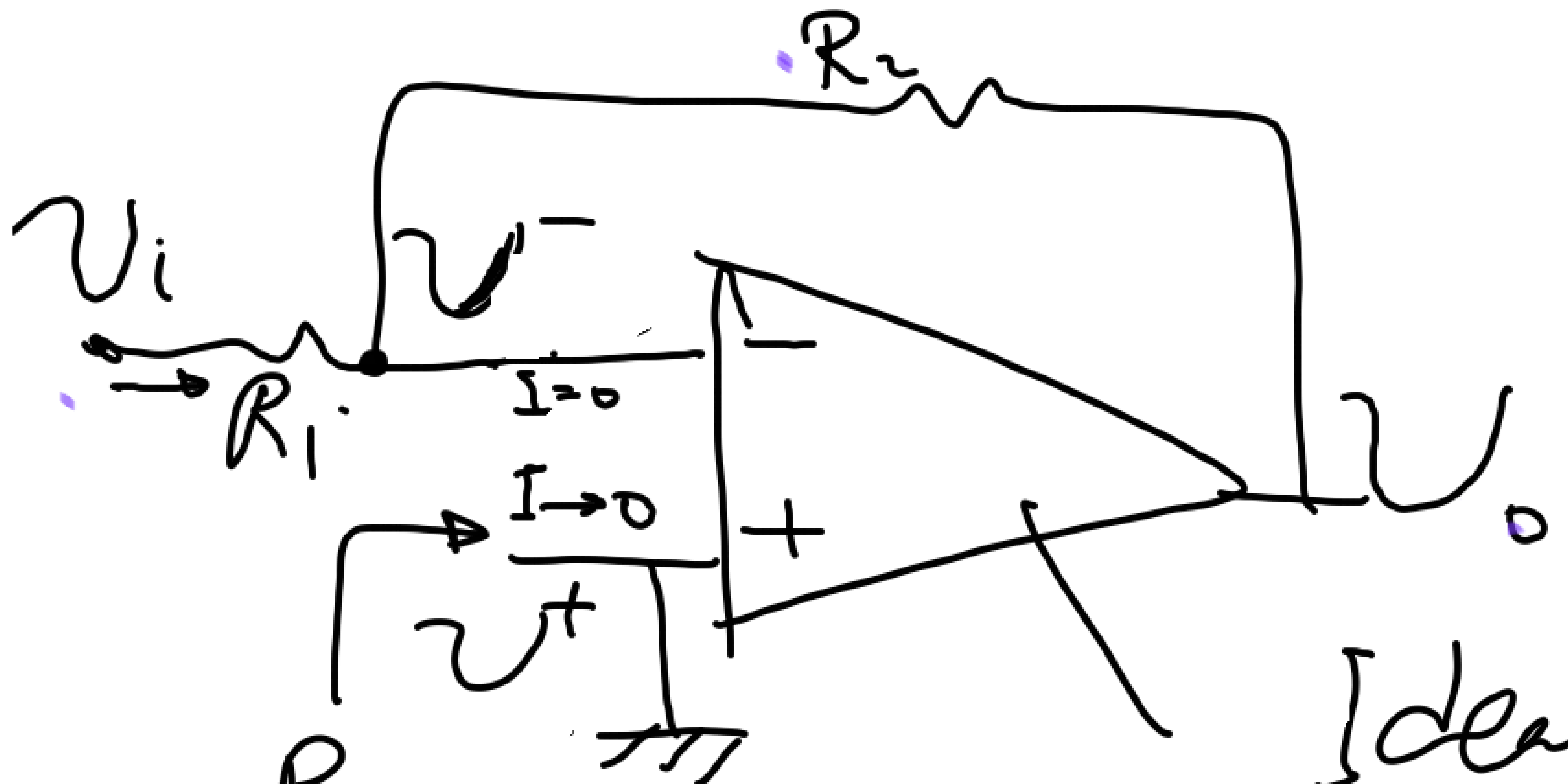


LTI

Ideal → $A \to \infty$ $\quad \frac{v_o}{\Delta V_i} \to \infty$

ΔV_i → zero

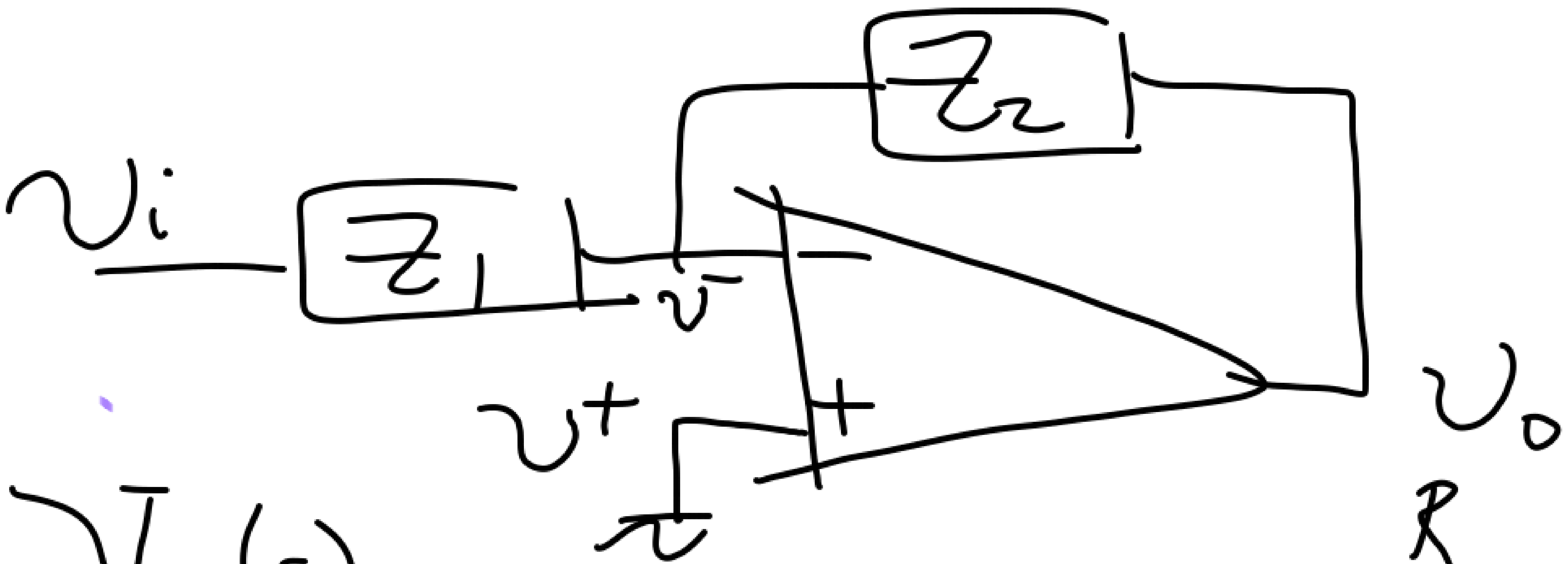
