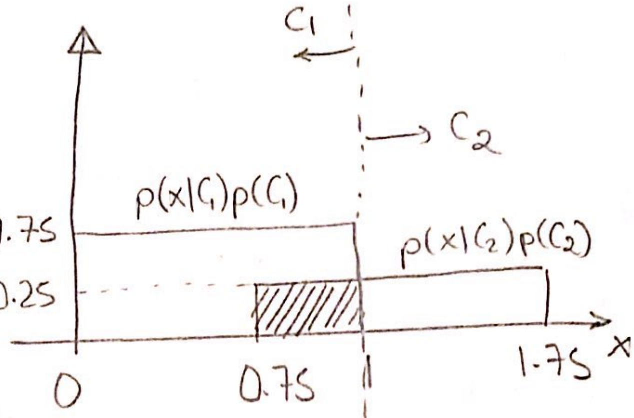
Sheet 3

(1) DB $\boxed{x_0 = 1}$

$$p(\text{error}) = \int_{-\infty}^{1} p(x|C_2)p(C_2)\,dx + \cancel{\int_{1}^{\infty} p(x|C_1)p(C_1)\,dx}$$



$= \text{Area } \boxed{////} = 0.25 \times 0.25 = 1/16$

$p(\text{correct}) = 15/16$.

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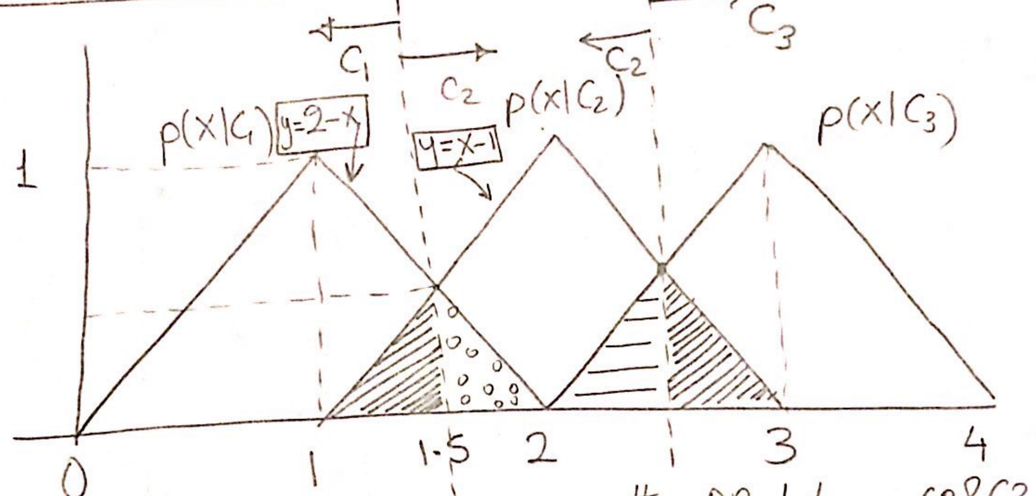
(2) $P(C_1) = P(C_2) = P(C_3)$



DB One DB between C1 & C2. and another DB between C2 & C3 (similar)

$$p(x|C_1)\cancel{p(C_1)} = p(x|C_2)\cancel{p(C_2)}$$

$$2 - x = x - 1$$

$$2x = 3$$

$$\boxed{x_0 = 1.5}$$

$$\boxed{x_1 = 2.5}$$

Decision Regions: Classify $x \in C1$ if $x \leqslant 1.5$

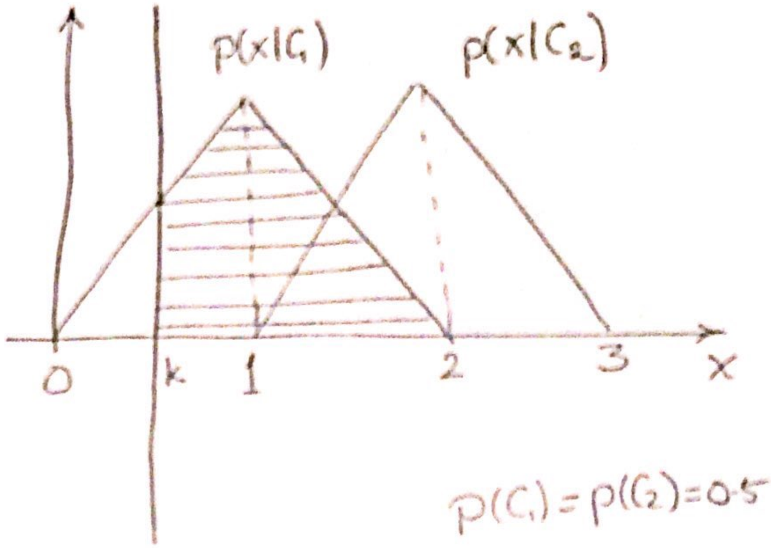
$x \in C_2$ if $1.5 < x \leqslant 2.5$

$x \in C_3$ if $x > 2.5$

$$\text{Probability of error} = \boxed{\int_{1.5}^{\infty} p(x|C_1)p(C_1)\,dx} + \boxed{\int_{-\infty}^{1.5} p(x|C_2)p(C_2)\,dx + \int_{2.5}^{\infty} p(x|C_2)p(C_2)\,dx} + \boxed{\int_{-\infty}^{2.5} p(x|C_3)p(C_3)\,dx}$$

$$= 4 * \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} * \frac{1}{3} = \boxed{\frac{1}{6}}$$

