Cairo University
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Information Theory
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Sheet 1

Problem#1

9.21 Consider the binary symmetric channel described in Figure 9.8. Let p_0 denote the probability of sending binary symbol $x_0 = 0$, and let $p_1 = 1 - p_0$ denote the probability of sending binary symbol $x_1 = 1$. Let p denote the transition probability of the channel.

(a) Show that the mutual information between the channel input and channel output is given by

$$I(\mathscr{X}; \mathscr{Y}) = \mathscr{H}(z) - \mathscr{H}(p)$$

where

$$H(z) = z \log_2\left(\frac{1}{z}\right) + (1 - z) \log_2\left(\frac{1}{1 - z}\right)$$
$$z = p_0 p + (1 - p_0)(1 - p)$$

and

$$H(p) = p \log_2\left(\frac{1}{p}\right) + (1 - p) \log_2\left(\frac{1}{1 - p}\right)$$

(b) Show that the value of p_0 that maximizes $I(\mathscr{X}; \mathscr{Y})$ is equal to 1/2.

(c) Hence, show that the channel capacity equals

$$C = 1 - H(p)$$

Problem#2

15.4-1 A binary channel matrix is given by

$$\begin{array}{ccc} & & \text{Outputs} \\ & & \begin{array}{cc} y_1 & y_2 \end{array} \\ \text{Inputs} & \begin{array}{c} x_1 \\[2ex] x_2 \end{array} & \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\[1ex] \frac{1}{10} & \frac{9}{10} \end{bmatrix} \end{array}$$

This means $P_{y|x}(y_1|x_1) = 2/3$, $P_{y|x}(y_2|x_1) = 1/3$, etc. You are also given that $P_x(x_1) = 1/3$ and $P_x(x_2) = 2/3$. Determine $H(x)$, $H(x|y)$, $H(y)$, $H(y|x)$, and $I(x; y)$.