Sheet 6 Signal Space Representation

Notes

① Gram-Schmidt

* Basis func are
  - Orthogonal → $\int \varphi_i \varphi_j \, dt = 0 \quad \forall i \neq j$
  - Normalized → $\int \varphi_i \varphi_i \, dt = 1$

* $\varphi_1(t) = \dfrac{s_1(t)}{\sqrt{E}}$

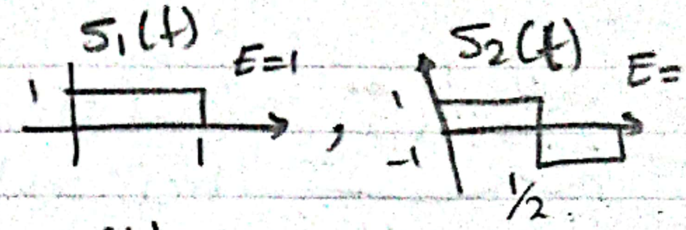
* $\psi_2(t) = s_2(t) - \overbrace{\left(\int s_2(t)\, \varphi_1(t)\, dt\right)}^{\text{correlation with } \varphi_1} \varphi_1(t)$

* لو طلعوا fully correlated، هتلاقي الـ ψ_2 طلعت بـ zero وهيكون في فرق بين s_1 و s_2 في الإشارة أو multiple من بعض! وديه مش هينفع تبقى basis func.
* لو كان الاتنين orth، الـ corr هيطلع zero، كده s_2 ممكن تبقى basis func.
* لو الـ corr اداني رقم، هنعمل الـ subtr كدي عادي ونجيب ψ_2 وال basis func، هنعملها Norm ونقسم على $\sqrt{E}$

* In General:

$$\psi_n(t) = s_N(t) - \int s_N(t) * \varphi_1(t)\, dt * \varphi_1(t) - \int s_N(t) * \varphi_n(t)\, dt * \varphi_n(t)$$

$$\varphi_n(t) = \frac{\psi_n(t)}{\sqrt{E}}$$

2) let 

E = Area under the function Ψ_N

$$\varphi_1(t) = \frac{S_1(t)}{\sqrt{E_1}} = S_1(t)$$

$\therefore$ $S_1(t) = 1 * \varphi_1(t)$

$$\Psi_2(t) = S_2(t) - \underbrace{\int S_2\,\varphi_1(t)\,dt}\,*\,\varphi_1(t)$$

$1*\frac{1}{2} - 1*\frac{1}{2} = 0$

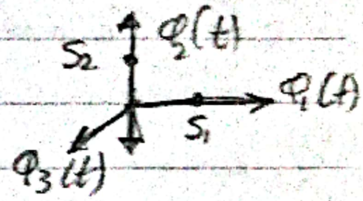
$\leftarrow$ $\therefore$ orthogonal

$$\Psi_2(t) = S_2(t)$$

$$\varphi_2(t) = \frac{\Psi_2(t)}{\sqrt{E_1}} = S_2(t)$$

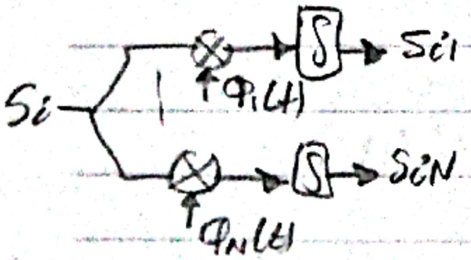
$$\therefore S_1 = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \varphi_1(t) \\ \varphi_2(t) \end{bmatrix}, \quad S_2 = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} \varphi_1(t) \\ \varphi_2(t) \end{bmatrix}$$

$$\therefore \underset{N}{S} = \underset{N \times M}{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}} \begin{bmatrix} \varphi_1(t) \\ \varphi_2(t) \end{bmatrix}_M$$



3) we know that

$\hookrightarrow S_i(t) = S_{i1}\,\varphi_1(t) + S_{i2}\,\varphi_2(t) + \dots + S_{iN}\,\varphi_N(t)$

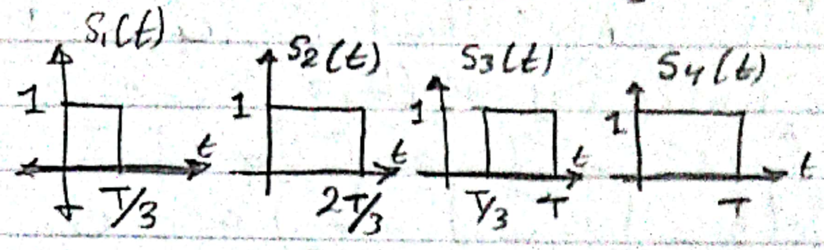


* لما أضرب في φ_1 ا كله هيبقى صفر ما عدا S_{11}
عشان هما orth مع بعض!