3 Assume $x = K$ Represents decision boundary



$p(C_1) = p(C_2) = 0.5$

Case 1 $\quad K \le 1$

$$p(\text{error}) = \frac{1}{2}\left(1 - \frac{K^2}{2}\right)$$

at $K = 0$

$P(\text{error}) = \frac{1}{2}$

at $K = 1$

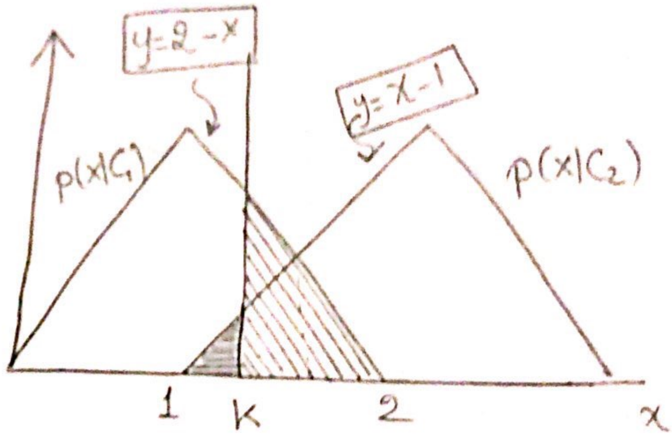
$P(\text{error}) = \frac{1}{4}$

Case 2 $\quad K \ge 2$ $\quad$ Symmetric to case 1

at $K = 2 \quad P(\text{error}) = \frac{1}{4}$

at $K = 3 \quad p(\text{error}) = \frac{1}{2}$

Case 3 $\quad 1 \le K \le 2$



$$P_{\text{error}} = \int_1^K p(x|C_2)\, p(C_2)\, dx + \int_K^2 p(x|C_1)\, p(C_1)\, dx$$

$$= \frac{(K-1)^2}{2} \cdot \frac{1}{2} + \frac{(2-K)^2}{2} \cdot \frac{1}{2} = \frac{1}{4}\left[(K-1)^2 + (2-K)^2\right]$$

$$\frac{dP_{\text{error}}}{dK} = 0 \implies \cancel{2}(K-1) - \cancel{2}(2-K) = 0$$

$$\Rightarrow K - 1 = 2 - K$$

$$2K = 3$$

$$\boxed{K = 1.5}$$

at $K = \frac{3}{2} \longrightarrow p(\text{error}) = \frac{1}{4}\left[0.5^2 + 0.5^2\right] = \boxed{\frac{1}{8}}$ Minimum

4 As μ_1 & μ_2 gets farther $\rightarrow$ p(error) $\downarrow$

As σ_1 & σ_2 increases $\longrightarrow$ p(error) $\uparrow$

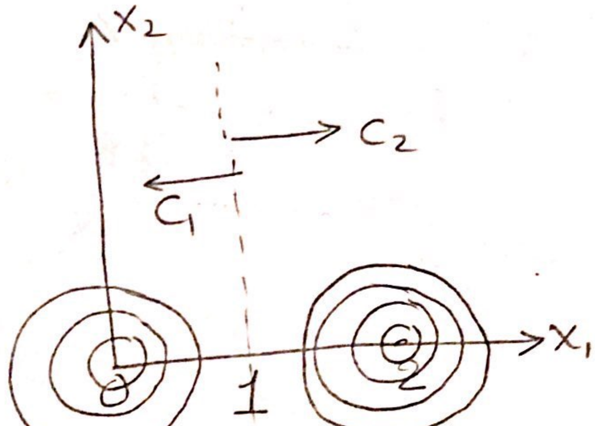
As $p(C_1)$ & $p(C_2)$ approaches 0.5 $\rightarrow$ p(error) $\uparrow$

---

5 $$p(x|C_i) = \frac{1}{(2\pi)^{\frac{N}{2}} |\Sigma_i|^{1/2}} \exp\left\{ \frac{-1}{2} (\underline{x} - \underline{\mu}_i)^T \Sigma_i^{-1} (\underline{x} - \underline{\mu}_i) \right\}$$

$$p(x|C_1) = \frac{1}{2\pi} e^{-\frac{1}{2}(x_1^2 + x_2^2)}$$

$$p(x|C_2) = \frac{1}{2\pi} e^{\frac{-1}{2}[(x_1-2)^2 + x_2^2]}$$



Decision boundary at $\boxed{x_1 = 1}$

or $\quad p(x|C_1)\cancel{p(C_1)} = p(x|C_2)\cancel{p(C_2)}$

$$x_1^2 = (x_1 - 2)^2$$

$$\therefore \cancel{x_1^2} = \cancel{x_1^2} - 4x_1 + 4$$

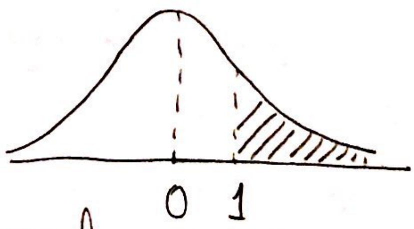
$$\boxed{x_1 = 1}$$

$$p(\text{error}) = \int_{\substack{\text{after}\\ \text{DB}}} p(x|C_1)\underline{p(C_1)}\, dx + \int_{\substack{\text{before}\\ \text{DB}}} p(x|C_2)p(C_2)\, dx$$

$$= \quad I_1 \quad + \quad I_2$$

$$I_1 = \frac{1}{2} \int_{x_2=-\infty}^{x_2=\infty} \int_{x_1=1}^{x_1=\infty} \frac{1}{2\pi} \cdot e^{\frac{-1}{2}(x_1^2 + x_2^2)}\, dx_1\, dx_2$$

$$= \frac{1}{2} \underbrace{\int_{x_2=-\infty}^{x_2=\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{x_2^2}{2}}\, dx_2}_{1} \cdot \int_{x_1=1}^{\infty} \frac{1}{\sqrt{2\pi}} e^{\frac{-x_1^2}{2}}\, dx_1$$



$$= \frac{1}{2}\left[0.5 - N(1) \right] \longrightarrow$$ has several definitions according to the table of Gaussian

$$p(\text{error}) = I_1 + I_2 = \boxed{0.5 - N(1).}$$

⑥ Refer to the slides. (You will find the detailed proof)

