



MASTER THESIS

DESIGN OF SINGLE PRECISION FLOAT ADDER (32-BIT NUMBERS) ACCORDING TO IEEE 754 STANDARD USING VHDL

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RESUM

La aritmètica de punt flotant és, amb diferència, el mètode més utilitzat d'aproximació a la aritmètica amb nombres reals per realitzar càlculs numèrics per ordinador.

Durant molt temps cada màquina presentava una aritmètica diferent: bases, mida dels significants i exponents, formats, etc. Cada fabricant implementava el seu propi model ,fet que dificultava la portabilitat entre diferents equips, fins que va aparèixer la norma IEEE 754 que definia un estàndard únic per a tothom.

L'objectiu d'aquest projecte és, a partir del estàndard IEEE 754, implementar un sumador/restador binari de punt flotant de 32 bits emprant el llenguatge de programació hardware VHDL.

ZHRNUTIE

Práca s číslami s pohyblivou desatinnou čiarkou je najpoužívanejší spôsob pre vykonávanie aritmetických výpočtov s reálnymi číslami na moderných počítačoch. Donedávna, každý počítač využíval rôzne typy formátov: báza, znamienko, veľkosť exponentu, atď. Každá firma implementovala svoj vlastný formát a zabraňovala jeho prenosu na iné platformy pokiaľ sa nevymedzil jednotný štandard IEEE 754. Cieľom tejto práce je implementovanie 32-bitovej sčítačky/odčítačky pracujúcej s číslami s pohyblivou desatinnou čiarkou podľa štandardu IEEE 754 a to pomocou jazyka na opis hardvéru VHDL.

ABSTRACT

Floating Point arithmetic is by far the most used way of approximating real number arithmetic for performing numerical calculations on modern computers.

Each computer had a different arithmetic for long time: bases, significant and exponents' sizes, formats, etc. Each company implemented its own model and it hindered the portability between different equipments until IEEE 754 standard appeared defining a single and universal standard.

The aim of this project is implementing a 32 bit binary floating point adder/subtractor according with the IEEE 754 standard and using the hardware programming language VHDL.

CHAPTER 1: INTRODUCTION

Many fields of science, engineering and finance require manipulating real numbers efficiently. Since the first computers appeared, many different ways of approximating real numbers on it have been introduced.

One of them, the floating point arithmetic, is clearly the most efficient way of representing real numbers in computers. Representing an infinite, continuous set (real numbers) with a finite set (machine numbers) is not an easy task: some compromises must be found between speed, accuracy, ease of use and implementation and memory cost.

Floating Point Arithmetic represent a very good compromise for most numerical applications.

1.1. Floating Point Numbers

The floating point numbers representation is based on the scientific notation: the decimal point is not set in a fixed position in the bit sequence, but its position is indicated as a base power.

All the floating point numbers are composed by three components:

- Sign: it indicates the sign of the number (0 positive and 1 negative)
- Mantissa: it sets the value of the number

- Exponent: it contains the value of the base power (biased)
- Base: the base (or radix) is implied and it is common to all the numbers (2 for binary numbers)

The free using of this format caused either designed their own floating point system. For example, Konrad Zuse did the first modern implementation of a floating point arithmetic in a computer he had built (the Z3) using a radix-2 number system with 14-bit significant, 7-bit exponents and 1-bit sign. On the other hand the PDP-10 or the Burroughs 570 used a radix-8 and the IBM 360 had radix-16 floating point arithmetic.

This led to the need for a standard which would make a clear and concise format to be used by all the developers.

1.2. The Standard IEEE 754

The first question that comes to mind is "What's IEEE?". The Institute of Electrical and Electronics Engineers (IEEE) is a non-profit professional association dedicated to advancing technological innovations and excellence.

It was founded in 1884 as the AIEE (*American Institute of Electrical Engineers*). The IEEE was formed in 1963 when AIEE merged with IRE (*Institute of Radio Engineers*).

One of its many functions is leading standards development organization for the development of industrial standards in a broad range of disciplines as telecommunications, consumer electronics or nanotechnology.

IEEE 754 is one of these standards.

1.2.1. Overview

Standard IEEE 754 specifies formats and methods in order to operate with floating point arithmetic.

These methods for computational with floating point numbers will yield the same result regardless the processing is done in hardware, software or a combination for the two or the implementation.

The standard specifies:

- Formats for binary and decimal floating point data for computation and data interchange
- Different operations as addition, subtraction, multiplication and other operations
- Conversion between integer-floating point formats and the other way around
- Different properties to be satisfied when rounding numbers during arithmetic and conversions
- Floating point exceptions and their handling (NaN, ±∞ or zero)

IEEE 754 specifies four different formats to representing the floating point values:

- Simple Precision (32 bits)
- Double precision (64 bits)
- Simple Extended Precision (≥43 bits but not too used)
- Double Extended Precision (≥79 bits, usually represented by 80)

1.2.2. Binary Interchange Format Encodings

Representations of floating point data in the binary interchange formats are encoded in k bits in the following three fields ordered as shown in Figure 1:

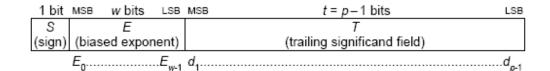


Figure 1. Floating Point format

If a Simple Precision format is used the bits will be divided in that way:

- The first bit (31st bit) is set the sign (S) of the number (0 positive and 1 negative)
- Next w bits (from 30th to 23rd bit) represents the exponent (E)
- The rest of the string, t, (from 22^{nd} to 0) is reserved to save the mantissa

The range of the enconding biased exponent is divided in three sections:

- Every integer between 1 and 2^w -2 (being w=8 \rightarrow 254₍₁₀₎ in order to encode the normal numbers
- The value 0 which encodes subnormal numbers and the zero value
- The reserved value 2^w-1 (being $w=8 \rightarrow 255_{(10)}$) to encode some special cases as NaN or $\pm \infty$

The exponent value has a bias of 127. It means the exponent value will be between -126 (00000000 $_{(2)}$) and +127 (111111110 $_{(2)}$) being zero at the value (011111111 $_{(2)}$).

Exponent and mantissa values determine the different number r cases that it can be had.

- If $E = 2^w 1$ and $T \neq 0$, then r is NaN regardless of S
- If $E = 2^w 1$ and T = 0, then r is \pm infinity according with the sign bit S
- If $1 \le E \le 2^w 2$, then *r* is a normal number
- If E=0 and $T\neq 0$, then r is a subnormal number
- If E = 0 and T = 0, then r is \pm zero according with S

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The mantissa value is 23 bits long but it contains an implicit bit depending on the type of data (1 for normal numbers and 0 for subnormal).

A number can be represented by different ways. As an example, the number $0.11 \cdot 2^5$ can be described as $110 \cdot 2^2$ or $0.011 \cdot 2^6$.

It is desirable to require unique representations. In order to reach this goal the finite non-zero floating point numbers may be normalized by choosing the representation for which the exponent is minimum.

To cope with this problem the standard provides a solution. The numbers will be standardized in two ways:

- Subnormal numbers will start with a zero an it has a form like $\pm 0.XX \cdot 2^0$
- Normal numbers MSB will be high $(\pm 1.XX \cdot 2^E)$ where 0 < E < 255

Both normal and subnormal numbers MSB will be implied but taken into account in order to get the proper value in decimal.

To calculate the value of the binary bit sequence in decimal this formula will be used:

$$M = \sum_{k=0}^{22} m_{22-k} \cdot 2^{-(1+k)} \tag{1}$$

Finally the different format parameters for simple and double precision are shown in *table 1*:

 Table 1. Binary interchange format parameters

Parameter	binary32	binary64
k, storage width in bits	32	64
p, precision in bits	24	53
emax, maximum exponent e	127	1023
Encoding parameters		
bias, E-e	127	1023
sign bit	1)	1
w, exponent field width in bits	8	11
t, trailing significand field width in bits	23	52
k, storage width in bits	32	64

1.2.3. Precision and Rounding

The number of values which can be represented by floating point arithmetic is finite because it has a finite number of bits.

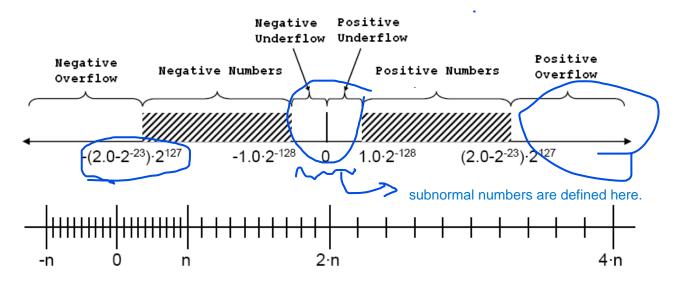


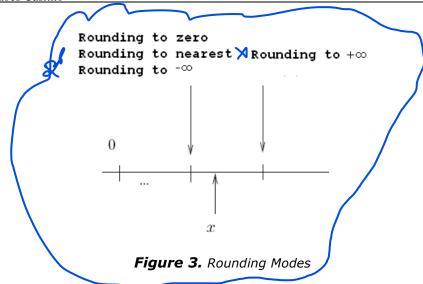
Figure 2. Floating Point values range

As it can be seen in the *figure 2*, the standardized numbers range is described as the values between the higher exponent and mantissa value and the lower ones. The <u>subnormal numbers</u> are between zero and the lowest number (positive or negative) which could be represented by normal numbers. However, these ranges are discontinuous because between two numbers there are also infinite real ones. The quantities of numbers, which can be represented, are the same than in fixed point but at the expense of increasing the distance between numbers a higher range is achieved.

The standard IEEE 754 requires that the operation result must be the same which would obtain if a calculation with absolute precision and rounded had been done.

Four types of rounding are described by the standard:

- Rounding to the nearest (to even number in case of tie) i the floating point number that is the closest to x.
- Rounding to $+\infty$ if the smallest floating point number (possibly $+\infty$) greater than or equal to x.
- Rounding to $-\infty$ is the largest floating point number (possibly $-\infty$) less than or equal to x.
- Rounding to zero is the closest floating point number to x that is no greater in magnitude than \underline{x} (it is equal to rounding to $-\infty$ if $x \ge 0$ and to $+\infty$ if $x \le 0$



The finite number of representing values and the rounding cause the appearance of errors in the result. This topic should be discussed when the results will be analyzed.

CHAPTER 2: CODE DEVELOPMENT

Once the standard IEEE 754 has been explained it is time to start with the implementation of the code. First of all thinking about the different steps we should do to perform the operation required is compulsory. It is because of this that this section will talk about the procedure in addition/subtraction operations and a first look at the code design in block diagram way.

A complete description will be done first and the subblocks will be explained immediately afterwards at successive subsections.

2.1. 32-bits Floating Point Adder Design

The main goal of this project is the implementation of a 32-bit Floating Point Adder with VHDL code. The format and the main features of the standard have been described before but nothing about the steps to achieve the target has been said.

The first logical step is trying to specify what operations should be done to obtain a proper addition or subtraction. Once the idea will be clear the block diagram of the entire code will be designed.

2.1.1. Addition/Subtraction Steps

Following the established plan, the way to do the operations (addition/subtraction) will be set.

This point will be also used to try to explain why these steps are necessary in order to make clearer and easier the explanation of the code in the next section.

The different steps are as follows:

1. Extracting signs, exponents and mantissas of both A and B numbers. As it has been said, the numbers format is as follows:

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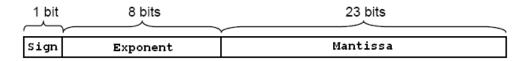


Figure 4. Floating Point Number format

Then the first step is finding these values.

- 2. Treating the special cases:
 - Operations with A or B equal to zero
 - Operations with ±∞
 - · Operations with NaN
- 3. Finding out what type of numbers are given:
 - Normal
 - Subnormal
 - Mixed
- 4. Shifting the lower exponent number mantissa to the right [Exp1-Exp2] bits. Setting the output exponent as the highest exponent.

A's Exponent \rightarrow 3 B's Exponent \rightarrow -1 Difference (A-B) \rightarrow 4 Number B:

$$1101001 \rightarrow 00001101001$$

5. Working with the operation symbol and both signs to calculate the output sign and determine the operation to do.

Table 1. Sign Operation

A's Sign	Symbol	B's Sign	Operation
+	+	+	+
+	+	-	-
+	-	+	-
+	-	-	+
-	+	+	ı
-	+	-	+
_	-	+	+
_	-	-	_

6. Addition/Subtraction of the numbers and detection of mantissa overflow (carry bit)

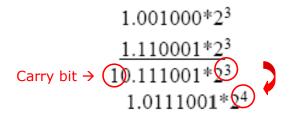


Figure 5. Example

7. Standardizing mantissa shifting it to the left up the first one will be at the first position and updating the value of the exponent according with the carry bit and the shifting over the mantissa.

$$0.1010101 \cdot 2^3 \rightarrow 1.010101 \cdot 2^2$$

8. Detecting exponent overflow or underflow (result NaN or $\pm \infty$)

This is the way forward to proper operation. Obviously there are some parts which have to be discussed because there will be more aspects to be taken into account but this will happen in next sections where the code will be explained.

The main idea has been described before. Once the different steps to follow have been explained it is time to start to think in the code implementation.

In this subsection a first block diagram –as a draft- will be made. It still does not go into the most difficult points because in the next section, once a division of the project in three parts will be done, a complete description of each step will be performed.

These three parts are as follows:

- Pre-Adder Block
- Adder Block
- Standardizing Block

They make reference to the three main processes of the project. First the numbers should be treated (*pre-adder*) in order to perform the operation properly (*adder*) and finally, standardizing the result according with the standard IEEE 754 (*standardizing*).

In *figure 6*, a first approximation of the design has been done:

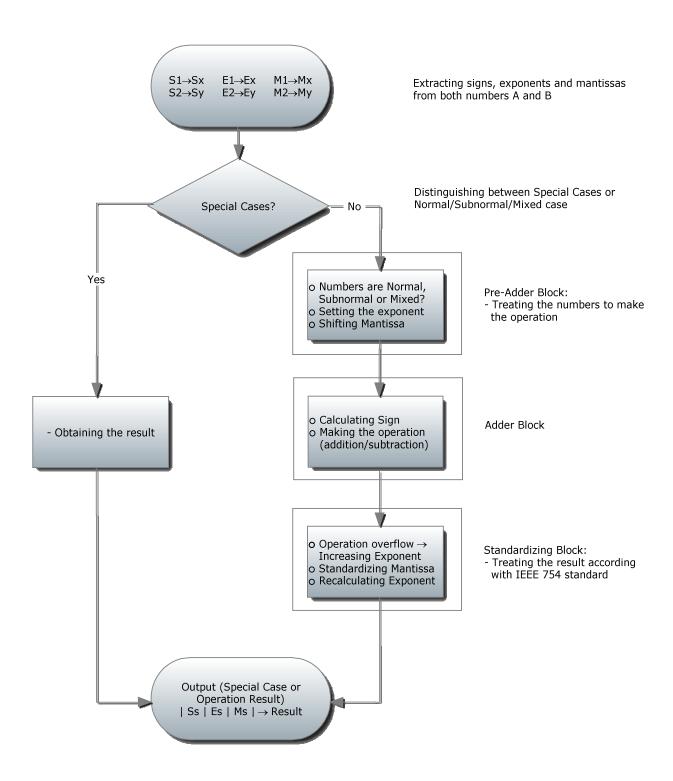


Figure 6. Block Diagram Code

2.2. Blocks Design

In this section the main blocks described in the previous block diagram will be explained.

The diagram has two branches:

- The special cases one is quiet simple because only the combination of the different exceptions are taken into account. This will be explained in the next chapter over the code directly
- The second one is more interesting. It includes the main operation of the adder. The different operations that should be done are divided in three big blocks: pre-adder, adder and normalizing block.

During the next subsections a first description of each block will be done. A block diagram will be designed to support the explanation and facilitate the comprehension. Moreover it will be used to design the different blocks in VHDL which form the 32-bit Floating Point Adder.

2.2.1. Pre-Adder Design

The first subblock is the Pre-Adder. The goals are:

- 1. Distinguishing between normal, subnormal or mixed (normal-subnormal combination) numbers.
- 2. Treating the numbers in order to be added (or subtracted) in the adder block.
 - Setting the Output's exponent
 - Shifting the mantissa
 - Standardizing the subnormal number in mixed numbers case to be treated as a normal case

The block diagram which display this behaviour is shown (figure 7) in the next page.

2.2.2. Adder Design

Adder is the easiest part of the blocks. This block only implements the operation (addition or subtraction). It can be said the adder block is the ALU (*Arithmetic Logic Unit*) of the project because it is in charge of the arithmetic operations.

Two functions are implemented in this part of the code:

- 1. Obtaining the output's sign
- 2. Implementing the desired operation

In this block two related problems should be taken into account. Firstly, the calculation symbol (+ or -) depends on itself and the A and B's signs. Secondly, positive or negative numbers addition gives the same result. The problem will appear when the signs are different. In these cases the positive number will be kept in the first operand and the negative one in the second operand. All these problems will be explained in detail in next sections.

As it is normal the easiest block has the easiest block diagram (figure 8).

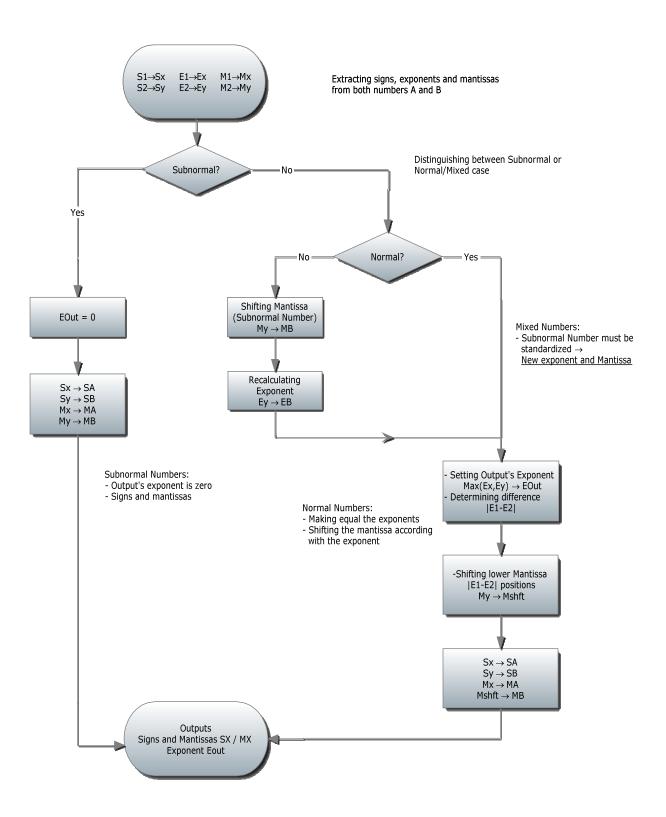


Figure 7. Pre-Adder Block Diagram

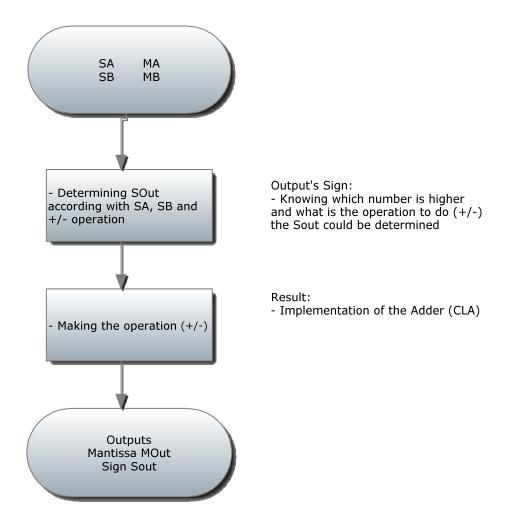


Figure 8. Adder Block Diagram

2.2.3. Standardizing Design

Finally the Standardizing Block takes the result of the addition/subtraction and gives it an IEEE 754 format.

The procedure is as follows:

- 1. Shifting the mantissa to standardize the result
- 2. Calculating the new exponent according with the addition/subtraction overflow (carry out bit) and the displacement of the mantissa.

The exponent value must be controlled when these steps are going to be made because it could be the number of positions the mantissa must be shifted are higher than the exponent value. In this case the result becomes subnormal. Another exception is when exponent and number of displacements are equal: mantissa will be shifted and exponent will be one.

As the previous subsections a block diagram with this description has been made. It can be seen in the *figure 8* where the different steps to standardize the value are shown.

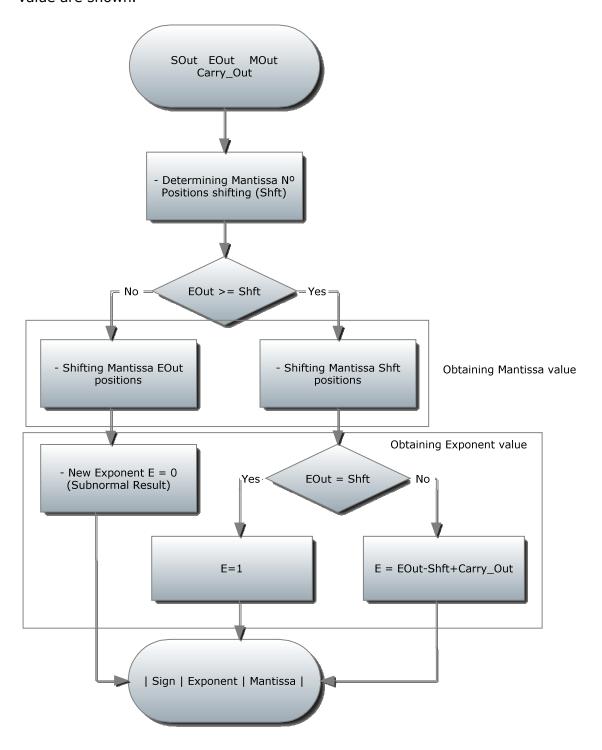


Figure 9. Standardizing Block Diagram

CHAPTER 3: PRE-ADDER

The first block is the Pre-adder. It is in the charge of distinguishing the type of numbers which are introduced as an input.

Four different cases are possible:

- 1. One of the different combinations which have been explained and labeled as special cases: NaN-Infinity, Infinity-Normal, Zero-Subnormal, etc.
- 2. A two subnormal numbers introduction.
- 3. A mixed option between normal and subnormal numbers.
- 4. A two normal numbers introduction

All this cases must be treated separately because of the process to achieve a successful operation must be different.

3.1. Special Cases

The adder is not always necessary to operate the numbers: there are some special cases which can be solved without it.

As it has been said, in addition to normal and subnormal numbers, infinity, NaN and zero are represented in IEEE 754 standard. Some possible combinations have a direct result, for example, if a zero and a normal number are introduced the output will be the normal number directly. Time and resources are saved implementing this block. The n_case block has been designed to run this behaviour.

3.1.1. n_case Block

Both number A and number B are introduced as inputs. Vector S is one of the outputs and it contains the result when there is a special case, otherwise

undefined. Finally, *enable* signal enables or disables the *adder* block if it is needed or not.

```
Number A (31:0)

S (31:0)

S (31:0)

S (31:0)

Port ( Number A : in std_logic_vector(31 downto 0); -- Number A Number B : in std_logic_vector(31 downto 0); -- Number B enable : out std_logic; -- Enable Adder S : out std_logic_vector(31 downto 0)); -- Output
```

Firstly the possible number values are coded (zero, infinity, NaN, normal and subnormal numbers) in two signals *outA* and *outB* according to the mantissa and exponent value as it can be seen in table 2.

```
outA <= "000" when EA = X"00" and MA = 0 else
                                                                       Zero
         "001" when EA = X"00" and MA > 0 else
                                                                        Subnormal
         "011" when (EA > X"00" and EA < X"FF") and MA > 0 else
                                                                       Normal
         "100" when EA = X"FF" and MA = 0 else
                                                                        Infinity
                                                   EA= Exp
         "110" when EA = X"FF" and MA > 0 else
                                                                       NaN
                                                   MA = Trailing
         "000";
outB <= "000" when EB = X"00" and MB = 0 else
                                                                    -- Zero
         "001" when EB = X"00" and MB > 0 else
                                                                     -- Subnormal
         "011" when (EB > X"00" and EB < X"FF") and MB > 0 else
                                                                    -- Normal
         "100" when EB = X"FF" and MB = 0 else
                                                                       Infinity
         "110" when EB = X"FF" and MB > 0 else
                                                                        NaN
         "000";
```

Table 2. Data coded

Exponent	Mantissa	Output	Output Coded
= 0	= 0	Zero	000
= 0	> 0	Subnormal	001
0 <e<255< td=""><td>> 0</td><td>Normal</td><td>011</td></e<255<>	> 0	Normal	011
= 255	= 0	Infinity	100
= 255	> 0	NaN	110

Once both A and B numbers have been coded the different signals combinations are taken into account.

Sign, mantissa and exponent are calculated depending on *outA* and *outB* values. For example, if *outA* is a zero and *outB* is a normal number, the result is the normal number coded in *outB*.