$\boxed{v^+ = v^-}$

$R_{in} \to \infty$

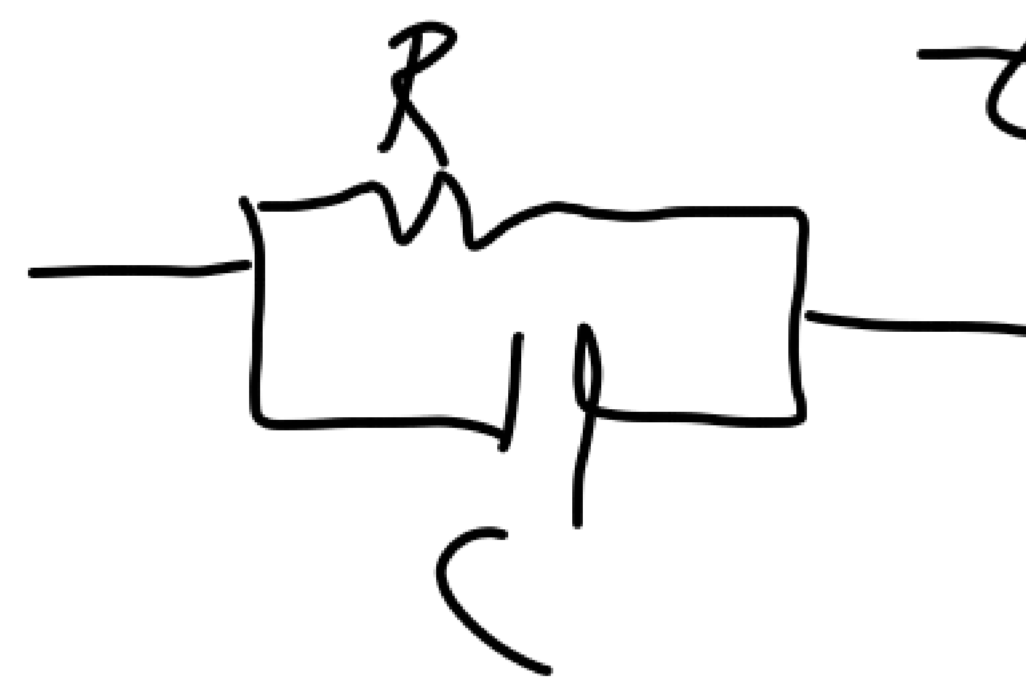
$$\frac{V_i(s) - V^-(s)}{R_1} = \frac{V^-(s) - V_o(s)}{R_2}$$

