CMP205: Computer Graphics

Lecture 1: Line Drawing

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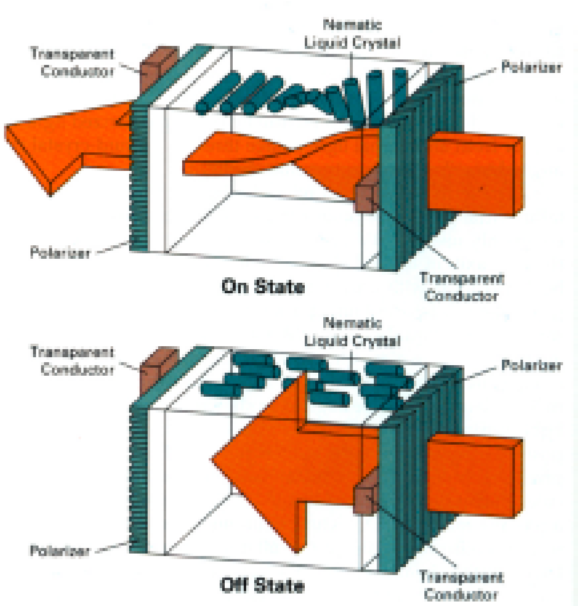
Agenda

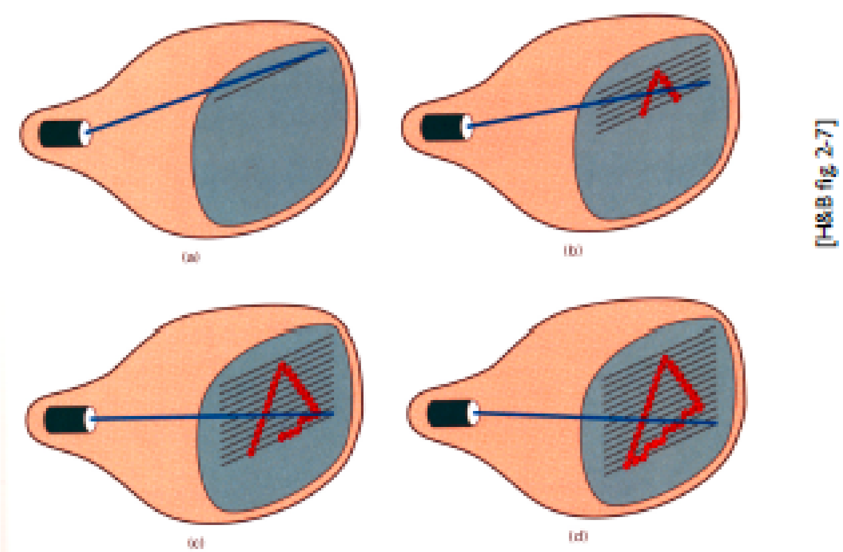
- Raster Displays
- Gamma Correction
- RGB Color & Alpha Channel
- Line Drawing

Acknowledgments: Some slides adapted from Steve Marschner and Fredo Durand.

Raster Displays

- Displays that present *raster* images to the user
- LCD, CRT, Projector, … etc



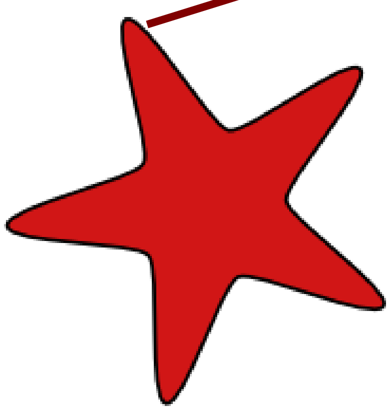


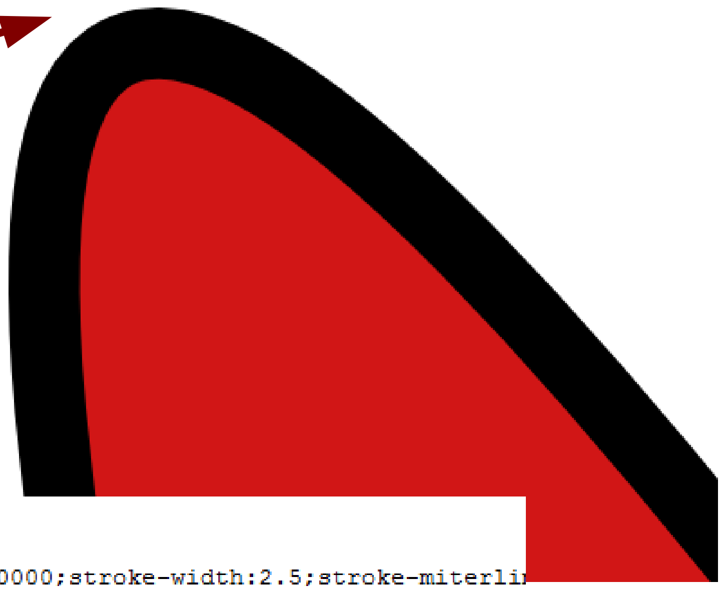
Raster Images Vs Vector Images

- Raster Images
  - Made up of pixels with a certain resolution
  - “Pixelization” effect on zoom in
  - E.g. JPEG, BMP, PNG, GIF

