Cognitive Robotics

09. FastSLAM

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Recap: Drawbacks of EKF SLAM

- Can only deal with a single mode
- Successful only in medium-scale scenes
- Computationally intractable for large maps

- **Idea**: Apply the particle filter principle to solve both problems simultaneously, localization and mapping

Particle Representation

- A set of weighted samples

$$\mathcal{X} = \left\{ \left\langle x^{[i]}, w^{[i]} \right\rangle \right\}_{i=1,\ldots,N}$$

- Each sample is a hypothesis about the state
- For feature-based SLAM:

$$x = (\underbrace{x_{1:t}}_{\textbf{poses}}, \underbrace{m_{1,x}, m_{1,y}, \ldots, m_{M,x}, m_{M,y}}_{\textbf{landmarks}})^T$$

Dimensionality Problem

- PF are effective in low-dimensional spaces
- The number of particles needed to represent a posterior grows exponentially with the dimension of the state space!

$$x = (x_{1:t}, m_{1,x}, m_{1,y}, \ldots, m_{M,x}, m_{M,y})^T$$

high-dimensional!

Dependencies

- Are there dependencies between the dimensions of the state space?
- Yes: The map depends on the robot poses from which measurements were obtained
- Use this dependency to solve the state estimation problem more efficiently

Exploit Dependencies Between the Different Dimensions of the State Space

$$x_{1:t}, m_1, \ \ldots, m_M$$

If We Know the Robot Poses, Mapping is Easy!

$$\underline{x_{1:t}},\ \underline{m_1,\ \ldots,\ m_M}$$

Key Idea

$$\underline{x_{1:t}}, \underline{m_1, \dots, m_M}$$

- Use the particle set only to model the robot's path
- For each sample, compute an individual map of landmarks

Rao-Blackwellization

- Use factorization to exploit dependencies between variables:

$$p(a, b) \;=\; p(b \mid a)\, p(a)$$

- If $p(b \mid a)$ can be computed efficiently, represent only $p(a)$ with samples and compute $p(b \mid a)$ for every sample

Rao-Blackwellization for SLAM

Factorization of the SLAM posterior

$$\underbrace{p(x_{0:t}}_{\text{poses}}, \underbrace{m_{1:M}}_{\text{map}} \mid \underbrace{z_{1:t}, u_{1:t}}_{\text{observations \& movements}}) =$$

[Murphy, 1999]

Rao-Blackwellization for SLAM

Factorization of the SLAM posterior

poses map observations & movements

$$p(x_{0:t}, m_{1:M} \mid z_{1:t}, u_{1:t}) =$$
$$p(x_{0:t} \mid z_{1:t}, u_{1:t}) \, p(m_{1:M} \mid x_{0:t}, z_{1:t})$$

path posterior map posterior

[Murphy, 1999]

Rao-Blackwellization for SLAM

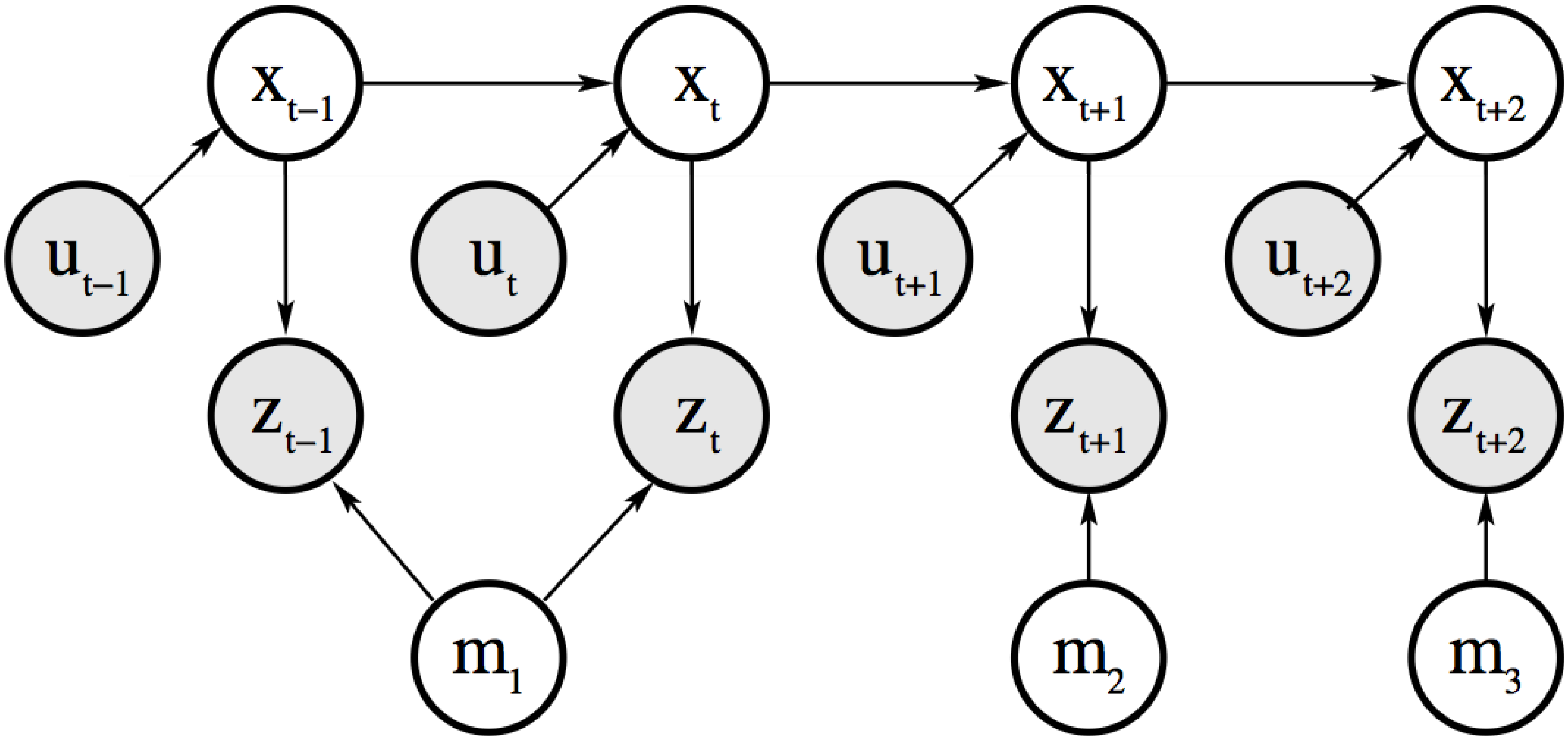
Factorization of the SLAM posterior

$$p(x_{0:t}, m_{1:M} \mid z_{1:t}, u_{1:t}) =$$
$$p(x_{0:t} \mid z_{1:t}, u_{1:t}) \, \underline{p(m_{1:M} \mid x_{0:t}, z_{1:t})}$$

How to compute this term efficiently?

