Sheet02: INDEXING STRUCTURES FOR FILES

Answers to Selected Exercises

14.14 Consider a disk with block size B=512 bytes. A block pointer is P=6 bytes long, and a record pointer is P R =7 bytes long. A file has r=30,000 EMPLOYEE records of fixed-length. Each record has the following fields: NAME (30 bytes), SSN (9 bytes), DEPARTMENTCODE (9 bytes), ADDRESS (40 bytes), PHONE (9 bytes), BIRTHDATE (8 bytes), SEX (1 byte), JOBCODE (4 bytes), SALARY (4 bytes, real number). An additional byte is used as a deletion marker.

(a) Calculate the record size R in bytes.

Record length R = (30 + 9 + 9 + 40 + 9 + 8 + 1 + 4 + 4) + 1 = 115 bytes

(b) Calculate the blocking factor bfr and the number of file blocks b assuming an unspanned organization.

Blocking factor bfr = floor(B/R) = floor(512/115) = 4 records per block
Number of blocks needed for file = ceiling(r/bfr) = ceiling(30000/4) = 7500

(c) Suppose the file is ordered by the key field SSN and we want to construct a primary index on SSN. Calculate

(i) the index blocking factor bfr

Index record size R i = (V SSN + P) = (9 + 6) = 15 bytes
Index blocking factor bfr i = fo = floor(B/R i) = floor(512/15) = 34

(ii) the number of first-level index entries and the number of first-level index blocks;

Number of first-level index entries r 1 = number of file blocks b = 7500 entries
Number of first-level index blocks b 1 = ceiling(r 1 /bfr i) = ceiling(7500/34)
= 221 blocks

(iii) the number of levels needed if we make it into a multi-level index;

We can calculate the number of levels as follows:
Number of second-level index entries r 2 = number of first-level blocks b 1
= 221 entries
Number of second-level index blocks b 2 = ceiling(r 2 /bfr i) = ceiling(221/34)
= 7 blocks
Number of third-level index entries r 3 = number of second-level index blocks b 2
= 7 entries
Number of third-level index blocks b 3 = ceiling(r 3 /bfr i) = ceiling(7/34) = 1
Since the third level has only one block, it is the top index level.
Hence, the index has x = 3 levels

(iv) the total number of blocks required by the multi-level index; and

Total number of blocks for the index b i = b 1 + b 2 + b 3 = 221 + 7 + 1
= 229 blocks

(v) the number of block accesses needed to search for and retrieve a record from the file--given its SSN value--using the primary index.

Number of block accesses to search for a record = x + 1 = 3 + 1 = 4

(d) Suppose the file is not ordered by the key field SSN and we want to construct a secondary index on SSN. Repeat the previous exercise (part c) for the secondary index and compare with the primary index.

(i) the index blocking factor bfr

Index record size R i = (V SSN + P) = (9 + 6) = 15 bytes
Index blocking factor bfr i = (fan-out) fo = floor(B/R i) = floor(512/15)
= 34 index records per block

(ii) the number of first-level index entries and the number of first-level index blocks;

Number of first-level index entries r 1 = number of file records r = 30000
Number of first-level index blocks b 1 = ceiling(r 1 /bfr i) = ceiling(30000/34)
= 883 blocks

(iii) the number of levels needed if we make it into a multi-level index;

We can calculate the number of levels as follows:
Number of second-level index entries r 2 = number of first-level index blocks b 1
= 883 entries
Number of second-level index blocks b 2 = ceiling(r 2 /bfr i) = ceiling(883/34)
= 26 blocks
Number of third-level index entries r 3 = number of second-level index blocks b 2
= 26 entries
Number of third-level index blocks b 3 = ceiling(r 3 /bfr i) = ceiling(26/34) = 1
Since the third level has only one block, it is the top index level.
Hence, the index has x = 3 levels

(iv) the total number of blocks required by the multi-level index; and

Total number of blocks for the index b i = b 1 + b 2 + b 3 = 883 + 26 + 1 = 910

(v) the number of block accesses needed to search for and retrieve a record from the file--given its SSN value--using the primary index.

