Generator matrix :

We start with K polynomials:

$g(x)$ & $K-1$ cyclic shifted versions of it:

$g(x)$

$x\,g(x)$

$\vdots$

$x^{K-1}g(x)$

These polynomials (their coefficients) are used to form the rows of the generator matrix.

Example: for the (7,4) Hamming code we have:

$g(x) = 1 + x + x^3$

$x\,g(x) = x + x^2 + x^4$

$x^2\,g(x) = x^2 + x^3 + x^5$

$x^3\,g(x) = x^3 + x^4 + x^6$

$$G' = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

This matrix is not in systematic form. We can get it in a systematic form by adding the 1st row to the 3rd row & adding the sum of the first 2 rows to the fourth row. These manipulations result in the desired gen matrix.

$$G = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$