All the possible values are shown in table 3 and also in the VHDL code added.

```
process (SA, SB, outA, outB)
 begin
                 ----- Zero
   if (outA = "000") then
                                                 -- Zero +/- Number B
    SS <= SB;
    ES <= EB;
    MS <= MB;
   elsif (outB = "000") then
                                                 -- Number A +/- Zero
    SS <= SA;
    ES <= EA;
    MS <= MA;
   end if:
    ----- Infinite
   if (outA(0) = '1' and outB = "100") then
                                                 -- Normal or Subnormal +/- Infinity
    SS <= SB:
    ES <= EB;
    MS <= MB:
   elsif (outB(0) = '1' and outA = "100") then
                                                 -- Infinity +/- Normal or Subnormal
    SS <= SA;
    ES <= EA:
    MS <= MA;
   end if:
   if ((outA and outB) = "100" and SA = SB) then
                                                 -- +/- Infinity +/- Infinity
    SS <= SA;
ES <= EA;
    MS <= MA;
    ----- Nan
   elsif ((outA and outB) = "100" and SA \neq SB) then -- + Infinity - Infinity
    SS <= '1';
     ES <= X"FF";
    if (outA = "110" or outB = "110") then
    SS <= '1';
     ES <= X"FF";
     MS <= "0000000000000000000000000001";
   end if;
          ----- Normal / Subnormal
   if((outA(0) and outB(0)) = '1') then
     SS <= '-';
     ES <= "----";
     MS <= "----";
   end if;
end process;
```

Table 3. Output coded

Sign	Out A	Out B	Sign Output	Output
Х	Zero	Number B	SB	Number B
Х	Number A	Zero	SA	Number A
Х	Normal / Subnormal	Infinity	SB	Infinity
Х	Infinity	Normal / Subnormal	SA	Infinity
SA=SB	Infinity	Infinity	SX	Infinity
SA≠SB	Infinity	Infinity	1	NaN
Х	NaN	Number B	1	NaN
Х	Number A	NaN	1	NaN

X: do not care SA: Number A's sign SB: Number B's sign SX: Sign A or B (it is the same)

Finally an enable signal has been made. If any normal or subnormal combination is had the enable signal is high, otherwise low.

```
-- If A and B are normal or subnormal numbers, enable = 1
-- If not, enable = 0
enable <= '1' when ((outA(0) and outB(0)) = '1') else '0';</pre>
```

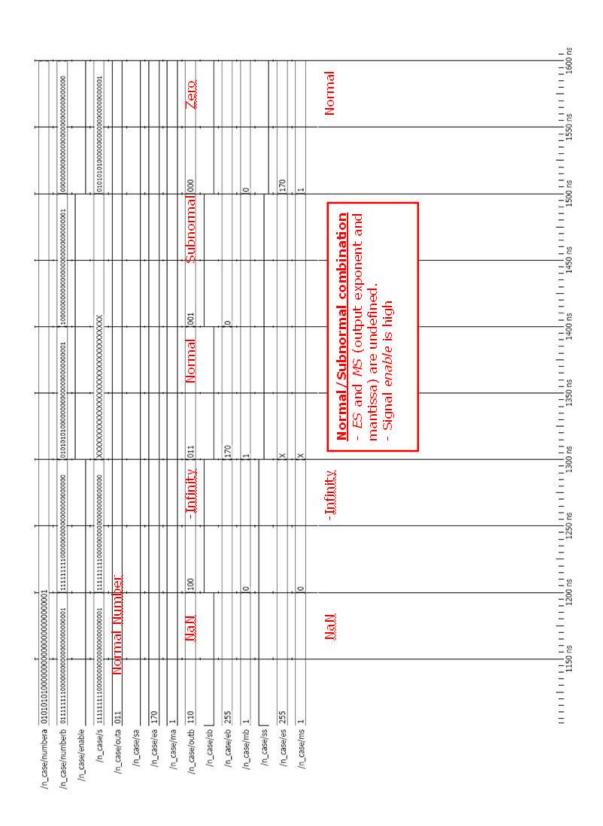


Figure 10. n_case Simulation

3.2. Subnormal Numbers

The operation using subnormal numbers is the easiest one.

It is designed in just one block and the procedure is as follows:

- 1. Obtaining the two sign bits and both mantissas
- 2. Making a comparison between both A and B numbers in order to acquire the largest number
- 3. Fixing the result exponent in zero

```
3.2.1. n_subn Block
```

Obviously Number A and B are the entries. The outputs are six. *SA-MA* and *SB-MB* contain the sign and mantissa of A and B respectively. *Comp* signal is referred to the result comparison and *EO* is the result exponent.

The code is so simply. Sign and mantissa of both numbers are obtained directly from the entries *NumberA* and *NumberB*. The outputs exponent *EO* is always zero because the input exponents are zero as well and *Comp* signal is high when A is bigger than B and low in the opposite case.

The comparison operation does not take into account the sign of the numbers. If the result is negative or positive it will be calculated in the Adder block using *SA*, *SB* and *Comp* signals.

```
SA <= NumberA(36);
                                 -- Sign A & B
SB <= NumberB(36);
MAa <= NumberA(27 downto 0);
                                  -- Mantissa A & B
MBb <= NumberB(27 downto 0);</pre>
        ----- Number Comparison
   <= '1' when MAa >= MBb else -- A > B
        '0' when MBb > MAa else
                                -- B > A
        T- T;
Comp <= C;
                            ----- Output's exponent
EO <= NumberA(35 downto 28);
MB <= MBb when C = '1' else
    MAa when C = '0' else
MA <= MAa when C = '1' else
    MBb when C = '0' else
```

Arturo Barrabés Castillo

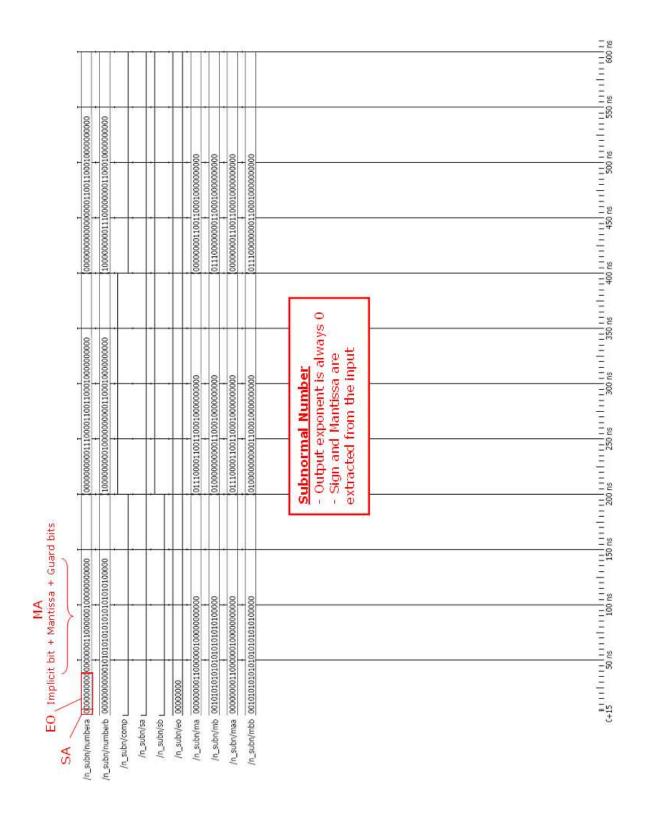


Figure 11. n_subn Entity

3.3. Mixed Numbers

When there is a mixed combination of numbers (one subnormal and other normal) the subnormal one must have a special treatment in order to be added or subtracted to the normal one.

The subnormal number treatment is going to be discuss in this block because once both numbers will be standardized the next block (normal numbers block) will be in charge of the operation between normal ones.

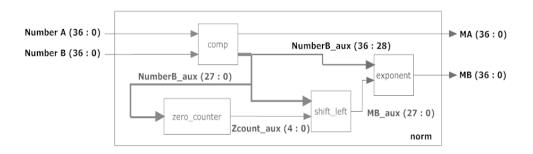


Figure 12. Mixed numbers block diagram

The work operation can be summarized in the following points:

- 1. Finding out what the subnormal number is
- 2. Counting the number of zeros the subnormal number has on the beginning
- 3. Shifting the vector and calculating the new exponent

This block is formed by three entities and each one is responsible for one of the points described.

3.3.1. comp Block

First block is *comp* entity. The block entries are both numbers and the outputs are the same numbers ordered as normal *NA* and subnormal *NB*.

```
Number A (36:0)

NA (36:0)

NA (36:0)

NA (36:0)

NA (36:0)

NA : out std_logic_vector(36 downto 0); -- Number B

NB (36:0)

NB : out std_logic_vector(36 downto 0); -- Normal number

NB : out std_logic_vector(36 downto 0); -- Subnormal number

end comp;
```

The code is not very extensive. A and B Mantissas are ordered according to the exponent: null exponent indicates what the subnormal number is and then this number is fixed in NB, leaving the normal one in NA.

```
EA <= NumberA(35 downto 28); -- Exponent & Mantissa
EB <= NumberB(35 downto 28);</pre>
process (NumberA, NumberB, EA, EB)
 begin
  if EA = X"00" then
                   -- If Number A is subnormal...
   NB <= NumberA;
   NA <= NumberB;
  NB <= NumberB;
   NA <= NumberA;
  else
   NA <= "----";
   NB <= "----";
  end if:
 end process;
```

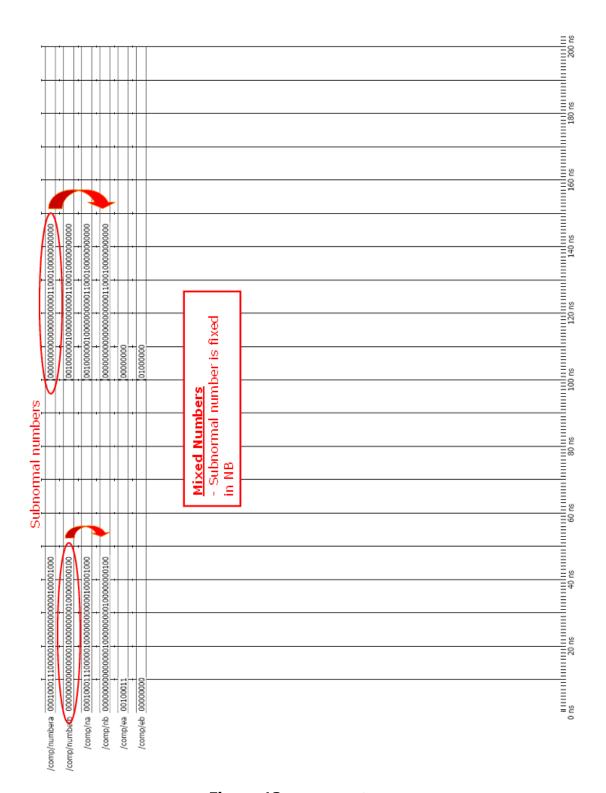


Figure 13. comp Simulation

3.3.2. zero Block

Counting zeros is the *zero* block target. The mantissa which is needed to shift is introduced as an entry in *T* vector and the output *Zcount* contains the number of zeros the mantissa has on the beginning which corresponds with the number of positions the vector must be shifted.

```
entity zero is

Port ( T : in std_logic_vector(27 downto 0); -- Significand

Zcount : out std_logic_vector(4 downto 0)); -- Number of Zeros end zero;
```

A zero vector is created ($Zero_vector$) and compared with the T vector. The Zcount value depends on the number of matches.

```
<= "----" when T(27 downto 27) = "-" else
aux
           X"10" when T(27 downto 0) = Zero vector(27 downto 0) else
           X"1B" when T(27 downto 1) = Zero vector(27 downto 1) else
           X"1A" when T(27 \text{ downto } 2) = Zero \text{ vector}(27 \text{ downto } 2) else
           X"19" when T(27 \text{ downto } 3) = Zero vector(27 \text{ downto } 3) else
           X"18" when T(27 \text{ downto } 4) = Zero \text{ vector}(27 \text{ downto } 4) else
           X"17" when T(27 \text{ downto } 5) = Zero \text{ vector}(27 \text{ downto } 5) else
           X"16" when T(27 downto 6) = Zero vector(27 downto 6) else
           X"15" when T(27 downto 7) = Zero vector(27 downto 7) else
           X"14" when T(27 downto 8) = Zero vector(27 downto 8) else
           X"13" when T(27 downto 9) = Zero vector(27 downto 9) else
           X"12" when T(27 \text{ downto } 10) = Zero \text{ vector}(27 \text{ downto } 10) \text{ else}
           X"11" when T(27 \text{ downto } 11) = Zero \text{ vector}(27 \text{ downto } 11) \text{ else}
           X"10" when T(27 downto 12) = Zero vector(27 downto 12) else
           X"OF" when T(27 downto 13) = Zero vector(27 downto 13) else
           X"OE" when T(27 downto 14) = Zero_vector(27 downto 14) else
           X"OD" when T(27 downto 15) = Zero_vector(27 downto 15) else
           X"OC" when T(27 downto 16) = Zero_vector(27 downto 16) else
           X"0B" when T(27 downto 17) = Zero vector(27 downto 17) else
           X"0A" when T(27 \text{ downto } 18) = Zero \text{ vector}(27 \text{ downto } 18) else
           X"09" when T(27 downto 19) = Zero vector(27 downto 19) else
           X"08" when T(27 downto 20) = Zero vector(27 downto 20) else
           X"07" when T(27 downto 21) = Zero vector(27 downto 21) else
           X"06" when T(27 downto 22) = Zero vector(27 downto 22) else
           X"05" when T(27 downto 23) = Zero vector(27 downto 23) else
           X"04" when T(27 \text{ downto } 24) = Zero \text{ vector}(27 \text{ downto } 24) else
           X"03" when T(27 downto 25) = Zero vector(27 downto 25) else
           X"02" when T(27 downto 26) = Zero vector(27 downto 26) else
           X"01" when T(27 downto 27) = Zero vector(27 downto 27) else
           X"00";
Zcount <= aux(4 downto 0);</pre>
```

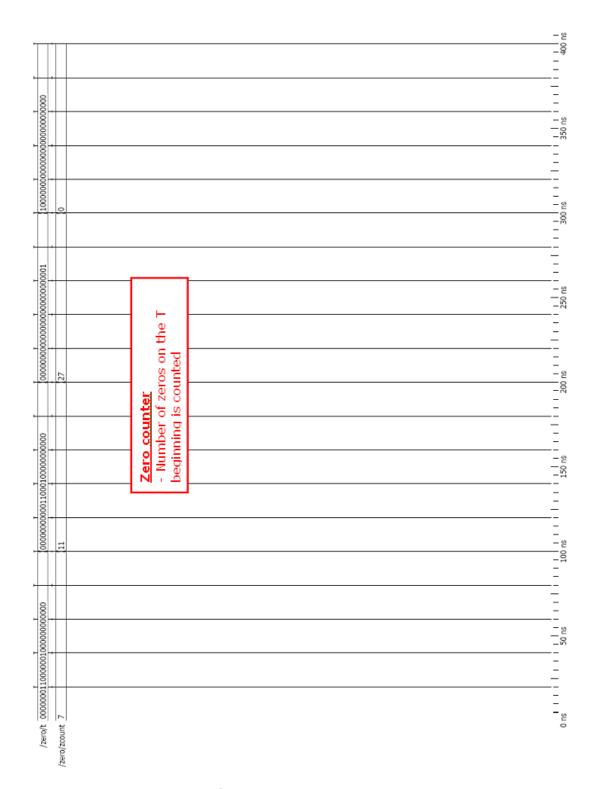


Figure 14. zero Simulation

3.3.3. shift_left/shift Block

Shifting is required to match the normal and mixed mantissas to perform the addition/subtraction properly.

A logarithmic shift schematic as the figure 15 is used but with some differences.

28 bits (1 implicit bit + 23 mantissa's bits + 4 guard bits) is had in the Floating point Adder design then up to 28 positions must be able to shift. Because of the fact that this shifter consists of 5 stages: the first stage shift one position, the second stage 2, the third one 4, the fourth one 8 and the last one 16. Using any combination 32 positions are able to shift which is big enough to the design purpose.

Both shifting left and shifting right are used in the Floating Point Adder implementation. In this chapter, the first one is explained but the code is quite similar to the second one. There is only a difference: the T vector order. If the bits order is changed from 0-27 to 27-0 a shifting right is achieved.

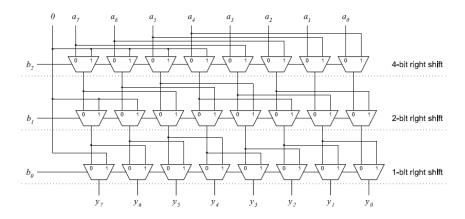


Figure 15. Logarithmic shift

The T vector and the number of positions to shift (Shft) are the entries of the shift entity. The shifted signal is set in S.

```
T (27:0) Shift Shift is Port ( T : in std_logic_vector(27 downto 0); -- Significand to shift is shift is shift is shift is sin std_logic_vector(4 downto 0); -- Exponent's subtraction s : out std_logic_vector(27 downto 0)); -- Output
```

The code is implemented as follows. A multiplexor has been designed and exported to this block. Afterwards a loop *for* has been used to generate the different 5 stages. Following the cascade design which has been shown before a 32 positions logarithmic shifter is implemented.

```
signal Z1, Z2, Z3, Z4, Z5 : std logic vector(27 downto 0);
begin
-- Components generation
Comp1: for i in 0 to 27 generate
  shifter0 0: if (i=0) generate
             shifter0 i: if ((i>0) and (i<28)) generate
             shifter0\_icomp: \ MUX \ port \ map \ (A \Rightarrow T((i-1)), \ B \Rightarrow T(i), \ Sel \Rightarrow Shft(0), \ Z \Rightarrow Zl(i));
             end generate;
  shifter1 0: if ((i>=0) and (i<2)) generate
             shifter1 \ Ocomp: MUX \ port \ map \ (A => '0', B => Z1(i), Sel => Shft(1), Z => Z2(i));
             end generate;
  shifter1_i: if ((i>1) and (i<28)) generate
             end generate;
  shifter2_0: if ((i>=0) and (i<4)) generate
             shifter2\_0comp: \ MUX \ port \ map \ (A \Rightarrow \ '0', \ B \Rightarrow \ Z2(i), \ Sel \Rightarrow \ Shft(2), \ Z \Rightarrow \ Z3(i));
             end generate;
  shifter2_i: if ((i>3) and (i<28)) generate
             shifter2 icomp: MUX port map (A => Z2((i-4)), B => Z2(i), Sel => Shft(2), Z => Z3(i));
             end generate;
  shifter3_0: if ((i>=0) and (i<8)) generate
             shifter3_0comp: MUX port map (A => '0', B => Z3(i), Sel => Shft(3), Z => Z4(i));
             end generate;
  shifter3 i: if ((i>7) and (i<28)) generate
             shifter3\_icomp: MUX port map (A => Z3((i-8)), B => Z3(i), Sel => Shft(3), Z => Z4(i));
             end generate;
 shifter4 0: if ((i>=0) and (i<16)) generate
             shifter4\_0comp: MUX port map (A => '0', B => Z4(i), Sel => Shft(4), Z => Z5(i));
             end generate:
  shifter4_i: if ((i>15) and (i<28)) generate
             shifter4\_icomp: MUX\_port\_map (A => Z4((i-16)), B => Z4(i), Sel => Shft(4), Z => Z5(i));
end generate;
S <= Z5;
```

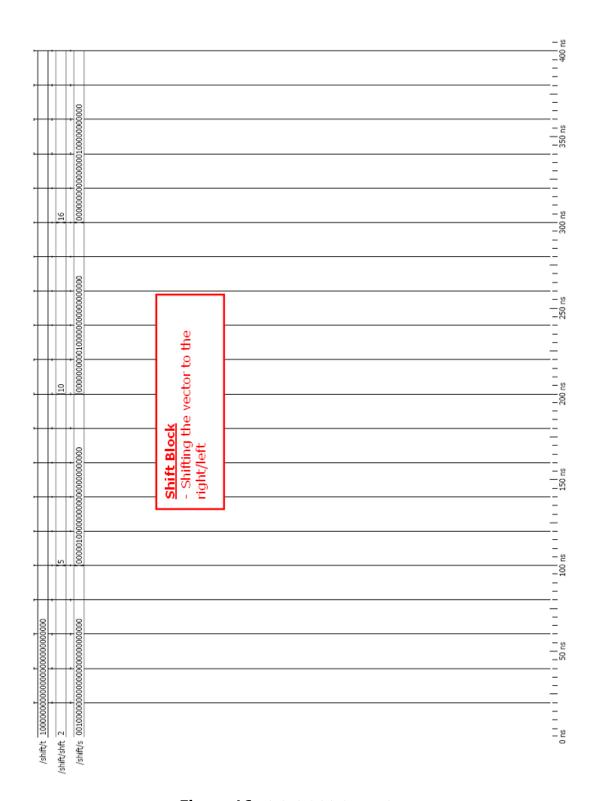
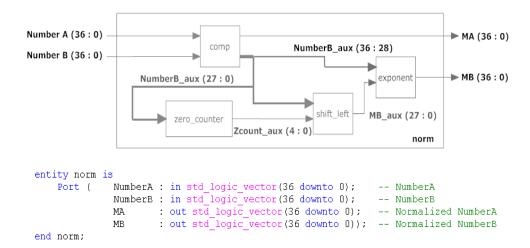


Figure 16. shift_left/shift Simulation

3.3.4. norm Block

Finally, the rest of the entities are all included in the *norm* block. It also performs the output exponent treatment.

The inputs are the numbers A and B. Once the subnormal one has been shifted it is fixed in MB. The normal number is set in MA.



The code could be divided in two parts. The first one implements the connection between the different blocks which the mixed numbers entity works with. The block diagram is coherent with the VHDL code.

```
comp0 : zero
   port map (T => NumberB_aux(27 downto 0), Zcount => Zcount_aux);

comp1 : shift_left
   port map (T => NumberB_aux(27 downto 0), shft => Zcount_aux, S => MB_aux);

comp2 : comp
   port map (NumberA => NumberA, NumberB => NumberB, NA => MA, NB => NumberB aux);
```

The second one is pretty interesting. As it has been explained before, negative prebiased exponents are not considered by the standard IEEE 754 but there is a possibility a normal and subnormal number may be operated. The number of positions the vector *MB* is shifted could be saved as a positive exponent but introducing a mark in the last guard bit which indicates the positive exponent is actually "negative".

So if a normal number with a quite small exponent is had it is possible that normal and subnormal numbers are able to be operated.

```
process(Zcount_aux, NumberB_aux, EB, MB_aux)
begin
  if Zcount_aux /= "-----" then
    EB <= "000" & Zcount_aux; -- Number shifted
    MB(27 downto 0) <= MB_aux(27 downto 1) & '1'; -- Bit 0 --> Mark
  else
    EB <= "------";
    MB(27 downto 0) <= MB_aux;
end if;
    MB(35 downto 28) <= EB;
    MB(36) <= NumberB_aux(36);
end process;</pre>
```

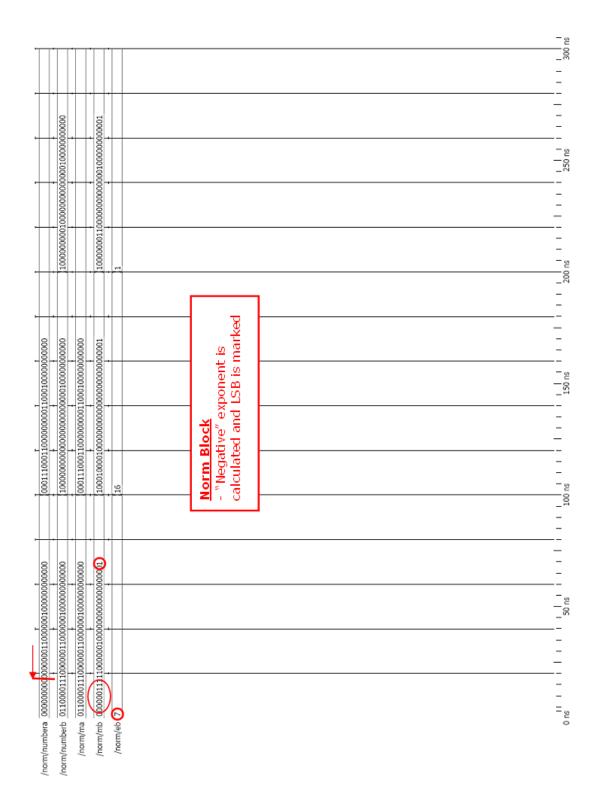


Figure 17. norm Simulation

3.4. Normal Numbers

Two normal numbers are the most common operation mode because it represents the main operation without any exception.

The procedure is as follows:

- 1. Making a comparison between both A and B numbers and obtaining the largest number
- 2. Obtaining the output exponent (the largest one)
- 3. Shifting the smallest mantissa to equal both exponents

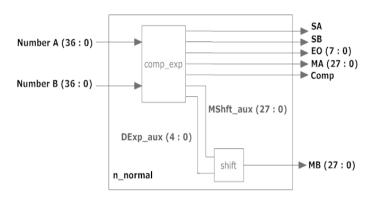


Figure 18. Normal numbers block diagram

3.4.1. comp exp Block

The *comp_exp* entries are the two introduced numbers again. There are several outputs: *SA* and *SB* are the sign of A and B respectively, *EMax* is the output exponent, *MMax* the largest mantissa, *Mshft* the mantissa to shift, *Dexp* the number of positions *Mshft* must be shifted and *Comp* indicates what number is the largest one.

```
SA
                                           SB
                                                                                 NumberA :
                                                                                                                                         Number A
Number A (36:0)
                                                                                NumberB : in std
SA : out st
                                                                                                                  (36 downto 0);
                                                                                                                                         Number B
                                             EMax (7:0)
                                                                                                                                         Sign A
Sign B
                                                                                 SA
SB
                                           MMax (27:0)
                            comp_exp
                                           ► MShft (27:0)
                                                                                 Emax
                                                                                            out
                                                                                                                  r(7 downto 0);
                                                                                                                                         Output exponent
                                                                                 Mmax
                                                                                                                  or(27 downto 0);
or(27 downto 0);
                                                                                                                                        Largest Mantissa
Mantissa to shift
Number B (36:0)
                                           DExp (4:0)
                                           Comp
                                                                                 Mshft
                                                                                                                 or(4 downto 0);
                                                                                                                                         Subtraction of exponents
                                                                                 Dexp
                                                                                 Comp
                                                                                                                                         Determine largest number
```

Exponents and signs are obtained from the introduced numbers directly. Once the exponents are fixed in *EA* and *EB* signals, these values are used to determine the largest number: if A is larger than B or number B's LSB (negative exponent mark) is high, *Comp* will be '1', otherwise '0'.