Number of block accesses to search for a record = x + 1 = 3 + 1 = 4

(e) Suppose the file is not ordered by the non-key field DEPARTMENTCODE and we want to construct a secondary index on SSN using Option 3 of Section 14.1.3, with an extra level of indirection that stores record pointers. Assume there are 1000 distinct values of DEPARTMENTCODE, and that the EMPLOYEE records are evenly distributed among these values. Calculate

(i) the index blocking factor bfr i

Index record size R i = (V DEPARTMENTCODE + P) = (9 + 6) = 15 bytes
Index blocking factor bfr i = (fan-out) fo = floor(B/R i) = floor(512/15)
= 34 index records per block

(ii) the number of blocks needed by the level of indirection that stores record pointers;

There are 1000 distinct values of DEPARTMENTCODE, so the average number of records for each value is (r/1000) = (30000/1000) = 30
Since a record pointer size P R = 7 bytes, the number of bytes needed at the level of indirection for each value of DEPARTMENTCODE is 7 * 30 =210 bytes, which fits in one block. Hence, 1000 blocks are needed for the level of indirection.

(iii) the number of first-level index entries and the number of first-level index blocks;

Number of first-level index entries r 1
= number of distinct values of DEPARTMENTCODE = 1000 entries
Number of first-level index blocks b 1 = ceiling(r 1 /bfr i) = ceiling(1000/34)
= 30 blocks

(iv) the number of levels needed if we make it a multi-level index;

We can calculate the number of levels as follows:
Number of second-level index entries r 2 = number of first-level index blocks b 1
= 30 entries
Number of second-level index blocks b 2 = ceiling(r 2 /bfr i) = ceiling(30/34) = 1
Hence, the index has x = 2 levels

(v) the total number of blocks required by the multi-level index and the blocks used in the extra level of indirection;

total number of blocks for the index b i = b 1 + b 2 + b indirection
= 30 + 1 + 1000 = 1031 blocks

vi) the approximate number of block accesses needed to search for and retrieve all records in the file having a specific DEPARTMENTCODE value using the index.

Number of block accesses to search for and retrieve the block containing the record pointers at the level of indirection = x + 1 = 2 + 1 = 3 block accesses

If we assume that the 30 records are distributed over 30 distinct blocks, we need an additional 30 block accesses to retrieve all 30 records. Hence, total block accesses needed on average to retrieve all the records with a given value for DEPARTMENTCODE = x + 1 + 30 = 33

(f) Suppose the file is ordered by the non-key field DEPARTMENTCODE and we want to construct a clustering index on DEPARTMENTCODE that uses block anchors (every new value of DEPARTMENTCODE starts at the beginning of a new block). Assume there are 1000 distinct values of DEPARTMENTCODE, and that the EMPLOYEE records are evenly distributed among these values. Calculate

(i) the index blocking factor bfr i (which is also the index fan-out fo);

Index record size R i = (V DEPARTMENTCODE + P) = (9 + 6) = 15 bytes
Index blocking factor bfr i = (fan-out) fo = floor(B/R i) = floor(512/15)
= 34 index records per block

(ii) the number of first-level index entries and the number of first-level index blocks;

Number of first-level index entries r 1
= number of distinct DEPARTMENTCODE values= 1000 entries

Number of first-level index blocks b 1 = ceiling(r 1 /bfr i)
= ceiling(1000/34) = 30 blocks

(iii) the number of levels needed if we make it a multi-level index;

We can calculate the number of levels as follows:
Number of second-level index entries r 2 = number of first-level index blocks b 1
= 30 entries
Number of second-level index blocks b 2 = ceiling(r 2 /bfr i) = ceiling(30/34) = 1
Since the second level has one block, it is the top index level.
Hence, the index has x = 2 levels

(iv) the total number of blocks required by the multi-level index; and

Total number of blocks for the index b i = b 1 + b 2 = 30 + 1 = 31 blocks

(v) the number of block accesses needed to search for and retrieve all records in the file having a specific DEPARTMENTCODE value using the clustering index (assume that multiple blocks in a cluster are either contiguous or linked by pointers).

Number of block accesses to search for the first block in the cluster of blocks
= x + 1 = 2 + 1 = 3

The 30 records are clustered in ceiling(30/bfr) = ceiling(30/4) = 8 blocks.
Hence, total block accesses needed on average to retrieve all the records with a given DEPARTMENTCODE = x + 8 = 2 + 8 = 10 block accesses

(g) Suppose the file is not ordered by the key field Ssn and we want to construct a B + -Tree access structure (index) on SSN. Calculate

(i) the orders p and p leaf of the B + -tree;

For a B + -tree of order p, the following inequality must be satisfied for each internal tree node:
(p * P) + ((p - 1) * V SSN) < B, or
(p * 6) + ((p - 1) * 9) < 512, which gives 15p < 521, so p=34