Raster Images Vs Vector Images

- Vector Images
  - Made up of “descriptions” of objects
  - Must be “rasterized” for display
  - Resolution independent
  - E.g. SVG





```
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   sodipodi:cy="448.07648"
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   inkscape:randomized="0.005"
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   inkscape:transform-center-y="4.4368255" />
```
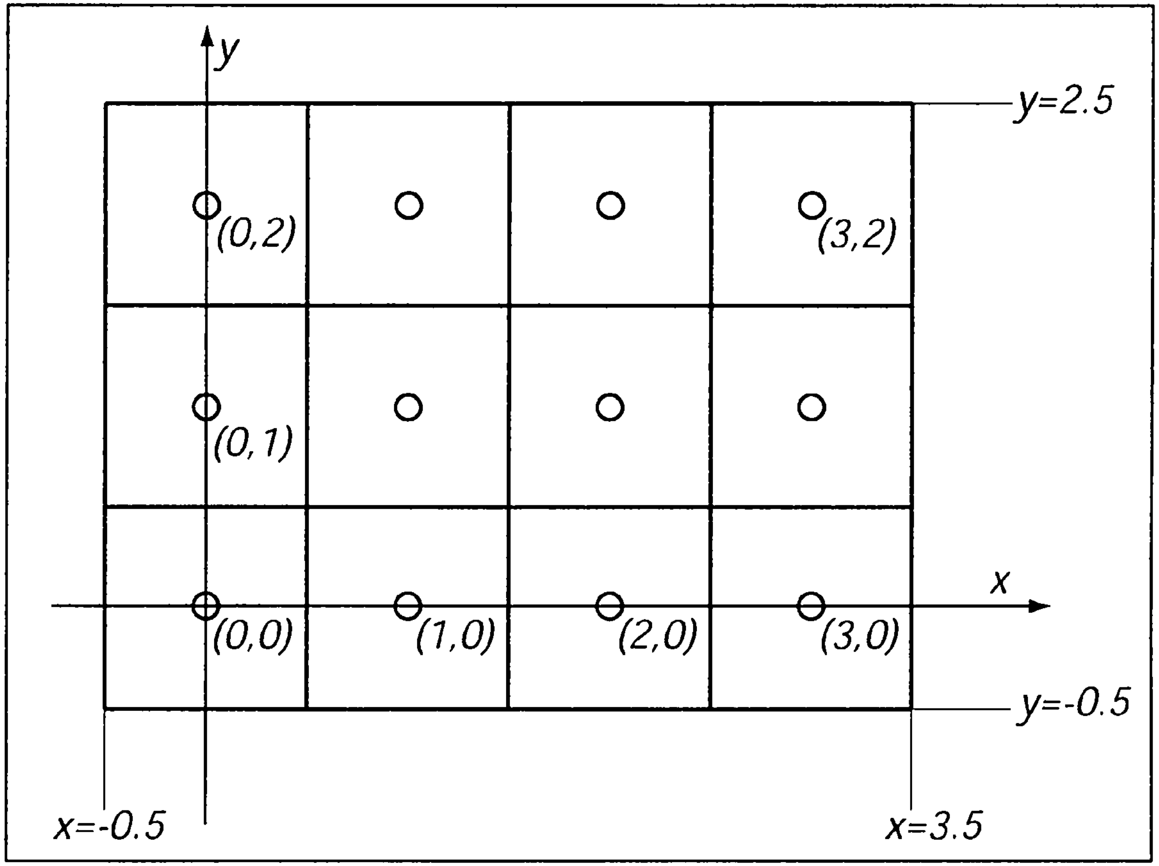

Pixels

- Square Grid of “pixels”
- Each pixel stores color values
- Resolution: size of raster



4 x 3 Pixel Grid

Pixels at integer coordinates

Display Intensity

- How many bits to represent each pixel?
  - Floating point number: 16 or 32 bits
  - Integers: 8, 10, 12, … bits
- Floating points
  - 0 → black (pixel off)
  - 1 → white (pixel fully on)
  - 0.5 → halfway gray
- Integers: quantized values e.g. 8-bits
  - 0 → black
  - 255 → white

Gamma Correction

- How do displays convert pixel *values* to *intensities*?
  - We would like the screen to:
    - display *white* (intensity 1.0) when we give it 1.0
    - display *black* (intensity 0.0) when we give it 0.0
    - display *grey* (intensity 0.5) when we give it 0.5

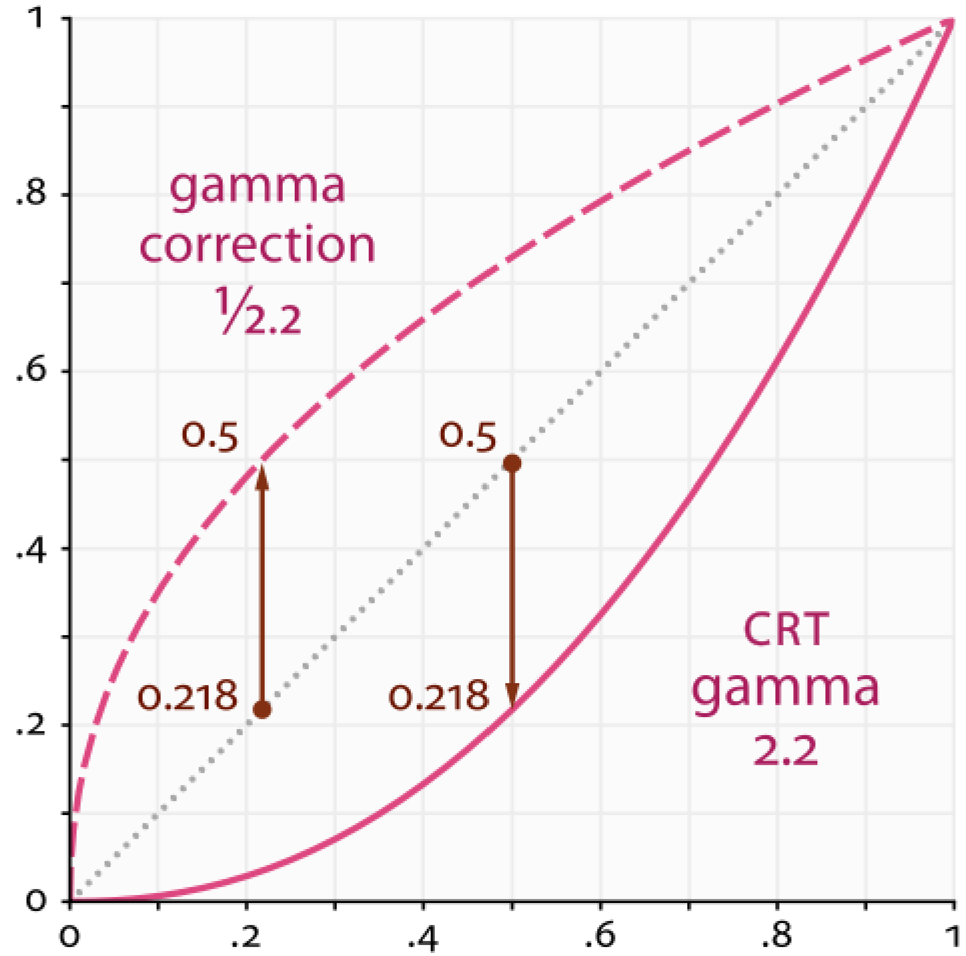
- What actually happens?
  - Screens respond non-linearly, e.g. they give intensity
    - 1.0 with input 1.0
    - 0.0 with input 0.0
    - 0.25 with input 0.5

Gamma Correction

This is modeled as:

$$I_{out} = I_{max} \times I_{\text{in}}^{\gamma}$$

$$\gamma = 2.2$$
$$I_{\text{in}} = 0 \rightarrow I_{out} = 0$$
$$I_{\text{in}} = 0.5 \rightarrow I_{out} = 0.218$$
$$I_{\text{in}} = 1 \rightarrow I_{out} = 1$$

