Faculty of Engineering
Cairo University

Communications and Computer Engineering Program
ELC N316 - Communications II

Midterm Exam

Thursday, April 23, 2009
10:00 AM−12:00 PM

- Answer **ALL** the questions in the spaces provided on the question sheets.
- A complementary error function (erfc) table of values is attached at the end of the exam if needed.
- Please make sure that your exam booklet has 14 pages in addition to the erfc table of values.
- All the hints provided are not mandatory to use but they are there to help you!

GOOD LUCK!

Name: ______________________________

Question	Points	Score
1	3	
2	4	
3	4	
4	4	
5	6	
6	6	
7	13	
8	10	
Total:	50	

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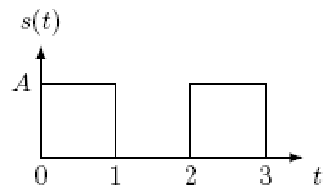
Problem 1 *(3 points)*

The received signal in a communication system is $r(t) = s(t) + n(t)$, where $s(t)$ is the pulse shown in the figure below, and $n(t)$ is AWGN with power-spectral density $N_0/2$ W/Hz.



(1) (1 point) Sketch the impulse response of the filter matched to $s(t)$.

(2) (2 points) Sketch the output of the matched filter to input $s(t)$.(**Hint:** No need to derive anything. Just sketch the output.)

Problem 2 *(4 points)*

An analog signal is sampled, quantized, and encoded into a binary PCM wave. The number of representation levels used is 128. A synchronizing pulse is added at the end of each code word representing a sample of the analog signal. The resulting PCM wave is transmitted over a channel of bandwidth 12 KHz using a quaternary PAM system with raised cosine spectrum. The rolloff factor is unity.

(1) (2 points) Find the rate in bits/sec at which information is transmitted through the channel.

(2) (2 points) Find the rate at which the analog signal is sampled. What is the maximum possible value for the highest frequency component of the analog signal?

Problem 3 *(4 points)*

Consider a binary PCM system which employs **polar** NRZ line code and is transmitting 100 Kbps. The noise power spectral density at the receiver is $N_o/2$, where $N_o = 2.5 \times 10^{-7}$ Volts^2/Hz.

(1) (2 points) What is the minimum average power of the transmitted signal that will produce a probability of bit error of 10^{-5} or less?

(2) (2 points) Now assume that **unipolar** NRZ line code is used instead. What is the minimum average power needed in this case?

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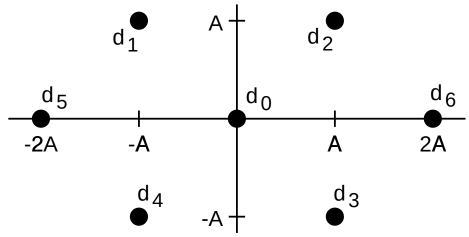
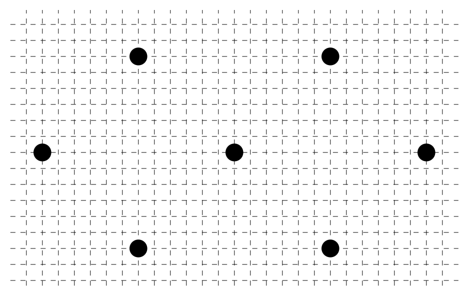
Problem 4 *(4 points)*

Consider the following constellation

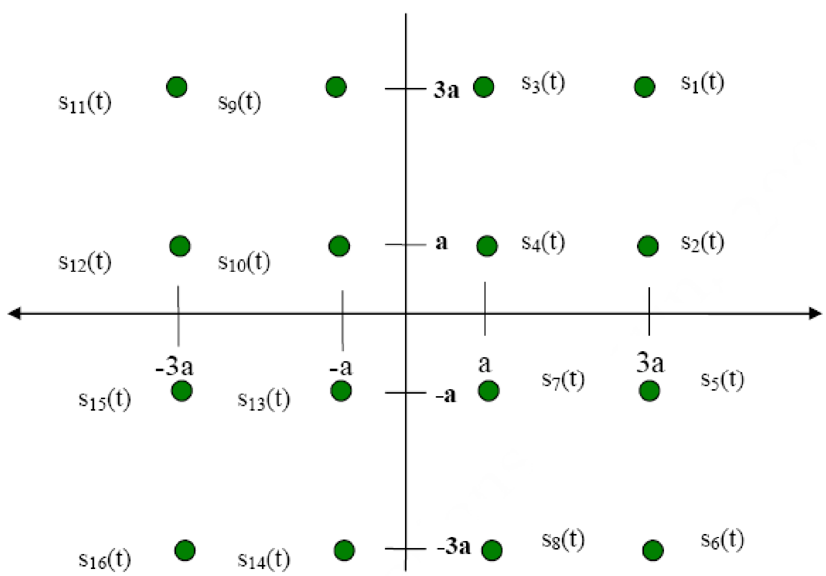


Sketch the decision regions using the figure below (assuming equiprobable symbols).



