Cognitive Robotics

05. Probabilistic Sensor Models

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Spring 2022

Acknowledgment

- These slides have been created by Wolfram Burgard, Dieter Fox, Cyrill Stachniss and Maren Bennewitz

State Estimation

- Estimate the state x of a system given observations z and actions u
- **Goal:** Determine $p(x \mid z, u)$

Recursive Bayes Filter (recap)

$$
\begin{aligned}
bel(x_t) = p(x_t \mid z_{1:t}, u_{1:t}) \\
&= p(z_t \mid x_t, z_{1:t-1}, u_{1:t})\, p(x_t \mid z_{1:t-1}, u_{1:t}) \,/\, p(z_t \mid z_{1:t-1}, u_{1:t}) \\
&= \eta\, p(z_t \mid x_t)\, p(x_t \mid z_{1:t-1}, u_{1:t}) \\
&= \eta\, p(z_t \mid x_t) \int p(x_t \mid x_{t-1}, z_{1:t-1}, u_{1:t}) \\
&\qquad\qquad p(x_{t-1} \mid z_{1:t-1}, u_{1:t})\, dx_{t-1} \\
&= \eta\, p(z_t \mid x_t) \int p(x_t \mid x_{t-1}, u_t)\, p(x_{t-1} \mid z_{1:t-1}, u_{1:t})\, dx_{t-1} \\
&= \eta\, p(z_t \mid x_t) \int p(x_t \mid x_{t-1}, u_t)\, p(x_{t-1} \mid z_{1:t-1}, u_{1:t-1})\, dx_{t-1} \\
&= \underbrace{\eta\, p(z_t \mid x_t)}_{\text{observation model}} \int \underbrace{p(x_t \mid x_{t-1}, u_t)}_{\text{motion model}}\, \underbrace{bel(x_{t-1})}_{\text{recursive term}}\, dx_{t-1}
\end{aligned}
$$

Motion and Observation Model

- Prediction step

$$\overline{bel}(x_t) = \int \underbrace{p(x_t \mid u_t, x_{t-1})}_{\text{motion model}} \, bel(x_{t-1}) \, dx_{t-1}$$

- Correction step

$$bel(x_t) = \eta \, \underbrace{p(z_t \mid x_t)}_{\text{sensor or observation model}} \, \overline{bel}(x_t)$$

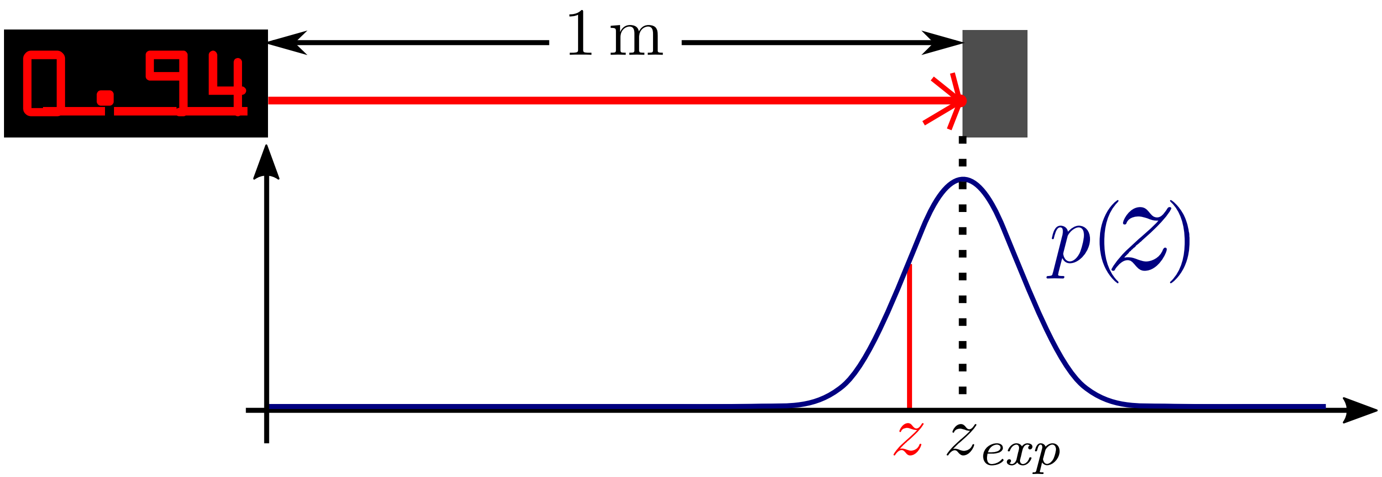
Previous Lecture: Probabilistic Motion Models

