



Error Detection and Correction

1) Sixteen-bit messages are transmitted using a single-bit Hamming code. a) Show the bit pattern transmitted for the payload 1101001100110101. b) How many check bits are needed to ensure that the receiver can detect and correct single bit errors?. Assuming that even parity is used in the Hamming code. c) After transmission, the bit number 20 -starting with index 1- of the packet is modified, show the error correction steps.

$$m=16$$

$$m+r+1 \leq 2^r$$

$$r=5$$

$$n = 16 + 5 = 21$$

bit#	value	bit#	value	bit#	value
1(p0)		9	0	17	1
2(p1)		10	0	18	0
3	1	11	1	19	1
4(p2)		12	1	20	0
5	1	13	0	21	1
6	0	14	0		
7	1	15	1		

8(p3)

16(p4)

p0, p1, p2, p3 and p4 don't have values yet

$$p0 = p0 \oplus b3 \oplus b5 \oplus b7 \oplus b9 \oplus b11 \oplus b13 \oplus b15 \oplus b17 \oplus b19 \oplus b21 = 0$$

$$p1 = p1 \oplus b3 \oplus b6 \oplus b7 \oplus b10 \oplus b11 \oplus b14 \oplus b15 \oplus b18 \oplus b19 = 1$$

$$p2 = p2 \oplus b5 \oplus b6 \oplus b7 \oplus b12 \oplus b13 \oplus b14 \oplus b15 \oplus b20 \oplus b21 = 1$$

$$p3 = p3 \oplus b9 \oplus b10 \oplus b11 \oplus b12 \oplus b13 \oplus b14 \oplus b15 = 1$$

$$p4 = p4 \oplus b17 \oplus b18 \oplus b19 \oplus b20 \oplus b21 = 1$$

$$n = 01111011001100111011$$

$$n^{\wedge} = 01111011001100111001$$

this time p0, p1, p2, p3 and p4 have actual values

$$p0 = p0 \oplus b3 \oplus b5 \oplus b7 \oplus b9 \oplus b11 \oplus b13 \oplus b15 \oplus b17 \oplus b19 \oplus b21 = 0$$

$$p1 = p1 \oplus b3 \oplus b6 \oplus b7 \oplus b10 \oplus b11 \oplus b14 \oplus b15 \oplus b18 \oplus b19 = 0$$

$$p2 = p2 \oplus b5 \oplus b6 \oplus b7 \oplus b12 \oplus b13 \oplus b14 \oplus b15 \oplus b20 \oplus b21 = 1$$

$$p3 = p3 \oplus b9 \oplus b10 \oplus b11 \oplus b12 \oplus b13 \oplus b14 \oplus b15 = 0$$

$$p4 = p4 \oplus b17 \oplus b18 \oplus b19 \oplus b20 \oplus b21 = 1$$

$$\text{error-ed bit } [p4 \ p3 \ p2 \ p1 \ p0] = [10100] = 16 + 4 = 20$$

corrected bit # 20

$$n = 01111011001100111011$$

2) Consider a datalink protocol that uses character count technique for framing. The data link protocol employs hamming error correction algorithm. Each frame consists of a one byte header consisting of the field “character count”. Assume that there can be no more than one bit error in any single frame. Ahmed claims that because we have at most one bit error per frame and we use hamming error correction algorithm, then it is possible to resync after any error because if the error occurs in the header, the receiver can correct that error and hence can recover the original value of the “character count” field. Prove or disprove this claim.

disprove:

*as for the hamming code to work well , we have to know n (message length) .
if the character count is modified then hamming code cannot work well.*

3) Show the steps of computing the internet checksum for the stream of data: (0001f203f4f5f6f7)hex. Show how the receiver can check the correctness of the received stream. Arrange data in 16-bit words.

$w_0=0001$, $w_1=f203$, $w_2=f4f5$, $w_3=f6f7$

0001 +
f203

f204 +
f4f5

e6f9 +
0001

e6fa +
f6f7

ddf1 +
0001

ddf2

1's complement = 220d

$n= 0001\ f203\ f4f5\ f6f7\ 220d$

receiver repeat the same steps but after adding the checksum word, the result should be all ones.

4) What is the remainder obtained by dividing $x^7 + x^5 + 1$ by the generator polynomial $x^3 + 1$?

using modulo 2 operations: $X^4 + X^2 + X(X^2 + X + 1) = 10110 (111)$

5) A bit stream 10011101 is transmitted using the standard CRC method. The generator polynomial is $x^3 + 1$. Show the actual bit string transmitted. Suppose the third bit from the left is inverted during transmission. Show that this error is detected at the receiver's end.

$m = (x^7 + x^4 + x^3 + x^2 + 1)(x^3) = (x^{10} + x^7 + x^6 + x^5 + x^3) = 10011101000$

using long division

$x^{10} + x^7 + x^6 + x^5 + x^3 \ / \ x^3 + 1 = x^7 + x^3 + x^2$ and remainder(x^2)

$n = 10011101 \text{ } 100$

$n^* = 10111101100$

$x^{10} + x^8 + x^7 + x^6 + x^5 + x^3 + x^2 \ / \ x^3 + 1 = x^7 + x^5 + x^3$ and remainder(x^2)

if the remainder is not 0 , then an error detected.

6) Data link protocols almost always put the CRC in a trailer rather than in a header. Why?

to start the division process while receiving and save time.

as the CRC is received at the end and used at the end of the division.