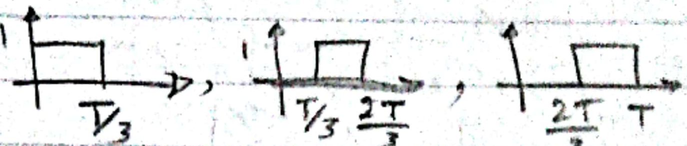
$\int_T 1^2 dt = \int$

Sheet 6

Q.1



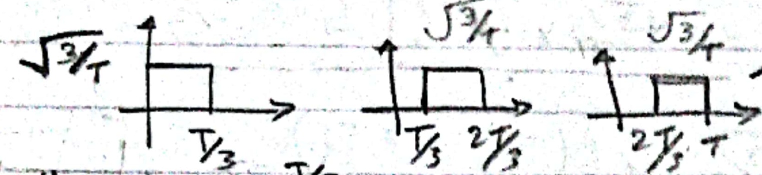
Method 1
بالفكر



همّا orth عشان حاصل
ضرب اي 2 بصفر

Normalize

Amp → $\sqrt{3/T}$

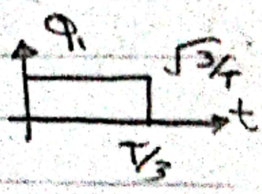


عشان
Calc E → Area
divide by $\sqrt{E}$ → Amp

OR
using Maths $\Rightarrow E = \int_0^{T/3} 1^2 dt = T/3$.

Method 2
Gram-Schmidt

1) $\phi_1(t) = \frac{s_1(t)}{\sqrt{E_{s_1}}} = \frac{s_1(t)}{\sqrt{\int_0^{T/3} 1^2 dt}} = \frac{s_1(t)}{\sqrt{T/3}} = \sqrt{\frac{3}{T}}\, s_1(t) \Rightarrow$

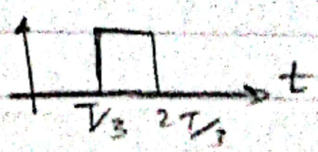


2) $\psi_2(t) = s_2(t) - \underbrace{\int s_2(t)\phi_1(t)\,dt}_{\text{Corr}} * \phi_1(t)$

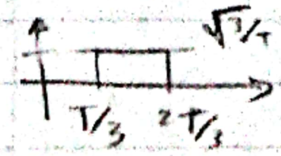
$= s_2(t) - \left[\int_0^{T/3} 1 * \sqrt{\tfrac{3}{T}}\, dt + \int_{T/3}^{2T/3} 1 * 0\, dt\right] * \phi_1(t)$

$= s_2(t) - \underbrace{\sqrt{\tfrac{3}{T}} * T/3 * \sqrt{3/T}}_{1} * \text{rect}_{\phi_1}$

$= s_2(t) - \text{rect}\,\phi_1 \Rightarrow$



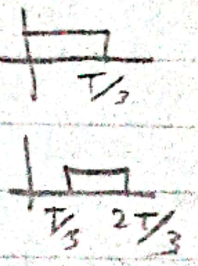
3) $\phi_2(t) = \frac{\psi_1(t)}{\sqrt{E}} \Rightarrow$



$\boxed{Q:1}$

$\int s_3 \varphi_1(t)$ conv

$\varphi_1 =$, $\varphi_2 =$



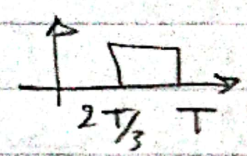
4) $\psi_3 = s_3 - s_{31}\varphi_1(t) - s_{32}\varphi_2(t)$

$s_{31} = \int_0^T s_3 \varphi_1(t)\,dt = \int_0^{T/3} 0 + \int_{T/3}^{2T/3} 0 + \int_{2T/3}^{T} 0 = 0$

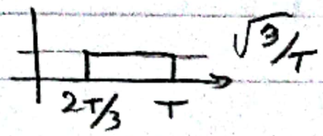
$s_{32} = \int_0^T s_3 \varphi_2(t)\,dt = \int_{T/3}^{2T/3} s_3 \varphi_2(t) + \int_{2T/3}^{T} 0\,dt$

$= \int_{T/3}^{2T/3} 1 \times \sqrt{3/T}\,dt = \sqrt{3/T} \times [T/3] = \sqrt{T/3}$