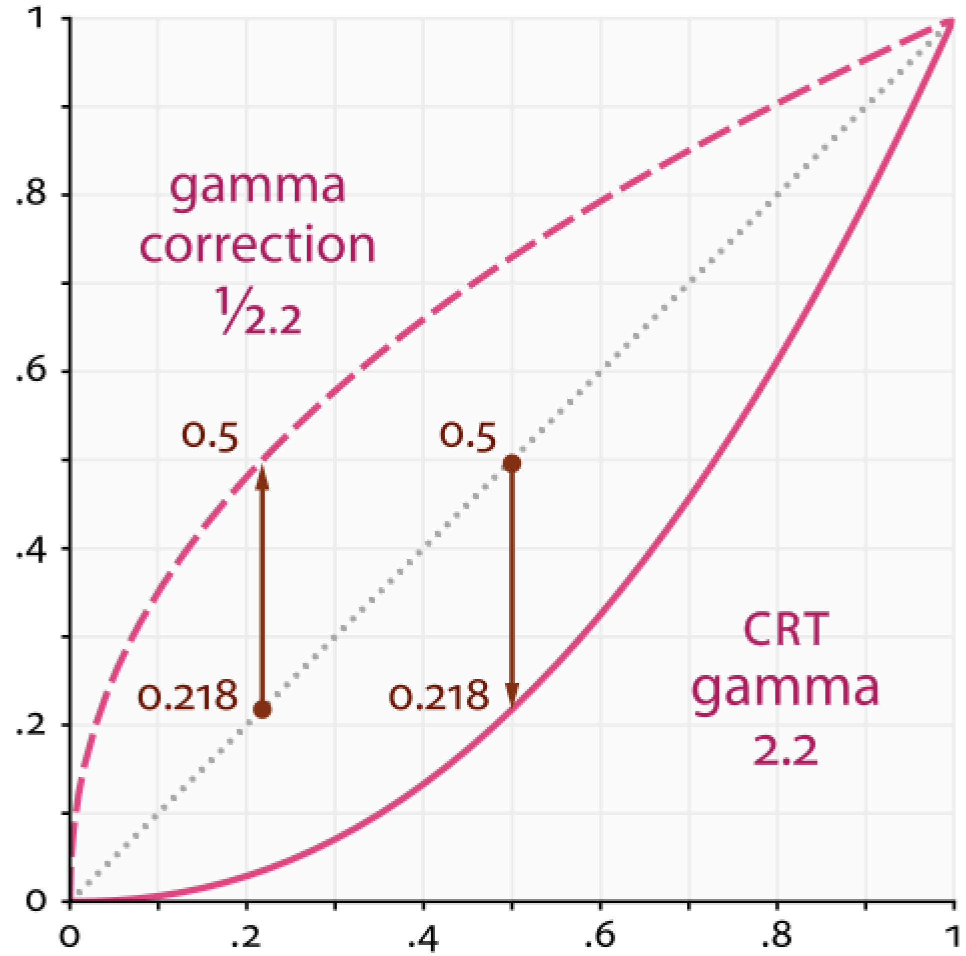


Gamma Correction

To get the correct intensity from the screen, we modify the input:

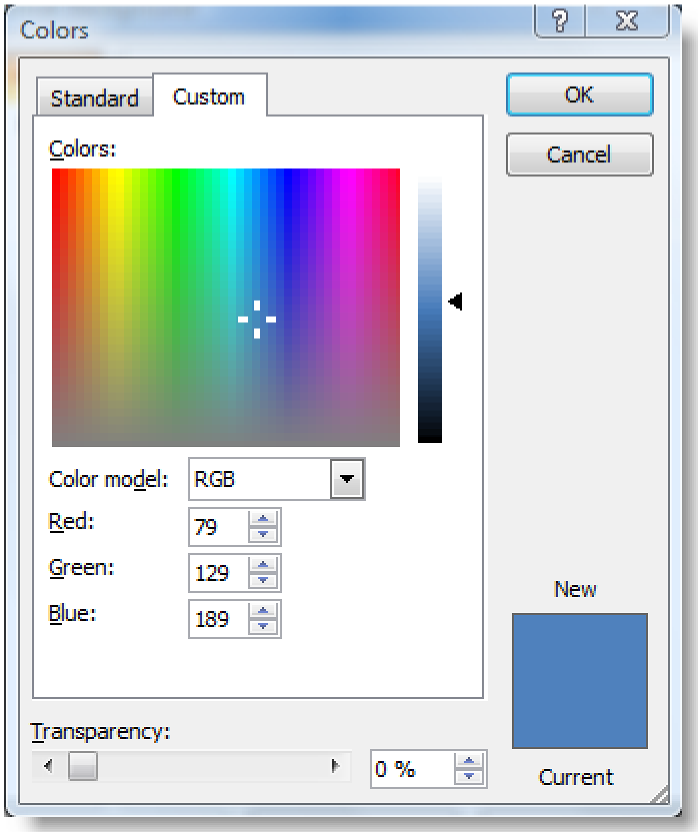
Correction

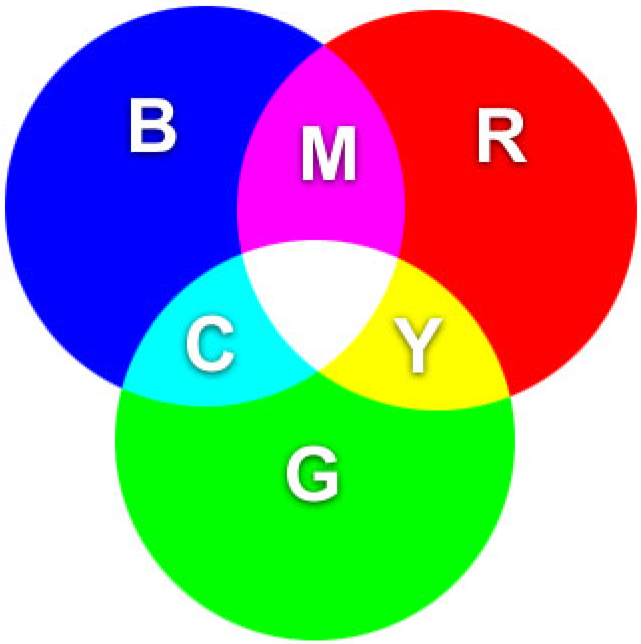
$$
\begin{aligned}
I_{out} &= I_{max} \times \left(I_{\text{in}}^{1/\gamma} \right)^{\gamma} \\
&= I_{max} \times I_{\text{in}}
\end{aligned}
$$



RGB Color

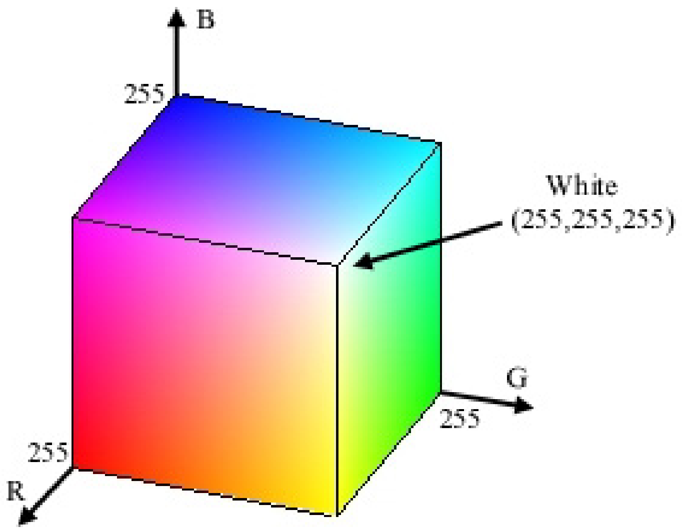
- For color images, each pixel has three values: Red, Green, and Blue Color
- Additive Property



