# NN Sheet 2

1) • Need decision boundary and regions

2D linear classifier takes the form

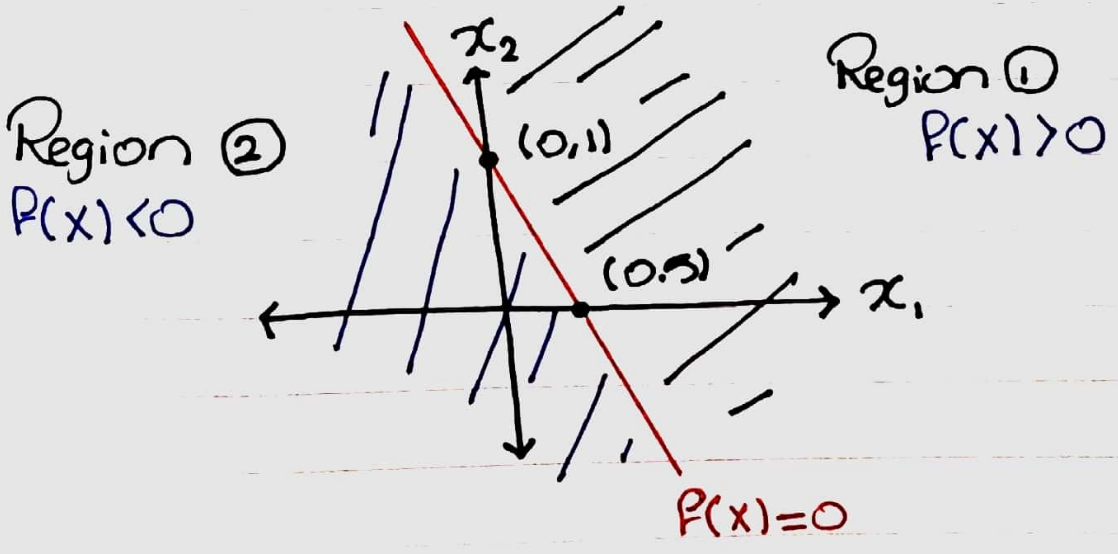
$$\underline{W}^T \underline{X} + W_0 = 0 \quad \text{where} \quad \underline{W} = \begin{pmatrix} W_1 \\ W_2 \end{pmatrix}, \underline{X} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

→ Here $\underline{W} = \begin{pmatrix} 1 \\ 0.5 \end{pmatrix}$ and $W_0 = -0.5$ so

$$(1 \quad 0.5) \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + -0.5 = 0$$

$$\underbrace{x_1 + 0.5 x_2 - 0.5}_{f(x)} = 0$$

- $x_1 = 0 \rightarrow x_2 = 1$
- $x_2 = 0 \rightarrow x_1 = 0.5$




- $x_2 = -2x_1 + 1$

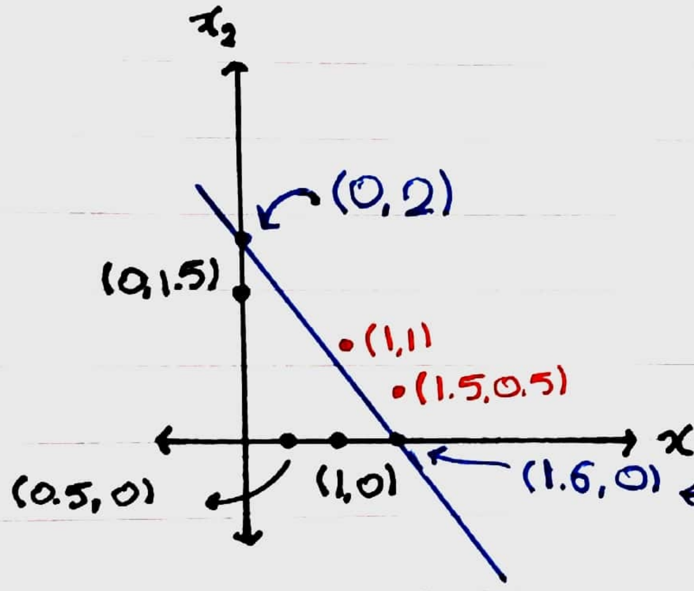
$f(x) > 0 \rightarrow x_2 > -2x_1 + 1$

(all points above the line)

2)

| Class ① | Class ② |
| --- | --- |
| (0, 1.5) | (1, 1) |
| (1, 0) | (1.5, 0.5) |
| (0.5, 0) | |

• $x(m)$ ↳ just an index




← measured using ruler.

