DC Sheet 1

\# Check 'DC Note'

$\Rightarrow$ We'll be working with f rather than ω.

Problem 1)

Recall,

$$tri\left(\frac{t}{5}\right) \leftrightarrow 5 . \, sinc^2(f5)$$

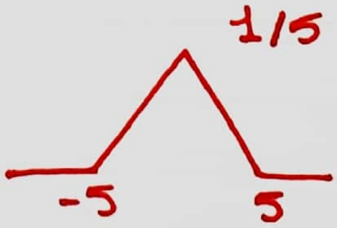
Hence,

$$sinc^2(t5) \leftrightarrow \frac{1}{5} . \, tri\left(\frac{f}{5}\right)$$

• Duality

Thus,

$$sinc^2(5t) \longleftrightarrow$$



• Has nulls whenever $t = \frac{n}{5}, \; n \in \mathbb{Z}^*$

Also recall

$$m(t)\,\delta_{T_s}(t) \longleftrightarrow f_s \sum_{k=-\infty}^{\infty} M(f - kf_s)$$

• Sampling in time domain

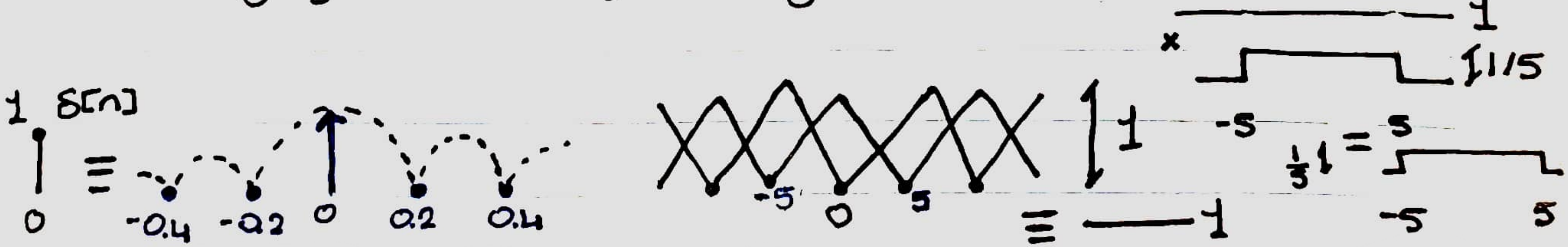
• Scaling by f_s and Periodic replication in frequency domain with Period f_s

Scanned with CamScanner

• At 5 Hz)

Sampled Signal | Spectrum | LPF (Gain=T_s)

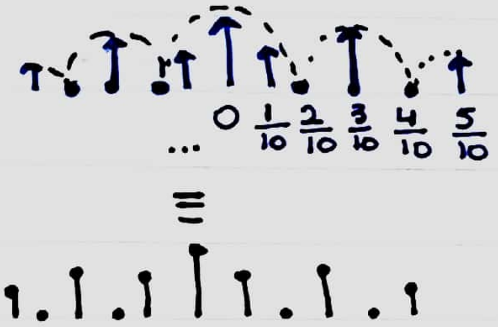
*every $\frac{1}{5}$ seconds ($t=\frac{n}{5}$) | *every 5 Hz



At 10 Hz)

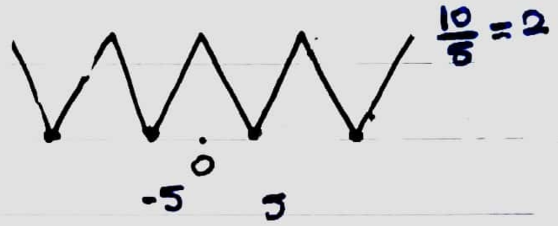
Sampled Signal

* every $\frac{1}{10}$ seconds



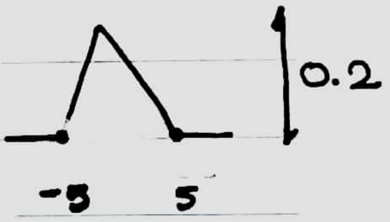
Spectrum

* every 10 Hz



- Can be recovered with an ideal LPF of $\omega_c = 10\pi$

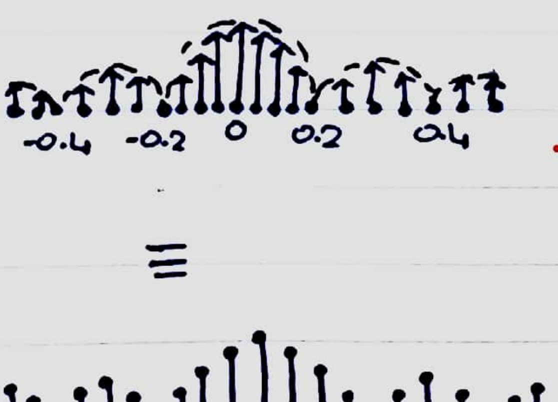
LPF (gain = T_s)



