5.1) a) Notice that z = 1, so we will consider only this case.

$$Bel(x) = p(z|x)p(x) * \eta$$
$$\eta = \frac{1}{\sum_i p(z|x_i)p(x_i)} = \frac{1}{(0.1 + 0.05 + 0.1)} = 4$$

	x=1	x=2	x=3	x=4	x=5	x=6	x=7
$p(x_i)$	0.1	0.2	0.2	0.2	0.2	0.1	0.0
$p(z = 1\|x_i)$	0.0	0.5	0.25	0.0	0.5	0.0	0.0
$p(z = 1\|x_i)p(x_i)$	0.0	0.1	0.05	0.0	0.1	0.0	0.0
$Bel(x_i)$	0.0	0.4	0.2	0.0	0.4	0.0	0.0

b) For each x_i:

$$\overline{Bel}(x_i) = 0.2 * Bel(x_i) + 0.6 * Bel(x_{i-1}) + 0.2 * Bel(x_{i-2})$$

	x=1	x=2	x=3	x=4	x=5	x=6	x=7
$Bel(x_i)$	0.0	0.4	0.2	0.0	0.4	0.0	0.0
$\overline{Bel}(x_i)$	0.0	0.08	0.28	0.2	0.12	0.24	0.08

5.2) (note to self: z = observation = m, u = action = a, x = state = v)

At t = 1, ai = -10, mi = 40

	V=20	v=30	v=40	v=50	V=60
$p(v_i)$	0.0	0.2	0.6	0.2	0.0
$\overline{Bel}(v_i) = \sum_i p(a = -10\|v_i) * p(v_i)$	0.7p(v=30)+ 0.3p(v=20)+ 0*p(v=10)= 0.7*0.2+0.3 *0= 0.14	0.7p(v=40)+ 0.3p(v=30)+ 0*p(v=20)= 0.48	0.7p(v=50)+ 0.3p(v=40)+ 0*p(v=30)= 0.32	0.7p(v=60)+ 0.3p(v=50)+ 0*p(v=40)= 0.06	0.7p(v=70)+ 0.3p(v=60)+ 0*p(50)= 0.0
$p(m = 40\|x_i)$	0	0.2	0.7	0.1	0
$p(m = 40\|x_i) * \overline{Bel}(v_i)$	0	0.096	0.224	0.006	0
$Bel(v_i) = \eta * p(m = 40\|x_i) * \overline{Bel}(v_i)$	0	0.29448	0.68712	0.0184	0

At t = 2, ai = 0, mi = 50

	V=20	v=30	v=40	v=50	V=60
$p(v_i)$	0	0.29448	0.68712	0.0184	0
$\overline{Bel}(v_i) = \sum_i p(a=0\|v_i) * p(v_i)$	0*p(v=30)+ 1*p(v=20)+ 0*p(v=10)= 0	0*p(v=40)+ 1*p(v=30)+ 0*p(v=20)= 0.29448	0*p(v=50)+ 1*p(v=40)+ 0*p(v=30)= 0.68712	0*p(v=60)+ 1*p(v=50)+ 0*p(v=40)= 0.0184	0*p(v=70)+ 1*p(v=60)+ 0*p(v=50)= 0
$p(m=50\|x_i)$	0	0	0.2	0.7	0.1
$p(m=50\|x_i) * \overline{Bel}(v_i)$	0	0	0.1374	0.01288	0
$Bel(v_i) = \eta * p(m=40\|x_i) * \overline{Bel}(v_i)$	0	0	0.91415	0.0857	0

At t = 3, ai = +10, mi = 50

	V=20	v=30	v=40	v=50	V=60
$p(v_i)$	0	0	0.91415	0.0857	0
$\overline{Bel}(v_i) = \sum_i p(a=10\|v_i) * p(v_i)$	0*p(v=30)+ 0.2p(v=20)+ 0.8p(v=10)= 0	0*p(v=40)+ 0.2p(v=30)+ 0.8p(v=20)= 0	0*p(v=50)+ 0.2p(v=40)+ 0.8p(v=30)= 0.18283	0*p(v=60)+ 0.2p(v=50)+ 0.8p(v=40)= 0.74846	0*p(v=70)+ 0.2p(v=60)+ 0.8p(v=50)= 0.06856
$p(m=50\|x_i)$	0	0	0.2	0.7	0.1
$p(m=50\|x_i) * \overline{Bel}(v_i)$	0	0	0.036566	0.523922	0.006856
$Bel(v_i) = \eta * p(m=40\|x_i) * \overline{Bel}(v_i)$	0	0	0.06445	0.92346	0.01208

5.3) In particle filter we have the probabilities of the locations in the map as particles spread in the map. And as we use our motion and sensor models, we update these particles. And to get the approximate true state x_t from a set of particles, we could:

1. Pick the particle with the largest weight and consider it the true state
2. Sort the particles with respect to their weight and take the average of the largest 5 particles.
3. Pick the average of the particles after removing outliers from the largest cluster of particles. In other words, say we have many particles accumulated in a small region, and other particles spread in the map. We would consider those accumulated in a specific region only, and take their average.

5.4) For a state sequence $x_{0:t} = x_0, x_1, \dots, x_t$, we have the posterior $bel(x_{0:t}) = p(x_{0:t}|u_{1:t}, z_{1:t})$, instead of $bel(x_t) = p(x_t|u_{1:t}, z_{1:t})$ (0:t instead of t to include the space over all the state sequence).

$$
\begin{aligned}
\because p(x_{0:t}|u_{1:t}, z_{1:t}) &= \eta\, p(z_t|x_{0:t}, z_{1:t-1}, u_{1:t-1})\, p(x_{0:t}|\, z_{1:t-1}, u_{1:t}) \\
&= \eta\, p(z_t|x_t)\, p(x_{0:t}|\, z_{1:t-1}, u_{1:t}) \\
&= \eta\, p(z_t|x_t)\, p(x_t|x_{0:t-1},\ z_{1:t-1},\ u_{1:t})\, p(x_{0:t-1}|\, z_{1:t-1},\ u_{1:t}) \\
&= \eta\, p(z_t|x_t)\, p(x_t|x_{0:t-1},\ u_{1:t})\, p(x_{0:t-1}|\, z_{1:t-1},\ u_{1:t-1})
\end{aligned}
$$

And $\because p(x_t|x_{t-1},\ u_t)\, bel(x_{0:t-1}) = p(x_t|x_{t-1}, u_t)\, p(x_{0:t-1}|\, z_{0:t-1},\ u_{0:t-1})$.

Hence, we can get the weights as:

$$
\begin{aligned}
\therefore w_t = \frac{target\ distribution}{proposal\ distribution} &= \frac{p(x_{0:t}|u_{1:t}, z_{1:t})}{p(x_t|x_{t-1},\ u_t)\, bel(x_{0:t-1})} \\
&= \frac{\eta\, p(z_t|x_t)\, p(x_t|x_{0:t-1},\ u_{1:t})\, p(x_{0:t-1}|\, z_{1:t-1},\ u_{1:t-1})}{p(x_t|x_{t-1}, u_t)\, p(x_{0:t-1}|\, z_{0:t-1},\ u_{0:t-1})} = \eta\, p(z_t|x_t)
\end{aligned}
$$

Which is easily calculated for the weights.