⑦ $P(C_1) = 0.7$

$P(C_2) = 0.3$

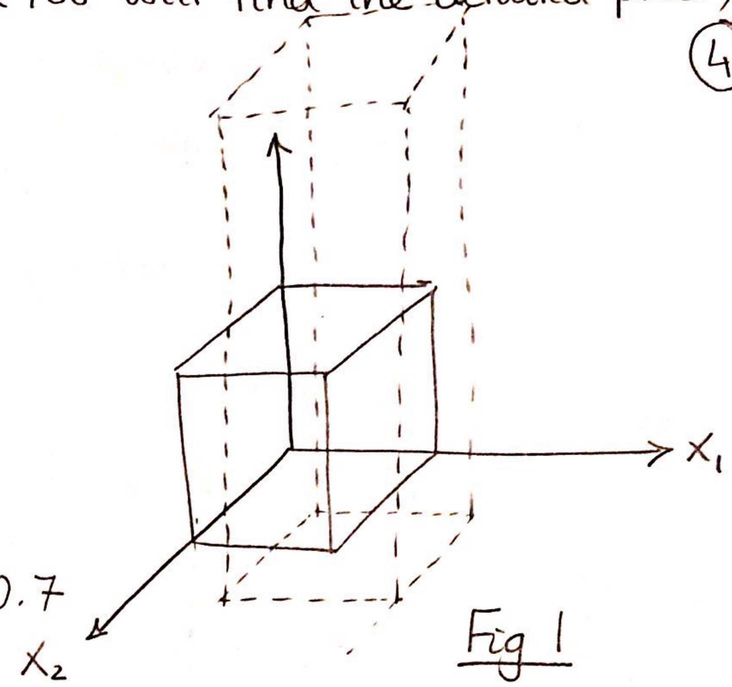


Fig 1

Decision boundary as indicated in Fig 2

$P(\text{error}) = 0.2 \times 0.2 \times 1 \times 0.7$

$= \boxed{0.028}$

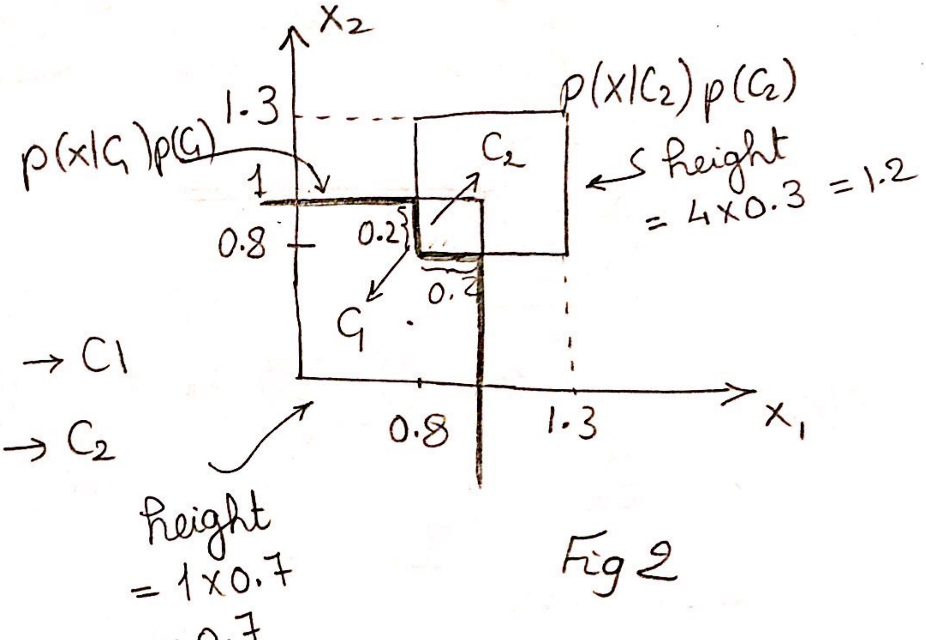


Fig 2

For the linear classifier

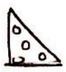
$x_1 + x_2 - 1.7 \leqslant 0 \rightarrow C_1$

