

Time: 1.5 Hours

1) Consider the following problem:

Class 1 patterns: $(2 \ 2)^T$, $(2 \ 3)^T$, $(3 \ 2)^T$, $(3 \ 3)^T$.

Class 2 patterns: $(0 \ 0)^T$, $(1 \ 0)^T$, $(0 \ 1)^T$, $(1 \ 1)^T$

a) Assume that we would like to use the nearest neighbor classifier. What would be the classification of the following pattern: $(1.5 \ 1.4)^T$?

b) Assume that we would like to use the K -nearest neighbor classifier, where $K = 3$. What would be the classification of the following pattern: $(1.1 \ 1.3)^T$?

2) a) State why the Bayes classifier is theoretically the optimal classifier. Discuss why in realistic situations it cannot be considered optimal.

b) The linear classifier is a classifier that has a decision boundary of the form of a (insert missing word).

c) Consider the two-dimensional three-class classification problem, where the class centers are given by the vectors $(0, \ 1)^T$, $(2, \ 0)^T$, $(2, \ 2)^T$. Plot the decision regions and the decision boundaries for the minimum distance classifier. What are the coordinates of the point of equal distance among all class centers.

3) a) Let x be a random variable having a Gaussian density with mean equal 0 and standard deviation equal 2. Write down the expression of the density $p(x)$. Compute $E(e^{-2x^2})$.

b) Consider the joint density

$$p(x, y) = \begin{cases} 2e^{-x}(1-y) & \text{if } 0 \leq x < \infty \text{ and } 0 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Find $p(x)$ (consider $x \geq 0$).

4) Consider a two-class single dimensional problem, where the a priori probabilities are given by $P(C_1) = 0.65$ and $P(C_2) = 0.35$, and the class-conditional densities are given by:

$$p(x|C_1) = \begin{cases} a(1-x) & \text{if } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

$$p(x|C_2) = \begin{cases} 1 & \text{if } 0.6 \leq x < 1.6 \\ 0 & \text{otherwise} \end{cases}$$

a) Find the value of a . b) Plot the densities, and plot the decision regions and the decision boundaries for the Bayes classifier. c) Find the probability error for the Bayes classifier.

b) c)