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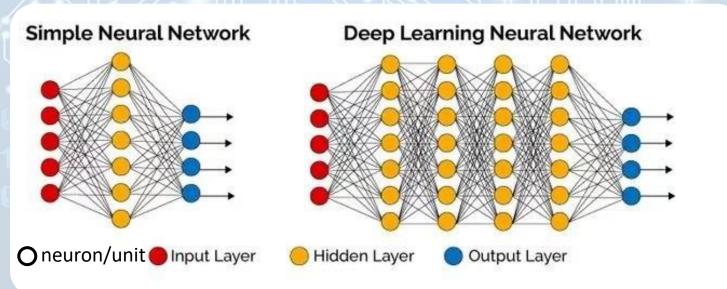
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Overview

- Neural networks are a fundamental computational tool for language processing.
- Their name and structure are inspired by the human brain, mimicking the way that biological neurons signal to one another.
- Feedforward Network: the computation proceeds iteratively from one layer of *units* to the next.
- Deep Learning: involves the use of modern neural nets that are often deep
 → have many layers.



Units/Neurons

- The building block of a neural network is a single computational unit.
 - A unit takes a set of real valued numbers as input, performs some computation on them, and produces an output.
- A neural unit takes a weighted sum of its inputs, with one additional term in the sum called a bias term.
 - Given a set of inputs $x_1, x_2, ... x_n$, a unit has a set of corresponding weights $w_1, w_2, ... w_n$ and a bias b, so the weighted sum z can be represented as: $z = b + \sum_{i} w_i x_i$ • Using vectors: $z = \mathbf{w} \cdot \mathbf{x} + b$ where \cdot is the dot product
- Finally, instead of using z (a linear function of x) as the output, neural units apply a non-linear function f to z.
 - The output of this function is known as the activation value for the unit, a.

$$a = f(z)$$

The final output of the network is referred to as y.

Example Activation Functions

• Sigmoid function:

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

- It maps the output into the range [0,1].
- It is useful in squashing outliers toward 0 or 1.
- It is differentiable.

tanh function:

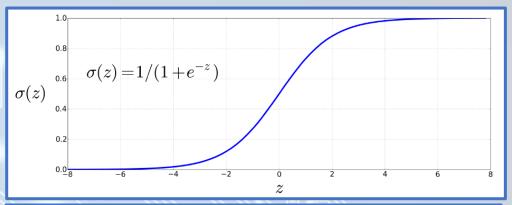
$$tanh(z) = \frac{e^z - e^{-z}}{e^z + e^{-z}}$$

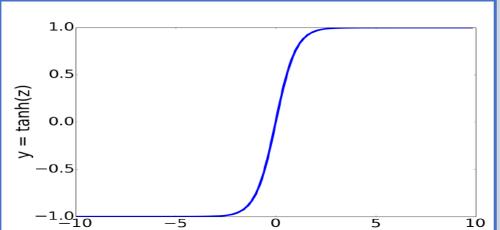
- It is a variant of the sigmoid that maps the output into the range [-1,1].
- It is smoothly differentiable.

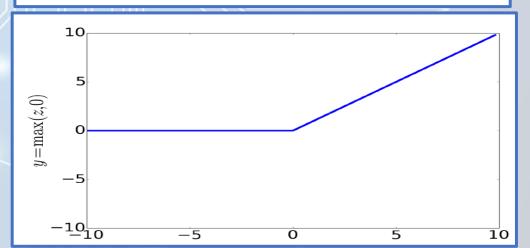
ReLU function:

$$ReLU(z) = max(z, 0)$$

- Rectified Linear Unit.
- It is just the same as z when z is positive, and 0 otherwise.
- It gives a result that is very close to linear.

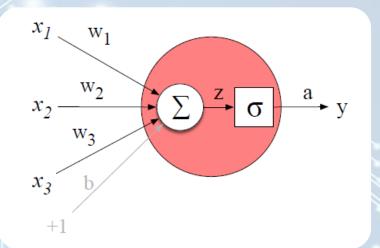






Basic Neural Unit Example

- The unit takes 3 input values x1, x2, and x3, and computes a weighted sum, multiplying each value by a weight (w1, w2, and w3, respectively), adds them to a bias term b, and then passes the resulting sum through a sigmoid function to result in a number between 0 and 1.
 - In this case the output of the unit *y* is the same as *a*, but in deeper networks we'll reserve *y* to mean the final output of the entire network, leaving *a* as the activation of an individual node.



 Using the following weight and bias values, what would this unit do with the following input vector x?

