$$\begin{pmatrix} 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{pmatrix}, \quad R_z(\theta) = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & \end{pmatrix}$$

1.5 | (20) | 9.5

Question 2: [5 points]

2.a) [3 pt] For the polygon shown in Figure 2, write down the corresponding edge table (including edge structure). [A(6,2), B (10,6), C(6,12), D(2,10), E(2,6)]

Edge Table

10	→ DC [12 | 2 | 2]
9	
8	
7	
6	→ DE [10 | 2 | 0] → BC [12 | 10 | $-\frac{2}{3}$]
5	
4	
3	
2	→ EA [6 | 6 | -1] → BA [6 | 6 | 1]
1	
0	

y_{max}

2.5/3

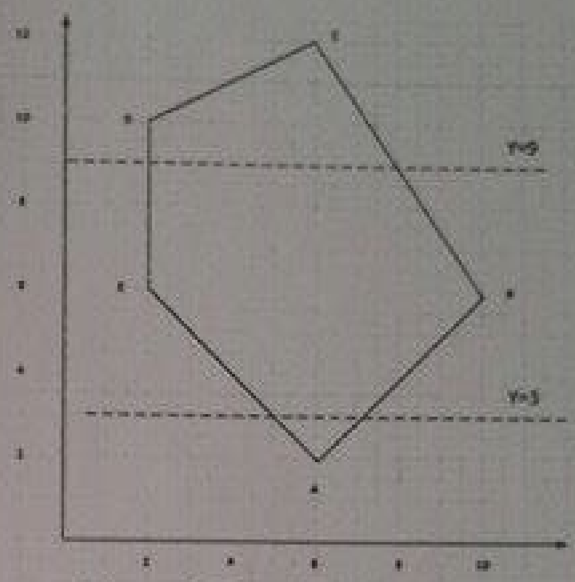


Figure 2 Question 2 (a and b)

2.b) [2 pt] Find out the AET for scan line values of Y=3 and Y = 9

y = 3: EA [6 | 5 | -1] → BA [6 | 7 | 1]

y = 9: DE [10 | 2 | 0] → BC [12 | 8 | $-\frac{2}{3}$]

1.5/2

Question 3: [5 points]

2.5/3

3.a) [3 pt] Write down the Bresenham's Line Drawing Algorithm steps for |m| < 1 for drawing a line from point (X_0,Y_0) to (X_1,Y_1).

```
void Bresenham (int x0 , int y0 , int x1 , int y1 )
{  dx = x1 - x0 ;
   dy = y1 - y0 ;
   set pixel (x0 , y0);
   if (abs (dx) > abs(dy))   // |m|<1
   {  offscaled frac = - dx;
      while ( x0 != x1 )
      {  x0 ± = 1;
         offscaled frac ± = 2dy;
         if ( offscaled frac >= 0)
         {  y0 ± = 1;
            offscaled frac + = 1;
         }
         set pixel (x0 , y0 )
      }
}
```

3.b) [2 pt] Use the non-zero winding number to determine if the following point P in Figure 3 below is an interior or exterior to the shown polygon. Show your steps

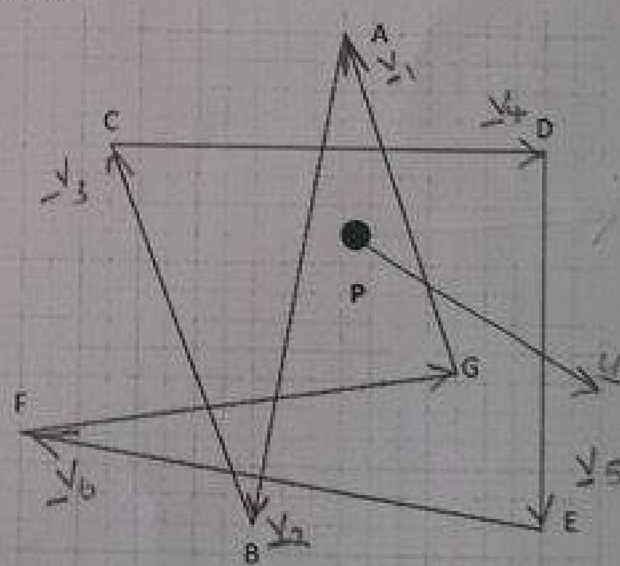


Figure 3: Question 3-b

① make a vector $\underline{u}$ from P intersect the edes (not vertcies)