For leaf nodes, assuming that record pointers are included in the leaf nodes, the following inequality must be satisfied:

(p leaf * (V SSN +P R)) + P < B, or
(p leaf * (9+7)) + 6 < 512, which gives 16p leaf < 506, so p leaf =31

(ii) the number of leaf-level blocks needed if blocks are approximately 69% full (rounded up for convenience);

Assuming that nodes are 69% full on the average, the average number of key values in a leaf node is 0.69*p leaf = 0.69*31 = 21.39. If we round this up for convenience, we get 22 key values (and 22 record pointers) per leaf node. Since the file has 30000 records and hence 30000 values of SSN, the number of leaf-level nodes (blocks) needed is b 1 = ceiling(30000/22) = 1364 blocks

(iii) the number of levels needed if internal nodes are also 69% full (rounded up for convenience);

We can calculate the number of levels as follows:
The average fan-out for the internal nodes (rounded up for convenience) is
fo = ceiling(0.69*p) = ceiling(0.69*34) = ceiling(23.46) = 24
number of second-level tree blocks b 2 = ceiling(b 1 /fo) = ceiling(1364/24)
= 57 blocks
number of third-level tree blocks b 3 = ceiling(b 2 /fo) = ceiling(57/24)= 3
number of fourth-level tree blocks b 4 = ceiling(b 3 /fo) = ceiling(3/24) = 1
Since the fourth level has only one block, the tree has x = 4 levels (counting the leaf level).

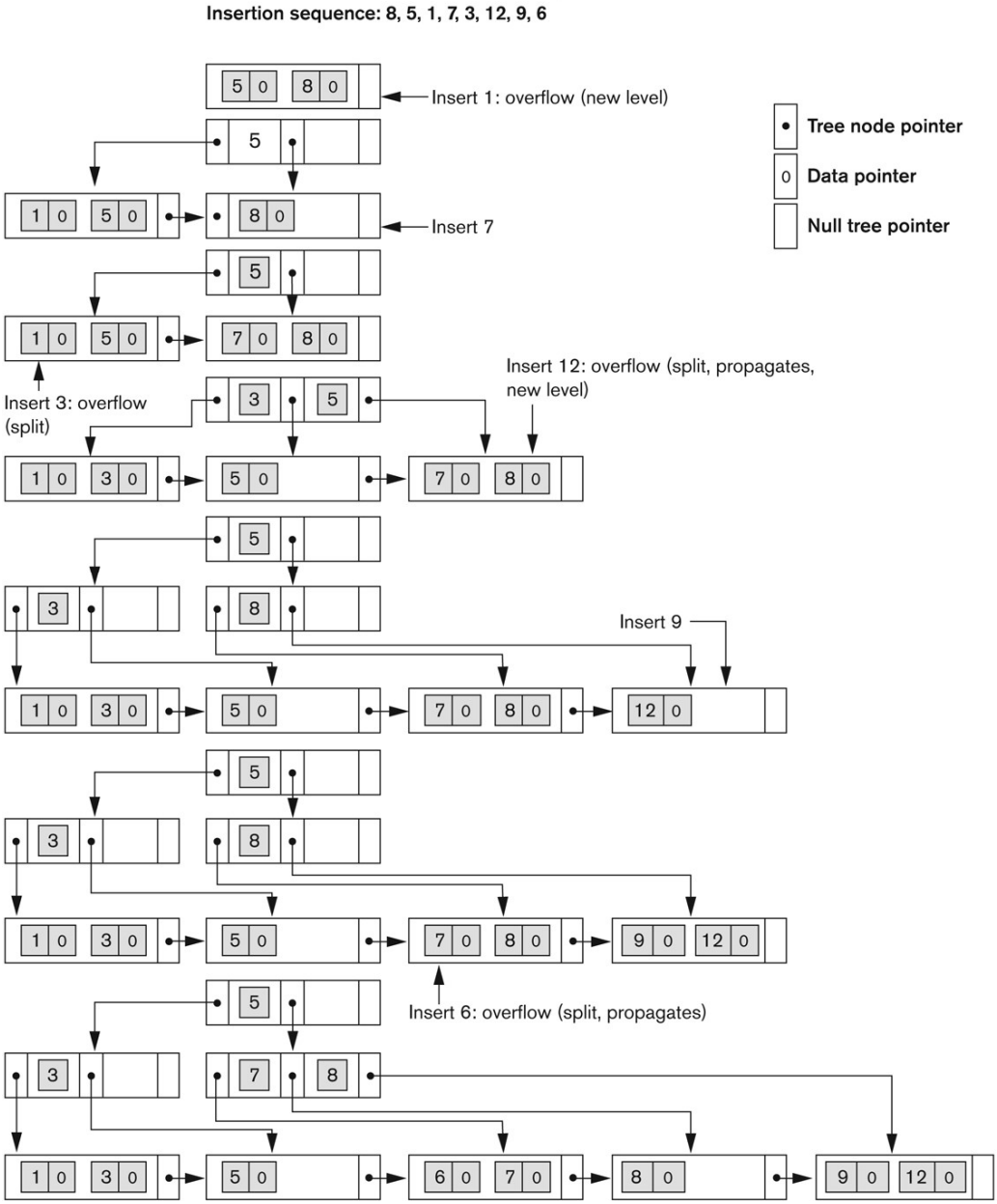
Note: We could use the formula:
$x = \text{ceiling}(\log_{fo} (b\ 1\ )) + 1 = \text{ceiling}(\log_{24}\ 1364) + 1 = 3 + 1 = 4$ levels