$x_1 + x_2 - 1.7 > 0 \rightarrow C_2$

D.B

$x_1 + x_2 - 1.7 = 0$

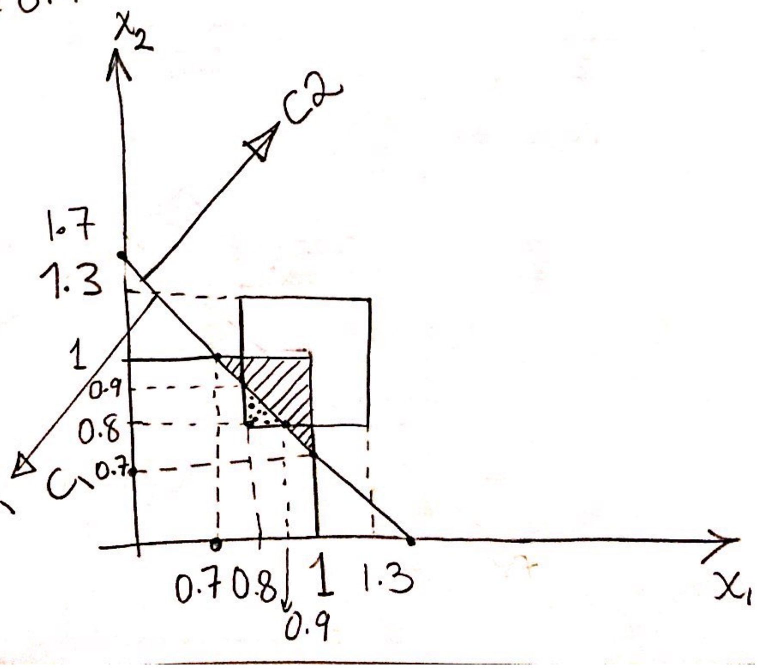
$x_2 = 1.7 - x_1$

P(error) =  + 

$= \underbrace{0.5 \times (0.3) \times (0.3) * 1}_{p(x|C_1)} \underbrace{* 0.7}_{p(C_1)}$

$+ \underbrace{0.5 * (0.1) \times (0.1) * 4}_{p(x|C_2)} \underbrace{\times 0.3}_{p(C_2)}$

$= \boxed{0.0375}$



1

Sheet 4

Hussein Fadl

$$\Sigma = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} \Rightarrow \begin{vmatrix} 2-\lambda & 1 \\ 1 & 3-\lambda \end{vmatrix} = 0$$

$$(2-\lambda)(3-\lambda) - 1 = 0$$

$$6 - 5\lambda + \lambda^2 - 1 = 0$$

$$\lambda^2 - 5\lambda + 5 = 0$$

$$\lambda = \frac{5 \pm \sqrt{25-20}}{2} = \frac{5 \pm \sqrt{5}}{2}$$

★ Compute the eigenvectors:

for $\lambda = \frac{5+\sqrt{5}}{2}$

$$(A - \lambda I)\underline{x} = 0$$

$$\begin{bmatrix} \frac{-1}{2} - \frac{\sqrt{5}}{2} & 1 \\ 1 & \frac{1}{2} - \frac{\sqrt{5}}{2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

Convert it to row echolon form

$$\begin{bmatrix} \frac{-1}{2}(1+\sqrt{5}) & 1 \\ 0 & \frac{1-\sqrt{5}}{1+\sqrt{5}} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} \frac{-1}{2}(1+\sqrt{5}) & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & \frac{-2}{1+\sqrt{5}} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$\therefore$ let $x_2 = t$

$$\therefore x_1 - \frac{2}{1+\sqrt{5}} x_2 = 0$$

$$x_1 = \frac{2}{1+\sqrt{5}} x_2 = \frac{2}{1+\sqrt{5}} t$$

$$\therefore \underline{u}_1 = \begin{bmatrix} \frac{2}{1+\sqrt{5}} \\ 1 \end{bmatrix}$$

$$\boxed{\hat{u}_1 = \begin{bmatrix} 0.5257 \\ 0.8507 \end{bmatrix}}$$

for $\lambda = \frac{5-\sqrt{5}}{2}$

$$(A - \lambda I)\underline{x} = 0$$

$$\begin{bmatrix} \frac{-1}{2} + \frac{\sqrt{5}}{2} & 1 \\ 1 & \frac{1}{2} + \frac{\sqrt{5}}{2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

Convert it to row echolon form

$$\begin{bmatrix} \frac{-1}{2}(1-\sqrt{5}) & 1 \\ 0 & \frac{1+\sqrt{5}}{1-\sqrt{5}} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} \frac{-1}{2}(1-\sqrt{5}) & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & \frac{-2}{1-\sqrt{5}} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

let $x_2 = t$

$$x_1 - \frac{2}{1-\sqrt{5}} x_2 = 0$$

$$x_1 = \frac{2}{1-\sqrt{5}} t$$

$$u_2 = \begin{bmatrix} \frac{2}{1-\sqrt{5}} \\ 1 \end{bmatrix}$$

$$\hat{u}_2 = \begin{bmatrix} -0.8507 \\ 0.5257 \end{bmatrix}$$

$$U = \begin{bmatrix} | & | \\ \hat{u}_1 & \hat{u}_2 \\ | & | \end{bmatrix} = \begin{bmatrix} 0.5257 & -0.8507 \\ 0.8507 & 0.5257 \end{bmatrix}$$

// Check your answer is correct by verifying

$u_1^T u_2 = 0 \longrightarrow \underline{u}_1 \cdot \underline{u}_2 = 0 \rightarrow \perp$ to each other

<u>Transform</u> : $Z = U^T Y$

<u>Select the best feature using the first principal component</u> : $Z = u_1^T Y$

<u>or</u> $Z = U^T Y$ and take z_1 only.

<u>Note</u> : Remember to sort the eigenvectors u_1 and u_2 according to their respective eigenvalues.

$\boxed{2}$ $\Sigma = ?$

$$U^T \Sigma U = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.2 \end{bmatrix} = \Omega$$

$$\cancel{U} \cancel{U^T} \Sigma \cancel{U} \cancel{U^T} = U \Omega U^T$$

$$\Sigma = U \Omega U^T$$

$$\Sigma = \begin{bmatrix} | & | & | \\ u_1 & u_2 & u_3 \\ | & | & | \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} \begin{bmatrix} — u_1 — \\ — u_2 — \\ — u_3 — \end{bmatrix}$$

$$= \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.2 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$= \checkmark$