At 20 Hz)

Sampled Signal

* every $\frac{1}{20}$ seconds



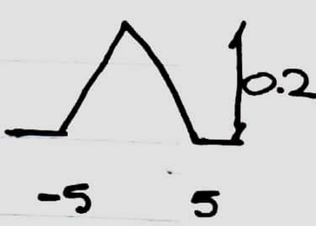
- Not to scale (equidistant samples)

Spectrum

* every 20 Hz

- Can be recovered with an ideal filter of $\omega_c \in [10\pi, 30\pi]$ $(5 \le f_c \le 15)$ or even a practical filter.

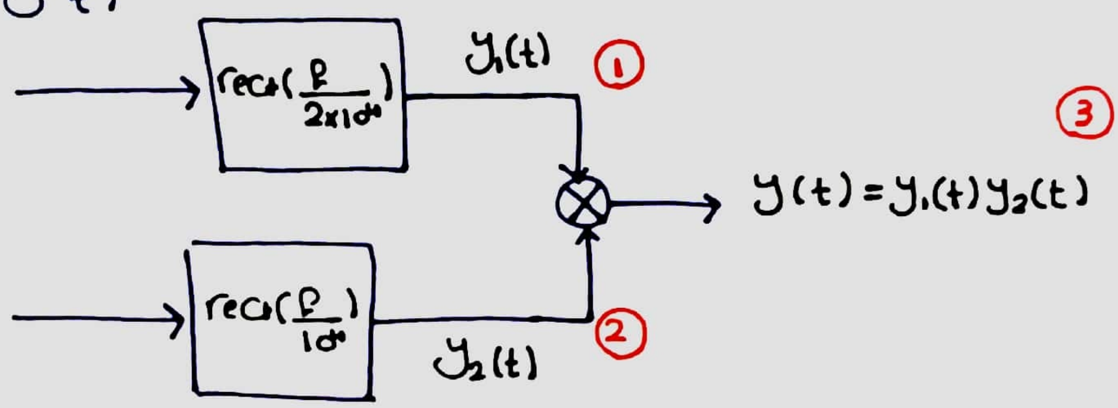
LPF (gain = T_s)



Problem 2)

$g_1(t) = 10^4 \, rect(10^4 t)$

$g_2(t) = \delta(t)$



• Need Nyquist rate of each of $y_1(t), y_2(t), y(t)$

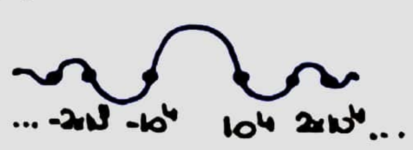
① Since

$$rect\left(\frac{t}{S}\right) \leftrightarrow S.\,Sinc(fS)$$

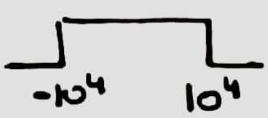
then

$$10^4 \, rect\left(\frac{t}{10^{-4}}\right) \leftrightarrow 10^4\left(10^{-4}\,Sinc(10^{-4}f)\right)$$

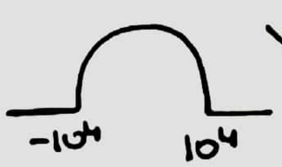
$$= Sinc(10^{-4}f)$$



→ when passed through a filter



yields



$Y_1(f)$

hence the bandwidth is 10^4 which results in a Nyquist rate of 2×10^4 Hz

② Since

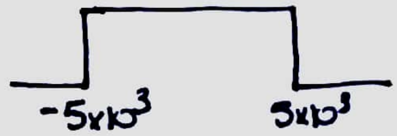
$$\delta(t) \longleftrightarrow 1$$

then

$$G_2(f) = 1$$

• Now let's pass it through the filter.

$$\bullet\ Y_2(f) = \text{rect}\left(\frac{f}{10^4}\right) \times \underbrace{G_2(f)}_{1} =$$



The BW in this case is 5×10^3 Hz which results in a Nyquist rate of 10^4 Hz

③ Since

$$y_1(t)\, y_2(t) \longleftrightarrow Y_1(f) * Y_2(f)$$

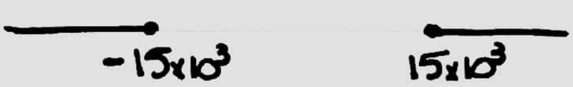
then

$$Y(f) = Y_1(f) * Y_2(f)$$

→ The starting/ending point of Y is the sum of starting/ending points of Y_1 and Y_2.

