
Cognitive Robotics

05. Probabilistic Sensor Models

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Acknowledgment

- These slides have been created by Wolfram Burgard, Dieter Fox, Cyrill Stachniss and Maren Bennewitz

State Estimation

- Estimate the state x of a system given observations z and actions u
- **Goal:** Determine $p(x | z, u)$

Recursive Bayes Filter (recap)

$$\begin{aligned} bel(x_t) &= p(x_t \mid z_{1:t}, u_{1:t}) \\ &= p(z_t \mid x_t, z_{1:t-1}, u_{1:t}) p(x_t \mid z_{1:t-1}, u_{1:t}) / p(z_t \mid z_{1:t-1}, u_{1:t}) \\ &= \eta p(z_t \mid x_t) p(x_t \mid z_{1:t-1}, u_{1:t}) \\ &= \eta p(z_t \mid x_t) \int p(x_t \mid x_{t-1}, z_{1:t-1}, u_{1:t}) \\ &\quad p(x_{t-1} \mid z_{1:t-1}, u_{1:t}) dx_{t-1} \\ &= \eta p(z_t \mid x_t) \int p(x_t \mid x_{t-1}, u_t) p(x_{t-1} \mid z_{1:t-1}, u_{1:t}) dx_{t-1} \\ &= \eta p(z_t \mid x_t) \int p(x_t \mid x_{t-1}, u_t) p(x_{t-1} \mid z_{1:t-1}, u_{1:t-1}) dx_{t-1} \\ &= \underline{\eta p(z_t \mid x_t)} \int \underline{p(x_t \mid x_{t-1}, u_t)} \underline{bel(x_{t-1})} dx_{t-1} \end{aligned}$$

observation model motion model recursive term

bnshof eh e7tmal en el z de
tetl3 given en el state xt 7asal.

Iw hya likely, fa handeha
kema kbera, lagn iw l 3ks
kemtha htba oyla, w da
by5lene a2dr a3ml correction
lel model bta3y,

Motion and Observation Model

- Prediction step

$$\overline{bel}(x_t) = \int \underline{p(x_t \mid u_t, x_{t-1})} \ bel(x_{t-1}) dx_{t-1}$$

motion model

- Correction step

$$bel(x_t) = \eta \underline{p(z_t \mid x_t)} \overline{bel}(x_t)$$

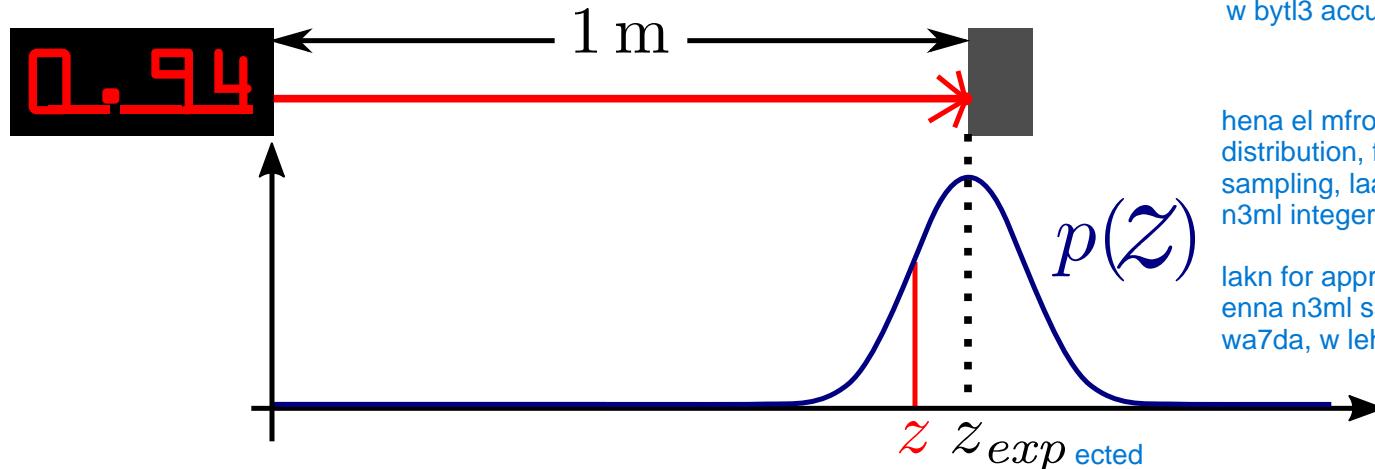
sensor or observation model

Previous Lecture: Probabilistic Motion Models

- Robots execute motion commands only inaccurately
- The motion model specifies the probability that action u_t carries the robot from pose x_{t-1} to x_t :
$$p(x_t \mid u_t, x_{t-1})$$
- Defined individually for each type or robot

Measurement Probability

el sensor hena ara 0.93 badal el 1m, 34an byb2a feh noise.



el errors fl sensors, dayman btb2a a2al bkter gedan mn el errors fl actuators.

7ata m3 el sensors el t3bana, brdu el filters btb2a shaghaala w bytl3 accuracy kwysa,

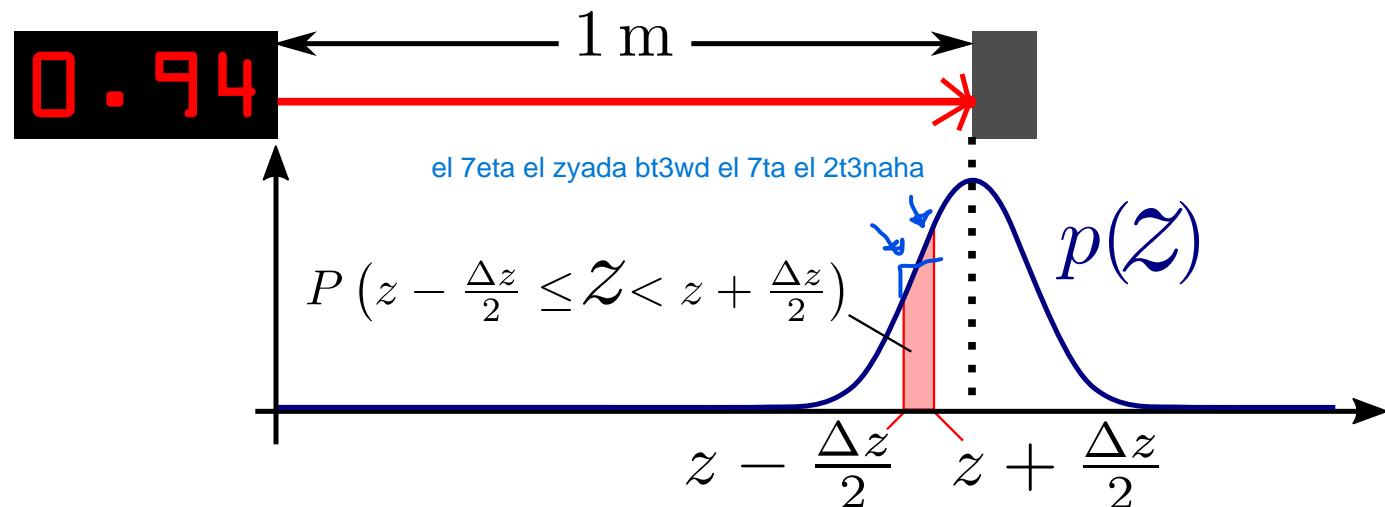
hena el mfrod da continous distribution, fa el mfrod mn3mlsh sampling, laa nakhud range w n3ml integeration 3leh.

Iakn for approximation, btmshy enna n3ml sampling 3la point wa7da, w leha proof.

bn3br 3n el noise b gauss distribution.

$$\begin{aligned} P(Z = z) &= 0 \\ &= P(Z = 0.9400000 \dots) \end{aligned}$$

Measurement Probability



$$P(0.935 \leq Z < 0.945) = P\left(z - \frac{\Delta z}{2} \leq Z < z + \frac{\Delta z}{2}\right)$$

$$= \int_{z - \frac{\Delta z}{2}}^{z + \frac{\Delta z}{2}} p(Z) dZ$$

$$\approx \Delta z \cdot p(Z) \quad (\text{for small } \Delta z)$$

Continuous vs. Discretized Random Variables

- Z is a **continuous** random variable with the probability density function

$$p(Z) = \lim_{\Delta z \rightarrow 0} \frac{P\left(z - \frac{\Delta z}{2} \leq Z < z + \frac{\Delta z}{2}\right)}{\Delta z}$$

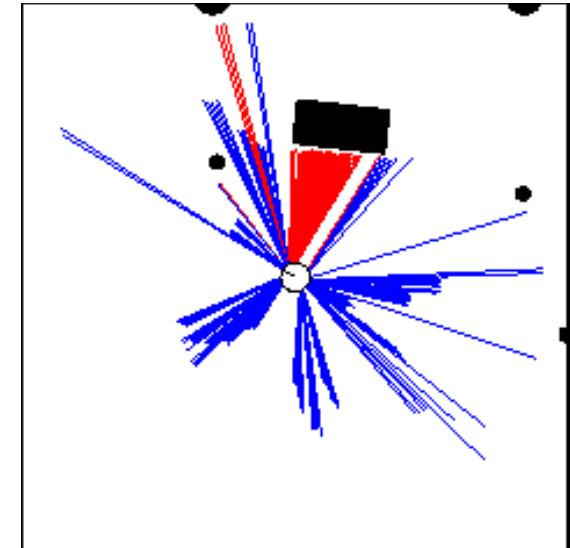
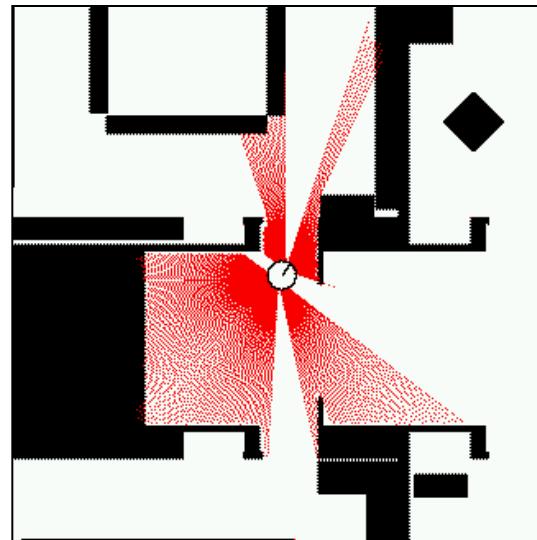
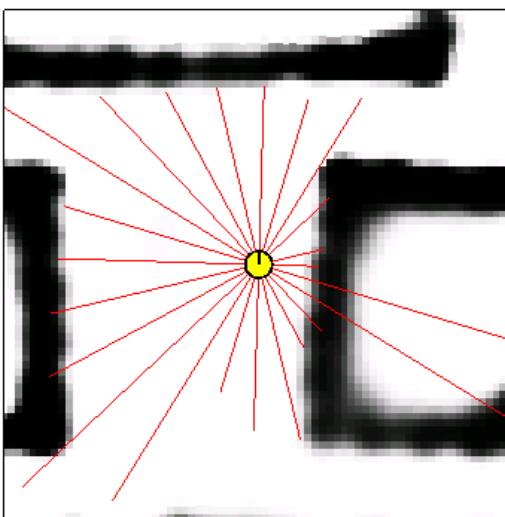
- We can only measure and represent Z in **discrete steps Δz**
- As Δz is constant for all measurements, we can ignore it when computing $bel(x_t)$ if we normalize $bel(x_t)$ in the end

el klam da valid in case of small delta z only, w bn7tag enna lazm n3ml normalization lama nkhs.

