Cairo University
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Natural Language Processing

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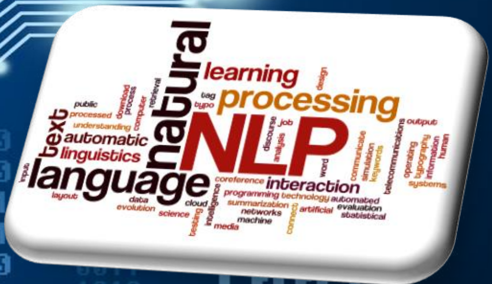


Word Embeddings

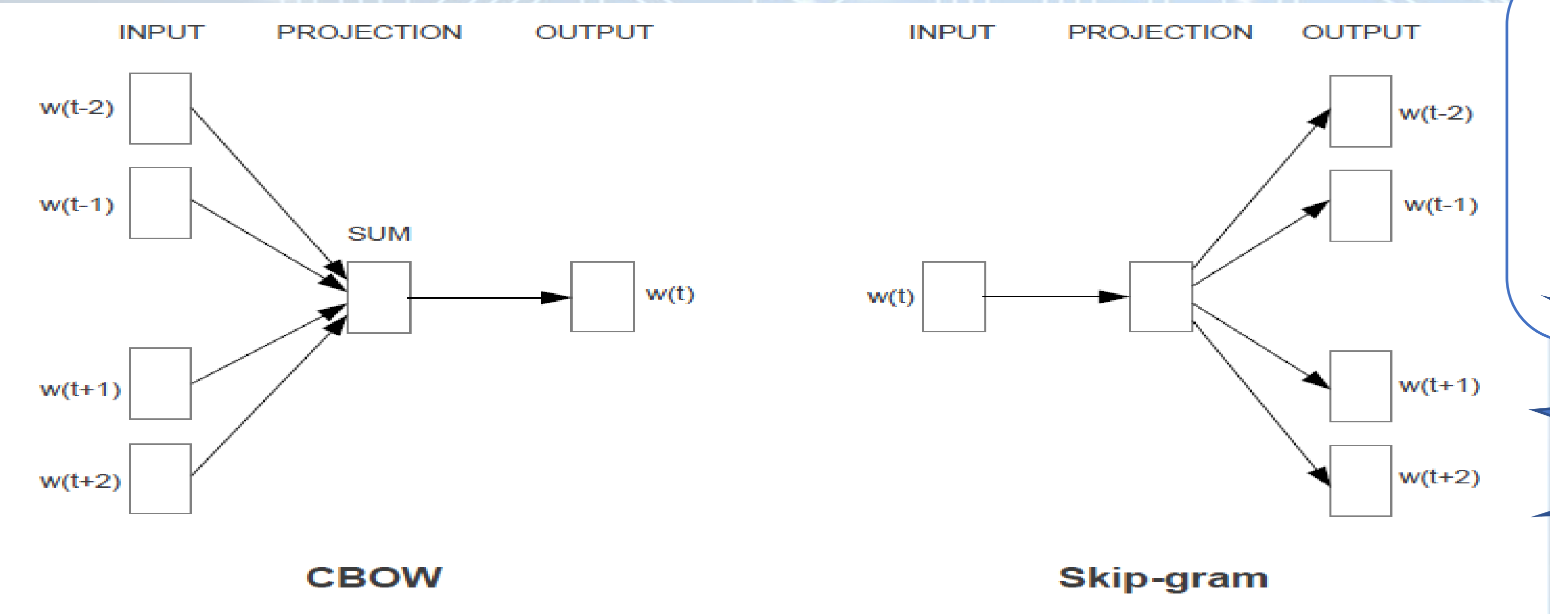
- Previously, we saw how to represent a word as a **sparse, long vector** with dimensions corresponding to words in the vocabulary or documents in a collection.
- We now introduce a more powerful word representation: **embeddings**, **short dense vectors**.
  - With number of dimensions *d* ranging from 50-1000.
  - The vectors are dense: instead of vector entries being sparse, mostly-zero counts or functions of counts, the values will be real-valued numbers that can be negative.
- It turns out that dense vectors work better in every NLP task than sparse vectors.
- One method for computing embeddings is Word2vec where embeddings are **static**: meaning that the method learns one fixed embedding for each word in the vocabulary.
- Other methods such as BERT, are for learning **dynamic contextual** embeddings in which the vector for each word is different in different contexts.

Word2vec

- Word2vec is developed by *Tomas Mikolov et al. at Google* where two models are proposed:

CBOW
- Continuous Bag-Of-Words
- where a word is predicted based on its context (the surrounding words).

SG
- Continuous Skip-Gram
- where the surrounding words are predicted given a current word.



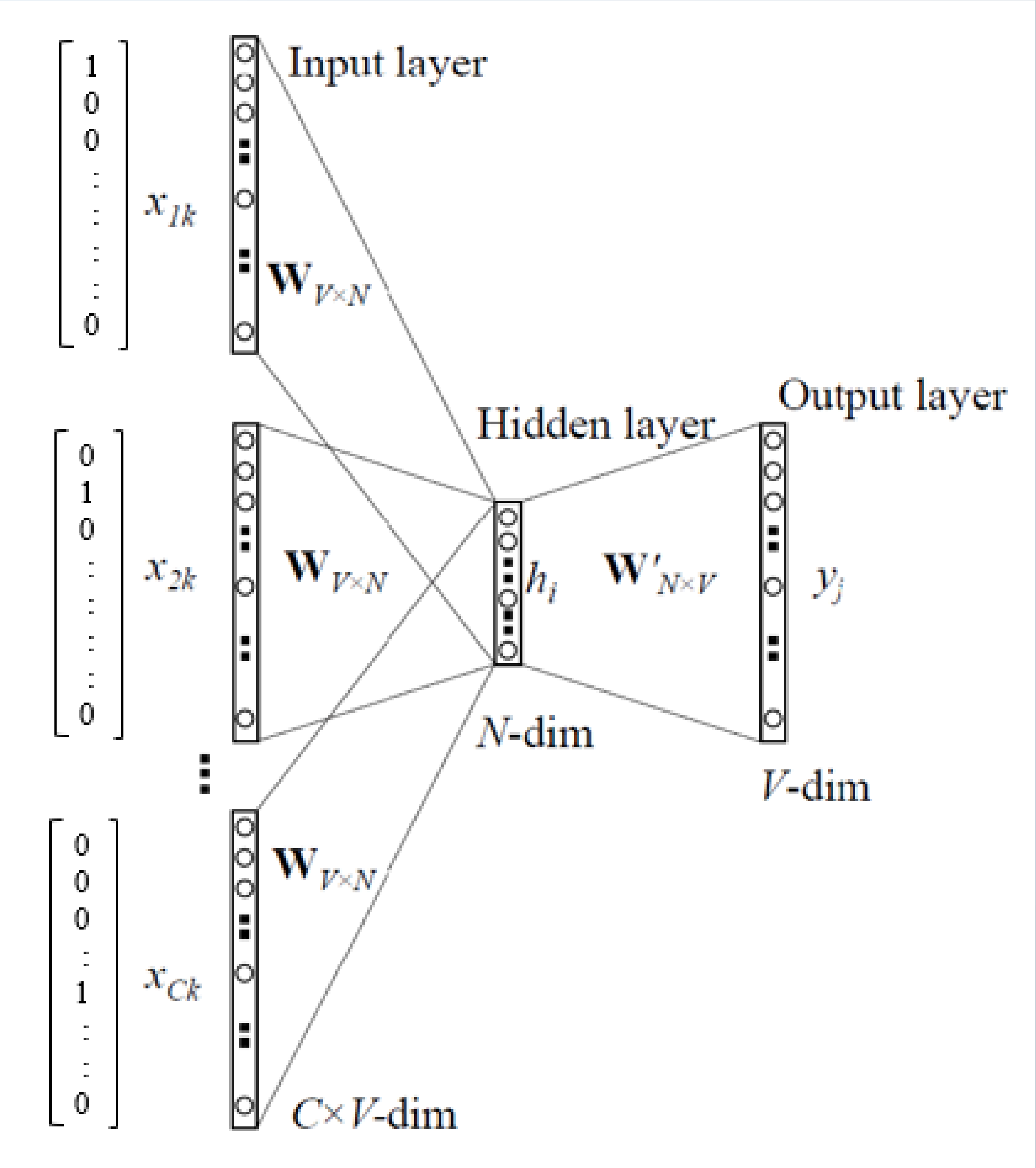
The models' architecture is a neural network with three layers: an input layer, a hidden layer and an output layer.
The models are trained using huge text corpora

Though, we won't use the neural network after training!!!

