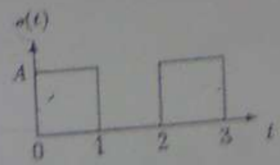
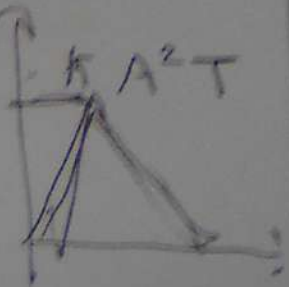
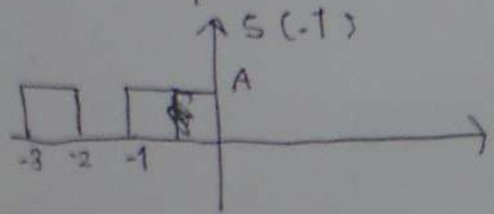
Problem 1 *(3 points)*
The received signal in a communication system is $r(t) = s(t) + n(t)$, where $s(t)$ is the pulse shown in the figure below, and $n(t)$ is AWGN with power-spectral density $N_0/2$ W/Hz.

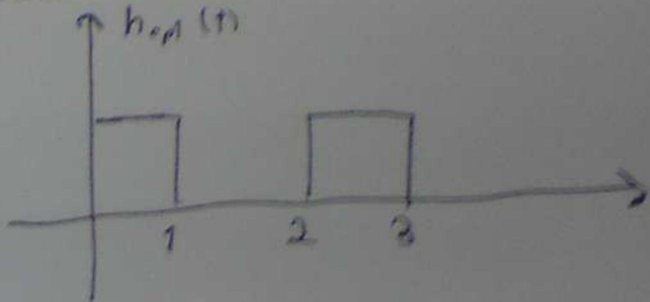




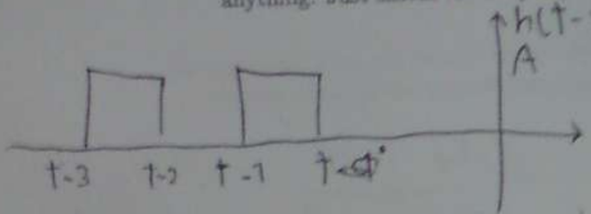
(1) (1 point) Sketch the impulse response of the filter matched to $s(t)$.

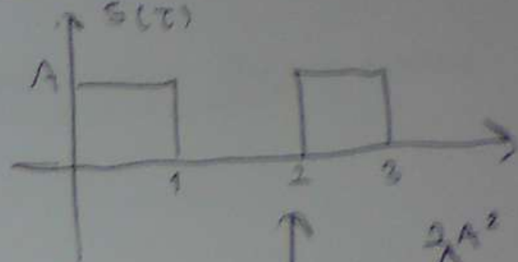
$h_{opt}(t) = s(T-t)$

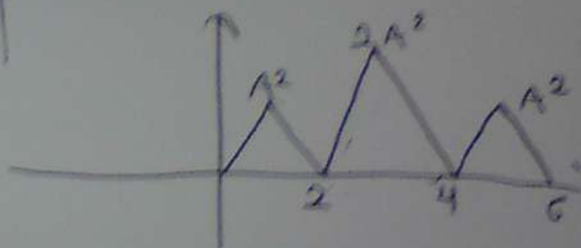




(2) (2 points) Sketch the output of the matched filter to input $s(t)$. (<u>**Hint:**</u> No need to derive anything. Just sketch the output.)







$0 < t < 1 \quad y(t) = A^2 t$

$1 < t < 2 \quad y(t) = A^2(2-t)$

$2 < t < 3 \quad y(t) = A^2[2t-4]$

$3 < t < 4 \quad y(t) = A^2[4-t+4-t] = A^2[8-2t]$

$4 < t < 5 \quad y(t) = A^2[t-4]$

$5 < t < 6 \quad y(t) = A^2[6-t]$

 Name:

Problem 2 *(4 points)*
An analog signal is sampled, quantized, and encoded into a binary PCM wave. The number of representation levels used is 128. A synchronizing pulse is added at the end of each code word representing a sample of the analog signal. The resulting PCM wave is transmitted over a channel of bandwidth 12 KHz using a quaternary PAM system with raised cosine spectrum. The rolloff factor is unity.

(1) (2 points) Find the rate in bits/sec at which information is transmitted through the channel.