Sheet 2

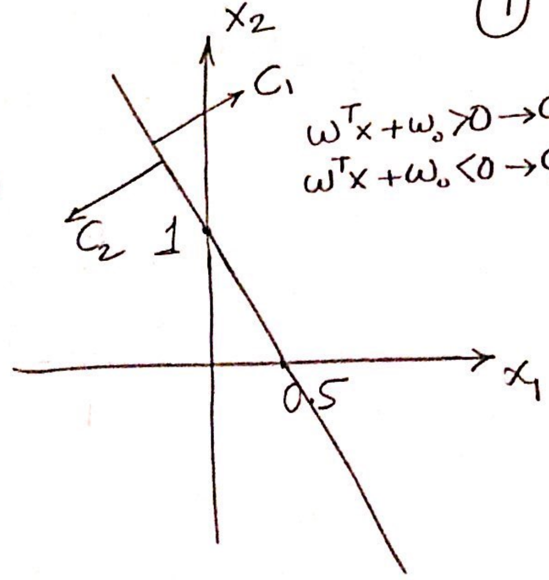
1 $w^T x + w_0 = 0$

$$\begin{bmatrix} 1 & 0.5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - 0.5 = 0$$

$$x_1 + 0.5\,x_2 - 0.5 = 0$$

$$\boxed{2x_1 + x_2 - 1 = 0}$$

$$\boxed{x_2 = -2x_1 + 1}$$



$w^T x + w_0 > 0 \rightarrow C_1$

$w^T x + w_0 < 0 \rightarrow C_2$

2

$$x_2 = m x_1 + c$$

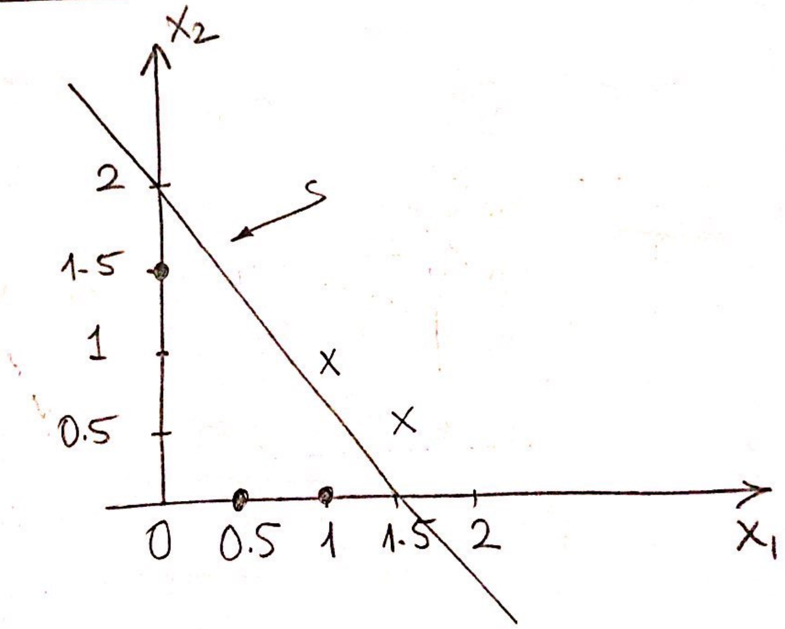
$$m = \frac{2-0}{0-1.5} = \frac{-4}{3}$$

$$x_2 = \frac{-4}{3} x_1 + c$$

at $(0, 2) \longrightarrow \underline{\underline{c = 2}}$

$$x_2 = \frac{-4}{3} x_1 + 2$$

$$\boxed{4x_1 + 3x_2 - 6 = 0}$$



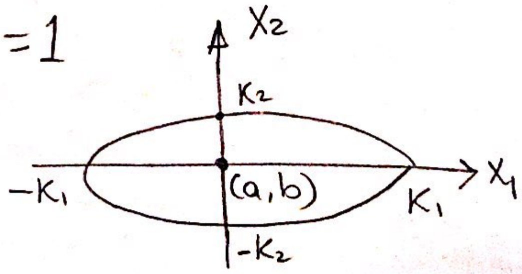
classify $x \in C_1$ if $4x_1 + 3x_2 - 6 \leq 0$

$x \in C_2$ if $4x_1 + 3x_2 - 6 > 0$.

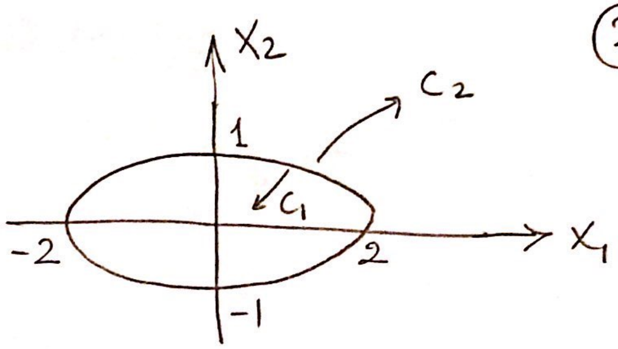
---

3 Eqn of circle: $(x_1 - a)^2 + (x_2 - b)^2 = r^2$ Center $= (a, b)$ radius $= r$

Eqn of ellipse: $\dfrac{(x_1 - a)^2}{k_1^2} + \dfrac{(x_2 - b)^2}{k_2^2} = 1$



Equ of DB = $\frac{x_1^2}{4} + x_2^2 = 1$



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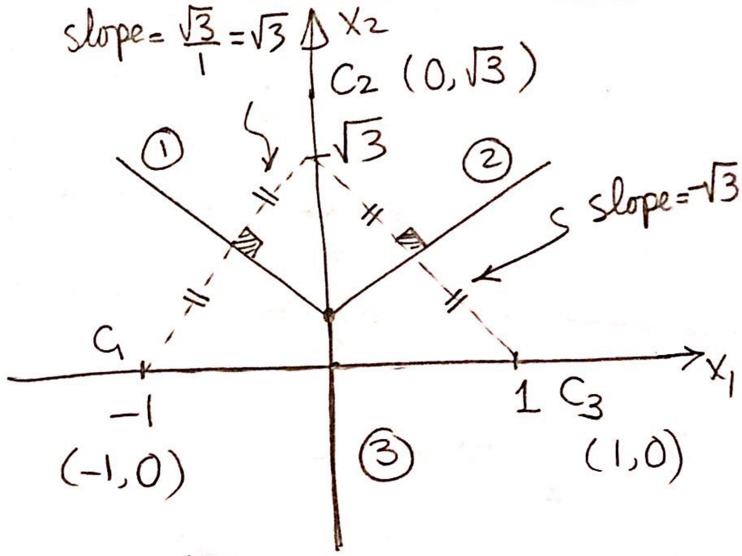
(4) DB1

