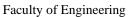
Cairo University





3rd Year Comp. MTH3251- Fall 2022 Number theory - Sheet 1

- Prove or disprove: If $a \mid (b + c)$, then either $a \mid b$ or $a \mid c$.
- Prove that for any integer a, one of the integers a, a + 2, a + 4 is divisible by 3.
- (3) Prove that if a and bare both odd integers, then if $16|a^4 + b^4 2$.
- (4) Prove or disprove that if a|bc, where a, b, and c are positive integers and $a \neq 0$, then a|b or a|c.
- (5) Show that if *n* is an integer then $n^2 \equiv 0$ or 1 (mod 4)
- (6) Prove that if n is an odd positive integer, then $n^2 \equiv 1 \pmod{8}$.
- (7) Show that if $\underline{n} \mid \underline{m}$, where n and m are integers greater than 1, and if $\underline{a} \equiv \underline{b} \pmod{\underline{m}}$, where a and b are integers, then $\underline{a} \equiv \underline{b} \pmod{\underline{n}}$.
- (8) Determine whether each of these integers is prime.
 - **d**) 19 b) 27
 - c) 93 d) 101
- (9) Determine whether the integers in each of these sets are pairwise relatively prime.
 - a) 21, 34, 55
- b) 14, 17, 85
- (10) How many zeros are there at the end of 100! ?
- (11) Prove that the product of any three consecutive integers is divisible by 6.7
- (12) We call a positive integer perfect if it equals the sum of its positive divisors other than itself.

 a) Show that 6 and 28 are perfect.
 - Show that $2^{p-1}(2^p 1)$ is a perfect number when 2^{p-1} is prime.