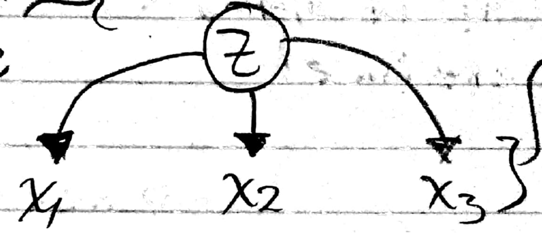
Sheet 9

14.1 a → 20%, b → 60%, C → 80%
- a coin is drawn randomly
- the coin is flipped 3-times (X_1, X_2, X_3)

ⓐ "the picked coin" may be a, b, c



"The possible outcomes" True if head

Bayesian Network

Z	P(Z)
a	1/3
b	1/3
c	1/3

— CPT —

Z	P(X/Z)
Z=a	0.2
Z=b	0.6
Z=C	0.8

ⓑ if X_1 = True, X_2 = True, X_3 = false → Z = ?
i.e. find Z with highest $P(Z / X_1, X_2, \neg X_3)$

$$* \; P(Z/X_1, X_2, \neg X_3) = \frac{P(X_1, X_2, \neg X_3 / Z) P(Z)}{P(X_1, X_2, \neg X_3)}$$

$$= \alpha P(X_1, X_2, \neg X_3 / Z) P(Z)$$

$$= \alpha P(X_1/Z) P(X_2/Z) P(\neg X_3/Z) P(Z)$$

∵ $X_{1,2,3}$ are Indep.

$* \; P(Z=a / X_1, X_2, \neg X_3) = \frac{0.032}{3}\alpha$, $P(Z=b/\sim) = \frac{0.144\alpha}{3}$, $P(Z=c/-) = \frac{0.128\alpha}{3}$

→ Z = b, ∵ it has the highest val

* to calc α, sum the 3-prob and equate with 1 → $\frac{0.304}{3}\alpha = 1$, $\alpha = \frac{3}{0.304}$

$* \; P(Z=a/\sim) = \frac{0.032}{0.304} \approx 0.105$, $P(Z=b/-) \approx 0.474$
$P(Z=c/-) \approx 0.421$

$H_x == G_x$

$\boxed{14.6}$ $H_x \xrightarrow{L} R$, $G_x \xrightarrow{L} R$, Same with prob (s), mutation prob (m)

(a) which of the Networks claim that $P(G_f, G_m, G_c) = P(G_f)\,P(G_m)\,P(G_c)$

a) Gm & Gf affect Gc i.e. Not indep.
b) Gm & Gf affect Gc i.e. Not indep.
c) Gm, Gf & Gc are indep. Ans. c)

(b) which of the Networks, is consistent with our hypothesis
Ans: a), b)

a) ✓
b) Doesn't violate any conditional indep → the extra arcs are unneeded
c) Assumes Gc is indep of Gf & Gm → Violation!

(c) which one is the best description of the hypothesis? Ans. c)

(D) what's the CPT of Gc in terms of s & m. from Network a)
Assume Left is the True value

Gm	Gf	P(Gc)
T	T	1-m
T	F	0.5
F	T	0.5
F	F	m

→

Gc	P(Hc)
T	s
F	1-s

(E) let $P(G_f=L) = P(G_m=L) = q$, in Network a), derive an expr. for $P(G_c=L)$ using m & q only.

$$* \; P(G_c) = \sum P(G_c \mid G_m, G_f)\, P(G_m)\, P(G_f)$$

$$= \overset{G_c=L}{(1-m)} * \overset{G_m=L}{q} * \overset{G_f=L}{q} + \left(0.5 * \overset{G_m=L\text{ or }R}{(1-q)} * \overset{G_f=L\text{ or }R}{q}\right) * 2 + \overset{G_c=L}{m}\,\overset{G_m=R}{(1-q)}\,\overset{G_f=R}{(1-q)}$$

$$= (1-m)\,q^2 + q(1-q) + m(1-q)^2$$

$$= \cancel{q^2} - q^2 m + q - \cancel{q^2} + m - 2mq + mq^2$$

$$= q + m - 2mq$$

(F) Under the conditions of Genetic Equ, we expect the distr of genes to be the same across generations [i.e $P(G_m) = P(G_f) = P(G_c) = q$] use this to calc q, explain why the hypothesis is wrong?!

$$* \therefore P(G_c) = P(G_f) = G(G_m) = q$$

$$\therefore \cancel{q} + m - 2mq = \cancel{q} \longrightarrow m - 2mq = 0$$

$$\longrightarrow m = 2mq \xrightarrow{m \neq 0} 1 = 2q \longrightarrow q = 1/2$$

