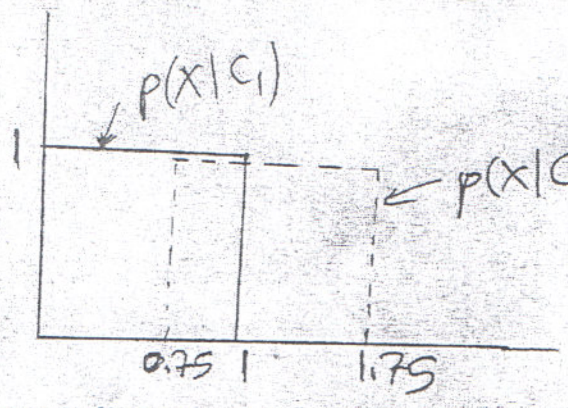
CAIRO UNIVERSITY
FACULTY OF ENGINEERING

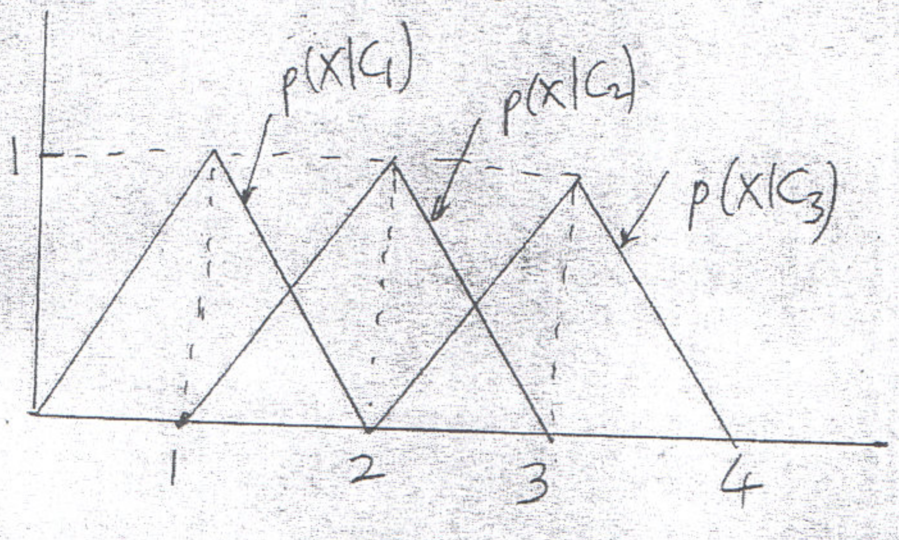
DEPT COMPUTER ENG
Excercize No.3

---

1) Consider the one-dimensional two-class problem given below. The class-conditional probability densities are as given below. The a priori probabilities are $P(C_1) = 0.75$, $P(C_2) = 0.25$. Find the classification regions and the classification boundary. Calculate the probability of correct classification and the probability of error in classification.



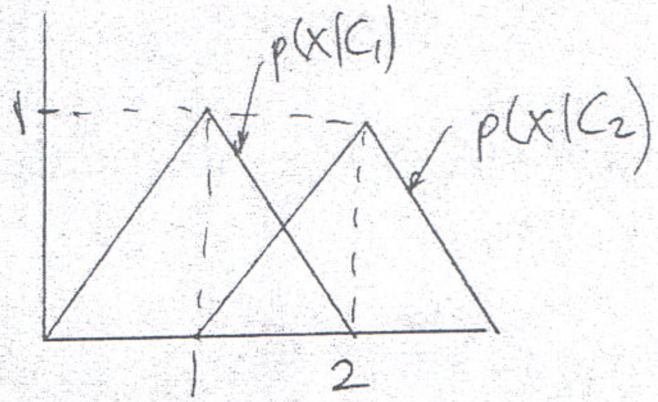
2) Consider the one-dimensional three-class problem given below. The class-conditional probability densities are as given below. The a priori probabilities are $P(C_1) = P(C_2) = P(C_3) = \frac{1}{3}$. Find the classification regions and the classification boundary. Calculate the probability of correct classification, and the probability of error in classification.



3) Consider the one-dimensional two-class problem given below. The class-conditional probability densities are as given below. The a priori probabilities are $P(C_1) = P(C_2) = 0.5$. It can be shown that the Bayes classifier is given by:

Classify x as Class 1 if $x < 1.5$, otherwise classify as Class 2.

Show that any other classification rule results in higher probability of error.



4) Consider the one-dimensional two-class case with the following Gaussian class-conditional density functions:

$$p(x|C_1) = N(\mu_1, \sigma_1)$$
$$p(x|C_2) = N(\mu_2, \sigma_2)$$

where μ_i means the mean of class C_i, and σ_i is the standard deviation of class C_i. Choose values for μ_i and σ_i, and sketch. When μ_1 gets farther from μ_2, will the probability of error increase or decrease? How about when σ_1 and σ_2 increase? How about when the a priori probabilities get closer to 0.5?

5) Consider the two-dimensional two-class case with the two-variate Gaussian densities, with parameters:

$$\text{Mean vector for Class 1}: \ \mu_1 = (0 \ \ 0)^T$$
$$\text{Mean vector for Class 2}: \ \mu_2 = (2 \ \ 0)^T$$
$$\text{Covariance matrix for Class 1}: \ \Sigma_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$
$$\text{Covariance matrix for Class 2}: \ \Sigma_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Write down the expressions for the densities. Plot the decision regions. Find the probability of correct classification.

6) Consider the N-dimensional two-class problem with general multi-variate Gaussian distributions. Let the mean vectors for the two classes be respectively μ_1 and μ_2. Let both covariance matrices be equal ($= \Sigma$). Show that the Bayes classifier is a linear classifier. Find a formula for that linear classifier.

7) Consider a two-dimensional pattern classification problem, with the a priori probabilities $P(C_1) = 0.7$ and $P(C_2) = 0.3$, and the class conditional probabilities:

$$p(\mathrm{x}|C_1) = \begin{cases} 1 & \text{if } 0 \leq x_1 \leq 1 \text{ and } 0 \leq x_2 \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

$$p(\mathrm{x}|C_2) = \begin{cases} 4 & \text{if } 0.8 \leq x_1 \leq 1.3 \text{ and } 0.8 \leq x_2 \leq 1.3 \\ 0 & \text{otherwise} \end{cases}$$

Plot the densities in a three dimensional plot. Plot the classification regions for the Bayes classification rule. Find its probability of error. Consider the linear classifier that classifies a pattern according to the following: If $x_1+x_2-1.7 \leq 0$ then $\mathrm{x} \in C_1$, and if $x_1+x_2-1.7 > 0$ then $\mathrm{x} \in C_2$. Find the probability of error for this classifier.