Problem 5 *(6 points)*

Consider the 16-symbol constellation shown in the figure below.



(1) (2 points) Calculate the average energy E_s for the constellation in terms of a. What is

the minimum distance between the transmitted symbols in terms of the average energy?

(2) (4 points) Assuming equiprobable symbols, use the union bound to calculate an approximate expression for the average probability of symbol error. Write your expression in terms of E_s and not a.

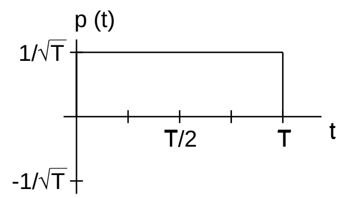
Problem 6 *(6 points)*

Consider the binary communications problem where we are transmitting a single bit, d, using the pulse $p(t)$. The transmitted signal $x(t)$ is given by

$$x(t) = \pm\, A\, p(t),$$

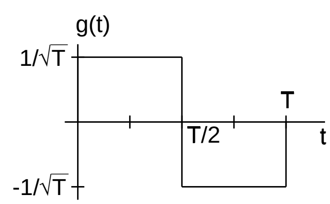
where A is a constant and $p(t)$ is given by,

$$p(t) \;=\; \begin{cases} \frac{1}{\sqrt{T}}, & 0 < t < T \\ 0, & \text{elsewhere} \end{cases}$$



The received signal is corrupted by a real additive white Gaussian noise (AWGN) process, $w(t)$, with variance $N_0/2$ and an interference signal, $g(t)$,

$$g(t) \;=\; \begin{cases} 1/\sqrt{T}, & 0 < t < T/2 \\ -1/\sqrt{T}, & T/2 < t < T \\ 0, & \text{elsewhere} \end{cases}$$



As a result, the received signal is given by

$$y(t) \;=\; x(t) + g(t) + w(t).$$

The received signal is then passed through a matched filter where its output $v(t)$ is sampled and a threshold detector is used to detect the transmitted bits.

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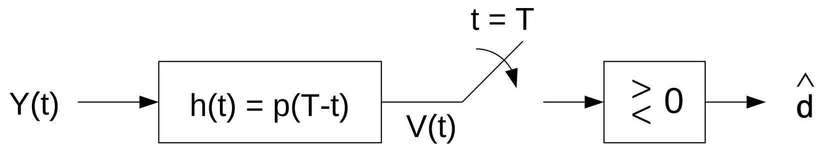
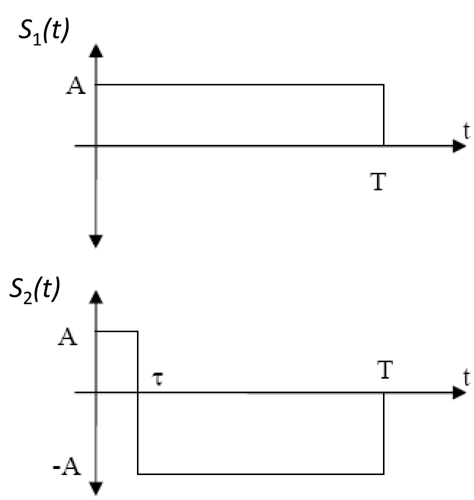


(1) (3 points) Find an expression for the sampled output of the matched filter, $v(T)$.(**Hint:** Remember the equivalence of matched filtering and sampling to the correlation process.)

(2) (3 points) Determine the average probability of error. What is your comment regarding the effect of the interfering signal?

Problem 7 *(13 points)*
Consider the following signal set used for binary modulation:



(1) (3 points) Determine a set of basis functions for this set in terms of A, T, and τ.(**Hint:**

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It is better to find the basis functions just by inspection. Remember the quiz!)

(2) (3 points) Plot the signal-space representation of this set for $\tau = T/4$. In your plot, write the projections in terms of the average signal energy (E_s).

(3) (4 points) Assume that matched filtering and the maximum likelihood decision rule are used. What is the probability of error in an AWGN channel for $\tau = T/2$ and $\tau = T/4$. Which of the two systems is better from the point of view of probability of error?

(4) (3 points) What value of τ will minimize the probability of error? Justify your answer.

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Problem 8 *(10 points)*

Assume a baseband communication system which utilizes binary modulation to transmit the information from a random binary source. The transmitted symbols (denoted by s) are $s = +1$ and $s = -1$ corresponding to information bits (denoted by b) $b = 1$ and $b = 0$, respectively. It is observed that the source generated bit 1 more often than bit 0 such that the probability of transmitting 1 is 0.6.