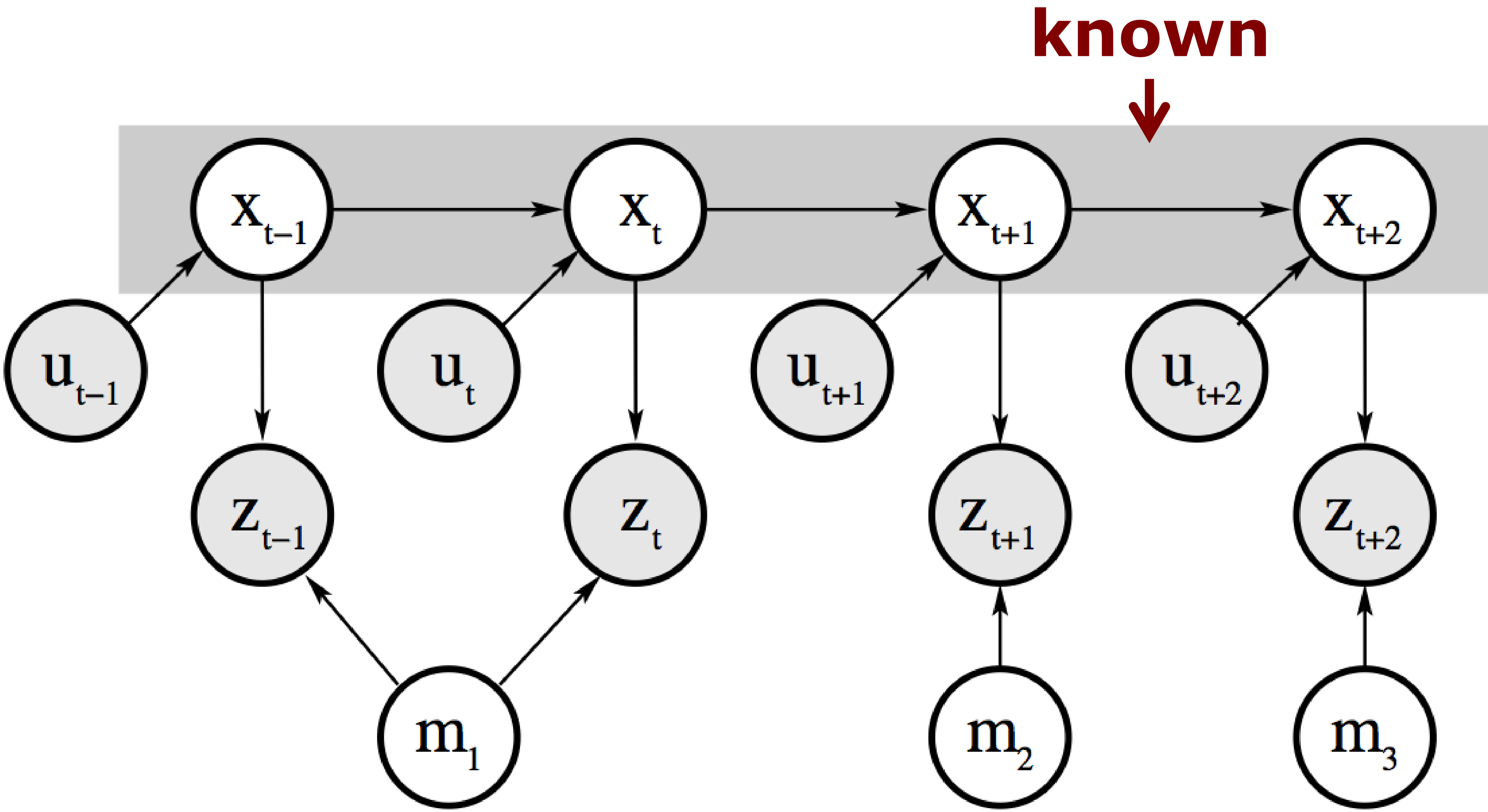
[Murphy, 1999]

Revisit the Graphical Model



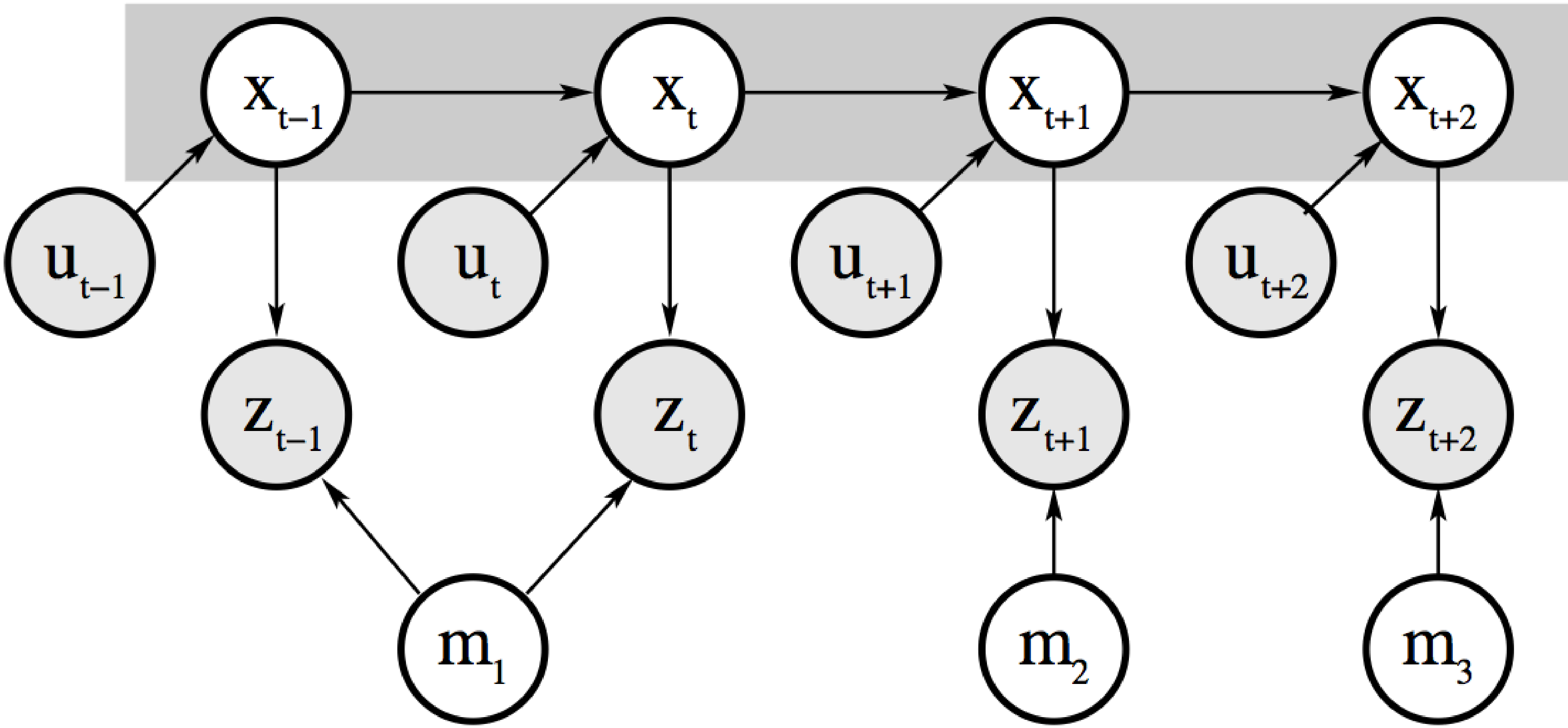
Courtesy: Thrun, Burgard, Fox

Revisit the Graphical Model



Courtesy: Thrun, Burgard, Fox

Landmarks are Conditionally Independent Given the Poses



Landmark variables are all disconnected (i.e. independent) given the robot's path

Rao-Blackwellization for SLAM

Factorization of the SLAM posterior

$$p(x_{0:t}, m_{1:M} \mid z_{1:t}, u_{1:t}) =$$
$$p(x_{0:t} \mid z_{1:t}, u_{1:t}) \, \underline{p(m_{1:M} \mid x_{0:t}, z_{1:t})}$$

Landmarks are conditionally independent given the poses

[Montemerlo et al., 2002]

Rao-Blackwellization for SLAM

Factorization of the SLAM posterior

$$p(x_{0:t}, m_{1:M} \mid z_{1:t}, u_{1:t}) =$$

$$p(x_{0:t} \mid z_{1:t}, u_{1:t})\, p(m_{1:M} \mid x_{0:t}, z_{1:t})$$

$$p(x_{0:t} \mid z_{1:t}, u_{1:t}) \prod_{i=1}^{M} p(m_i \mid x_{0:t}, z_{1:t})$$

[Montemerlo et al., 2002]

Rao-Blackwellization for SLAM

Factorization of the SLAM posterior

$$p(x_{0:t}, m_{1:M} \mid z_{1:t}, u_{1:t}) =$$

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$$p(x_{0:t} \mid z_{1:t}, u_{1:t}) \prod_{i=1}^{M} \underline{p(m_i \mid x_{0:t}, z_{1:t})}$$

2-dimensional EKFs!

[Montemerlo et al., 2002]

Rao-Blackwellization for SLAM

Factorization of the SLAM posterior

$$p(x_{0:t}, m_{1:M} \mid z_{1:t}, u_{1:t}) =$$

$$p(x_{0:t} \mid z_{1:t}, u_{1:t})\; p(m_{1:M} \mid x_{0:t}, z_{1:t})$$

$$\underline{p(x_{0:t} \mid z_{1:t}, u_{1:t})} \prod_{i=1}^{M} \underline{p(m_i \mid x_{0:t}, z_{1:t})}$$

particle filter similar to MCL

2-dimensional EKFs!