4 symbols $\rightarrow m = \log_2 M$

$= 2$

Bit rate $= m R_s$

$= 2 \times 12\,K$

$\boxed{R_B = 24\ kb/s}$

(2) (2 points) Find the rate at which the analog signal is sampled. What is the maximum possible value for the highest frequency component of the analog signal?

$$\text{rate} = \frac{R_B \text{ bits/s}}{\text{Number of bits/Sample}}$$

$$= \frac{24000}{(7+1)}$$

128 level $n=7$; synchronization pulse

$f_s = 3\ KHz$ $\quad\therefore$ Highest freq component $= \frac{R_B}{2}$

 Name: ____________

Problem 3 (4 points)

Consider a binary PCM system which employs **polar** NRZ line code and is transmitting 100 Kbps. The noise power spectral density at the receiver is $N_o/2$, where $N_o = 2.5 \times 10^{-7}$ Volts^2/Hz.

(1) (2 points) What is the minimum average power of the transmitted signal that will produce a probability of bit error of 10^{-5} or less?

'0' sent $P_{10} = \frac{1}{2}\, erfc\left(\frac{\lambda - u_1}{\sqrt{2}\,\sigma}\right) = \frac{1}{2}\, erfc\left(\frac{\lambda + A}{\sqrt{2}\sqrt{\frac{N_o}{2T_b}}}\right)$

'1' sent $P_{01} = \frac{1}{2}\, erfc\left(\frac{u_2 - \lambda}{\sqrt{2}\,\sigma_3}\right) = \frac{1}{2}\, erfc\left(\frac{A - \lambda}{\sqrt{\frac{N_o}{T_b}}}\right)$

$P_{error} = P_0 P_{10} + P_1 P_{01}$

at $P_0 = P_1$ $\quad \lambda_{opt} = 0$

$P_{error} = \frac{1}{2}\, erfc\sqrt{\frac{E_b}{N_o}} = 10^{-5}$

$\sqrt{\frac{E_b}{N_o}} = 3.02$

$E_b = 2.2801 \times 10^{-6}$

$P_{av\,min} = \frac{E_b}{T_b} = E_b \times R_B$

$= 0.228$ W

(2) (2 points) Now assume that **unipolar** NRZ line code is used instead. What is the minimum average power needed in this case?

'0' sent $P_{10} = \frac{1}{2}\, erfc\left(\frac{\lambda}{\sqrt{\frac{N_o}{T_b}}}\right)$

'1' sent $P_{01} = \frac{1}{2}\, erfc\left(\frac{A - \lambda}{\sqrt{\frac{N_o}{T_b}}}\right)$