The transmitted signals are distorted by an additive noise (**<u>not Gaussian</u>**). At the receiver, a normalized matched filter is used to filter the received signal. The matched filter output is sampled at the symbol rate. These samples, denoted by x, are used by the detector to estimate the transmitted information bit. Assume that the statistics of samples x are known by the following conditional probability density function (pdf):

$$p(x|s \text{ is transmitted}) \; = \; \begin{cases} \frac{4}{9}(x-s)+\frac{2}{3}, & s-\frac{3}{2} < x \leq s; \\ -\frac{4}{9}(x-s)+\frac{2}{3}, & s < x \leq s+\frac{3}{2}; \\ 0, & \text{otherwise} \end{cases}$$

(1) (4 points) Apply the maximum a posteriori (MAP) decision rule to determine the optimum decision threshold. (**<u>Hint:</u>** For the above case, the MAP rule is given by: say 1 is transmitted if $p(s=1|x) > p(s=-1|x)$ and say 0 if the inequality sign is reversed. Use Bayes rule to proceed.)

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continue your solution here....

(2) (4 points) What is the average probability of symbol error if the MAP rule is used?

(3) (2 points) If the maximum likelihood (ML) decision rule is used instead, do you expect the obtained average probability of symbol error to decrease or increase? Why?

End of exam.