Hence,

Y(f) is nonzero here.



} No need to actually convolve to see how it looks
→ just need BW

$$BW = 15\times10^3 \text{ Hz}$$

$$f_{s_{min}} = 30\times10^3 \text{ Hz}$$ // Nyquist rate

Problem 3)

- CD Records audio using PCM
- Audio Signal bandwidth is 15 KHz

a) $f_{s_{min}} = 2 \times f_m = 30\ KHz$

b) $L = 2^n$ ← no. of bits

↳ no. of levels

Thus, $n = \log_2 65.536 = 16$ bit (Per sample)

c) $R_b = f_s \times n = 30K \times 16 = 480\ K$ bit/s

(Per audio channel)

- CD Audio has two channels, hence total bit rate is 960 Kbit/s

d) $f_s = 44.1\ KHz$

$n = \log_2 65.536 = 16$ bit

$R_b = 44.1K \times 16 = 705.6$ Kbit/s (Per audio channel)

(1411.2 Kbit/s for both)

$BW_{min} = \frac{R_b}{2} = 705.6\ KHz$ (352.8 KHz for each channel)

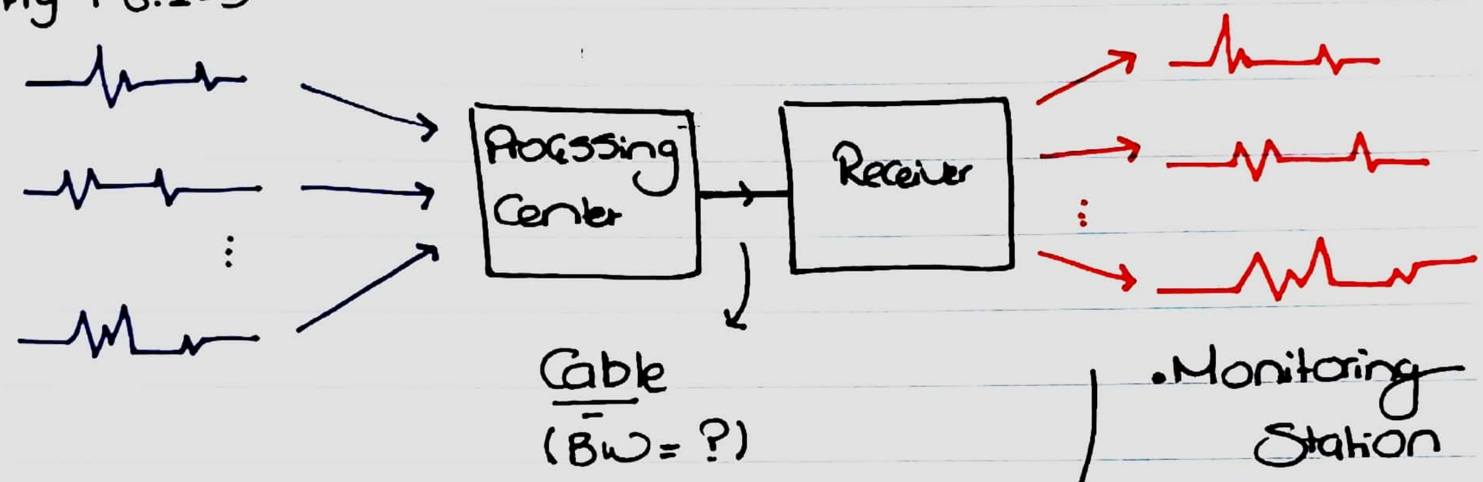
$\underline{\underline{2}}$ → Min BW implies $\eta = 2$ "Ideal Nyquist channel"

⇒ Can send 2 bits for each Hz in the spectrum

$\eta = 2$ bits/s/Hz

Problem 4)