- Robots execute motion commands only inaccurately
- The motion model specifies the probability that action u_t carries the robot from pose x_{t-1} to x_t:

$$p(x_t \mid u_t, x_{t-1})$$

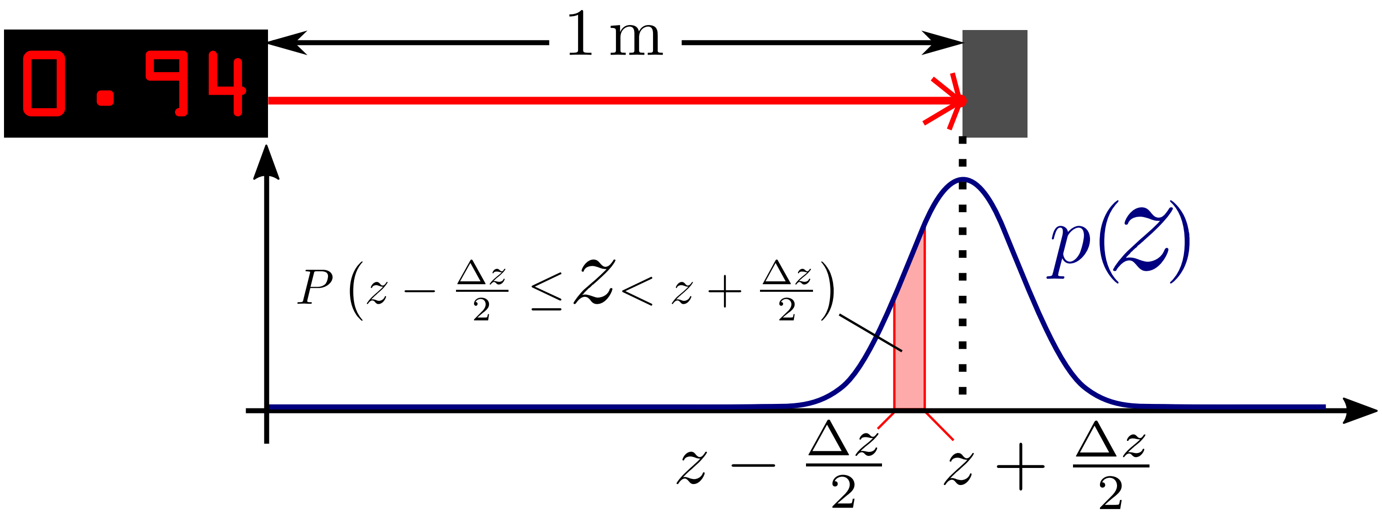
- Defined individually for each type or robot

Measurement Probability



$$P(Z = z) = 0$$
$$= P(Z = 0.9400000\ldots)$$

Measurement Probability



$$P(0.935 \leq Z < 0.945) = P\left(z - \frac{\Delta z}{2} \leq Z < z + \frac{\Delta z}{2}\right)$$

$$= \int_{z-\frac{\Delta z}{2}}^{z+\frac{\Delta z}{2}} p(Z)\,\mathrm{d}Z$$

$$\approx \Delta z \cdot p(Z) \quad \text{(for small } \Delta z\text{)}$$

Continuous vs. Discretized Random Variables

- Z is a continuous random variable with the probability density function

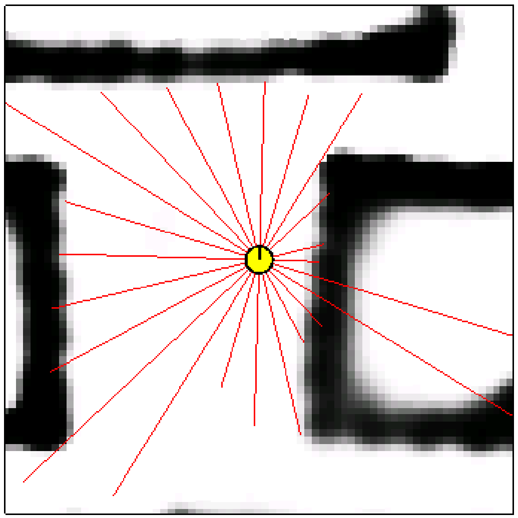
$$p(Z) = \lim_{\Delta z \to 0} \frac{P\left(z - \frac{\Delta z}{2} \leq Z < z + \frac{\Delta z}{2}\right)}{\Delta z}$$