(iv) the total number of blocks required by the B + -tree; and (v) the number of block accesses needed to search for and retrieve a record from the file--given its SSN value--using the B + -tree.

total number of blocks for the tree b i = b 1 + b 2 + b 3 + b 4
= 1364 + 57 + 3 + 1 = 1425 blocks

v. number of block accesses to search for a record = x + 1 = 4 + 1 = 5

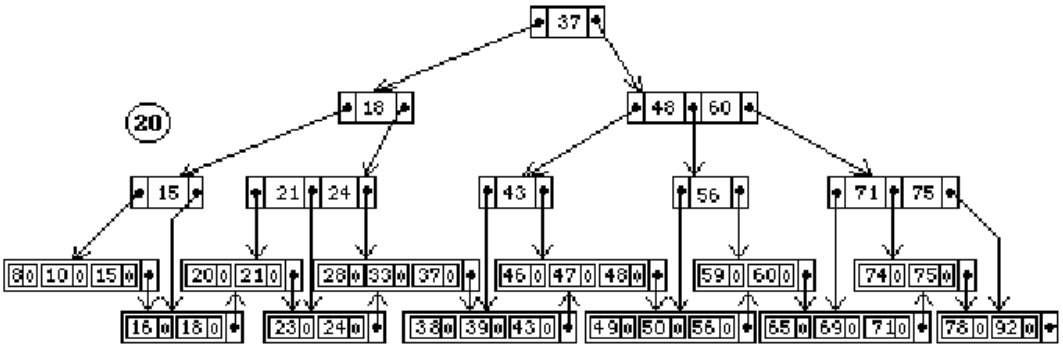
Figure 14.12
An example of insertion in a B+-tree with $p = 3$ and $p_{leaf} = 2$.



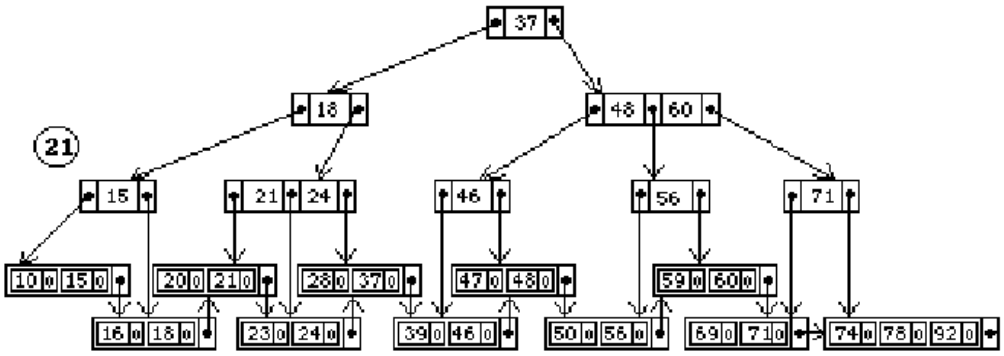
14.17 Suppose the following search field values are deleted in the given order from the B + -tree of Exercise 14.15, show how the tree will shrink and show the final tree. The deleted values are: 65, 75, 43, 18, 20, 92, 59, 37.