$$\frac{V_i(s) - 0}{R_1} = \frac{0 - V_o(s)}{R_2}$$

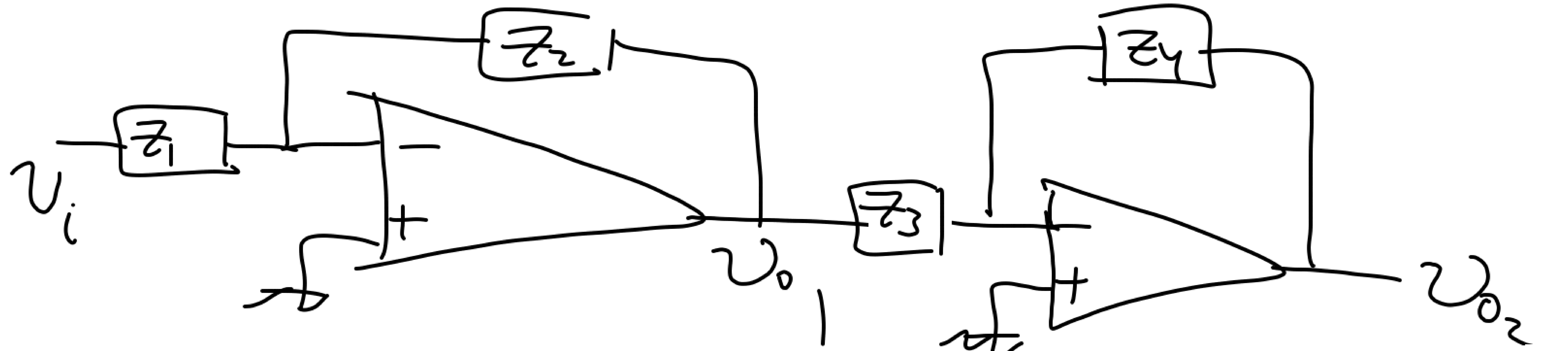
$$\frac{V_o(s)}{V_i(s)} = -\frac{R_2}{R_1}$$



$$\frac{V_o(s)}{V_i(s)} = -\frac{Z_2}{Z_1}$$



$$Z = R \,//\, \frac{1}{sC} = \frac{1}{\frac{1}{R} + sC} = \frac{R}{sCR+1}$$

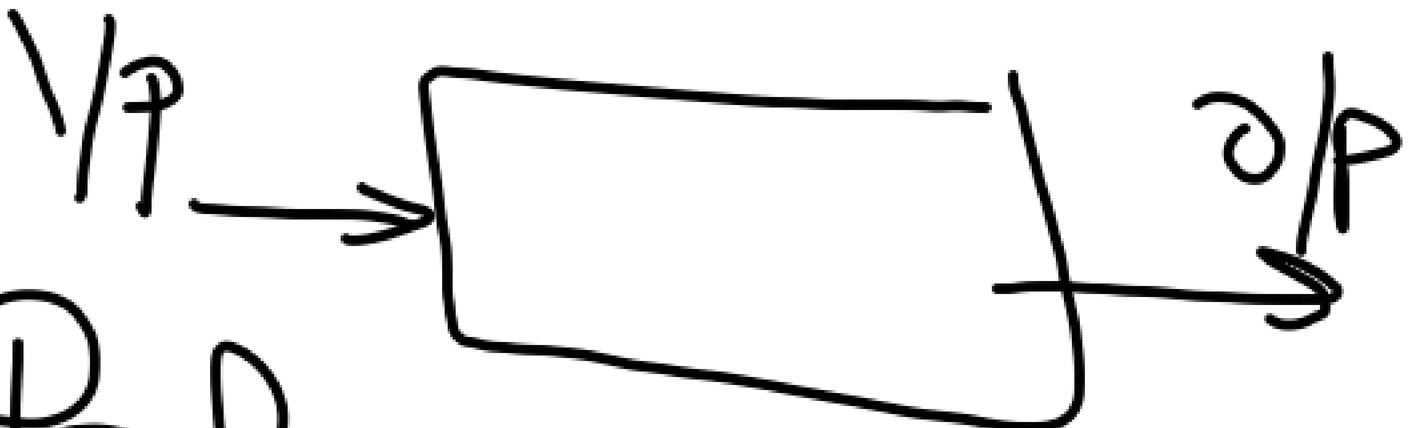


$$\frac{V_{o2}}{V_i(s)} = \frac{V_{o2}}{V_{o1}} \times \frac{V_{o1}}{V_i}$$

$$= \frac{-Z_4}{Z_3} \times -\frac{Z_2}{Z_1}$$

Model $\longrightarrow$ T.F

- Soln (o/p(t))
- Final Value
- Initial Value
- Analysis



Reference ( )

Const

(Set-Point)

S.S.

(Tracking Performance)

S.S. ↳ $t \to \infty$

$$\frac{Y}{R} = G\frac{E}{R}$$

Block Diagram [-ve Unity f.b]

$$e_{s.s.} = \lim_{s \to 0} sE(s) = \lim_{t \to \infty} e(t) = r_{ss} - y_{ss}$$

↳ $\frac{Y(s)}{R(s)}$: Closed-loop T.F

$$= \frac{f.f.}{1 + \text{Loop}} = \frac{G}{1 + G \times 1}$$

Loop: f.f. · f.b = 1

$$\therefore \frac{E}{R} = \left(\frac{G}{1+G}\right) \Big/ G = \frac{1}{1+G}$$

$$\frac{E(s)}{R(s)} = \frac{1}{1+G(s)}$$

Open loop $= G \times H$

$$E(s) = \frac{R(s)}{1+G(s)}$$

$$e_{s.s.} = \lim_{s\to 0} \frac{sR(s)}{1+\underbrace{G(s)}_{\text{Open loop}}}$$

Step: (1)

Ramp: (t)