(between C1 & C3)

midpoint = $\left(\frac{-1}{2}, \frac{\sqrt{3}}{2}\right)$

slope = $\frac{-1}{\sqrt{3}}$



Eqn: $x_2 = \frac{-1}{\sqrt{3}} x_1 + C$

$\frac{\sqrt{3}}{2} = \frac{-1}{\sqrt{3}} \cdot \frac{-1}{2} + C \rightarrow C = \frac{1}{\sqrt{3}}$

$x_2 = \frac{-1}{\sqrt{3}} x_1 + \frac{1}{\sqrt{3}}$

$\boxed{x_2 = \frac{1}{\sqrt{3}}(1 - x_1).}$

DB 2 between (C2) and (C3)

midpoint = $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

slope = $\frac{1}{\sqrt{3}}$

$x_2 = \frac{1}{\sqrt{3}} x_1 + C$

$\frac{\sqrt{3}}{2} = \frac{1}{2\sqrt{3}} + C \rightarrow C = \frac{1}{\sqrt{3}}$

$\boxed{x_2 = \frac{1}{\sqrt{3}}(x_1 + 1)}$

DB3 $\boxed{x_1 = 0}$

The three decision boundaries are

DB1 $x_2 = \frac{1}{\sqrt{3}}(1 - x_1)$

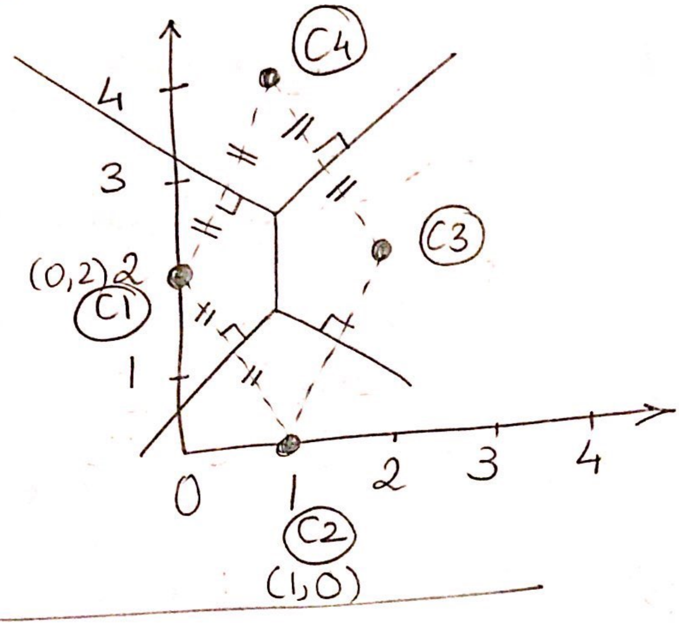
DB2 $x_2 = \frac{1}{\sqrt{3}}(x_1 + 1)$

DB3 $x_1 = 0$

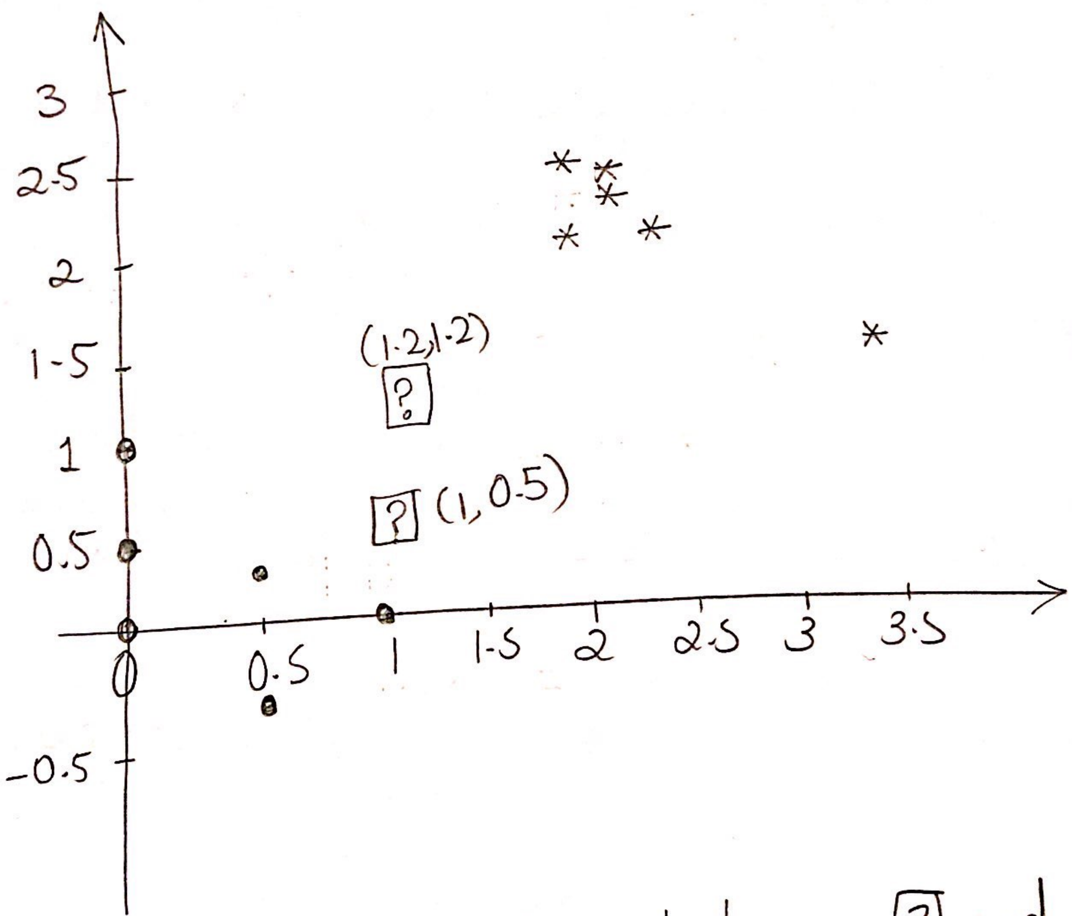
They all intersect at $x_1 = 0$ & $x_2 = \frac{1}{\sqrt{3}}$