Complementary Error Function Table

x	erfc(x)	x	erfc(x)	x	erfc(x)	x	erfc(x)	x	erfc(x)	x	erfc(x)	x	erfc(x)
0	1.000000	0.5	0.479500	1	0.157299	1.5	0.033895	2	0.004678	2.5	0.000407	3	0.00002209
0.01	0.988717	0.51	0.470756	1.01	0.153190	1.51	0.032723	2.01	0.004475	2.51	0.000386	3.01	0.00002074
0.02	0.977435	0.52	0.462101	1.02	0.149162	1.52	0.031587	2.02	0.004281	2.52	0.000365	3.02	0.00001947
0.03	0.966159	0.53	0.453536	1.03	0.145216	1.53	0.030484	2.03	0.004094	2.53	0.000346	3.03	0.00001827
0.04	0.954889	0.54	0.445061	1.04	0.141350	1.54	0.029414	2.04	0.003914	2.54	0.000328	3.04	0.00001714
0.05	0.943628	0.55	0.436677	1.05	0.137564	1.55	0.028377	2.05	0.003742	2.55	0.000311	3.05	0.00001608
0.06	0.932378	0.56	0.428384	1.06	0.133856	1.56	0.027372	2.06	0.003577	2.56	0.000294	3.06	0.00001508
0.07	0.921142	0.57	0.420184	1.07	0.130227	1.57	0.026397	2.07	0.003418	2.57	0.000278	3.07	0.00001414
0.08	0.909922	0.58	0.412077	1.08	0.126674	1.58	0.025453	2.08	0.003266	2.58	0.000264	3.08	0.00001326
0.09	0.898719	0.59	0.404064	1.09	0.123197	1.59	0.024538	2.09	0.003120	2.59	0.000249	3.09	0.00001243
0.1	0.887537	0.6	0.396144	1.1	0.119795	1.6	0.023652	2.1	0.002979	2.6	0.000236	3.1	0.00001165
0.11	0.876377	0.61	0.388319	1.11	0.116467	1.61	0.022793	2.11	0.002845	2.61	0.000223	3.11	0.00001092
0.12	0.865242	0.62	0.380589	1.12	0.113212	1.62	0.021962	2.12	0.002716	2.62	0.000211	3.12	0.00001023
0.13	0.854133	0.63	0.372954	1.13	0.110029	1.63	0.021157	2.13	0.002593	2.63	0.000200	3.13	0.00000958
0.14	0.843053	0.64	0.365414	1.14	0.106918	1.64	0.020378	2.14	0.002475	2.64	0.000189	3.14	0.00000897
0.15	0.832004	0.65	0.357971	1.15	0.103876	1.65	0.019624	2.15	0.002361	2.65	0.000178	3.15	0.00000840
0.16	0.820988	0.66	0.350623	1.16	0.100904	1.66	0.018895	2.16	0.002253	2.66	0.000169	3.16	0.00000786
0.17	0.810008	0.67	0.343372	1.17	0.098000	1.67	0.018190	2.17	0.002149	2.67	0.000159	3.17	0.00000736
0.18	0.799064	0.68	0.336218	1.18	0.095163	1.68	0.017507	2.18	0.002049	2.68	0.000151	3.18	0.00000689
0.19	0.788160	0.69	0.329160	1.19	0.092392	1.69	0.016847	2.19	0.001954	2.69	0.000142	3.19	0.00000644
0.2	0.777297	0.7	0.322199	1.2	0.089686	1.7	0.016210	2.2	0.001863	2.7	0.000134	3.2	0.00000603
0.21	0.766478	0.71	0.315335	1.21	0.087045	1.71	0.015593	2.21	0.001776	2.71	0.000127	3.21	0.00000564
0.22	0.755704	0.72	0.308567	1.22	0.084466	1.72	0.014997	2.22	0.001692	2.72	0.000120	3.22	0.00000527
0.23	0.744977	0.73	0.301896	1.23	0.081950	1.73	0.014422	2.23	0.001612	2.73	0.000113	3.23	0.00000493
0.24	0.734300	0.74	0.295322	1.24	0.079495	1.74	0.013865	2.24	0.001536	2.74	0.000107	3.24	0.00000460
0.25	0.723674	0.75	0.288845	1.25	0.077100	1.75	0.013328	2.25	0.001463	2.75	0.000101	3.25	0.00000430
0.26	0.713100	0.76	0.282463	1.26	0.074764	1.76	0.012810	2.26	0.001393	2.76	0.000095	3.26	0.00000402
0.27	0.702582	0.77	0.276179	1.27	0.072486	1.77	0.012309	2.27	0.001326	2.77	0.000090	3.27	0.00000376
0.28	0.692120	0.78	0.269990	1.28	0.070266	1.78	0.011826	2.28	0.001262	2.78	0.000084	3.28	0.00000351
0.29	0.681717	0.79	0.263897	1.29	0.068101	1.79	0.011359	2.29	0.001201	2.79	0.000080	3.29	0.00000328
0.3	0.671373	0.8	0.257899	1.3	0.065992	1.8	0.010909	2.3	0.001143	2.8	0.000075	3.3	0.00000306
0.31	0.661092	0.81	0.251997	1.31	0.063937	1.81	0.010475	2.31	0.001088	2.81	0.000071	3.31	0.00000285
0.32	0.650874	0.82	0.246189	1.32	0.061935	1.82	0.010057	2.32	0.001034	2.82	0.000067	3.32	0.00000266
0.33	0.640721	0.83	0.240476	1.33	0.059985	1.83	0.009653	2.33	0.000984	2.83	0.000063	3.33	0.00000249
0.34	0.630635	0.84	0.234857	1.34	0.058086	1.84	0.009264	2.34	0.000935	2.84	0.000059	3.34	0.00000232
0.35	0.620618	0.85	0.229332	1.35	0.056238	1.85	0.008889	2.35	0.000889	2.85	0.000056	3.35	0.00000216
0.36	0.610670	0.86	0.223900	1.36	0.054439	1.86	0.008528	2.36	0.000845	2.86	0.000052	3.36	0.00000202
0.37	0.600794	0.87	0.218560	1.37	0.052688	1.87	0.008179	2.37	0.000803	2.87	0.000049	3.37	0.00000188
0.38	0.590991	0.88	0.213313	1.38	0.050984	1.88	0.007844	2.38	0.000763	2.88	0.000046	3.38	0.00000175
0.39	0.581261	0.89	0.208157	1.39	0.049327	1.89	0.007521	2.39	0.000725	2.89	0.000044	3.39	0.00000163
0.4	0.571608	0.9	0.203092	1.4	0.047715	1.9	0.007210	2.4	0.000689	2.9	0.000041	3.4	0.00000152
0.41	0.562031	0.91	0.198117	1.41	0.046148	1.91	0.006910	2.41	0.000654	2.91	0.000039	3.41	0.00000142
0.42	0.552532	0.92	0.193232	1.42	0.044624	1.92	0.006622	2.42	0.000621	2.92	0.000036	3.42	0.00000132
0.43	0.543113	0.93	0.188437	1.43	0.043143	1.93	0.006344	2.43	0.000589	2.93	0.000034	3.43	0.00000123
0.44	0.533775	0.94	0.183729	1.44	0.041703	1.94	0.006077	2.44	0.000559	2.94	0.000032	3.44	0.00000115
0.45	0.524518	0.95	0.179109	1.45	0.040305	1.95	0.005821	2.45	0.000531	2.95	0.000030	3.45	0.00000107
0.46	0.515345	0.96	0.174576	1.46	0.038946	1.96	0.005574	2.46	0.000503	2.96	0.000028	3.46	0.00000099
0.47	0.506255	0.97	0.170130	1.47	0.037627	1.97	0.005336	2.47	0.000477	2.97	0.000027	3.47	0.00000092
0.48	0.497250	0.98	0.165769	1.48	0.036346	1.98	0.005108	2.48	0.000453	2.98	0.000025	3.48	0.00000086
0.49	0.488332	0.99	0.161492	1.49	0.035102	1.99	0.004889	2.49	0.000429	2.99	0.000024	3.49	0.00000080