

Time: 2 Hours

1) Consider the 2-D two-class classification problem, given by the following vectors:

Class 1 patterns:  $(2 \ 2)^T$ ,  $(2 \ 2.5)^T$ ,  $(2.8 \ 2)^T$ ,  $(3 \ 3)^T$ .

Class 2 patterns:  $(0 \ 0)^T$ ,  $(0.5 \ 0)^T$ ,  $(0 \ 1)^T$ ,  $(1 \ 1)^T$

a) Assume that we would like to use the nearest neighbor classifier. What would be the classification of the following pattern:  $(1.4 \ 1.5)^T$ ?

b) Assume that we would like to apply the Bayes classifier to classify the same pattern:  $(1.4 \ 1.5)^T$ . What would be its classification? Assume that the class conditional densities are estimated using the Parzen window estimator, with  $h = 1$ , and assume equal a priori probabilities.

2) Consider the 2-D two-class classification problem, given by the following vectors:

Class 1 :  $x(1) = (0 \ 0)^T$ ,  $x(2) = (1 \ 0)^T$ ,  $x(3) = (1.5 \ 0)^T$ .

Class 2 :  $x(4) = (0 \ 1)^T$ ,  $x(5) = (0 \ 1.5)^T$ ,  $x(6) = (1 \ 0.5)^T$ .

Plot the classification regions and classification boundary for the nearest neighbor classifier.

3) a) Explain in details what is the concept of overfitting. Explain in details what factors affect the overfitting, in what way, and why.

b) Explain in details how a high value and a low value of  $h$  (the width of the Gaussian bump) affect the Parzen window density estimate.

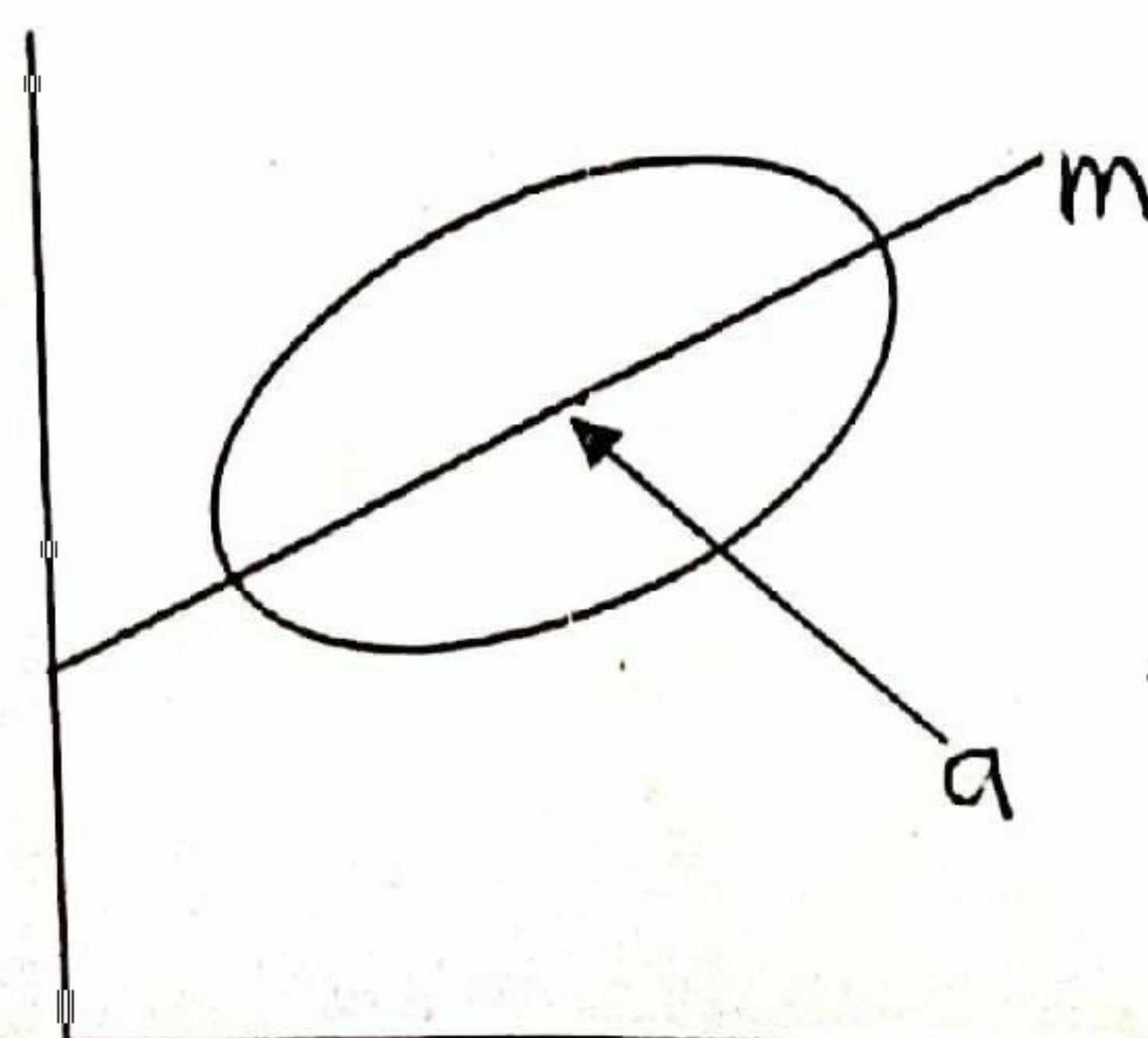
c) Explain in details the sequential forward feature selection algorithm. Is it a wrapper-type or a filter-type?

4) a) Consider a two-dimensional vector  $x = (x_1 \ x_2)^T$  with mean vector  $\mu = (1, -2)^T$ , and covariance matrix given by

$$\Sigma = \begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix}$$

Find the standard deviation of  $x_2$ . Also find  $E(x_1)$ ,  $E(x_1^2)$  and  $E(x_1 x_2)$ .

b) Consider the figure below. The ellipse shown is the (equal density) contour of a two-dimensional Gaussian density. What is the point  $a$  in terms of the parameters of the density? Also, express the equation of line  $m$  in terms of the parameters of the density.





5) Consider the 3-D two-class classification problem, given by the following vectors:

Class 1 patterns:  $(0 \ 0 \ 0.3)^T$ ,  $(1 \ 0 \ 0.5)^T$ ,  $(0 \ 1 \ 0.1)^T$ ,  $(1 \ 1 \ 0.8)^T$ ,  $(0.5 \ 0.6 \ 0.4)^T$ .

Class 2 patterns:  $(2 \ 1.5 \ 1.1)^T$ ,  $(2.7 \ 1.7 \ 1.4)^T$ ,  $(2.7 \ 2.4 \ 1.7)^T$ ,  $(2.6 \ 2.1 \ 1.9)^T$ ,  $(2.2 \ 2.2 \ 1.3)^T$ .

a) Consider that we would like to apply support vector machine classifier. The optimization algorithm determined that the support vectors are the following:  $(1 \ 1 \ 0.8)^T$ , and  $(2 \ 1.5 \ 1.1)^T$ . Find the weights  $w$  and  $w_0$ .

b) Find the Lagrange multipliers  $\alpha_i$  for all ten points.

c) What is the classification error rate for this classifier?

d) Explain what is the concept of support vectors, and explain their significance and the effect do they have on the solution.