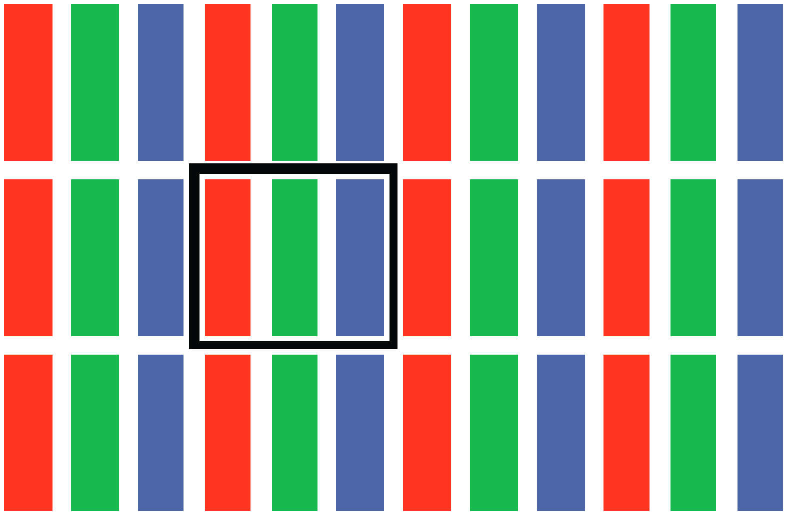


RGB Colors

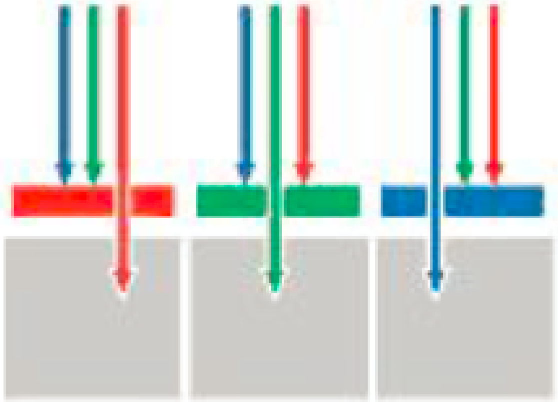
- Represent colors as 3 dimensional vectors (r, g, b)
  - Black = (0, 0, 0)
  - Red = (1, 0, 0)
  - …
  - White = (1, 1, 1)