People has 50 % chance to be left or right handed → False

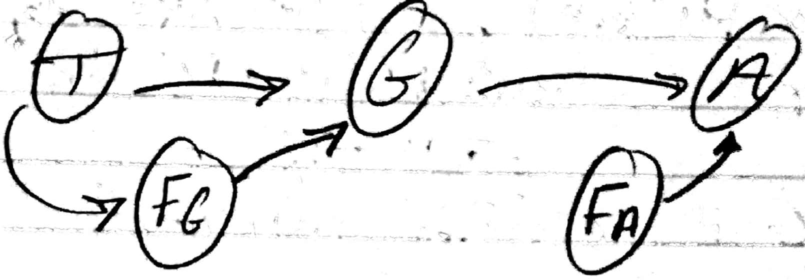
14.11 Alarm (A) if Gauge (G) > ?, TEMP (T)

measured by

Faulty Alarm (FA)

Faulty Gauge (FG) likely to fail if TPP

ⓐ



Poly tree

ⓑ Is it a Poly tree? No, there are 2-Paths from T → G i.e. General Graph

[Binary tree with N-Branches]

ⓒ

faulty

T	FG	P(G)
Normal	True	Y
N	False	X
High	True	1-Y
H	False	1-X

→ Assume True is normal

* Gauge gives the correct temp by Prob x If Not faulty (working)
* Gauge gives the correct temp by Prob Y if faulty

ⓓ

True is faulty

G	FA	P(A)
Normal	T	F
Normal	F	F
high	T	F
high	F	T

→ Assume true is ringing. "sounds"

* The Alarm works correctly unless Faulty

No Sound

follow 14.11

(f) FA = false, FG = false, A = True, P(T = High | ---)

Recall: $P(x|e) = \alpha P(x,e) = \alpha \sum P(x,e,y)$ hidden var

query var, evidence

* Let $t \to T=H$, $g \to G=H$, $f_G \to FG = faulty$, $f_A \to FA = faulty$, $a \to A = Sound$

* $P(t|a,\neg f_G,\neg f_A) = \alpha P(t,a,\neg f_G,\neg f_A)$

$= \alpha \sum_G P(t,a,\neg f_G,\neg f_A,G)$

$= \alpha [P(a|G,\neg f_A) * P(t) * P(\neg f_A) * P(G|\neg f_G,t) * P(\neg f_G|t) +$
$P(a|\neg G,\neg f_A) * \text{---} * \text{---} * P(\neg G|\neg f_G,t) * \text{---}]$

* for $\neg G \Rightarrow P(a|\neg G,\neg f) = 0 \to$ the 2nd term = 0

* $P(t|a,\neg f_G,\neg f_A) = \alpha P(a|G,\neg f_A) * P(t) * P(\neg f_A) * \cancel{P(G|\neg f_G,t)}^{1-x} * P(\neg f_G|t)$

$= \alpha [1 * (1-x) * P(t) * P(\neg f_A) * P(\neg f_G|t)]$ (1)

* $P(\neg t|a,\neg f_G,\neg f_A) = \alpha [1 * (x) * P(\neg t) * P(\neg f_G|\neg t) * P(\neg f_A)]$ (2)

(1)+(2) = 1 $\Rightarrow \alpha P(\neg f_A) [(1-x) P(t) P(\neg f_G|t) + (x) P(\neg t) P(\neg f_G|\neg t)] = 1$

$\alpha = 1/z$, $z = P(\neg f_A)[\text{---} + \text{---}]$

Subst α in (1)

$$P(t|a,\neg f_G,\neg f_A) = \frac{(1-x) P(t) \cancel{P(\neg f_A)} * P(\neg f_G|t)}{\cancel{P(\neg f_A)} [(1-x) P(t) P(\neg f_G|t) + \text{---}]}$$

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(a)

i) No, bcz I is depend on B & M

ii) is (J) indep of (I) Given (G)? Yes

iii) is (M) indep of (J) Given (G, B, I)? Yes

(b) $P(b, i, \neg m, g, j) = P(b)\,P(\neg m)\,P(i \mid b, \neg m)\,P(j \mid g)\,P(g \mid b, i, \neg m)$

$= 0.9 * 0.9 * 0.5 * 0.9 * 0.8$

(c) $P(j \mid b, i, m) = \sum P(j \mid b, i, m, g)$

$= P(j \mid g)\,P(g \mid b, i, m) + P(j \mid \neg g)\,P(\neg g \mid b, i, m)$

$= 0.9 * 0.9 + 0$

$= 0.81$

(D) Context-spec

* ال indep Context-spec يعني حاجات indep في حالة قيمة معينة، فلاحظ
كدا ال (G) بيساوي دايماً zero لو ال (I) بـ false بغض النظر عن قيمة
B & M

↳ G is indep on B & M Give I = false, since, $P(G \mid B, \neg I, M) = 0 = P(G \mid \neg I)$

(E) Add var P(Presidential Pardon عفو رئاسي)

← أعمل أي assumption برا نطاق المسألة بس يكون منطقي!