CBOW

Input Layer:

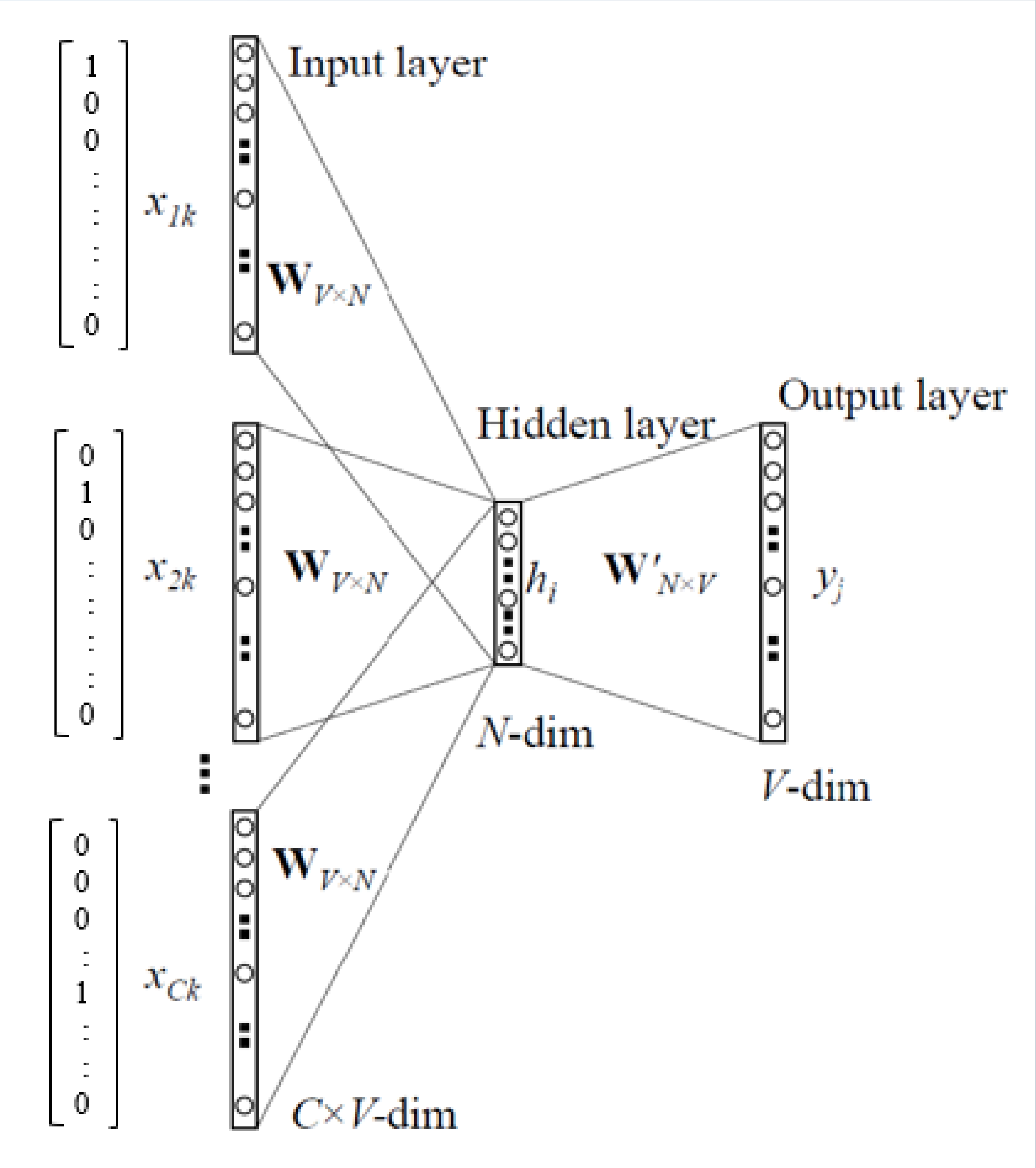
- Its size is CV
  - C=number of context words
  - V=number of words in the vocabulary
- The inputs are **C context vectors**:
  - Each context vector is one-hot encoded vector of size V (it is a vector having 1 in the position encoding the word, and 0 otherwise).



CBOW

Hidden Layer:

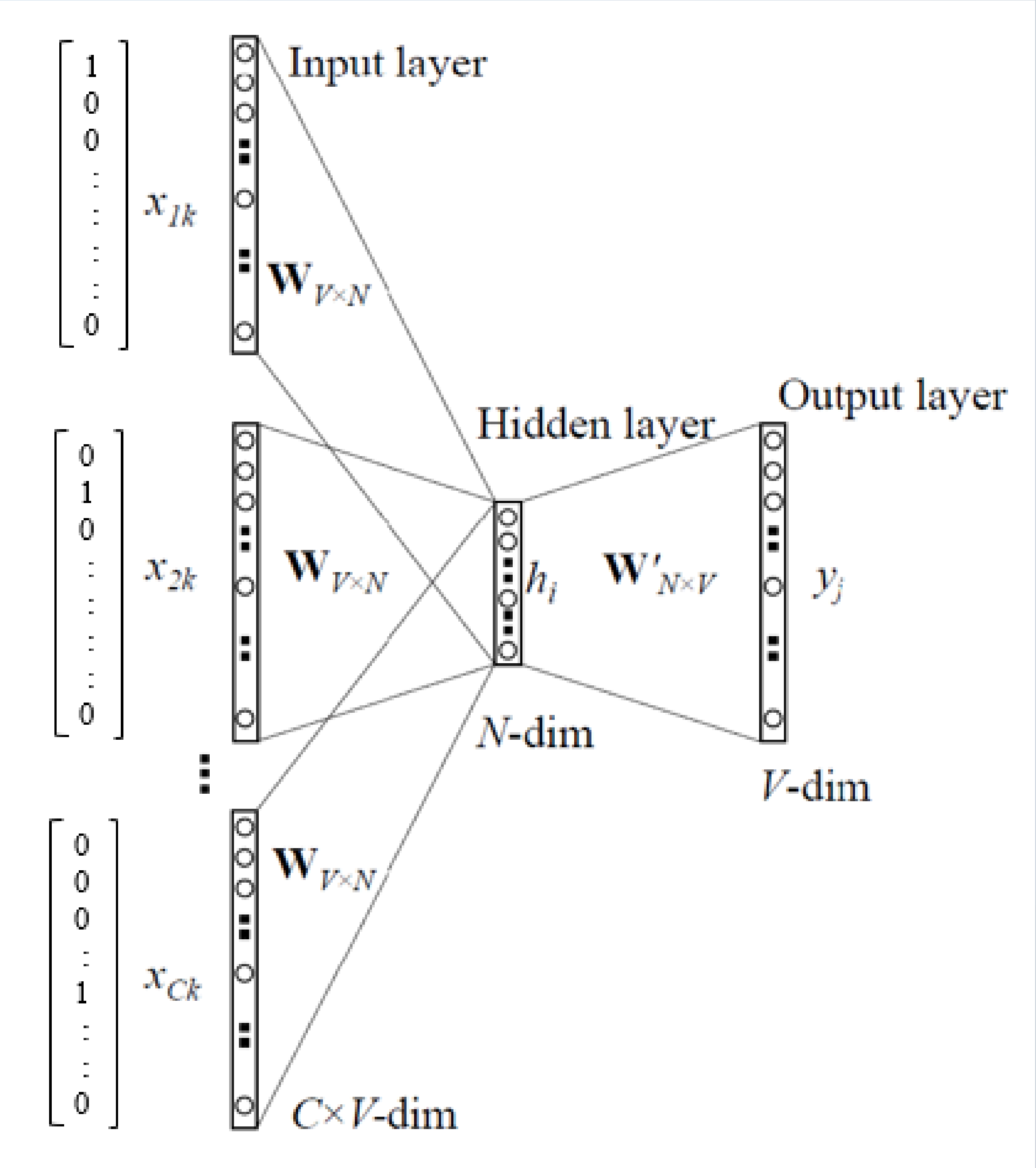
- Its size is N
  - N=number of neurons in the hidden layer
    =the dimension of embeddings
    (it is a hyper parameter)
- Fully-connected (dense) layer whose weights are the word embeddings.