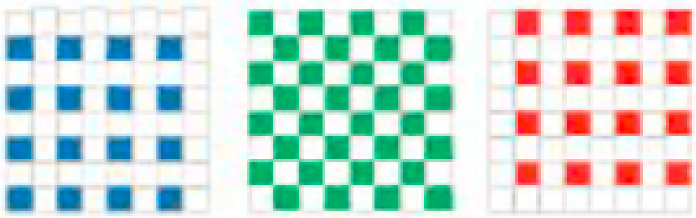


Displays and Cameras



LCD pixels have 3 sub-pixels



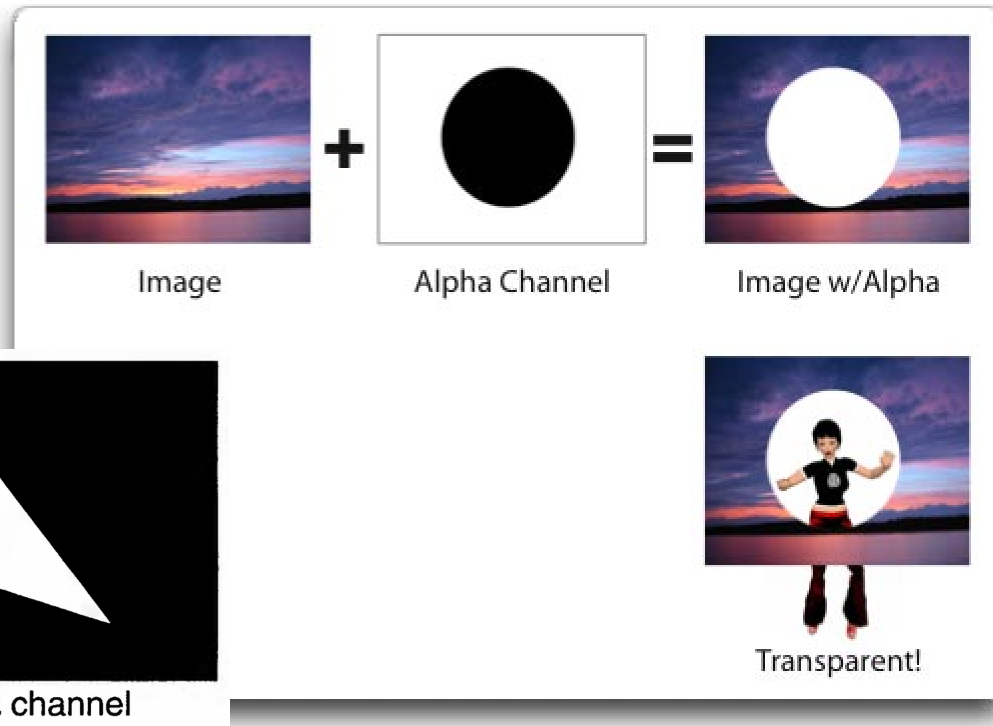


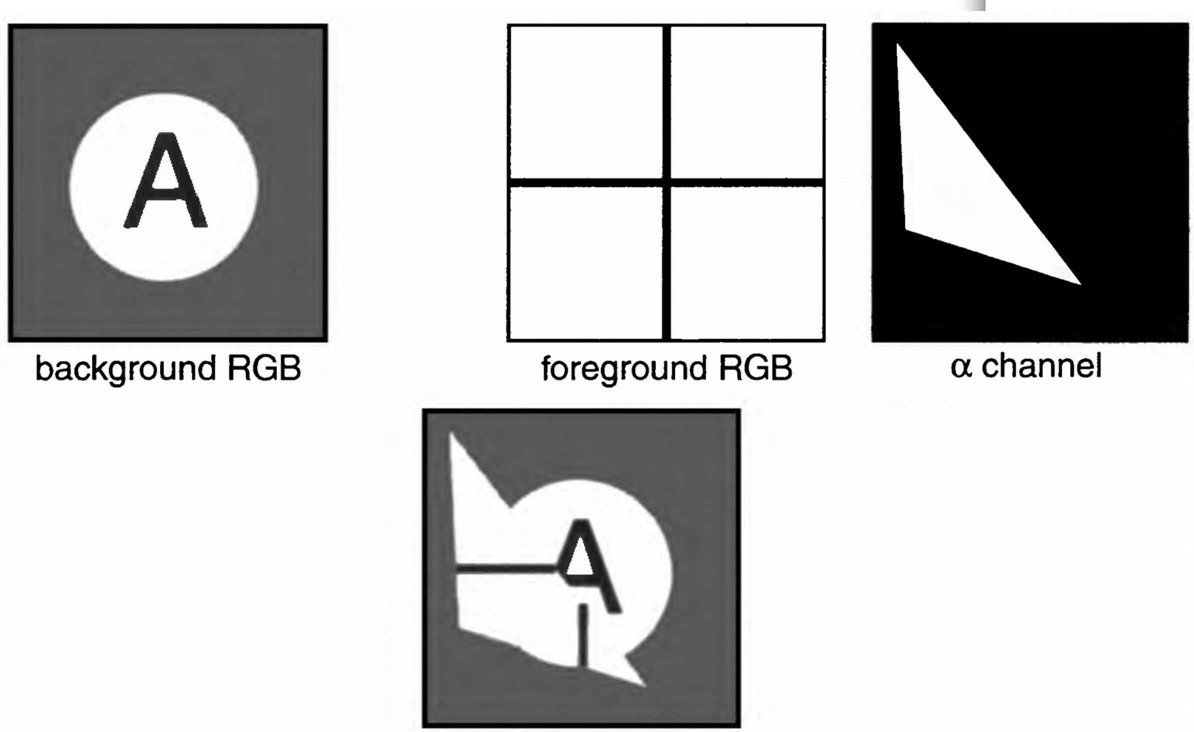
Bayer Mosaic in Digital Cameras

Alpha Channel

- Handles (partially) transparent objects

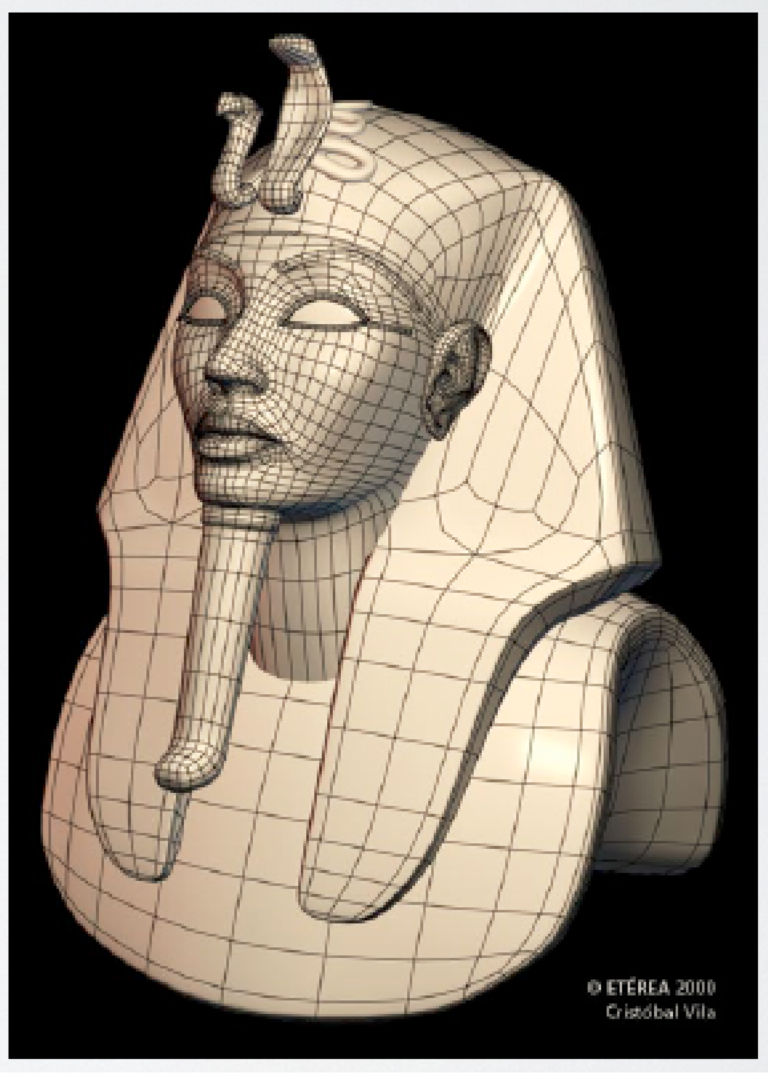
$$c = \alpha c_f + (1 - \alpha) c_b$$





Line Drawing

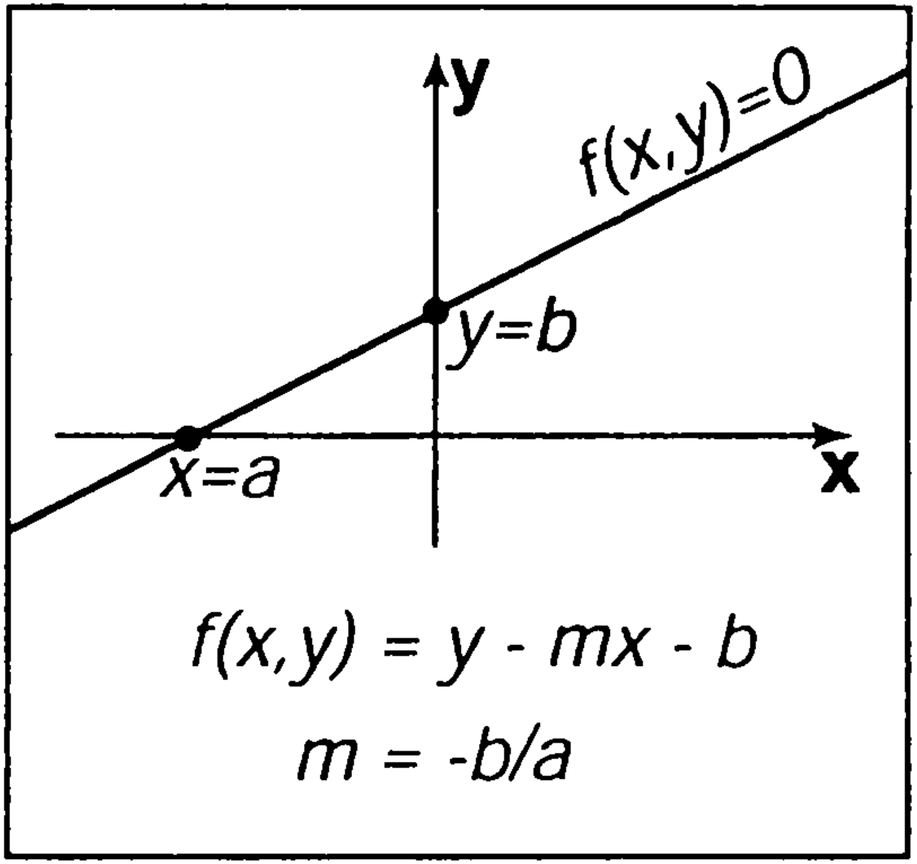
- Basic Operation



Line Representation

- Slope-Intercept
- Problem?
  - Vertical lines!
  - $m = \infty$
- Solution?