Sensors for Mobile Robots

- Proprioceptive sensors:
 - Accelerometers
 - Gyroscopes
 - Compasses
- Typical proximity sensors:
 - Sonars
 - Laser range-finders
- Visual sensors:
 - (Stereo) Cameras
 - Structured light (RGBD cameras)
- Infrastructure-based sensors: GPS, WLAN

Proximity Sensors



Question: How can we calculate the likelihood of such a measurement given the robot pose?

Beam-Based Sensor Model

- Sensor data consists of K measurements

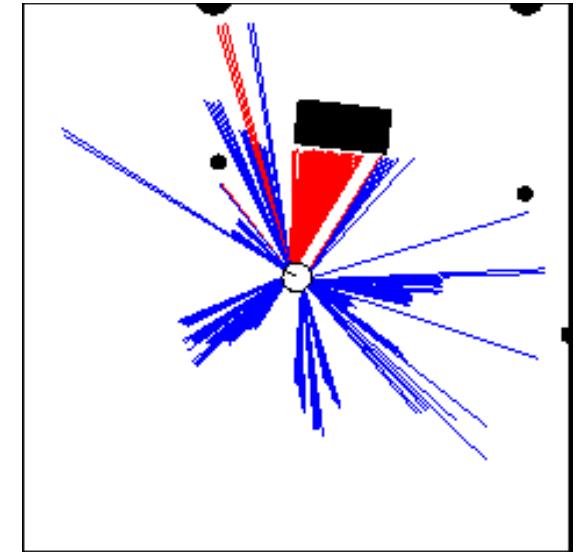
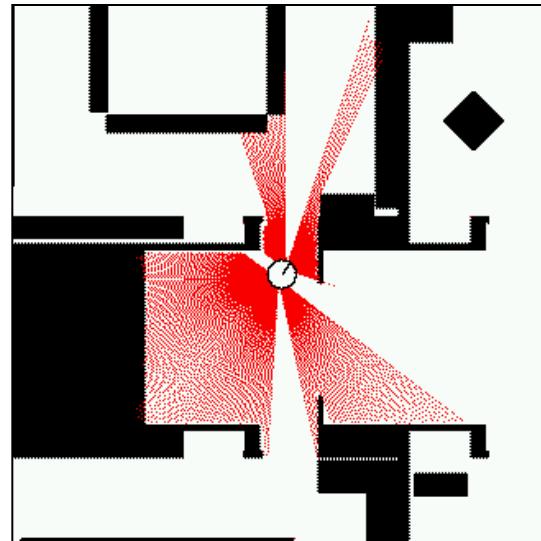
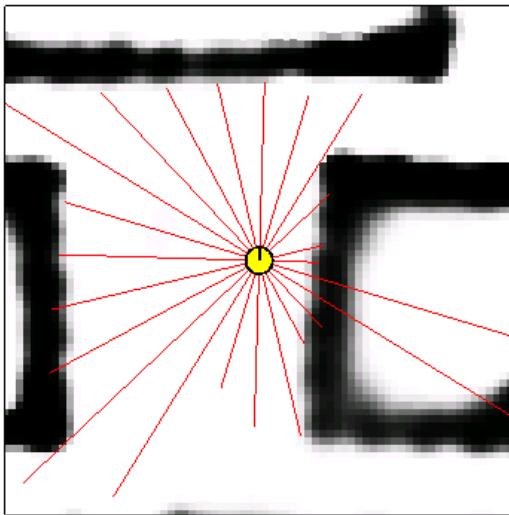
$$z = \{z_1, \dots, z_K\}$$

- Assumption: The individual measurements are independent given the robot's pose:

$$p(z \mid x, m) = \prod_{k=1}^K p(z_k \mid x, m)$$

- "How well can the distance measurements be explained given the pose (and the map)"

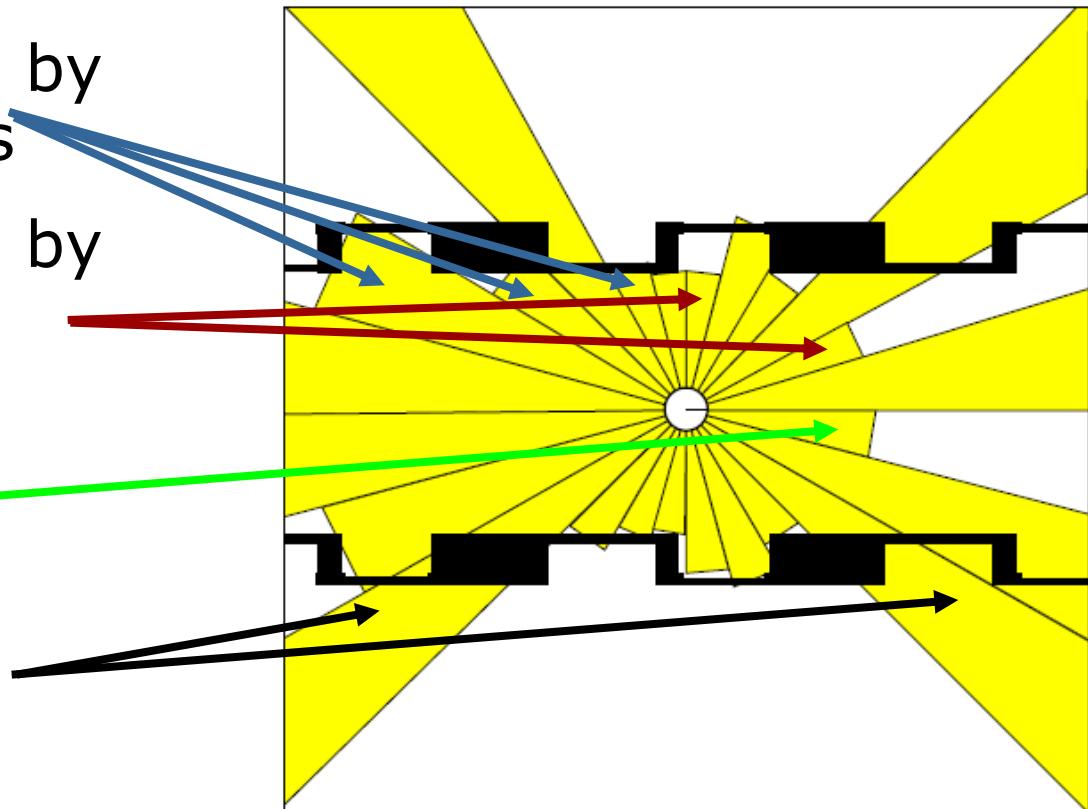
Beam-Based Sensor Model



$$p(z \mid x, m) = \prod_{k=1}^K p(z_k \mid x, m)$$

Typical Measurement Errors of Range Measurements

1. Beams reflected by known obstacles
2. Beams reflected by people / objects
3. Random measurements
4. Maximum range measurements

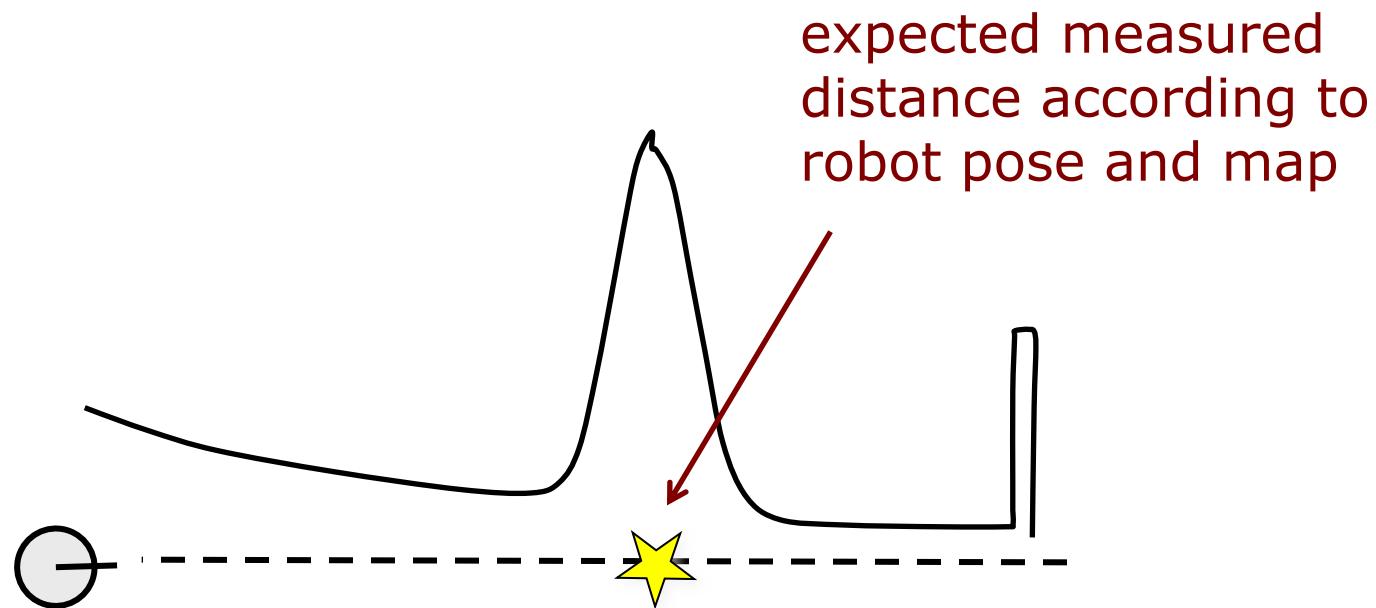


Proximity Measurements

- **A measurement can be caused by:**
 - a known obstacle
 - an unexpected obstacle (people, furniture, ...)
 - random measurements, **cross-talk** (sonars)
 - missing all obstacles
- **Noise is due to uncertainty:**
 - in measuring distance to known obstacle (sensor noise)
 - in the position of known obstacles ("map noise")
 - in the position of additional objects
 - whether an obstacle is missed

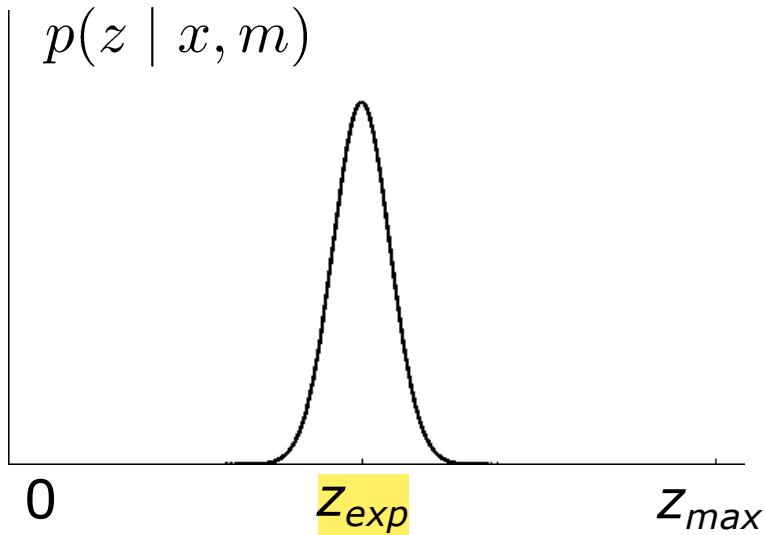
Beam-Based Proximity Model

- Considers the first obstacle along the line of sight
- Mixture of four components