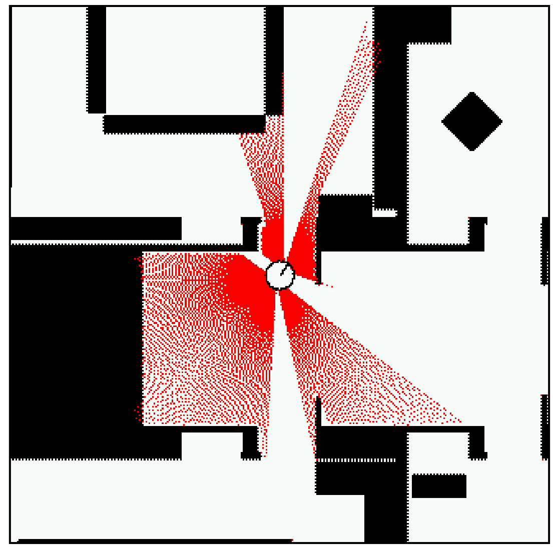
- We can only measure and represent Z in discrete steps Δz
- As Δz is constant for all measurements, we can ignore it when computing $bel(x_t)$ if we normalize $bel(x_t)$ in the end

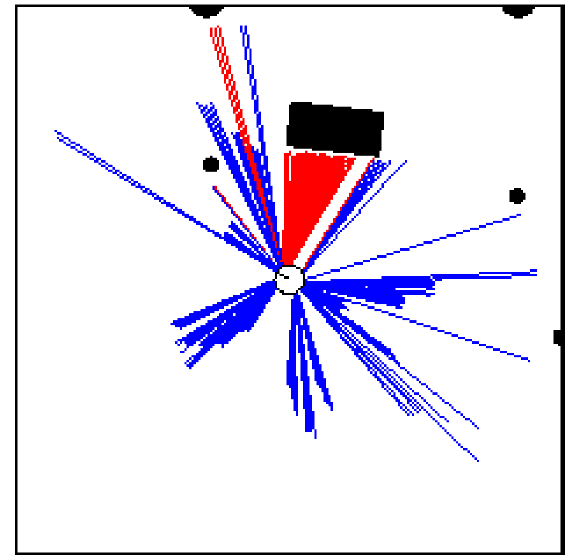
Sensors for Mobile Robots

- Proprioceptive sensors:
  - Accelerometers
  - Gyroscopes
  - Compasses
- Typical proximity sensors:
  - Sonars
  - Laser range-finders
- Visual sensors:
  - (Stereo) Cameras
  - Structured light (RGBD cameras)
- Infrastructure-based sensors: GPS, WLAN

Proximity Sensors







Question: How can we calculate the likelihood of such a measurement given the robot pose?