CBOW

Output layer:

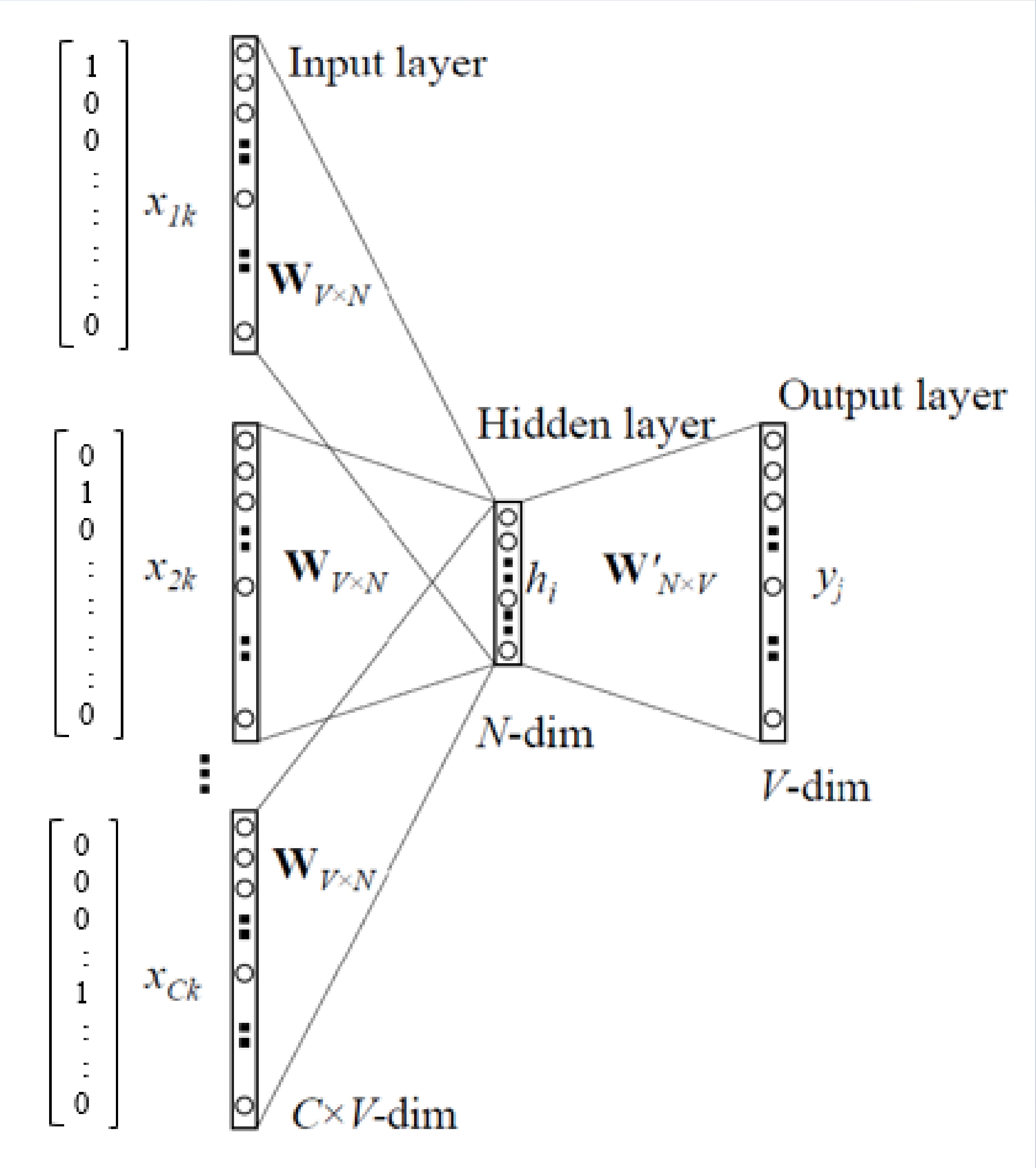
- Its size is V
    - Where the gold output is the one-hot vector encoding the **"middle/target word"**.
- It is a softmax layer which is designed so that the sum of the probabilities obtained in the output layer equals 1.
    - The network is going to tell us the probability for every word in our vocabulary of being the **"target word"** for the given input(s) **"context word(s)"**



CBOW

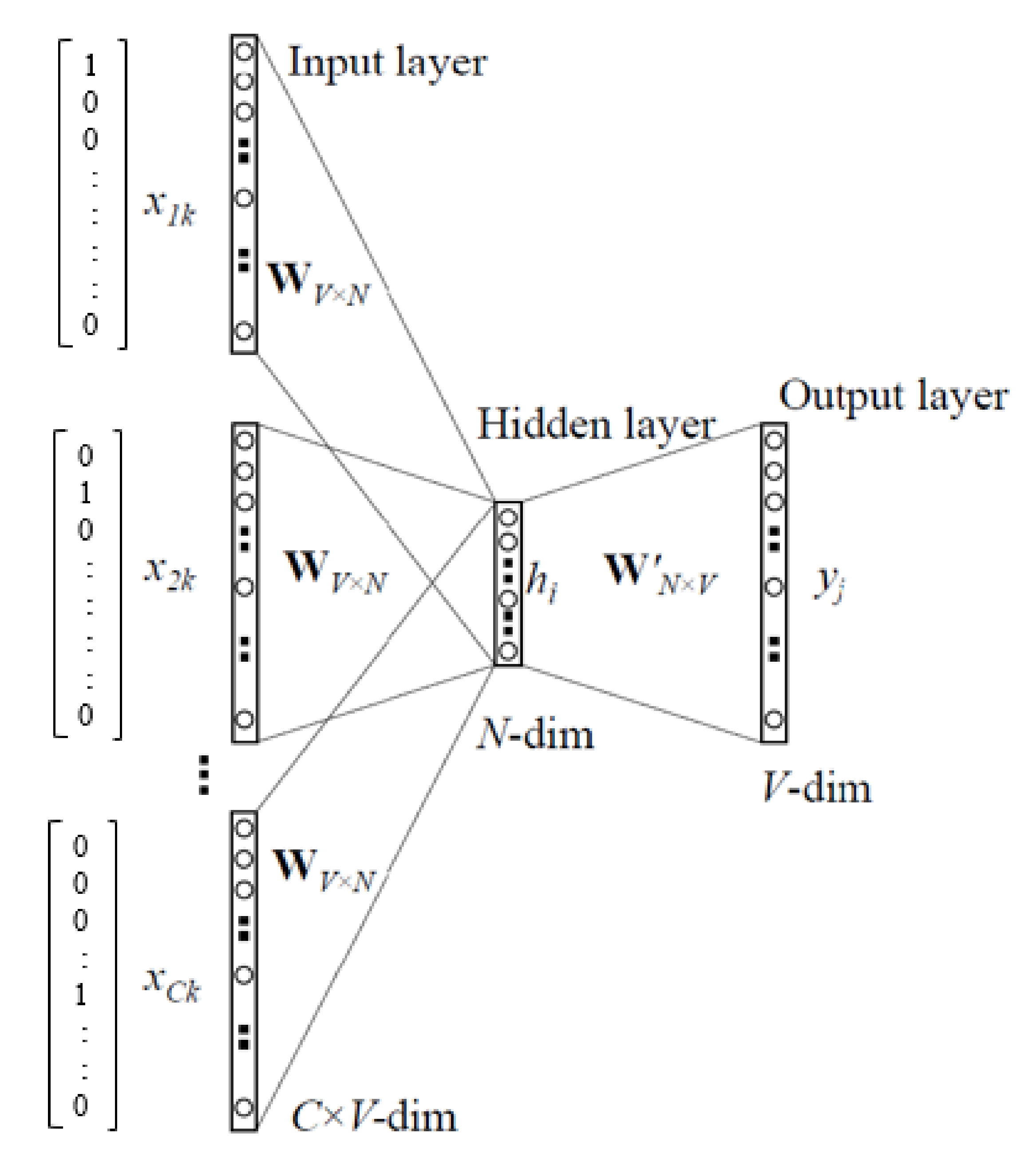
Weights:

- There are two sets of weights:
  - one is between the input and the hidden layer:
    Input-Hidden layer matrix
    consisting of C W[V ×N] matrices
  - second is between hidden and output layer:
    Hidden-Output layer matrix size =[N × V]



CBOW

- There is a no activation function between any layers (linear activation):
  - Linear network except for the softmax at the output layer.
- The input-to-hidden-layer weight matrix is multiplied with the input vector to produce the hidden layer output.
- The hidden input gets multiplied by hidden-output weights to produce the output then pass it to softmax.
- Error between output and target is calculated and back propagated to re-adjust the weights.
- The weights between the input layer and the hidden layer are taken as the word vectors after training.