② make the edges as a vectors anti clock wise

③ set counter = 0;

④ $\underline{u} \times \underline{v}_1$ -ve

counter --;   // counter = -1

$\underline{u} \times \underline{v}_5$ -ve

Counte --   // counter = -2

Counter != 0

then p is inside the polygon

1.5/2

Question 1: [5 points]

1.5/2

1.a) [2 pt] Explain the main differences between the following algorithms (how each algorithm works, complexity, and efficiency)

A. The four-connected and 8-connected boundary fill algorithms.

- both works like each other, the four connected it chooses a point check if it's inside the polygon then check if its colored or not if its not colored, it color it and push its four connected in a stack (x+1,y), (x-1,y), (x,y+1), (x,y-1). when it color a pixel it pop it.
- The difference between 4-connected and 8-connected that, the 8-connected checks for (x+1,y), (x-1,y), (x+1,y+1), (x-1,y-1), (x,y+1), (x,y-1) (x+1,y-1), (x-1,y+1). its also more efficient because it can colour the whole polygon but the four-connected might get stop because it doesn't check the corners. both of them cause stack overflow so we creat our own stack.

B. Boundary fill and active edge table (AET) algorithms.

I descriped the work of boundry fill in question 1(a)

The AET: The edge table contains each edge of the polygon orderd with y_{min}.

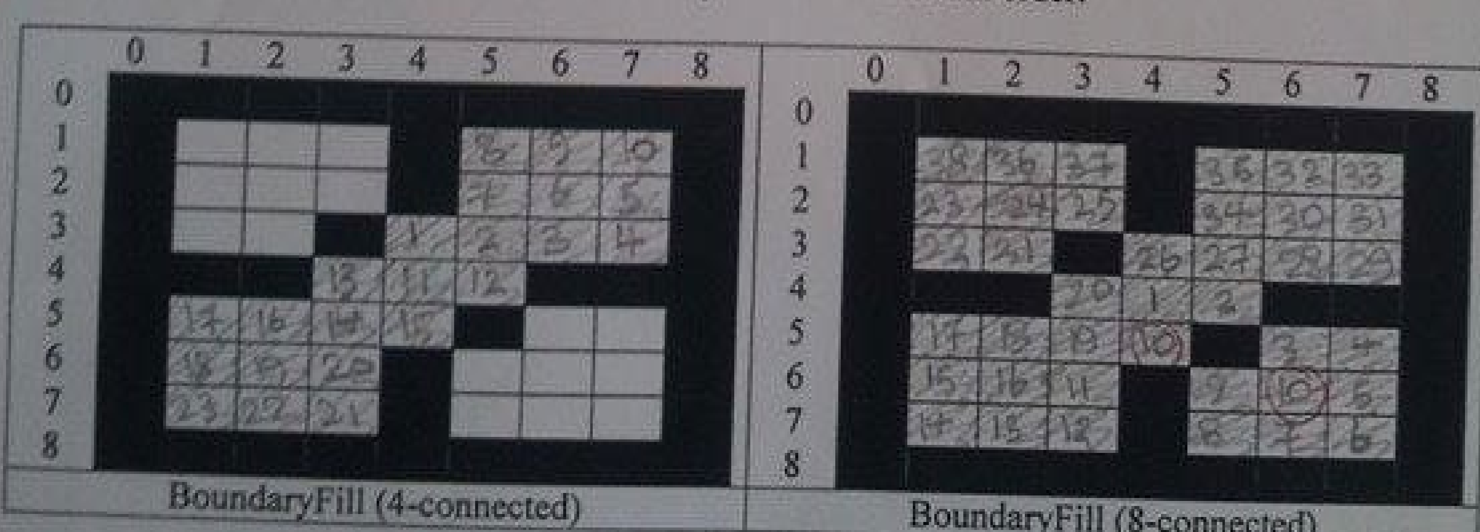
each edge contain y_{max}, X of y_{min} and dx/dy