• Draw Points to scale
• Draw line, see what 2 Points it Passes through → expression

• The line has equation ← $f(x)$

$$2x_2 + 1.6x_1 + (0 - 2 \times 1.6) = 0$$

$2x_2 + 1.6x_1 - 3.2 > 0$ | $2x_2 + 1.6x_1 - 3.2 < 0$

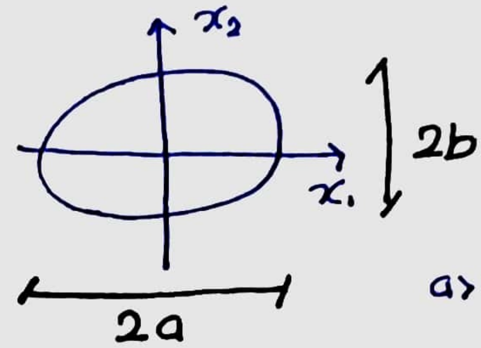
Class 2 | Class 1

• given (1, 1.5) we have $f(x) = 1.4$ thus class 2.

* Since data should be linearly separable, a line through (1.1, 0) and (0, 1.6) would've also worked

$1.1 \to 1 + \Delta$, $1.6 \to 1.5 + \Delta$

3. $\frac{x_1^2}{a^2} + \frac{x_2^2}{b^2} = 1 \longleftrightarrow$




$a > b > 0$

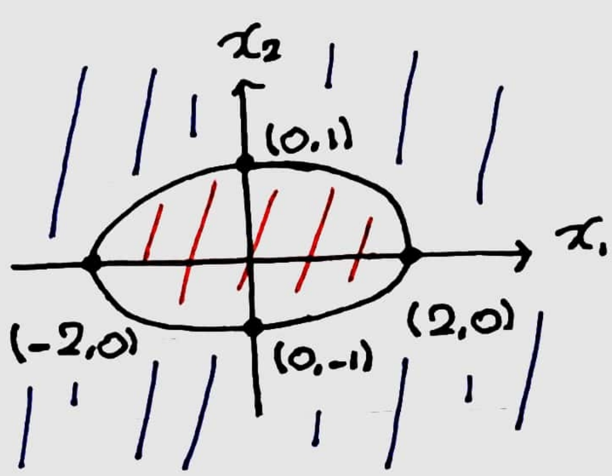
given $\frac{x_1^2}{(2^2)} + \frac{x_2^2}{1^2} = 1$

$\frac{x_1^2}{2^2} + \frac{x_2^2}{1^2} < 1$

- inside the ellipse (eqn. of all smaller ellipses of same a/b ratio)

$\frac{x_1^2}{2^2} + \frac{x_2^2}{1^2} > 1$

- outside the ellipse




- class 1
- class 2

* there's no 3rd class.

4.

* Class Centers are given

$m_1$ $m_2$ $m_3$
$(-1,0)$, $(1,0)$, $(0,\sqrt{3})$

→ let $\underline{m}_i$ be a class center, then the distance between an arbitrary point $X$ and $m_i$ squared is

$$\|\underline{X}-\underline{m}_i\|^2 = (\underline{X}-\underline{m}_i)^T(\underline{X}-\underline{m}_i)$$

$$= X^T X - X^T m_i - m_i^T X + m_i^T m_i$$
$$= X^T X - X^T m_i . 2 + m_i^T m_i$$

• $X^T m_i = m_i^T X$

→ For two classes $m_i$, $m_j$ the decision boundary is wherever $\|\underline{X}-\underline{m}_i\|^2 = \|X-m_j\|^2$

$$f(\underline{X}) = \|\underline{X}-\underline{m}_i\|^2 - \|\underline{X}-\underline{m}_j\|^2$$

$$= -2X^T m_i + m_i^T m_i$$
$$+ 2X^T m_j - m_j^T m_j$$

$$= 2X^T(m_j - m_i) - (\|m_j\|^2 - \|m_i\|^2)$$

• can also ÷ 2

• $f(\underline{X}) > 0$ means $\|X-m_i\| > \|X-m_j\|$ so classify as $m_j$.

Between $m_1 = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$, $m_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$$F(\underline{x}) = (x_1 \quad x_2)\begin{pmatrix} -2 \\ 0 \end{pmatrix} - \frac{1}{2}(1 - 1)$$

$$= -2x_1$$

- Hence, $x_1 = 0$ is the decision boundary.

$-2x_1 > 0 \rightarrow m_1$

Between $m_1 = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$, $m_3 = \begin{pmatrix} 0 \\ \sqrt{3} \end{pmatrix}$

$$F(\underline{x}) = (x_1 \quad x_2)\begin{pmatrix} -1 \\ -\sqrt{3} \end{pmatrix} - \frac{1}{2}(1 - 3)$$

$$= -x_1 - \sqrt{3}\, x_2 + 1$$

- $-x_1 - \sqrt{3}\, x_2 + 1 > 0 \rightarrow m_1$

Between $m_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $m_3 = \begin{pmatrix} 0 \\ \sqrt{3} \end{pmatrix}$

$$F(\underline{x}) = (x_1 \quad x_2)\begin{pmatrix} 1 \\ -\sqrt{3} \end{pmatrix} - \frac{1}{2}(1 - 3)$$

$$= x_1 - \sqrt{3}\, x_2 + 1$$

- $x_1 - \sqrt{3}\, x_2 + 1 > 0 \rightarrow m_2$

- Could've also used the fact that
  - $\rightarrow$ line due to MDC is $\perp$ on line joining two centers ($m_t m_n = -1$)
  - $\longrightarrow$ line due to MDC passes by midpoint.

| $-2x_1 = 0$ | $-x_1 - \sqrt{3}\,x_2 + 1 = 0$ | $x_1 - \sqrt{3}\,x_2 + 1 = 0$ |
|---|---|---|
| $>$ $m_1$    $<$ $m_2$ | $>$ $m_1$    $<$ $m_3$ | $>$ $m_2$    $<$ $m_3$ |
| $x_1 = 0$ | $x_2 = \dfrac{1 - x_1}{\sqrt{3}}$ | $x_2 = \dfrac{1 + x_1}{\sqrt{3}}$ |
| $>$ $m_2$    $<$ $m_1$ | $>$ $m_3$    $<$ $m_1$ | $>$ $m_3$    $<$ $m_2$ |