CBOW

- Let the available corpus be this one sentence *"word vectors are widely used"*.
- The unique words in the vocabulary are 5 words (V = 5) as follows:
  ["word", "vectors", "are", "widely", "used"].
- Training data for C=1 (context words)

training sample	context word	target word
1	word	vectors
2	vectors	are
3	are	widely
4	widely	used

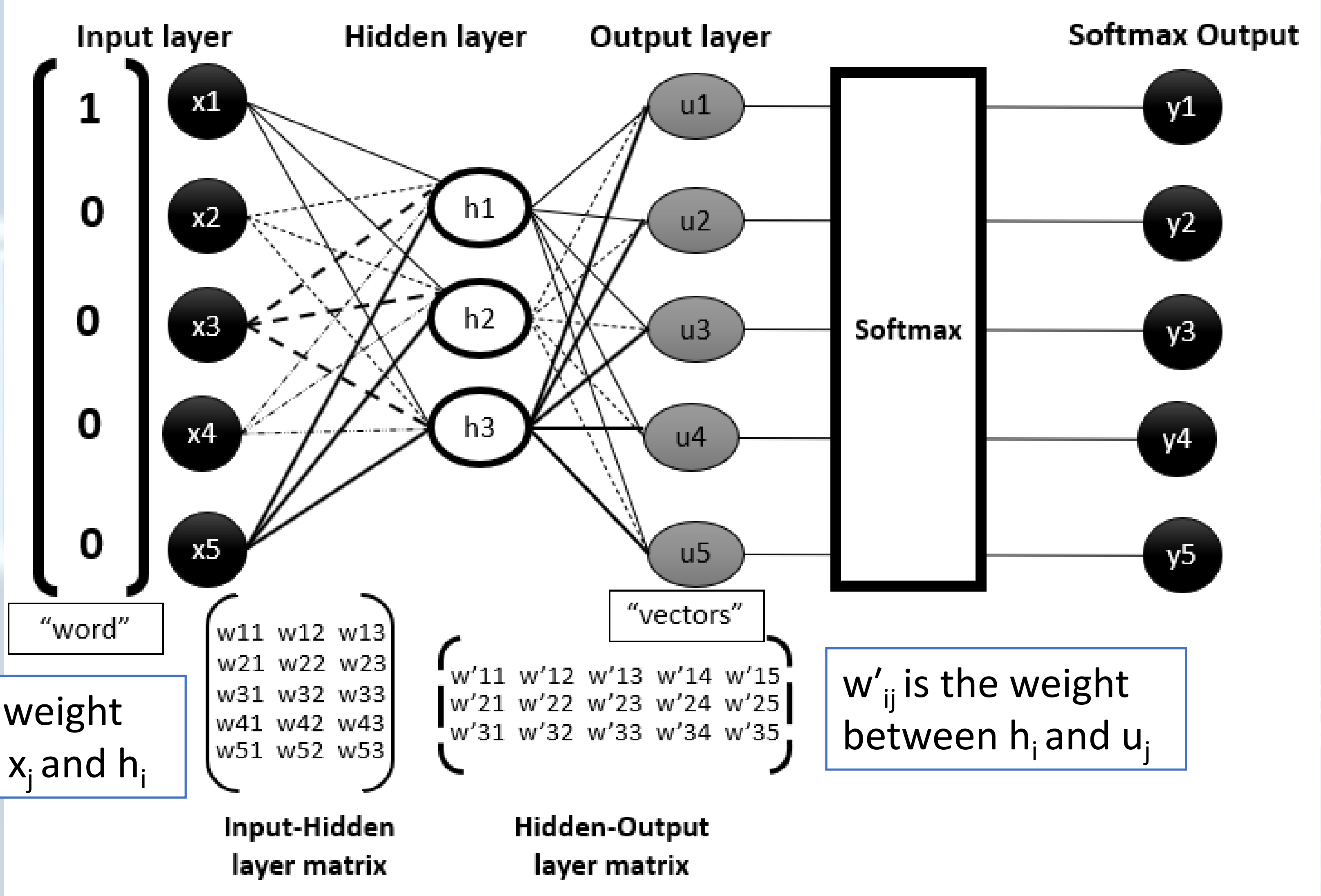
- Encoded training data:

training sample	context word	target word
1	[1,0,0,0,0]	[0,1,0,0,0]
2	[0,1,0,0,0]	[0,0,1,0,0]
3	[0,0,1,0,0]	[0,0,0,1,0]
4	[0,0,0,1,0]	[0,0,0,0,1]

CBOW

- Let N = 3, consider the first training sample:

training sample	context word	target word
1	[1,0,0,0,0]	[0,1,0,0,0]



w_{ji} is the weight between x_j and h_i

w'_{ij} is the weight between h_i and u_j

CBOW

- Equations:

$$h_i = \sum_j w_{ji} * x_j \text{ where } i = 1,2,3 \text{ and } j = 1,2,3,4,5 \qquad (1)$$

$$u_j = \sum_i w'_{ij} * h_i \text{ where } i = 1,2,3 \text{ and } j = 1,2,3,4,5 \qquad (2)$$

$$y_j = Softmax(u_j) \text{ where } j = 1,2,3,4,5 \qquad (3)$$

- Matrix Forms:

x1	x2	x3	x4	x5

×

w11	w12	w13
w21	w22	w23
w31	w32	w33
w41	w42	w43
w51	w52	w53

=

h1	h2	h3

$$[0 \quad 0 \quad 0 \quad \boxed{1} \quad 0] \times \begin{bmatrix} 17 & 24 & 1 \\ 23 & 5 & 7 \\ 4 & 6 & 13 \\ \boxed{10 \quad 12 \quad 19} \\ 11 & 18 & 25 \end{bmatrix} = [10 \quad 12 \quad 19]$$

The output of the hidden layer is just the “word vector” for the input word.

h1	h2	h3

×

w’11	w’12	w’13	w’14	w’15
w’21	w’22	w’23	w’24	w’25
w’31	w’32	w’33	w’34	w’35

=

u1	u2	u3	u4	u5

CBOW

- The error calculations are performed given the target vector: [0,1,0,0,0] through **back propagation** where the weight values (w and w') are updated.
- The training continues for **many iterations** until the weights are adjusted for all the training samples.
- Finally, the vectors of the words in the vocabulary are weights in the Input-Hidden layer matrix such that each row in the matrix is the word vector representation of the equivalent word.

The word in the vocabulary	The equivalent word vector
word	[w11 w12 w13]
vectors	[w21 w22 w23]
are	[w31 w32 w33]
widely	[w41 w42 w43]
used	[w51 w52 w53]

CBOW

- For C>1 (context words), the only difference will be in handling the C input vectors.
  - Each vector will be multiplied with the input-hidden layer matrix W[V ×N] returning C [1 × N] vectors.
  - Finally, to obtain a single vector → all C [1 × N] vectors will be averaged element-wise.
- This computation is equivalent to initially computing a **mean vector** for the C input one-hot encoded vectors then proceeding as in the case of C = 1.
- For C=2 → window size=1

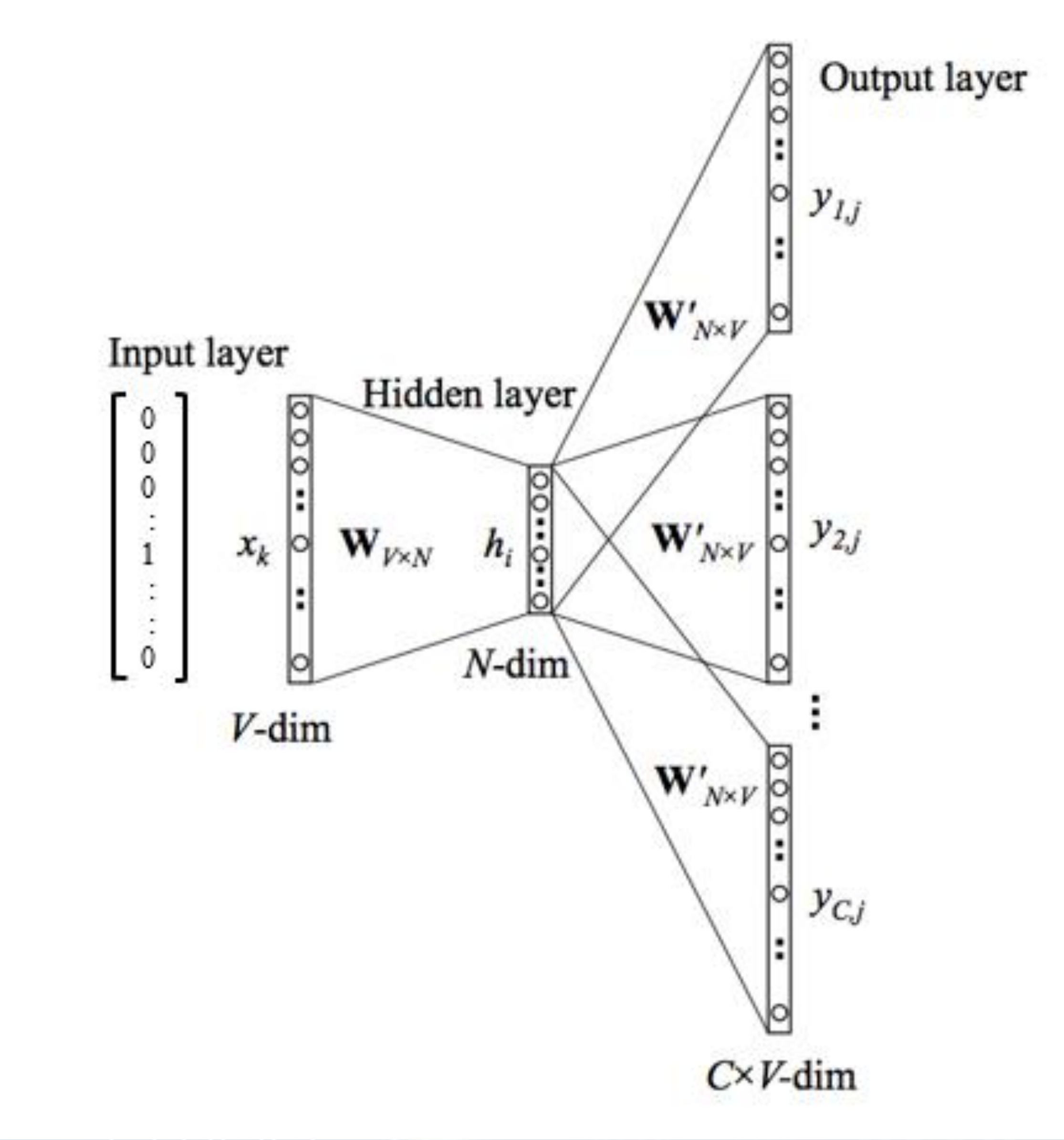
training sample	context word	target word
1	(word,are)	vectors
2	(vectors,widely)	are
3	(are,used)	widely
4	widely	used