Parabolic signal (t^2)

$$\frac{P(s)}{s^{N} Q(s)} \rightarrow \text{Type}$$

① Step: $R(s) = \frac{1}{s}$

$$e_{ss}\Big|_{step} = \lim_{s \to 0} \cancel{s} \frac{1/\cancel{s}}{1+G(s)}$$

$$= \frac{1}{1 + \lim_{s \to 0} G(s)} \rightarrow k_p\text{: Position Error Gain}$$

$$= \frac{1}{1+k_p}$$

$$k_p = \lim_{s \to 0} G(s)$$

Type zero: $G(s) = \frac{P(s)}{Q(s)}$

$$k_p = \lim_{s \to 0} \frac{P(s)}{Q(s)} = \text{Const}$$

$$e_{ss}\Big|_{\substack{\text{Type zero} \\ \text{step}}} \rightarrow \text{Const.}$$

<u>Type 1:</u> $G(s) = \frac{P(s)}{s\, Q(s)}$

$k_p = \lim_{s \to 0} \frac{P(s)}{s\, Q(s)} \to \infty$

$e_{ss}\Big|_{\substack{\text{step} \\ \text{Type-1}}} = \text{Zero}$

<u>② Ramp I/p: $R(s) = \frac{1}{s^2}$</u>

$e_{ss} = \lim_{s \to 0} \cancel{s} \frac{\frac{1}{s^{\cancel{2}}}}{1 + G(s)}$

$= \lim_{s \to 0} \frac{1}{s + s\,G(s)} = \frac{1}{0 + \lim_{s \to 0} s\,G(s)}$

k_v

$e_{ss}\Big|_{\text{ramp}} = \frac{1}{k_v}$

$k_v = \lim_{s \to 0} s\,G(s)$

Type-zero:

$$k_v = \lim_{s \to 0} s \frac{P(s)}{Q(s)} = \text{Zero}$$

$$e_{ss}\Big|_{\substack{\text{ramp} \\ \text{Type-zero}}} \longrightarrow \infty$$

Type-1:

$$k_v = \lim_{s \to 0} \cancel{s} \frac{P(s)}{\cancel{s}\, Q(s)} = \text{Const}$$

$$e_{ss}\Big|_{\substack{\text{ramp} \\ \text{Type-1}}} \longrightarrow \text{Const}$$

Type ≥ 2: $k_v = \lim_{s \to 0} \cancel{s} \frac{P(s)}{s^{\cancel{2}} Q(s)}$ ∞

$$e_{ss}\Big|_{\substack{\text{ramp} \\ \text{Type} \geq 2}} \longrightarrow \text{Zero}$$

③ Parabolic Signal $\left(R(s)=\dfrac{1}{s^3}\right)$

$r(t)=\frac{1}{2}t^2$

$$e_{ss}=\lim_{s\to 0} s\,\frac{\frac{1}{s^3}}{1+G}$$

$$=\frac{1}{0+\lim_{s\to 0} s^2 G(s)}$$

$$=\frac{1}{k_a}$$ Acceleration

$$k_a=\lim_{s\to 0} s^2 G(s)$$

Type zero, 1:

$$k_a=\lim_{s\to 0} s^2 \frac{P(s)}{Q(s)}=\text{zero}$$

$e_{ss}\Big|_{\text{Para., Type 0, 1}}\to\infty$

Type 2:

$$k_a=\lim_{s\to 0} \cancel{s^2}\,\frac{P(s)}{\cancel{s^2}Q(s)}=\text{Const}$$

$e_{ss}\Big|_{\text{Para., Type 2}}\to\text{Const.}$

Type 3 :