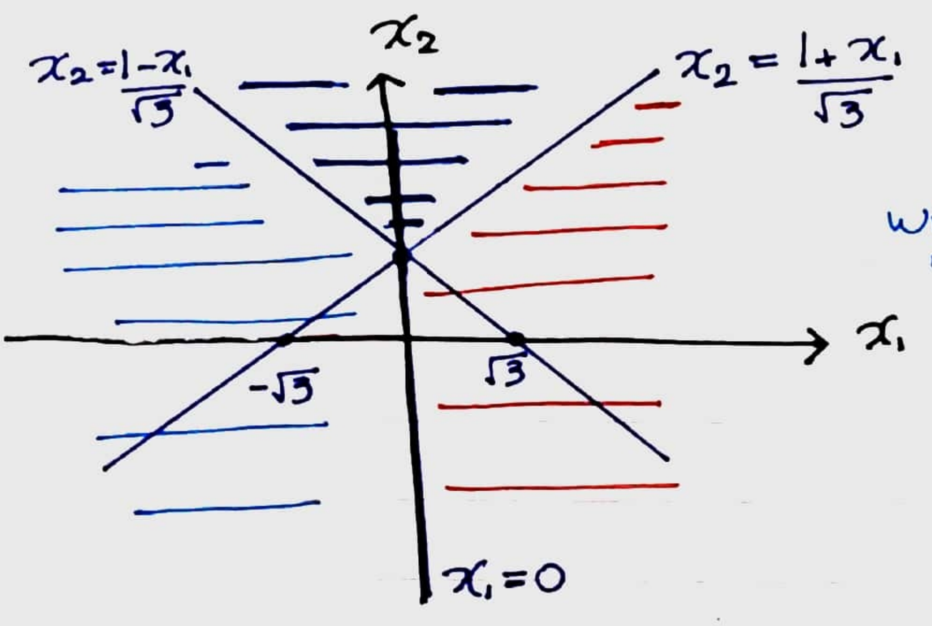
• To make $x_2$ vertical axis

$x_2 > \ldots$ means above

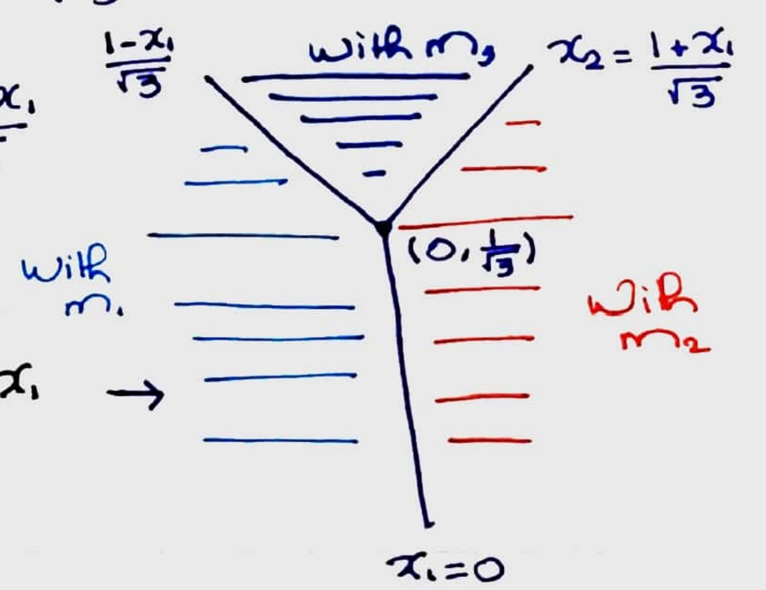
• $m_1$ whenever $x_1 < 0$ and $x_2 < \dfrac{1 - x_1}{\sqrt{3}}$ teal

• $m_2$ whenever $x_1 > 0$ and $x_2 < \dfrac{1 + x_1}{\sqrt{3}}$ red

• $m_3$ whenever $\dfrac{1 - x_1}{\sqrt{3}} < x_2$, $\dfrac{1 + x_1}{\sqrt{3}} < x_2$ Blue




$\rightarrow$



5) Centers

$$\begin{pmatrix}0\\2\end{pmatrix}, \begin{pmatrix}1\\0\end{pmatrix}, \begin{pmatrix}1\\4\end{pmatrix}, \begin{pmatrix}2\\2\end{pmatrix}$$

$m_1 \quad m_2 \quad m_3 \quad m_4$

• ${}^4C_2$ boundaries

• Between $m_1$ and $m_2$

$$(x_1 \;\; x_2)\begin{pmatrix}-1\\2\end{pmatrix} - \frac{1}{2}(4-1)$$

• $f(x) > 0 \rightarrow m_1$

$$= 2x_2 - x_1 - 1.5$$

• Between $m_1$ and $m_3$

• $f(x) > 0 \rightarrow m_1$

$$(x_1 \;\; x_2)\begin{pmatrix}-1\\-2\end{pmatrix} - \frac{1}{2}(4-(1+4^2))$$

$$= -x_1 - 2x_2 + 6.5$$

• Between $m_1$ and $m_4$

• $f(x) > 0 \rightarrow m_1$

$$(x_1 \;\; x_2)\begin{pmatrix}-2\\0\end{pmatrix} - \frac{1}{2}(4-(4+4))$$

$$= -2x_1 + 2$$

Between $m_2$ and $m_3$

$$(x_1 \ x_2)\begin{pmatrix}0\\-4\end{pmatrix} - \frac{1}{2}(1-(1+16)$$

$$= -4x_2 + 8$$ $P(\underline{x})>0 \rightarrow m_2$

Between $m_2$ and $m_4$

$$(x_1 \ x_2)\begin{pmatrix}-1\\-2\end{pmatrix} - \frac{1}{2}(1-(4+4))$$ $P(\underline{x})>0 \rightarrow m_2$

$$= -x_1 - 2x_2 + 3.5$$

Between $m_3$ and $m_4$

$$(x_1 \ x_2)\begin{pmatrix}-1\\2\end{pmatrix} - \frac{1}{2}((1+16)-(4+4))$$ $P(\underline{x})>0 \rightarrow m_3$

