Cairo University
Faculty of Engineering
Computer Engineering

Pattern Recognition and Neural Networks
Problem Set 1: A Review on Probability.

Question (1):

There are 5 boxes of mixed fruits with 1 of the boxes is red, 1 is blue and the rest are green. Each box color has a mixture of fruits according to the table:

	Red Box	Blue Box	Green Box
Apples	3	5	3
Oranges	4	5	3
Limes	3	0	4

Apply the **sum and product rules** to compute the following:

a) Compute p(Fruit = *Orange* | Box = *Red*).
b) Compute p(Fruit = *Orange*).
c) Compute p(Box = *Red* | Fruit = *Orange*).
d) Compute p(Box = *Green* | Fruit = *Apples*).

Question (2):

A coin is flipped twice. Assuming that all four points in the sample space S = {(h, h), (h, t), (t, h), (t, t)} are equally likely, what is the conditional probability that both flips land on heads, given that:

(a) the first flip lands on heads?
(b) at least one flip lands on heads?

Question (3):

Two coins are flipped, and all 4 outcomes are assumed to be equally likely. If E is the event that the first coin lands on heads and F the event that the second lands on tails. Prove that E and F are independent.

Question (4):

Suppose the random variable X has a probability mass function given by:

$$p(1) = \frac{1}{4}, p(2) = \frac{1}{2}, p(3) = \frac{1}{8}, p(4) = \frac{1}{8}.$$

a) Verify that the given function is a PMF.
b) Compute the expected value of the random variable X.
c) Compute the variance and standard deviation of the random variable X.
d) Compute the cumulative density function and plot it.
e) Let the random variable Y = ln(x). Compute the expectation, variance and standard deviation of Y.

Question (5):

If $\mathrm{E[X]} = 1$ and $\mathrm{Var(X)} = 5$, find:

(a) $\mathrm{E}[(2 + \mathrm{X})^2]$.
(b) $\mathrm{Var}(4 - 3\mathrm{X})$.

Question (6):

A box contains 5 green and 5 blue balls. Two balls are selected at random without replacement. If the two balls have the same color, then your team earns 5.5 points. If they are different colors, then you win -5 points. (That is, you lose 5 points)
Calculate:
(a) the expected value of the points your team earns.
(b) the variance of the points your team earns.

Question (7):

The amount of time in hours that a computer functions before breaking down is a continuous random variable with probability density function given by

$$f(x) = \begin{cases} \lambda \mathrm{e}^{-\frac{\mathrm{x}}{100}} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

What is the probability that:
(a) a computer will function between 50 and 150 hours before breaking down?
(b) it will function for fewer than 100 hours?

Question (8):

The density function of X is given by

$$f(x) = \begin{cases} \mathrm{a} + \mathrm{bx}^2 & 0 \leq x \leq 1 \\ 0 & otherwise \end{cases}$$

If $\mathrm{E[X]} = 35$, find a and b.

Question (9):

Prove the following identities:

a) E[aX + b] = a E[X] + b.
b) Var(aX + b) = a^2 Var(X).
c) Var(X + Y) = Var(X) + Var(Y) + 2 Cov(X, Y).
d) Cov(X, Y) = E[XY] – E[X]E[Y].

Question (10):

If X and Y are found to be independent random variables, which of the following variables will be computed to zero? There may be more than one correct answer. Explain why they are correct and the other choices are wrong.

a) E[XY]
b) Var(X+Y)
c) Cov(X, Y)
d) Corr(X, Y)
e) E[XY] – E[X+Y]
f) E[XY] – E[X]E[Y]
g) Var(XY) – Var(X)Var(Y)
h) Var(X+Y) – Var(X) – Var(Y)

Question (11):

Consider the Gaussian distribution shown as: $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp(-\frac{(x-\mu)^2}{2\sigma^2})$ where x represents a normal random variable, prove the following identities:

a) $E[X] = \mu$
b) $Var(X) = \sigma^2$

If $Z = \frac{X-\mu}{\sigma}$ represents the standard normal random variable. Prove the following identities:

a) $E[Z] = 0$
b) $Var(Z) = 1$

Question (12):

For the Gaussian density in the previous question with parameters $\mu = 0$ and $\sigma = 1$, compute $E[e^{-\alpha x}]$, where E(.) means expectation over the Gaussian density described.

Question (13):

Using the fact that:

$$E\big(f(x) + g(x)\big) = E\big(f(x)\big) + E(g(x))$$

Find using the results obtained in the previous question:

$$E(e^{-x} + e^{-2x} + x^2)$$