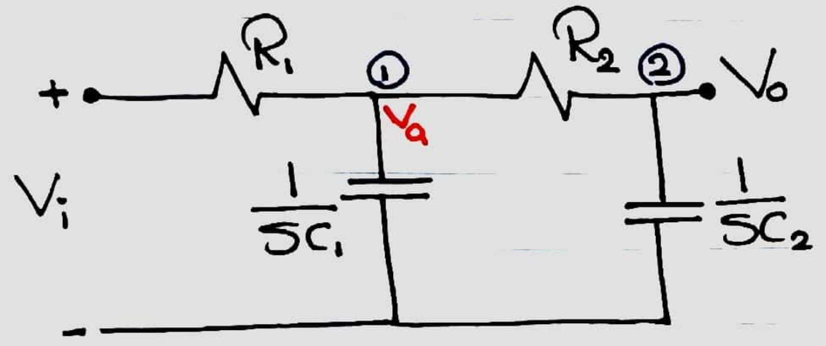
CE Sheet 2

• Capital letters → S domain

1) Find $\frac{V_o(S)}{V_i(S)}$ for



Recall

• $\sum I_{in} = \sum I_{out}$

• 

$I = \frac{V_x - V_y}{Z}$

• Node analysis (@ ① and ②) // can generalize

① $$\frac{V_i - V_a}{R_1} = \frac{V_a - 0}{\left(\frac{1}{SC_1}\right)} + \frac{V_a - V_o}{R_2}$$

$$V_i\left(\frac{1}{R_1}\right) = V_a\left(\frac{1}{R_1} + SC_1 + \frac{1}{R_2}\right) - V_o\left(\frac{1}{R_2}\right) \quad ①$$

② $$\frac{V_a - V_o}{R_2} = \frac{V_o}{\left(\frac{1}{SC_2}\right)} \rightarrow V_a\left(\frac{1}{R_2}\right) = V_o\left(\frac{1}{R_2} + SC_2\right) \quad ②$$

- The underlying system is then

$$\left.\begin{aligned} aV_i &= bV_a - cV_o \\ cV_a &= eV_o \end{aligned}\right\} \text{ Seek } \frac{V_o}{V_i} \text{ from this}$$

- Clearly $V_a = \frac{e}{c} . V_o$ and hence $aV_i = \frac{be}{c} V_o - cV_o$

which yields $\frac{V_o}{V_i} = \frac{a}{\frac{be}{c} - c} = \frac{ac}{be - c^2}$

$$\frac{V_o(S)}{V_i(S)} = \frac{(1/R_1)(1/R_2)}{\left(\frac{1}{R_1} + SC_1 + \frac{1}{R_2}\right)\left(\frac{1}{R_2} + SC_2\right) - \left(\frac{1}{R_2}\right)^2}$$

- Yes, same as the TA's answer (mathematically)
- No negative powers of S but can $\frac{\times(R_1R_2).R_2}{\times(R_1R_2).R_2}$

$$\frac{V_o(S)}{V_i(S)} = \frac{R_2}{(R_1 + R_2 + SC_1R_1R_2)(1 + SC_2R_2) - R_1}$$

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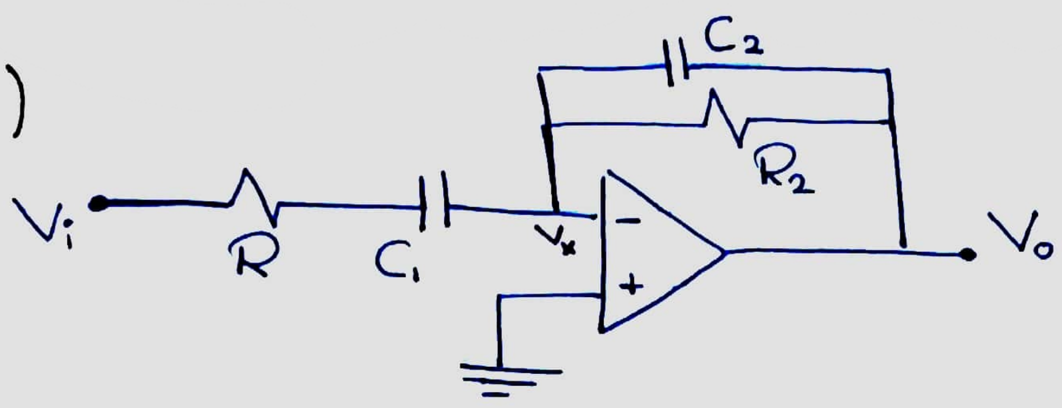
2)