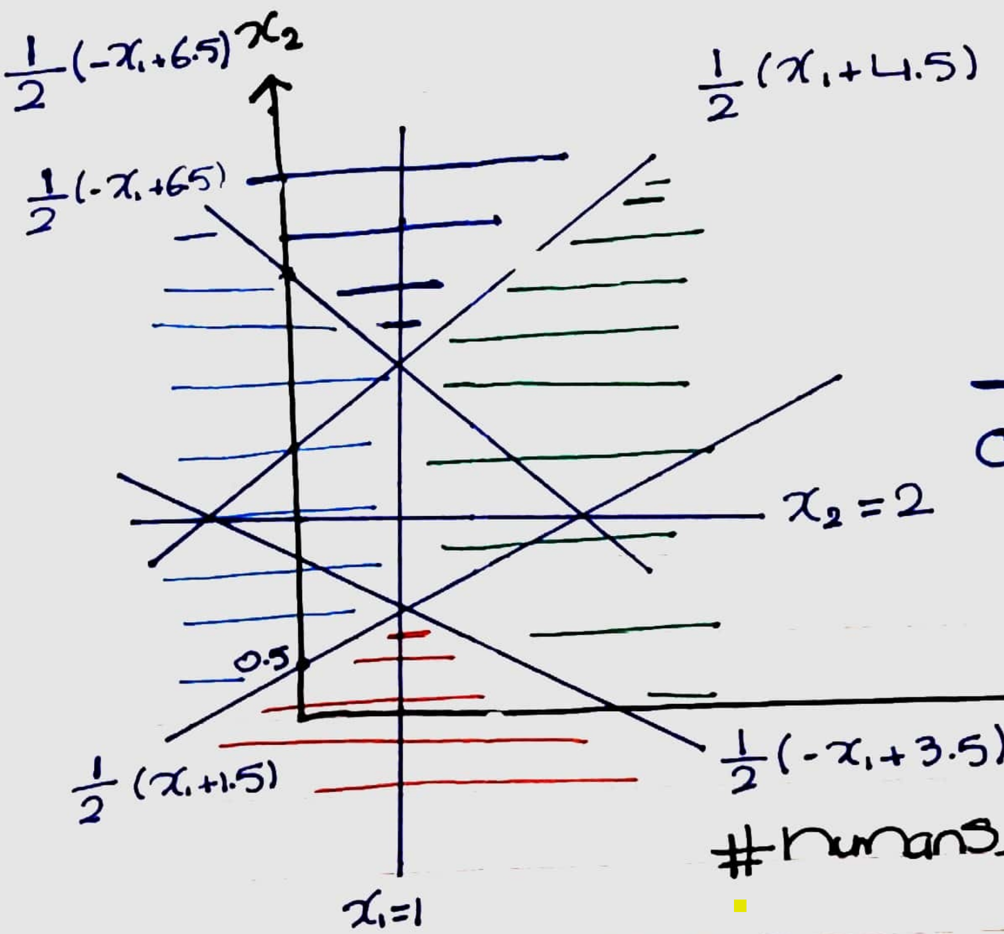
$$= -x_1 + 2x_2 - 4.5$$

$\Rightarrow$ The decision bandaries are

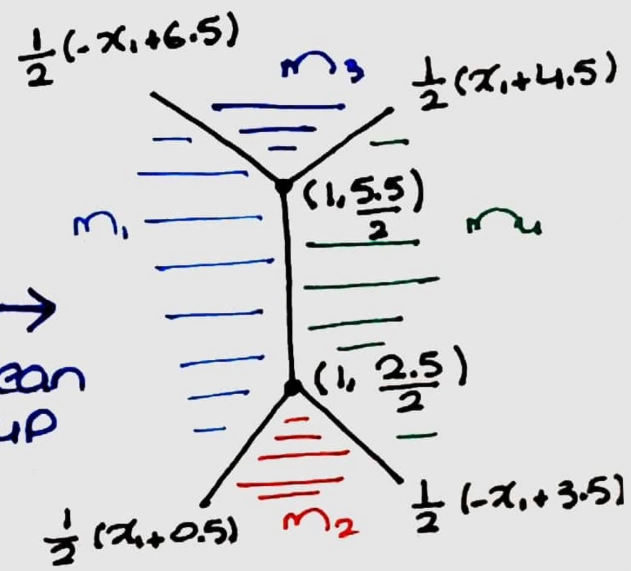
- $2x_2 - x_1 - 1.5 \rightarrow x_2 = \frac{1}{2}(x_1 + 1.5)$ $\quad > m_1$, $< m_2$
- $-x_1 - 2x_2 + 6.5 \rightarrow x_2 = \frac{1}{2}(-x_1 + 6.5)$ $\quad > m_3$, $< m_1$
- $-2x_1 + 2 \rightarrow x_1 = 1$ $\quad > m_4$, $< m_1$
- $-4x_2 + 8 \rightarrow x_2 = 2$ $\quad > m_3$, $< m_2$
- $-x_1 - 2x_2 + 3.5 \rightarrow x_2 = \frac{1}{2}(-x_1 + 3.5)$ $\quad > m_4$, $< m_2$
- $-x_1 + 2x_2 - 4.5 \rightarrow x_2 = 1/2(x_1 + 4.5)$ $\quad > m_3$, $< m_4$

- $m_1$ whenever
  $x_2 > \frac{1}{2}(x_1 + 1.5)$, $x_2 < \frac{1}{2}(-x_1 + 6.5)$, $x_1 < 1$
  teal
- $m_2$ whenever $x_2 < \frac{1}{2}(x_1 + 1.5)$, $x_2 < 2$, $x_2 < \frac{1}{2}(-x_1 + 3.5)$
  red
- $m_3$ whenever $x_2 > \frac{1}{2}(-x_1 + 6.5)$, $x_2 > 2$, $x_2 > \frac{1}{2}(x_1 + 4.5)$
  blue
- $m_4$ whenever $x_1 > 1$, $x_2 > \frac{1}{2}(-x_1 + 3.5)$, $x_2 < \frac{1}{2}(x_1 + 4.5)$
