Lis as:-

[Sheet (1) solutions].

a) • A person recieves a positive outcome on a first stage test. for a serious but rare dieses.

• the test reports a false positive with probability of 0.005

• we assume that there are no false negatives.

• what is the probability that the person actually suffers from the dieses, if the test was positive.

• Given that, out of 50,000 people, only one suffers.

Analysis:

[1] lets Define our variables.

T: Person is Tested Positive for dieses

D: Person has the dieses actually.

[2] Analyze the given

a) the false positive test probability implies that we know that he has not the dieses but he is tested positive → $P(T \mid D^c) = 0.005$

from here we can get another probability which is, if we know that he has not the dieses the probability that he will be tested negative = 0.99...

$P(T^c \mid D^c)$

P:: as:-

b) the probability of the dieses is $\frac{1}{50,000} = P(D)$

c) the requirement is, ~~if the~~ knowing that the test is positive, what is the probability that the person actually has the dieses → $P(D|T)$.

Solution steps:-

we know Bayes theorm :-

$$P(D|T) = \frac{P(T|D)\,P(D)}{P(T)}$$

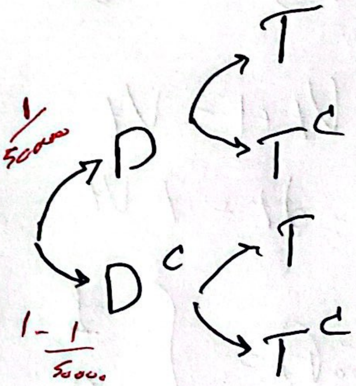
usually the problem with the Denominator

∵ we have $P(T|D) = 1$

∵ we have no false Negatives, so $P(T^c|D) = 0$

∴ $P(T|D) = 1 - P(T^c|D) = \boxed{1}$

∵ we have $P(D) = \frac{1}{50,000}$



∴ we just need T, By this tree and By using the total law of probability

$$P(T) = P(D)\,P(T|D) + P(D^c)\,P(T|D^c)$$

$$= \frac{1}{50,000}(1) + \left(1 - \frac{1}{50,000}\right)(0.005)$$

$\therefore \cancel{P(D|T)} = \cancel{P(D|T)\,P(D)}$

$$\therefore P(D|T) = \frac{P(T|D)\,P(D)}{P(T)}$$

$$\therefore P(D|T) = \frac{(1-0)\left(\frac{1}{50,000}\right)}{\frac{1}{50,000}(1-0) + \left(1-\frac{1}{50,000}\right)(0.005)} = \boxed{3.98 \times 10^{-3}}$$

2) Robot is equiped with an unreliable person detector,

. It outputs $\begin{cases}\to \text{Person.} \\ \to \text{No person,}\end{cases}$

. If there is a person in front of the robot it indicates Person with probability of "0.8"

. If there is no person, it also indicates a Person with probability of "0.2"

. the Prior belif of the robot about the person being in front is 0.5

. What is the Postirior probability of a person being in front of the robot, but it says "No person"?

3

Analysis:-

[1] lets Define events:

EF: the Robot says Person exist in front of me

AE: Person is actually standing in front of R

$P(EF \mid AE)$, we know that the person is actually standing, And depending on that the robot will say "Person" with probability of 0.7

$P(EF \mid AE^{C})$, we " " " " is " not standing, However the Robot says Person 0.2

- we want to compute $P(AE \mid EF^{C})$?
- $P(AE) = 0.5$ → prior probability

P (EF | AE) = 0.7
P (!EF | AE) = 0.3

P(EF | !AE) = 0.2
P(!EF | !AE) = 0.8

P (AE) = 0.5

RTF
P(!EF | AE)

Solution steps:

1. we all know Bayes theorm:-

$$P(AE \mid EF^C) = \frac{P(EF^C \mid AE)\, P(AE)}{P(EF^C)}$$ ← Problem is here

. we know $P(AE) = \underline{0.5}$

. we can get $P(EF^C \mid AE) = 1 - P(EF \mid AE)$

$\therefore P(EF^C \mid AE) = 1 - 0.7 = \boxed{0.3}$ ①

Now to get $P(EF^C)$, we get it from total law of probability as

EF < AE, AE^C
EF^C < AE, AE^C

$P(EF^C) = P($
$= P(AE)\, P(EF^C \mid AE) + P(AE^C)$ ~~$P(EF^C \mid AE)$~~ $P(EF^C \mid AE^C)$

$P(AE) = 0.5$
$P(AE^C) = 0.5$

$P(EF^C \mid AE) = 1 - P(EF \mid AE) = \boxed{0.3}$

$P(EF^C \mid AE^C) = 1 - P(EF \mid AE^C) = 1 - 0.2 = \boxed{0.8}$

$$\therefore P(EF^C) = \frac{(0.3)\cancel{(0.5)}}{(0.3)\cancel{(0.5)} + (0.8)\cancel{(0.5)}} = \boxed{\frac{3}{11}}$$

Correct first time $\frac{06}{11}$

Kingsoo

[3] ~~[illegible]~~

3) Having two Dimensional space $X = (X_1, X_2)$

X_1: position of a cart (in meters)

X_2: the cart's velocity in (m/s)

the distance between two time steps is $\boxed{0.1\,s}$

Describe the matrix A that maps X_t to X_{t+1} in the noiseless case.