training sample	context words	mean context word	target word
1	([1,0,0,0,0],[0,0,1,0,0])	[0.5,0,0.5,0,0]	[0,1,0,0,0]
2	([0,1,0,0,0],[0,0,0,1,0])	[0,0.5,0,0.5,0]	[0,0,1,0,0]
3	([0,0,1,0,0],[0,0,0,0,1])	[0,0,0.5,0,0.5]	[0,0,0,1,0]
4	[0,0,0,1,0]	[0,0,0,1,0]	[0,0,0,0,1]

SG

<u>Input Layer:</u>

- Its size is V
  - V=number of words in the vocabulary
- The input is **1 middle/target vector**:
  - the target vector is a one-hot encoded vector (a vector having 1 in the position encoding the word, and 0 otherwise).



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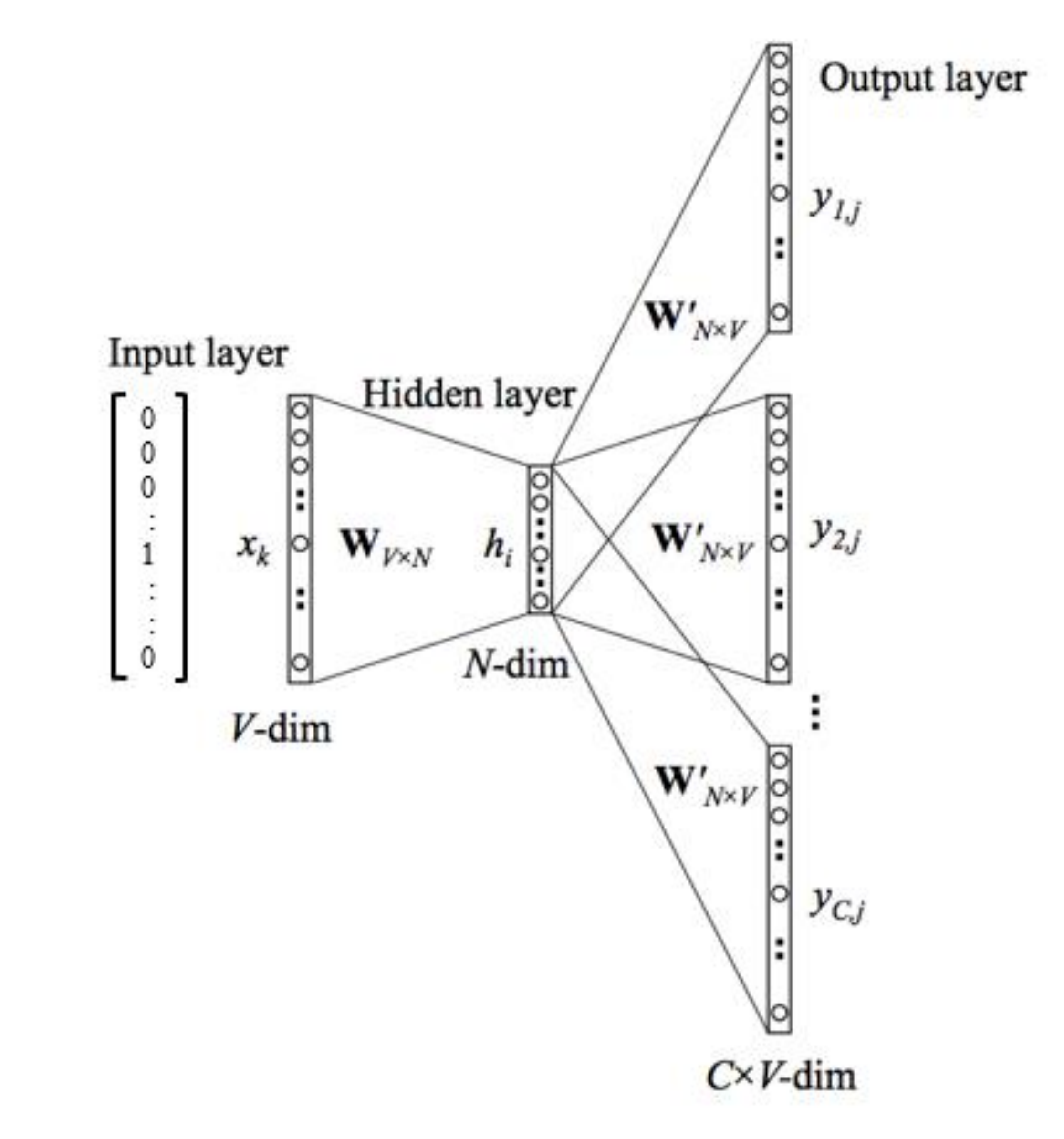
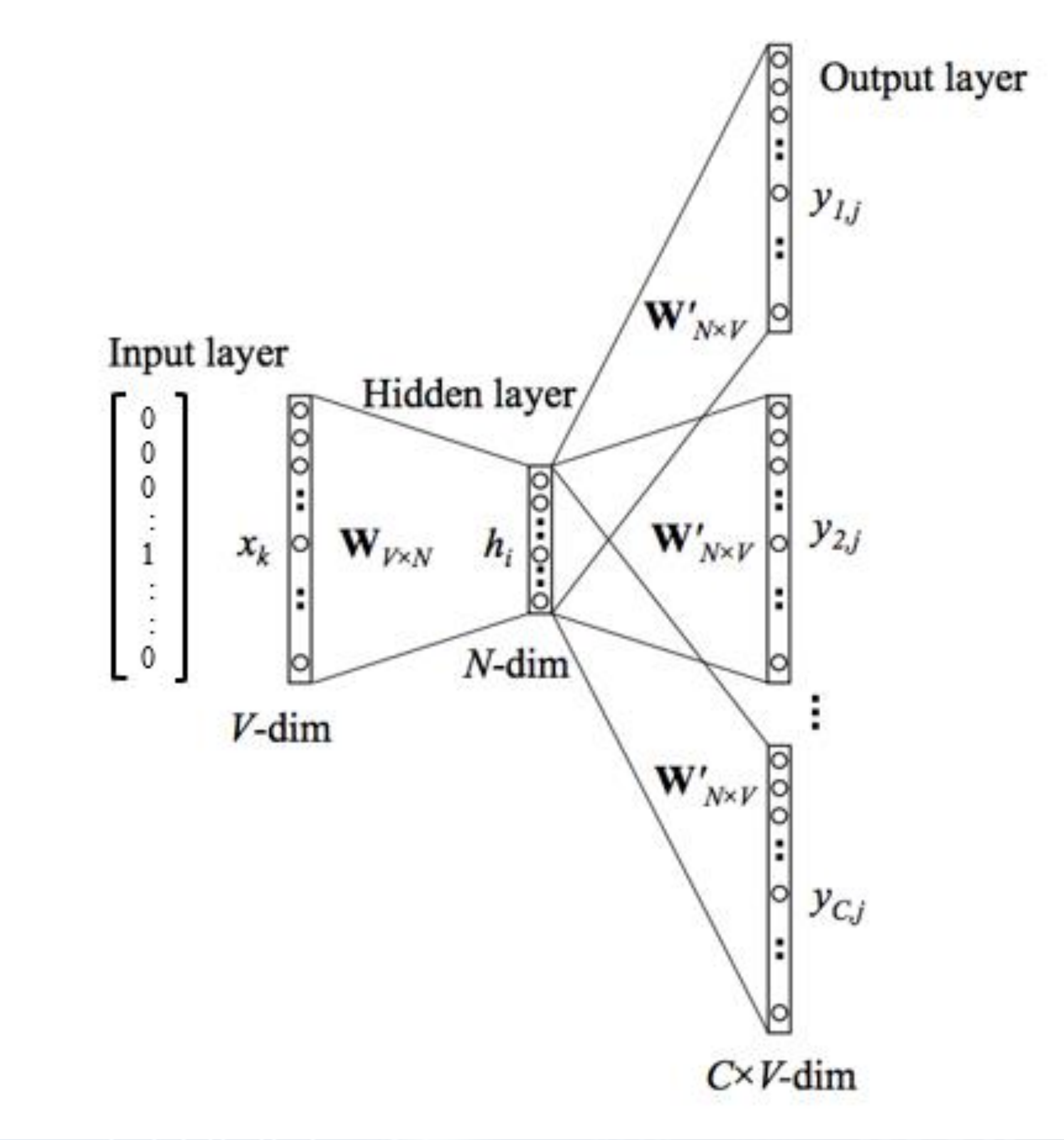