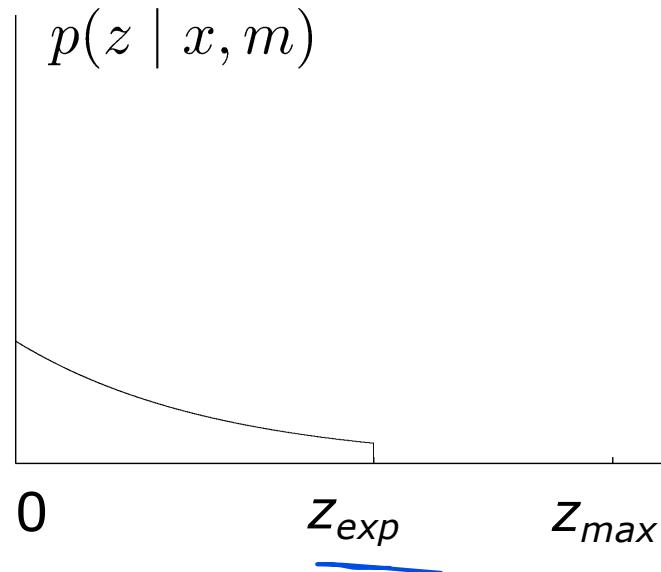


Beam-Based Proximity Model

Measurement noise



Unexpected objects



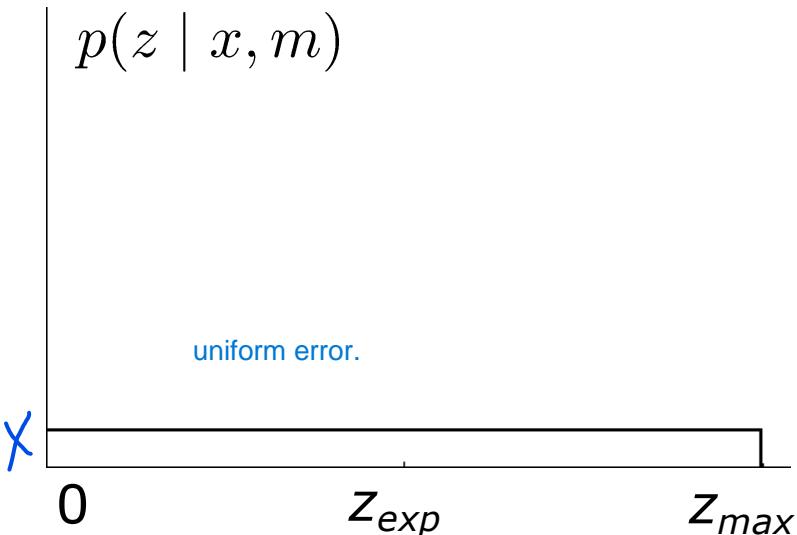
$$p_{hit}(z | x, m) = \begin{cases} \frac{\eta}{\sqrt{2\pi\sigma^2}} e^{-\frac{(z-z_{exp})^2}{2\sigma^2}} & \text{if } z \leq z_{max} \\ 0 & \text{otherwise} \end{cases}$$

$$p_{unexp}(z | x, m) = \begin{cases} \eta \lambda e^{-\lambda z} & \text{if } z < z_{exp} \\ 0 & \text{otherwise} \end{cases}$$

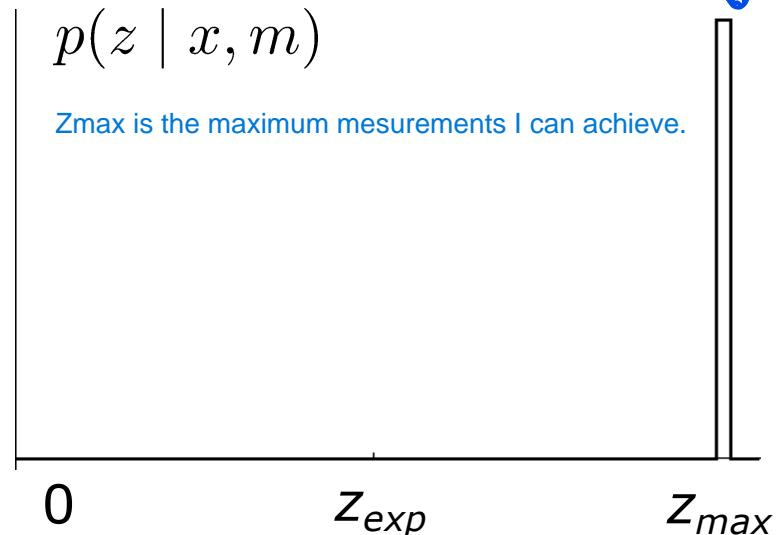
Note: The expected distance z_{exp} is computed by ray casting in the map, starting from the pose x

Beam-Based Proximity Model

Random measurement



Max range

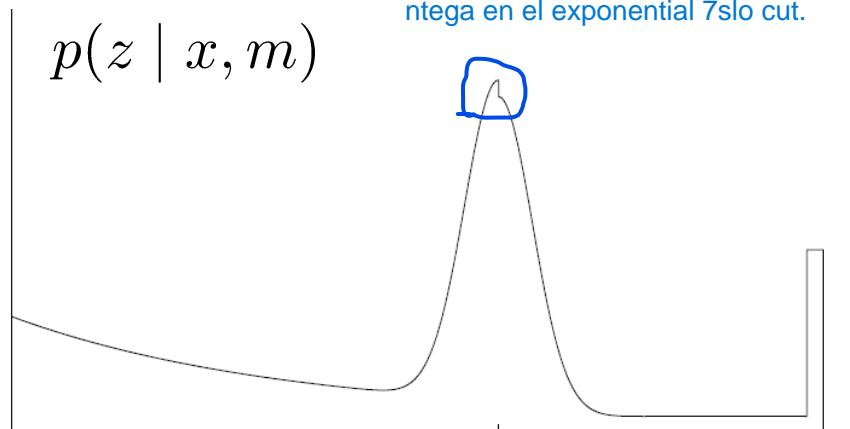


$$p_{rand}(z | x, m) = \begin{cases} \frac{1}{z_{max}} & \text{if } z < z_{max} \\ 0 & \text{otherwise} \end{cases}$$

$$p_{max}(z | x, m) = \begin{cases} \frac{1}{z_{small}} & \text{if } z \in [z - z_{small}, z_{max}] \\ 0 & \text{otherwise} \end{cases}$$

Note: The expected distance z_{exp} is computed by ray casting in the map, starting from the pose x

Resulting Mixture Density



e7na bn7tag net3lm el parameters bta3t el alpha, 34an a3rf on average eh el by7sl.

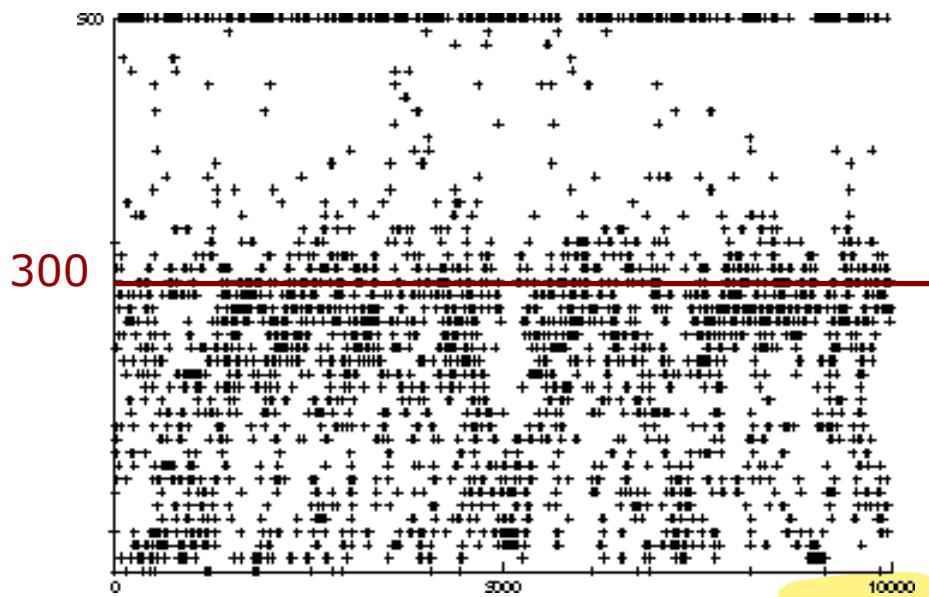
$$p(z | x, m) = \begin{pmatrix} \alpha_{\text{hit}} \\ \alpha_{\text{unexp}} \\ \alpha_{\text{max}} \\ \alpha_{\text{rand}} \end{pmatrix}^T \cdot \begin{pmatrix} p_{\text{hit}}(z | x, m) \\ p_{\text{unexp}}(z | x, m) \\ p_{\text{max}}(z | x, m) \\ p_{\text{rand}}(z | x, m) \end{pmatrix}$$

How can we determine the model parameters?

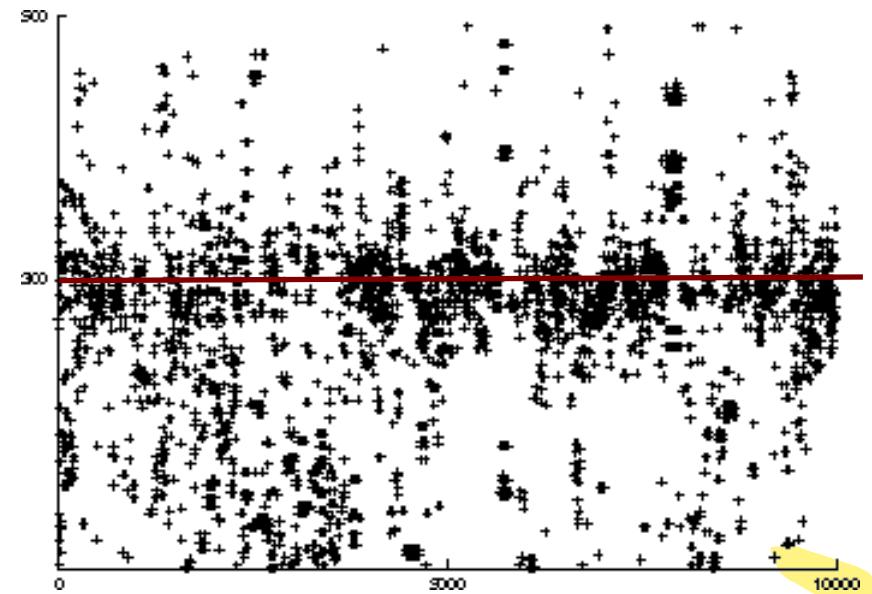
Raw Sensor Data

Measured distances for the “true” expected distance of 300 cm (maximum range 500 cm)

el sonar bytl3 max mesurement bnsba akbur bkteer mn el laser, natega eno msh akwa 7aga,



Sonar



Laser

fe 7alet el laser, el error bb2a a2al bkter.

bw2f el model odam 7eta maslan w bakhud 10000 eraya mokhtalefa w ashuf eh el 7sl.