[Montemerlo et al., 2002]

Modeling the Robot's Path

- Sample-based representation for
  $p(x_{0:t} \mid z_{1:t}, u_{1:t})$
- Each sample represents a path hypothesis

$$x_0 \qquad x_1 \qquad x_2 \qquad \ldots$$

starting location

pose hypothesis
at time t=1

- But: Past poses of a sample are not revised
- Thus, no need to maintain past poses in the sample set

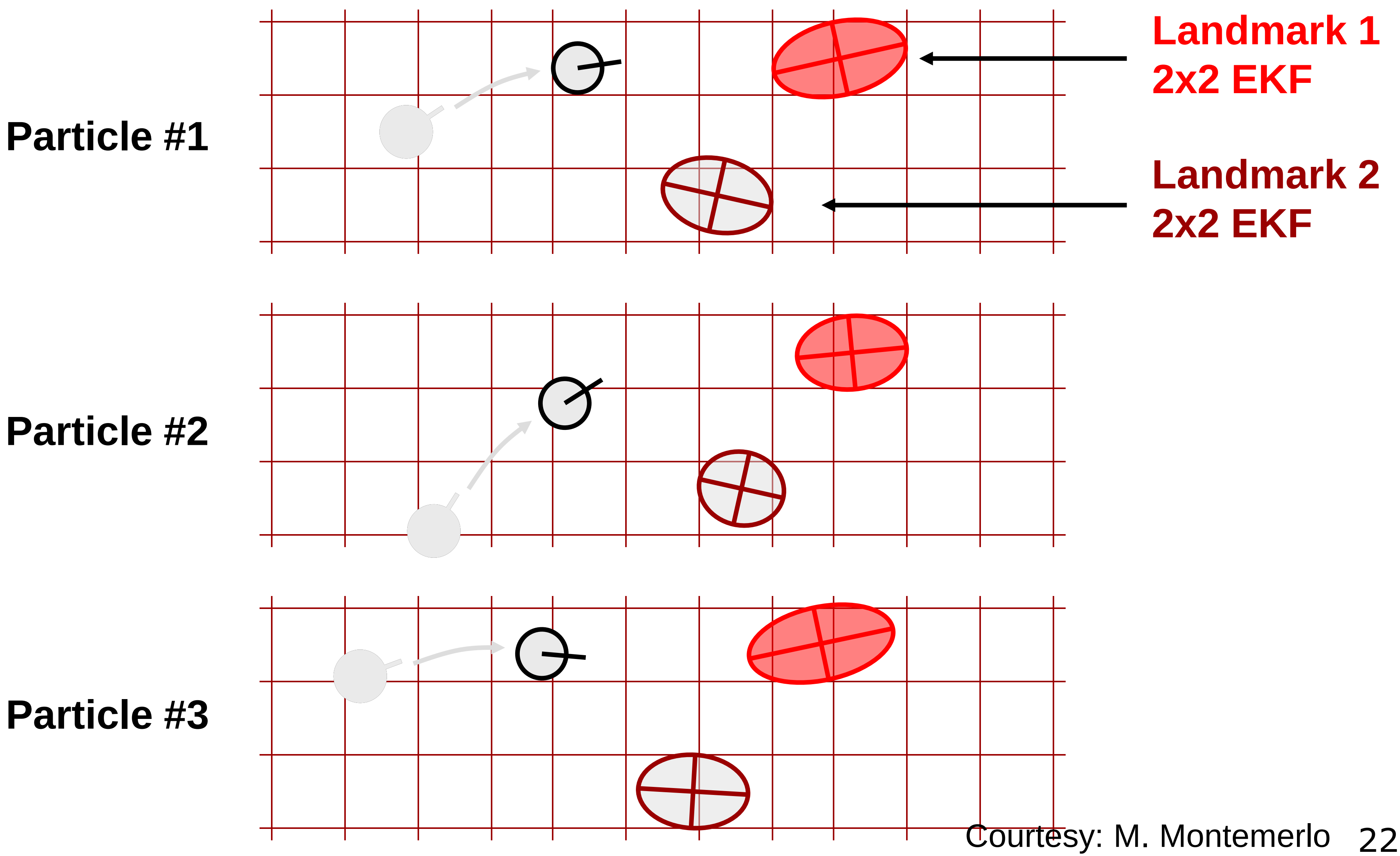
FastSLAM

- Each landmark is represented by a 2x2 EKF
- Thus, each particle has to maintain M individual EKFs

Particle 1	x, y, θ	**Landmark 1**	**Landmark 2**	**...**	**Landmark M**
Particle 2	x, y, θ	**Landmark 1**	**Landmark 2**	**...**	**Landmark M**
⋮					
Particle N	x, y, θ	**Landmark 1**	**Landmark 2**	**...**	**Landmark M**

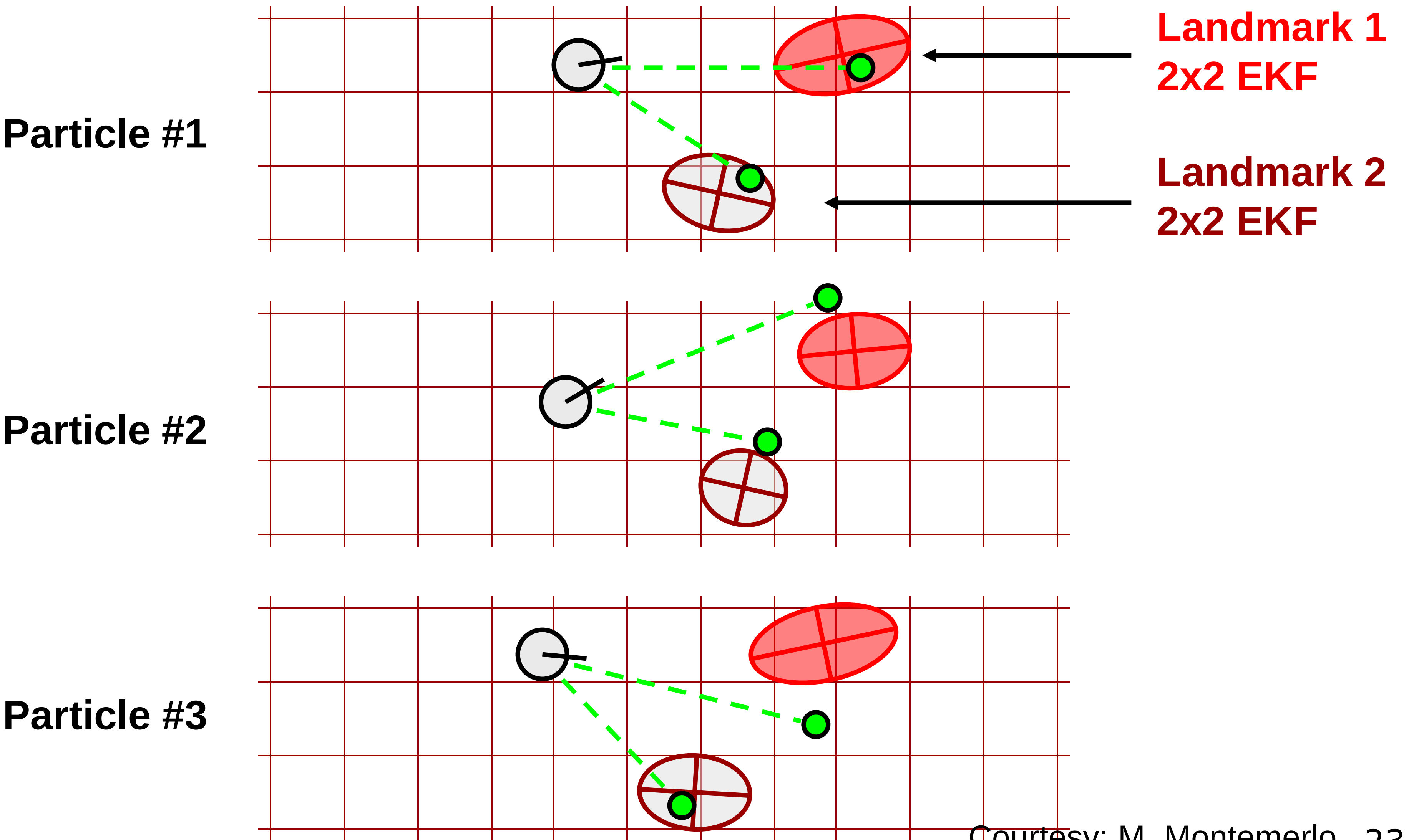
[Montemerlo et al., 2002]

