# Sheet 4

**Q.1**

$$\Sigma = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix} \rightarrow \begin{vmatrix} 2-\lambda & 1 \\ 1 & 3-\lambda \end{vmatrix} = 0$$

$B = \Sigma$

$\Sigma U = U \Omega$

$\Sigma \underline{u_i} = \underline{u_i} \lambda_i$

$\boxed{(\Sigma - \lambda_i I)\underline{u_i} = 0}$

$$\therefore (2-\lambda)(3-\lambda) - 1 = 0 \rightarrow 6 - 5\lambda + \lambda^2 - 1 = 0$$

$$\lambda^2 - 5\lambda + 5 = 0 \rightarrow \lambda = \frac{5 \pm \sqrt{5}}{2}$$ ] eigen values

eigenvectors: $\Sigma u_i = u_i \lambda$ (step2)

$$\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{5+\sqrt{5}}{2} x \\ \frac{5+\sqrt{5}}{2} y \end{bmatrix} \xrightarrow{i=1} \begin{matrix} 2x + y = \frac{5+\sqrt{5}}{2} x \\ x + 3y = \frac{5+\sqrt{5}}{2} y \end{matrix} \rightarrow u_1 = \begin{bmatrix} 0.5257 \\ 0.8507 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} \text{———} \begin{bmatrix} \frac{5-\sqrt{5}}{2} x \\ \frac{5-\sqrt{5}}{2} y \end{bmatrix} \xrightarrow{i=2} \text{———} \rightarrow u_2 = \begin{bmatrix} -0.8507 \\ 0.5257 \end{bmatrix}$$

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$$\therefore \left.\begin{matrix} U_1^T U_2 = 0 \\ \underline{U_1} \cdot \underline{U_2} = 0 \end{matrix}\right\} \rightarrow U_1 \perp U_2 \rightarrow ✓✓$$

$$U = \begin{array}{c} \begin{bmatrix} 0.5257 & -0.8507 \\ 0.8507 & 0.5257 \end{bmatrix} \\ \quad \underline{U_1} \qquad\quad \underline{U_2} \end{array}$$

(step3) $\therefore \lambda_1 > \lambda_2 \rightarrow$ choose $\underline{u_1} \Rightarrow \underset{1\times1}{z} = \underset{1\times2}{\underline{u_1}^T}\, y_{2\times1}$

Transform

$\underset{\Rightarrow}{\text{OR}}\ z = U^T Y$ & take $z_1$ only

$\boxed{\text{step1}}$ $\hat{\mu} = \frac{1}{M} \sum_{1}^{M} \underline{x}(m)$

$$\underline{Y} = \underline{X} - \hat{\mu}$$

$$\hat{\Sigma} = \frac{1}{M} \sum_{1}^{M} \underline{Y}(m)\, \underline{Y}(m)^T$$

Q.2 $\underline{u}_1 = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{pmatrix}$, $\underline{u}_2 = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{pmatrix}$, $\underline{u}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$\lambda_1 = 2$ , $\lambda_2 = 0.5$ , $\lambda_3 = 0.2$

$\Sigma = ?$

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$$U^T \Sigma U = \Omega = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.2 \end{bmatrix}$$

∴ orthogonal

$$\underbrace{UU^T}_{I} \Sigma \underbrace{UU^T}_{I} = U \Omega U^T \rightarrow \Sigma = U \Omega U^T$$

$$\Sigma = \begin{bmatrix} | & | & | \\ u_1 & u_2 & u_3 \\ | & | & | \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} \begin{bmatrix} - u_1 - \\ - u_2 - \\ - u_3 - \end{bmatrix}$$

$$= \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$= \_\_\_$

Q.3 Neural Networks

To be Contu!