Beam-Based Sensor Model

- Sensor data consists of K measurements

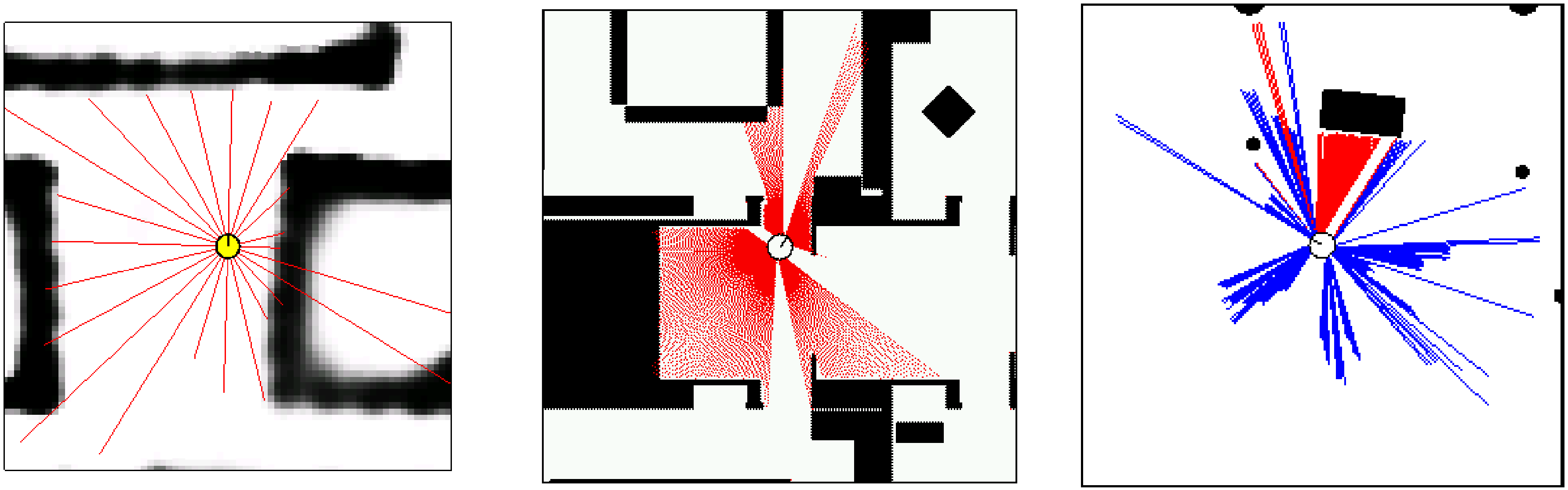
$$z = \{z_1, \ldots, z_K\}$$

- Assumption: The individual measurements are independent given the robot's pose:

$$p(z \mid x, m) = \prod_{k=1}^{K} p(z_k \mid x, m)$$

- "How well can the distance measurements be explained given the pose (and the map)"

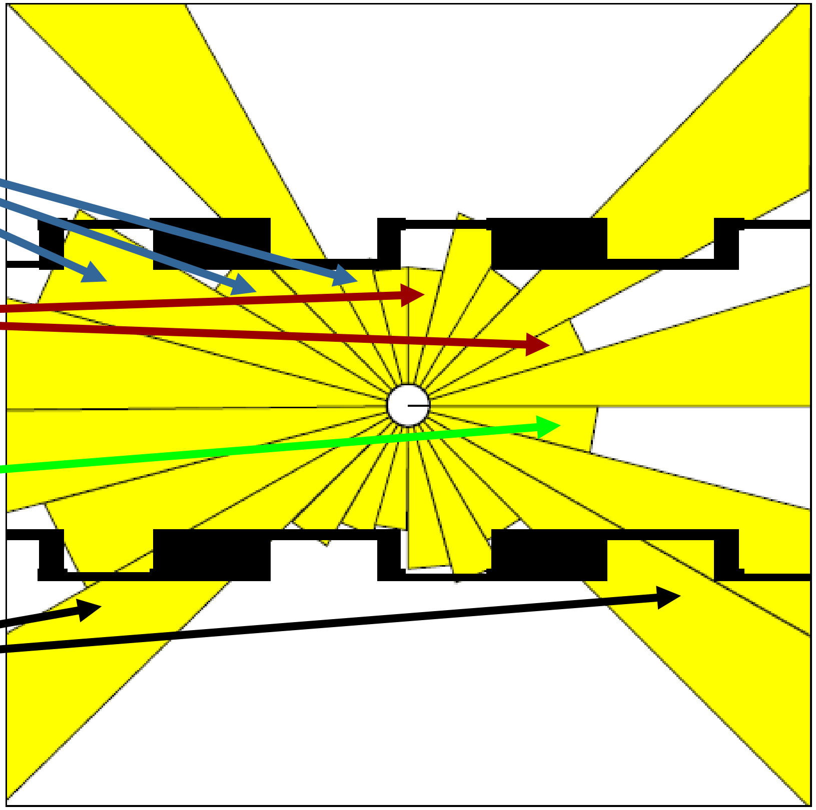
Beam-Based Sensor Model



$$p(z \mid x, m) = \prod_{k=1}^{K} p(z_k \mid x, m)$$

Typical Measurement Errors of an Range Measurements

1. Beams reflected by known obstacles
2. Beams reflected by people / objects
3. Random measurements
4. Maximum range measurements