FastSLAM – Motion Update



Courtesy: M. Montemerlo

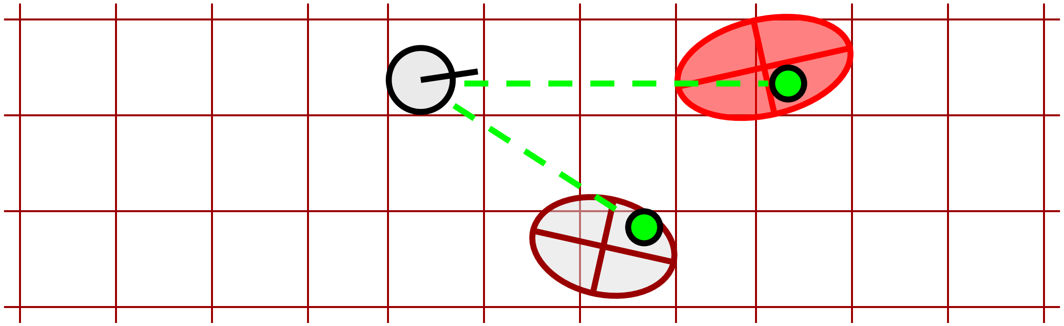
FastSLAM – Sensor Update



Courtesy: M. Montemerlo

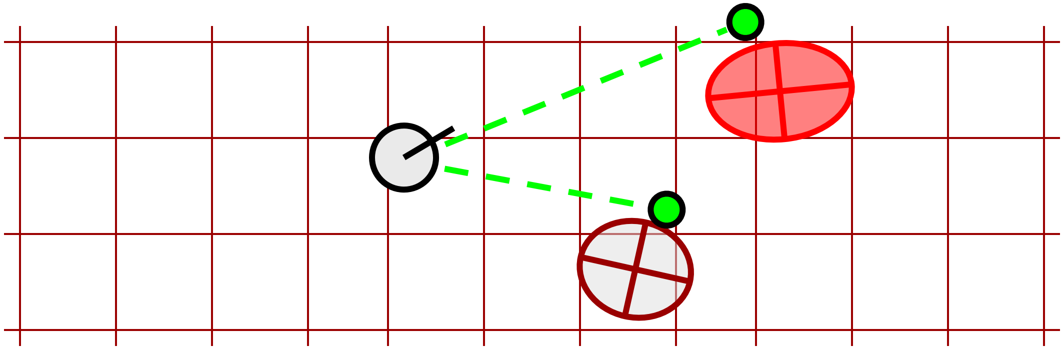
FastSLAM – Sensor Update

Particle #1



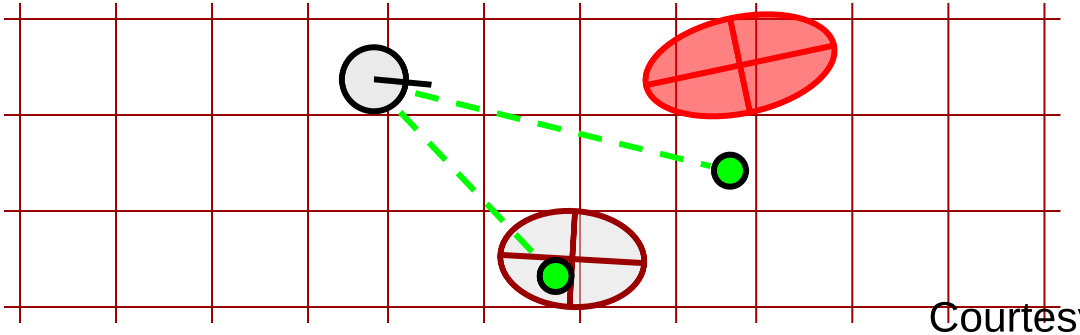
Weight = 0.8

Particle #2



Weight = 0.4

Particle #3



Weight = 0.1

Courtesy: M. Montemerlo

FastSLAM – Sensor Update

Particle #1

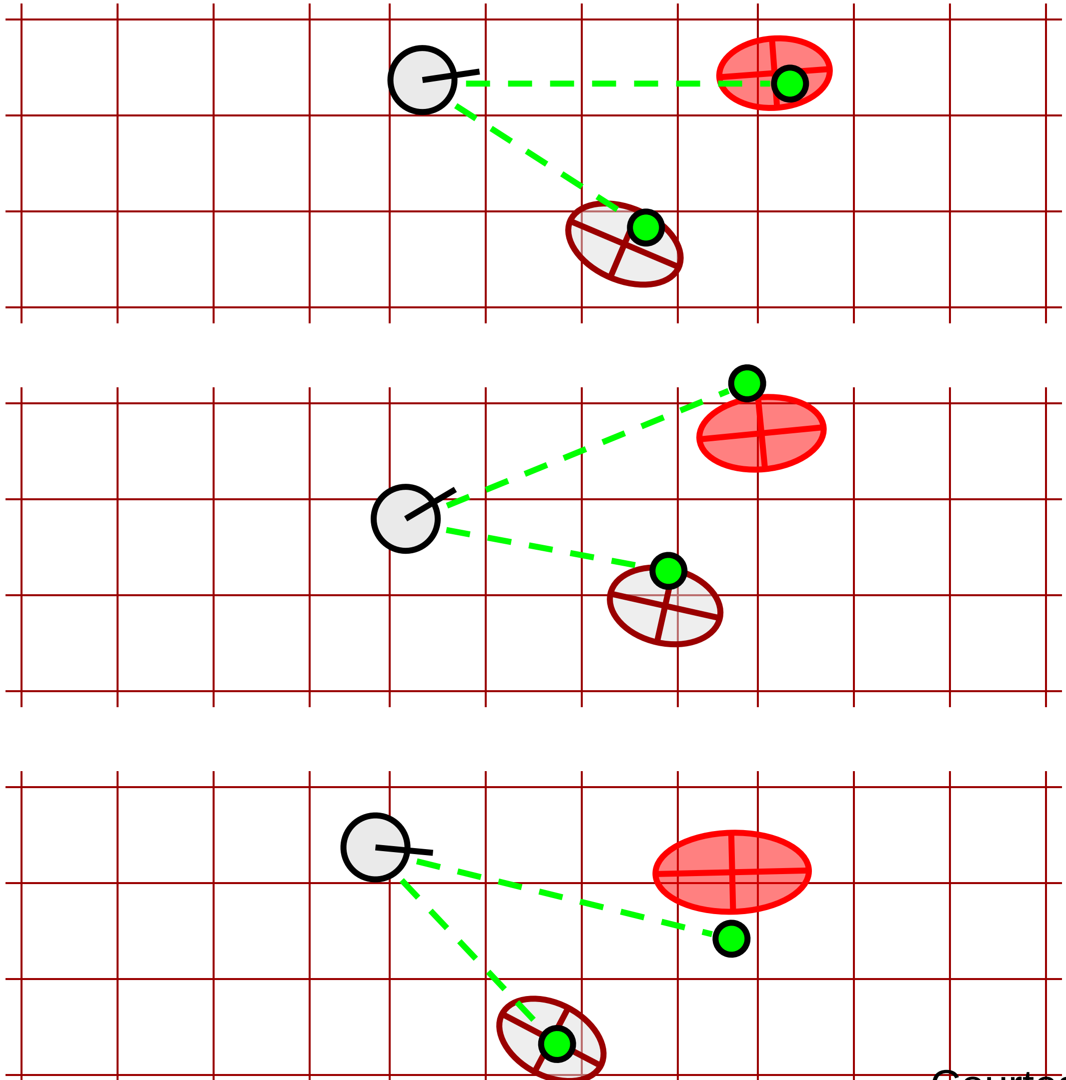
Update map of particle 1

Particle #2

Update map of particle 2

Particle #3

Update map of particle 3



Courtesy: M. Montemerlo

Key Steps of FastSLAM 1.0

- Sample a new pose for each particle