→ Fig P6.2-5



• Each Signal (ElectroCardiodiagram) has bandwidth 100 Hz

⇒ Hence, $f_{s_{min}} = 200\,Hz \times 2 = 400\,Hz$ (minimum sample rate to sample each at the Processing Center)
// double the Nyquist rate (200 Hz)

• $q_{max} = \frac{0.25}{100} m_p$

⇒ Since $q_{max} = \frac{\Delta_{max}}{2}$ Then $\Delta_{max} = \frac{0.5}{100} m_p$

⇒ Since $\Delta_{max} = \frac{2m_p}{L_{min}}$ Then $\frac{0.5}{100} m_p = \frac{2m_p}{L_{min}}$

$L_{min} = 400$ ← To guarantee not going over 0.25%

There's no integer n for which $2^n = 400$

• At the very least $n = \lceil \log_2 400 \rceil = 9$ (which corresponds to $L = 519$ ← lowest L we can 'actually' use without going beyond .25%.

Thus the bit rate is

$$R_{b_{min}} = \underbrace{400}_{f_{s_{min}}} \times \underbrace{9}_{n_{min}} = 3.6 \text{ Kbit/s}$$

(Per ECG Signal)

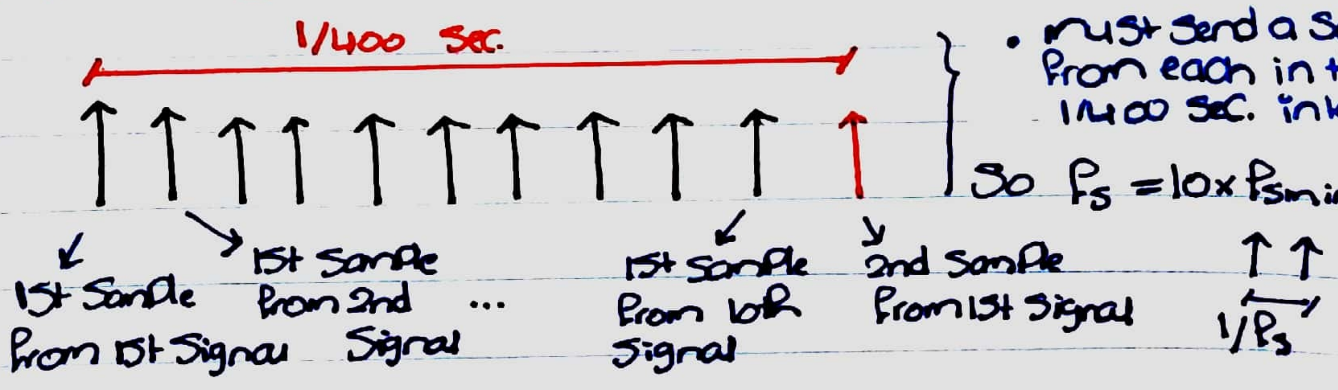
$$R_{b_{min\,(tot)}} = 36 \text{ Kbit/s}$$

(For all ECG Signals)

$$BW_{min} = \frac{36K}{2} = 18 \text{ KHz}$$

Justifying why multiply by 10

• In time-division multiplexing we send the 10 signals on the same channel (cable) by sending a sample from the 1st, 2nd, ..., 10th signal then the next sample from the 1st, 2nd, ..., 10th signal and so on



• must send a sample from each in the 1/400 sec. interval.

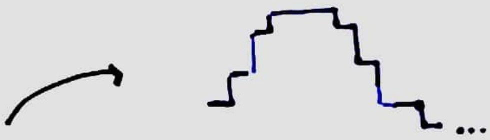
So $f_s = 10 \times f_{s_{min}}$

Problem 5)