Using this signal the output exponent could be determined.

```
SA \leftarrow NumberA(36);
                                  -- Sign A & B
SB <= NumberB(36);
EA <= NumberA(35 downto 28);
                                  -- Exponent & Mantissa
EB <= NumberB(35 downto 28);
MA <= NumberA(27 downto 0);
MB <= NumberB(27 downto 0);
----- Exponent Comparison
C \leftarrow '1' when (EA > EB) or (MB(0) = '1') else
                                               -- Exponent A > Exponent B
     '0' when EA < EB else
                                               -- Exponent B > Exponent A
     '1' when MA >= MB else
                                               -- EA = EB --> A > B
     '0' when MA < MB else
                                               -- EA = EB --> B > A
     T - T;
Comp <= C;
----- Largest exponent
Emax <= EA when C = '1' else
      EB when C = '0' else
       "-----";
```

Next step is determining the difference between both exponents. Once more time *comp* signal fixes the largest exponent and determines the subtraction order.

If B's LSB is high a negative exponent is had. In this case EA and EB are added.

```
----- Difference between exponents
dif \leftarrow EA-EB when (C = '1') and (MB(0) = '0') else
      EB-EA when C = '0' else
      EA+EB when (C = '1') and (MB(0) = '1') else
      "----";
process (dif)
 begin
  Dexp <= "11100";
                         -- The difference is 28
  else
   Dexp <= "----";
  end if:
 end process;
----- Mantissa
Mshft <= MB when C = '1' else
     MA when C = '0' else
      "_____":
Mmax <= MA when C = '1' else
     MB when C = '0' else
```

The mantissa to shift corresponds with the smallest number (using *comp* again).

Finally a maximum value is set if the difference between exponents is greater than 28 which is the maximum number of bits that the mantissa has.

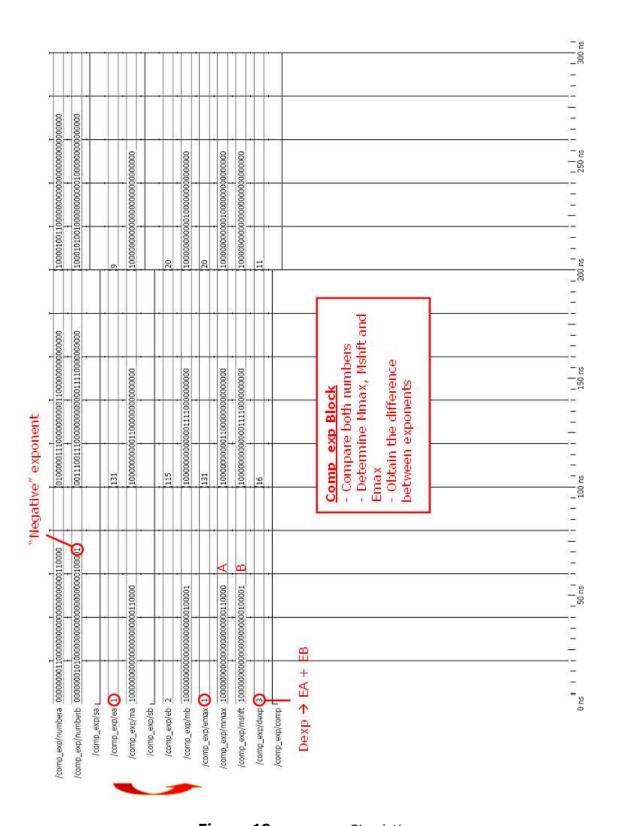


Figure 19. comp_exp Simulation

3.4.2. shift Block

A shifter is needed to match the exponents. The entity is the same than in the mixed case. The vector T is the input which contains the mantissa to shift, shft fix the number of positions to move and S is the output with the result of the operation.

```
Shift (4:0)

Shift (4:0)

Shift (4:0)

Shift (4:0)

Shift (4:0)

Shift (4:0)

Shift : in std_logic_vector(27 downto 0); -- Significand to shift shift : in std_logic_vector(4 downto 0); -- Exponent's subtraction shift:
```

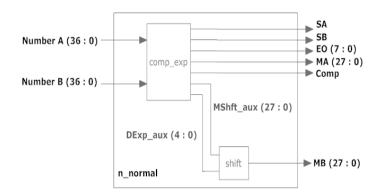
The code is quite similar. Only a part is added because it is enough to see its operation.

```
-- Components generation
Comp1: for i in 0 to 27 generate
  shifter0_0: if (i=0) generate
                shifter0_0comp: MUX port map (A => '0', B => T(27),
                                                                            Sel => Shft(0), Z => Z1(27-i);
                end generate;
  shifter0_i: if ((i>0) and (i<28)) generate
                shifter0_icomp: MUX port map (A =>
                                                        T(27-(i-1)), B => T(27-i), Sel => Shft(0), Z => Z1(27-i));
                end generate;
  shifter1 0: if ((i>=0)) and (i<2)) generate
                shifter1\_0comp: \ MUX \ port \ map \ (A \Rightarrow \ '0', \ B \Rightarrow \ Z1(27-i), \ Sel \Rightarrow \ Shft(1), \ Z \Rightarrow \ Z2(27-i));
                end generate:
  shifter1 i: if ((i>1) and (i<28)) generate
                shifter1 icomp: MUX port map (A \Rightarrow Z1(27-(i-2)), B \Rightarrow Z1(27-i), Sel \Rightarrow Shft(1), Z \Rightarrow Z2(27-i));
                end generate;
```

Changing the order of the vector, a displacement in the other direction is achieved. The simulation is not required because the result is the same but on the right.

3.4.3. n_normal Block

The n_normal block includes the two blocks which have been explained above. The entries are NumberA and NumberB and the outputs are both sign A (SA) and sign B (SB), the result exponent (EO), the Comp signal and the two mantissas (MA and MB).



The VHDL code implements just the interconnection between the different blocks.

Comp_exp fix the mantissa which has to been shifted and the number of positions it must be displaced.

Shift block collects these two signals and gives the mantissa in order to be operated in the next block: the Adder block.

```
component comp_exp port (NumberA, NumberB : in std_logic_vector(36 downto 0);
                                      : out std_logic;
                           SA, SB
                                           cout std_logic_vector(7 downto 0);
cout std_logic_vector(27 downto 0);
cout std_logic_vector(4 downto 0);
cout std_logic);
                           Emax
                           Mmax, Mshft
                           Dexp
                           Comp
end component;
component shift port (T
                            : in std_logic_vector(27 downto 0);
                       shft : in std_logic_vector(4 downto 0);
s : out std_logic_vector(27 downto 0));
end component;
comp0 : comp exp
    port map (NumberA => NumberA, NumberB => NumberB,
               SA => SA, SB => SB, Emax => EO, Mmax => MA, Mshft => Mshft_aux, Dexp => Dexp_aux, Comp => Comp);
comp1 : shift
    port map (T => Mshft_aux, shft => Dexp_aux,
               S => MB);
```

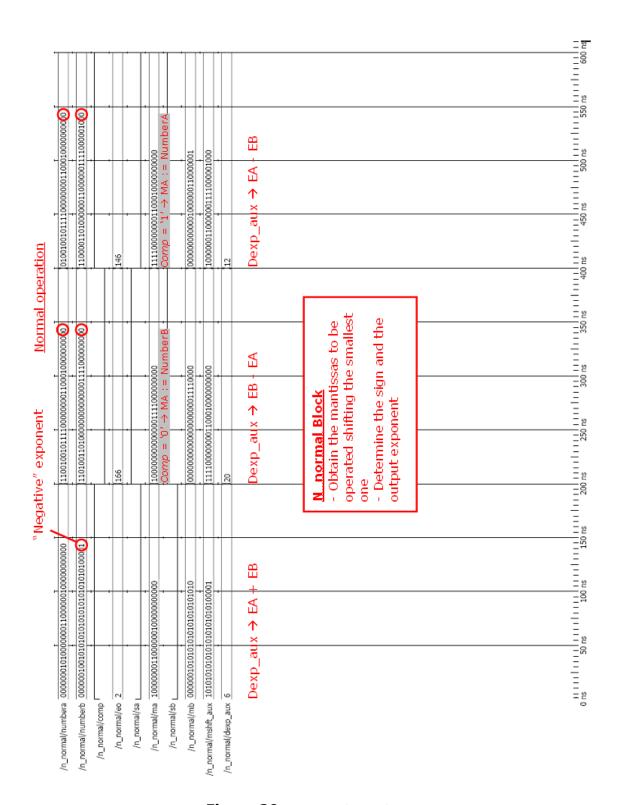


Figure 20. n_normal Simulation

3.5. Pre-Adder

Finally a complete Pre-Adder block diagram will be shown and explained. As it could be seen there are some blocks which are not discussed. These blocks are 4:

- 1. The first one is so important: Selector block
- 2. A demultiplexor (demux) to route the signal in the correspondent block
- 3. A multiplexor (*mux_ns*) to choose between the mixed numbers or the normal ones
- 4. A multiplexor (mux_adder) to choose between the normal or subnormal numbers

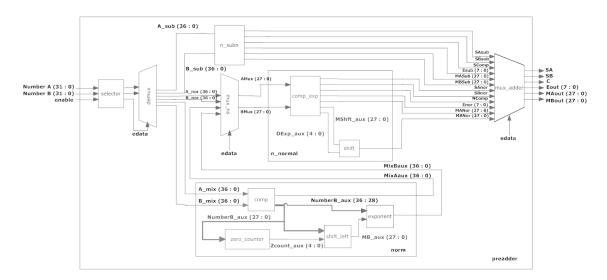


Figure 21. preadder block diagram

These blocks are going to be grouped in two different chapters. First one includes only the *selector* block which is more important and has more complexity.

The second group contains the different multiplexors and demultiplexors. They are going to be treated all together because of code's simplicity.

3.5.1. selector Block

Selector block prepares the numbers: the entries are shorter than outputs because the implicit bit (high if the number is normal and low in subnormal's case) and the guard bits are added in this block. Enable signal enables this block (and therefore the entire preadder block) when we do not have a special case.

The outputs are the two numbers with the added bits and the e_data signal which distinguish between normal, subnormal and mixed numbers.

```
Number A (31:0)
Number B (31:0)
enable

NA (36:0)
NB (36
```

If *enable* signal is high it means we do not have a special case. Then the outputs signals *NA* and *NB* are made: first the sign and exponent bits are placed in its positions.

Next step, the implicit bit is fixed according with the exponent value. The mantissa and the guard bits are added too.

```
SA <= NumberA(31);
SB <= NumberB(31);
EA \le NumberA(30 downto 23):
EB <= NumberB(30 downto 23);
MA <= NumberA(22 downto 0);
MB <= NumberB(22 downto 0);
process (SA, SB, EA, EB, MA, MB, enable)
begin
  if enable = '1' then
    NA(36) \le SA;
                                  -- Exponent & sign A
    NA(35 downto 28) <= EA;
    NB(36) <= SB;
                                  -- Exponent & sign B
    NB(35 downto 28) <= EB;
                                                                   If the exponent is bigger
        <u>-----</u>---- Mantissa A
                                                                   than 0 \rightarrow Normal Number
    if (EA > X"00") then
NA(27) <= '1';
                                                                   Implicit bit → '1'
                               -- Implicit bit
      NA(26 downto 4) <= MA; -- Mantissa
NA(3 downto 0) <= X"0"; -- Guard bits

ightharpoonup If the exponent is 0 
ightharpoonup
    elsif EA = X"00" then ----
      NA(27) <= '0'; -- Implicit bit
NA(26 downto 4) <= MA; -- Mantissa
      NA(27) <= '0';
                                                                   Subnormal Number
                                                                  Implicit bit → '0'
      NA(3 downto 0) <= X"0";
                                -- Guard bits
      NA <= "----";
     end if:
    ----- Mantissa B
   if (EB > X"00") then
     NB(27) <= '1';
                               -- Implicit bit
     NB(26 downto 4) <= MB; -- Mantissa
NB(3 downto 0) <= X"0"; -- Guard bits
   elsif EB = X"00" then
     NB(27) <= '0';
                               -- Implicit bit
     NB(26 downto 4) <= MB;
                               -- Mantissa
     NB(3 downto 0) <= X"0"; -- Guard bits
   else
    NB <=
   end if;
   NB <= "----";
 end if:
end process;
```

Finally the *e_data* signal is fixed as follows:

- 1. Subnormal numbers → e_data := "00"
- 2. Normal numbers → e_data := "01"
- 3. Mixed numbers → e_data := "10"

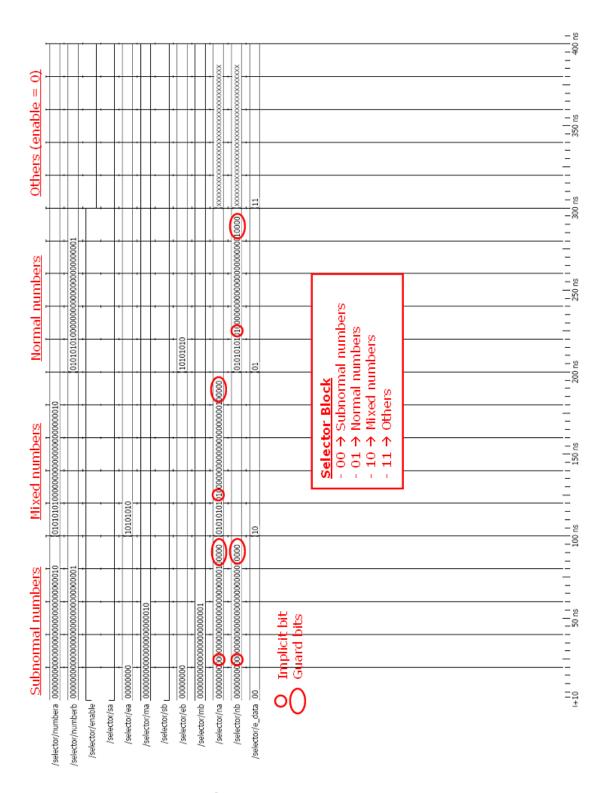


Figure 22. selector Simulation

3.5.2.

MUX/DEMUX Blocks

The operation of the *demux* demultiplexor is routing the A and B numbers to the subnormal, normal or mixed block according with the *e_data* value.

NumberA, NumberB and the enable signal e_data are the entries and the outputs are 3 pairs of signals but only one pair is activated in each time. The typical demultiplexor's behaviour.

```
entity demux is
                               NumberA : in std_logic_vector(36 downto 0);
                          Port (
                                                     -- Number A
                               NumberB : in std_logic_vector(36 downto 0);
e_data : in std_logic_vector(1 downto 0);
                                   : out std_logic_vector(36 downto 0); -- Number A1
NumberB (36:0)
                                  : out std_logic_vector(36 downto 0); -- Number B1
: out std_logic_vector(36 downto 0); -- Number A2
: out std_logic_vector(36 downto 0); -- Number B2
: out std_logic_vector(36 downto 0); -- Number A3
                               NBU
                                  : out std_logic_vector(36 downto 0)); -- Number B3
                               NB2
         e data
                        end demux;
  process (NumberA, NumberB, e data)
  begin
    case e data is
                                 ----- Subnormals
      when "00"
               => NAO <= NumberA;
                  NBO <= NumberB;
                  NA1 <= "----";
                        "----";
                  NB1 <=
                  NA2 <= "----";
                  NB2 <= "----";
                           ---- Normals
      when "01"
               => NAO <= "-----
                  NBO <= "----";
                  NA1 <= NumberA;
                  NB1 <= NumberB;
                  NA2 <= "-----
                  NB2 <= "----":
              ----- MiX
               => NAO <= "-----
      when "10"
                  NBO <= "----":
                  NA1 <= "----";
                  NB1 <= "-----
                  NA2 <= NumberA;
                  NB2 <= NumberB;
      when others => NAO <=
                        "_____
                        "_____
                  NA1 <=
                  NB1 <= "----";
    end case:
```

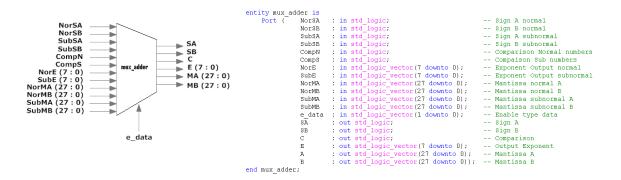
The *mux_ns* multiplexor's target is selecting which signal must be introduced in the normal numbers block: normal numbers or a standardized numbers from the mixed numbers block.

The entries are the two pairs of numbers and e_data signal and the outputs are the A and B numbers according with e_data value.



Finally *mux_adder* multiplexor is in charge of selecting which data are going to be introduced in the adder.

The entries are the comp signal, the two mantissas, signs and exponents and all of them multiplied by two: one for the subnormal numbers and another for the normal/mixed numbers. The output is one of the pair's members according with e_data .

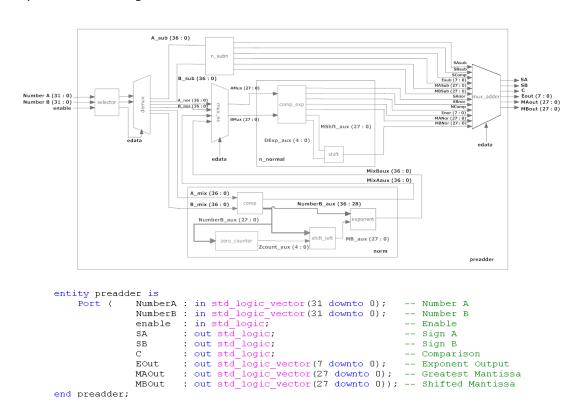


```
A <= NorMA when e data = "01" or e data = "10" else
                                                           -- Normal/Mix numbers
      SubMA when e_data = "00" else
                                                            -- Subnormal numbers
      "----";
B <= NorMB when e data = "01" or e data = "10" else
                                                           -- Normal/Mix numbers
      SubMB when e data = "00" else
                                                            -- Subnormal numbers
 <= CompN when e data = "01" or e data = "10" else</pre>
                                                           -- Normal/Mix numbers
      Comps when e data = "00" else
                                                            -- Subnormal numbers
SA <= NorSA when e data = "01" or e data = "10" else
                                                            -- Normal/Mix sign A
      SubSA when e_data = "00" else
                                                            -- Subnormal sign A
SB <= NorSB when e data = "01" or e data = "10" else
                                                            -- Normal / Mix sign B
      SubSB when e data = "00" else
                                                            -- Subnormal sign B
E <= NorE when e data = "01" or e data = "10" else
                                                           -- Normal / Mix exponent
      SubE when e data = "00" else
                                                            -- Subnormal exponent
```

3.5.3. preadder Block

Finally the *preadder* block is going to be explained. The special cases block is not considered in this block diagram because it will be added next to *adder* block in a complete block diagram.

NumberA, NumberB and enable are the inputs. A and B sign (SA and SB), the C signal, output's exponent Eout, and both MAout and MBout mantissas are the outputs of the design.



The components description is shown in this part of the code. Normal numbers block (n_normal) , subnormal numbers block (n_subn) , mixed numbers block (norm), the multiplexor and demultiplexors $(mux_ns, mux_adder \text{ and } demux)$ and the selector entity are added there.

```
component n normal port (NumberA, NumberB : in std logic vector(36 downto 0);
                          Comp : out std_logic;
                          : out std_logic;
SB : out std_logic;
EO
                         EO : out std_logic_vector(7 downto 0);
MA, MB : out std_logic_vector(27 downto 0));
end component;
end component;
: in std_logic_vector(1 downto 0);
                      e data
                      NAO, NBO, NA1, NB1, NA2, NB2 : out std logic vector(36 downto 0));
end component;
component mux adder port (NorSA, NorSB, SubSA, SubSB : in std logic;
                          CompN, CompS : in std_logic;
NorE, SubE : in std_logic_vector(7 downto 0);
                          NorMA, NorMB, SubMA, SubMB: in std_logic_vector(27 downto 0);
e_data : in std_logic_vector(1 downto 0);
SA, SB, C : out std_logic;
E : out std_logic_vector(7 downto 0);
A, B : out std_logic_vector(27 downto 0);
end component;
                                                ----- Mixed Numbers
component norm port (NumberA, NumberB : in std logic vector(36 downto 0);
                 MA, MB : out std_logic_vector(36 downto 0));
end component:
                                                                         ----- Subnormal Numbers
component n subn port (NumberA, NumberB : in std logic vector(36 downto 0);
                       Comp : out std_logic; SA : out std_logic;
                       SA : out std_logic;
SB : out std_logic;
EO : out std_logic_vector(7 downto 0);
                       MA, MB : out std_logic_vector(27 downto 0));
end component;
component selector port (NumberA, NumberB : in std_logic_vector(31 downto 0);
                       enable : in std_logic;
e_data : out std_logic_vector(1 downto 0);
NA, NB : out std_logic_vector(36 downto 0));
end component;
```

Finally the connection between the different components is described in the second part of the code.

```
comp0 : n_normal
     port map (NumberA => Amux, NumberB => Bmux,
                  \texttt{Comp} \implies \texttt{NComp}, \texttt{ SA} \implies \texttt{SAnor}, \texttt{ SB} \implies \texttt{SBnor}, \texttt{ EO} \implies \texttt{Enor}, \texttt{ MA} \implies \texttt{MAnor}, \texttt{ MB} \implies \texttt{MBnor});
comp1 : n_subn
      port map (NumberA => A_sub, NumberB => B_sub,
Comp => SComp, SA => SAsub, SB => SBsub, EO => Esub, MA => MAsub, MB => MBsub);
comp2 : norm
      port map (NumberA => A_mix, NumberB => B_mix,
                  MA => MixAaux, MB => MixBaux);
comp3 : demux
      comp4 : mux_ns
      port map (NorA => A nor, NorB => B nor, MixA => MixAaux, MixB => MixBaux, e data => edata,
                  NA => Amux, NB => Bmux);
comp5 : selector
      port map (NumberA => NumberA, NumberB => NumberB, enable => enable,
                  e_data => edata, NA => NA_out_select, NB => NB_out_select);
comp6 : mux adder
      port map (NorSA => SAnor, NorSB => SBnor, SubSA => SAsub, SubSB => SBsub, CompN => NComp, CompS => SComp,
                  NorE => Enor, SubE => Esub, NorMA => MANOr, NorMB => MBNOr, SubMA => MASub, SubMB => MBsub, e_data => edata, SA => SA, SB => SB, C => C, E => Eout, A => MAOut, B => MBout);
```

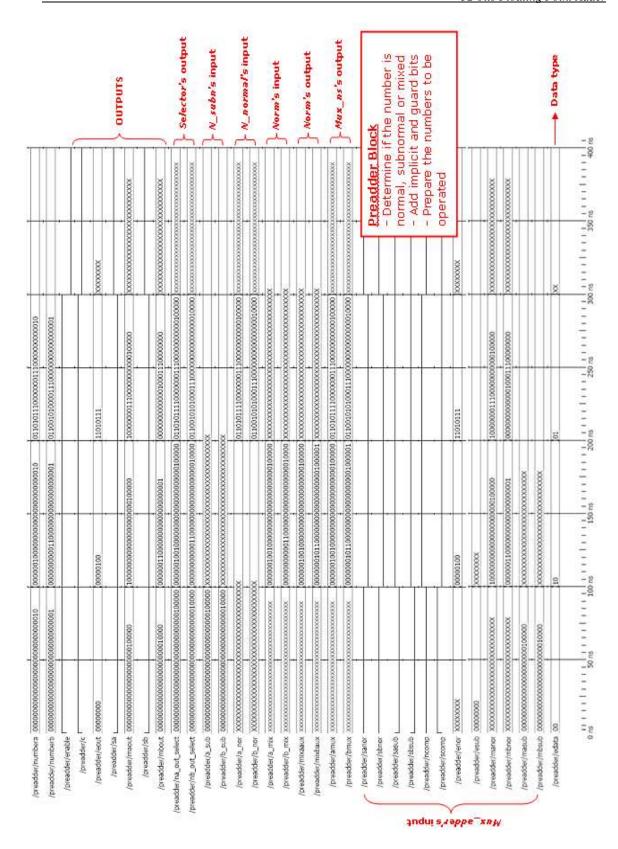


Figure 23. preadder Simulation

CHAPTER 4: ADDER

This chapter will deal with the adder and the standardizing block. The first one is in charge of operating the numbers which have been prepared in the Pre-adder block. The second one will standardize the result according with standard IEEE 754.

The block procedure is as follows:

- 1. Calculating the output's sign according to the sign numbers and the operation symbol
- 2. Addition/Subtraction of the both A and B numbers
- 3. Standardizing the result as IEEE 754 standard says
- 4. Grouping sign, exponent and mantissa in a single vector

The result is reached and the last step is multiplexing this value with the other one obtained as a special cases event explained in previous chapters.

4.1. Adder

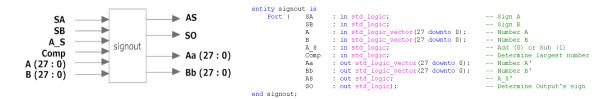
The adder is a fundamental piece of the design because it implements the addition/subtraction operation, main purpose of the 32 bit Floating Point Adder.

The Adder block is composed by two entities: *signout* and *adder*. *Signout* is responsible for the sign operation and the *adder* is the adder strictly speaking.

4.1.1. Signout Block

Signout entity has six inputs: numbers A and B, both signs SA and SB, signal A_S which indicates if we add or subtract and the bit Comp (high if A is greater than B otherwise low).

The outputs are the two numbers Aa and Bb, the outputs sign SO and the signal AS that has the same function than A_S : determine the sign of the operation.



Three different parts are visible in the code.

Firstly the outputs sign will be determined using the bits A_S , Comp, SA and SB, A's sign and B respectively.

An exclusive OR operand performs the function that is shown in table 4.

Table 4. SB xor A_S

A_S	SB	SB_aux
+	+	+
+	-	-
-	+	-
-	-	+

Basically, it does the mathematical combination between the operation's symbol and the B number sign. Once the "new" B's sign is found out, the outputs sign SO is determined with the aid of SA and the bit Comp.

Table 5. SO determined

SA	SB_aux	Comp	so
+	+	0	+
+	+	1	+
+	-	0	-
+	ı	1	+
-	+	0	+
1	+	1	-
	=	0	-
ı	-	1	-

First of all, the two vectors A and B are reordered according with their original value (remember the numbers have been exchanged –or not- in *preadder* block when the exponents have been made equal)

When A is greater than B, the outputs sign *SO* will be equal to A's sign, *SA*. Otherwise, if B is greater than A, the output will keep the sign of the number B (the "new B's sign" one, *SB aux*).