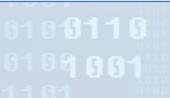
$$\mathbf{w} = [0.2, 0.3, 0.9]$$
 $b = 0.5$
 $\mathbf{x} = [0.5, 0.6, 0.1]$

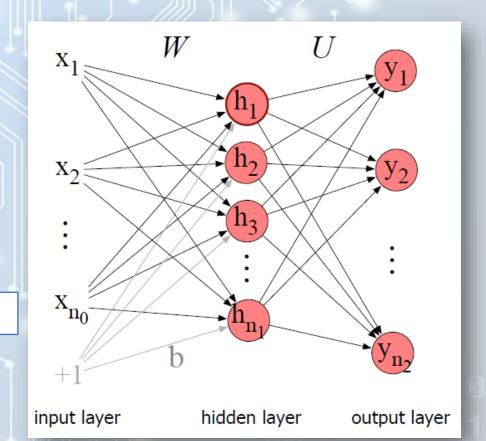
$$y = \sigma(\mathbf{w} \cdot \mathbf{x} + b) = \frac{1}{1 + e^{-(\mathbf{w} \cdot \mathbf{x} + b)}} = \frac{1}{1 + e^{-(.5*.2 + .6*.3 + .1*.9 + .5)}} = \frac{1}{1 + e^{-0.87}} = .70$$

Feedforward Neural Networks

- A feedforward network:
 - is a multilayer network
 - the units are connected with no cycles
 - the outputs from units in each layer are passed to units in the next higher layer
 - no outputs are passed back to lower layers
 - has three kinds of nodes:
 - input units, hidden units, and output units.
- A simple 2-layer feedforward network:
 - one hidden layer
 - one output layer
 - one input layer

the input layer is usually not counted when enumerating layers

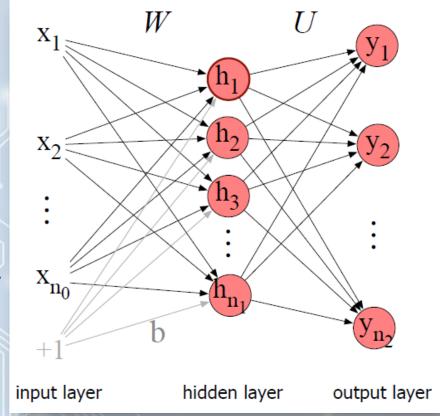




Feedforward Neural Networks

- In the standard architecture, each layer is fullyconnected:
 - each unit in each layer takes as input the outputs from all the units in the previous layer.
 - there is a link between every pair of units from two adjacent layers.
- A single hidden unit has as parameters a weight vector and a bias.
 - We represent the parameters for the entire hidden layer as a single weight matrix W: with dimensionality [n1,n0] and a single bias vector b: with dimensionality [n1,1]
 - Each element **Wji** of the weight matrix **W** represents the weight of the connection from the *ith* input unit *xi* to the *jth* hidden unit *hj*.

 Why??

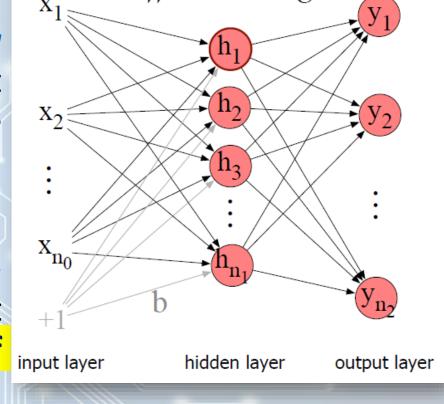


the hidden layer computation for a feedforward network can be done very efficiently with simple matrix operations.

 $\mathbf{h} = \sigma(\mathbf{W}\mathbf{x} + \mathbf{b})$

Feedforward Neural Networks

- Like the hidden layer, the output layer has a weight matrix U: with dimensionality [n2,n1] but some models don't include a bias vector b in the output.
- The intermediate output z is given by: z = Uh
- However, z can't be the output of a classifier, since it's a vector of real-valued numbers → while what we need for classification is a vector of probabilities → we need to perform normalization
 - A popular function used is softmax



$$\operatorname{softmax}(\mathbf{z}_i) = \frac{\exp(\mathbf{z}_i)}{\sum_{j=1}^d \exp(\mathbf{z}_j)} \ 1 \le i \le d$$

$$\mathbf{z} = [0.6, 1.1, -1.5, 1.2, 3.2, -1.1],$$

softmax(
$$\mathbf{z}$$
) = [0.055, 0.090, 0.0067, 0.10, 0.74, 0.010]

General Terminology

- Superscripts in square brackets to mean layer numbers, starting at 0 for the input layer.
 - W^[1]: weight matrix for the first hidden layer.
 - **b**^[1]: bias vector for the first hidden layer.
- n_i: number of units at layer j.
- g(.): activation function.
 - (ReLU or tanh for intermediate layers and sigmoid or softmax for output layers).
- a[i]: output from layer i.
 - a^[0] refers to the inputs x.
- $z^{[i]}$: combination of weights and biases $W^{[i]}a^{[i-1]}+b^{[i]}$.