$$k_a = \lim_{s \to 0} s^2 \frac{P(s)}{s^3 Q(s)} \rightarrow \infty$$

$e_{ss} \Big|_{\substack{Para \\ Type\ 3}} \longrightarrow$ zero

---

$r(t) = 3 + 5t + 7t^2$

$$R(s) = \frac{3}{s} + \frac{5}{s^2} + \frac{7 \times 2}{s^3}$$

$$e_{ss} = \frac{3}{1+k_p} + \frac{5}{k_v} + \frac{14}{k_a}$$

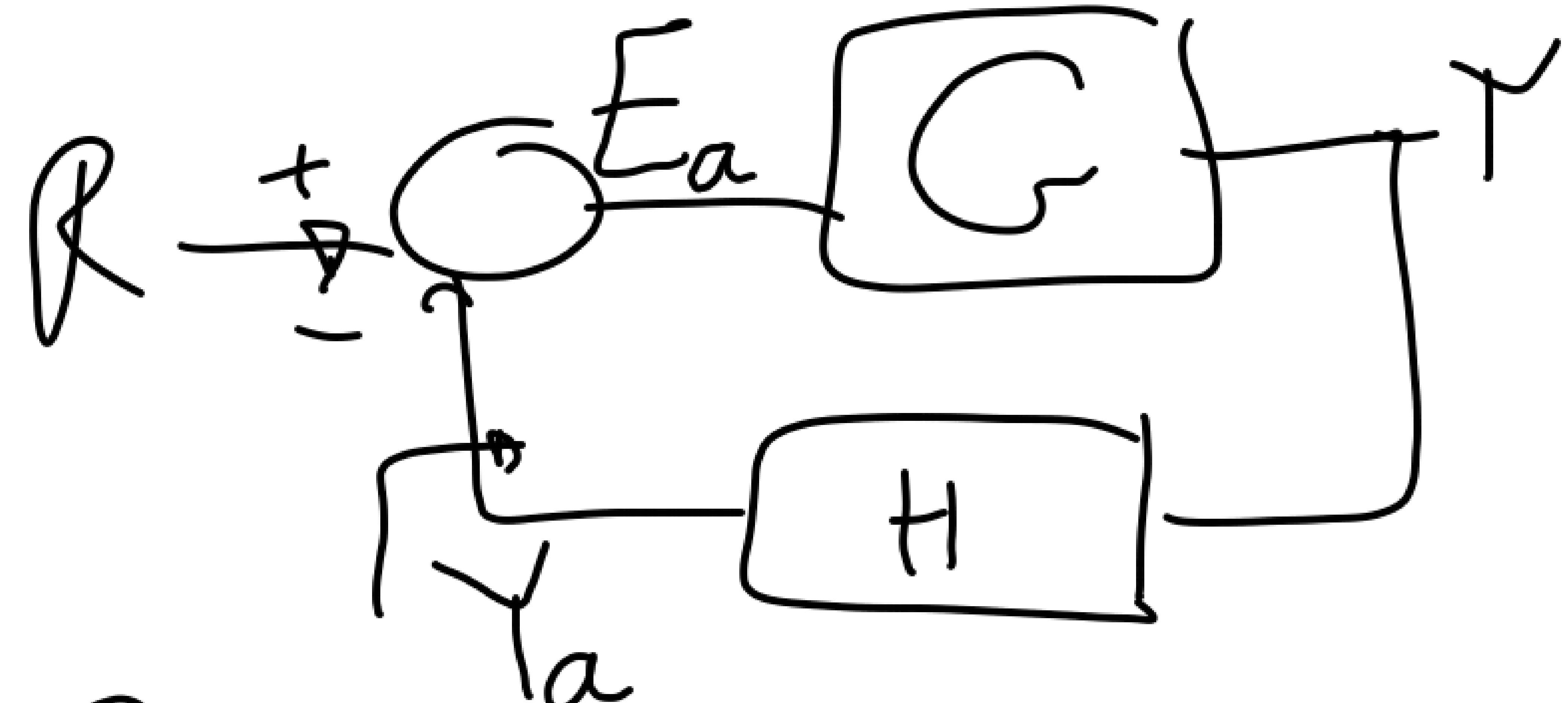
$t^{\boxed{n}}$

e_{ss}

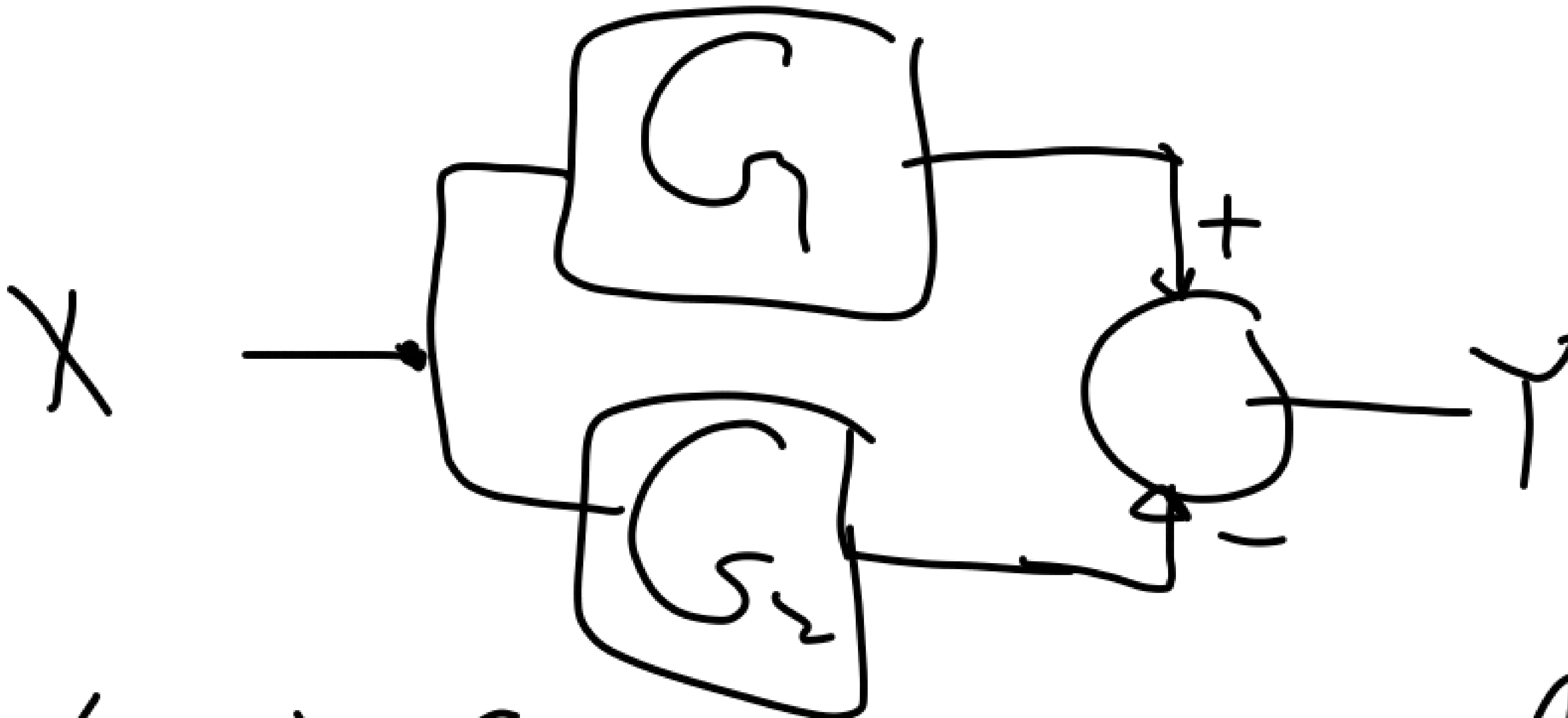
Type(N)	Step(n=0) (K_p)	Ramp(n=1) (K_v)	Parabolic Signal (n=2) (K_a)
Zero	Const.	∞	∞
1	Zero	Const	∞
2	Zero	Zero	Const
≥ 3	Zero	Zero	Zero

