$ax + by = c$

such that

$x, y \in \mathbb{Z}$

general solution.

- $\gcd(a,b) \mid c$
- (x_0, y_0) is a solution
- $(x_0 + tq, y_0 - tp)$

$t \in \mathbb{Z}$

$q = \frac{b}{\gcd(a,b)}, \; p = \frac{a}{\gcd(a,b)}$

$\boxed{7}$

- $6x + 51y = 22$

$\gcd(6, 51)$

$= \gcd(6, 3)$

$= 3$

$3 \nmid 22$

Cannot be solved

$\boxed{8}$ a) $56x + 72y = 40$

$\gcd(72, 56)$

$= \gcd(56, 16)$

$= \gcd(16, 8)$

$= 8$

$8 \mid 40$

$\therefore$ there are solutions

$72 = 56 + 16$

$56 = 3 \cdot 16 + 8$

$56 = 3(72 - 56) + 8$

$56 = 3 \cdot 72 - 3 \cdot 56 + 8$

$8 = 4 \cdot 56 - 3 \cdot 72 \quad \times 5$

$40 = 20 \cdot 56 - 15 \cdot 72$

$x = 20, \ y = -15$

all solutions are

$(20 + t \cdot 9, -15 - t \cdot 7)$

$t \in \mathbb{Z}$

$\boxed{10}$

4.55 \$

dimes = 10 cents

quarters = 25 cents

- maximum & minimum number of coins
- # of dimes = # of quarters?

x: # of dimes

y: # of quarters

$$10x + 25y = 455$$

$$\gcd(25, 10) = 5$$

$$5 = 25 - 2 \cdot 10 \quad \times 91$$

$$455 = -2 \cdot 91 \cdot 10 + 91 \cdot 25$$

$$x = -182, \quad y = 91$$

all solutions are

$$(-182 + t \cdot 5, \ 91 - t \cdot 2)$$

$t=37$, $(3,17)$, 20

$t=38$, $(8,15)$, 23

$(13,13)$, 26

$t=39$,

$t=40$, $(18,11)$, 1

$t=41$, $(23,9)$, 1

$t=42$, $(28,7)$, 1

$t=43$, $(33,5)$

$t=44$, $(38,3)$, 44

$t=45$, $(43,1)$,

1.8 \$ Adults

0.75 \$ Child

sum 90 \$

x : # of Adults

y : # of kids.

$180x+75y=9000$

$\gcd(180,75)$

$=\gcd(75,30)$

$=\gcd(30,15)=15$

$180=2\cdot 75+30$

$75=2\cdot 30+15$

$15=75-2(180-2\cdot 75)$

$15=5\cdot 75-2\cdot 180$

1

① $\times 600$

$9000 = 3000 \cdot 75 - 1200 \cdot 180$

$x = -1200, \; y = 3000$

all solutions

$(-1200 + t \cdot 5, \; 3000 - t \cdot 12)$

. since $x > y$

$-1200 + 5t > 3000 - 12t$

$17t > 4200$

$t > 247. -$

$t = 248, \; (40, 24)$

$t = 249, \; (45, 12)$

$t = 250, \; (50, 0)$

x: # of sixes

y: # of nines

$6x + 9y = 126$ ✓

$6y + 9x = 114$

How many of 6s & 9s?
sol

$\gcd(9, 6) = 3$

$3 = 9 - 6$

$3 = -6 + 9 * 42$

$126 = -42 \cdot 6 + 42 \cdot 9$

$x = -42, \quad y = 42$

$(-42 + t \cdot 3, 42 - t \cdot 2)$

$t = 14, \ (0, 14)$

$t = 15, \ (3, 12)$

$\boxed{t = 16, \ (6, 10)}$ ✓

$t = 17, \ (9, 8)$

$t = 18, \ (12, 6)$

$t = 19, \ (15, 4)$

$t = 20, \ (18, 2)$

$t = 21$

$t = 21$

$(21, 0)$

- $ma \equiv mb \pmod{mn}$

$\longrightarrow a \equiv b \pmod{n}$

proof

let $ma \equiv \quad \pmod{mn}$

$\therefore mn \mid (ma - mb)$

$\therefore \cancel{m}a - \cancel{m}b = k\cancel{m}n$

$\therefore a - b = kn$

$n \mid (a-b)$

$a \equiv b \pmod{n}$

- $ma \equiv mb \pmod{n} \wedge \gcd(m,n) = 1$

$\longrightarrow a \equiv b \pmod{n}$

proof

let

$ma \equiv mb \pmod{n}$

& $\gcd(m,n) = 1$

$n \mid m(a-b)$ let $a-b = c$

$n \mid mc \Rightarrow mc = kn$

$1 = xm + yn \quad * \, c$

$c = x \cdot cm + ycn$

$c = knx + ycn \Rightarrow c = (kx + yc)n$

$\therefore n \mid c \rightarrow n \mid (a-b)$

$\therefore a \equiv b \pmod{n}$

$\bullet\ a \equiv b \pmod{n}$

$\longrightarrow a \equiv (b+kn) \pmod{n}$

proof

let $a \equiv b \pmod{n}$

$\therefore n \mid (a-b),\ n \mid -kn$

$\therefore n \mid (a-b-kn)$

$n \mid (a-(b+kn))$

$a \equiv (b+kn) \pmod{n}$

$ax \equiv b \pmod{n}$

$\gcd(a,n)=1$ $\quad 5 \mid (6+n8)$

$\boxed{13}$

$5x \equiv 6 \pmod{8}$

$\gcd(5,8)=1$

$5x \equiv 14 \pmod{8}$

$5x \equiv 22 \pmod{8}$

$5x \equiv 30 \pmod{8}$

$x \equiv 6 \pmod{8}$

$5x \equiv 4 \pmod{6}$

$\gcd(5, 6) = 1$

$5x \equiv 10 \pmod{6}$

$x \equiv 2 \pmod{6}$

$3x - 2 \equiv 0 \pmod{11}$

$3x \equiv 2 \pmod{11}$

$\gcd(3, 11) = 1$

$3x \equiv 13 \pmod{11}$

$3x \equiv 24 \pmod{11}$

$x \equiv 8 \pmod{11}$