$\psi_3 = s_3 - \sqrt{T/3} * \sqrt{3/T}\ \text{rect}(T/3,\ 2T/3) \Rightarrow$



5) $\varphi_3 = \psi_3 / \sqrt{E} \Rightarrow$



6) $\psi_4 = s_4 - s_{41}\varphi_1(t) - s_{42}\varphi_2 - s_{43}\varphi_3$

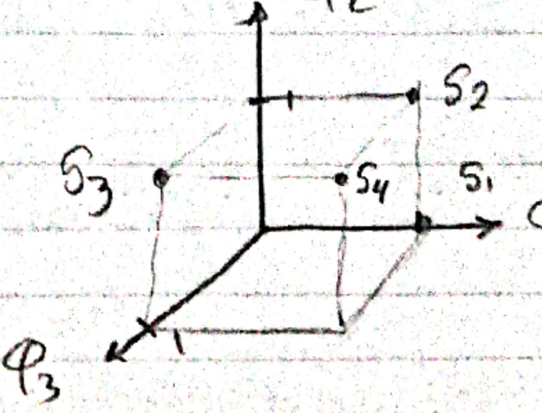
$\underbrace{- \int_0^{T/3} \quad - \int_{T/3}^{2T/3} \quad - \int_{2T/3}^{T}}_{s_4}$

$\psi_4 = s_4 - s_4 = \text{zero} \rightarrow \text{stop}$

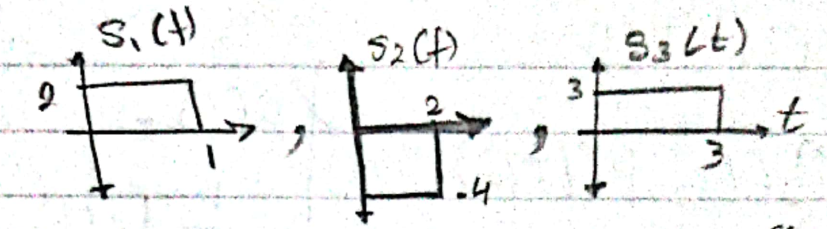
* في الامتحان طلب بالآخر، زي ما طلبنا في الأول خارجي !

$\boxed{\text{Part 2}}$

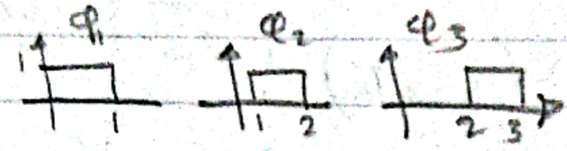
$$S = \begin{array}{c} \begin{array}{ccc} \varphi_1 & \varphi_2 & \varphi_3 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ +1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \end{array} \begin{bmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \end{bmatrix}$$



$\boxed{Q:2}$

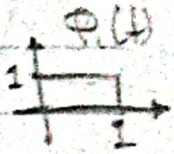


بالنظر



$$\Rightarrow \begin{array}{c} \begin{array}{ccc} \varphi_1 & \varphi_2 & \varphi_3 \end{array} \\ \begin{pmatrix} 2 & 0 & 0 \\ -4 & -4 & 0 \\ 3 & 3 & 3 \end{pmatrix} \end{array} \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \end{pmatrix}$$

Maths $\varphi_1(t) = \dfrac{s_1(t)}{\sqrt{E_{s_1}}} = \dfrac{s_1(t)}{\sqrt{4}} = \dfrac{2\,\text{rect}(0,1)}{2} = \text{rect}(0,1)$

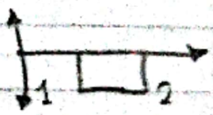


$\psi_2 = S_2 - \int S_{21}\varphi_1\,dt\;\varphi_1(t)$

$= S_2 - \int_0^1 -4 * 1 \;\; dt * \varphi_1(t)$

$= -4\,\text{rect}(0,2) + 4\,\text{rect}(0,1) = -4\,\text{rect}(1,2)$

$\varphi_2 = \dfrac{-4\,\text{rect}(1,2)}{\sqrt{16}} = -\text{rect}(1,2) \Rightarrow$



$\varphi_2 = \dfrac{S_2 - S_{21}\varphi_1}{4} \Rightarrow S_2 = \sqrt{E}\,\varphi_2 + S_{21}\varphi_1$

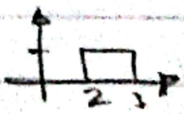
$= 4\,\varphi_2 - 4\,\varphi_1$

$\psi_3 = S_3 - S_{31}\varphi_1 - S_{32}\varphi_2$

$S_{31} = \int_0^1 3\,\varphi_1\,dt = 3 \quad , S_{32} = \int_1^2 3\,\varphi_2\,dt = -3$