$$O.L. = G \times 1$$

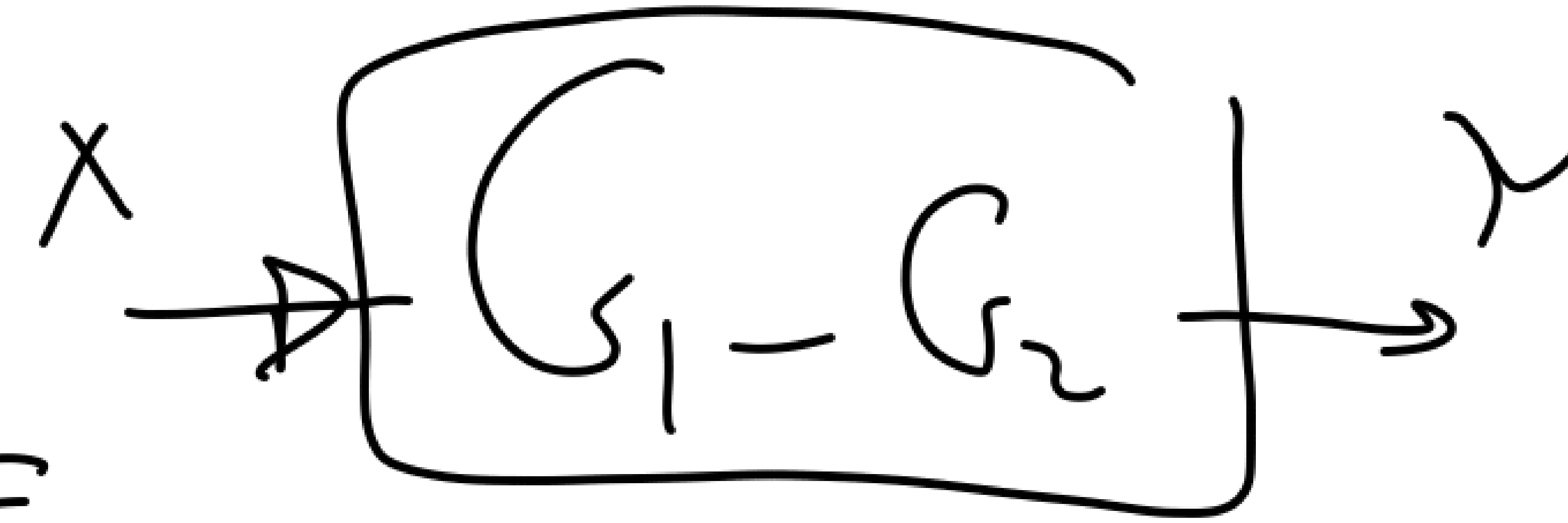
$$e_{ss} = r_{ss} - y_{ss}$$



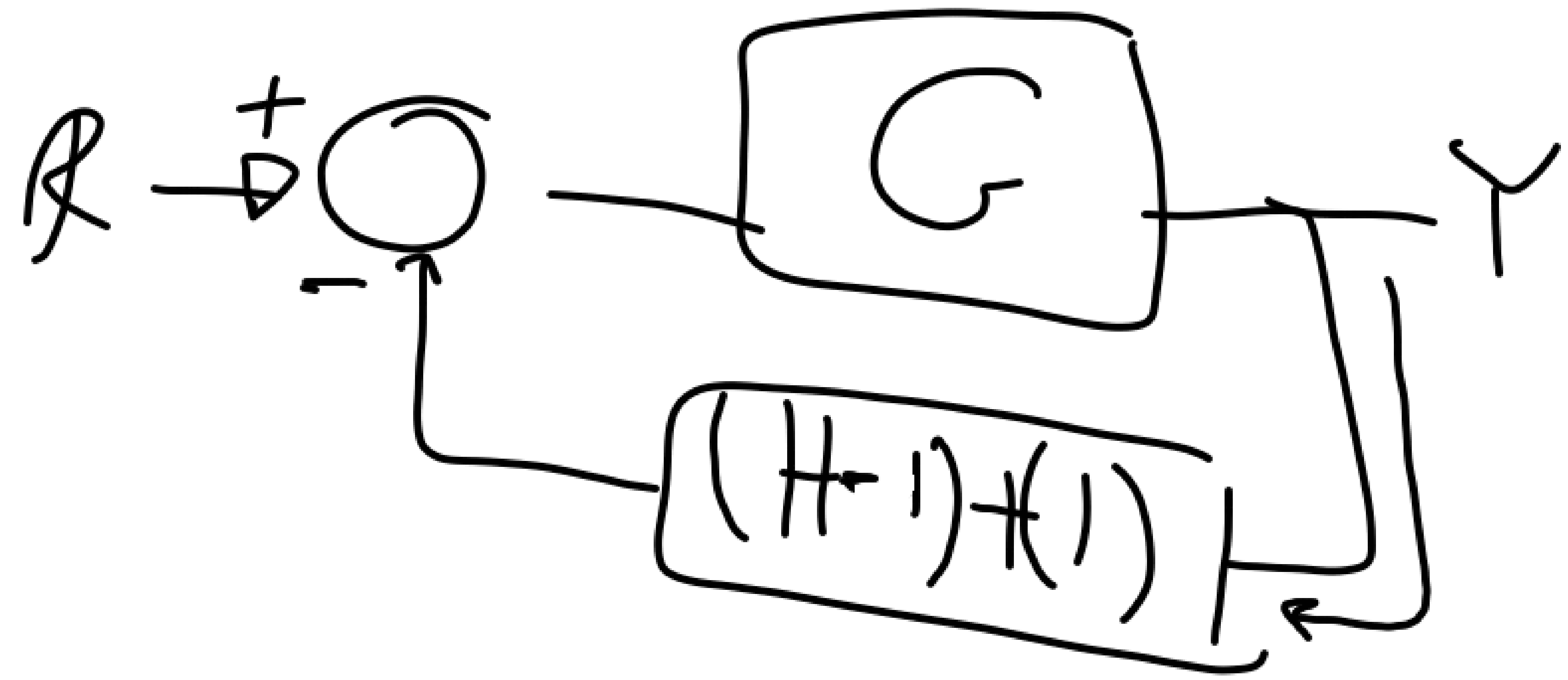
$$Ca_{ss} \longrightarrow O.L. = G \times H$$