5 You will have to construct

1. DB between C1, C2
2. DB between C1, C3
3. DB between C1, C4
4. DB between C2, C3
5. DB between C3, C4



$\boxed{6}$



$\boxed{\text{NN}}$

$\boxed{\text{KNN}}$

Compute the distance between $\boxed{?}$ and every point.

In NN, select the smallest distance → classify according to the closest point

In KNN, classify according to the closest $\underline{\underline{K}}$ points

Pattern Recognition
Sheet 1

1 a) $p(F = \text{Orange} \mid B = \text{Red}) = \frac{4}{10} = 0.4$

b) $p(F = \text{Orange}) = \sum\limits_{\substack{\text{all} \\ \text{boxes} \\ b \in B}} p(F = \text{Orange}, B = b)$

$\sum\limits_{b \in B} p(F = \text{Orange} \mid B = b)\, p(B = b)$

$= p(F = \text{Orange} \mid B = R)\, p(B = R) + p(F = \text{Orange} \mid B = \text{Blue})\, p(B = \text{Blue}) + \dots$

$= \frac{4}{10} * \frac{1}{5} + \frac{5}{10} \times \frac{1}{5} + \frac{3}{10} \times \frac{3}{5} = \frac{4+5+9}{50} = \frac{18}{50}$

c) $p(B = R \mid F = O) = \frac{p(B = R,\ F = O)}{p(F = O)} = \frac{p(F = O \mid B = R)\, p(B = R)}{p(F = O)}$

$= \frac{0.4 \times 1/5}{18/50} = \frac{4}{18}.$

2 A = event that both flips land on heads $\{(h,h)\}$

B = event that first flip lands on heads $\{(h,h), (h,t)\}$

C = event that at least one flip lands on heads $\{(h,h), (h,t), (t,h)\}$

$p(A \mid B) = \frac{p(A, B)}{p(B)} = \frac{1/4}{2/4} = \frac{1}{2}$

$p(A \mid C) = \frac{p(A, C)}{p(C)} = \frac{1/4}{3/4} = \frac{1}{3}$

3 $E = \{(h,t), (h,h)\}$

$F = \{(h,t), (t,t)\}$

$p(E, F) = \frac{1}{4} \qquad p(E) = \frac{1}{2} \qquad p(F) = \frac{1}{2}$

$\therefore p(E,F) = p(E)\,p(F)$

$\therefore$ independent

④ a) $\sum_{i=1}^{4} p(i) = \frac{1}{4} + \frac{1}{2} + \frac{1}{8} + \frac{1}{8} = 1$

b) $E[X] = 1 \times \frac{1}{4} + 2 \times \frac{1}{2} + 3 \times \frac{1}{8} + 4 \times \frac{1}{8} \rightarrow \frac{17}{8}$

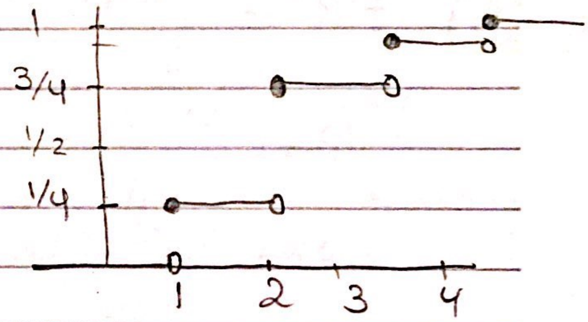
c) $E[X^2] = (1)^2 \times \frac{1}{4} + (2)^2 \times \frac{1}{2} + (3)^2 \times \frac{1}{8} + (4)^2 \times \frac{1}{8} = \frac{43}{8}$

$Var(X) = E[X^2] - E[X]^2 = \frac{43}{8} - \left(\frac{17}{8}\right)^2 = 0.86$

$\sigma(x) = 0.927$

d) $F(1) = \frac{1}{4}$ $F(2) = ..$

$$F(x) = \begin{cases} 0 & , x < 1 \\ 0.1/4 & , 1 \leq x < 2 \\ 3/4 & , 2 \leq x < 3 \\ 7/8 & , 3 \leq x < 4 \\ 1 & , x \geq 4 \end{cases}$$



e) $y = \ln(x)$ ✓

⑤ - $E[4 + 4X + X^2] = 4 + 4E[X] + E[X^2]$
$= 4 + 4(1) + (5 + 1^2) = 14$

- $Var(4 - 3X) = 9\,Var(x) = 9 \times 5 = 45$

⑥ same colour (2 green , 2 blue)
different color (1 green, 1 blue).