Then the active edge Table have every scan line with its intersected pixels with the edges orderd by increasing edge, and while $y < y_{max}$ it fill the pixels after checking that the number of intersected edges is odd (the pixel is inside the polygon).

Then it Retire the edges with $y > y_{max}$.

2.5/3

1.b) [3 pt] Use BoundaryFill (4-connected) & (8-connected) to fill the shape shown in Figure 1. Start from the centre point (4,4) and label the pixels to be filled in order.



specify the order

Figure 1: Question 1-b

Question 4: [5 points]

4.a) [2 pt] Given a 2D rotation transformation matrix R(θ). Is $R(\theta_1)R(\theta_2)=R(\theta_1+\theta_2)$? Prove your answer.

2/2

$$L.H.S = R(\theta_1)R(\theta_2) = \begin{bmatrix}\cos\theta_1 & -\sin\theta_1 & 0\\ \sin\theta_1 & \cos\theta_1 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\cos\theta_2 & -\sin\theta_2 & 0\\ \sin\theta_2 & \cos\theta_2 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$= \begin{bmatrix}\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2 & -(\cos\theta_1\sin\theta_2+\cos\theta_2\sin\theta_1) & 0\\ \sin\theta_1\cos\theta_2+\sin\theta_2\cos\theta_1 & \cos\theta_1\cos\theta_2-\sin\theta_1\sin\theta_2 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$= \begin{bmatrix}\cos(\theta_1+\theta_2) & -\sin(\theta_1+\theta_2) & 0\\ \sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$R.H.S = R(\theta_1+\theta_2) = \begin{bmatrix}\cos(\theta_1+\theta_2) & -\sin(\theta_1+\theta_2) & 0\\ \sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) & 0\\ 0 & 0 & 1\end{bmatrix}$$

L.H.S = R.H.S

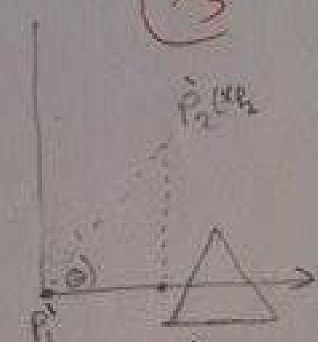
Then $R(\theta_1)R(\theta_2) = R(\theta_1+\theta_2)$

4.b) [3 pt] Explain using proper transformation matrices how to reflect an object around a line P_1P_2 ~~parallel to Y axis~~ as shown in Figure 4.

3/3

① Translate P_1 to the origin

$$T = \begin{bmatrix}1 & 0 & -x_{P_1}\\ 0 & 1 & -y_{P_1}\\ 0 & 0 & 1\end{bmatrix}$$