$\equiv$



$$Y = XG_1 - XG_2$$
$$= (G_1 - G_2)X$$



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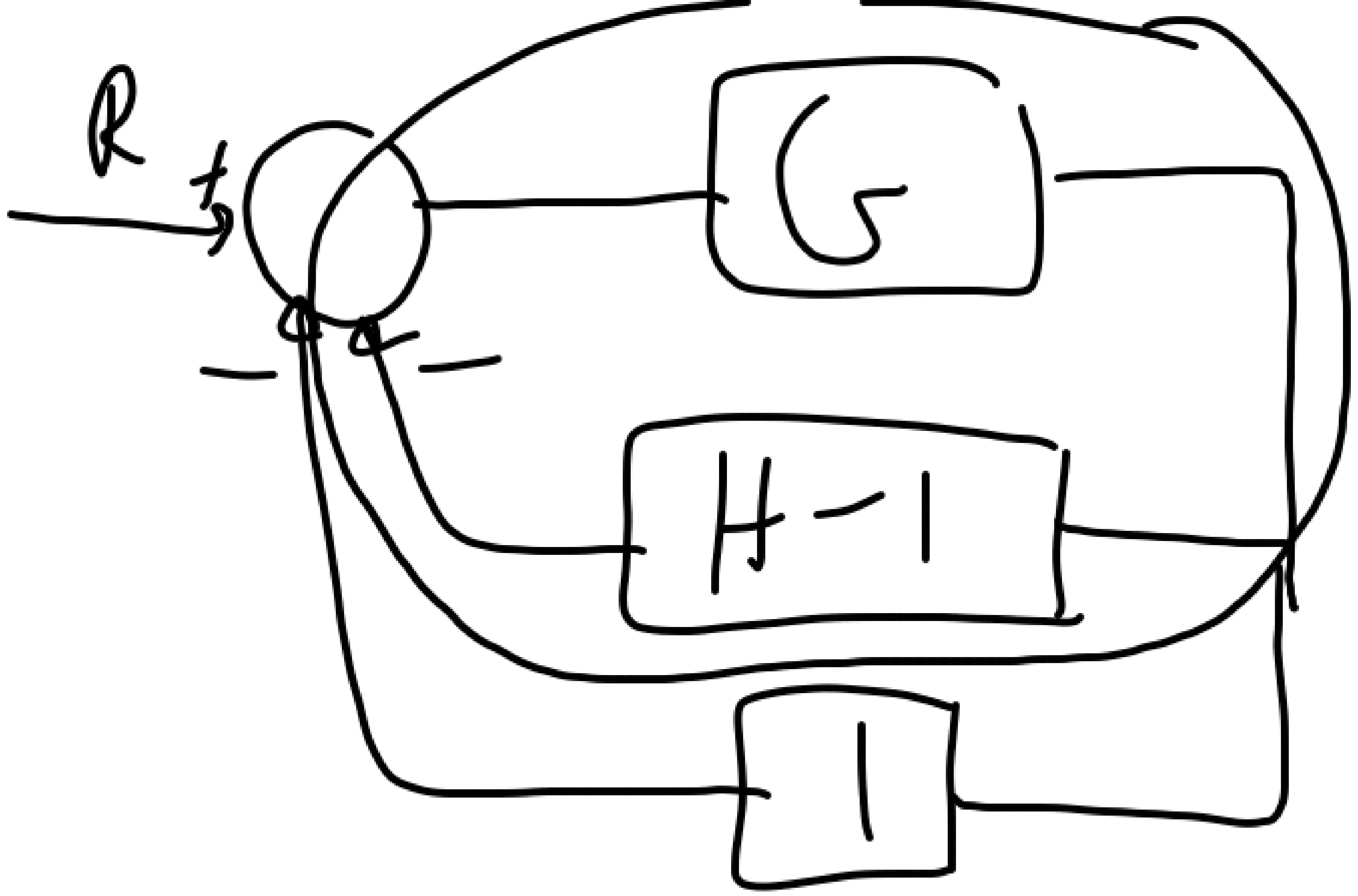