$$x_t^{[k]} \sim p(x_t \mid x_{t-1}^{[k]}, u_t)$$

- Compute the particle weights

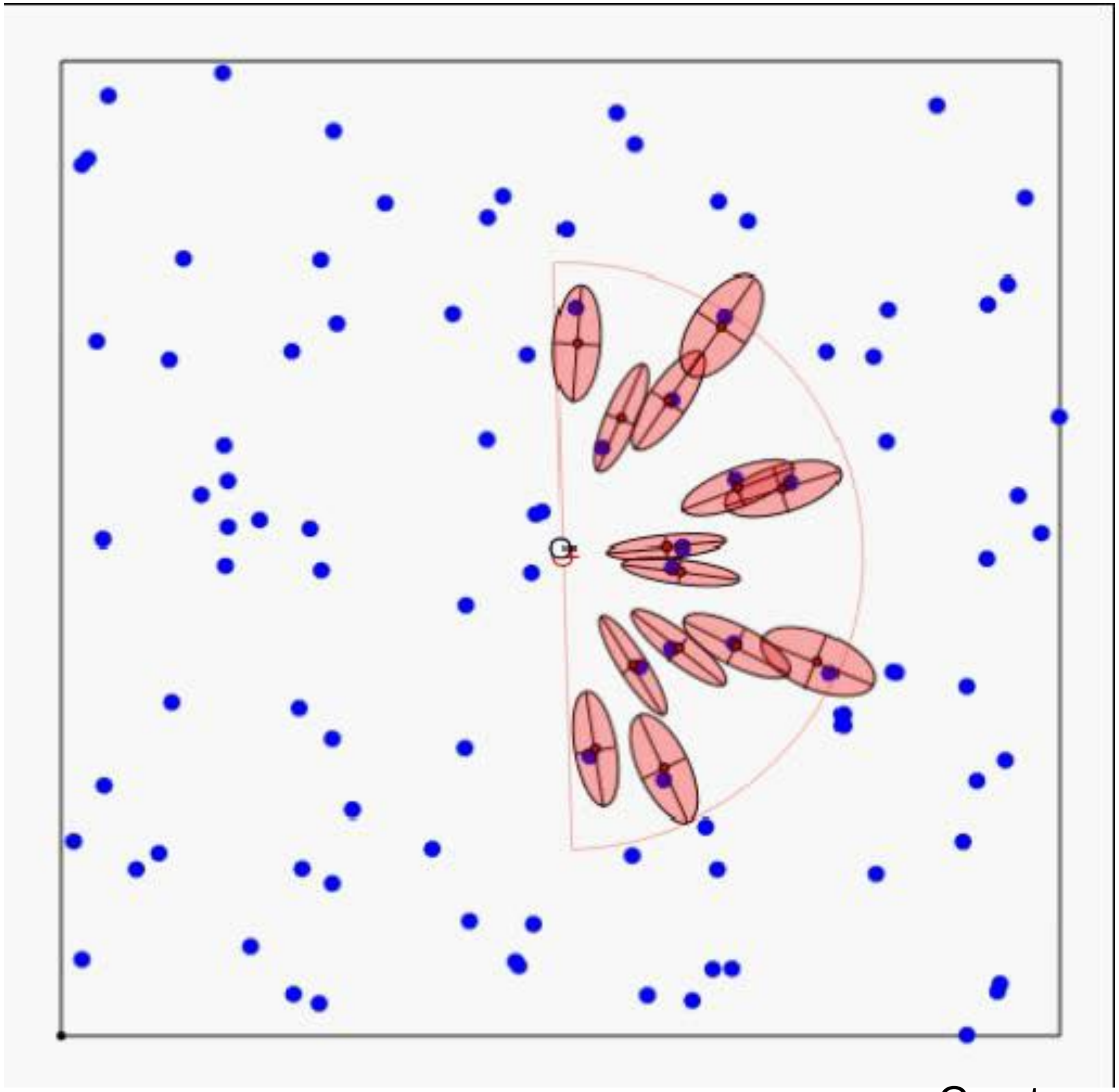
exp. observation

$$w^{[k]} = |2\pi Q|^{-\frac{1}{2}} \ \exp\left\{-\frac{1}{2}(z_t - \hat{z}^{[k]})^T Q^{-1} \ (z_t - \hat{z}^{[k]})\right\}$$

measurement covariance

- Update belief of observed landmarks (EKF update rule)
- Resample

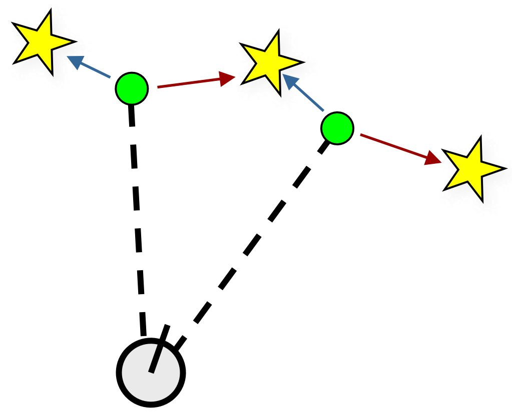
FastSLAM in Action



Courtesy: M. Montemerlo

Data Association Problem

- Which observation belongs to which landmark?

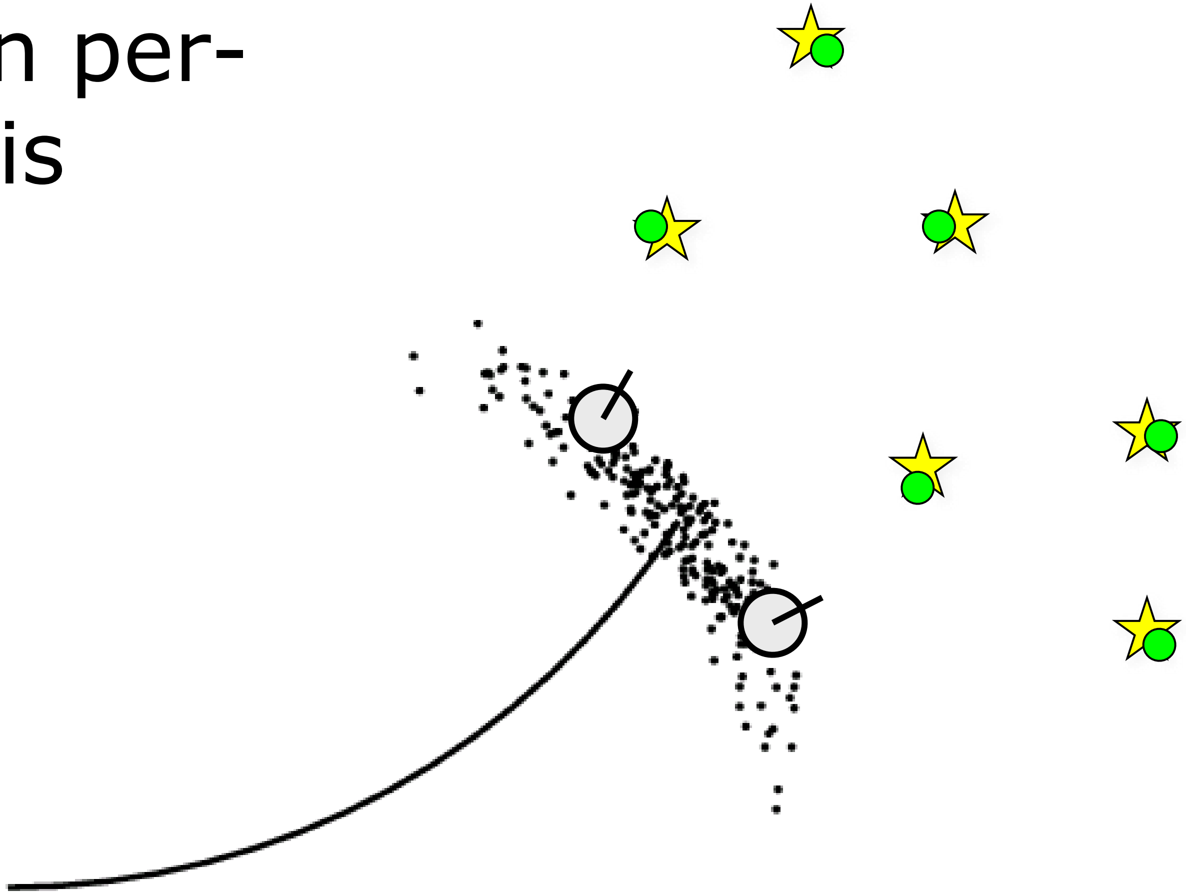


- More than one possible association
- **Potential data associations depend on the pose of the robot**