Analysis:

- we are given vector in this form $\begin{bmatrix} d_t \\ V_t \end{bmatrix}$
- we are looking for Matrix A, such that $A \begin{bmatrix} d_t \\ V_t \end{bmatrix}$ will provide us with our new vector $\begin{bmatrix} d_{t+1} \\ V_{t+1} \end{bmatrix}$

$$\begin{bmatrix} d_{t+1} \\ V_{t+1} \end{bmatrix} = A \begin{bmatrix} d_t \\ V_t \end{bmatrix}$$

- Knowing that $\Delta t = 0.2$

Solution steps:-

1. the A diminsions must be 2x2, so that we can multiply it by 2x1 vector & get $\boxed{2\times1}$ new vector ✓

2. $A = \begin{bmatrix} i & j \\ K & w \end{bmatrix}$ so → $\boxed{\begin{aligned} d_{t+1} &= d_t\, i + V_t\, j \\ V_{t+1} &= d_t\, K + V_t\, w \end{aligned}}$ eq needed

• the solution now will require you to remember some dynamics rules:-

a) ∵ there is no acceleration, so we have a constant velocity → so $\boxed{V_{t+1} = V_t}$

$\boxed{b)\ d_{t+1} = d_t + V_t\,t}$

the new distance is equal to the previous distance plus the velocity multiplied by the time passed, so this will provide us with the amount of covered distance in that duration.

We need to remember these rules, because we're gonna use them a lot!

Now try to get (i, θ, K, ω)

a) ∵ $V_{t+1} = V_t$ ∴ $K = 0$, $\omega = 1$

∵ $d_{t+1} = d_t + t\,V_t$ ∴ $i = 1$, $\theta = t = 0.1$

$$\therefore A = \begin{bmatrix} 1 & 0.1 \\ 0 & 1 \end{bmatrix}$$ ✓✓ Correct!

Now I assume that you can solve the next problem easily. give it a try ☺

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4) Consider there is a new matrix U_t, that accelerate the cart,

How should B looks like so $\boxed{X_{t+1} = A X_t + B u_t}$

Solution trial:

$\because$ there is an acceleration, so

$$\boxed{\begin{aligned} d_t &= \underbrace{d_{t-1} + t\, V_{t-1}}_{} + \frac{t^2\, a_{t-1}}{2} \\ V_t &= V_{t-1} + t\, a_{t-1} \\ a_t &= a_{t-1} \rightarrow \text{constant acceleration \#} \end{aligned}}$$

$$\cancel{X_{t+1} = \begin{bmatrix} d_{t+1} \\ V_{t+1} \\ a_{t+1} \end{bmatrix}}$$

$$\cancel{X_{t+1} = A x_t + B u_t}$$

Dimensions of B_t is $\underline{3 \times 1}$ as u_t is $\underline{1 \times 1}$, so we can it up with 3×1 vector

so $X_{t+1} = A \begin{bmatrix} d_t \\ V_t \\ a_t \end{bmatrix} + B\, [u_t]$

$$B \rightarrow \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Rewriting the motion equations:-

$$\left\{ \begin{aligned} \cdot\, d_{t+1} &= d_t + t V_t + \boxed{\frac{\Delta t^2}{2} a_t} \\ \cdot\, V_{t+1} &= (0) d_t + V_t + \boxed{\Delta t\, a_t} \\ \cdot\, a_{t+1} &= (0) d_t + (0) V_t + a_t \end{aligned} \right.$$

$\rightarrow$ so from here we can just conclude that the B matrix should be 2×1 as u_t is $\underline{1 \times 1}$

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So we can write this as:-

$$X_{t+1} = \underset{2\times 1}{A X_t} + \underset{2\times 1}{B} \overset{2\times 1}{\ } \underset{1\times 1}{u_t}, \quad B_t = \begin{bmatrix} i \\ j \end{bmatrix}$$

$$\therefore \begin{bmatrix} d_{t+1} \\ v_{t+1} \end{bmatrix} = \begin{bmatrix} 1 & 0.1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} d_t \\ v_t \end{bmatrix} + \begin{bmatrix} i \\ j \end{bmatrix} [u_t]$$

$$\text{so } d_{t+1} = (1) d_t + (0.1) v_t + (u_t)(i)$$

$$v_{t+1} = 0(d_t) + (1) v_t + (u_t)(j)$$

this is the chosed part

so by comparison:-

$$j = 0.1 \qquad i = \frac{0.1^2}{2} = \boxed{0.005}$$

$$\therefore B = \begin{bmatrix} 0.005 \\ 0.1 \end{bmatrix} \#$$

first part of sheet 1 is now Done #