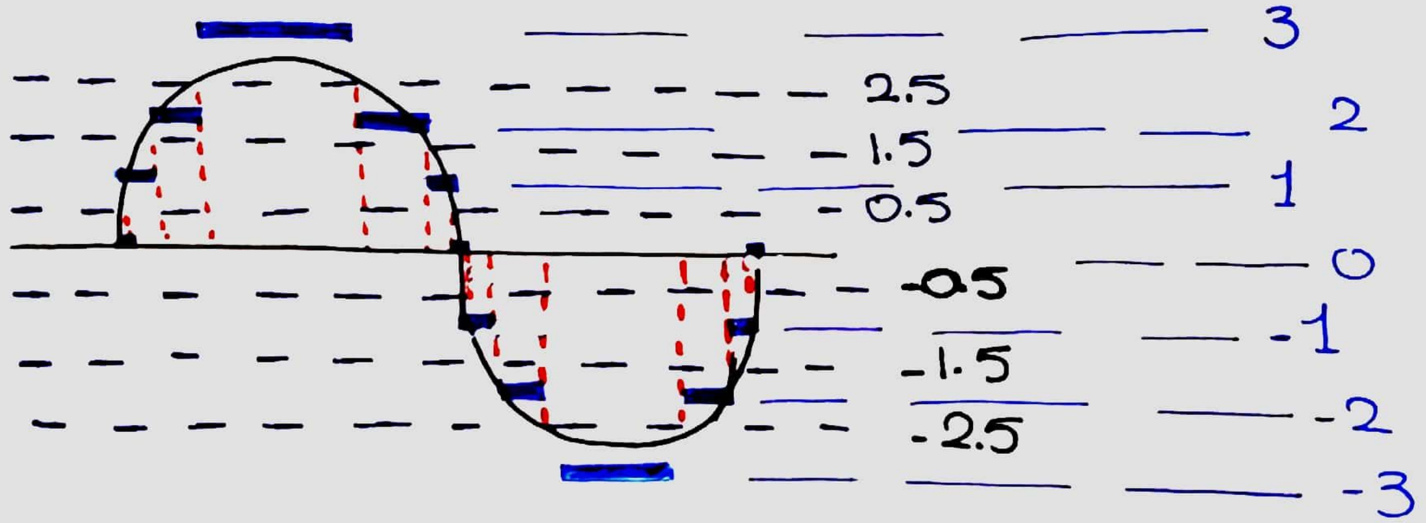
-3 -2 -1 0 1 2 3 — Quantized Output

-3.5 -2.5 -1.5 -0.5 0.5 1.5 2.5 3.5 — Analog Input Range

Mid-tread Quantizer



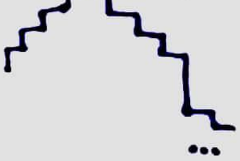
$y(t) = 3.25 \sin t$



- ▬ Denotes the quantized signal (for lack of another color)
- Remember y(t) is our analog input (for midtread it maps 0, 0.5 → 1, 0.5, 1.5 → 2, ...)

1. Draw input levels
2. Draw red lines (curve hits input level)
3. Draw output levels
4. Choose output level between every 2 red lines depending curve (input range)

Midrise Quantizer



-3.5 -2.5 -1.5 -0.5 0.5 1.5 2.5 3.5 (Quantized output)

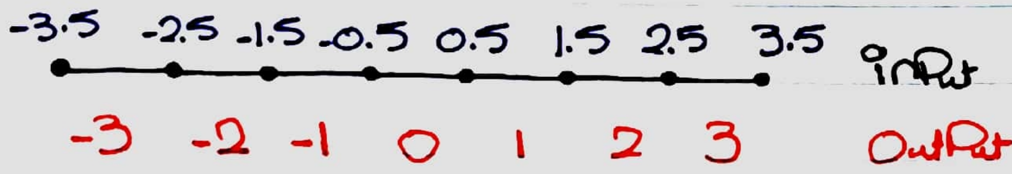
-4 -3 -2 -1 0 1 2 3 4 (Analog input range)

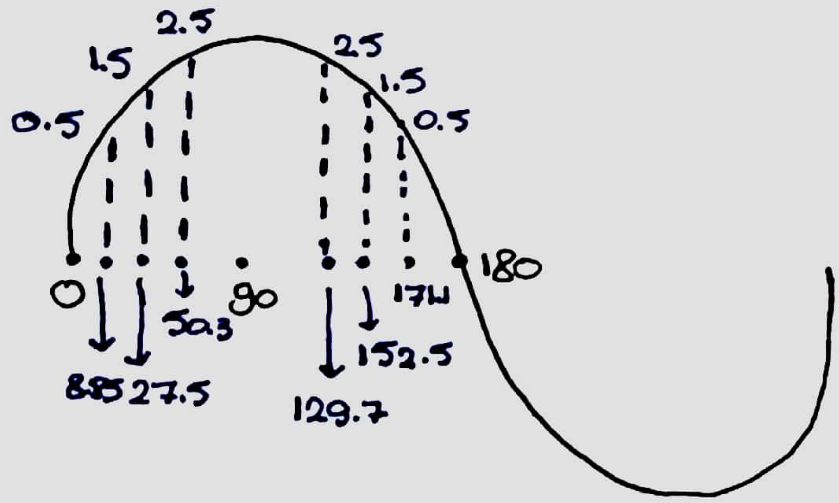
// Left for you.