Courtesy: M. Montemerlo

Particles Support Multi-Hypotheses Data Association

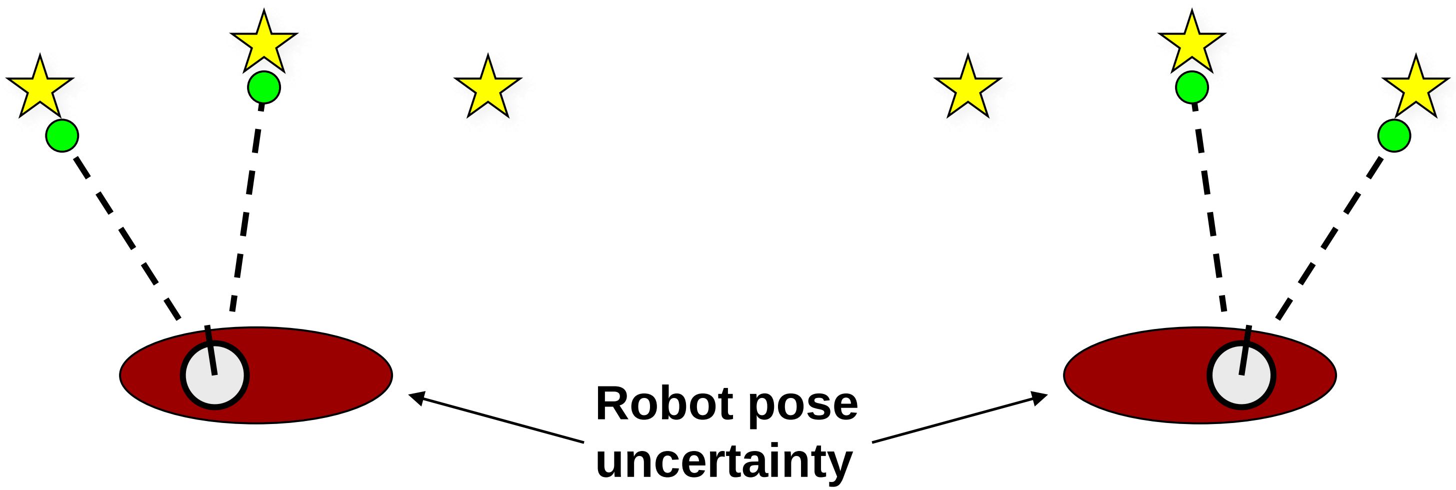
Decisions on per-particle basis



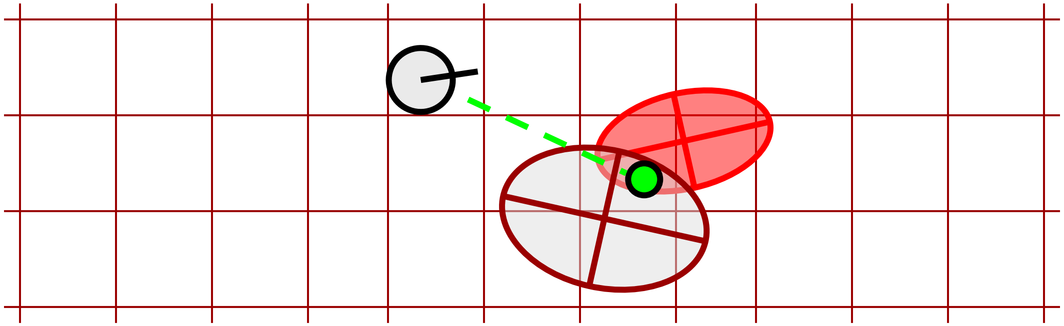
Courtesy: M. Montemerlo

Recap: Data Association in EKF SLAM

The best data association is not obvious even within the error ellipse of the pose estimate



Per-Particle Data Association

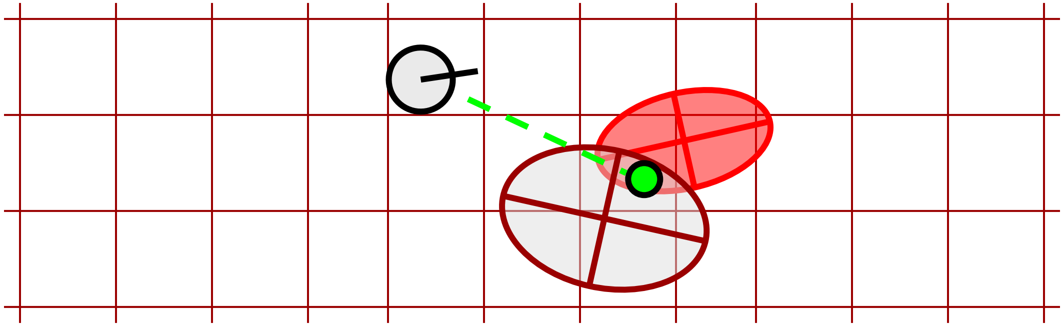


Is the observation generated by the **red** or by the **brown** landmark?

P(observation|red) = 0.3 P(observation|brown) = 0.7

Courtesy: M. Montemerlo

Per-Particle Data Association



Is the observation generated by the **red** or by the **brown** landmark?

P(observation|red) = 0.3 P(observation|brown) = 0.7

- Options:
  - Pick the most probable match
  - Pick a random association weighted by the observation likelihoods
- If the probability for an assignment is too low, generate a new landmark

Per-Particle Data Association

- Multi-modal belief supports multi-hypotheses data association
- Simple but effective data association applicable
- **Big advantage of FastSLAM over EKF**