- Have shown in the lecture that [Lec. 2, 2]

$$\frac{V_o(s)}{V_i(s)} = -\frac{Z_2(s)}{Z_1(s)}$$

Here

$$Z_1(s) = R + \frac{1}{sC_1}, \quad Z_2(s) = R_2 \parallel \frac{1}{sC_2}$$

$$= \frac{R_2 \times \frac{1}{sC_2}}{R_2 + \frac{1}{sC_2}} = \frac{R_2}{1+sC_2R_2}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{-R_2}{1+sC_2R_2} \cdot \frac{1}{R+\frac{1}{sC_1}} \times \frac{sC_1}{sC_1}$$

$$= \frac{-sC_1R_2}{(1+sC_2R_2)(1+sC_1R)}$$

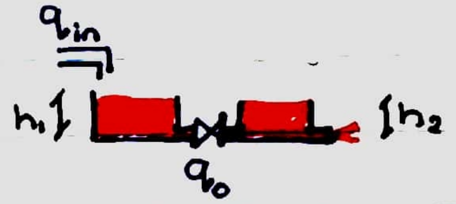
- Alternatively, can directly solve

$$\frac{V_i - V_x}{R + (sC_1)^{-1}} = \frac{V_x - V_o}{R_2 \parallel (C_2 s)^{-1}}, \quad V_x = 0 \quad (\text{ideal amplifier has } V^+ = V^-)$$

3.

Be reminded that for tank systems

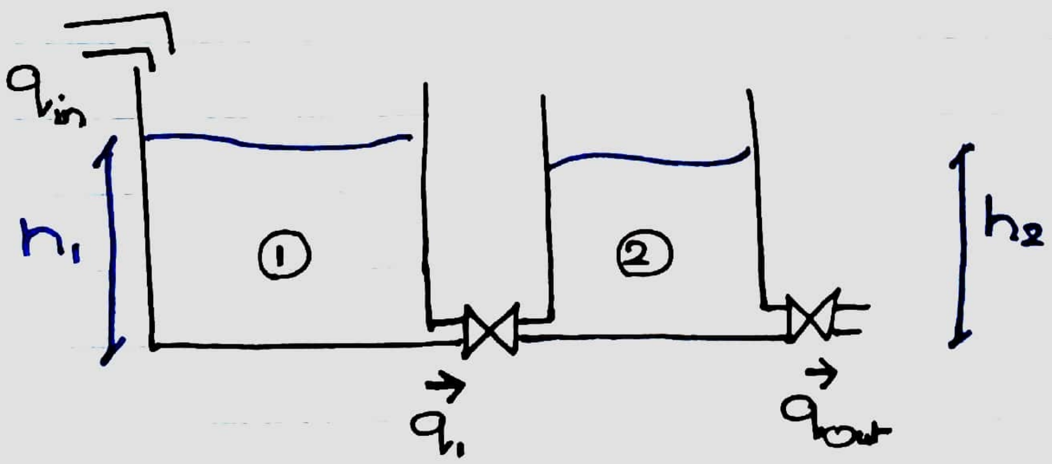
- $q_i(t) - q_o(t) = A\dfrac{dh(t)}{dt}$
- $q_o(t) = \dfrac{h_1(t) - h_2(t)}{R}$



→ Flow goes from tank of higher height to tank of lower height.

→ No flow if liquid level kept at the same height for both

→ If there's no further tank $h_2(t) = 0$ (it's an infinite sink)



① $q_{in}(t) - q_1(t) = A_1\dfrac{dh_1(t)}{dt}$, $q_1(t) = \dfrac{1}{R}(h_1(t) - h_2(t))$

② $q_1(t) - q_o(t) = A_2\dfrac{dh_2(t)}{dt}$, $q_o(t) = \dfrac{1}{R}h_2(t)$

① $Q_i - Q_1 = A_1 . S H_1$

$Q_1 = \frac{1}{R}(H_1 - H_2)$

② $Q_1 - Q_o = A_2 S H_2$

$Q_o = \frac{1}{R} H_2$

Need to Eliminate Q_1, H_1, H_2

- Add 1st and 3rd eqns

$$Q_i - Q_o = S(A_1 H_1 + A_2 H_2)$$

- Plug with last eqn

$$Q_i - Q_o = S(A_1 H_1 + A_2 Q_o R) \ominus$$ Simplify

- Now need a relation between H_1 and Q_i, Q_o
  → Plug with 2nd eqn. In 1st (and note $H_2 = Q_o R$)