$$y = mx + b$$



Line Representation

- Implicit Form $f(x,y) = Ax + By + C = 0$

Both points are on the line:

$$Ax_0 + By_0 + C = 0$$

$$Ax_1 + By_1 + C = 0$$

The normal vector on the line is:

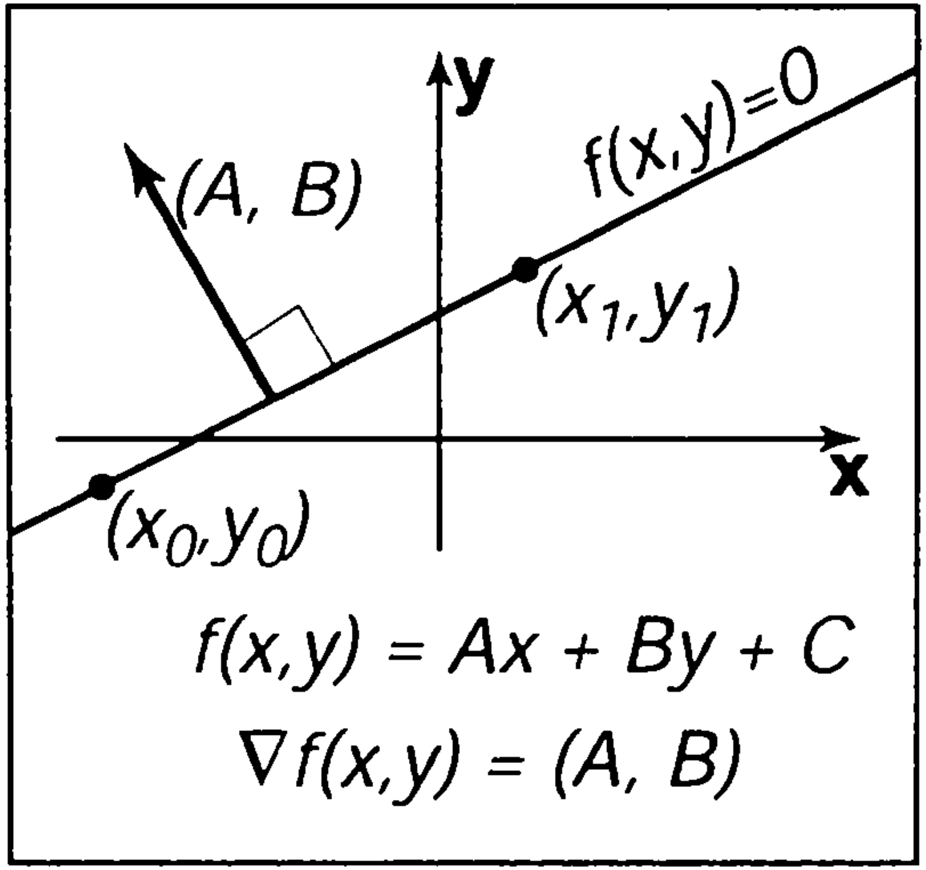
(A, B)

$$(A, B)^T (x_1 - x_0, y_1 - y_0) = 0$$

why?

One possible (A, B) is:

$$(A, B) = (y_0 - y_1, x_1 - x_0)$$



Line Representation

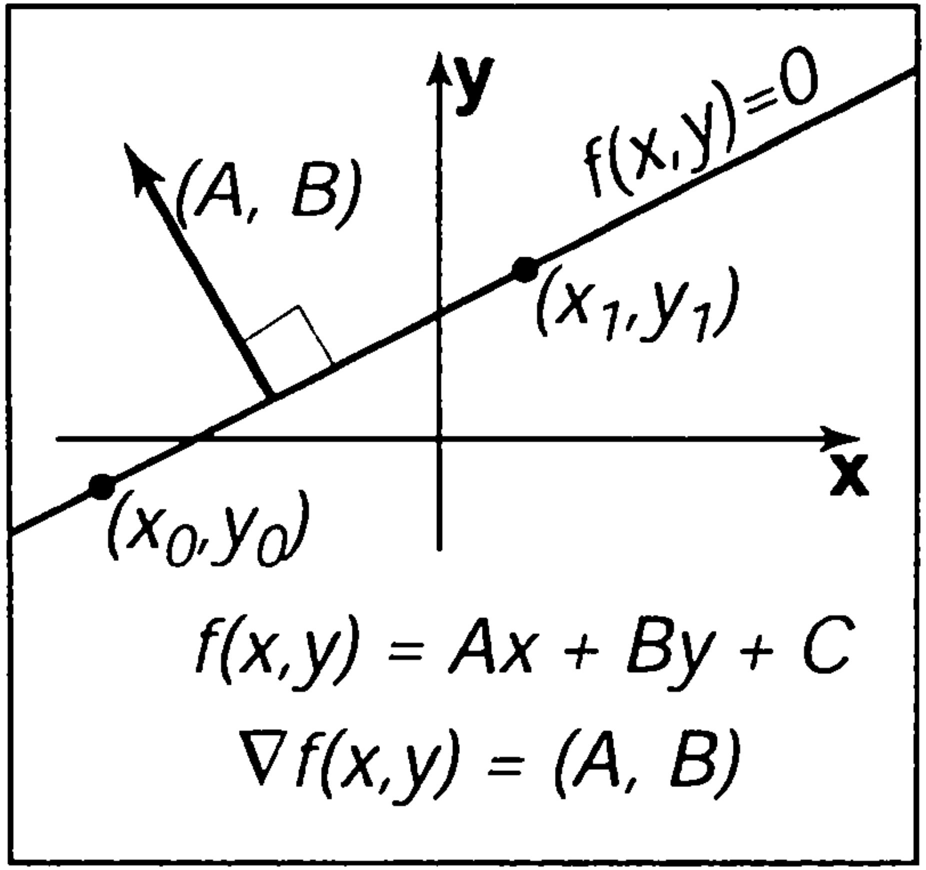
- Implicit Form $\quad f(x,y) = Ax + By + C = 0$