Approximation

- Maximize log likelihood of the data

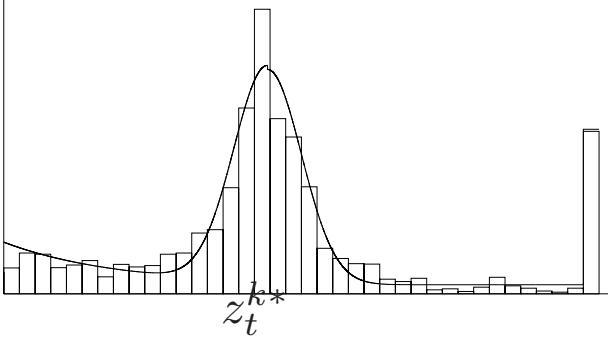
$$p(z \mid z_{\text{exp}})$$

- Search space of $n-1$ parameters
 - Hill climbing
 - Gradient descent
 - Genetic algorithms
 - ...
- Deterministically compute the n -th parameter to satisfy normalization constraint

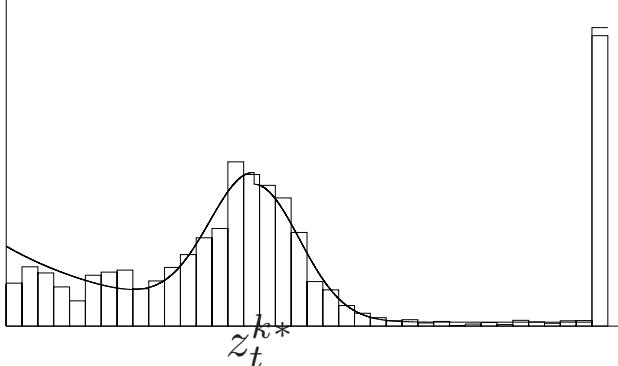
b3ml el 7sab da, 3la kol Zexp 3ndy, mn awl el 0 to Zexp.

Approximation Results

e7na el bn3mlo da, 34an net3lm el probability distribution 34an yeb2a 3ndk model te2dr test5dmo b3d keda.



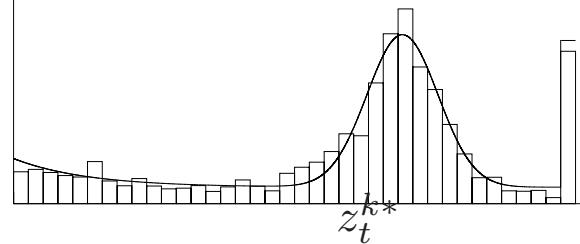
Laser



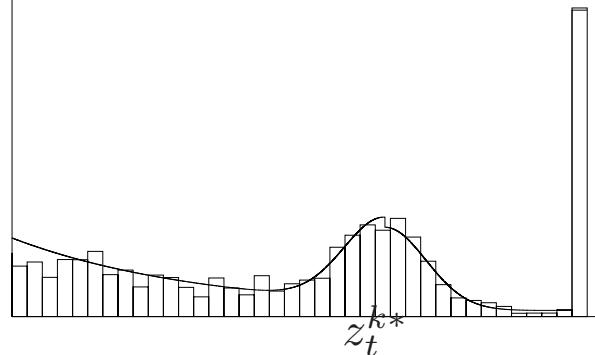
Sonar

300cm

kol da byb2a fl learning phase, w b3d keda bn7oto gowa look-up table, 34an lama ntl3o lel inference, lama agy a2es ay msafa, kol el b3mlo eny bro7 abos fl look-up table bta3y w ageb el corresponding probability.



lw el mesurement elly



400cm

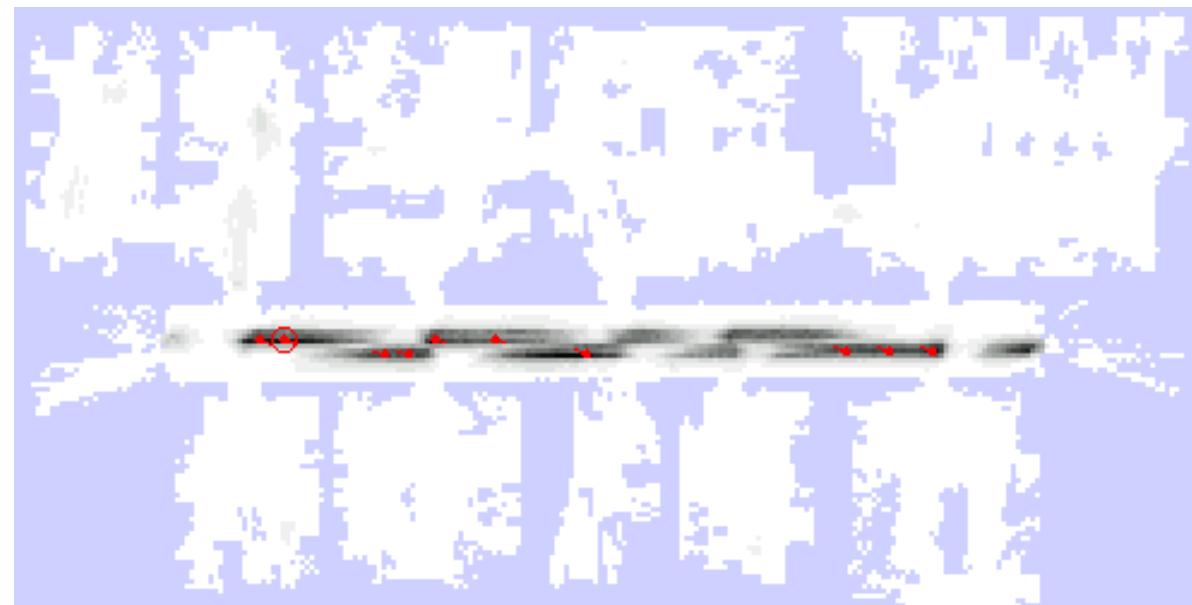
Example

da goz2 mn el map



z

de el map kamla.



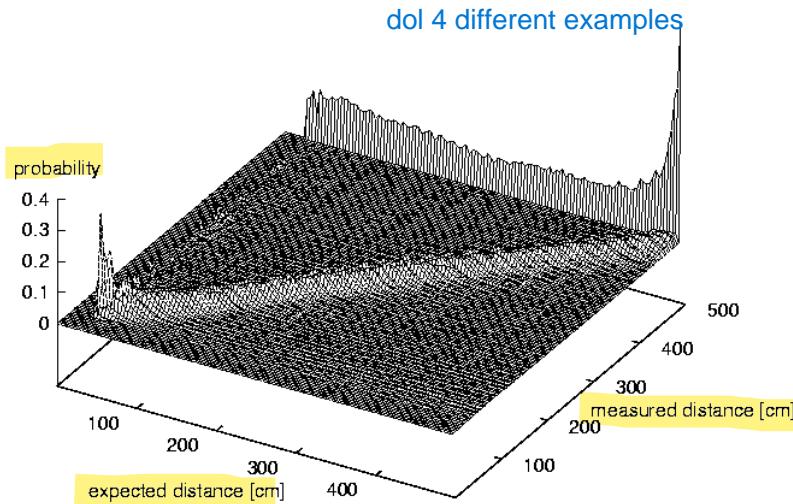
$$p(z|x,m)$$

fkeda el ana 2dert mn el mesurement elly m3aya el hya 3la el shmal, ageb el location
elly 3ndo most likelihood, enha ttl3 el part da mn el map.

w lama akhud el max, hytl3ly el no2ta el mt3lm 3leha b dayra 7amra de.

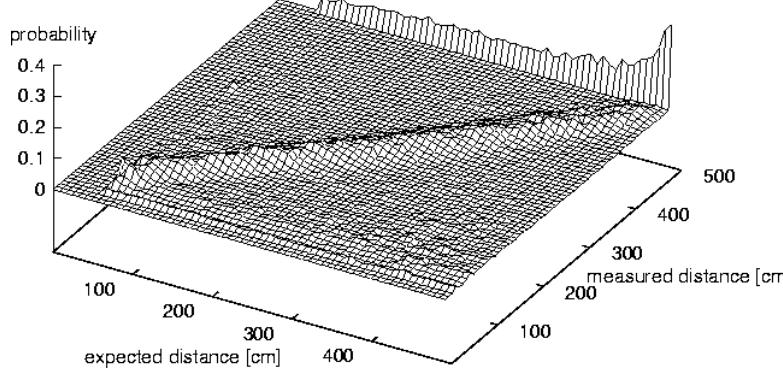
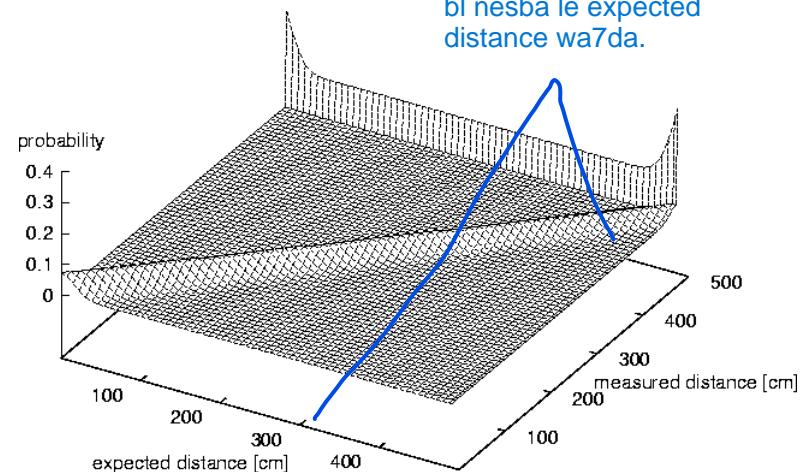
Approximation Results