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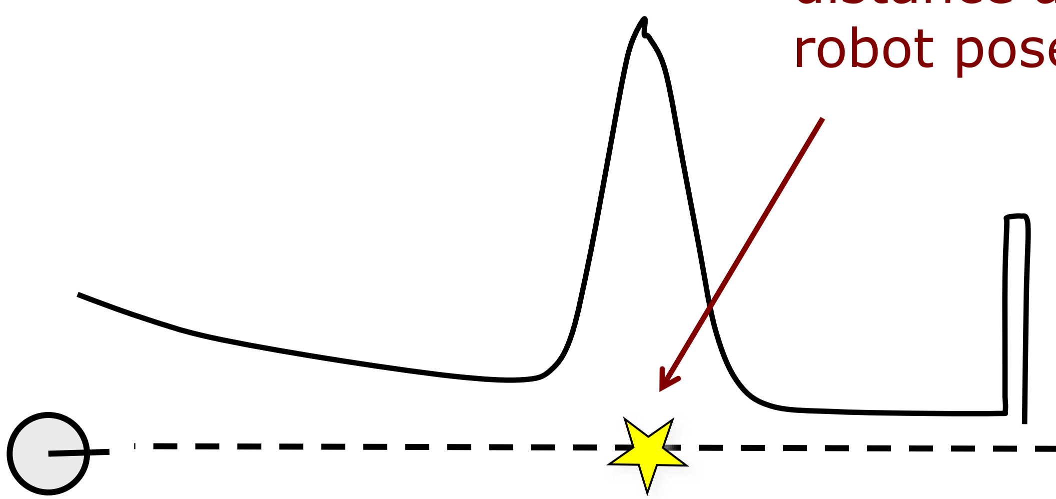
Proximity Measurements

- **A measurement can be caused by:**
  - a known obstacle
  - an unexpected obstacle (people, furniture, …)
  - random measurements, cross-talk (sonars)
  - missing all obstacles
- **Noise is due to uncertainty:**
  - in measuring distance to known obstacle (sensor noise)
  - in the position of known obstacles (“map noise”)
  - in the position of additional objects
  - whether an obstacle is missed

Beam-Based Proximity Model

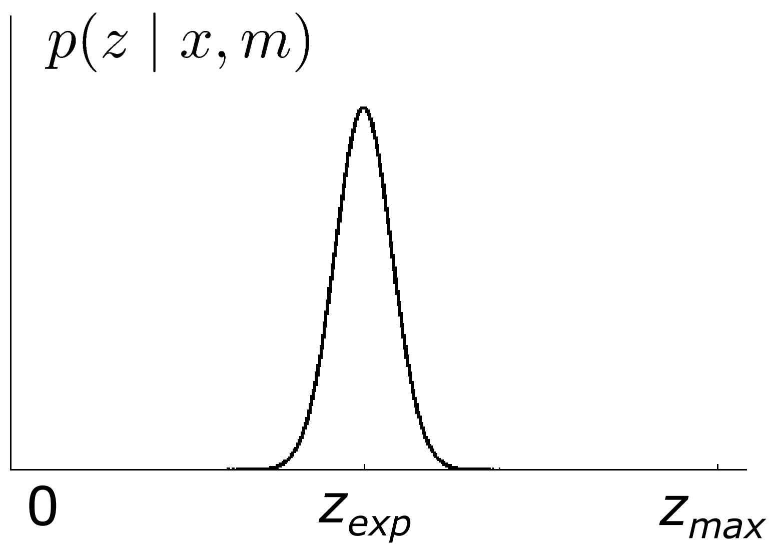
- Considers the first obstacle along the line of sight
- Mixture of four components