Secondly, if both *SA* and *SB* signs are equal is realized that both number A and number B will be added with the only difference of the sign. On the other hand, if *SA* and *SB* are different, what number is the negative one will be determined in order to simplify the adder implementation.

```
Aaux <= A when Comp = '1' else
       B when Comp = '0' else
Baux <= B when Comp = '1' else
      A when Comp = '0' else
process (SA, SB aux, A, B)
 ----- if Sign A is equal to Sign B
   if (SA xor SB aux) = '0' then
    Aa <= A;
                               -- Nothing changes
    Bb <= B;
 ----- if Sign A is 1 and Sign B is 0
   elsif SA = '1' and SB aux = '0' then
    Aa \leftarrow B;
                                -- A is changed by B
    Bb \le A;
 ----- if Sign A is O and Sign B is 1
   elsif SA = '0' and SB aux = '1' then
    Aa <= A;
                               -- Nothing changes
    Bb \le B;
   else
    Aa <= "----";
    Bb <= "----";
   end if:
end process;
```

As it is seen in the code, the negative number when we have two different signs always will be in the vector B called Bb setting the positive one in vector Aa. In another way it does not care: Aa will be A and Bb will be B.

Finally a bit indicating when a subtraction is produced is needed in order to achieve a properly operation in the adder. If A and B signs are equal that means an addition will be calculated (AS low). In the other hand, if A and B are different, the number A Aa and the negative number B, which had being moved to the vector Bb, are going to be subtracted (AS high).

```
AS <= '1' when SA /= SB_aux else -- Complement to 1 is needed when '0'; -- the signs are different
```

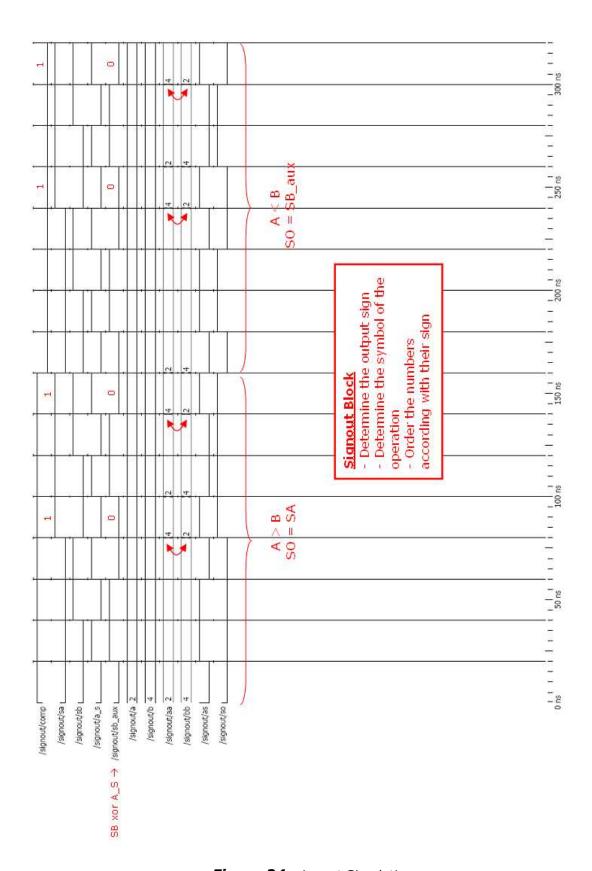


Figure 24. signout Simulation

4.1.2. Adder Block

The Adder block is in charge of the addition/subtraction operation. The two numbers A and B and the bit A_S which indicates the operation's symbol are the entries. The outputs are the result vector S and the Carry bit Co which shows if there was an overflow.

```
entity adder is
                                                             : in std_logic_vector(27 downto 0);
                                                 Port ( A
     A_S
                                ► S (27:0)
                                                                : in std_logic_vector(27 downto 0);
                                                                                                     -- Number B
   Comp
                   adder
                                                               : in std_logic;
                                                                                                     -- Add(0) / Sub (1)
A (27:0)
                                                                : out std logic vector(27 downto 0);
                                                                                                     -- Output
                                CO
B (27:0)
                                                               : out std_logic);
                                                                                                     -- Carry out
                                              end adder;
```

First of all the type of adder will be explained. A Carry Look Ahead structure has been implemented. This structure allows a faster addition than other structures. It improves by reducing the time required to determine carry bits. This is achieved calculating the carry bits before the sum which reduces the wait time to calculate the result of the large value bits.

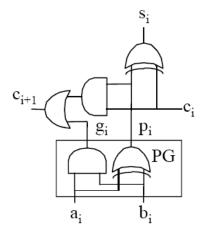


Figure 25. 1-bit Carry Look Ahead Structure

The implementation of the Carry Look Ahead structure is shown at the figure above. The idea is to obtain the carry generation and the carry propagation independently of each bit in order to obtain last carry faster.

The code has been designed implementing a 1-bit CLA structure and generating the other components up to 28 (the number of bits of the adder) by the function generate. Before that the A_S signal is used to determine if the second operand should be in complement to 1 (subtraction) or not (addition) using an exclusive or gate.

Finally the $\it Co$ bit is fixed by the carry of the last component.

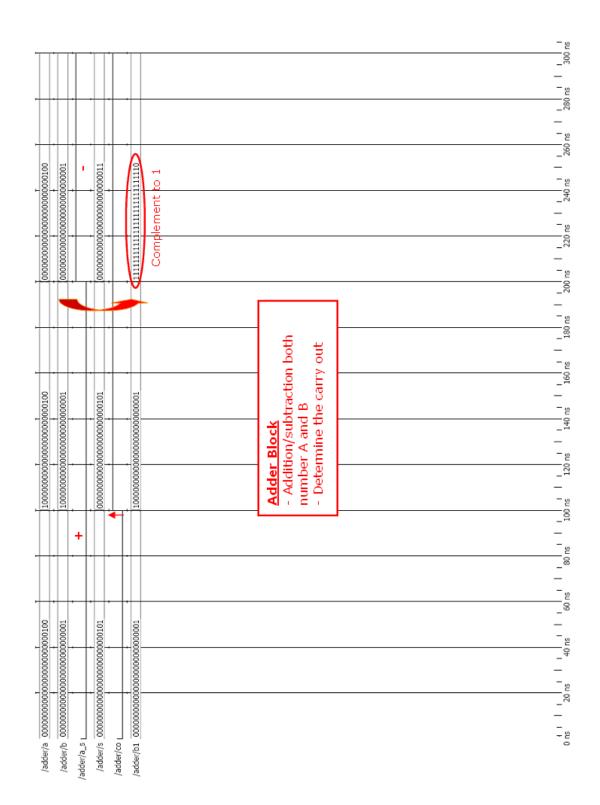
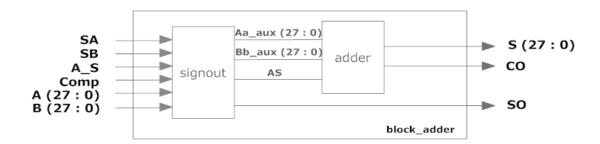


Figure 26. adder Simulation

4.1.3. Block_Adder Block

Finally the *Block_Adder* block joins both *signout* and *adder* entities to implement the complete adder. The inputs are the signs and the value of A and B (*SA-SB* and *A-B* respectively), the bit *Comp* and *A_S*. Moreover the outputs are the result *S*, the carry *CO* and the outputs sign *SO*.



```
entity block adder is
              SA : in std logic;
                                                             -- Sign A
    Port (
                  : in std logic;
              SB
                                                             -- Sign B
                    : in std_logic_vector(27 downto 0);
                                                             -- Number A
                    : in std_logic_vector(27 downto 0);
              В
                                                             -- Number B
              A_S : in std_logic;
Comp : in std_logic;
                                                             -- Add(0) / Sub (1)
                                                             -- Comparison
                    : out std logic vector(27 downto 0);
                                                             -- Output
              SO
                    : out std logic;
                                                             -- Output's sign
                   : out std_logic);
                                                             -- Carry out
              Co
end block adder;
```

The code is quite brief. Basically the connection of the different blocks is done in this block.

```
component adder port (A, B : in std logic vector(27 downto 0);
                     A_S : in std_logic;
                     s_
                          : out std_logic_vector(27 downto 0);
                     Co : out std_logic);
end component;
component signout port (SA, SB : in std logic;
                       A, B : in std_logic_vector(27 downto 0);
                       A_S, Comp : in std_logic;
                       Aa, Bb : out std logic vector(27 downto 0);
                       AS, SO
                               : out std_logic);
end component;
signal Aa_aux, Bb_aux, S_aux : std_logic_vector(27 downto 0);
signal AS_aux, SO_aux, Co_aux : std_logic;
begin
component00: signout port map (SA => SA, SB => SB, A => A, B => B, A S => A S, Comp => Comp,
                              Aa => Aa aux, Bb => Bb aux, AS => AS aux, SO => SO aux);
componentO1: adder port map (A => Aa aux, B => Bb aux, A S => AS aux, S => S aux, Co => Co aux);
----- If a complement to 1 is used and Output's sign is 1 a C2 is needed
S <= (S aux xor X"FFFFFFF")+'1' when ((AS aux and SO aux) = '1') else
      S_aux;
Co <= '0' when ((SB xor A_S) /= SA) else
     Co aux;
SO <= SO_aux;
```

The most interesting part is on the bottom: if a subtraction operation is done and the outputs sign is set (that means negative number is greater than the positive) that means a complement to 2 is needed over the result because as it has been explained the negative number always is moved to the vector B and the result is "negative" (C to 2) when truly it is not. An example is shown:

Table 6. Example correct operation

SA	A	SB	В	A_s	so	Result
-	101	+	100	+	-	001

At the table $\bf 6$ it is shown the correct and theoretical operation of the adder. The signs are not taken into account because they have their own bits. The result is 1-decimal positive with SO negative.

Table 7. Example wrong operation

Aa	Bb	AS	so'	Result
100	101	1	-	111

At table 7 the operation of the adder is shown without the complement to 2 part. The negative vector is move to Bb then a negative binary result (-1) is obtained not being correct according to the IEEE 754 standard. *AS* is recalculated in *signout* according to the sign values. Then if *AS* (subtraction) and *SO* (negative) are set a complement to 2 is necessary to reach a correct result.

Note the complement to 2 is not necessary when we have two negative numbers because it has been considered like an addition of two positive numbers.

Finally the carry value is also corrected in the same circumstances: when a subtraction is operated and we have a negative number at the output it will always have a carry out high. If the complement to 2 is needed that implies a carry low to obtain a proper result.

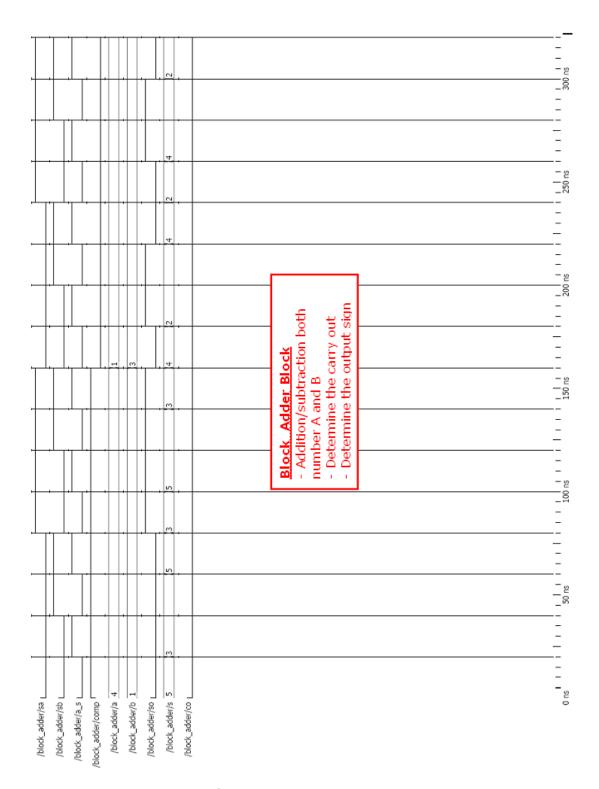


Figure 27. Block_Adder Simulation

4.2. Standardizing Block

The Standardizing block, as its name suggest, is responsible for displaying the addition/subtraction operation value according to the IEEE 754 standard.

This block is composed of four entities. Shift_left and zero blocks have been explained in the previous chapter. Round and vector are the two new ones. Basically they are in charge of dealing with the result obtained from the adder and showing it in the same format as the numbers had been introduced

4.2.1. round Block

Round block provides more accuracy to the design. Four bits at the end of the vector had been added in the Pre-Adder block. Now it is time to use these bits in order to round the result.

This block has only one input and one output. The input is the vector *Min* and the output *Mout*. Note *Min* is larger than *Mout* (27 bits against 22). The reason is *Min* contains the implicit and round bits that will be treated during the *round* code execution.

```
Min (27:0) Mout (22:0) Mout (22:0)

Hentity round is

Port ( Min : in std_logic_vector(27 downto 0); -- Input's mantissa

Mout : out std_logic_vector(22 downto 0)); -- Output's mantissa

end round;
```

The process to round is chosen arbitrarily: if the round bits are greater than the value "1000" the value of the mantissa will be incremented by one. Otherwise the value keeps the same value.

4.2.2. shift_left/zero Block

Both *shift_left* and *zero* blocks are completely reused from the mixed number block. It has been explained in the last chapter and it is not going to be commented again.

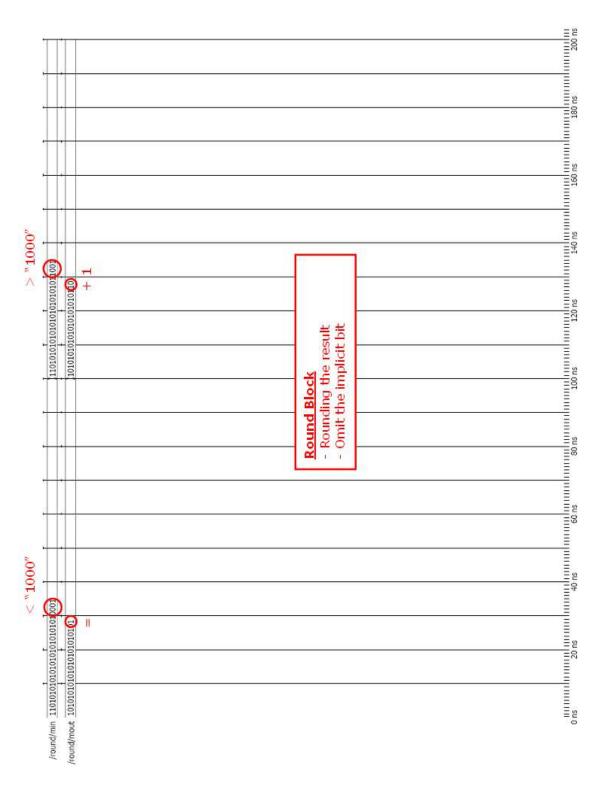
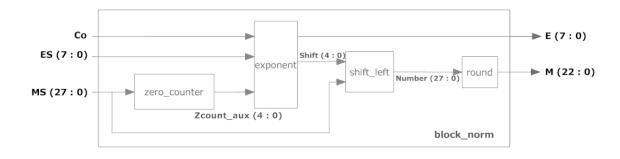


Figure 28. Round Simulation

4.2.3. block_norm Block

The block_norm implement the standardizing function. It is composed by the entities signout, $shift_left$ and $zero_counter$. The entries are the result's mantissa (MS) and exponent (ES) and the add's carry Co. The outputs will be the standardized result (its mantissa (M) and exponent (E)).



The first part of the code implements the connection between the different components which compose the block. Zero, shift_left and round are connected as shown in the code and the block diagram.

```
comp0 : zero
   port map (T => MS, Zcount => Zcount_aux);

comp1 : shift_left
   port map (T => MS, shft => Shift, S => Number);

comp2 : round
   port map (Min => Number, Mout => M);
```

The second part of the code refers to the exponent treatment. Three different cases have been taken into account:

- 1. If the exponent is larger than the number of zeros (number of positions the vector should be shifted) it means the number is normal and the standardized exponent will be the exponent minus the positions shifted plus the carry.
- 2. If the exponent is shorter than the number of zeros it means the output will be subnormal, only the value which marks the exponent could be shifted and the final exponent will be zero.
- 3. Last case referred when the exponent is equal to the number of zeros. On this occasion the vector will be shifted the number of positions the exponent marks (or the signal zero_counter) and the result will be normal with exponent one.

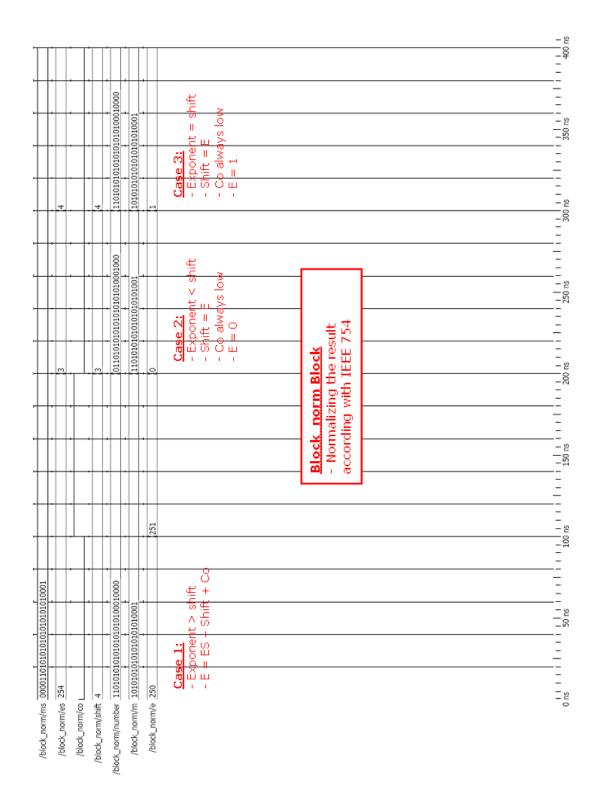


Figure 29. norm_block Simulation

4.2.4. vector Block

This block has an easy function: regrouping the sign, exponent and mantissa in a single vector to be consistent with the format adopted for data entry.

It has three inputs: sign S which it comes from the adder block, mantissa M and exponent E. The output is the vector N which keeps the format of numbers A and B (the main entries of the system).

```
S Port ( S: in std_logic; -- Sign
E(7:0) N(31:0) M: in std_logic_vector(22 downto 0); -- Exponent
N(22:0) end vector;
```

The code is so simple. Sign, mantissa and exponent are set in the proper position as follows:

```
N(31) <= S;
N(30 downto 23) <= E;
N(22 downto 0) <= M;
```

Table 8. Bit positions

31	3023	220
Sign	Exponent	Mantissa

Note this entity will be out of the standardizing block because it uses signals from two different blocks. However it is part of the standardizing process and it is clearer to be explained in this chapter.

Two simulations are added: the first one test the entity operation and the second one perform the behaviour when it is joined to the *block_norm* entity.

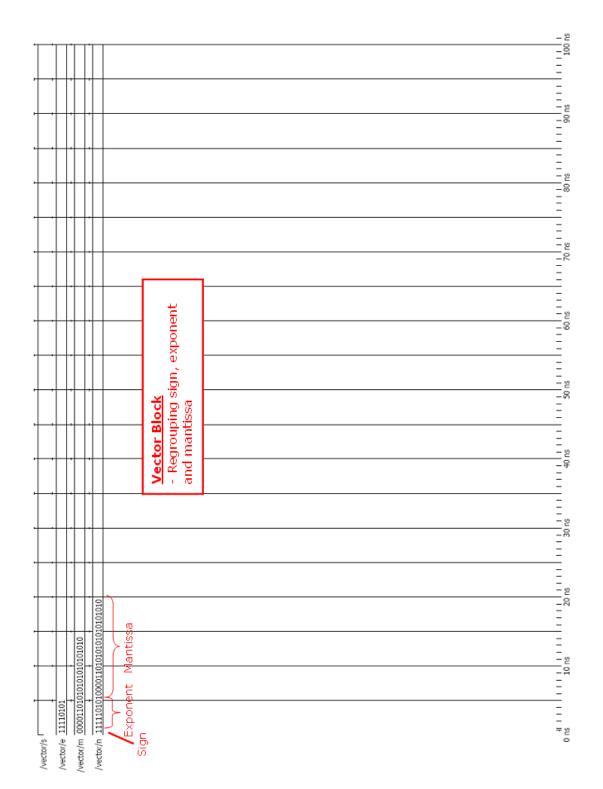


Figure 30. vector Simulation

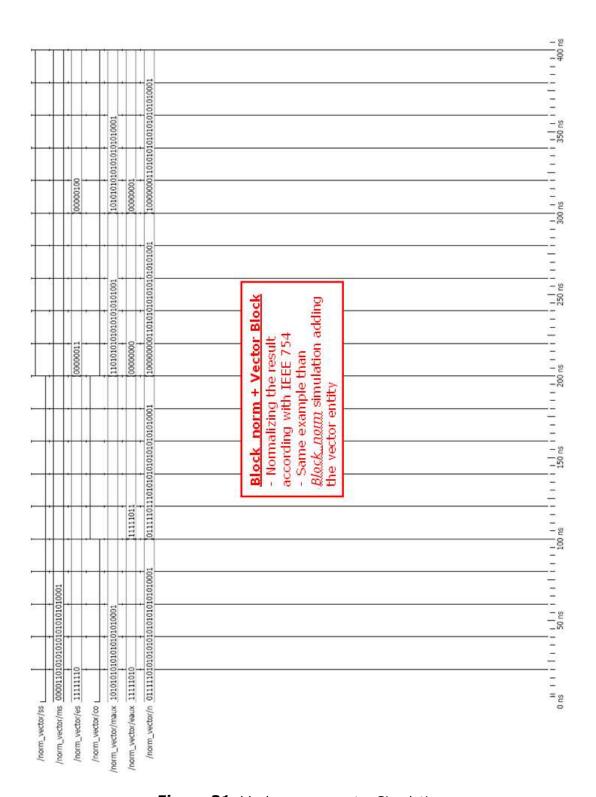


Figure 31. block_norm + vector Simulation

CHAPTER 5: 32-BITS FLOATING POINT ADDER

Finally all the different entities and sub-blocks has been described and explained. In this chapter the blocks will be joined in order to test totally the Floating Point Adder. The procedure will be as follows:

- 1. *n_case* Entity sort the data type according with the standard IEEE 754 and enable the adder if it is needed to operate the numbers. Otherwise (special cases) the result is done by it.
- 2. If the numbers are normal, subnormal or a mix, the Pre-Adder sub-block deals with the treatment of the numbers in order to be added / subtracted.
- 3. The Adder Block adds or subtracts the two numbers given
- 4. It is time to standardize the result according with the standard: shifting the mantissa and recalculating the new exponent.
- 5. Finally the result will be choose between the special case or the operated one depending on the input values

Two more entities will be explained in this section: the multiplexor which takes care of the last step of the list and the *fpadder* grouping all this points and making them work together.

On this occasion, the simulations will not be added to the code. Being the complete Floating Point Adder it is considered make another point in this chapter to demonstrate the proper functioning.

Moreover a table will be used to collect the different binary values, convert to decimal and as similar or different the results are.

5.1. Floating Point Adder

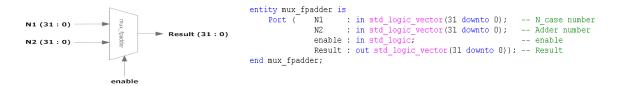
As it has been said, this first section will contain two entities: the multiplexor that is in charge of setting the correct result in the output (special cases or operated result) and the entity which groups all the blocks.

Continuing with the format used before, the ports and the block diagram are explained at the beginning and immediately afterwards the behaviour.

5.1.1. Mux_fpadder Block

The multiplexor is not so complicated. It has three inputs: N1, N2 and enable. The first signal refers to the n_case result and the second one to the vector obtained in the adder block. Enable decides which one will be at the output.

Finally, the output is *Result* which contains the 32 bits (Sign, exponent and mantissa) result.



The code is pretty simple. If *enable* is high it means the numbers were normal, subnormal or mixed and then the vector which comes from the adder is the correct result. Otherwise, if *enable* is low, a special case combination is had and the block n_case is who has the proper value.

```
Result <= N1 when enable = '0' else -- N_case number

N2 when enable = '1' else -- Adder number
```

5.1.2. fpadder Block

Finally, the entire Floating Point Adder is designed. The last entity is *fpadder* which joins all the different blocks previously described.

The inputs are both NumberA and NumberB numbers and the operand A_S . Obviously, the output is the final result of the operation according with the standard.