dol 4 different examples

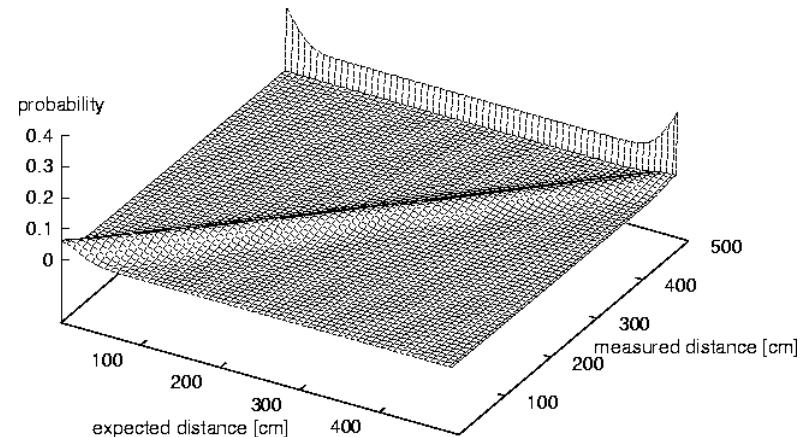


Laser

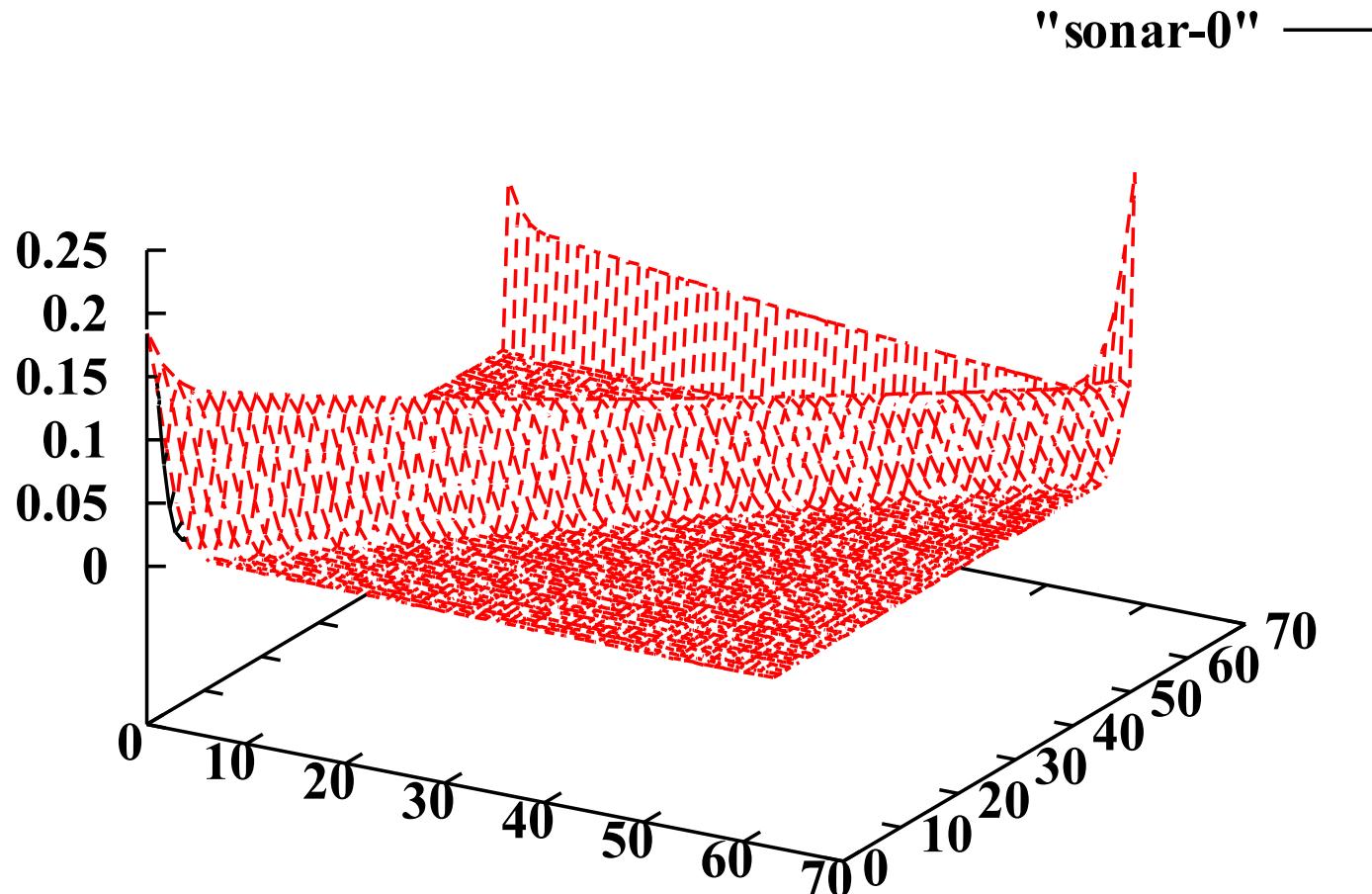
Iw 5dn4 el goz2 da ka cross-section, da keda akny bgeb kol el mesured distance bl nesba le expected distance wa7da.