Answer:

An important note about a deletion algorithm for a B + -tree is that deletion of a key value from a leaf node will result in a reorganization of the tree if: (i) The leaf node is less than half full; in this case, we will combine it with the next leaf node (other algorithms combine it with either the next or the previous leaf nodes, or both), (ii) If the key value deleted is the rightmost (last) value in the leaf node, in which case its value will appear in an internal node; in this case, the key value to the left of the deleted key in the left node replaces the deleted key value in the internal node. Following is what happens to the tree number 19 after the specified deletions (not tree number 20):
Deleting 65 will only affect the leaf node. Deleting 75 will cause a leaf node to be less than half full, so it is combined with the next node; also, 75 is removed from the internal node leading to the following tree:

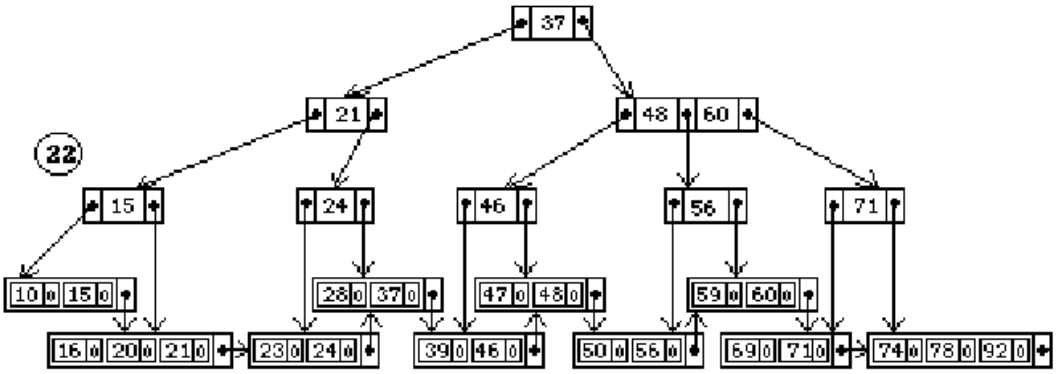


Deleting 43 causes a leaf node to be less than half full, and it is combined with the next node. Since the next node has 3 entries, its rightmost (first) entry 46 can replace 43 in both the leaf and internal nodes, leading to the following tree:

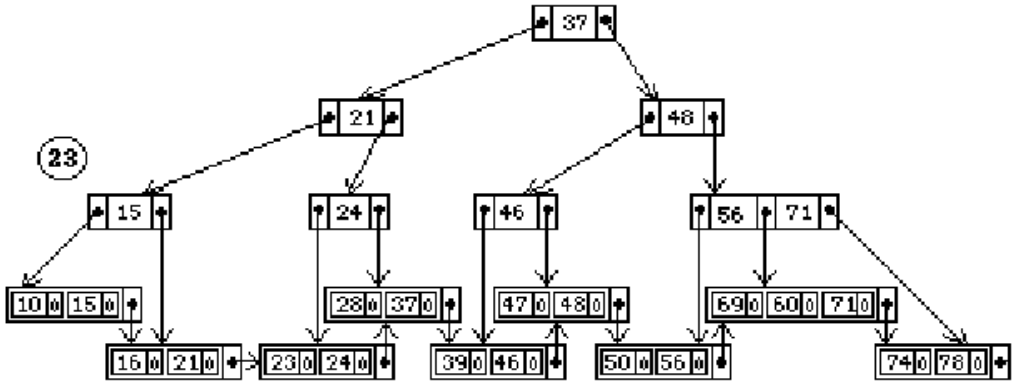


Next, we delete 18, which is a rightmost entry in a leaf node and hence appears in an internal node of the B + -tree. The leaf node is now less than half full, and is combined with the next node. The value 18 must also be removed from the internal node, causing underflow in the internal node. One approach for dealing with underflow in internal

nodes is to reorganize the values of the underflow node with its child nodes, so 21 is moved up into the underflow node leading to the following tree:



Deleting 20 and 92 will not cause underflow. Deleting 59 causes underflow, and the remaining value 60 is combined with the next leaf node. Hence, 60 is no longer a rightmost entry in a leaf node and must be removed from the internal node. This is normally done by moving 56 up to replace 60 in the internal node, but since this leads to underflow in the node that used to contain 56, the nodes can be reorganized as follows:



Finally, removing 37 causes serious underflow, leding to a reorganization of the whole tree. One approach to deleting the value in the root node is to use the rightmost value in the next leaf node (the first leaf node in the right subtree) to replace the root, and move this leaf node to the left subtree. In this case, the resulting tree may look as follows: