

Information Theory 3<sup>rd</sup> year 2015 – 2016

## **Sheet 1**

## Problem#1

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9.21 Consider the binary symmetric channel described in Figure 9.8. Let  $p_0$  denote the probability of sending binary symbol  $x_0 = 0$ , and let  $p_1 = 1 - p_0$  denote the probability of sending binary symbol  $x_1 = 1$ . Let p denote the transition probability of the channel.

(a) Show that the mutual information between the channel input and channel output is given by

$$I(\mathcal{X}; \mathcal{Y}) = \mathcal{H}(z) - \mathcal{H}(p)$$

where

$$H(z) = z \log_2\left(\frac{1}{z}\right) + (1-z) \log_2\left(\frac{1}{1-z}\right)$$
$$z = p_0p + (1-p_0)(1-p)$$

and

$$H(p) = p \log_2\left(\frac{1}{p}\right) + (1-p) \log_2\left(\frac{1}{1-p}\right)$$

Show that the value of  $p_0$  that maximizes  $I(\mathcal{X}; \mathcal{Y})$  is equal to 1/2. (c) Hence, show that the channel capacity equals

$$C = 1 - H(p)$$

## Problem#2

15.4-1 A binary channel matrix is given by



Outputs
$$y_{1} \quad y_{2}$$

$$x_{1} \left[\begin{array}{cc} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{10} & \frac{9}{10} \end{array}\right]$$
Inputs

This means  $P_{y|x}(y_1|x_1) = 2/3$ ,  $P_{y|x}(y_2|x_1) = 1/3$ , etc. You are also given that  $P_x(x_1) = 1/3$  and  $P_x(x_2) = 2/3$ . Determine H(x), H(x|y), H(y), H(y|x), and I(x;y).