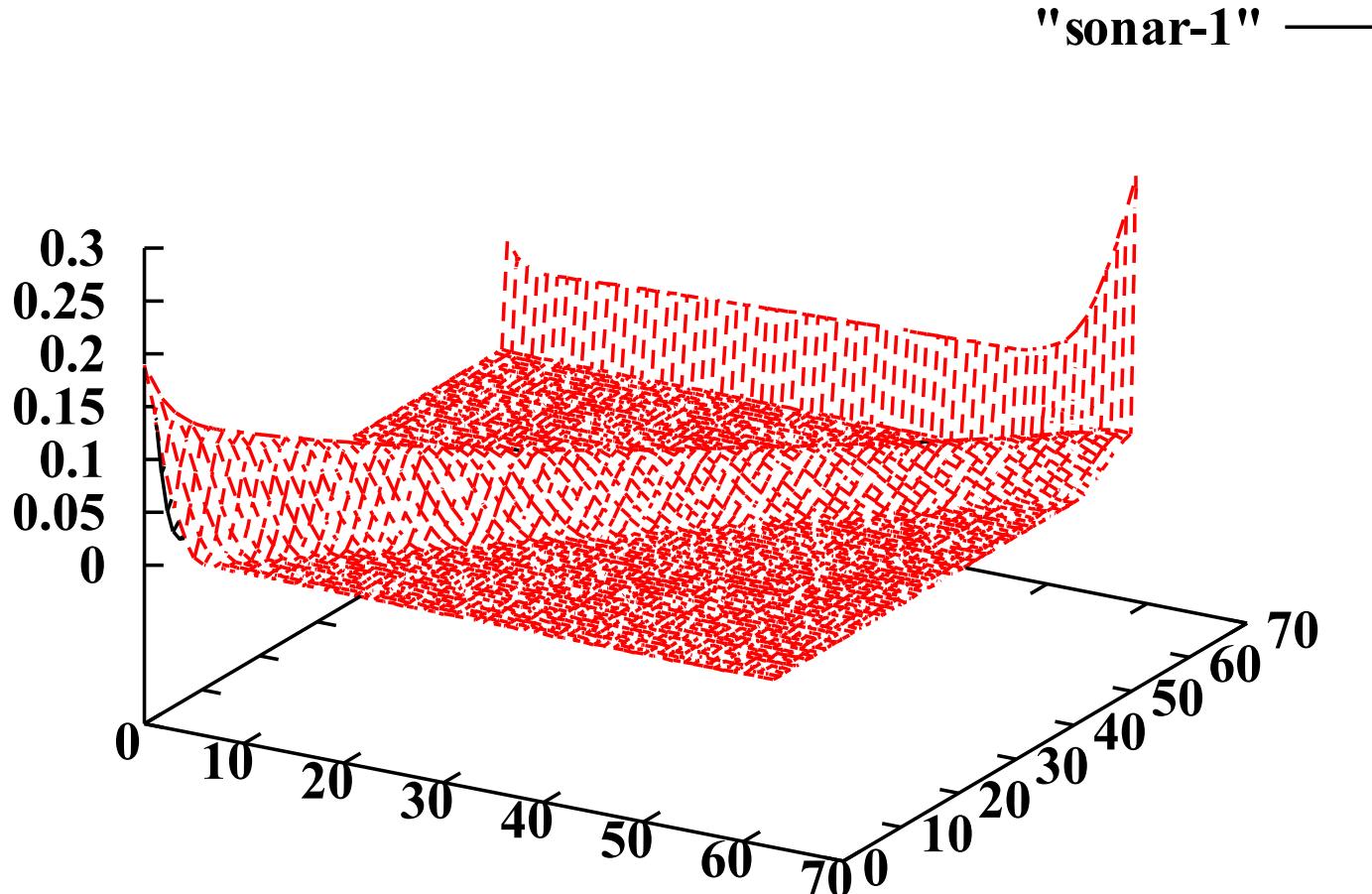
Sonar



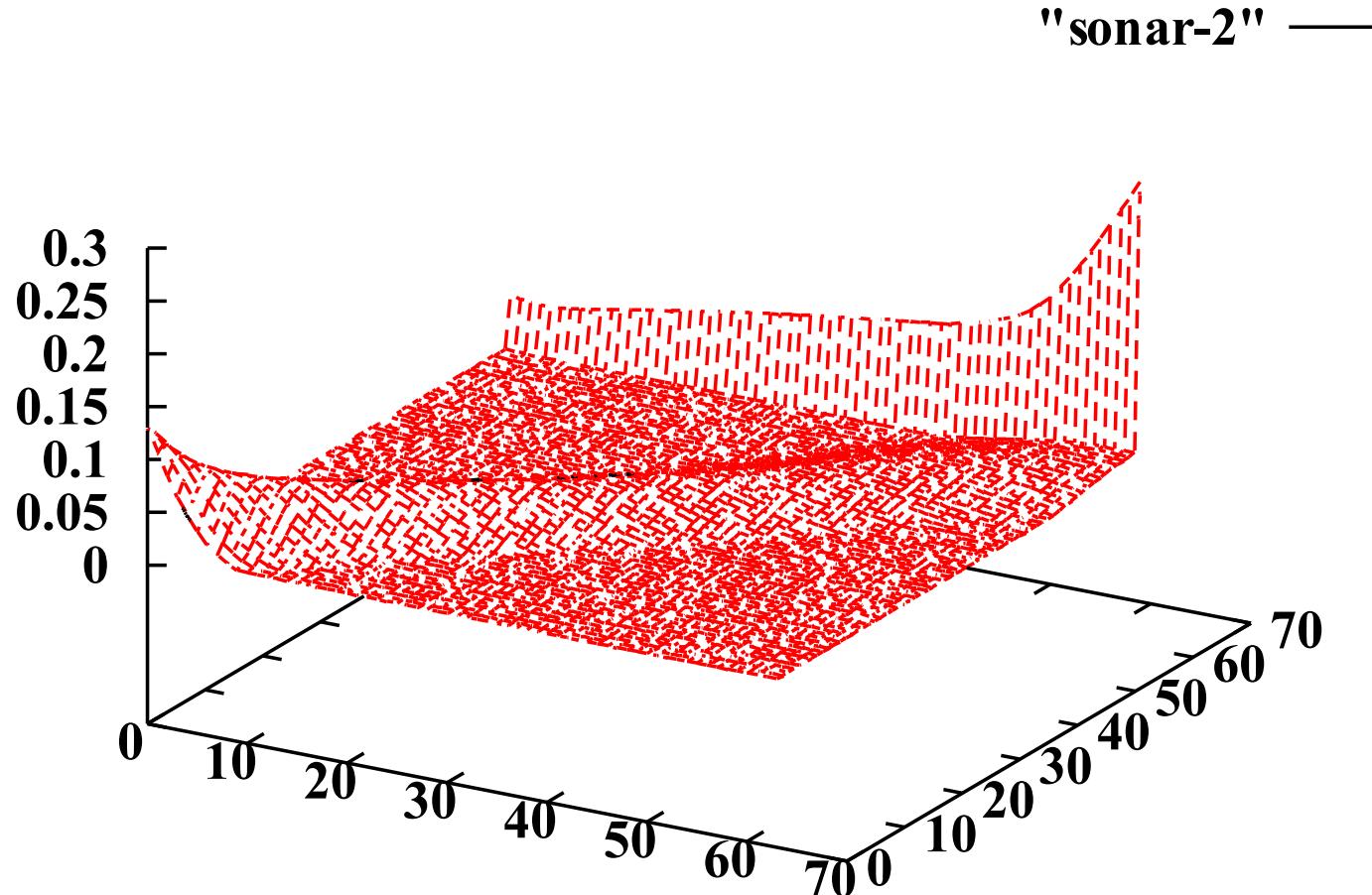
Influence of Angle to Obstacle



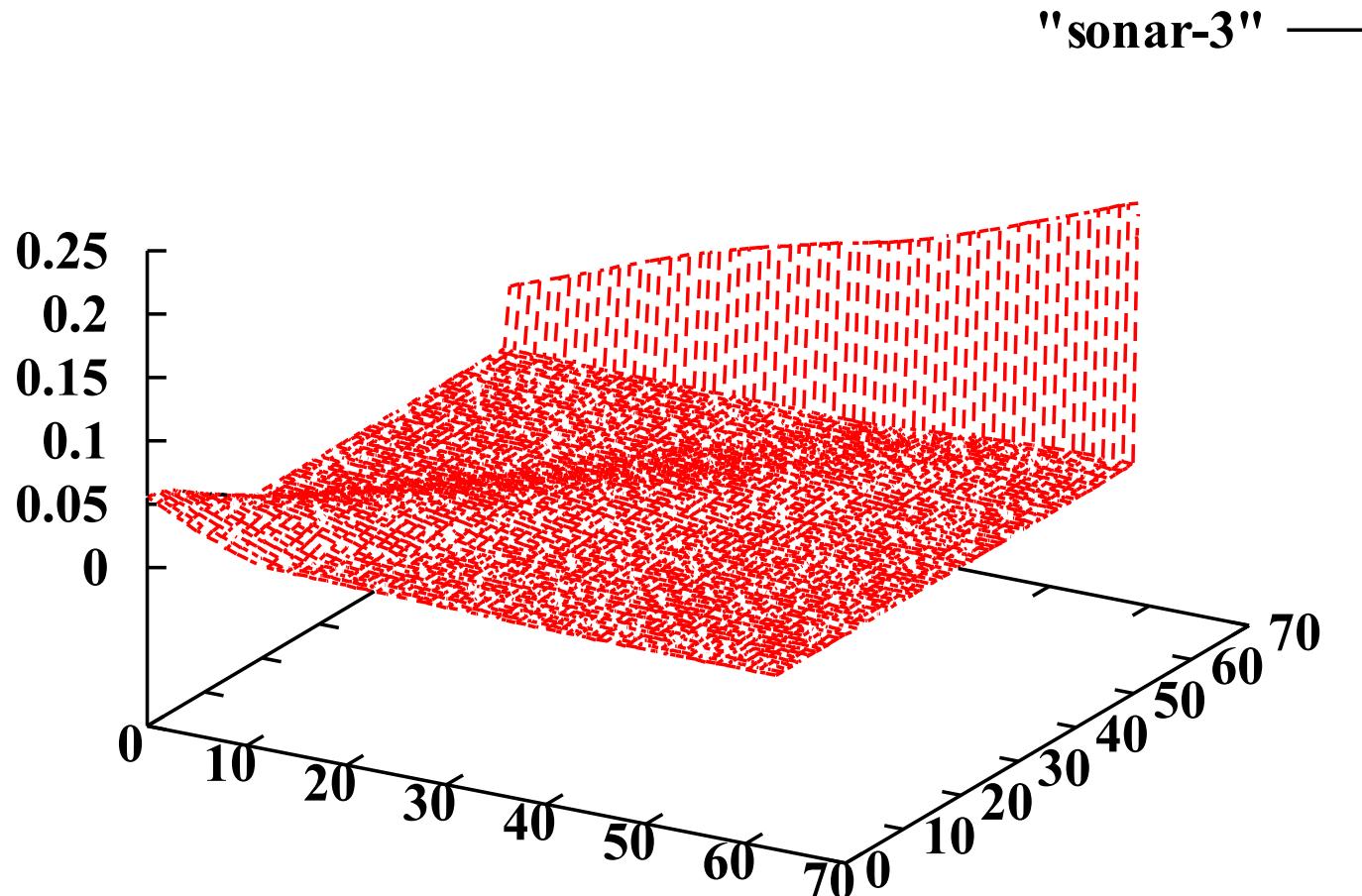
Influence of Angle to Obstacle



Influence of Angle to Obstacle



Influence of Angle to Obstacle



Summary Beam-Based Model (1)

- Assumes independence between beams
 - Justification?
 - Problem: Overconfident estimates
- Models physical causes for measurements
 - Mixture of densities for these causes
 - Assumes independence between causes

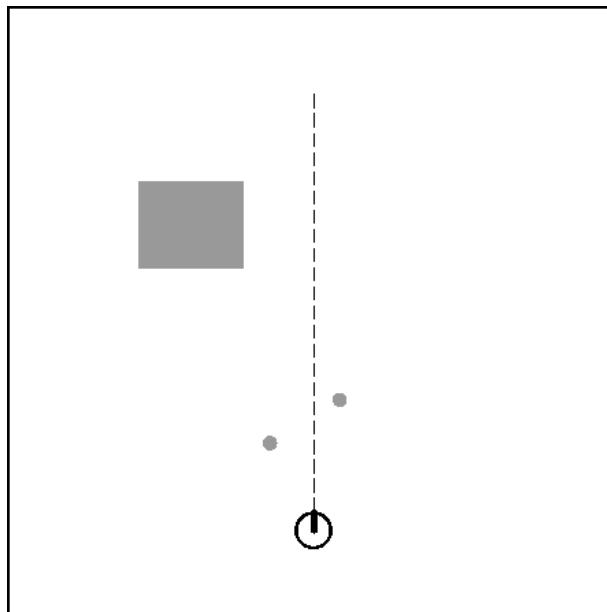
Summary Beam-Based Model (2)

- **Implementation**

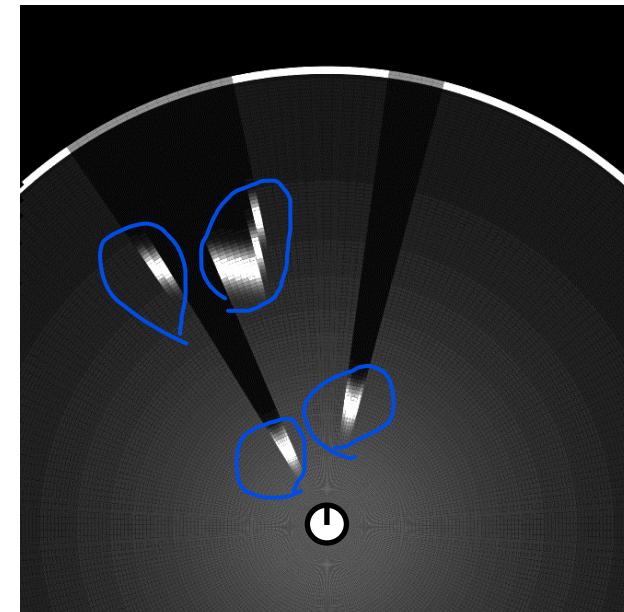
- Learn parameters based on real data
- Different models should be learned for different angles at which the sensor beam hits the obstacle (sonar)
- Determine expected distances by ray casting
- Expected distances can be pre-computed

Summary Beam-Based Model (3)

- **Disadvantages:** The beam-based model is
 - not smooth at edges
 - not very efficient (ray casting or precomputed lookup tables)



Map m



Likelihood field

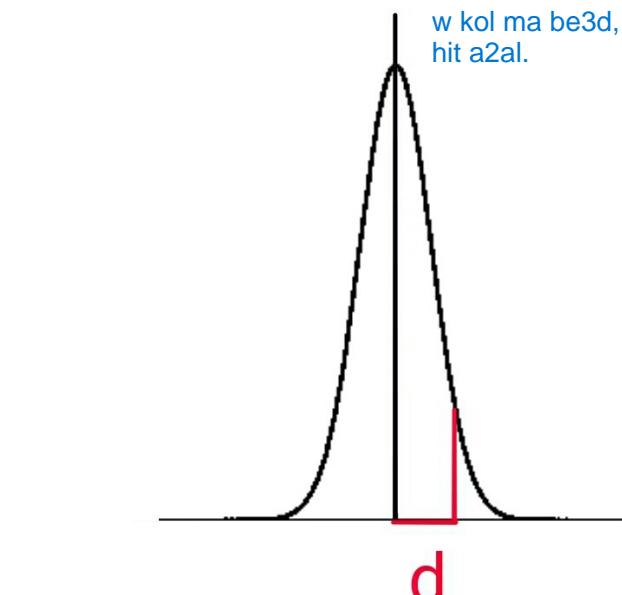
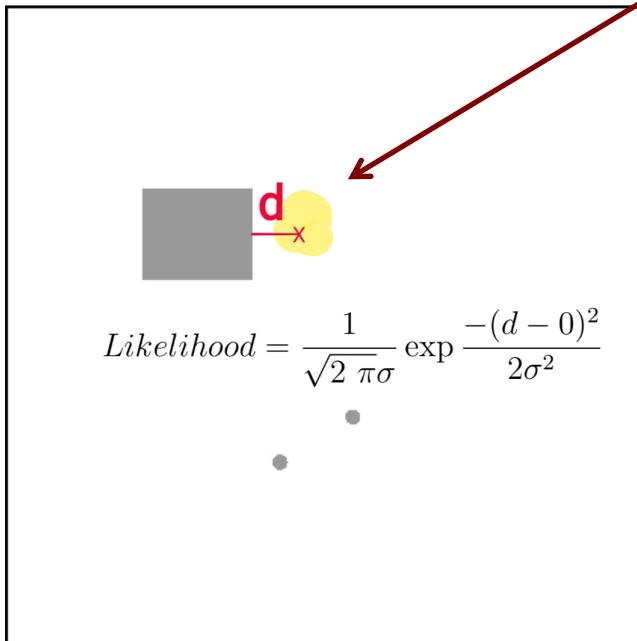
End-Point Model

- Idea of the end-point model: Instead of following along the beam, just check the end point of the beam
- Precompute a so-called likelihood field (distance grid)