$$Q_i - \frac{1}{R}(H_1 - Q_o R) = A_1 S H_1$$

Simplify

$$Q_i + Q_o = H_1\left(\frac{1}{R} + A_1 S\right)$$

$$Q_i = H_1(SA_1) + Q_o(1 + A_2 R S) \ominus$$

• The underlying system is hence

$$Q_i + Q_o = H_1 a$$
$$Q_i = H_1 b + Q_o c$$

→ From the 1st $H_1 = \frac{Q_i + Q_o}{a}$, Plugging in the second

$$Q_i = (Q_i + Q_o)\frac{b}{a} + Q_o c$$

This implies $Q_i\left(1 - \frac{b}{a}\right) = Q_o\left(c + \frac{b}{a}\right)$

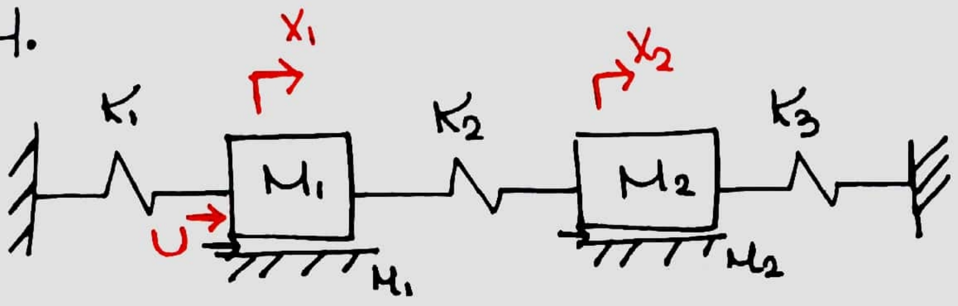
yielding $\frac{Q_o}{Q_i} = \frac{1 - b/a}{c + b/a} = \frac{a - b}{ac + b}$

$$\frac{Q_o(s)}{Q_i(s)} = \frac{\left(\frac{1}{R} + A_1 s\right) - sA_1}{\left(\frac{1}{R} + A_1 s\right)(1 + A_2 R s) + sA_1} \times \frac{R}{R}$$

$$= \frac{1}{(1 + RA_1 s)(1 + A_2 R s) + sA_1 R}$$

• Also equal to the TA's result besides looking different.

4.



- U is an external force
- Sheet treats F_1, F_2 as friction consts

• m_1's equation of motion:

$$u - K_1 x_1 - K_2(x_1 - x_2) - \mu_1 \dot{x}_1 = m_1 \ddot{x}_1$$

$\mathcal{L}$

$$U - K_1 X_1(S) - K_2(X_1(S) - X_2(S)) - \mu_1 S X_1(S) = m_1 S^2 X_1(S)$$

$$U - X_1(S)(K_1 + K_2 + \mu_1 S + m_1 S^2) = X_2(S)(-K_2) \quad (1)$$

• m_2's equation of motion:

$$-K_3 x_2 - K_2(x_2 - x_1) - \mu_2 \dot{x}_2 = m_2 \ddot{x}_2$$

$\mathcal{L}$

$$-K_3 X_2(S) - K_2(X_2(S) - X_1(S)) - \mu_2 S X_2(S) = m_2 S^2 X_2(S)$$

$$-X_2(S)(K_3 + K_2 + \mu_2 S + m_2 S^2) = X_1(S)(-K_2) \quad (2)$$

⇒ From (1), (2) the underlying system clearly is

$$U - X_1 . a = X_2 . b$$
$$-X_2 . c = X_1 . d$$
} want $\frac{X_1}{U}$, $\frac{X_2}{U}$

• For $\frac{X_1}{U}$, Plug $X_2 = -X_1 \frac{d}{c}$ in the 1st eqn.

$$U - X_1 . a = -X_1 \frac{d}{c} . b$$
$$U = X_1 \left(a - \frac{db}{c}\right) \rightarrow \frac{X_1}{U} = \frac{1}{a - \frac{db}{c}} = \frac{c}{ac - db}$$

$$T \left\{ \frac{X_1(S)}{U(S)} = \frac{(K_3 + K_2 + M_2 S + m_2 S^2)}{(K_1 + K_2 + M_1 S + m_1 S^2)(K_3 + K_2 + M_2 S + m_2 S^2) - K_2^2} \right.$$

• For $\frac{X_2}{U}$, can multiply $\frac{X_1}{U}$ by $\frac{-d}{c}$ which gives

$$\frac{X_2(S)}{U(S)} = \frac{K_2}{(K_1 + K_2 + M_1 S + m_1 S^2)(K_3 + K_2 + M_2 S + m_2 S^2) - K_2^2}$$

$\Rightarrow$ Now let $u(t) = 1$ newton $\leftrightarrow U(S) = \frac{1}{S}$

* Assume it's a stable system

• $\lim_{t \to \infty} x_1(t) = \lim_{S \to 0} S . X_1(S) = \lim_{S \to 0} S . U(S) . TF_1(S) = \lim_{S \to 0} TF_1(S)$

• $\lim_{t \to \infty} x_1(t) = (K_2 + K_3) / ((K_1 + K_2)(K_3 + K_2) - K_2^2)$, likewise

• $\lim_{t \to \infty} x_2(t) = K_2 / ((K_1 + K_2)(K_3 + K_2) - K_2^2)$

Thank you <3

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