  green

Bondaries)
$x_1 = 1$, $x_2 = 2$
$x_2 = \frac{1}{2}(x_1 + 1.5)$, $x_2 = \frac{1}{2}(x_1 + 4.5)$
$x_2 = \frac{1}{2}(-x_1 + 3.5)$, $x_2 = \frac{1}{2}(-x_1 + 6.5)$




Clean up




#humans_are_not_machines.

- ~~Perhaps~~ its Possible to only find the decision bondaries (don't Keep track of inequalities) → Plot all lines → Place centers → use logic to decide regions
  - There should be 4 of them.

6.

$$d_1 = (1.2 - x)^2 + (1.2 - y)^2$$

$$d_2 = (1 - \underset{x_1}{x})^2 + (0.5 - \underset{x_2}{y})^2$$

| Point | $d_1$ | $d_2$ |
|---|---|---|
| (0,0) | 2.88 | 1.25 |
| (0.5, -0.1) | 2.18 | 0.61 3rd |
| (0.5,0.25) | 1.3925 | 0.3125 2nd |
| (1,0) | 1.48 3rd | 0.25 1st |
| (0,0.5) | 1.93 | 1 |
| (0,1) | 1.48 2nd | 1.25 |
| (2,2) | 1.28 1st | 3.25 |
| (2,2.5) | 2.33 | 5 |
| (2.5,2) | 2.33 | 4.5 |
| (2.25, 2.25) | 2.205 | 4.625 |
| (2.1, 2.5) | 2.5 | 5.21 |
| (3.5,1.5) | 5.38 | 7.25 |

- For 3NN Blue Blue
- For NN red Blue