On the next page a complete block diagram is shown:

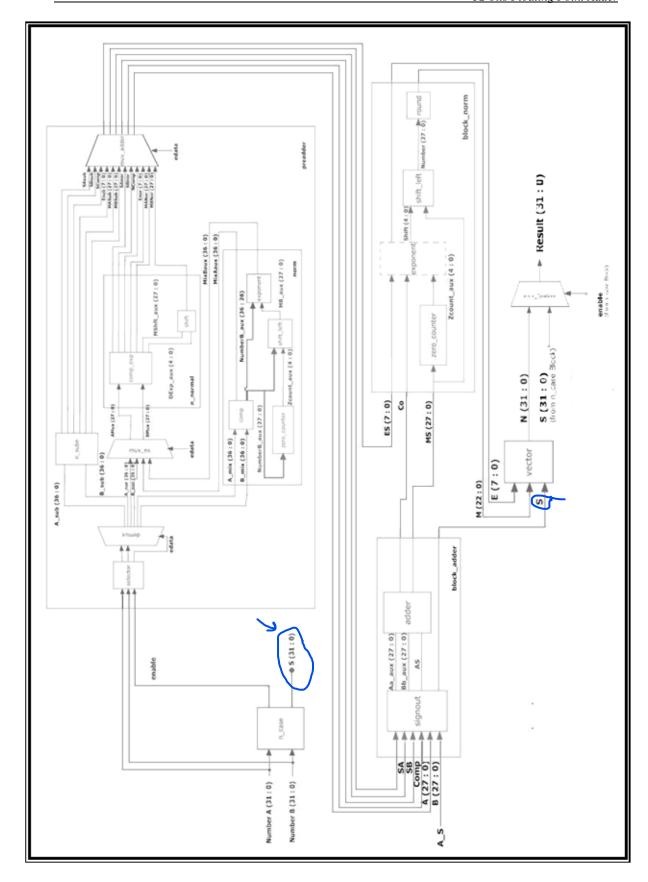


Figure 32. Block Diagram

The first part of the code includes all the component declarations. As it can be seen all the main blocks are here: n_case , preadder, $block_adder$, $norm_vector$ ($norm_vector_ve$

```
component n_case port (NumberA, NumberB : in std_logic_vector(31 downto 0);
                     enable : out std_logic;
s : out std_logic_vector(31 downto 0));
end component;
component preadder port (NumberA, NumberB : in std_logic_vector(31 downto 0);
                       enable : in std_logic;
SA, SB : out std_logic;
                       C : out std_logic;
EOut : out std_logic_vector(7 downto 0);
MAOut, MBOut : out std_logic_vector(27 downto 0));
end component;
______ Adder
A S : in std logic;
                          Comp : in std_logic;
                          s : out std logic_vector(27 downto 0);
so : out std_logic;
                          Co : out std logic);
end component;
component norm vector port (SS : in std logic;
                          MS : in std_logic_vector(27 downto 0);
ES : in std_logic_vector(7 downto 0);
                          Co : in std logic;
                          N : out std_logic_vector(31 downto 0));
end component;
                                                      ----- Mux fpadder
component mux fpadder port (N1, N2 : in std logic vector(31 downto 0);
                           enable : in std_logic;
                           Result : out std logic vector(31 downto 0));
end component;
```

The second part of the code is responsible for connecting the different blocks properly as it is represented in the block diagram.

5.2. Simulations

At this point the simulations to test the operation will be comment. As it has been done before four different cases could happen: special case, normal, subnormal or mixed numbers.

All the different possibilities must be tested and this is the reason why the different data types will be treated separately.

The procedure will be as follows:

- 1. Enough different cases for each data type to demonstrate the correct working will be taken into account. The binary values of the entries and the output will be grouped in a table.
- 2. Using the simulation the result will be obtained and added to the table.
- 3. Decimal value of the numbers and the result will be calculated with the formula which had been explained at the standard IEEE 754 chapter.
- 4. Simulation value will be compared with the arithmetic value in order to see as similar or different the numbers will be.

The discussion about the accuracy of the 32-bit Floating Point Adder and the general standard IEEE 754 will be carried out in the next chapter.

5.2.1. Special Cases

Recovering the table added in the second chapter, 8 different cases are possible.

Zero-NumberX, NaN-NumberX and Normal/Subnormal-Infinity cases can be taken into account only one time (Zero-NumberX or NumberX-Zero tests the same result). Then the simulation will contain 5 combinations.

Out A Out B Sign Output Sign **Output** Number B Χ Zero SB Number B Χ Number A Zero SA Number A Χ Normal / Subnormal SB Infinity Infinity Χ Infinity Normal / Subnormal SA Infinity SA=SB SX Infinity Infinity Infinity SA≠SB 1 Infinity Infinity NaN Χ NaN Number B 1 NaN Χ Number A 1 NaN NaN

Table 9. Special Cases combination

X: do not care SA: Number A's sign SB: Number B's sign SX: Sign A or B (it is the same)

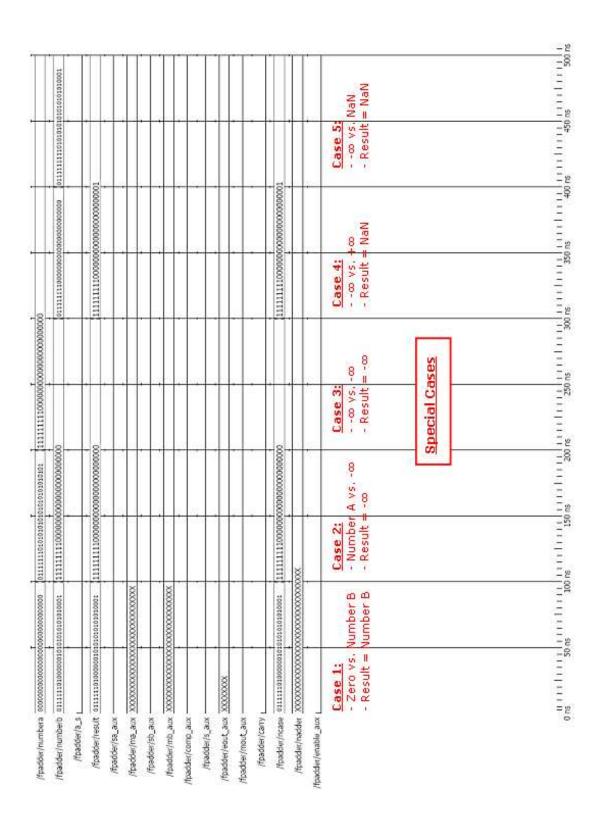


Figure 33. Special Cases Simulation

The results are collected in the next table:

Table 10. Special Cases Results

SA	EA	Σ	MA	A ₍₁₀	SB	EB	MB		B ₍₁₀
0	00000000	000000000000000000000000000000000000000	000000000000	0	0	11111101	1010101010101010101010101	101010101	+1.417843161699894e+038
0	11111110	101010101010	10101010101010101010101	+1.417843161699894e+038	1	11111111	000000000000000000000000000000000000000	000000000	87
1	11111111	000000000000000000000000000000000000000	000000000000	ω-	1	11111111	000000000000000000000000000000000000000	000000000	87
1	11111111	000000000000000000000000000000000000000	00000000000	φ-	0	11111111	000000000000000000000000000000000000000	000000000	8+
н	11111111	000000000000000000000000000000000000000	0000000000000	8	0	11111111	1010101010101010101010001	100010101	NaN
		SS	ES	MS		S ₍₁₀		A(10 ± B(10	
		0	11111101	1010101010101010101010101		1,417843161	+1.417843161699894e+038		
		Ħ	11111111	000000000000000000000000000000000000000		8-		-	
		н	11111111	000000000000000000000000000000000000000	Щ	8			
		н	11111111	000000000000000000000000000000000000000		NaN	Z		
		H	11111111	000000000000000000000000000000000000000		NeN	Z	-	

SX: Sign Number X EX: Exponent Number x MX: Mantissa Number X XS: Result X(10: Base-10 Number

As it can be seen the results are consistent with the theoretical explanation.

5.2.2.

Normal Numbers

 n_case block is only a combination between the entries and no more blocks are involved in this operation. Normal numbers have more complexity.

In this section the blocks which are responsible for normal numbers are tested.

/fpadder	/fpadder/numbera_011111101101010101	01111110	110101010101010101010101	TI	1010101010101	01	111111010	10011001	1001100	111110110110011001100110011011111	1111000001111	11111000010111011101110111011101	011111001110111011101110111011101	110111011101
/fpadder	/fpadder/numberb 011111101101010101	01111110	110101010101010101010001		0101101010101	01111010101010101010101010101010101	011111010	001010100	1010101	011110101010101010101010101011110	111100001111	111110000101110011100111011100	1111101101111001110011011100	110011011100
/fp	/fpadder/a_s					300			Д,	1			<u> </u>	2 10
/fpac	fpadder/result 011101000000000000	01110100	000000000000000000000000000000000000000		00110101011111	11111001101010111111111111010101100	111111000	0001001	1011100	1111100000010011101110110010001000	1111000101110	111110001011101010101110101111001	0111110011111001011111001011111001	10011111001
/fpadd	/fpadder/sa_aux			-					L					
/fpadde	/fpadder/ma_aux 11010101010101010	1101010	1010101010101010000	П	1010101010101	110101010101010101010101010000	11001100	01100110	001100	110011001100110011001100110011	10111011101	110111011101110110100000	•	
/fpadd	/fpadder/sb_aux_											-		
/fpadde	/fpadder/mb_aux 11010101010101010	1101010	101010101010100010000		1010101010101	101010101010101010101000001	1010101	0010101	110101	1010101010101010101010100001010101010101	101110011101	1101110011011100111000000	000110110011101110111001	00111011000
/fpad	//badder/s_aux _			<u> </u>					L					
/fpadder	fpadder/eout_aux 11111101	1111110		11110101	1010		11111010		Ш		11110000		111111001	
/fpadder/	fpadder/mout_aux 000000000000000	0000000	000000100000000000000000000000000000000		101011111111	00010101011111111110101010000	00100010	0011101	100100	0000010001001110111001000100	0111010101111	101110101011110101110010000	11111001011111001011110001000	11110001000
'fpadder/	fpadder/comp_aux_		•											
/gback	/fpadder/carry_		e:			2.0								
/fpad	Ider/ncase X	000000	/fpadder/ncase xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx	0000000000										
pped4/	/fpadder/nadder o11101000000000000	011110100	000000000000000000000000000000000000000		0011010101111	1111100110101011111111111111101011100	111111000	00010011	1011100	11111000000100111011100100010111111	0111000101110	1111100010111010101011101010111001	100111100101111100111110011111001	100101111001
e/sapped	padder/enable_aux											r		
SA	EA		MA			A(10		AS	SB	EB		MB	B	B ₍₁₀
0	11111101		10101010101010	101010101010101	+1.4178	+1.417843161699894e+038	8607	1	0	11111101	10101010	10101010101010101010001		+1.417842756051702e+038
0	11110101		01010101010101010101010	110101010	+4.42213	+4.422139697774293e+035	+035	-	0	11110101	10101010	10101010101010101010101		+5.538449850390212e+035
1	11111010		10011001100110	100110011001100	-1.70141	-1.701411733192644e+037	-037	+	0	11111010	01010100	01010101010101010101010		+1,415084703287774e+037
1	11110000		10111011101110	101110111011101	-1.79999	-1.799996137003937e+034	-034	+	П	11110000	10111001	10111001101110011011100		-1.791851358103467e+034
0	11111001	-	10111011101110	101110111011101	+9.21598	+9.215980221460157e+036	+036	-	н	111101111	10111001	10111001101110011011100	Щ	-2.293569738372437e+036
	J,	SS	ES		MS				S	S (10		A ₍₁₀ :	A ₍₁₀ ± B ₍₁₀	
		0	11101000	00000000	0000000	000000000000000000000000000000000000000	+ 4.0)5648	1920	+4.056481920730334e+031	\models	-4.05648192C	+4.056481920730334e+031	
		H	111100111	0101011	1111111	0101011111111101010100	-1.1	1631	0152	-1.116310152615919e+035	믬	1.116310152	-1.116310152615919e+035	
		П	11111000	0001001	1101110	000100011101110010001000	-2.8	6327	0299	-2.863270299048707e+036		2.863270299	-2.863270299048707e+036	
		п	11110001	0111010	1011101	0111010101110101111001	-3.0	2985	7503	-3.029857503386945e+034		3.591847495	-3.591847495107403e+034	
		0	11111001	1111001(0111100	11110010111100101111001	+1.0	3627	6540	+1.036276540755903e+037	H	1.150954995	+1.150954995983259e+037	

Figure 34. Normal Numbers Simulation

5.2.3.

Subnormal Numbers

Turn to the subnormal numbers. Different possibilities with the sign of the numbers and the operation symbol will be treated in order to test more combinations.

/fpadder/	fpadder/numbera 000000000101010101	000000000	1001010101010101010001	TI	00000000011111111111111111111111	00000000	001010100	1010101	000000000000000000000000000000000000000	100000000010111011101110111011101	101110111011101	1000000001011100110011011011100	100111011100
/fpadder/	fpadder/numberb 000000000101010101	000000000	1010101010101010101010101		100000000000000000000000000000000000000	10000000011111111111111111111111111	11111111	1	П	10000000000011011101110111011101	101110111011101	00000000000001110111011101110111011101	10111011101
Edy/	/fpadder/a_s_L		_		-								
/fpadi	der/result	000000000	/fpadder/result 00000000101010101010101010101010101010	1,2	000000000000000000000000000000000000000	1000000000101010101010101010101	01010101	0101010		00000010000000	100000000000000000000000000000000000000	1000000010111010101110101111001	01010111001
/fpadde	/fpadder/sa_aux_												
/fpadder	/fpadder/ma_aux 01010101010101010	01010101	00010101010101010000		001111111111111111111111111111111111111	0111111	1111111	1	0000111111111111111111111000	101110111011101	010111011101110111010000	010111011101110111011010000	0111010000
/fpadde	/fpadder/sb_aux_							L					
/fpadder	xne_dm/.	01010101	/fpadder/mb_aux 01010101010101010101000010000		00101010101010101010100000	0010101	0010101	110101	001010101010101010101000000	00111011101110	000111011101110111010000	01011100110111001111000000	0111000000
/fpado	/tpadder/s_aux_			.		L		L					
/fpadder/	/fpadder/eout_aux 00000000	0000000						Ц					
/fpadder/n	/fpadder/mout_aux 10101010101010101	10101010	0101010101010100000		0101010101010101010101010000	0101010	1101010	1001010	01010101010101010101010101010	100000000000001	010000000000000000000000000000000000000	101110101011101011110010000	1110010010
/fpadder/c	/fpadder/comp_aux_											•	
ped _J /	/fpadder/carry _		10			4						•	
/fpadc	ler/ncase	XXXXXXX	/fpadder/ncase XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX	+ XXXXXXXXXX		4							
/fpadde	/fpadder/nadder 000000001010101010101	00000000	0100101010101010101010		000000000000000000000000000000000000000	1000000001010101010101010101010101	101010101	0101010		100000000000000000000000000000000000000	0000000000000000	1000000001011101010111010111000	01010111001
/fpadder/enable_aux	able_aux					_			T				
Š	SA	EA	MA		A(10		AS	SB	EB		MB	B ₍₁₀	
	0000	00000000	101010101010101	010101010101010001	+7.836622933188571e-039	1e-039	+	0	00000000	1010101010	10101010101010101010101	+7.836628538382429e-039	82429e-039
٦	0 0000	00000000	11111111111111	111111111111111111	+1.175494210692441e-038	1e-038	+	1	00000000	0101010010	01010101010101010101010	-3.887821313958274e-039	58274e-039
٦	0 0000	00000000	010101010101010110101010	10110101010	3.887821313958274e-039	e-039	+	1	00000000	111111111	1111111111111111111111111	-1.175494210692441e-038	92441e-038
-7	1 0000	000000000	10111011101110111011101	10111011101	-8.620290691571439e-039	e-039	1	1	00000000	0011101110	00111011101110111011101	-2.742818937460002e-039	50002e-039
	1 0000	00000000	1011100110111	001101110011011100	-8.528095061708117e-039	'e-039	-	0	00000000	1011101110	10111011101110111011101	8.620290691571439e-039	1439e-039
		SS	ES		MS			S	S ₍₁₀		A ₍₁₀ ± B ₍₁₀	B ₍₁₀	
		0	00000001	0101010:	01010101010101010100110		1.5673	32514	+1.567325147157100e-038	\sqsubseteq	1.567325147	+1.567325147157100e-038	
		0	00000000	1010101	10101011010101010101010101		7.867	1207	+7.867120792966137e-039	\sqsubseteq	7.867120792	+7.867120792966137e-039	
		T	000000000	1010101	10101011010101010101010101		.8671	2079	-7.867120792966137e-039		-7.867120792966137e-039	966137e-039	
		Ħ	00000000	10000001	100000000000000000000000000000000000000		.8774	7175	-5.877471754111438e-039	H	-5.877471754111438e-039	.11438e-039	
		Ħ	00000001	0111010:	01110101011101010111001	Щ	.7 148	3857	-1.714838575327956e-038		-1.714838575327956e-038	327956e-038	

Figure 35. Subnormal Numbers Simulation

5.2.4. Mixed Numbers

Finally, the mixed numbers. The other combinations will be tried.

-lpadd/	r/numbera	000000100	/fpadder/numbera 000000100101010101010101010001		1000010010101010101010101010101010101		01010101010101010101010101010000	00101010	, []	010101000000001	1000000001010101010101010101010101010101	1	***************************************
/fpadde	r/numberb	0000000	/fpadder/numberb 00000000010101010101010101010101010101	1010101		100001	100000000011111111111111111111111111111			100000101011101110111011101	101110111011101	01010101010101010101010111101010	01010101010
1/4	/fpadder/a_s L	70	_	L		-		<u> </u>					
/fpa	/fpadder/result 00000010010110101	000000100	0110101010101010101010	TT	1000010001010101010101010101010101010101	TT	0000010101010101010101010101010101010101	00111010		100001001100110011001100110010	011001100110010	00000111101010101010101111111111	00111111111
/fpadk	/fpadder/sa_aux_		-		•	1		4					
/fpadd	/fpadder/ma_aux 1101010101010101	1101010	101010101010100010000	T	1101010101010101010101010000		010010010	1011010	10101010101010101010100000	1001110111011101110111010000	1110111010000	1010101010101010101010100000	000001010
/pedk	/fpadder/sb_aux_	1578								K Ja		-	
/ppedd/	/fpadder/mb_aux 0000010101010101	0000010	101010101010101010101		000000000000000000000000000000000000000		000000000000011111111111111111111111111	1		01010101010101010101010101010	0101010101010	000000000000000000000000000000000000000	010101010
/¿bac	/kpadder/s_aux_			_		. 		.		74 5 27		•	
/ppedd/	/fpadder/eout_aux 00000100	0000010		00010000	00	00001010	010			00000101		00001111	
/fpadder	/fpadder/mout_aux 11011010101010101	1101101	010101010101001100101	П	1101010101010101010101010101	M	1010101010111010111010101011111	1011010		1001101100110011001100100110	10011001100110	10101010101010100111111111110110	1111111111
/fpadder/	/fpadder/comp_aux F					-		L					
ed/	/fpadder/camy L	100	•	• ;		-						•	
//pac	der/ncase	XXXXXXX	/fpadder/ncase XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX	XXXXXXXXX									
/fpadc	/fpadder/nadder_00000010010110101	000000100	01101010101010101010101010	10000100	010101010101010101010101010101010101		01010101101101011110101101010101010	00111010		1000001001100110011001100110010	011001100110010	000001111010101010101010111111111	11111111100
//padder/e	/fpadder/enable_aux				,								
			-	-	-	-		- -	-				
n	NA H	E A	ΜM		A(10		n P	92	EB		MB	D(10	
	0000	000000100	101010101010101	01010101010100001	+1,567325315312916e-037	.916e-037	+	0	00000000	1010101010	1010101010101010101010101	+7.836628538382429e-039	32429e-039
	1 0000	000010000	101010101010101	010101010101010101	-2.507721221965479e-036	479e-036	'	0	00000000	1010101010	1010101010101010101010101	+7.836628538382429e-039	32429e-039
	0000 0	00001010	010101010101010110101010	0110101010	+8.009095588956748e-036	748e-036	-	1	00000000	1111111111	111111111111111111111111	-1.175494210692441e-038	2441e-038
	1 0000	00000000	101010101010101	010101010101010101	-7.836628538382429e-039	429e-039	'	н	000000101	00111011110	00111011101110111011101	-2,319641991309260e-037	9260e-037
	1 0000	00000000	101010101010101	010101010101010101	-7,836628538382429e-039	429e-039	+	0	00001111	0101010101	0101010101010101010101010	+2,567906439457154e-034	57154e-034
		SS	ES		MS			3	S ₍₁₀		A(10 ± B(10	B ₍₁₀	
		0	000000100	10110101	1011010101010101010100110		+1.606	5084	+1.606508422972366e-037	\vdash	.645691600	+1.645691600696740e-037	
		H	000010000	1010101	10101011010101010101010		-2.511	53947	511639476679485e-036	.e-036 -2.	.5155578505	515557850503861e-036	
		0	000001010	0101010	0101010111010110101010		+8.014	9730	+8.014973060710860e-036		.997340646	+7.997340646849824e-036	
		Ţ	000000101	00110110	001101110011001100110010		-2.280	45877	-2.280458771545933e-037		-2.398008276693085e-037	93085e-037	
		0	00001111	0101010	01010101010101011111111		+2.567	8671	+2.567867179782547e-034	\sqsubseteq	567828073	+2.567828073171770e-034	

Figure 36. Mixed Numbers Simulation

CHAPTER 6: RESULTS

Finally in the last chapter the results, they have been obtained before, will be evaluated.

Firstly a theorical and brief introduction about floating point errors is compulsory because this information is important to understand the behaviour of the results achieved.

At last, the report will finished with a conclusion where the main goals of the adder will be discussed.

6.1. Errors

There is a lot of literature which speaks about errors in a floating point system. The most of these errors are produced in the conversion between the internal binary format and the external decimal one or conversely.

Usually the computers use a fixed quantity of memory to represent each sort of number. This representation makes the electronic design easier but it involves rounding and it can lead to erroneous values.

This project focuses on the design of the binary floating point adder. Hence this type of errors will not be taken into account unless when a decimal representation with MATLAB is used (we will see it later).

The floating point format is discontinuous. It means not all real numbers have representation and this is another error source especially important with high numbers where the gap between them is largest.

6.1.1. Gap between Numbers

Once again, the real numbers could have an infinite number of digits and the floating point format is used to represent it with a computer.

The accuracy of the number is represented by the number of digits of the mantissa. A 24bits mantissa could be represented by 7 decimal digits.

In numerical analysis, errors are very often expressed in terms of relative errors. And yet, when errors of "nearly atomic" function are expressed, it is more adequate and accurate to express errors in terms of the last bit of the significant: the last significant weight give us the precision of the system. Let us define that notion more precisely. William Krahan coined the term *ulp* (unit in the last place) in 1960 and its definition was as follows:

Ulp(x) is the gap between the two floating point numbers nearest to x, even if x is one of them.

Mathematically the *ulp* could be defined as follows:

$$ulp = \beta^{-p+e} \tag{2}$$

The value in our system will be (when $e = e_{min}$) $ulp = 2^{-24+1} = 1.1921 \cdot 10^{-7}$

As it has been said, the floating point format is discontinuous that means not all the real numbers have a representation in this format. The *ulp* represents the step between two consecutive numbers. Using *MATLAB* with p=6, $\beta=2$ and 0<e<3 (simplifying results) a representation of this discontinuous format has been obtained:

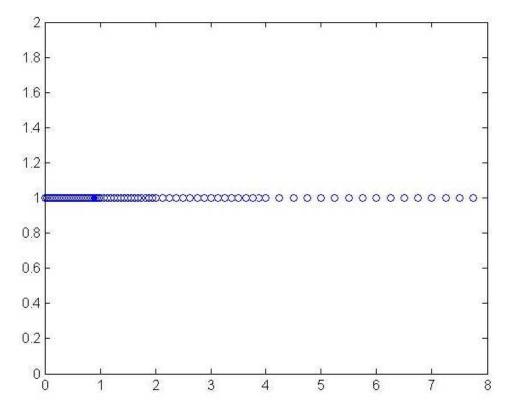


Figure 37. ulp representation

The *ulp* is doubled as the exponent increases by one because of the base (power of two).

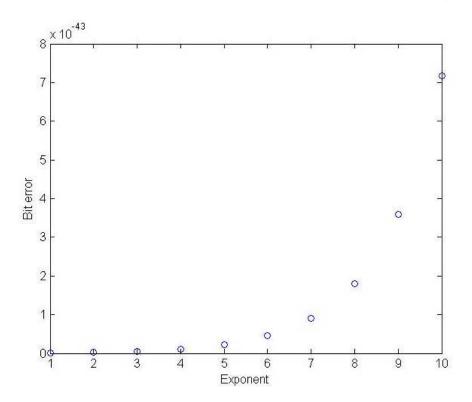


Figure 38. ulp increase

6.1.2. Rounding or Truncation

To represent a real number in a computer an adequate floating point number must be chosen. At first glance it seems the nearest one will be the best choice but sometimes this will not be an option unless all the number digits are known.

We need a procedure which requires only one digit more to represent the number. Using this condition rounding and truncation are our two possibilities.

Truncation consists of chosen the m more significant bits/digits of a number x. On the other hand, rounding needs to know the next bit/digit too and according with it add one to the LSB/last digit (5, 6, 7, 8 and 9 cases) or not (1, 2, 3, 4).

We use an example to find out what method is better and why.

Rounding
$$(2/3) = (0.666...)$$
 \rightarrow 0.667
Truncation $2/3) = (0.666...)$ \rightarrow 0.666

If a mantissa of 3 bits is considered we have a maximum truncation error when the number ends with a periodic 9 as 0.666**99999**. The error is $0.999 \cdot 10^{-4} \approx 10^{-4}$ which matches up with the bit error (or the ulp).

The maximum rounding error is produced when the numbers have a decimal part ended in 4 plus infinity 9 digits as 0.66649999. The error is calculated rounding the maximum error $4.9999 \cdot 10^{-4} \approx 5 \cdot 10^{-4}$. Then the relative error is equal to half the last digit (1/2ulp).

In conclusion we have demonstrated the relative error caused by truncation is equal to the ulp and the rounding one $\frac{1}{2}ulp$.

As it has said the error increases according with the exponent of the number.

6.1.3. Floating Point Addition

Finally a brief comment about the floating point addition/subtraction will be done.

The basic arithmetic operations have their equivalent in floating point format. The goal is noting these operations always have errors.

Let see an example:

Being $x=1867=0.1867\cdot10^4$ and $y=0.32=0.3200\cdot10^0$, then:

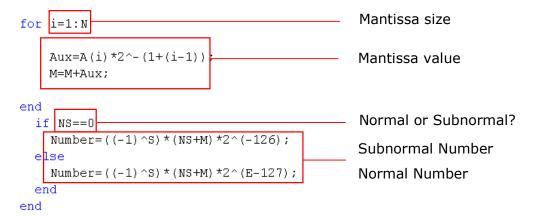
$$fl(x + y) = fl(0.1867 \cdot 10^4 + 0.000032 \cdot 10^4) = 0.1867 \cdot 10^4 \neq 1867.32 = x + y$$

The floating point representation is correct but anyway we have an error. A mantissa shifting is necessary to add two numbers with different exponents and we can lose some "information" during the procedure.