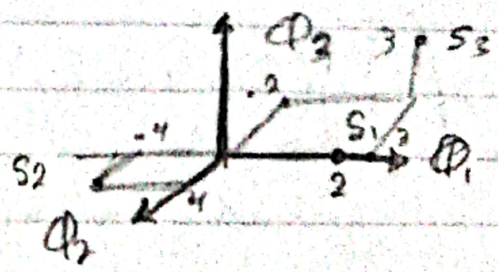
$\psi_3 = S_3 - 3\,\text{rect}(0,1) - 3\,\text{rect}(1,2) = 3\,\text{rect}(2,3)$

$\varphi_3 = \text{rect}(2,3) \Rightarrow$



$\Big|\; S_3 = \underset{S_{31}}{3\varphi_1} \underset{S_{32}}{-3\varphi_2} + \underset{\sqrt{E}}{3\varphi_3}$

$$\begin{array}{c} \begin{array}{ccc} \varphi_1 & \varphi_2 & \varphi_3 \end{array} \\ \begin{pmatrix} 2 & 0 & 0 \\ -4 & 4 & 0 \\ 3 & -3 & 3 \end{pmatrix} \end{array} \begin{bmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \end{bmatrix}$$



$\boxed{Q:3}$ Given $\to \varphi_1(t) = p(t)$

$\to \varphi_2(t) = p(t - T_0)$

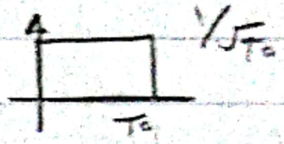
$\to \varphi_3(t) = p(t - 2T_0)$

$p(t) = \frac{1}{\sqrt{T_0}} [u(t) - u(t - T_0)]$

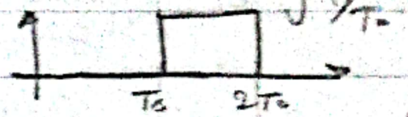
Sketch

$$S_i = \begin{pmatrix} 1 & 1 & 0 \\ 2 & -1 & 1 \\ 3 & 2 & -1/2 \\ -1/2 & -1 & 1 \end{pmatrix} \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \end{pmatrix}$$

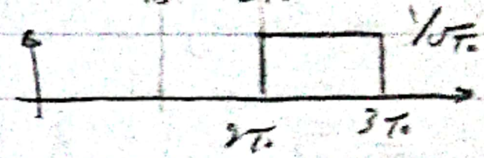
$\varphi_1 =$



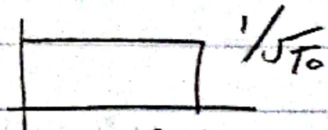
$\varphi_2 =$



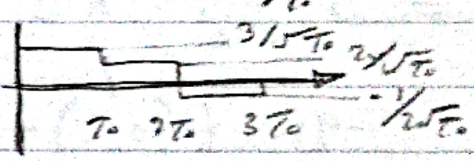
$\varphi_3 =$



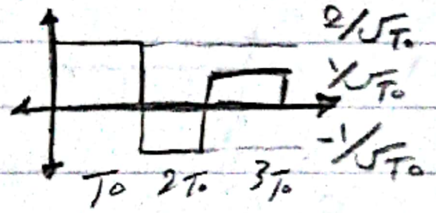
$S_1 \Rightarrow$



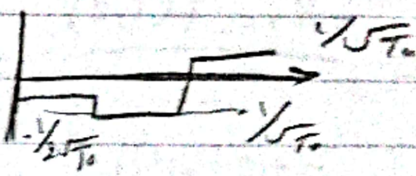
, $S_3 \Rightarrow$



$S_2 \Rightarrow$



, $S_4 \Rightarrow$



$\boxed{Q:4}$ $\varphi_k = p[t - (k-1)T_0]$, $k = 1, 2, 3, 4, 5.$

a) as Q3

b) Find Area Under each signal

e.g

$S_1 \Rightarrow (-1, 2, 3, 1, 4) \Rightarrow \sqrt{\Sigma v^2} = 1 + 4 + 9 + 1 + 16 = 31$

c) orthogonal $\Rightarrow S_i^T \cdot S_k = 0$

let $i = 3$, $k = 4$ $\Rightarrow \begin{pmatrix} 3 \\ -2 \\ 3 \\ 4 \\ 1 \end{pmatrix} (-2, 4, 2, 2, 0) = -6 - 8 + 6 + 8 + 0$

$= \text{zero}$