SG

<u>Output Layer:</u>

- Its size is CV
  - C=number of context words
  - V=number of words in the vocabulary
  - Where the gold outputs are C one-hot vectors each encoding one of the **"context words"**.
- It is a softmax layer which is designed so that the sum of the probabilities obtained in the output layer equals 1.
  - The network is going to tell us the probability for every word in our vocabulary of being the **"context word"** for the given input **"target word"**
- **C separate errors** are calculated and are added element-wise to obtain a final error vector which is back propagated to update the weights.



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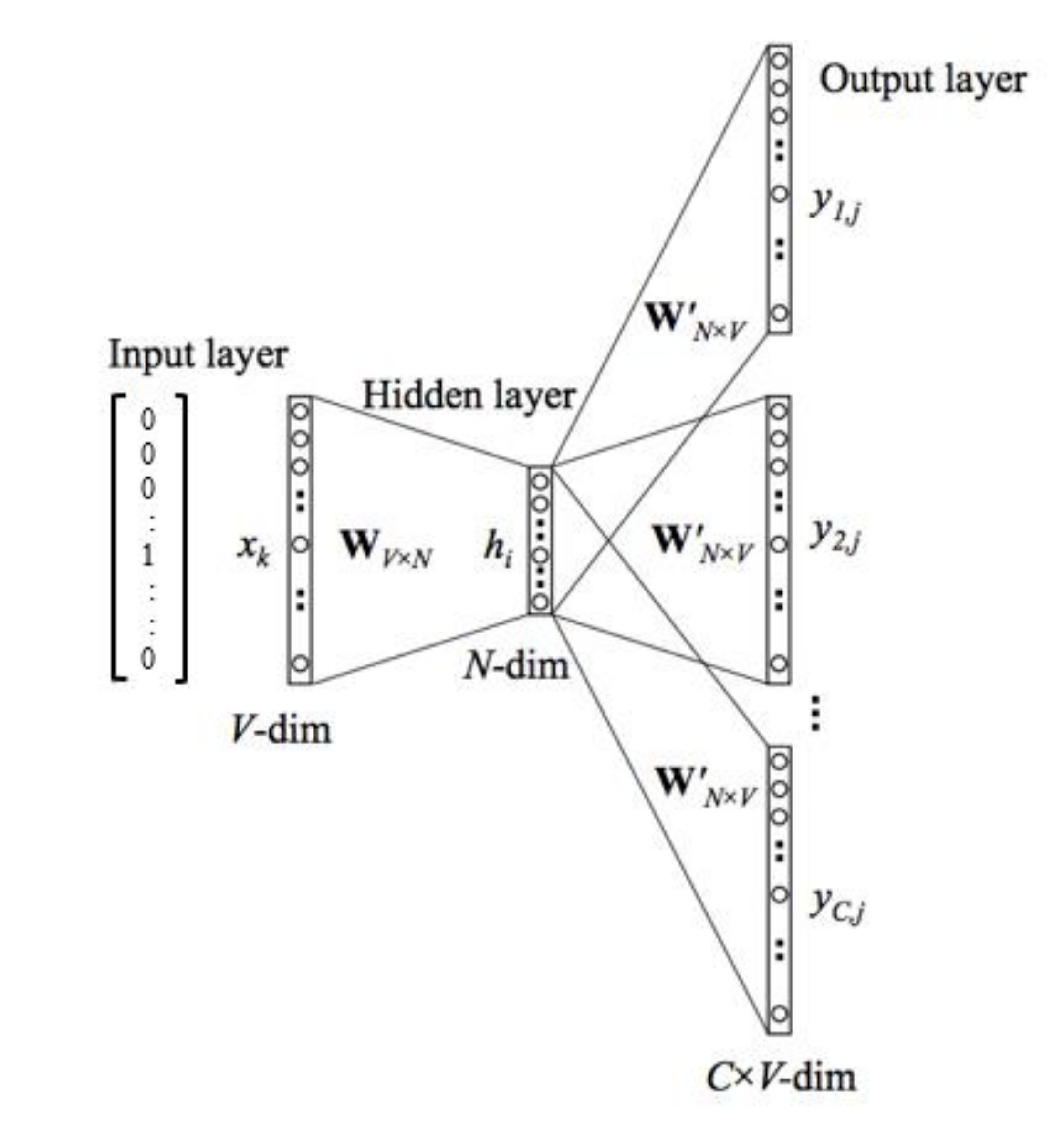

SG

- There is a no activation function between any layers (linear activation):
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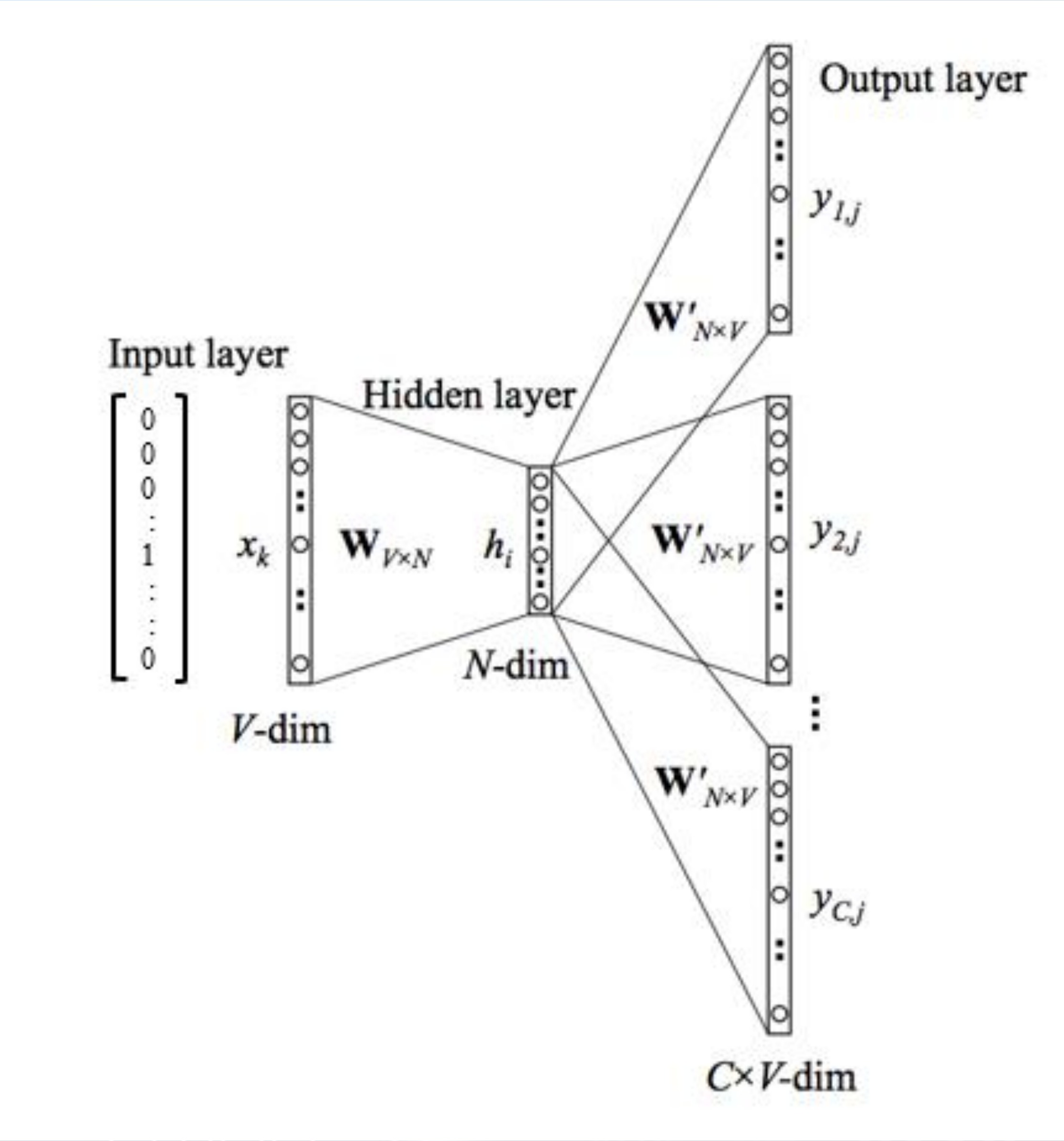
- The weights between the input layer and the hidden layer are taken as the word vectors after training.