expected measured distance according to robot pose and map



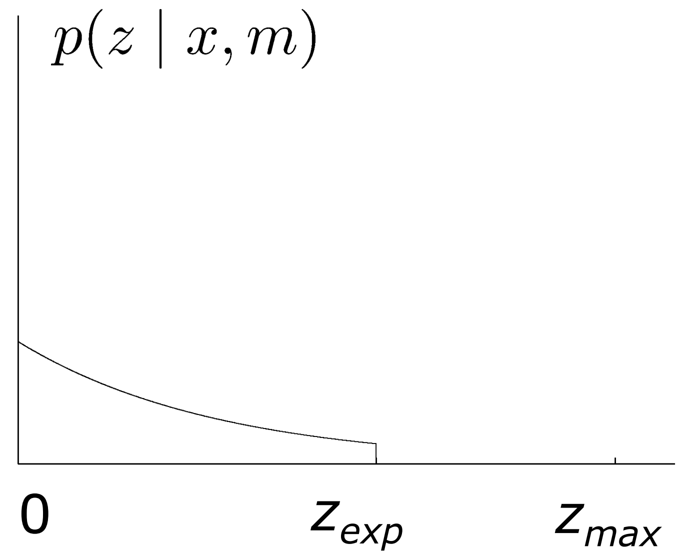
Beam-Based Proximity Model

Measurement noise



$$p_{hit}(z \mid x, m) = \begin{cases} \frac{\eta}{\sqrt{2\pi\sigma^2}} e^{-\frac{(z - z_{exp})^2}{2\sigma^2}} & \text{if } z \leq z_{max} \\ 0 & \text{otherwise} \end{cases}$$

Unexpected objects

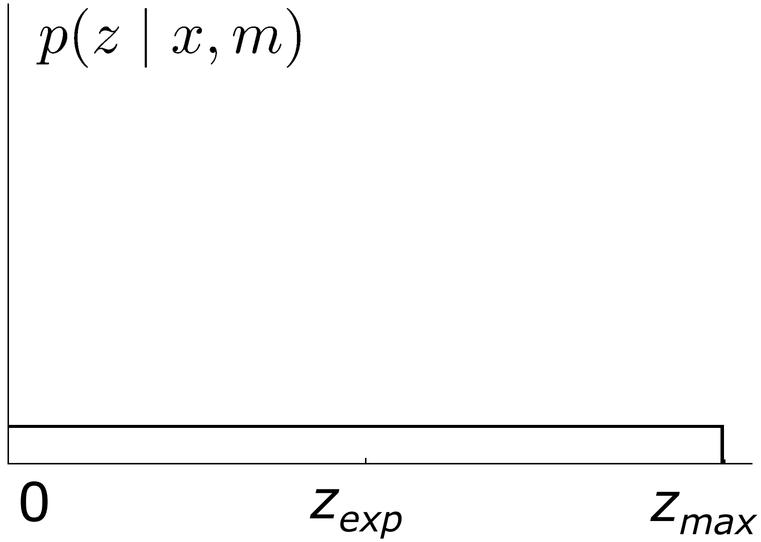


$$p_{unexp}(z \mid x, m) = \begin{cases} \eta \lambda e^{-\lambda z} & \text{if } z < z_{exp} \\ 0 & \text{otherwise} \end{cases}$$

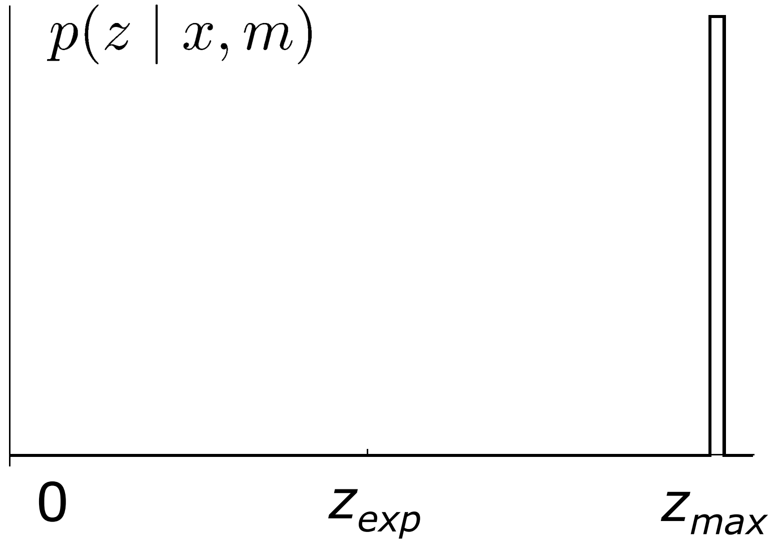
Note: The expected distance z_{exp} is computed by ray casting in the map, starting from the pose x