$$\sin\theta = \frac{y'_{P_2}}{\sqrt{y'^2_{P_2}+x'^2_{P_2}}}$$

$$\cos\theta = \frac{x'_{P_2}}{\sqrt{y'^2_{P_2}+x'^2_{P_2}}}$$

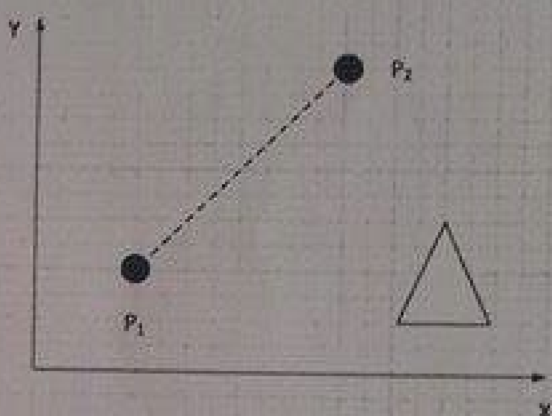
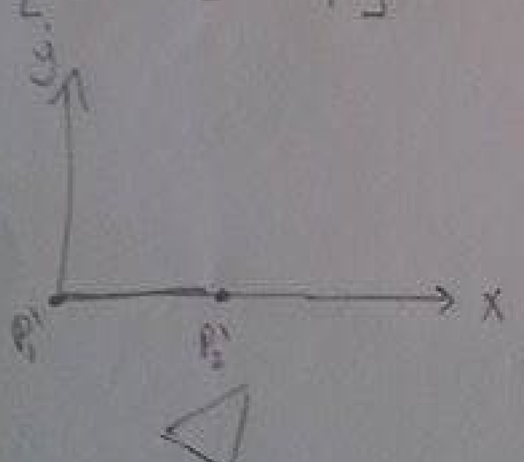


Figure 4: Question 4 - b

② Rotate with angle θ

+ve

$$R(\theta) = \begin{bmatrix}\cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}$$



③ reflect about x axis

$$S_x(-1) = \begin{bmatrix}1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

④ Rotate with $-\theta$

$$R^{-1}(\theta) = R(-\theta) = \begin{bmatrix}\cos(-\theta) & -\sin(-\theta) & 0\\ \sin(-\theta) & \cos(-\theta) & 0\\ 0 & 0 & 1\end{bmatrix}$$

⑤ Translate with x_{P_1}, y_{P_1}

$$T^{-1} = \begin{bmatrix}1 & 0 & x_{P_1}\\ 0 & 1 & y_{P_1}\\ 0 & 0 & 1\end{bmatrix}$$

$$\text{Transformation} = T^{-1}R^{-1}(\theta)\,S_x(-1)\,R(\theta)\,T$$

$$= \begin{bmatrix}1 & 0 & x_{P_1}\\ 0 & 1 & y_{P_1}\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\cos\theta & \sin\theta & 0\\ -\sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\cos\theta & -\sin\theta & 0\\ \sin\theta & -\cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$\begin{bmatrix}1 & 0 & -x_{P_1}\\ 0 & 1 & -y_{P_1}\\ 0 & 0 & 1\end{bmatrix}$$

Question 2: [5 points]

2.a) [3 pt] For the polygon shown in Figure 2, write down the corresponding edge table (including edge structure). [A(6,2), B (10,6), C(6,12), D(2,10), E(2,6)]

12
11
10 → DC → | 12 | 2 | 2 |
9
8
7
6 → ED' → | 9 | 2 | 0 | → BC → | 12 | 10 | $-\frac{4}{6}$ |
5
4
3
2 → AE' → | 5 | 6 | -1 | → AB' → | 5 | 6 | 1 |
1
0

y_{min}

$\frac{6-2}{12-10} = \frac{4}{2} = 2$

$\frac{10-6}{6-12} = \frac{4}{-6} = -\frac{2}{3}$

$\frac{6-2}{2-6} = \frac{4}{-4}$

$\frac{6-10}{2-6} = \frac{-4}{-4} = 1$

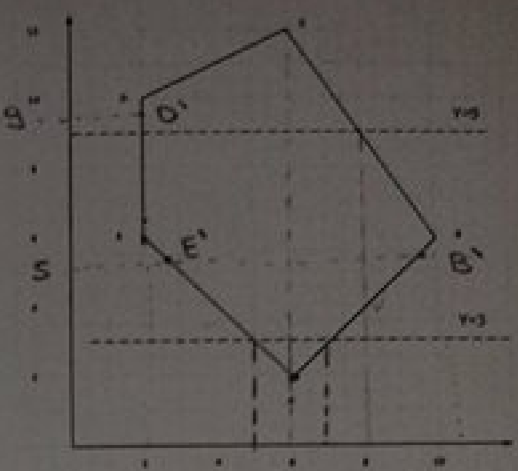


Figure 2 Question 2 (a and b)