6.2. Results analysis

Once a small introduction to the errors is done we are going to use the values from last section tables.

To show the result we have implemented a simple program using MATLAB. First, inside the *for* loop we calculate the mantissa value and set its value in *Aux*. Then according with the data type (normal or subnormal) we use the appropriate formula for each case.



6.2.1. Subnormal Numbers

In the table 14 we have grouped all the results obtained during a new simulation with subnormal numbers.

As it can be seen the relative error is always zero. The error for a subnormal bits is $2^{(-23-126)} = 1.4012 \cdot 10^{-45}$. The number is small enough. In addition there is not any shifting and the double precision format of MATLAB provides this great accuracy.

Table 11. Subnormal Numbers Results

MA/MB	A(10/ B(10	AS	A(10 ± B(10	s	5(10	E,
000000000000000000000000000000000000000	-7.052964983894954e-039		000 40000000000000000000000000000000000		000 10000 10000 1	
0T0T0T0T0T0T0T0T0T0T0T0 00000000 T	-3.918313568541982e-039	ı	-I.U9/12/055245594e-U50	1 000000000 111011101110111011101100110	-1.09/12/05524559446-050	-
T0T00000000000000000000000000000000000	7.006492321624085e-045		100 40000000000000000000000000000000000		160 -00000000000000000000000000000000000	c
T0000000000000000000000000000000000000	-1.401298464324817e-045	1	0.407790702409026-045	077000000000000000000000000000000000000	0.40//90/039409026-043	-
00111011001110110011101100000000011	-8.528095061708117e-039		000000000000000000000000000000000000000		0000	ď
101110111011101110111011000000000000000	8.620290691571439e-039	ı	-1./140303/332/930E-030	1 000000001 011010101010101010000001	-1./140303/332/930E-U30	-
T0TTT0TTT0TTT0TTT0TTT0T 000000000 T	-8.620290691571439e-039		00000		0000	ď
10111011101110111011101100000000011	-2.742818937460002e-039	ı	-5.0//4/1/54111450E-059	T I nonnonnonnonnonnonnonnonnonnonnonnon	-5.0//4/1/341114306-039	-
010101011010101010101010100000000000000	3.887821313958274e-039	-	000 -501330005001630 5		000 4001220000000000000000	-
1 00000000 1111111111111111111111	-1.175494210692441e-038	+	-1.00/120/3230013/6-033	T 000000000 TOTOTOTOTOTOTOTOTOTOTOTOTOTOTOTOTOTOT	-/-00/120/9290013/6-039	>
0 00000000 1111111111111111111111	1.175494210692441e-038	-	000 000133000000001030 0		000 -001230000001020 0	-
010101011010101001010101010000000011	-3.887821313958274e-039	+		101010101010101010101010101000000000000	/·00/120/9230013/6-039	,
0 0000000000000000000000000000000000000	7.836622933188571e-039	+	1 6672261471671000-020		1 6679961471671006-099	-
0 0000000000000000000000000000000000000	7.836628538382429e-039	+	000-000-001-01-000-00-1		000-000T/0T/#T090/00*T	,
100000000000000000000000000000000000000	-4.778427763347626e-043	+	000000000000000000000000000000000000000		000000000000000000000000000000000000000	-
010101010000000000000000000000000000000	-2.382207389352189e-043	+		110000000000000000000000000000000000000	-/-Топородовороворого	-

6.2.2.

Mixed Numbers

Table 12. Mixed Numbers Results

	A(10/ B(10	ΑS	A(10 ±B(10	s	S(10	Ē,
110000000001010101010101010101010101	-7.836628538382429e-39	-	00 000000000000000000000000000000000000	100010110011001100110011	00 000000000000000000000000000000000000	r 4000000000000000000000000000000000000
110001001100110011001100110011001	-9.860760727515472e-33	+	-9.86U/68364144Ulle-33		-9.860/644009353196-53	-4.2ZI99ZU94ZU/b96e-/
101010101010101010101010101000000000000	+1.959157204660530e-38		00 9300160306030 11		00-00043944400043007 [1]	001000000000000000
0100000000110011001100110011001	+7.052964983894954e-39	1	00-00-00-00-00-00-00-00-00-00-00-00-00-	111101110111011101110110100000000000000	00-00-00-00-00-00-00-00-00-00-00-00-00-	TO: COTOZOOPECEPSOTOS: OT
10000100001010101010100001000011	-2.507721221965479e-36		20-0130000000000000000000000000000000000		20-030007320003113 0-	-0 00155255017055
01000000000101010101010101010101010	+7.836628538382429e-39		-2. 0100000000000000000000000000000000000	11000010000101010101010101010101010	00-2004670074600110.7-	0.0001000000000000000000000000000000000
01000010101010010010101010101000010	+8.009095588956748e-36		20 40030301030100000 01		30 000012000020010 01	2 0000000000000000000000000000000000000
110000000001111111111111111111111111	-1.175494210692441e-38	1	To: 0200303310030736-30		TO: 0143730007100005-50	
1100000000010101010101010101010101010	-7.836628538382429e-39		20 000000000000000000000000000000000000	010011001100110011001000000000000000000	70 0003k3 17703k000 C	000000000000000000000000000000000000000
1/000000101/001110111011100111011101	-2.319641991309260e-37	1	T6.6416/0/00360400E-0/		-2.2004007/1040300E-07	TO. 01 / 40440332433
1100000000010101010101010101010101010	-7.836628538382429e-39	+	No -00551515500000533 64		VC	1 500040000400000
010000111110101010101010101010101010	+2.567906439457154e-34	+			F0-04F030404100400434	
110000000010010010010010010000000000000	-6.717110175756367e-39	-	200000000000000000000000000000000000000	001100000110000110000110	70 00000000000000000000000000000000000	200000000000000000000000000000000000000
1010000010101000010101010101010101	+7.836628706538244e-37	+	+/*./0343/0004/00001E=3/		+/. 00304040404	+0.00456706705450
01000101010101010101010101010101010101	+2.567906531292650e-33	+	00 000000000000000000000000000000000000	1010111010101010101010	66-530KK3F00KC10F33 61	2 0000000000000000000000000000000000000
010000000000000000000000000000000000000	+1.175493930432748e-38	٢	T4.00/9106006019006-00	101011111111111111111111111111111111111	44.00/014400/044006-00	-4. 40000045550555605-0

The smaller the number, the greater the relative errors we have because each time, we are nearer to the bit error value.

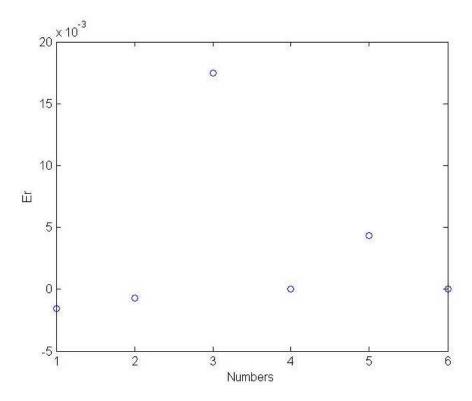


Figure 39. Relative error

The reason to get this error is that using mixed numbers we have to shift the subnormal number, first to standardize it and later to make equal the exponents causing a big displacement because the other number is normal and its exponent will be greater.

6.2.3. Normal Numbers

Finally the normal numbers turn. The results have been grouped in the table 13.

The relative error is not so big in most cases but there are two values where it increase so much. Two possible causes: the first one, as we have said, the greater the number, the greater the error we can have. The second one, as in the mixed numbers, we must shift one of the numbers losing some bits in the operation and increasing the error if the exponents are very different.

Table 13. Normal Numbers Results

MA/MB	A(10/ B(10	AS	A(10 ±B(10	s	S(10	Ēŗ
0 1111110101010101010101010111111101	+1.417843161699894e+38					¢
10001010101010101010101010111111110	+1.417842756051702e+38	ı	+4.055481920730334e+31	0 11101000 0000000000000000000000000000	+4.US648I9ZU73U334e+3I	П
0101010101010100100101010101010101010	+4.422139697774293e+35		201-010313031010311 1		3017010313031010311 1	c
0 11110101101010101010101010101010101	+5.538449850390212e+35	ı	-1:1103101030104131	0077070707777777777777070707077777	-1.1103101340134134133	Þ
01111111011100110011100110011001100	-1.701411733192644e+37					c
011111010101010010100101010101010101010	+1.415084703287774e+37	+	-2.0032702990467076+36		-2.8632702990487076+36	Þ
10111101111011101110111011101111111101	+1.799996137003937e+34		000000000000000000000000000000000000000			c
11111100001100111001100110110011000	-1.791851358103467e+34	+	+0.1447709004703026+31		+0.144//09004/09026+31	Þ
0 11111001 10111011101110111011101	+9.215980221460157e+36		601.00038300001360300 11	100111101001111110	E01400033E0003500 11	000 -250000101201030 61
1/111101100111001110011011011011100	-1.146784869186219e+36	ı	+1:0302/03030040300437		TI.0304/0340/33903ET3/	+3.0301001010442/0E-000
0 11111101 1010101010101010101010101010	+1.417843161699894e+38		0010000132210300000 61	0110010101010101010101010101010101010101	001000000000000000000000000000000000000	073126006000006 0-
1111111101110101010101010101010101010101	-1.417842756051702e+38		TC: 000000011/1/100000000		74. 20003401002300005130	0507/700700007:0-
0 10011001101110111011101110111011	+98426320	+	-13421752	000111010011001100110110110110110110110	-13421752	o
11100111001110101010101010101010010	-111848072	-	300		300 1131	ò
0 1011011011010101010101010101010001	+5.284222818320384e+16	+	3(10300)222222300375		9110900643806608336 91	036037018207011 07
0 10110011 101010101010101010101010001	+7.505997052510208e+15	+	011100011000100000000000000000000000000		011110000000000000000000000000000000000	6076040404040404

6.3. Conclusions

The importance and usefulness of floating point format nowadays does not allow any discussion. Any computer or electronic device which operates with real numbers implements this type of representation and operation.

During this report I have tried to explain the operation and benefits of using this notation against the fixed point. The main feature is that it can represent a very large or small numbers with a relatively small and finite quantity of memory.

The clear utility of the floating point format was the main reason why I decide to do this work. The other reason was the possibility of implementing it in VHDL. I could work on something that I like as programming and design something which has a current use. What is the result?

The reached goal is the implementation of a 32bits adder/subtractor based on floating point arithmetic according with the IEEE 754 standard.

This design works with all the numbers defined by the standard: normal and subnormal. Furthermore, all the exceptions are taken into account as NaN, zero or infinity.

The VHDL code has been implemented so that all the operations are carried out with combinational logic which reaches a faster response because there are not any sequential devices as flip-flops which delays the execution time.

If I have to defend this project I will appoint two features.

The first one deals with type of architecture used. For example, the adder or the shifter is implemented with a known structure. Predetermined operations as addition (+) or shifting (SLL or SLR) are allow but I decided using a *generate* function and designing my own device which improves the time response.

Finally the mixed numbers option. IEEE 754 does not say anything about the operations between subnormal and normal numbers. I have designed a trick which allows the operation. I standardize the subnormal number and set a "false positive subnormal exponent" (truly it is negative but IEEE 754 does not allow negative exponent prebiased). Adding the normal exponent and the false subnormal one I know the number of positions I must shift the mantissa and then the operation is done properly.

Obviously the design is not perfect. The accuracy is not optimal. In the future using a double precision format would be an improvement. The execution time would increase but the accuracy also would be better.

Maybe a complete FPU design would be another good improvement. Multiplication and division have an easier implementation than the addition/subtraction and it would do the project more complete.

Finally using the code over a FPGA and testing it physically over a board would be the last aim which would leave the design completely finished.

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<!-- The state of the state o

ANNEX: VHDL CODE

PRE-ADDER BLOCK: n_case Block

```
2
      -- Floating point adder (32 bits)
 3
      -- Block 01 --> n case
      -- Description: Identify the types of data between:
          -NaN
 б
                   --> E=255 & T>0
          -Infinity --> E=255 & T=0
      -- -Normal
                     --> 0<E<255 & T>0
 8
      -- -Subnormal --> E=0 & T>0
 9
10
          -Zero
                  --> E=0 & T=0
      -- and solve the operations which we can made without the adder
11
12
13
14
      library IEEE;
15
      use IEEE.STD_LOGIC_1164.ALL;
16
      use IEEE.STD LOGIC ARITH.ALL;
      use IEEE.STD_LOGIC_UNSIGNED.ALL;
17
18
19
20
      -- Entity declaration
21
22
23
      entity n case is
        Port ( Number A : in std_logic_vector(31 downto 0); -- Number A Number B : in std_logic_vector(31 downto 0); -- Number B
24
25
26
                    enable : out std logic;
                                                                   -- Enable Adder
27
                    s : out std_logic_vector(31 downto 0)); -- Output
28
      end n case;
29
30
31
      -- Architecture description
32
33
34
      architecture behavioral of n_case is
35
36
      -- Signals declaration
37
38
39
        signal outA, outB : std logic vector(2 downto 0);
40
41
        signal EA, EB
                           : std_logic_vector(7 downto 0);
                            : std_logic_vector(22 downto 0);
42
        signal MA, MB
                            : std_logic;
        signal SA, SB
43
44
        signal SS)
45
                             : std logic;
                             : std_logic_vector(7 downto 0);
        signal ES
46
        signal MS
47
                             : std logic vector(22 downto 0);
48
49
        begin
5.0
51
          SA <= NumberA(31);
52
          \overline{SB} \ll NumberB(31);
          EA <= NumberA(30 downto 23);
53
          EB <= NumberB(30 downto 23);
54
55
          MA <= NumberA(22 downto 0);</pre>
56
         MB <= NumberB(22 downto 0);
57
58
          outA \leftarrow "000" when EA = X"00" and MA = 0 else
                                                                                 -- Zero
                   "001" when EA = X"00" and MA > 0 else
59
                                                                                 -- Subnormal
60
                    "011" when (EA > X"00" and EA < X"FF") and MA > 0 else
                                                                                 -- Normal
                    "100" when EA = X"FF" and MA = 0 else
61
                                                                                 -- Infinity
62
                    "110" when EA = X"FF" and MA > 0 else
                                                                                  -- NaN
                    "000";
63
64
          outB <= "000" when EB = X"00" and MB = 0 else
65
                                                                                  -- Zero
                   "001" when EB = X"00" and MB > 0 else
66
                                                                                 -- Subnormal
                    "011" when (EB > X"00" and EB < X"FF") and MB > 0 else
67
                                                                               -- Normal
                    "100" when EB = X"FF" and MB = 0 else
68
                                                                                 -- Infinity
                    "110" when EB = X"FF" and MB > 0 else
                                                                                 -- NaN
69
                    "000";
```

```
71
 72
          -- If A and B are normal or subnormal numbers, enable = 1
          -- If not, enable = 0
enable <= '1' when ((outA(0) and outB(0)) = '1') else '0';</pre>
 73
 74
 75
 76
          process (SA, SB, outA, outB)
 77
           begin
             Zero (outA = "000") then
 78
 79
                                                               -- Zero +/- Number B
 80
                SS <= SB;
 81
                ES <= EB;
 82
               MS <= MB;
 83
              elsif (outB = "000") then
                                                               -- Number A +/- Zero
 84
               SS <= SA;
 85
                ES <= EA;
 86
               MS <= MA;
 87
             end if;
 88
              ----- Infinite
            if (outA(0) = '1' and outB = "100") then
89
                                                               -- Normal or Subnormal +/- Infinity
                SS <= SB;
 90
                ES <= EB;
 91
92
93
               MS <= MB;
              elsif (outB(0) = '1' and outA = "100") then
                                                               -- Infinity +/- Normal or Subnormal
94
                SS <= SA;
 95
                ES <= EA;
96
               MS <= MA;
97
              end if;
98
99
              if ((outA and outB) = "100" and SA = SB) then
                                                               -- +/- Infinity +/- Infinity
100
              SS <= SA;
101
                ES <= EA;
102
               MS <= MA;
103
               ----- NaN
              elsif ((outA and outB) = "100" and SA /= SB) then -- + Infinity - Infinity
104
105
                SS <= '1';
                ES <= X"FF";
106
               107
              end if:
108
109
              if (outA = "110" or outB = "110") then
                SS <= '1';
110
                ES <= X"FF";
111
               MS <= "0000000000000000000000000001";
112
            end if;
113
               ----- Normal / Subnormal
114
              if((outA(0) and outB(0)) = '1') then

SS <= '-';
ES <= "----";
115
116
117
               MS <= "----";
118
119
            end if:
120
          end process;
121
122
          S(31) <= SS;
          S(30 downto 23) <= ES;
123
          S(22 downto 0) <= MS;
124
125
126
        end behavioral;
```

PRE-ADDER BLOCK: Select Block

```
-- Floating point adder (32 bits)
     -- Block 02 --> Pre - Adder
     -- Sub Block 1 --> Selector
5
6
     -- Description: Identify the type of the numbers between:
     -- - Normal & Subnormal (or Subnormal & Normal)
-- - Normal & Normal
-- Subnormal & Subnormal
8
9
10
     -- and activate the correspondent block.
11
     -- Moreover, I add the implicit and the guard bits
12
13
     library IEEE;
14
15
     use IEEE.STD_LOGIC_1164.ALL;
16
     use IEEE.STD LOGIC ARITH.ALL;
17
     use IEEE.STD LOGIC UNSIGNED.ALL;
18
19
20
     -- Entity declaration
21
22
23
     entity selector is
24
      Port ( NumberA : in std_logic_vector(31 downto 0);
                  NumberB : in std logic vector(31 downto 0); -- Number B
25
                  26
27
                 NA : out std logic vector(36 downto 0); -- Number A'
NB : out std_logic_vector(36 downto 0)); -- Number B'
28
29
30
     end selector;
31
32
33
     -- Architecture description
34
35
36
     architecture behavioral of selector is
37
38
     -- Signals declaration
39
      40
41
42
43
44
       begin
45
          SA <= NumberA(31);
46
         SB <= NumberB(31);
47
48
         EA <= NumberA(30 downto 23);
49
         EB <= NumberB(30 downto 23);</pre>
50
         MA <= NumberA(22 downto 0);
51
         MB <= NumberB(22 downto 0);</pre>
52
53
         process (SA, SB, EA, EB, MA, MB, enable)
54
         begin
          if enable = '1' then
55
56
             NA(36) \le SA;
                                          -- Exponent & sign A
57
             NA (35 downto 28) <= EA;
58
             NB(36) \le SB;
                                          -- Exponent & sign B
59
             NB(35 downto 28) <= EB;
60
              ----- Mantissa A
61
             if (EA > X"00") then
               NA(27) <= '1';
62
                                          -- Implicit bit
63
               NA(26 \text{ downto } 4) \le MA;
                                          -- Mantissa
               NA(3 downto 0) <= X"0";
                                          -- Guard bits
64
             elsif EA = X"00" then
65
66
               NA(27) <= '0';
                                          -- Implicit bit
67
               NA(26 \text{ downto } 4) \le MA;
                                          -- Mantissa
68
               NA(3 downto 0) <= X"0";
                                          -- Guard bits
69
             else
70
               NA <= "----";
71
             end if:
72
              ----- Mantissa B
```

```
74
75
76
77
78
79
        NB (26 downto 4) <= MB;
                      -- Mantissa
                     -- Guard bits
80
        NB(3 downto 0) <= X"0";
81
       else
       NB <= "----";
82
83
       end if:
84
85
       NB <= "----";
86
87
      end if:
88
     end process;
89
     90
91
92
93
94
95
    end behavioral;
```

PRE-ADDER BLOCK: Normal Numbers: Comp_Exp Block

```
2
      -- Floating point adder (32 bits)
 3
      -- Block 02 --> Pre - Adder
 4
           Sub Block 2 --> Normal Numbers
 5
      -- Description: Prepare normal numbers for addition or subtraction operation
 8
      -- Comp_exp: Calculate the difference between exponents and the largest one
           Determine the largest number (to calculate the output's sign)
 g
10
                  Determine the shortest mantissa to shift in the next block
11
12
13
      library IEEE;
14
      use IEEE.STD_LOGIC_1164.ALL;
15
      use IEEE.STD_LOGIC_ARITH.ALL;
16
      use IEEE.STD_LOGIC_UNSIGNED.ALL;
17
18
19
      -- Entity declaration
20
21
22
      entity comp exp is
23
                    NumberA : in std_logic_vector(36 downto 0); -- Number A
                    NumberB : in std logic vector(36 downto 0); -- Number B
24
25
                    SA : out std_logic;
                                                  -- Sign A
26
                    SB
                            : out std logic;
                                                                   -- Sign B
27
                    Emax : out std logic vector(7 downto 0); -- Output exponent
                    Mmax : out std_logic_vector(27 downto 0); -- Largest Mantissa
Mshft : out std_logic_vector(27 downto 0); -- Mantissa to shift
28
29
30
                     Dexp : out std logic vector(4 downto 0); -- Subtraction of exponents
31
                           : out std logic);
                                                                    -- Determine largest number
                    Comp
32
      end comp_exp;
33
34
35
      -- Architecture description
36
37
38
      architecture behavioral of comp exp is
39
40
      -- Signals declaration
41
       signal EA, EB : std_logic_vector(7 downto 0);
signal MA, MB : std_logic_vector(27 downto 0);
42
43
44
        signal dif : std_logic_vector(7 downto 0);
signal C : std_logic;
45
46
47
48
        begin
49
50
         SA \ll NumberA(36);
                                                -- Sign A & B
51
          SB <= NumberB(36);
52
          EA <= NumberA(35 downto 28);
53
                                               -- Exponent & Mantissa
54
          EB <= NumberB(35 downto 28);
          MA <= NumberA(27 downto 0);
55
56
          MB <= NumberB(27 downto 0);</pre>
57
58
           ----- Exponent Comparison
59
          C \ll 1' when (EA > EB) or (MB(0) = 1') else
60
                                                             -- Exponent A > Exponent B
61
               '0' when EA < EB else
                                                             -- Exponent B > Exponent A
               '1' when MA >= MB else
                                                             -- EA = EB --> A > B
62
               '0' when MA < MB else
                                                             -- EA = EB --> B > A
63
               T- T;
64
65
66
          Comp <= C;
67
68
               ----- Largest exponent
          Emax <= EA when C = '1' else
70
                EB when C = '0' else
                  "----"":
71
72
```

```
73
         ----- Difference between exponents
        dif <= EA-EB when (C = '1') and (MB(0) = '0') else
74
               EB-EA when C = '0' else
75
               EA+EB when (C = '1') and (MB(0) = '1') else
76
77
78
79
       process (dif)
80
         begin
81
82
           if dif <= X"1B" then
                                     -- If the difference is less than or equal to 27...
            83
           elsif dif > X"1B" then
84
                                     -- The difference is 28
85
             Dexp <= "11100";
86
           else
            Dexp <= "----";
87
88
           end if:
89
90
          end process;
91
92
        ----- Mantissa
93
94
        Mshft <= MB when C = '1' else
95
              MA when C = '0' else
96
97
        Mmax <= MA when C = '1' else
98
               MB when C = '0' else
99
100
101
    end behavioral;
```

PRE-ADDER BLOCK: Normal Numbers: Shift Block: MUX Entity

```
-- Logarithmic Shifter
3
     ______
4
    library IEEE;
     use IEEE.STD_LOGIC 1164.ALL;
6
     use IEEE.STD_LOGIC_ARITH.ALL;
7
    use IEEE.STD LOGIC UNSIGNED.ALL;
8
g
10
    -- Entity declaration
11
    -----
12
     entity MUX is port (
13
     A, B, Sel : in std_logic; Z : out std_logic);
14
15
    end MUX;
16
17
18
     -- Architecture description
19
20
     architecture behavioral of MUX is
21
22
23
     Z <= A when Sel = '1' else
24
25
26
27
     end behavioral;
```

PRE-ADDER BLOCK: Normal Numbers: Shift Block

```
2
     -- Floating point adder (32 bits)
     -- Block 02 --> Pre - Adder
 4
5
          Sub Block 2 --> Normal Numbers
     -- Description: Prepare normal numbers for addition or subtraction operation
 6
8
     -- Shift: Shift the shortest significand to do the addition/subtraction
g
10
11
     library IEEE;
     use IEEE.STD_LOGIC_1164.ALL;
12
13
     use IEEE.STD LOGIC ARITH.ALL;
     use IEEE.STD LOGIC UNSIGNED.ALL;
14
15
16
17
     -- Entity declaration
18
19
20
    entity shift is
                       : in std_logic_vector(27 downto 0); -- Significand to shift
: in std_logic_vector(4 downto 0); -- Exponent's subtraction
      Port ( T
21
                 Shft : in std logic vector(4 downto 0); -- Exponer
s : out std_logic_vector(27 downto 0)); -- Output
22
23
24
    end shift;
25
26
2.7
     -- Architecture description
28
     _____
29
30
     architecture behavioral of shift is
31
32
     -- Signals and components declaration
33
34
35
       component MUX port (A, B, Sel : in std logic; Z : out std logic); end component;
36
37
       signal Z1, Z2, Z3, Z4, Z5 : std logic vector(27 downto 0);
38
39
       begin
41
      -- Components generation
43
      Compl: for i in 0 to 27 generate
       46
       shifter0_i: if ((i>0) and (i<28)) generate
48
                50
                end generate;
       shifter1_0: if ((i>=0) and (i<2)) generate
52
53
                shifter1_0comp: MUX port map (A => '0', B => Z1(27-i), Sel => Shft(1), Z => Z2(27-i));
54
55
       end generate;
shifter1_i: if ((i>1) and (i<28)) generate
                shifter1_icomp: MUX port map (A => Z1(27-(i-2)), B => Z1(27-i), Sel => Shft(1), Z => Z2(27-i));
57
                end generate;
       59
60
       end generate;
shifter2 i: if ((i>3) and (i<28)) generate
61
                64
                end generate;
       66
       end generate;
shifter3 i: if ((i>7) and (i<28)) generate
68
70
71
                shifter3\_icomp: MUX port map (A => Z3(27-(i-8)), B => Z3(27-i), Sel => Shft(3), Z => Z4(27-i));
                end generate;
       shifter4_0: if ((i>=0) and (i<16)) generate
73
                shifter4_0comp: MUX port map (A => '0', B => Z4(27-i), Sel => Shft(4), Z => Z5(27-i));
75
       end generate;
shifter4_i: if ((i>15) and (i<28)) generate
                77
                end generate;
79
80
      end generate;
81
82
      s <= 25:
83
84
    end behavioral;
```