$p(2\text{ green}) \longrightarrow \frac{5}{10} \times \frac{4}{9} = \frac{2}{9}$ $p(2\text{ blue}) = \frac{5}{10} \times \frac{4}{9} = \frac{2}{9}$

$p(1\text{ green} \,\&\, 1\text{ blue}) = \frac{5}{10} \times \frac{5}{9} = \frac{5}{18}$

$E[X] = 5.5 \times \frac{2}{9} \times 2 - 5 \times \frac{5}{18} = \boxed{1.056}$

$Var(X) = E[X^2] - E[X]^2.$

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⑦ ⓐ $\int f(x)\,dx = 1$

$f(x) = \begin{cases} \frac{1}{100} e^{\frac{-x}{100}}, & x \geq 0 \\ 0 & x < 0 \end{cases}$

$-100\lambda \int_0^{\infty} \frac{-1}{100} e^{-\frac{x}{100}}\,dx = 1$

$-100\lambda\, e^{\frac{-x}{100}} \Big|_0^{\infty} = 1$

$\Rightarrow \boxed{\lambda = \frac{1}{100}}$

a) $-\int_{50}^{150} \frac{-1}{100} e^{\frac{-x}{100}}\,dx$

$= e^{\frac{-x}{100}} \Big|_{150}^{50}$

$= e^{\frac{-1}{2}} - e^{-3/2} = 0.383$

b) $-\int_0^{100} \frac{-1}{100} e^{\frac{-x}{100}}\,dx = 1 - e^{-1}$

$= 0.632$

⑧ $f(x) = \begin{cases} a + bx^2 & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$

$\int_0^1 (a + bx^2)\,dx = 1$

$ax + \frac{bx^3}{3} \Big|_0^1 = 1 \Rightarrow a + \frac{b}{3} = 1 \rightarrow ①$

$\int_0^1 (ax + bx^3)\,dx = 35$

$\frac{ax^2}{2} + \frac{bx^4}{4} \Big|_0^1 = 35$

$\frac{a}{2} + \frac{b}{4} = 35 \rightarrow ②$

⑩ $corr(x,y) = \frac{cov(x,y)}{\sigma_x \sigma_y} = 0$

$cov(x,y) = 0$

$E[XY] - E[X]E[Y] = 0$

$Var(X+Y) = Var(X) + Var(Y) + 2Cov(X,Y)$

a) ✗ b) ✗ c) ✓ d) ✓ e) ✗ f) ✓ g) ✗ h) ✓

⑪ $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$

$y = x - \mu$

$dy = dx$

$\int_{-\infty}^{\infty} x f(x)\,dx = \frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{\infty} x e^{\frac{-(x-\mu)^2}{2\sigma^2}}\,dx$

$= \frac{-\sigma^2}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{\infty} \frac{-y}{\sigma^2} e^{\frac{-y^2}{2\sigma^2}}\,dy + \cancel{\frac{\mu}{\sqrt{2\pi\sigma}} \int e^{\frac{-y^2}{2\sigma^2}}\,dy}$ 1

$= \cancel{\frac{-\sigma^2}{\sqrt{2\pi\sigma^2}} e^{\frac{-y^2}{2\sigma^2}} \Big|_{-\infty}^{\infty}} + \mu$

0

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$$E[X^2] = \frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{\infty} (y+\mu)^2 e^{\frac{-y^2}{2\sigma^2}} dy$$

$$= \frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{\infty} y^2 e^{\frac{-y^2}{2\sigma^2}} dy + \frac{2\mu}{\sqrt{2\pi\sigma^2}} \underbrace{\cancel{\int_{-\infty}^{\infty} y\, e^{\frac{-y^2}{2\sigma^2}} dy}}_{0}$$

$$+ \mu^2$$

$u = y \qquad dv = \int_{-\infty}^{\infty} y\, e^{\frac{-y^2}{2\sigma^2}} dy$

$du = 1 \qquad v = -\sigma^2 e^{\frac{-y^2}{2\sigma^2}}$

$$= \frac{1}{\sqrt{2\pi\sigma^2}} \left[\underbrace{\cancel{-\sigma^2 y\, e^{\frac{-y^2}{2\sigma^2}} \Big|_{-\infty}^{\infty}}}_{0} + \sigma^2 \int_{-\infty}^{\infty} \frac{\cancel{e^{\frac{-y^2}{2\sigma^2}}}}{1} \right] + \mu^2 = \sigma^2 + \mu^2$$

$$= \sigma^2 + \mu^2$$

$\boxed{12}$ $$E[e^{-aX}] = \int_{-\infty}^{\infty} e^{-ax} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{\frac{-1}{2}[x^2+2ax]} dx = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{\frac{-1}{2}\left([x+a]^2 - a^2\right)}$$

$$= \frac{1}{\sqrt{2\pi}} e^{\frac{a^2}{2}} \int_{-\infty}^{\infty} e^{\frac{-1}{2}(x+a)^2} dx$$

$$= \frac{1}{\sqrt{2\pi}} \cdot e^{\frac{a^2}{2}} \int_{-\infty}^{\infty} e^{\frac{-1}{2}y^2} dy$$

$$= e^{a^2/2}$$

$\mathrm{var}(X) + E[X]^2$

$1 + 0$

$$E[e^{-x} + e^{-2x} + x^2] = E[e^{-X}] + E[e^{-2X}] + E[X^2]$$

$$= e^{1/2} + e^{2} + 1.$$