• For a 2-layer net:

$$\mathbf{z}^{[1]} = \mathbf{W}^{[1]} \mathbf{a}^{[0]} + \mathbf{b}^{[1]}$$
 $\mathbf{a}^{[1]} = g^{[1]} (\mathbf{z}^{[1]})$
 $\mathbf{z}^{[2]} = \mathbf{W}^{[2]} \mathbf{a}^{[1]} + \mathbf{b}^{[2]}$
 $\mathbf{a}^{[2]} = g^{[2]} (\mathbf{z}^{[2]})$
 $\hat{\mathbf{y}} = \mathbf{a}^{[2]}$

 The algorithm for computing the forward step in an n-layer feedforward network:

for
$$i$$
 in 1,...,n
 $\mathbf{z}^{[i]} = \mathbf{W}^{[i]} \mathbf{a}^{[i-1]} + \mathbf{b}^{[i]}$
 $\mathbf{a}^{[i]} = g^{[i]}(\mathbf{z}^{[i]})$
 $\hat{\mathbf{y}} = \mathbf{a}^{[n]}$

Why non-linear activation functions?

- If we did not use them, the resulting network is exactly equivalent to a single-layer network.
- How??
 - Consider the first two layers of such a network of purely linear layers:

$$\mathbf{z}^{[1]} = \mathbf{W}^{[1]}\mathbf{x} + \mathbf{b}^{[1]}$$

 $\mathbf{z}^{[2]} = \mathbf{W}^{[2]}\mathbf{z}^{[1]} + \mathbf{b}^{[2]}$

• Rewriting:

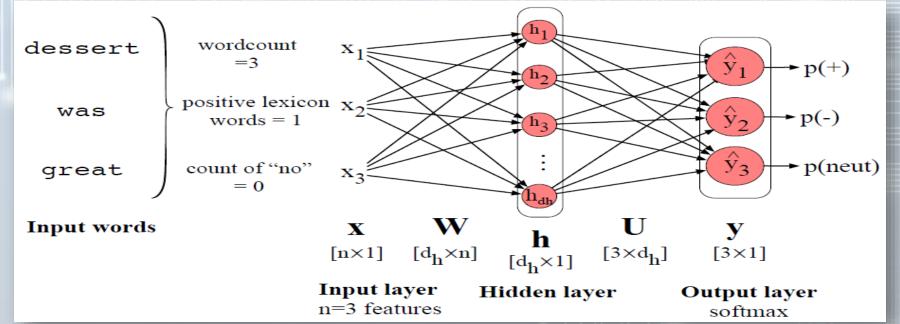
$$\mathbf{z}^{[2]} = \mathbf{W}^{[2]}\mathbf{z}^{[1]} + \mathbf{b}^{[2]}
= \mathbf{W}^{[2]}(\mathbf{W}^{[1]}\mathbf{x} + \mathbf{b}^{[1]}) + \mathbf{b}^{[2]}
= \mathbf{W}^{[2]}\mathbf{W}^{[1]}\mathbf{x} + \mathbf{W}^{[2]}\mathbf{b}^{[1]} + \mathbf{b}^{[2]}
= \mathbf{W}'\mathbf{x} + \mathbf{b}'$$

This generalizes to any number of layers

without non-linear activation functions, a multilayer network is just a single layer network with a different set of weights, and we lose all the representational power of multilayer networks.

NLP Example

- A 2-layer sentiment classifier.
- The inputs are scalar features
 - e.g.: x_1 =count(words in doc), x_2 =count(positive lexicon words in doc), x_3 = 1 if "no" in doc, and so on.
- The output layer \hat{y} has 3 nodes (positive, negative, neutral):
 - $\widehat{y_1}$ the estimated probability of positive sentiment
 - $\widehat{y_2}$ the estimated probability of negative sentiment
 - $\widehat{y_3}$ the estimated probability of neutral sentiment.



Training Neural Nets

- A feedforward neural net is an instance of supervised machine learning in which we
 know the correct output y for each observation x.
- The goal of the training procedure is to learn parameters $\mathbf{W}^{[i]}$ and $\mathbf{b}^{[i]}$ for each layer i that make \hat{y} (network output) for each training observation as close as possible to the true y.
- Components:
- 1. Loss function: to model the distance between the network output and the gold output.
- 2. Gradient descent optimization: to find the parameters that minimize this loss function.

$$w_{ji(new)} = w_{ji(old)} - \eta \frac{\partial L}{\partial w_{ji}}$$
 where η is the learning rate

- 3. Error backpropagation: gradient descent requires knowing the gradient of the loss function, the vector that contains the partial derivative of the loss function with respect to each of the parameters.
 - For neural networks, with millions of parameters in many layers, it's much harder to see how to compute the partial derivative of some weight in layer 1 when the loss is attached to some much later layer > The answer is the algorithm called error backpropagation or backward differentiation.
 - The gradients are calculated starting from the final layer and then through use of the *chain rule*, the gradients can be passed backwards to calculate the gradients in the previous layers.
 - Chain rule: given y=g(u), and u=f(x)

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