PRE-ADDER BLOCK: Normal Numbers: n_normal Block

```
-- Floating point adder (32 bits)
       -- Block 02 --> Pre - Adder
              Sub Block 2 --> Normal Numbers
       -- Description: Prepare normal numbers for addition or subtraction operation
       library IEEE;
       use IEEE.STD_LOGIC_1164.ALL;
use IEEE.STD_LOGIC_ARITH.ALL;
10
11
12
       use IEEE.STD_LOGIC_UNSIGNED.ALL;
13
14
15
       -- Entity declaration
16
17
18
       entity n normal is
                        NumberA : in std_logic_vector(36 downto 0); -- Number A
NumberB : in std_logic_vector(36 downto 0); -- Number B
Comp : out std_logic; -- A & B Com
19
20
21
                                                                                 -- A & B Comparison
                                : out std_logic; -- A & B Comparison
: out std_logic; -- Sign A
: out std_logic; -- Sign B
: out std_logic_vector(7 downto 0); -- Exponent Output
: out std_logic_vector(27 downto 0); -- Greatest Mantissa
: out std_logic_vector(27 downto 0)); -- Shifted Mantissa
23
                        SB
24
                        EO
25
26
27
       end n_normal;
28
29
30
       -- Architecture description
31
32
33
       architecture behavioral of n_normal is
34
35
       -- Signals and components declaration
36
37
38
          component comp_exp port (NumberA, NumberB : in std_logic_vector(36 downto 0);
                                         (NumberA, NumberB : In Starlogic vector(7 downto 0);

Emax : out std_logic_vector(7 downto 0);
39
40
                                                          : out std logic vector(27 downto 0);
: out std_logic_vector(4 downto 0);
: out std_logic);
41
                                         Mmax, Mshft
42
                                         Dexp
43
                                         Comp
44
          end component;
45
          46
47
48
49
          end component;
50
          51
52
53
54
55
          comp0 : comp_exp
             port map (NumberA => NumberA, NumberB => NumberB,
56
                          SA => SA, SB => SB, Emax => EO, Mmax => MA, Mshft => Mshft aux, Dexp => Dexp aux, Comp => Comp);
57
58
59
          comp1 : shift
              port map (T => Mshft_aux, shft => Dexp_aux,
60
61
63
       end behavioral;
64
```

PRE-ADDER BLOCK: Subnormal Numbers: n_subn Block

```
-- Floating point adder (32 bits)
      -- Block 02 --> Pre - Adder
            Sub Block 3 --> SubNormal Numbers
 6
      -- Description: Prepare subnormal numbers for addition or subtraction operation
 8
      library IEEE;
10
      use IEEE.STD LOGIC 1164.ALL;
11
      use IEEE.STD_LOGIC_ARITH.ALL;
12
      use IEEE.STD LOGIC UNSIGNED.ALL;
13
14
15
      -- Entity declaration
16
17
18
      entity n_subn is
                     NumberA : in std_logic_vector(36 downto 0);
NumberB : in std_logic_vector(36 downto 0);
                                                                      -- Number A
19
20
                     Comp : out std_logic;
21
                                                                       -- Comparison A & B
                             : out std_logic;
: out std_logic;
22
                                                                       -- Sign A
                                                                       -- Sign B
23
                         : out std logic vector(7 downto 0); -- Exponent Ou

: out std logic vector(27 downto 0); -- Mantissa A

: out std_logic_vector(27 downto 0)); -- Mantissa B
                                                                      -- Exponent Output
-- Mantissa A
24
                     EO
25
26
                     MB
27
      end n_subn;
28
29
30
      -- Architecture description
31
32
33
      architecture behavioral of n subn is
34
35
      -- Signals declaration
36
37
        signal MAa, MBb : std_logic_vector(27 downto 0);
signal C : std_logic;
38
39
40
41
       begin
42
          SA <= NumberA(36);
43
                                                    -- Sign A & B
44
          SB <= NumberB(36);
45
          MAa <= NumberA(27 downto 0);</pre>
46
                                                    -- Mantissa A & B
47
          MBb <= NumberB(27 downto 0);</pre>
48
49
         ----- Number Comparison
              50
                   '0' when MBb > MAa else
5.1
52
53
54
          Comp <= C;
55
                                        ----- Output's exponent
56
57
          EO <= NumberA(35 downto 28);</pre>
58
59
          MB <= MBb when C = '1' else
60
                MAa when C = '0' else
61
62
          MA <= MAa when C = '1' else
63
64
               MBb when C = '0' else
65
66
        end behavioral;
```

PRE-ADDER BLOCK: Mixed Numbers: Comp Block

```
-- Floating point adder (32 bits)
 3
     -- Block 02 --> Pre - Adder
 5
          Sub Block 4 --> Mixed numbers
      -- Description: Prepare a mix of numbers (normal & subnormal) for addition
                      or subtraction operation
 8
 9
      -- Comp : Determine the subnormal number
10
11
12
      library IEEE;
13
      use IEEE.STD LOGIC 1164.ALL;
     use IEEE.STD_LOGIC_ARITH.ALL;
use IEEE.STD_LOGIC_UNSIGNED.ALL;
14
15
16
17
18
      -- Entity declaration
19
20
21
      entity comp is
       Port ( NumberA : in std_logic_vector(36 downto 0); -- Number A

NumberB : in std_logic_vector(36 downto 0); -- Number B

NA : out std_logic_vector(36 downto 0); -- Normal number
22
23
24
25
                        : out std_logic_vector(36 downto 0)); -- Subnormal number
26
     end comp;
27
28
29
      -- Architecture description
30
31
32
     architecture behavioral of comp is
33
34
      -- Signals declaration
35
36
       signal EA, EB : std logic vector(7 downto 0);
37
38
       begin
39
40
         EA <= NumberA(35 downto 28); -- Exponent & Mantissa
41
         EB <= NumberB(35 downto 28);
42
43
        process (NumberA, NumberB, EA, EB)
44
           begin
45
              if EA = X"00" then
46
                                        -- If Number A is subnormal...
47
              NB <= NumberA;
                NA <= NumberB;
48
              elsif EB = X"00" then
49
                                             -- If Number B is subnormal...
50
                NB <= NumberB;
51
                NA <= NumberA;
52
              else
53
                NA <= "----";
54
                NB <= "----";
55
             end if:
56
57
            end process;
59
    end behavioral;
```

PRE-ADDER BLOCK: Mixed Numbers: Zero Block

```
2
      -- Floating point adder (32 bits)
 3
      -- Block 02 --> Pre - Adder
 4
         Sub Block 4 --> Mixed numbers
 6
      -- Description: Prepare subnormal numbers to be operated with the normal ones
 Я
      -- Zero: Count the number of zeros to shift the subnormal number
10
11
      library IEEE;
12
      use IEEE.STD_LOGIC_1164.ALL;
13
      use IEEE.STD LOGIC ARITH.ALL;
14
      use IEEE.STD LOGIC UNSIGNED.ALL;
15
17
      -- Entity declaration
18
19
20
      entity zero is
       Port ( T : in std_logic_vector(2/ downto 0), -- Number of Zeros
                    T : in std_logic_vector(27 downto 0);
21
22
23
      end zero;
24
25
26
      -- Architecture description
27
28
29
      architecture behavioral of zero is
30
31
      -- Signals and components declaration
32
33
34
        signal Zero_vector : std_logic_vector(27 downto 0);
35
        signal aux
                       : std logic vector(7 downto 0);
36
37
38
        Zero vector <= X"0000000";</pre>
39
40
                 <= "----" when T(27 downto 27) = "-" else
41
42
                   X"1C" when T(27 \text{ downto } 0) = Zero \text{ vector}(27 \text{ downto } 0) else
43
                    X"1B" when T(27 downto 1) = Zero_vector(27 downto 1) else
                    X"1A" when T(27 \text{ downto } 2) = Zero\_vector(27 \text{ downto } 2) else
44
45
                    X"19" when T(27 downto 3) = Zero_vector(27 downto 3) else
                    X"18" when T(27 \text{ downto } 4) = Zero\_vector(27 \text{ downto } 4) else
46
                    X"17" when T(27 \text{ downto } 5) = Zero vector(27 \text{ downto } 5) else
47
                    X"16" when T(27 downto 6) = Zero_vector(27 downto 6) else
48
49
                    X"15" when T(27 \text{ downto } 7) = Zero \text{ vector}(27 \text{ downto } 7) else
                    X''14'' when T(27 \text{ downto } 8) = Zero vector(27 \text{ downto } 8) else
50
51
                    X"13" when T(27 downto 9) = Zero_vector(27 downto 9) else
                    X"12" when T(27 \text{ downto } 10) = Zero_vector(27 \text{ downto } 10) else
52
                    X"11" when T(27 downto 11) = Zero vector(27 downto 11) else
53
                    X"10" when T(27 downto 12) = Zero_vector(27 downto 12) else
54
                    X"0F" when T(27 downto 13) = Zero_vector(27 downto 13) else
55
                    X"0E" when T(27 downto 14) = Zero vector(27 downto 14) else
                    X"OD" when T(27 \text{ downto } 15) = Zero\_vector(27 \text{ downto } 15) else
57
                    X"OC" when T(27 \text{ downto } 16) = Zero_vector(27 \text{ downto } 16) else
58
                    X"0B" when T(27 downto 17) = Zero_vector(27 downto 17) else
59
                    X"0A" when T(27 \text{ downto } 18) = Zero\_vector(27 \text{ downto } 18) else
60
                    X"09" when T(27 downto 19) = Zero_vector(27 downto 19) else
61
                    X"08" when T(27 \text{ downto } 20) = Zero_vector(27 \text{ downto } 20) else
62
63
                    X"07" when T(27 downto 21) = Zero_vector(27 downto 21) else
                    X"06" when T(27 \text{ downto } 22) = Zero \text{ vector}(27 \text{ downto } 22) else
64
                    X"05" when T(27 \text{ downto } 23) = Zero_vector(27 \text{ downto } 23) else
65
                    X"04" when T(27 downto 24) = Zero_vector(27 downto 24) else
66
67
                    X"03" when T(27 \text{ downto } 25) = Zero \text{ vector}(27 \text{ downto } 25) else
                    X"02" when T(27 downto 26) = Zero_vector(27 downto 26) else
68
                    X"01" when T(27 downto 27) = Zero_vector(27 downto 27) else
69
70
                    x"00";
71
72
        Zcount <= aux(4 downto 0);</pre>
73
      end behavioral;
```

PRE-ADDER BLOCK: Mixed Numbers: shift_left Block

```
2
      -- Floating point adder (32 bits)
 3
      -- Block 02 --> Pre - Adder
 5
           Sub Block 4 --> Mixed Numbers
 6
      -- Description: Prepare normal numbers for addition or subtraction operation
 8
      -- Shift left: Shift the subnormal significand to get a normal one
9
10
11
      library IEEE;
      use IEEE.STD LOGIC 1164.ALL;
12
13
      use IEEE.STD_LOGIC_ARITH.ALL;
14
      use IEEE.STD LOGIC UNSIGNED.ALL;
15
16
17
      -- Entity declaration
18
19
20
      entity shift_left is
          Port ( T : in std_logic_vector(27 downto 0); -- Significand to shift
Shft : in std_logic_vector(4 downto 0); -- Exponent's subtraction
21
22
                            : out std_logic_vector(27 downto 0)); -- Output
23
                     S
      end shift_left;
2.4
25
26
27
      -- Architecture description
28
29
30
      architecture behavioral of shift left is
31
32
      -- Signals and components declaration
33
34
35
        component MUX port (A, B, Sel : in std logic; Z : out std logic); end component;
36
       signal Z1, Z2, Z3, Z4, Z5 : std_logic_vector(27 downto 0);
37
38
39
       begin
40
41
        -- Components generation
42
43
       Compl: for i in 0 to 27 generate
44
45
         shifter0 0: if (i=0) generate
                     shifter0 Ocomp: MUX port map (A \Rightarrow 0, B \Rightarrow T(0), Sel \Rightarrow Shft(0), Z \Rightarrow Z1(i);
46
47
                     end generate:
         shifter0 i: if ((i>0) and (i<28)) generate
48
                     49
50
                     end generate;
51
52
         shifter1 0: if ((i>=0)) and (i<2)) generate
53
                     shifter1 0comp: MUX port map (A \Rightarrow '0', B \Rightarrow Z1(i), Sel \Rightarrow Shft(1), Z \Rightarrow Z2(i));
54
                     end generate;
55
         shifter1 i: if ((i>1) and (i<28)) generate
56
                     shifter1 icomp: MUX port map (A \Rightarrow Z1((i-2)), B \Rightarrow Z1(i), Sel \Rightarrow Shft(1), Z \Rightarrow Z2(i));
                     end generate;
         shifter2 0: if ((i>=0) and (i<4)) generate
                     shifter2 Ocomp: MUX port map (A \Rightarrow '0', B \Rightarrow Z2(i), Sel \Rightarrow Shft(2), Z \Rightarrow Z3(i));
                     end generate;
61
          shifter2 i: if ((i>3) and (i<28)) generate
                     shifter2 icomp: MUX port map (A \Rightarrow Z2((i-4)), B \Rightarrow Z2(i), Sel \Rightarrow Shft(2), Z \Rightarrow Z3(i));
                     end generate:
65
66
         shifter3 0: if ((i>=0) and (i<8)) generate
                     end generate;
69
         shifter3_i: if ((i>7) and (i<28)) generate
70
                     shifter3\_icomp: MUX port map (A => Z3((i-8)), B => Z3(i), Sel => Shft(3), Z => Z4(i));
71
                     end generate;
72
73
         shifter4 0: if ((i>=0) and (i<16)) generate
                     shifter4 0comp: MUX port map (A \Rightarrow '0', B \Rightarrow Z4(i), Sel \Rightarrow Shft(4), Z \Rightarrow Z5(i));
                     end generate;
         shifter4_i: if ((i>15) and (i<28)) generate
77
                     end generate;
78
```

```
79
80 end generate;
81
82 s <= Z5;
83
84 end behavioral;
```

PRE-ADDER BLOCK: Mixed Numbers: norm Block

```
-- Floating point adder (32 bits)
     -- Block 02 --> Pre - Adder
          Sub Block 4 --> Mixed numbers
     -- Description: Prepare the subnormal number for addition or subtraction
                  with the normal one
 8
9
     -- Norm: Normalize the subnormal number to operate like a normal number
10
11
     library IEEE;
12
     use IEEE.STD_LOGIC_1164.ALL;
use IEEE.STD_LOGIC_ARITH.ALL;
13
14
     use IEEE.STD_LOGIC_UNSIGNED.ALL;
15
16
17
18
     -- Entity declaration
19
20
21
     entity norm is
      22
23
2.4
25
26
     end norm;
27
28
29
     -- Architecture description
30
31
32
     architecture behavioral of norm is
33
34
     -- Signals and components declaration
35
36
37
       component zero port (T : in std_logic_vector(27 downto 0); Zcount : out std_logic_vector(4 downto 0));
38
       end component;
39
40
       41
42
43
                                       : out std logic vector(27 downto 0));
44
        end component;
45
        component comp port (NumberA : in std_logic_vector(36 downto 0);
46
47
                             NumberB : in std_logic_vector(36 downto 0);
                                  : out std_logic_vector(36 downto 0);
: out std_logic_vector(36 downto 0));
48
49
50
        end component;
51
                                  : std logic vector(4 downto 0);
52
        signal Zcount aux
53
                                  : std_logic_vector(7 downto 0);
: std_logic_vector(36 downto 0);
54
        signal EB
55
        signal NumberB_aux
56
        signal MB_aux
                                   : std_logic_vector(27 downto 0);
57
58
        begin
59
60
                      ----- Components declaration
61
        comp0 : zero
62
           port map (T => NumberB_aux(27 downto 0), Zcount => Zcount_aux);
63
64
        comp1 : shift_left
           port map (T => NumberB_aux(27 downto 0), shft => Zcount_aux, S => MB_aux);
65
66
67
       comp2 : comp
68
           port map (NumberA => NumberA, NumberB => NumberB, NA => MA, NB => NumberB aux);
69
70
        ----- New Exponent
```

```
71
72
        process(Zcount_aux, NumberB_aux, EB, MB_aux)
73
        begin
           if Zcount aux /= "----" then
74
            EB <= "000" & Zcount_aux; -- Number shifted

MB(27 downto 0) <= MB_aux(27 downto 1) & '1'; -- Bit 0 --> Mark
75
76
77
           else
            EB <= "----";
78
79
             MB(27 downto 0) <= MB_aux;
80
           end if:
           MB(35 downto 28) <= EB;
81
82
             MB(36) \le NumberB aux(36);
83
      end process;
84
85 end behavioral;
```

PRE-ADDER BLOCK: MUX/DEMUX

```
2
    -- Floating point adder (32 bits)
3
    -- Block 02 --> Pre - Adder
        Sub Block 5 --> DEMUX
5
6
    -- Description: Demultiplexor
8
9
    library IEEE;
10
    use IEEE.STD_LOGIC_1164.ALL;
11
    use IEEE.STD LOGIC ARITH.ALL;
    use IEEE.STD LOGIC UNSIGNED.ALL;
12
13
14
15
    -- Entity declaration
16
17
    entity demux is
18
              NumberA : in std_logic_vector(36 downto 0);
19
      Port (
                                               -- Number A
20
              NumberB : in std_logic_vector(36 downto 0); -- Number B
              e_data : in std_logic_vector(1 downto 0); -- Enable type data
NAO : out std_logic_vector(36 downto 0); -- Number A1
21
22
23
              NB0
                   : out std_logic_vector(36 downto 0); -- Number B1
24
              NA1
                    : out std logic vector(36 downto 0); -- Number A2
                   : out std logic vector(36 downto 0); -- Number B2
25
              NB1
                   : out std_logic_vector(36 downto 0); -- Number A3
26
              NA2
                    : out std logic vector(36 downto 0)); -- Number B3
27
              NB2
28
    end demux;
29
30
31
    -- Architecture description
32
33
34
    architecture behavioral of demux is
35
36
     begin
37
       process (NumberA, NumberB, e_data)
38
39
       begin
40
41
         case e_data is
                   ----- Subnormals
42
          when "00" => NA0 <= NumberA;</pre>
43
                      NBO <= NumberB;
44
45
                      NB1 <= "----";
46
                      NA2 <= "----";
47
                      NB2 <= "----";
48
49
                            ----- Normals
          when "01" => NA0 <= "-----
50
                      NBO <= "----";
51
52
                      NA1 <= NumberA;
53
                      NB1 <= NumberB;
                      NA2 <= "----":
54
                      NB2 <= "----";
55
                            ----- MiX
56
          when "10" => NA0 <= "------
57
                      NBO <= "----";
58
                      NA1 <= "----";
59
                      NB1 <= "----";
60
61
                      NA2 <= NumberA;
62
                      NB2 <= NumberB;
          when others => NAO <= "------
63
                      NBO <= "----";
64
                      NA1 <= "-----
65
                      NB1 <= "----";
66
                      NA2 <= "----";
67
68
                      NB2 <= "----";
69
         end case;
70
71
       end process;
72
73
      end behavioral:
```

```
2
      -- Floating point adder (32 bits)
 3
 4
      -- Block 02 --> Pre - Adder
 5
      -- Sub Block 6 --> MUX
 6
      -- Description: Multiplexor
 8
 9
      library IEEE;
10
      use IEEE.STD LOGIC 1164.ALL;
      use IEEE.STD LOGIC ARITH.ALL;
11
12
      use IEEE.STD_LOGIC_UNSIGNED.ALL;
13
14
15
      -- Entity declaration
16
17
18
      entity mux_ns is
                            : in std_logic_vector(36 downto 0); -- Normal A
: in std_logic_vector(36 downto 0); -- Normal B
19
         Port ( NorA
20
                    NorB
21
                    MixA : in std_logic_vector(36 downto 0); -- Mixed A
                    MixB : in std_logic_vector(36 downto 0); -- Mixed B
e_data : in std_logic_vector(1 downto 0); -- Enable type data
22
                    MixB
23
                     NA : out std logic vector(36 downto 0); -- Number A'
24
25
                    NB
                            : out std_logic_vector(36 downto 0)); -- Number B'
26
      end mux_ns;
27
28
29
      -- Architecture description
30
31
32
      architecture behavioral of mux ns is
33
34
       begin
35
          NA <= NorA when e_data = "01" else
36
                                                                -- Normal numbers
                MixA when e_data = "10" else
37
                                                                -- Mixed numbers
38
39
40
         NB <= NorB when e data = "01" else
                                                                 -- Normal numbers
                MixB when e_data = "10" else
41
                                                                 -- Mixed numbers
                 "_____":
42
43
44
        end behavioral;
45
```

```
-- Floating point adder (32 bits)
     -- Block 02 --> Pre - Adder
 5
      -- Sub Block 7 --> MUX
 6
      -- Description: Multiplexor
      library IEEE;
10
      use IEEE.STD_LOGIC_1164.ALL;
11
      use IEEE.STD LOGIC ARITH.ALL;
12
      use IEEE.STD LOGIC UNSIGNED.ALL;
13
14
15
      -- Entity declaration
16
17
18
      entity mux adder is
                           : in std logic;
                                                                     -- Sign A normal
19
       Port ( NorSA
20
                    NorsB
                            : in std logic;
                                                                     -- Sign B normal
                            : in std logic;
                                                                    -- Sign A subnormal
21
                    SubSA
22
                    SubSB
                            : in std_logic;
                                                                     -- Sign B subnormal
23
                    CompN
                            : in std logic;
                                                                     -- Comparison Normal numbers
                    CompS
24
                            : in std logic;
                                                                    -- Compaison Sub numbers
25
                                                                    -- Exponent Output normal
                    NorE
                            : in std logic vector(7 downto 0);
                                                                    -- Exponent Output subnormal
26
                    SubE
                            : in std logic vector(7 downto 0);
                           : in std_logic_vector(27 downto 0);
: in std_logic_vector(27 downto 0);
27
                    NorMA
                                                                    -- Mantissa normal A
                                                                    -- Mantissa normal B
28
                    NorMB
29
                    SubMA : in std_logic_vector(27 downto 0);
                                                                    -- Mantissa subnormal A
                    SubMB : in std logic vector(27 downto 0);
e_data : in std_logic_vector(1 downto 0);
30
                                                                    -- Mantissa subnormal B
31
                                                                    -- Enable type data
                           : out std_logic;
32
                    SA
                                                                     -- Sign A
                            : out std_logic;
                                                                    -- Sign B
33
                    SB
34
                                                                    -- Comparison
                    C
                           : out std_logic;
35
                    Е
                            : out std_logic_vector(7 downto 0);
                                                                    -- Output Exponent
36
                            : out std logic vector(27 downto 0); -- Mantissa A
                           : out std_logic_vector(27 downto 0)); -- Mantissa B
37
                    В
38
      end mux_adder;
39
40
41
      -- Architecture description
42
43
44
      architecture behavioral of mux adder is
45
46
       begin
47
48
          A <= NorMA when e_data = "01" or e_data = "10" else
                                                                         -- Normal/Mix numbers
49
                SubMA when e data = "00" else
                                                                         -- Subnormal numbers
50
51
          B <= NorMB when e_data = "01" or e_data = "10" else
                                                                         -- Normal/Mix numbers
52
                SubMB when e_data = "00" else
53
                                                                         -- Subnormal numbers
54
55
          C <= CompN when e_data = "01" or e_data = "10" else
56
                                                                         -- Normal/Mix numbers
57
                Comps when e data = "00" else
                                                                         -- Subnormal numbers
58
59
60
          SA <= NorSA when e_data = "01" or e_data = "10" else
                                                                         -- Normal/Mix sign A
                SubSA when e_data = "00" else
                                                                         -- Subnormal sign A
61
62
63
64
          SB <= NorSB when e_data = "01" or e_data = "10" else
                                                                         -- Normal / Mix sign B
65
                SubSB when e data = "00" else
                                                                         -- Subnormal sign B
66
67
          E <= NorE when e_data = "01" or e_data = "10" else
                                                                         -- Normal / Mix exponent
68
                SubE when e_data = "00" else
69
                                                                         -- Subnormal exponent
70
71
72
        end behavioral;
```