Plug in $(x_{0,} y_0)$ we get:

$$(y_0 - y_1)x_0 + (x_1 - x_0)y_0 + C = 0$$

Solve for C:

$$C = x_0 y_1 - x_1 y_0$$



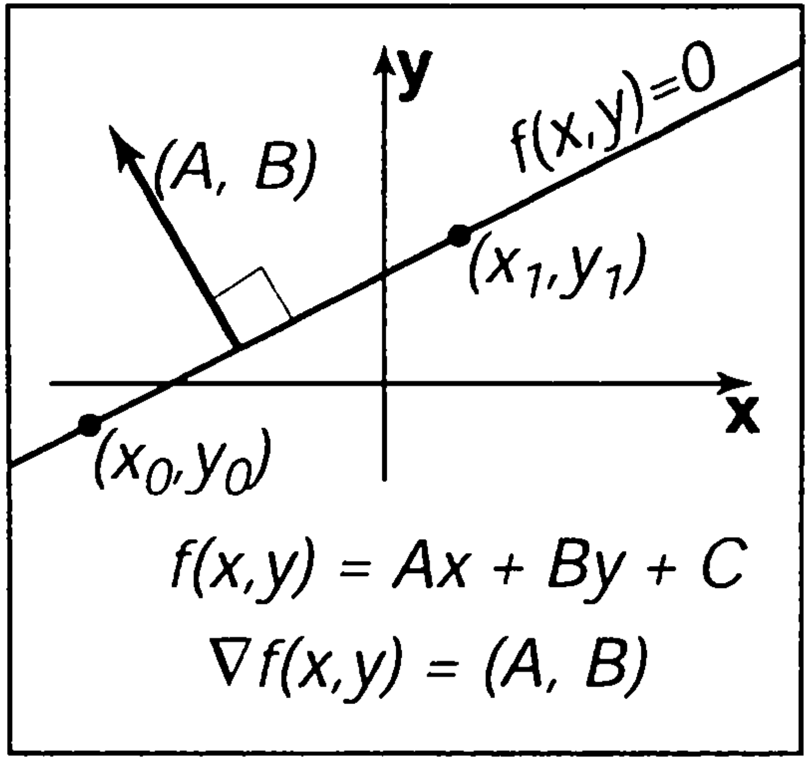
$$A = y_0 - y_1$$
$$B = x_1 - x_0$$
$$C = x_0 y_1 - x_1 y_0$$

Line Representation

$$f(x,y) = Ax + By + C = 0$$

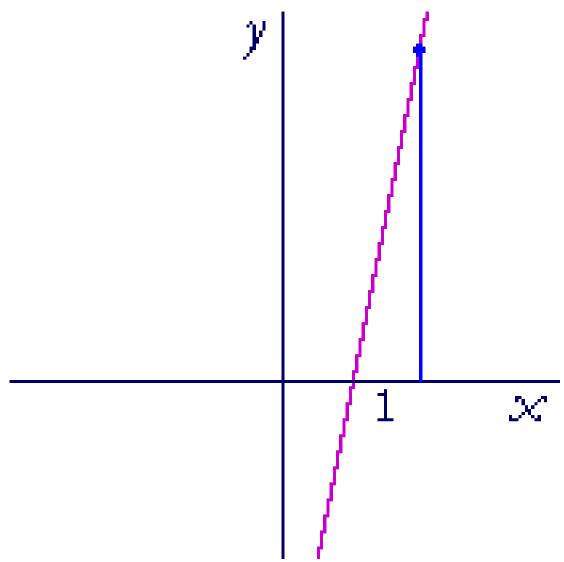
f (x, y) is the signed scaled distance from the point (x, y) to the line

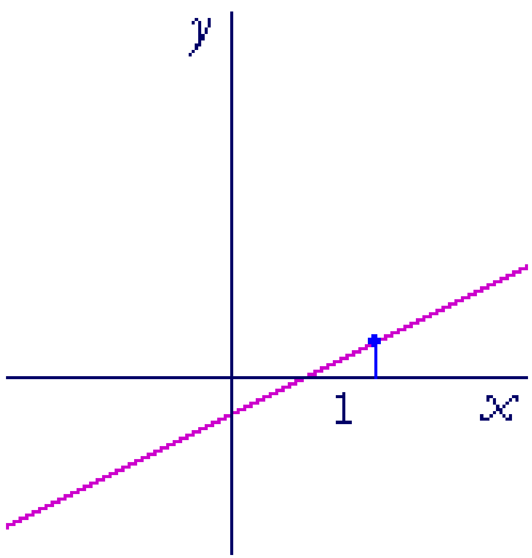
$f(x,y) = 0$ → On the line
$f(x,y) > 0$ → Above the line
$f(x,y) < 0$ → Below the line



Midpoint Algorithm

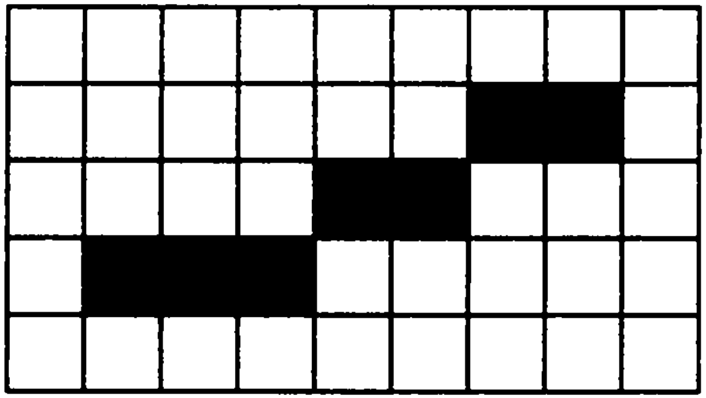
- Line Drawing Algorithm
- Uses the Implicit Equation
- Similar to Bresenham's Algorithm
- Integer end-points
- Four Cases:
  - $0 < m \leqslant 1$
  - $1 < m < \infty$
  - $-1 < m \leqslant 0$
  - $-\infty < m \leqslant -1$