End-Point Model

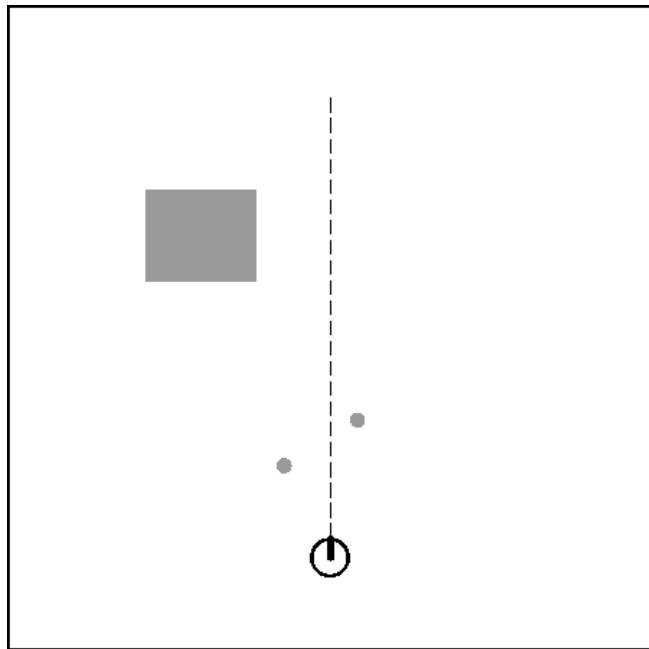
- Probability is a mixture of:
 - a Gaussian distribution evaluating the **distance to the closest obstacle**,
 - a uniform distribution for random measurements, and
 - a small uniform distribution for max range measurements
- Again, independence between different components is assumed

Gaussian Used within the Likelihood of a Measurement

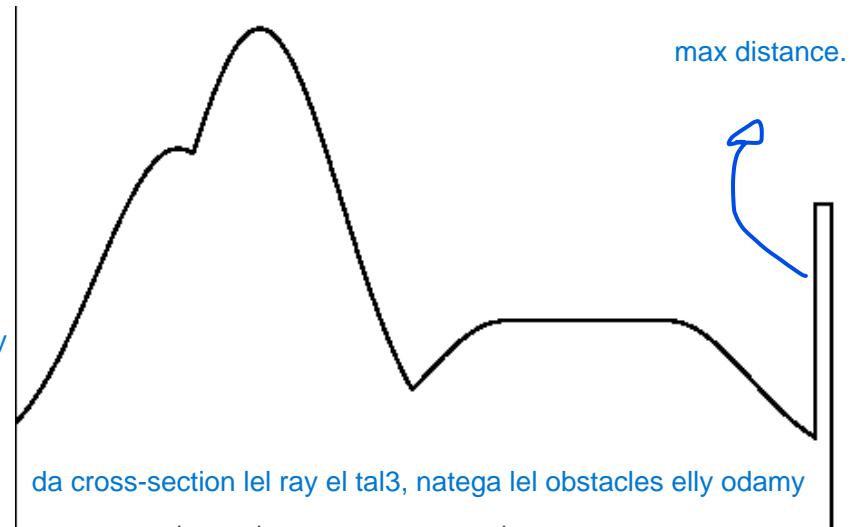
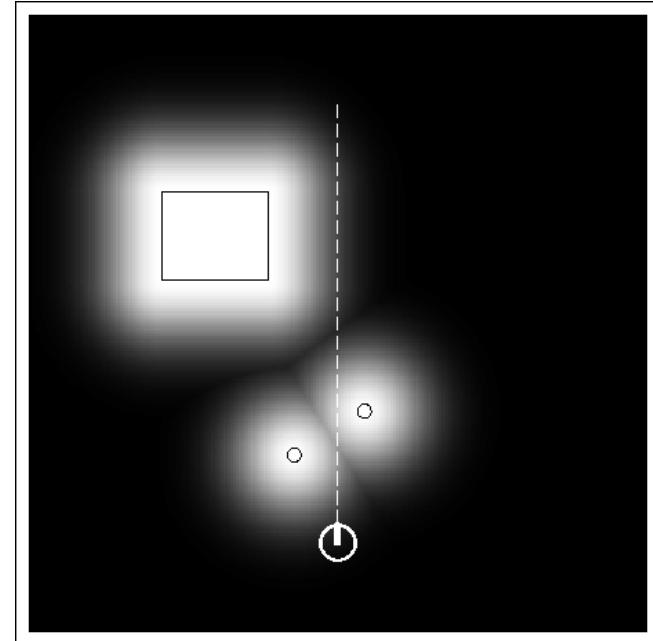


Gaussian to evaluate the distance

Example



Map m

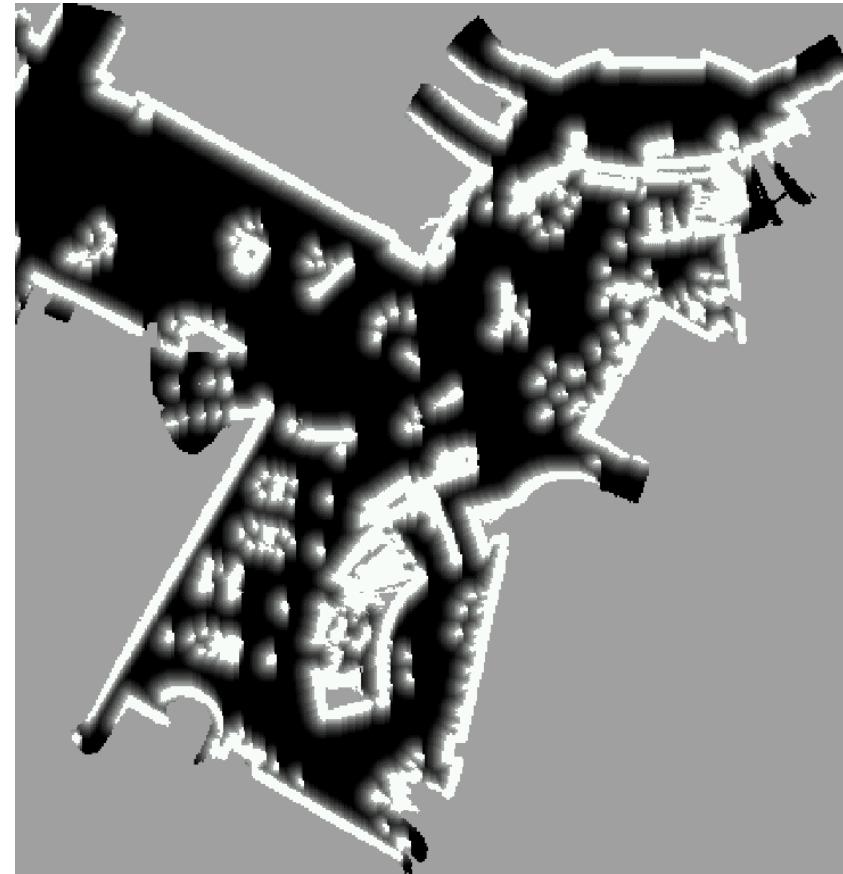


moshkelt el end-point eno msh by-consider el physical properties of objects, by-assume en el ray momken y5tarek el objects.
el mushkela fe keda, eno bybos 3la akher no2ta bs, msh el inerval, y3ny lama a2olo $z = 2$, byro7 ybos at $z = 2$, mlosh d3wa baa b-ay 7aga fl nos.

San Jose Tech Museum



Occupancy grid map



Likelihood field

Note: Precomputed
independently of robot pose

Properties End-Point Model

- Highly efficient, uses 2D tables only
- Distance grid is smooth w.r.t. to small changes in robot position
- Ignores physical properties of beams
- Treats sensor as if it can see through walls

hwa asr3 bs el accuracy a2al.

Landmarks

akny b3lm el amakn.

- Active **beacons** (e.g., radio, GPS)
- **Passive markers** (e.g., visual, retro-reflective)
- Standard approach: **triangulation**
- Sensor provides
 - distance, or
 - bearing, or
 - distance and bearing

Distance and Bearing



Probabilistic Model

1. Algorithm **landmark_detection_model(z,x,m)**:

$$z = \langle i, d, \alpha \rangle, x = \langle x, y, \theta \rangle$$

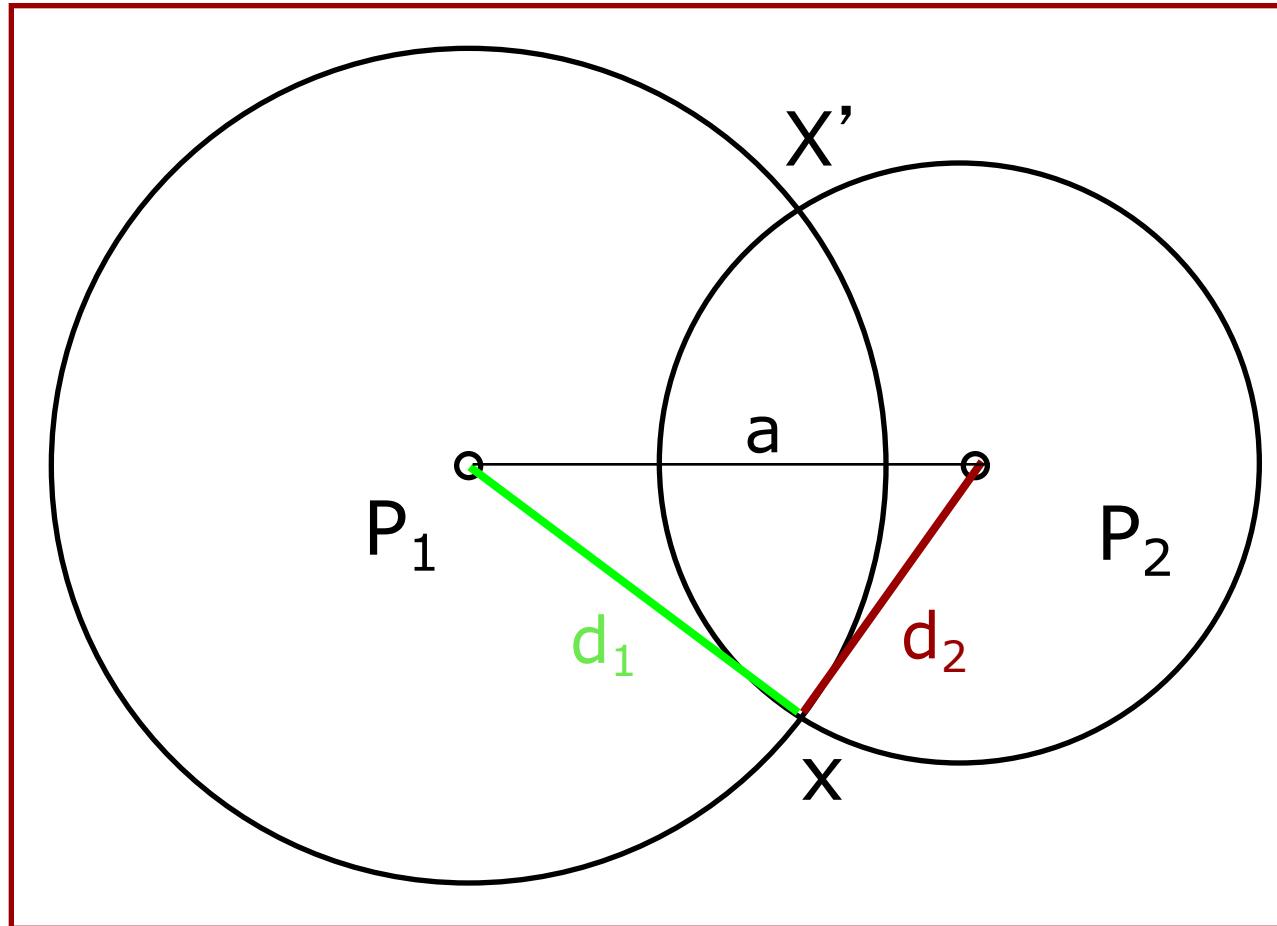
2. $\hat{d} = \sqrt{(m_x(i) - x)^2 + (m_y(i) - y)^2}$ ————— Expected distance
3. $\hat{\alpha} = \text{atan2}(m_y(i) - y, m_x(i) - x) - \theta$ ————— Expected angle