2.b) [2 pt] Find out the AET for scan line values of Y=3 and Y = 9

$Y=3$

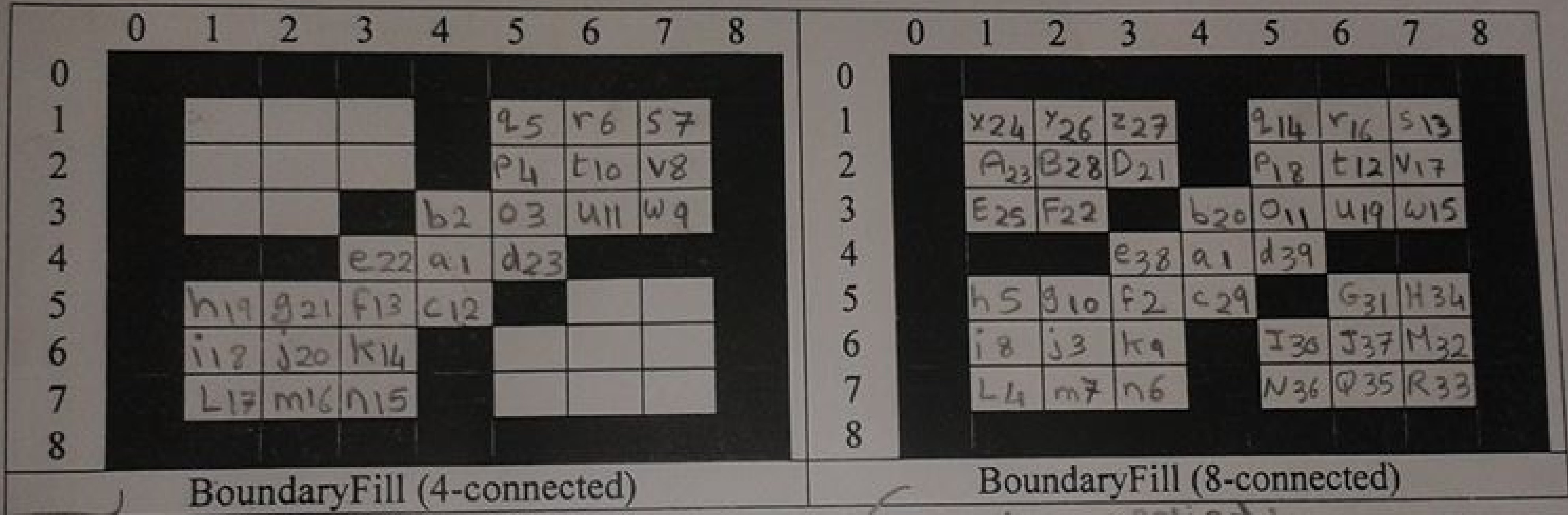
AE' | 5 | 5 | -1 | → AB' | 5 | 7 | 1 |

$Y=9$

ED' | 9 | 2 | 0 | → BC | 12 | 8 | $-\frac{2}{3}$ |

scanline.

1.b) [3 pt] Use BoundaryFill (4-connected) & (8-connected) to fill the shape shown in Figure 1. Start from the centre point (4,4) and label the pixels to be filled in order.



order applied:-
(x+1,y),(x-1,y),(x,y+1),(x,y-1)

Figure 1: Question 1-b

order applied:-
(x+1,y),(x-1,y),(x,y+1),(x,y-1),
(x+1,y+1),(x-1,y-1),(x+1,y-1),(x-1,y+1)