Results – Victoria Park

- 4 km traverse
- < 2.5 m RMS position error
- 100 particles

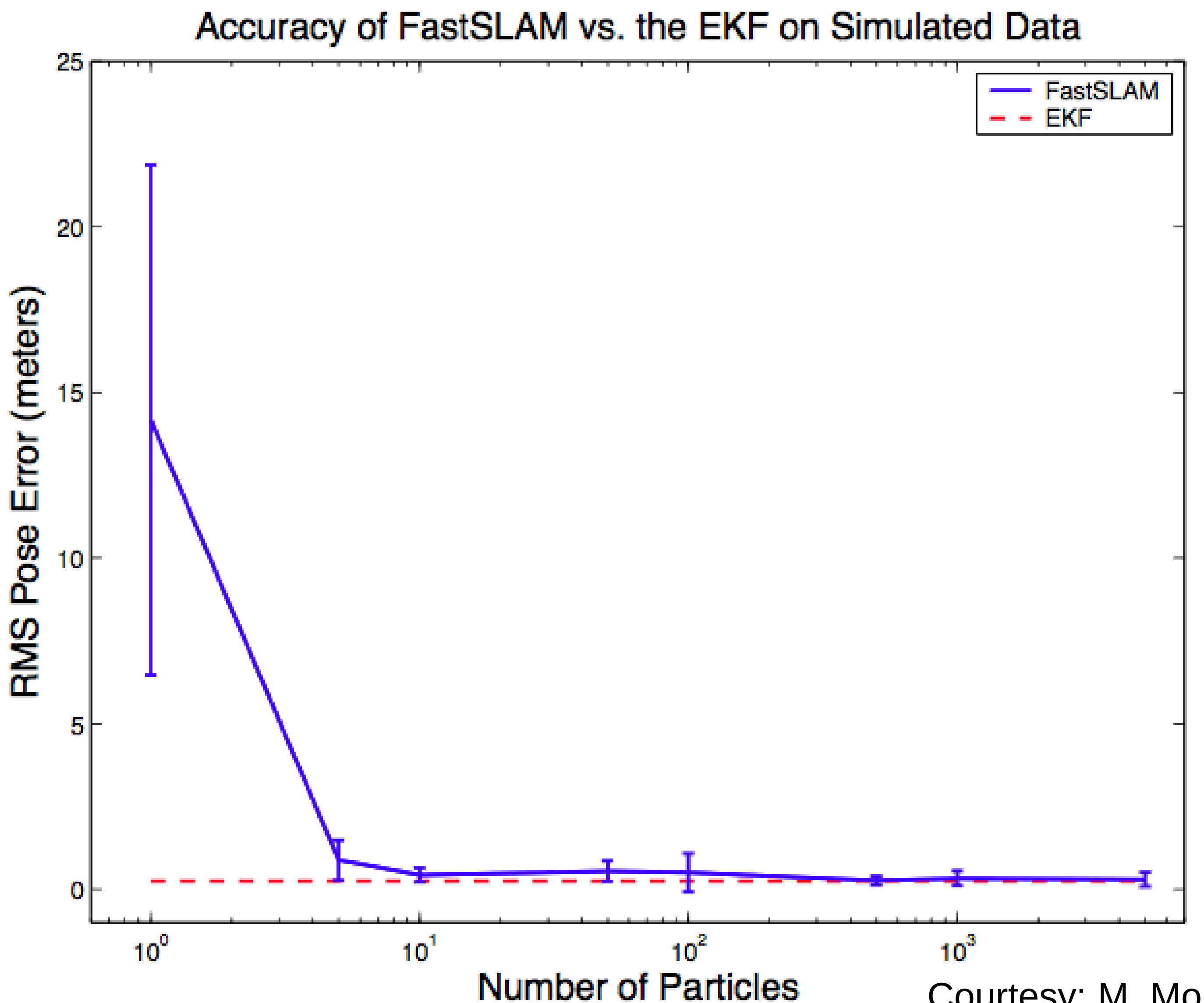
Blue = GPS
Yellow = FastSLAM

Courtesy: M. Montemerlo

Results – Victoria Park (Video)

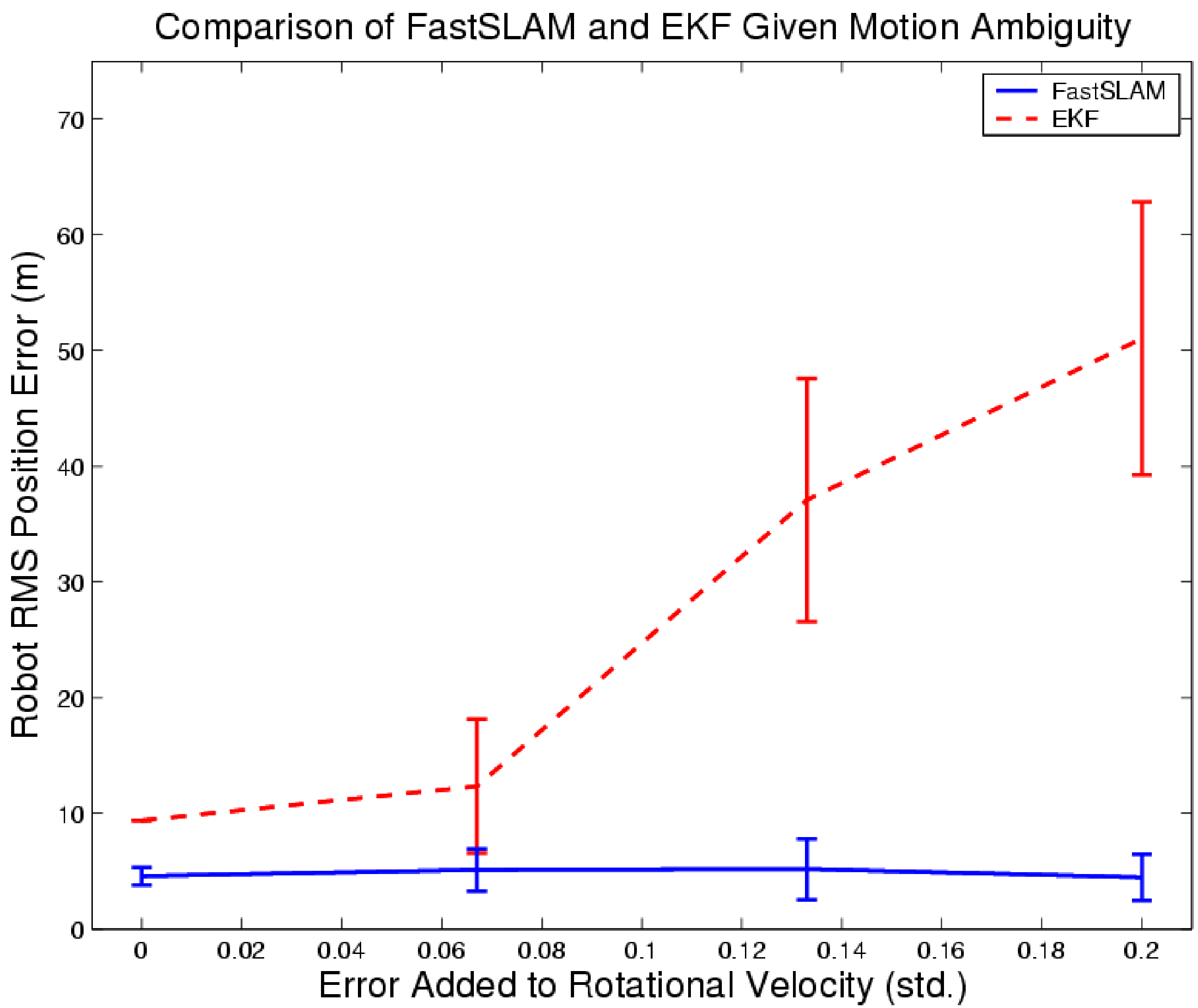
Courtesy: M. Montemerlo

Results (Sample Size)



Courtesy: M. Montemerlo

Results (w. Motion Uncertainty)



Courtesy: M. Montemerlo

Feature-Based FastSLAM 1.0

- Factors the SLAM posterior into low-dimensional estimation problems
- Models the robot's path by samples
- Computes the landmarks given particle pose
- Per-particle data association
- No robot pose uncertainty in the data association

FastSLAM Complexity – Simple Implementation

- Update robot pose particles based on the control $\mathcal{O}(N)$
- Incorporate an observation into the Kalman filters $\mathcal{O}(N)$
- Resample particle set $\mathcal{O}(NM)$

$$\frac{}{\mathcal{O}(NM)}$$

N = Number of particles
M = Number of map features

Summary: FastSLAM so far

- Feature-Based SLAM with particle filter
- Rao-Blackwellization: model the robot's path by sampling and compute the landmarks given the robot poses
- Data association on per-particle basis
- Robust to ambiguities in the data association
- Scales well (1 million+ features)
- Advantages compared to EFK SLAM

Now: FastSLAM for Grid Maps

- So far, we addressed landmark-based SLAM (KF-based SLAM, FastSLAM)
- We learned how to build grid maps assuming "known poses"

Grid-Based Mapping With Raw Odometry

Courtesy: Dirk Hähnel

Grid-Based Mapping

- Assuming known poses fails

Questions

- Can we solve the SLAM problem if no pre-defined landmarks are available?
- Can we use the ideas of FastSLAM to build grid maps?

Recap: Rao-Blackwellization for SLAM

Factorization of the SLAM posterior

poses map observations movements

$$p(x_{0:t}, m \mid z_{1:t}, u_{1:t})$$

[Murphy, 1999]

Recap: Rao-Blackwellization for SLAM

Factorization of the SLAM posterior

poses map observations movements

$$\underbrace{p(x_{0:t}, m \mid z_{1:t}, u_{1:t})}$$

$$= \underbrace{p(x_{0:t} \mid z_{1:t}, u_{1:t})}_{\text{path posterior (particle filter)}} \; \underbrace{p(m \mid x_{1:t}, z_{1:t})}_{\text{map posterior (given the path)}}$$

path posterior
(particle filter)

map posterior
(given the path)

[Murphy, 1999]