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- Training data for C=2 (context words)→ window size=1

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- Encoded training data:

training sample	context words	target word
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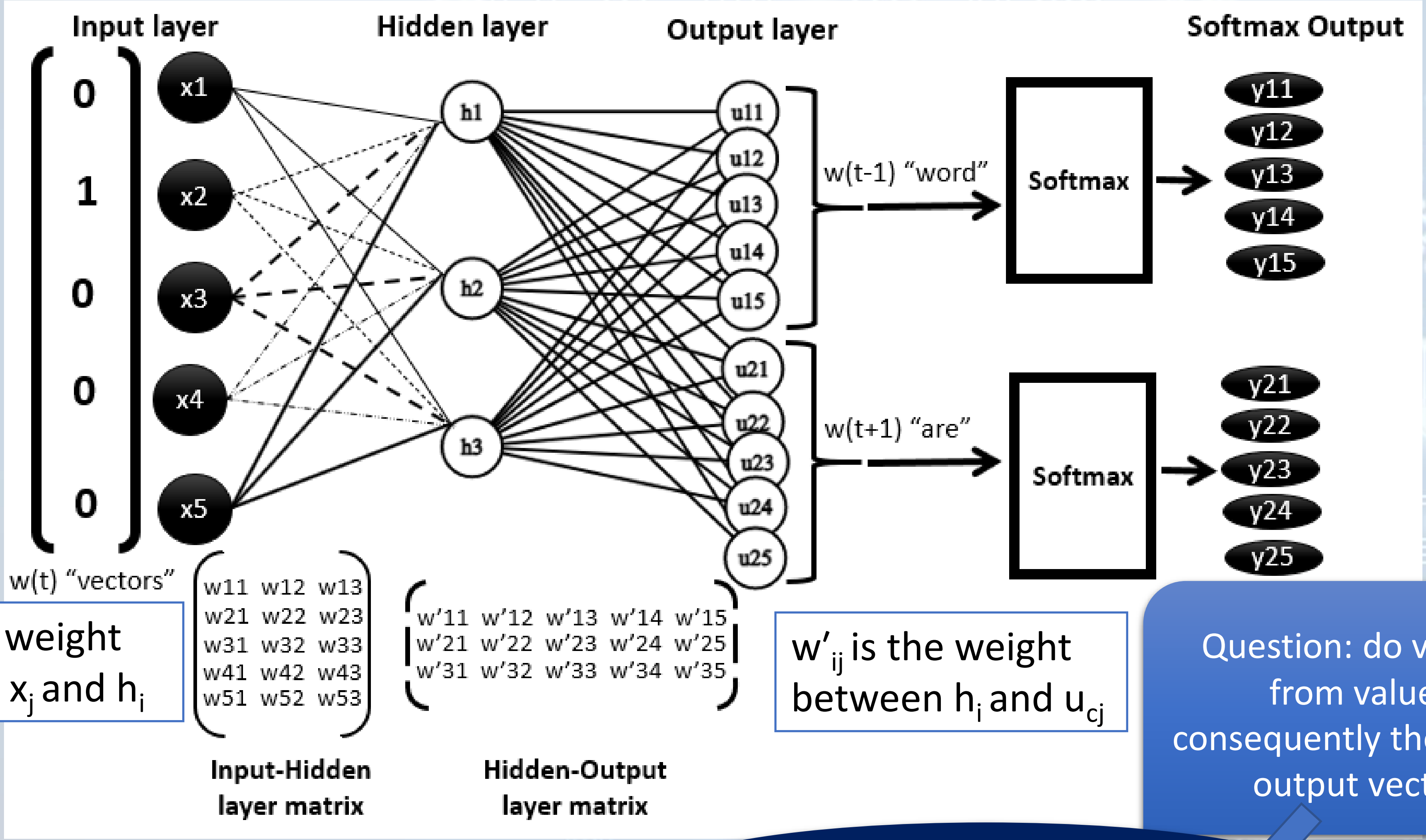

w_{ji} is the weight between x_j and h_i

w'_{ij} is the weight between h_i and u_{cj}

Question: do values **u11->u15** **differ** from values **u21→u25** (and consequently the y) i.e. do we have **one** output vector or **C** vectors???

Only **one** since **W'** matrix is the same

SG

- The training process is similar to CBOW except that the error calculation differs as there are C gold outputs. Therefore, the final error is computed as the summation of the error for each context word as follows:

$$E = \sum_{c=1}^{C} E_c$$

- Finally, the vectors of the words in the vocabulary are weights in the Input-Hidden layer matrix such that each row in the matrix is the word vector representation of the equivalent word.

The word in the vocabulary	The equivalent word vector
word	[w11 w12 w13]
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used	[w51 w52 w53]

SG Calculations

- For input x with xk=1 and xk'=0 for all k'≠k, the outputs of the hidden layer will be row k in W: $\mathbf{h} = \mathbf{x}^T\mathbf{W} = \mathbf{W}_{(k,.)} := \mathbf{v}_{w_I}$
- The input to the jth node of the cth output word is: $u_{c,j} = \mathbf{v'}^T_{w_j}\mathbf{h}$ jth column in W' matrix
- Since the output layers for each output word share the same weights so $u_{c,j} = u_j$
- Computing the jth node of the cth output word via softmax function:

$$p(w_{c,j} = w_{O,c}|w_I) = y_{c,j} = \frac{exp(u_{c,j})}{\sum_{j'=1}^{V} exp(u_{j'})}$$

This is the probability that the output of the jth node of the cth output is equal to the actual value of the jth index of the cth output vector (one-hot encoded)

- This way the forward pass is done so next the weights W and W' need to be learned with backpropagation and stochastic gradient descent.

SG Calculations

- First, a loss function is defined as:

$$E = -\log p(w_{O,1}, w_{O,2}, \cdots, w_{O,C} | w_I)$$
$$= \boxed{-\log \prod_{c=1}^{C}} \frac{exp(u_{c,j_c^*})}{\sum_{j'=1}^{V} exp(u'_j)}$$
$$= -\sum_{c=1}^{C} u_{j_c^*} + C \cdot \log \sum_{j'=1}^{V} exp(u_{j'})$$

$$-\sum_{c=1}^{C} log$$

Negative log likelihood
→ objective is to minimize it

The probability of the output words (context words) given the input (target/middle) word

jc* is the index of the cth output

- Take the derivative w.r.t. uc,j:

$$\frac{\partial E}{\partial u_{c,j}} = y_{c,j} - t_{c,j}$$

tc,j=1 if the jth node of the cth true output word is equal to 1 and 0 other wise.

How is this computed??