at $P_{10} = P_{01} \Rightarrow \lambda_{opt} = \frac{A}{2}$

$P_{error} = \frac{1}{2}\, erfc\left(\sqrt{\frac{E_b}{2N_o}}\right)$

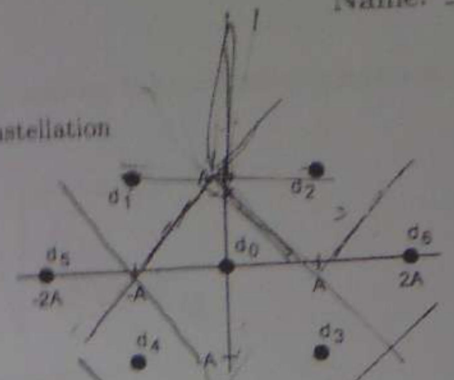
$\sqrt{\frac{E_b}{2N_o}} = 3.02$

$E_b = 4.5602 \times 10^{-6}$

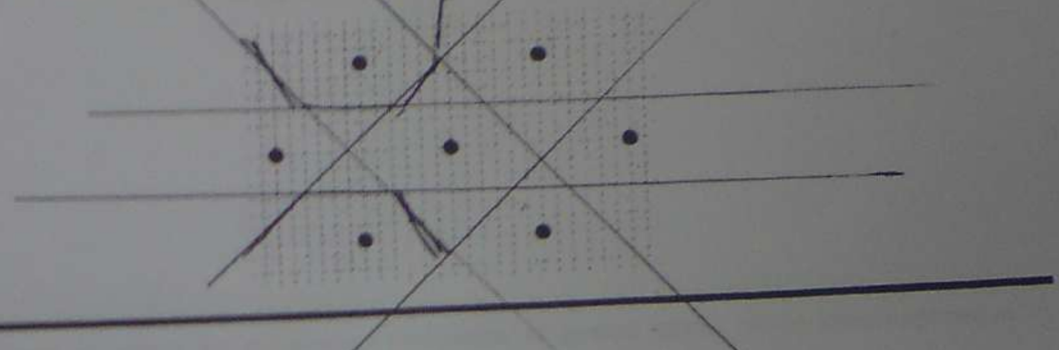
$P_{av} = E_b R_B = 0.45602$

Name: ______________

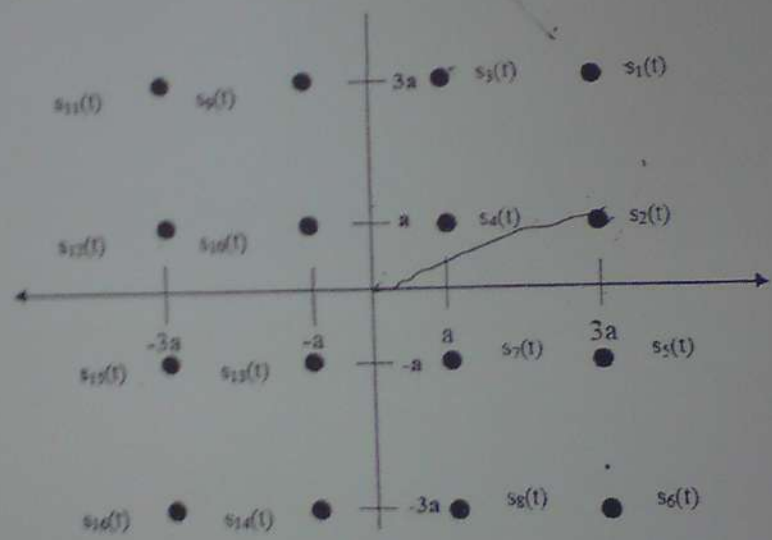
Problem 4 *(4 points)*
Consider the following constellation



Sketch the decision regions using the figure below (assuming equiprobable symbols).



Problem 5 *(6 points)*
Consider the 16-symbol constellation shown in the figure below.



(1) (2 points) Calculate the average energy E_s for the constellation in terms of a. What is

$$E_s = \frac{4\left[E_{s1} + E_{s2} + E_{s3} + E_{s4}\right]}{16}$$ Due to symmetry

$$= \frac{4\left[18a^2 + 10a^2 + 10a^2 + 2a^2\right]}{16}$$

$$= 10a^2$$

 Name: ____________

the minimum distance between the transmitted symbols in terms of the average energy?

$= 2a$

$= 2\sqrt{\frac{E_s}{10}}$

$= \frac{2}{\sqrt{10}}\sqrt{E_s}$

(2) (4 points) Assuming equiprobable symbols, use the union bound to calculate an approximate expression for the average probability of symbol error. Write your expression in terms of E_s and not a.

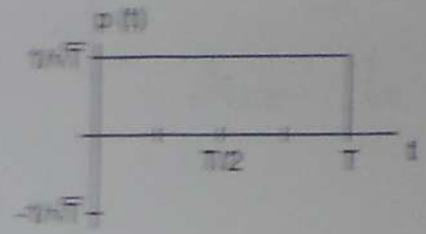
Problem 6 *(6 points)*

Consider the binary communications problem where we are transmitting a single bit, d, using the pulse $p(t)$. The transmitted signal $x(t)$ is given by

$$x(t) = \pm A\, p(t),$$

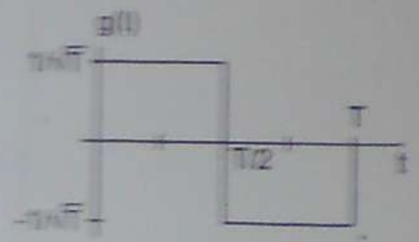
where A is a constant and $p(t)$ is given by,

$$p(t) = \begin{cases} \frac{1}{\sqrt{T}}, & 0 < t < T \\ 0, & \text{elsewhere} \end{cases}$$



The received signal is corrupted by a real additive white Gaussian noise (AWGN) process, $w(t)$, with variance $N_0/2$ and an interference signal, $g(t)$.

$$g(t) = \begin{cases} 1/\sqrt{T}, & 0 < t < T/2 \\ -1/\sqrt{T}, & T/2 < t < T \\ 0, & \text{elsewhere} \end{cases}$$



As a result, the received signal is given by

$$y(t) = x(t) + g(t) + w(t).$$

The received signal is then passed through a matched filter where its output $v(t)$ is sampled and a threshold detector is used to detect the transmitted bits.