→ Finish in 1 min with a quick sketch.

Problem 5) // In case you don't like all the line mess and would rather solve for t.

• A mid-tread quantizer rounds the input to the nearest output level.

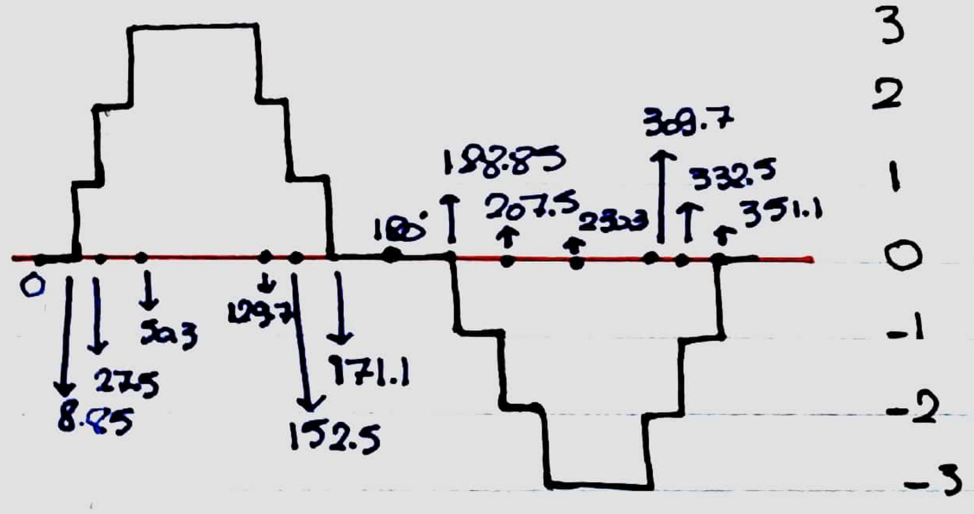




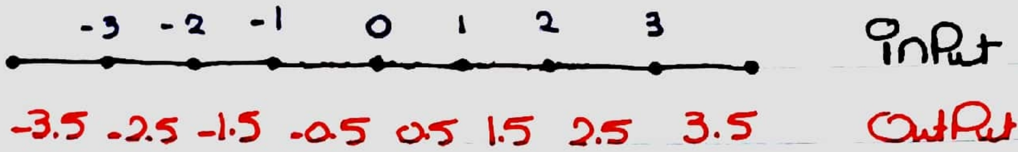
• Keep solving

$3.25 \sin x =$ input threshold

← The quantizer's characteristics are symmetric



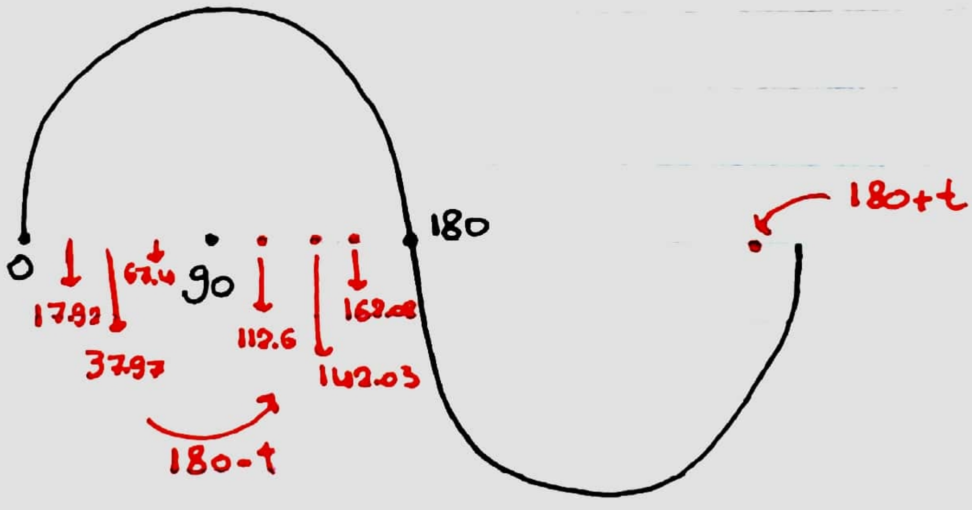
Midrise Quantizer

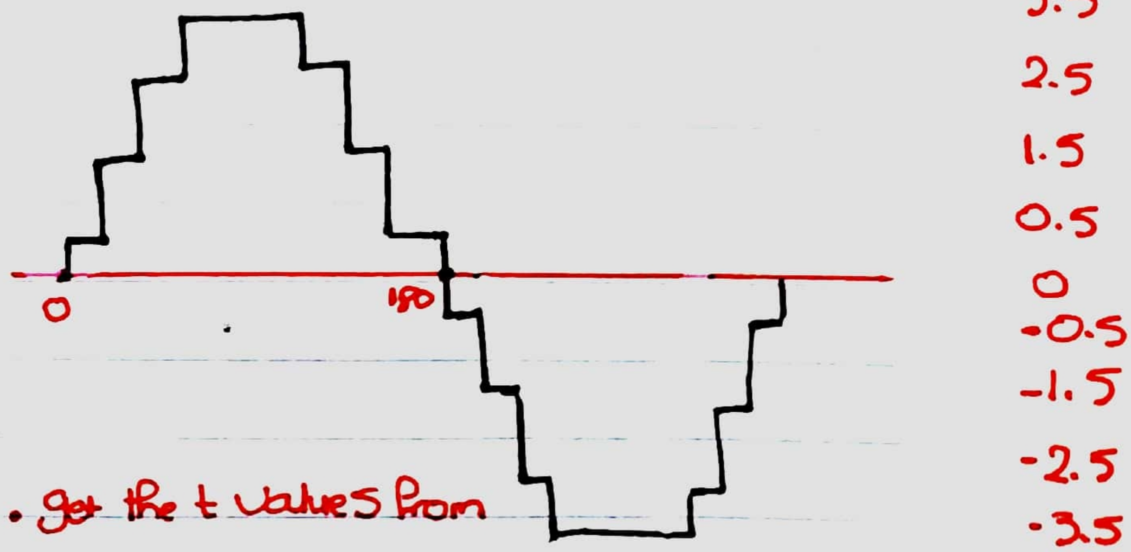