4. $p_{\text{det}} = \text{prob}(\hat{d} - d, \varepsilon_d) \cdot \text{prob}(\hat{\alpha} - \alpha, \varepsilon_\alpha)$ ————— Independence assumption, two Gaussians
5. Return p_{det}

Assumption: Correspondences are known

Distances Only No Uncertainty

$$x = (a^2 + d_1^2 - d_2^2) / 2a$$
$$y = \pm \sqrt{(d_1^2 - x^2)}$$

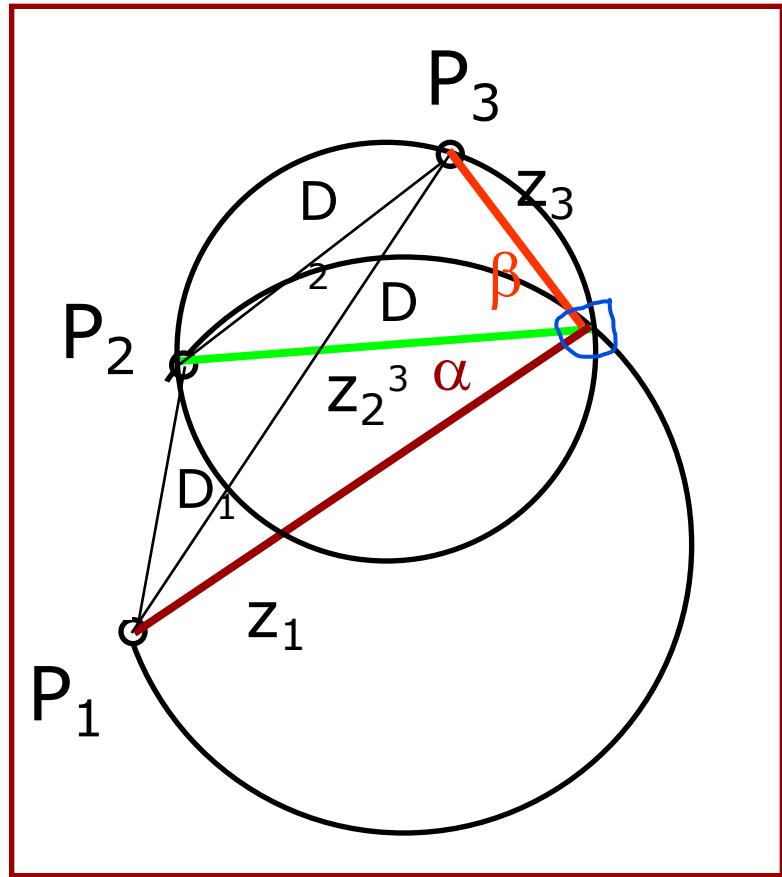
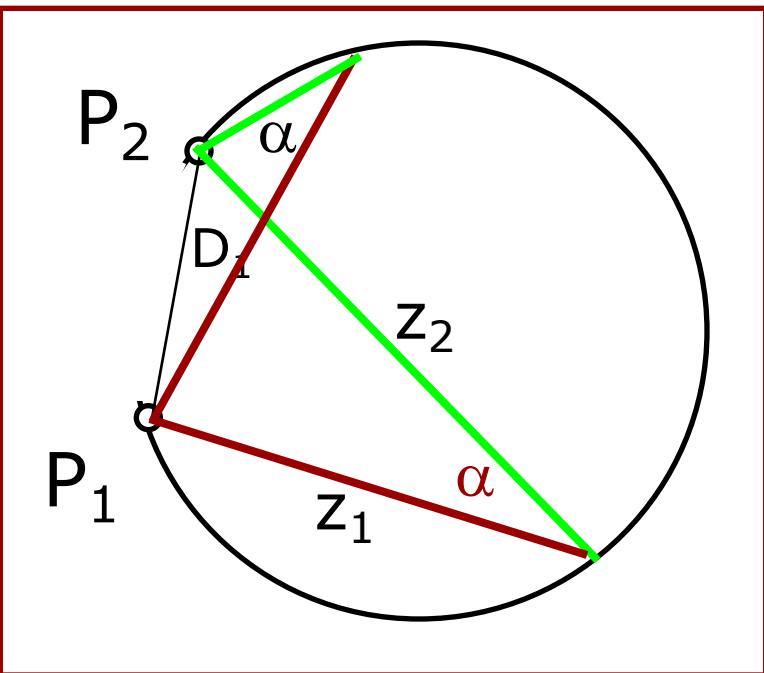


el amakn elly momken ab2a fehom hwa el intersections of two circles.

$$P_1 = (0,0)$$

$$P_2 = (a,0)$$

Bearings Only No Uncertainty



Law of cosine

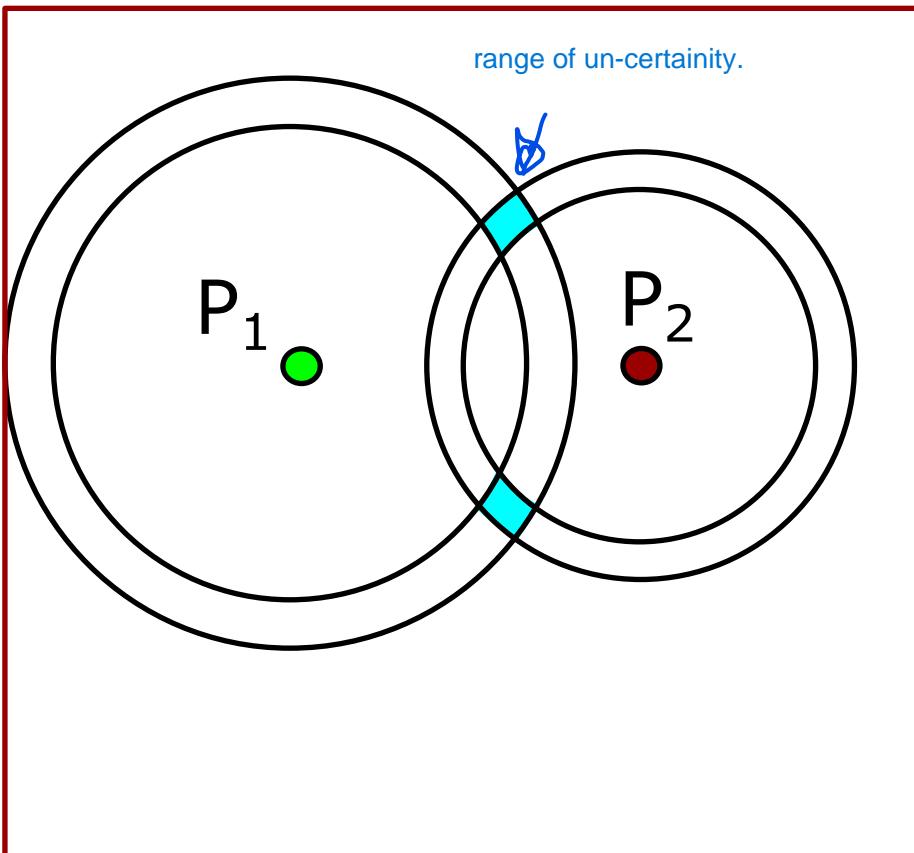
$$D_1^2 = z_1^2 + z_2^2 - 2 z_1 z_2 \cos \alpha$$

$$D_1^2 = z_1^2 + z_2^2 - 2 z_1 z_2 \cos(\alpha)$$

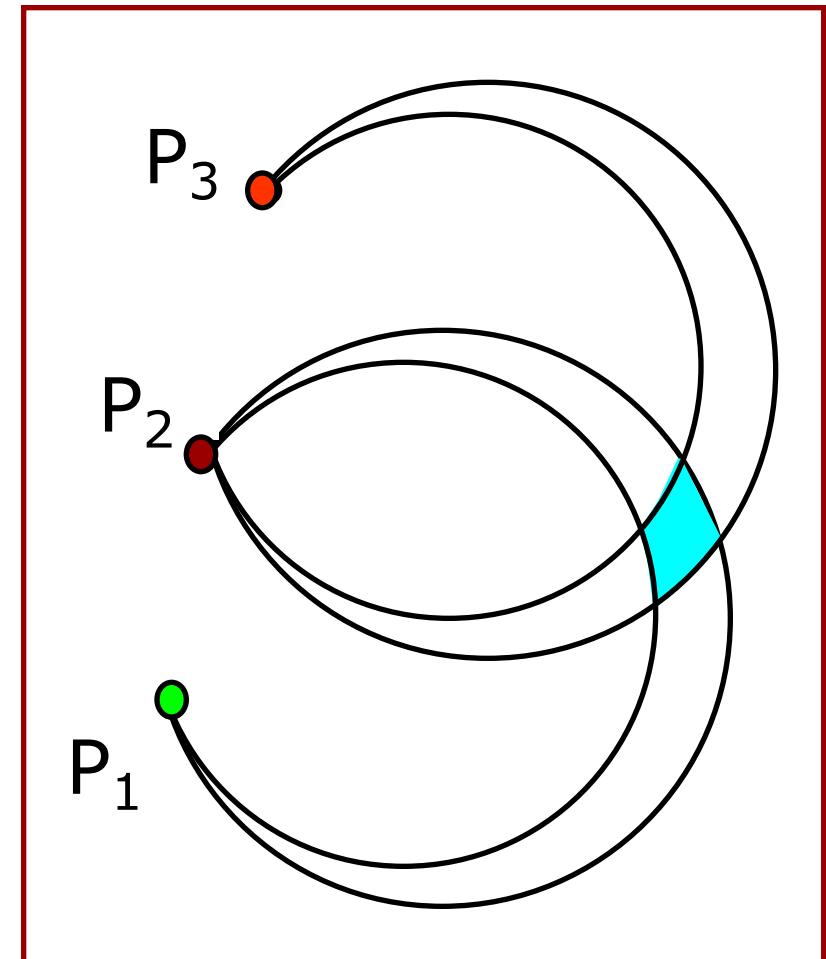
$$D_2^2 = z_2^2 + z_3^2 - 2 z_1 z_2 \cos(\beta)$$

$$D_3^2 = z_1^2 + z_3^2 - 2 z_1 z_2 \cos(\alpha + \beta)$$

Landmark Measurements with Uncertainty



Distance only



Bearings only

Summary (1)

- **Explicitly modeling uncertainty in sensing is key to robustness**
- In many cases, good models can be found by the following approach:
 - Determine the **parametric model** of a **noise-free measurement**
 - Analyze the **individual sources of noise**
 - Add **adequate noise** to parameters (add densities for noise)
 - **Learn parameters** by **fitting a model** to the **data**

Summary (2)

- The likelihood of a measurement is given by “probabilistically comparing” the actual with the expected measurement
- It is extremely important to be aware of the underlying assumptions!

Midterm

- This lecture is included in the Midterm



Acknowledgment

- These slides have been created by Wolfram Burgard, Dieter Fox, Cyrill Stachniss and Maren Bennewitz