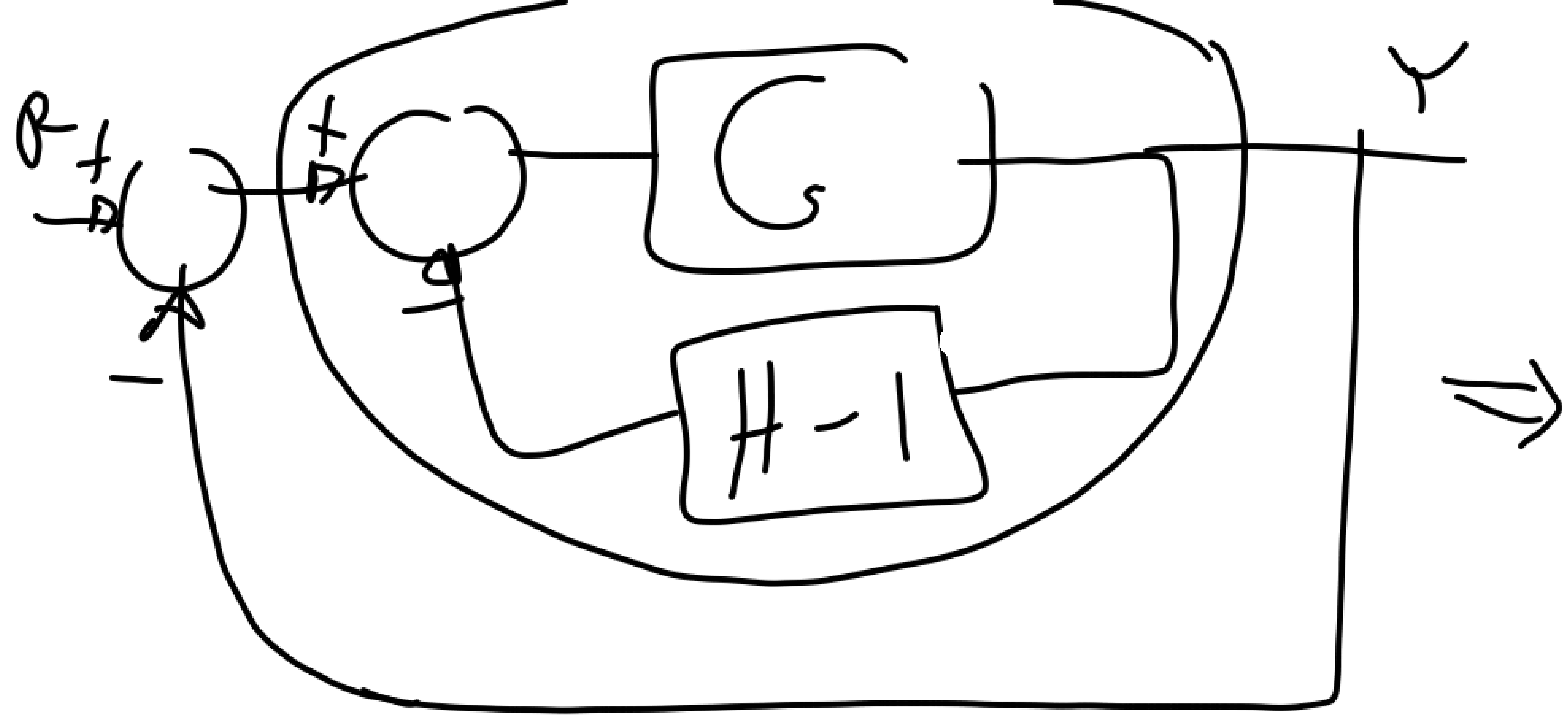

$\Rightarrow$

O.L.: $G' = \dfrac{G}{1 + G \times (H - 1)}$

e_{ss}