}}$$

$$= \frac{\partial L}{\partial z^{[1]}} * 1$$

d) $\underline{x} = \begin{bmatrix} 3 \\ 0.2 \\ -0.1 \end{bmatrix}$

$$\underline{z}^{[1]} = \underline{W}^{[1]} \underline{x} + b^{[1]}$$

$$= \begin{bmatrix} 0.1 & 0.2 & 0.3 \\ -0.1 & 0 & 0.2 \end{bmatrix} \begin{bmatrix} 3 \\ 0.2 \\ -0.1 \end{bmatrix} = \begin{bmatrix} 0.31 \\ -0.32 \end{bmatrix}$$

$$a^{[1]} = \begin{bmatrix} 0.5769 \\ 0.4207 \end{bmatrix}$$

$$z^{[2]} = \begin{bmatrix} 0.2 & -0.2 \end{bmatrix} \begin{bmatrix} 0.5769 \\ 0.4207 \end{bmatrix} = 0.03124$$

$$a^{[2]} = 0.5070 \Rightarrow \hat{y}$$

e) $\hat{y} < 0.6 \Rightarrow$ output: 0

f) Cost: $\mathcal{L} = -(\cancel{y \log \hat{y}} + (1-y) \log(1-\hat{y}))$

$$= -(1 * \ln(1 - 0.5070))$$

$$= 0.70887$$

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9) $$\frac{\partial L}{\partial W^{[2]}} = (a_i^{[2]} - y)(a_i^{[1]})^T$$

$$= (0.5078 - 0) * [0.5769 \quad 0.4207]$$

$$= [0.2929 \quad 0.2136] \ \#$$

$$\frac{\partial L}{\partial b^{[2]}} = a^{[2]} - y = 0.5078 \ \#$$

$$\partial L / \partial a^{[1]} = (0.5078 * [0.2 \quad -0.2])^T = \begin{bmatrix} 0.10156 \\ -0.10156 \end{bmatrix}$$

element wise

$$\frac{\partial L}{\partial Z^{[1]}} = \begin{bmatrix} 0.10156 \\ -0.10156 \end{bmatrix} * \begin{bmatrix} 0.5769 \\ 0.4207 \end{bmatrix} * \begin{bmatrix} 1-0.5769 \\ 1-0.4207 \end{bmatrix}$$

$$= \begin{bmatrix} 0.0247 \\ -0.0247 \end{bmatrix}$$

$$\frac{\partial L}{\partial W^{[1]}} = \begin{bmatrix} 0.0247 \\ -0.0247 \end{bmatrix} [3 \quad 0.2 \quad -0.1]$$

$$= \begin{bmatrix} 0.0741 & 4.94 \times 10^{-3} & -2.47 \times 10^{-3} \\ -0.0741 & -4.94 \times 10^{-3} & 2.47 \times 10^{-3} \end{bmatrix} \ \#$$

$$\frac{\partial L}{\partial b^{[1]}} = \begin{bmatrix} 0.0247 \\ -0.0247 \end{bmatrix} \ \#$$

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h) Gradient descent :-

$$W^{[2]} := W^{[2]} - \alpha \frac{\partial L}{\partial W^{[2]}}$$

$$b^{[2]} := b^{[2]} - \alpha \frac{\partial L}{\partial b^{[2]}}$$

$$W^{[1]} := W^{[1]} - \alpha \frac{\partial L}{\partial W^{[1]}}$$

$$b^{[1]} := b^{[1]} - \alpha \frac{\partial L}{\partial b^{[1]}}$$

α is the learning rate $\alpha = 0.05$

$$W^{[2]} := [0.2 \quad -0.2] - 0.05\,[0.2929 \quad 0.2136]$$
$$= [0.1854 \quad 2.136 \times 10^{-3}]$$

$$b^{[2]} = 0 - 0.05 * 0.5078 = -0.02539$$

$$W^{[1]} = \begin{bmatrix} 0.1 & 0.2 & 0.3 \\ -0.1 & 0 & 0.2 \end{bmatrix} - 0.05 \begin{bmatrix} 0.0741 & 4.94 \times 10^{-3} & -2.47 \times 10^{-3} \\ -0.0741 & -4.94 \times 10^{-3} & 2.47 \times 10^{-3} \end{bmatrix}$$

$$= \begin{bmatrix} 0.0963 & 0.199753 & 0.300125 \\ -0.0963 & 2.47 \times 10^{-4} & 0.19987 \end{bmatrix}$$

$$b^{[1]} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} - 0.05 * \begin{bmatrix} 0.0247 \\ -0.0247 \end{bmatrix} = \begin{bmatrix} -1.235 \times 10^{-3} \\ 1.235 \times 10^{-3} \end{bmatrix}$$

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