PRE-ADDER BLOCK: preadder Block

```
2
      -- Floating point adder (32 bits)
      -- Block 02 --> Pre - Adder
      -- Prepare the numbers to be used by the adder
 6
 8
      library IEEE;
      use IEEE.STD_LOGIC_1164.ALL;
 9
      use IEEE.STD_LOGIC_ARITH.ALL;
10
11
      use IEEE.STD LOGIC UNSIGNED.ALL;
12
13
14
      -- Entity declaration
15
16
17
      entity preadder is
          Port ( NumberA : in std_logic_vector(31 downto 0); -- Number A NumberB : in std_logic_vector(31 downto 0); -- Number B enable : in std_logic; -- Enable
18
19
20
                      SA : out std_logic;
SB : out std_logic;
21
                                                                        -- Sign A
22
                                                                        -- Sign B
                     C : out std_logic; -- Comparison

EOut : out std_logic_vector(7 downto 0); -- Exponent Output

MAOut : out std_logic_vector(27 downto 0); -- Greatest Mantissa

MBOut : out std_logic_vector(27 downto 0)); -- Shifted Mantissa
23
24
25
26
27
      end preadder;
28
29
30
       -- Architecture description
31
32
33
      architecture behavioral of preadder is
34
35
      -- Signals and components declaration
36
37
38
                                    ----- Normal Numbers
        component n_normal port (NumberA, NumberB : in std_logic_vector(36 downto 0);
39
                                   Comp : out std_logic; SA : out std_logic;
40
                                   SA
41
42
                                   SB
                                                      : out std logic;
                                                   : out std_logic_vector(7 downto 0);
: out std_logic_vector(27 downto 0));
43
                                   EO
                                   MA, MB
44
45
        end component;
46
47
48
        component mux ns port (NorA, NorB, MixA, MixB : in std logic vector(36 downto 0);
                                             : in std_logic_vector(1 downto 0);
49
                                 e_data
50
                                                           : out std logic vector(36 downto 0));
51
        end component;
52
53
        component demux port (NumberA, NumberB : in std_logic_vector(36 downto 0);
                                                                : in std_logic_vector(1 downto 0);
54
                                e data
                                NAO, NBO, NA1, NB1, NA2, NB2 : out std logic vector(36 downto 0));
55
56
        end component;
57
58
        component mux adder port (NorSA, NorSB, SubSA, SubSB : in std logic;
                                    CompN, CompS : in std logic;
59
'nП
                                    NorE, SubE : in std_logic_vector(7 downto 0);
NorMA, NorMB, SubMA, SubMB : in std_logic_vector(27 downto 0);
61
62
                                                    : in std_logic_vector(1 downto 0);
                                     e data
63
                                     SA, SB, C
                                                                  : out std_logic;
64
                                                                  : out std_logic_vector(7 downto 0);
                                    Е
65
                                    A, B
                                                                  : out std_logic_vector(27 downto 0));
66
        end component;
67
68
69
        component norm port (NumberA, NumberB : in std_logic_vector(36 downto 0);
70
                              MA, MB : out std_logic_vector(36 downto 0));
71
        end component;
72
73
         ------ Subnormal Numbers
        component n_subn port (NumberA, NumberB : in std_logic_vector(36 downto 0);
74
                                 Comp : out std_logic; SA : out std_logic;
75
76
```

```
: out std logic;
  78
                                                                                                 : out std_logic_vector(7 downto 0);
                                                                EO
  79
                                                                MA, MB
                                                                                                 : out std logic vector(27 downto 0));
  80
                  end component:
  81
  82
                                                                                                                                 ----- Selector
                  83
  84
  85
                                                                     e data
                                                                                                      : out std_logic_vector(1 downto 0);
  86
                                                                    NA, NB
                                                                                                      : out std logic vector(36 downto 0));
  87
                  end component;
  88
  89
                  signal NA_out_select, NB_out_select : std_logic_vector(36 downto 0);
  90
                  signal A sub, B sub : std logic vector(36 downto 0);
                  signal A nor, B nor : std logic vector(36 downto 0);
                  signal A mix, B mix : std logic vector(36 downto 0);
signal MixAaux, MixBaux : std logic_vector(36 downto 0);
  92
  93
  94
                  signal Amux, Bmux : std logic vector (36 downto 0);
  95
                  signal SAnor, SBnor, SAsub, SBsub, NComp, SComp : std_logic;
                  signal Enor, Esub : std_logic_vector(7 downto 0);
  96
  97
                  signal MAnor, MBnor, MAsub, MBsub : std_logic_vector(27 downto 0);
  98
  qq
                  signal edata : std_logic_vector(1 downto 0);
100
101
                  begin
102
103
                  comp0 : n_normal
104
                          port map (NumberA => Amux, NumberB => Bmux,
105
                                                  Comp => NComp, SA => SAnor, SB => SBnor, EO => Enor, MA => MAnor, MB => MBnor);
106
107
                  comp1 : n subn
108
                           port map (NumberA => A_sub, NumberB => B sub,
109
                                                 Comp => SComp, SA => SAsub, SB => SBsub, EO => Esub, MA => MAsub, MB => MBsub);
110
                  comp2 : norm
111
112
113
                           114
115
                comp3 : demux
116
                         port map (NumberA => NA_out_select, NumberB => NB_out_select, e_data => edata,
                                            NA0 \Rightarrow A \text{ sub, } NB0 \Rightarrow B \text{ sub, } NA1 \Rightarrow A \text{ nor, } NB1 \Rightarrow B \text{ nor, } NA2 \Rightarrow A \text{ mix, } NB2 \Rightarrow B \text{ mix)};
117
118
119
                comp4 : mux_ns
120
                         port map (NorA => A_nor, NorB => B_nor, MixA => MixAaux, MixB => MixBaux, e_data => edata,
121
                                            NA => Amux, NB => Bmux);
122
123
                comp5 : selector
                         port map (NumberA => NumberA, NumberB => NumberB, enable => enable,
124
125
                                            e_data => edata, NA => NA_out_select, NB => NB_out_select);
126
127
                comp6 : mux adder
128
                          port map (NorSA => SAnor, NorSB => SBnor, SubSA => SAsub, SubSB => SBsub, CompN => NComp, CompS => SComp,
129
                                            NorE => Enor, SubE => Esub, NorMA => MAnor, NorMB => MBnor, SubMA => MAsub, SubMB => MBsub,
130
                                             e_{a} = a_{a} = a_{a
131
            end behavioral;
```

ADDER BLOCK: Signout Block

```
2
      -- Floating point adder (32 bits)
      -- Block 03 --> Adder
 5
      -- Description: Prepare normal numbers for addition or subtraction operation
 6
 7
     -- Signout: Calculate the B's and output's sign because of addition or subtraction
 8
 g
10
      library IEEE;
      use IEEE.STD LOGIC 1164.ALL;
11
12
      use IEEE.STD_LOGIC_ARITH.ALL;
13
      use IEEE.STD_LOGIC_UNSIGNED.ALL;
14
15
16
      -- Entity declaration
17
18
19
      entity signout is
20
                          : in std_logic;
                                                                   -- Sign A
         Port ( SA
                    sb : in std_logic;
21
                                                                   -- Sign B
                    A
                          : in std logic vector(27 downto 0); -- Number A
: in std_logic_vector(27 downto 0); -- Number B
22
23
                    В
24
                    A_S : in std_logic;
                                                                   -- Add (0) or Sub (1)
                    Comp : in std_logic;
25
                                                                   -- Determine largest number
                           : out std logic vector(27 downto 0);
                                                                   -- Number A'
2.6
                    Aa
                         : out std_logic_vector(27 downto 0);
27
                                                                   -- Number B'
                         : out std_logic;
: out std_logic);
28
                    AS
                                                                   -- A S *
                                                                   -- Determine Output's sign
29
                    SO
30
      end signout;
31
32
33
      -- Architecture description
34
35
36
      architecture behavioral of signout is
37
38
      -- Signals declaration
39
40
41
        signal SB aux : std logic;
        signal Aaux, Baux : std logic vector(27 downto 0);
42
43
44
       begin
45
46
         SB aux <= SB xor A S;
                                                 -- Sign B because of the operation
47
          SO <= SA when Comp = '1' else
48
                                                 -- A > B --> Sign A
                SB_aux when Comp = '0' else
                                                 -- B > A --> Sign B
49
50
51
         AS <= '1' when SA /= SB_aux else
                                                -- Complement to 1 is needed when
52
               101;
53
                                                 -- the signs are different
54
          Aaux <= A when Comp = '1' else
55
                B when Comp = '0' else
56
57
5.8
59
          Baux <= B when Comp = '1' else</pre>
60
                 A when Comp = '0' else
61
62
63
          process (SA, SB_aux, Aaux, Baux)
64
65
            ----- if Sign A is equal to Sign B
66
            if (SA xor SB aux) = '0' then
67
               Aa <= Aaux;
                                                -- Nothing changes
68
               Bb <= Baux;
69
           ----- if Sign A is 1 and Sign B is 0
70
             elsif SA = '1' and SB_aux = '0' then
71
               Aa <= Baux;
                                                -- A is changed by B
               Bb <= Aaux;
72
```

```
73
74
        ----- if Sign A is 0 and Sign B is 1
         elsif SA = '0' and SB_aux = '1' then
75
         Aa <= Aaux;
76
                                  -- Nothing changes
77
          Bb <= Baux;
78
         else
           Aa <= "----";
79
           Bb <= "----";
80
81
         end if:
82
      end process;
83
     end behavioral;
```

ADDER BLOCK: Adder Block: CLA Entity

```
-- Adder Carry LookAhead
 2
 3
 4
     library IEEE;
     use IEEE.STD_LOGIC_1164.ALL;
 5
 6
     use IEEE.STD LOGIC ARITH.ALL;
     use IEEE.STD_LOGIC_UNSIGNED.ALL;
 8
 9
10
     -- Entity declaration
11
     entity CLA is port (
12
      A, B, Cin : in std_logic;
S, Cout : out std_logic);
13
14
15
     end CLA;
16
17
18
     -- Architecture description
19
20
     architecture behavioral of CLA is
21
22
     -- Signals declaration
23
      signal c_g, c_p : std_logic;
24
25
26
     begin
27
                                 -- Carry generation
28
      cg \ll AandB;
29
      c_p <= A xor B;
                                       -- Carry propagation
30
       Cout <= c_g or (c_p and Cin); -- Carry out
31
32
      S <= c_p xor Cin;
                                       -- Bit's sum
33
34
    end behavioral;
```

ADDER BLOCK: Adder Block

```
2
       -- Floating point adder (32 bits)
 3
 4
       -- Block 03 --> Adder
 5
       -- Description: Implement the addition with a CLA adder
       library IEEE;
 8
       use IEEE.STD_LOGIC_1164.ALL;
 9
       use IEEE.STD_LOGIC_ARITH.ALL;
10
11
       use IEEE.STD_LOGIC_UNSIGNED.ALL;
12
13
14
       -- Entity declaration
15
16
17
       entity adder is
18
                              : in std_logic_vector(27 downto 0);
                                                                             -- Number A
                       B : in std_logic_vector(27 downto 0);
A S : in std_logic;
                                                                           -- Number B
19
20
                                                                              -- Add(0) / Sub (1)
                              : out std_logic_vector(27 downto 0);
: out std_logic);
                                                                              -- Output
21
                        S
                                                                              -- Carry out
22
                       Co
       end adder;
23
2.4
25
26
       -- Architecture description
27
28
29
       architecture behavioral of adder is
30
31
       -- Signals and components declaration
32
33
34
         component CLA port (A, B, Cin : in std_logic; S, Cout : out std_logic); end component;
35
        signal B1, aux, S_aux : std_logic_vector(27 downto 0);
36
37
38
39
40
        -- Components generation
42
        Comp1: for i in 0 to 27 generate
43
         B1(i) <= B(i) xor A_S;
45
         sumador_0: if (i=0) generate
46
                     sumador\_0comp: CLA port map (A \Rightarrow A(i), B \Rightarrow B1(i), Cin \Rightarrow A\_S, S \Rightarrow S\_aux(i), Cout \Rightarrow aux(i));
         end generate;
sumador_i: if ((i>0) and (i<28)) generate
48
49
50
                     sumador\_icomp: CLA port map (A \Rightarrow A(i), B \Rightarrow B1(i), Cin \Rightarrow aux(i-1), S \Rightarrow S\_aux(i), Cout \Rightarrow aux(i));
                     end generate;
51
52
53
54
55
        end generate;
       S <= S aux;
56
57
58
       Co \leq aux(27);
     end behavioral;
```

ADDER BLOCK: Block_Adder Block

```
2
      -- Floating point adder (32 bits)
 3
      -- Block 03 --> Adder
 5
      -- Description: Implement the addition with a CLA adder
 6
 8
      library IEEE;
 q
      use IEEE.STD LOGIC 1164.ALL;
      use IEEE.STD LOGIC ARITH.ALL;
10
11
      use IEEE.STD_LOGIC_UNSIGNED.ALL;
12
13
14
      -- Entity declaration
15
16
17
      entity block_adder is
          Port ( SA : in std_logic;
18
                                                                    -- Sign A
                                                                    -- Sign B
19
                    SB
                          : in std logic;
20
                          : in std logic vector(27 downto 0);
                                                                    -- Number A
                         : in std_logic_vector(27 downto 0);
: in std_logic;
21
                    В
                                                                    -- Number B
                                                                    -- Add(0) / Sub (1)
22
23
                    Comp : in std logic;
                                                                    -- Comparison
24
                          : out std_logic_vector(27 downto 0);
                    S
                                                                    -- Output
                         : out std_logic;
                                                                    -- Output's sign
25
                    SO
26
                         : out std logic);
                    Co
                                                                    -- Carry out
27
      end block_adder;
28
29
30
      -- Architecture description
31
32
33
      architecture behavioral of block adder is
34
35
      -- Signals and components declaration
36
37
38
       component adder port (A, B : in std logic vector(27 downto 0);
39
                            A_S : in std_logic;
40
                            s : out std logic vector(27 downto 0);
41
                            Co : out std_logic);
42
       end component:
43
44
       45
46
47
                              A_S, Comp : in std_logic;
                              Aa, Bb : out std_logic_vector(27 downto 0);
As, so : out std_logic);
48
49
50
       end component;
51
52
       signal Aa_aux, Bb_aux, S_aux : std_logic_vector(27 downto 0);
53
       signal AS_aux, SO_aux, Co_aux : std_logic;
54
55
       begin
56
       component00: signout port map (SA => SA, SB => SB, A => A, B => B, A_S => A_S, Comp => Comp,
57
58
                                    Aa => Aa_aux, Bb => Bb_aux, AS => AS_aux, SO => SO_aux);
59
60
       component01: adder port map (A => Aa_aux, B => Bb_aux, A_S => AS_aux, S => S_aux, Co => Co_aux);
61
       ----- If a complement to 1 is used and Output's sign is 1 a C2 is needed
62
       S <= (S aux xor X"FFFFFFF")+'1' when ((AS_aux and SO_aux) = '1') else
63
64
             S aux;
65
       Co <= '0' when ((SB xor A_S) /= SA) else
66
67
            Co_aux;
68
69
       SO <= SO aux;
70
71
     end behavioral;
```

STANDARDIZING BLOCK: Round Block

```
2
      -- Floating point adder (32 bits)
      -- Block 04 --> Normalize
 5
      -- Description: Normalize the result
 6
 7
     -- round: Round the result deleting the guard bits
 8
      library IEEE;
10
11
      use IEEE.STD_LOGIC_1164.ALL;
12
      use IEEE.STD_LOGIC_ARITH.ALL;
     use IEEE.STD LOGIC UNSIGNED.ALL;
13
14
15
16
      -- Entity declaration
17
18
19
      entity round is
      Port ( Min : in std_logic_vector(27 downto 0); -- Input's mantissa
20
21
                   Mout : out std logic vector(22 downto 0)); -- Output's mantissa
22
23
24
25
      -- Architecture description
26
27
28
      architecture behavioral of round is
29
30
      -- Signals and components declaration
31
32
33
       signal M aux : std logic vector(22 downto 0);
34
35
      begin
36
      process (Min)
37
38
        begin
39
          if Min(3 downto 0) = "----" then
40
             M_aux <= "----";
41
          elsif Min(3 downto 0) >= "1000" then
42
                                                   -- Round Mantissa
43
            M aux <= Min(26 downto 4) + '1';</pre>
44
          else
           M aux <= Min(26 downto 4);</pre>
45
46
          end if:
47
        end process;
48
49
         Mout <= M_aux;
50
     end behavioral;
```

STANDARDIZING BLOCK: Vector Block

```
2
      -- Floating point adder (32 bits)
 3
 4
      -- Block 04 --> Normalize
 5
      -- Description: Normalize the result
 6
      -- vector: Regroup sign, exponent and mantissa in a single vector
 8
 9
10
      library IEEE;
      use IEEE.STD LOGIC 1164.ALL;
11
      use IEEE.STD_LOGIC_ARITH.ALL;
12
13
      use IEEE.STD_LOGIC_UNSIGNED.ALL;
14
15
16
     -- Entity declaration
17
```

```
18
19
     entity vector is
20
      Port ( S : in std logic;
                                                       -- Sign
                  21
22
23
                  N : out std_logic_vector(31 downto 0)); -- Vector
24
     end vector;
25
26
27
     -- Architecture description
28
29
30
     architecture behavioral of vector is
31
32
     -- Signals and components declaration
33
34
35
     begin
36
     N(31) \le S;
37
38
       N(30 downto 23) <= E;
39
      N(22 \text{ downto } 0) \le M;
40
     end behavioral;
```

STANDARDIZING BLOCK: Block_norm Block

```
-- Floating point adder (32 bits)
       -- Block 04 --> Normalize
            Sub Block 4 --> Normalize the result
 8
       library IEEE;
 9
       use IEEE.STD_LOGIC_1164.ALL;
10
       use IEEE.STD_LOGIC_ARITH.ALL;
11
       use IEEE.STD_LOGIC_UNSIGNED.ALL;
12
13
14
       -- Entity declaration
15
16
17
       entity block_norm is
18
                      MS : in std_logic_vector(27 downto 0);
ES : in std_logic_vector(7 downto 0);
                                                                      -- Number S
19
                                                                       -- Exponent S
                                                                       -- Carry out
20
                       Co : in std logic;
                       M : out std logic vector(22 downto 0); -- Output's Mantissa
E : out std_logic_vector(7 downto 0)); -- Output's Exponent
21
22
23
       end block_norm;
24
25
26
       -- Architecture description
27
28
29
       architecture behavioral of block norm is
30
31
       -- Signals and components declaration
32
33
34
         component zero port (T : in std_logic_vector(27 downto 0); Zcount : out std_logic_vector(4 downto 0));
35
         end component;
36
37
                                        (T : in std_logic_vector(27 downto 0);
Shft : in std_logic_vector(4 downto 0);
S : out std_logic_vector(27 downto 0));
38
         component shift_left port (T
39
40
41
         end component;
42
43
         component round port (Min : in std logic vector(27 downto 0);
44
                                 Mout : out std_logic_vector(22 downto 0));
45
         end component;
46
47
         signal Zcount_aux, Shift : std_logic_vector(4 downto 0);
                             : std logic vector(27 downto 0);
48
         signal Number
```

```
49
50
       begin
51
       ----- Components declaration
52
53
       comp0 : zero
54
        port map (T => MS, Zcount => Zcount aux);
5.5
56
      comp1 : shift_left
        port map (T => MS, shft => Shift, S => Number);
57
58
59
       comp2 : round
60
          port map (Min => Number, Mout => M);
61
       ----- Normal or Subnormal Number
62
63
                                               -- & New Wxponent
64
       process (MS, ES, Shift, Zcount_aux, Co)
65
66
67
          if Zcount aux = "----" then
           Shift <= "----";
E <= "----";
68
69
70
          elsif ES > Zcount aux then
                                          -- If the number is normal...
          Shift <= Zcount aux;
                                          -- ... the number is shifted --> Output normal
71
72
           E <= ES - Shift + Co;
73
         elsif ES < Zcount aux then
                                           -- If the number is normal...
74
           Shift <= ES(4 downto 0);
                                          -- ... the number is shifted --> Output subnormal
75
           E <= X"00";
76
          elsif ES = Zcount aux then
                                          -- If N° Zeros = Exponent...
77
          Shift <= Zcount_aux;
                                          -- ... the mantissa is shifted and EO = 1
78
           E <= X"01";
79
          end if:
80
81
        end process;
82
     end behavioral;
```

STANDARDIZING BLOCK: norm+vector Block

```
2
     -- Floating point adder (32 bits)
3
 4
     -- Block 04 --> Normalize
5
     -- Sub Block 4 --> Normalize the result
6
8
     library IEEE;
     use IEEE.STD_LOGIC_1164.ALL;
9
10
     use IEEE.STD LOGIC ARITH.ALL;
11
     use IEEE.STD LOGIC UNSIGNED.ALL;
12
13
14
     -- Entity declaration
15
                          ______
16
17
     entity norm_vector is
18
                 SS : in std logic;
                                                            -- Sign S
                   MS: in std_logic_vector(27 downto 0); -- Number S
ES: in std_logic_vector(7 downto 0); -- Exponent S
19
20
21
                   Co : in std logic;
                                                            -- Carry out
                   N : out std_logic_vector(31 downto 0)); -- Output Number
22
23
     end norm_vector;
24
25
26
      -- Architecture description
27
28
29
     architecture behavioral of norm vector is
30
31
     -- Signals and components declaration
```

```
33
34
        component block norm port (MS : in std logic vector(27 downto 0);
35
                                    ES : in std_logic_vector(7 downto 0);
36
                                    Co : in std_logic;
                                    M : out std_logic_vector(22 downto 0);
E : out std_logic_vector(7 downto 0));
37
38
39
        end component;
40
41
        component vector port (S : in std logic;
42
                                E : in std_logic_vector(7 downto 0);
M : in std_logic_vector(22 downto 0);
43
44
                                N : out std_logic_vector(31 downto 0));
45
46
        end component;
47
48
        signal Maux
                        : std_logic_vector(22 downto 0);
49
        signal Eaux
                        : std_logic_vector(7 downto 0);
50
51
52
53
        ----- Components declaration
54
       comp0 : block norm
55
         port map (MS => MS, ES => ES, Co => Co, M => Maux, E => Eaux);
56
57
       comp1 : vector
58
            port map (S => SS, E => Eaux, M => Maux, N => N);
59
60
      end behavioral;
61
```

32bit Floating Point Adder BLOCK: MUX

```
2
      -- Floating point adder (32 bits)
3
 4
 5
      library IEEE;
 6
      use IEEE.STD LOGIC 1164.ALL;
 7
      use IEEE.STD_LOGIC_ARITH.ALL;
      use IEEE.STD LOGIC UNSIGNED.ALL;
8
9
10
11
      -- Entity declaration
12
13
14
      entity mux_fpadder is
      Port ( N1 : in std_logic_vector(31 downto 0); -- N_case number N2 : in std_logic_vector(31 downto 0); -- Adder number
15
16
17
                     enable : in std_logic;
                     Result : out std_logic_vector(31 downto 0)); -- Result
18
19
      end mux_fpadder;
20
21
22
      -- Architecture description
23
24
25
      architecture behavioral of mux fpadder is
26
27
        begin
28
        Result <= N1 when enable = '0' else
29
                                                                  -- N_case number
30
                   N2 when enable = '1' else
                                                                   -- Adder number
31
32
        end behavioral;
```

32bit Floating Point Adder BLOCK:

```
-- Floating point adder (32 bits)
 3
 4
 5
      library IEEE;
      use IEEE.STD_LOGIC_1164.ALL;
     use IEEE.STD_LOGIC_ARITH.ALL;
use IEEE.STD_LOGIC_UNSIGNED.ALL;
10
11
      -- Entity declaration
12
13
14
      entity fpadder is
      Port ( NumberA : in std_logic_vector(31 downto 0); -- Number A NumberB : in std_logic_vector(31 downto 0); -- Number B
15
16
17
                    A_S : in std_logic;
                                                                  -- Add / Sub
18
                    Result : out std logic vector(31 downto 0)); -- Result
19
      end fpadder;
20
21
22
      -- Architecture description
23
24
25
      architecture behavioral of fpadder is
26
27
      -- Signals and components declaration
28
29
         n case
30
31
       component n_case port (NumberA, NumberB : in std_logic_vector(31 downto 0);
32
                              enable : out std logic;
s : out std logic_vector(31 downto 0));
33
34
35
       end component;
```

```
36
37
38
39
          component preadder port (NumberA, NumberB : in std_logic_vector(31 downto 0);
 40
                                      enable
                                                 : in std_logic;
 41
                                                         : out std logic;
                                      SA, SB
 42
                                      С
                                                        : out std logic;
                                                       : out std_logic_vector(7 downto 0);
: out std_logic_vector(27 downto 0));
 43
                                      EOut
 44
                                      MAOut, MBOut
 45
          end component;
 46
 47
 48
 49
         component block_adder port (SA, SB : in std_logic;
                                         A, B : in std_logic_vector(27 downto 0);
A_S : in std_logic;
50
51
 52
                                         Comp
                                                : in std logic;
53
                                                 : out std logic vector(27 downto 0);
54
                                                 : out std logic;
55
                                         Co
                                                : out std logic);
 56
         end component;
57
58
          component norm_vector port (SS : in std_logic;
59
                                         MS : in std_logic_vector(27 downto 0);
                                         ES : in std_logic_vector(7 downto 0);
Co : in std_logic;
 60
 61
 62
                                         N : out std_logic_vector(31 downto 0));
63
          end component;
 64
65
           ------ Mux fpadder
 66
 67
          component mux_fpadder port (N1, N2 : in std_logic_vector(31 downto 0);
                                          enable : in std logic;
 68
                                          Result : out std_logic_vector(31 downto 0));
 69
 70
          end component;
 71
 72
         signal MA_aux, MB_aux, MOut_aux : std_logic_vector(27 downto 0);
 73
         signal EOut_aux : std_logic_vector(7 downto 0);
        signal Comp_aux, Carry : std logic;
signal SA_aux, SB_aux, S_aux : std_logic;
signal enable_aux : std_logic;
 74
 75
 76
 77
         signal Ncase, Nadder : std_logic_vector(31 downto 0);
 78
 79
 80
         comp0 : preadder
81
             port map (NumberA => NumberA, NumberB => NumberB, enable => enable_aux,
SA => SA_aux, SB => SB_aux, C => Comp_aux, EOut => EOut_aux, MAOut => MA_aux, MBOut => MB_aux);
82
83
 84
 85
         comp1 : block adder
 86
               port map (SA => SA aux, SB => SB aux, A => MA aux, B => MB aux, A S => A S, Comp => Comp aux,
 87
                         S => Mout_aux, SO => S_aux, Co => Carry);
 88
 89
         comp2 : norm_vector
               port map (SS => S_aux, MS => MOut_aux, ES => EOut_aux, Co => Carry,
911
                        N => Nadder);
 91
 92
 93
         comp3 : mux_fpadder
 94
              port map (N1 => Ncase, N2 => Nadder, enable => enable_aux,
 95
                         Result => Result);
96
97
         comp4 : n_case
              port map (NumberA => NumberA, NumberB => NumberB,
98
99
                         enable => enable aux, S => Ncase);
100
      end behavioral;
```