- it floors then adds 1/2

$y = 3.25 \sin(t)$

t	y
0	0
17.92	1
37.97	2
67.38	3





- get the t values from the graph above.

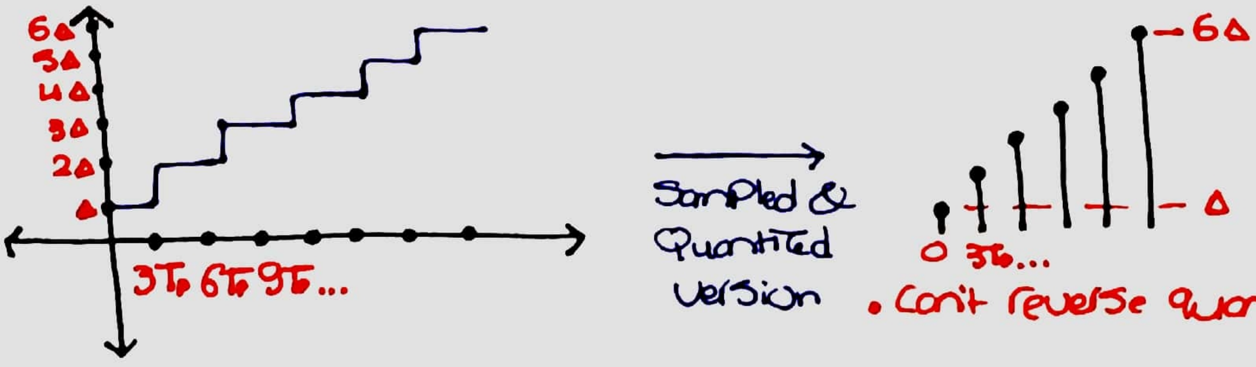
Problem 6)

- Code word is 3 bits (8 Possible levels)
- $-1 \to 0$ and $1 \to 1$

Thus the code is

001	010	011	100	101	110
1	2	3	4	5	6

The quantized version of the signal is thus



- Can't reverse quantization

- Recall that Δ (distance between 2 levels) depends on the signal $\Delta = \frac{2m_p}{L}$

thank you! <3