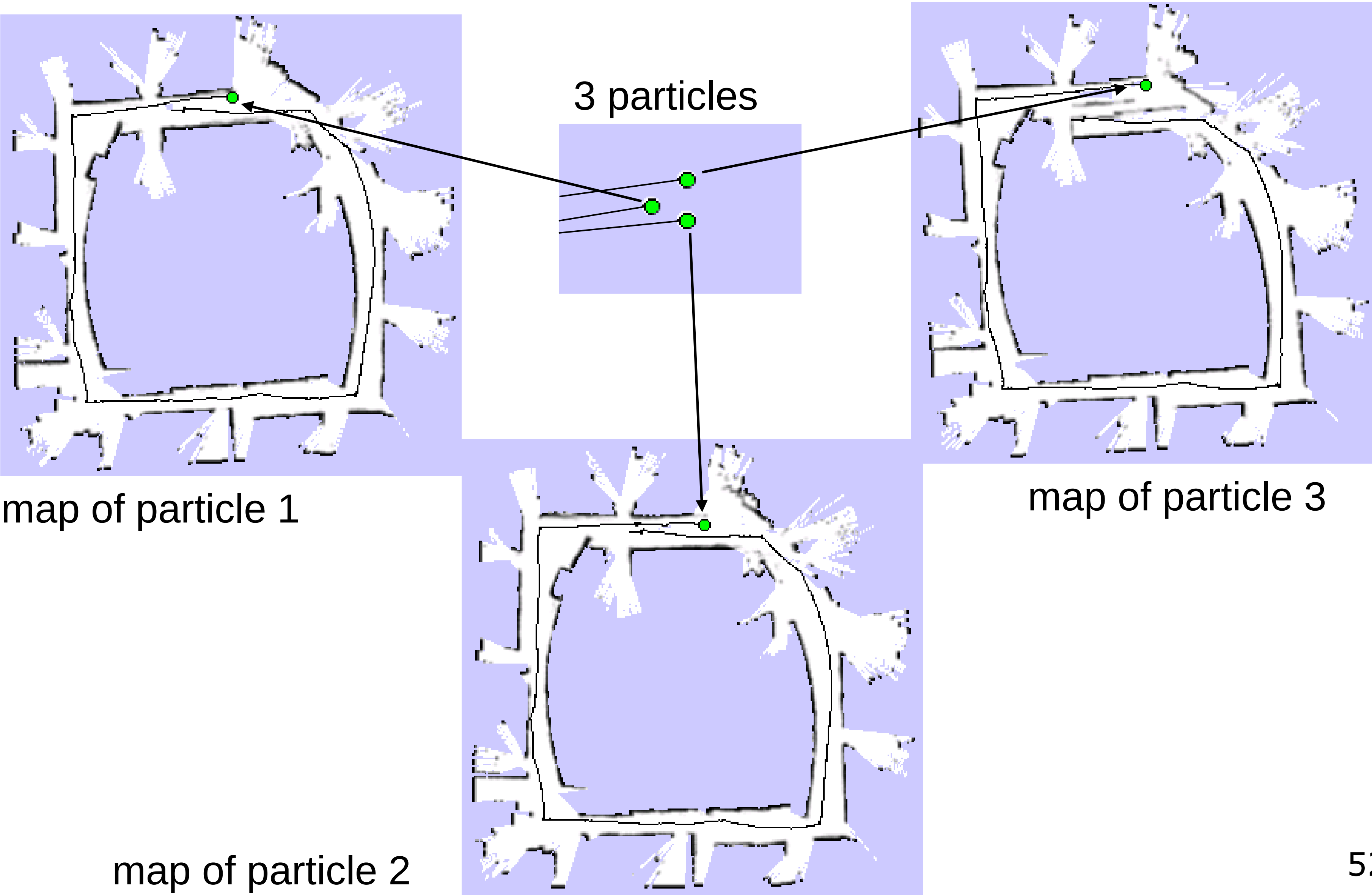
Grid-Based SLAM

- As with landmarks, the map depends on the poses of the robot during data acquisition
- If the poses are known, grid-based mapping is easy

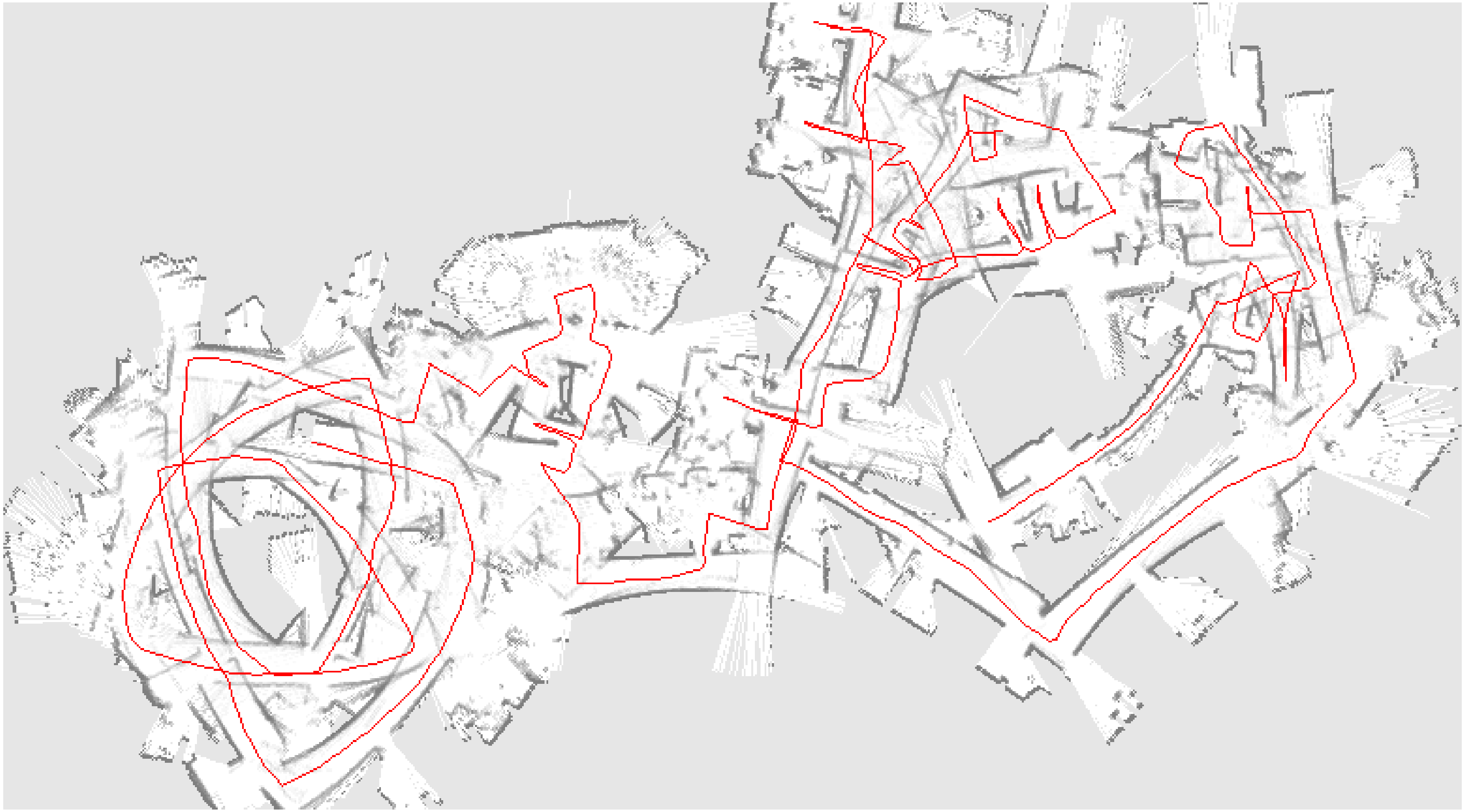
Grid-Based Mapping with Rao-Blackwellized Particle Filters

- Each particle represents a possible trajectory of the robot
- Each particle maintains its own map
- Each particle updates it upon "mapping with known poses"

Particle Filter Example



Performance of Grid-Based FastSLAM 1.0



Problem

- Too many samples are needed to sufficiently model the motion noise
- Increasing the number of samples is difficult as each map is quite large

- **Idea:** Improve the pose estimate **before** applying the particle filter

Recap: Pose Correction Using Scan Matching

Maximize the likelihood of the **current** pose relative to the **previous** pose and map

$$x_t^* = \underset{x_t}{\operatorname{argmax}} \left\{ p(z_t \mid x_t, m_{t-1}) \, p(x_t \mid u_t, x_{t-1}^*) \right\}$$

current measurement

robot motion

map constructed so far

Mapping Using Scan Matching

Courtesy: Dirk Hähnel

Grid-Based FastSLAM with Improved Odometry

- Scan matching provides a **locally consistent** pose correction
- **Idea:** Pre-correct short odometry sequences using scan matching and use them as input to FastSLAM
- Fewer particles are needed, since the error in the input in smaller

[Hähnel et al., 2003]

Acknowledgment

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