- Compute the derivative w.r.t. W':

$$\frac{\partial E}{\partial w'_{ij}} = \sum_{c=1}^{C} \frac{\partial E}{\partial u_{c,j}} \cdot \frac{\partial u_{c,j}}{\partial w'_{ij}}$$
$$= \sum_{c=1}^{C} (y_{c,j} - t_{c,j}) \cdot h_i$$

Using the chain rule

- The gradient descent update equation for W':

$$w'^{(new)}_{ij} = w'^{(old)}_{ij} - \eta \cdot \sum_{c=1}^{C} (y_{c,j} - t_{c,j}) \cdot h_i$$

SG Calculations

- Computing for W, begin with taking the derivative of the error w.r.t. hi:

$$\frac{\partial E}{\partial h_i} = \sum_{j=1}^{V} \frac{\partial E}{\partial u_j} \cdot \frac{\partial u_j}{\partial h_i}$$

Using the chain rule

$$= \sum_{j=1}^{V} \sum_{c=1}^{C} (y_{c,j} - t_{c,j}) \cdot w'_{ij}$$

- Compute the derivative w.r.t. W:

$$\frac{\partial E}{\partial w_{ki}} = \frac{\partial E}{\partial h_i} \cdot \frac{\partial h_i}{\partial w_{ki}}$$

Using the chain rule

$$= \sum_{j=1}^{V} \sum_{c=1}^{C} (y_{c,j} - t_{c,j}) \cdot w'_{ij} \cdot x_k$$

- The gradient descent update equation for W:

$$w_{ji}^{(new)} = w_{ji}^{(old)} - \eta \cdot \sum_{j=1}^{V} \sum_{c=1}^{C} (y_{c,j} - t_{c,j}) \cdot w'_{ij} \cdot x_j$$

- Matrix forms:

[V*N]=[V*N]-[V*1] ([N*V][V*1])^T

$$W_{input}^{(new)} = W_{input}^{(old)} - \eta \cdot x \cdot (W_{output} \sum_{c=1}^{C} e_c)^T$$

[N*V]=[N*V]-[N*1] [1*V]

$$W_{output}^{(new)} = W_{output}^{(old)} - \eta \cdot h \cdot \sum_{c=1}^{C} e_c$$

Each gradient descent update requires a **summation over the entire vocabulary →** **Computationally expensive**. So techniques such as hierarchical softmax and negative sampling are used to make this computation more efficient.

Other Kinds of Static Embeddings

- FastText
- GloVe
- Sphere

FastText

- An extension of word2vec developed by *Piotr Bojanowski et al. at Facebook*.
- Addresses a problem with word2vec "**unknown words**"—words that appear in a test corpus but were unseen in the training corpus.
- A related problem is **word sparsity**, such as in languages with rich morphology, where some of the many forms for each noun and verb may only occur rarely.

- FastText deals with these problems by using **subword models**, representing each word as **itself plus a bag of constituent n-grams**, with special boundary symbols < and > added to each word.
- For example, with n = 3 the word *where* would be represented by the sequence *<where>* plus the character n-grams: *<wh, whe, her, ere, re>*
  - Note that the sequence *<her>,* corresponding to the word *her* is different from the tri-gram *her* from the word *where*.
- Now, the vocabulary is the union of the subwords of all words (subwords refer to the words themselves + n-grams). Then a skip-gram/cbow embedding is learned for each subword.
- The vector of a word is the sum of the whole word vector in addition to the sum of its subwords' vectors.

- Unknown words can then be presented only by the sum of the constituent n-grams.

GloVe

- Short for Global Vectors, because the model is based on capturing global corpus statistics developed by *Jeffrey Pennington et al. at Stanford University*.

- Unlike word2vec that captures local statistics of a corpus, GloVe captures both local and global statistics of a corpus.
  - In word2vec, the vector learnt for a certain word is only affected by the surrounding context words.
  - In GloVe, the global statistics of the corpus are considered as it operates on **co-occurrence matrices** that are built across the entire corpus.

- GloVe is based on **ratios of probabilities** from the word-word co-occurrence matrix.

- GloVe is a global log-bilinear regression model.

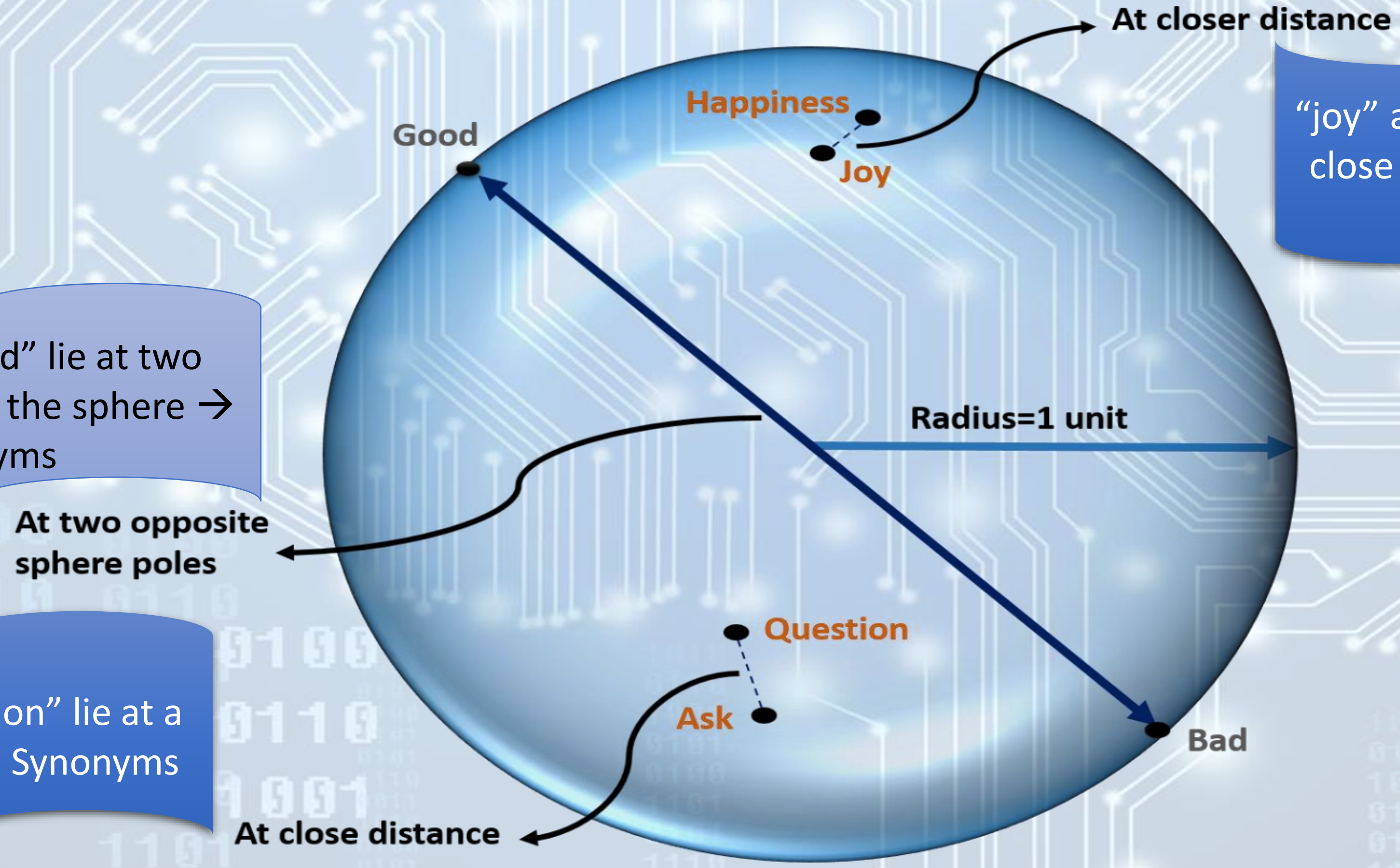
Sphere

- The approaches such as word2vec, fastText and GloVe are **distributional** approaches:
  - Antonyms *(good, bad)* can lie in close proximity **(cosine similarity close to 1)** → as antonyms can often occur in similar contexts.
  - Synonyms *(happiness, joy)* lie in close proximity **(cosine similarity close to 1).**
  - Unrelated words *(car, apple)* lie far away **(cosine similarity close to 0).**
  - Other antonyms can lie far away **(cosine similarity close to 0).**

There is **no clear distinction** between the different semantic relations among words such as synonyms, antonyms and unrelated words.

Sphere

- Sphere approach developed by *Sandra Rizkallah et al. at Cairo University* solves this problem by embedding the vectors of opposite or antonyms words at opposite poles of the sphere **(cosine similarity close to -1)**.
- Sphere is a semi-supervised relaxation algorithm that optimizes a devised error function.



“joy” and “happiness” lie at a close distance → Synonyms

“good” and “bad” lie at two opposite poles of the sphere → Antonyms

“ask” and “question” lie at a close distance → Synonyms

Evaluating Vector Models

- The most important evaluation metric for vector models is **extrinsic** evaluation on tasks.

- It is also useful to have **intrinsic** evaluations:
  1. **Similarity**: computing the correlation between an algorithm's word similarity scores and word similarity ratings assigned by humans.

  2. **Semantic Textual Similarity**: evaluates the performance of sentence-level similarity algorithms, consisting of a set of pairs of sentences, each pair with human-labeled similarity scores.

  3. **Analogy**: the system has to solve problems of the form *"a is to b as c is to ___" e.g.: "Cairo is to Egypt as Rome is to ___"* → *"Italy"*. Need to find *d* such that the associated word vectors *va, vb, vc, vd* are related to each other in the following relationship:

     "vb – va = vd – vc" where the *vd* is obtained to be the vector with highest cosine similarity to the vector *(vb-va+vc).*

Thank You