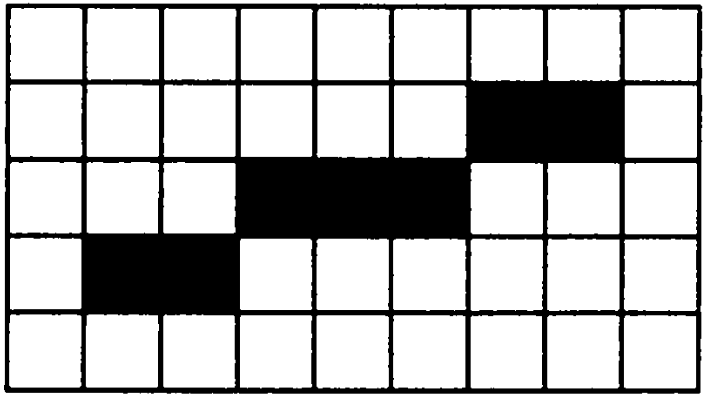


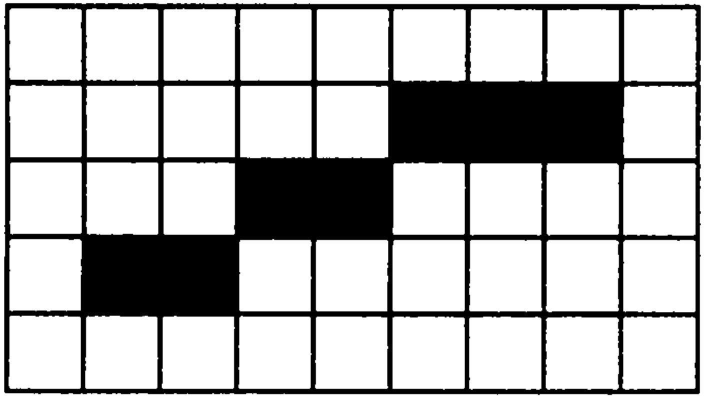


Midpoint Algorithm

- First case:
  - more “run” than “rise”
  - Assume moving right







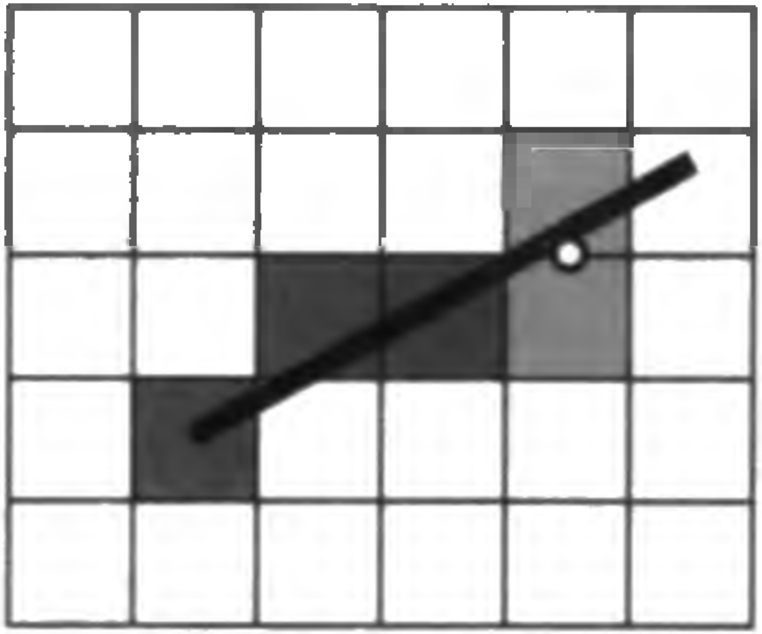
- Which one is better?

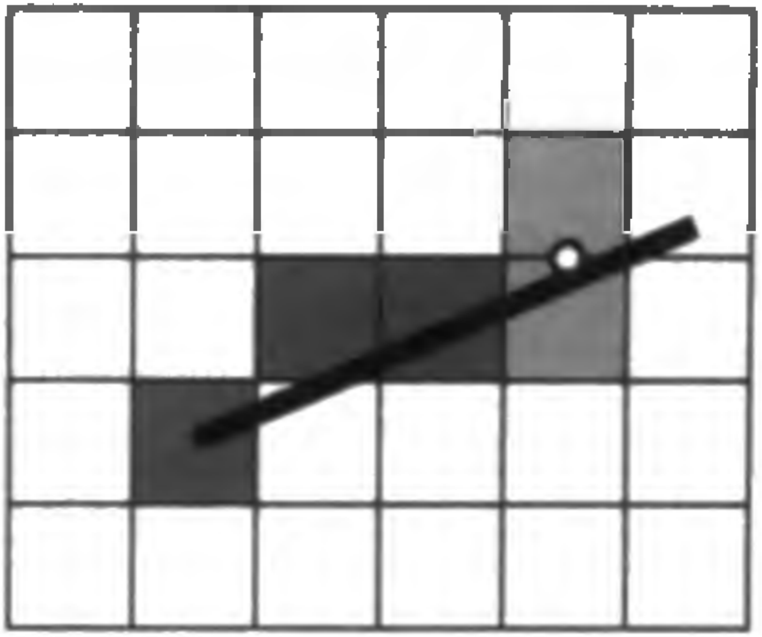
Midpoint Algorithm

- First version

$$
\begin{aligned}
&y = y_0 \\
&\text{for } x = x_0 : x_1 \\
&\quad draw(x, y) \\
&\quad \text{if } f(x+1, y+0.5) < 0 \\
&\quad\quad y = y+1
\end{aligned}
$$

- Optimizations?





Midpoint Algorithm

- Optimizations
  - Incremental Calculations
  - Integer Operations

$$
\begin{aligned}
& y = y_0 \\
& \text{for } x = x_0 : x_1 \\
& \quad draw(x, y) \\
& \quad \text{if } \; f(x+1, y+0.5) < 0 \\
& \quad\quad y = y + 1
\end{aligned}
$$

Midpoint Algorithm

- Incremental Calculations
  - Note that:

$$f(x,y)=(y_0-y_1)x+(x_1-x_0)y+C=0$$

  - Which implies that:

$$f(x+1,y)=f(x,y)+(y_0-y_1)$$
$$f(x+1,y+1)=f(x,y)+(y_0-y_1)+(x_1-x_0)$$

Midpoint Algorithm

- Second Pass

$$
\begin{aligned}
& y = y_0 \\
& d = f(x_0 + 1, y_0 + 0.5) \\
& \text{for } x = x_0 : x_1 \\
& \quad draw(x, y) \\
& \quad \text{if } d < 0 \\
& \qquad y = y + 1 \\
& \qquad d = d + (x_1 - x_0) + (y_0 - y_1) \\
& \quad \text{else} \\
& \qquad d = d + (y_0 - y_1)
\end{aligned}
$$

Midpoint Algorithm

- Integer Operations
  - Note that:

$$f(x,y) = 2f(x,y) = 0$$

- Which implies:

$$d = 2f(x_0 + 1, y_0 + 0.5)$$

$$\downarrow$$

$$d = 2(y_0 - y_1)(x_0 + 1) + (x_1 - x_0)(2y_0 + 1) + 2x_0 y_1 - 2x_1 y_0$$

Midpoint Algorithm

- Third Pass

$$
\begin{aligned}
&y = y_0 \\
&d = 2(y_0 - y_1)(x_0 + 1) + (x_1 - x_0)(2y_0 + 1) + 2x_0 y_1 - 2x_1 y_0 \\
&\text{for } x = x_0 : x_1 \\
&\quad draw(x, y) \\
&\quad \text{if } d < 0 \\
&\qquad y = y + 1 \\
&\qquad d = d + 2(x_1 - x_0) + 2(y_0 - y_1) \\
&\quad \text{else} \\
&\qquad d = d + 2(y_0 - y_1)
\end{aligned}
$$

Recap

- Raster Displays
- Alpha Channels
- Gamma Correction
- Line Drawing