Beam-Based Proximity Model

Random measurement



Max range

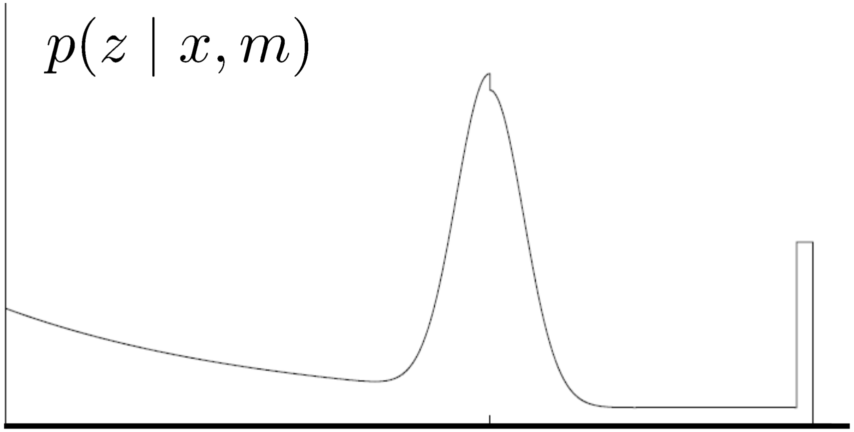


$$p_{rand}(z \mid x, m) = \begin{cases} \frac{1}{z_{max}} & \text{if } z < z_{max} \\ 0 & \text{otherwise} \end{cases}$$

$$p_{max}(z \mid x, m) = \begin{cases} \frac{1}{z_{small}} & \text{if } z \in [z - z_{small}, z_{max}] \\ 0 & \text{otherwise} \end{cases}$$

Note: The expected distance z_{exp} is computed by ray casting in the map, starting from the pose x

Resulting Mixture Density

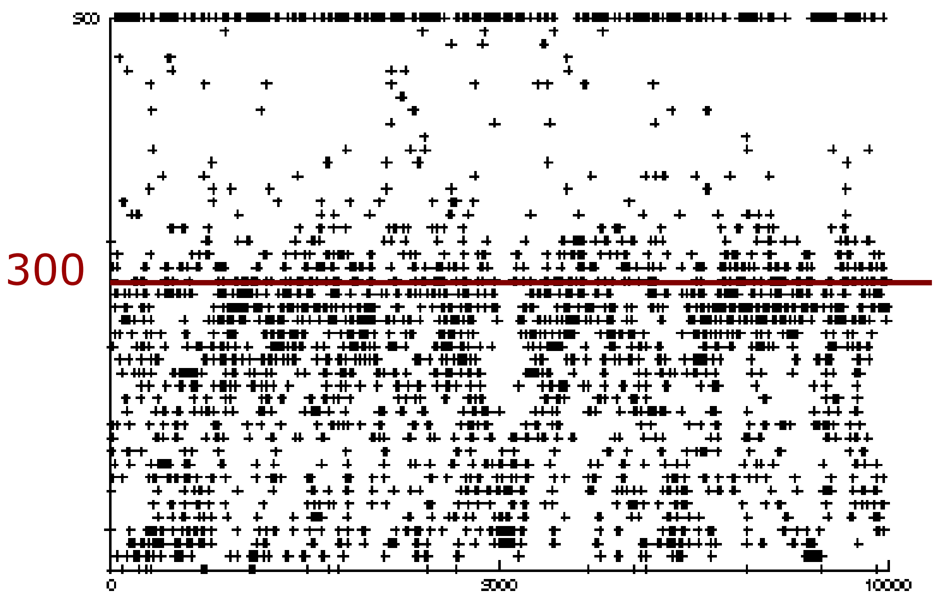


$$p(z \mid x, m) = \begin{pmatrix} \alpha_{\text{hit}} \\ \alpha_{\text{unexp}} \\ \alpha_{\text{max}} \\ \alpha_{\text{rand}} \end{pmatrix}^T \cdot \begin{pmatrix} p_{\text{hit}}(z \mid x, m) \\ p_{\text{unexp}}(z \mid x, m) \\ p_{\text{max}}(z \mid x, m) \\ p_{\text{rand}}(z \mid x, m) \end{pmatrix}$$

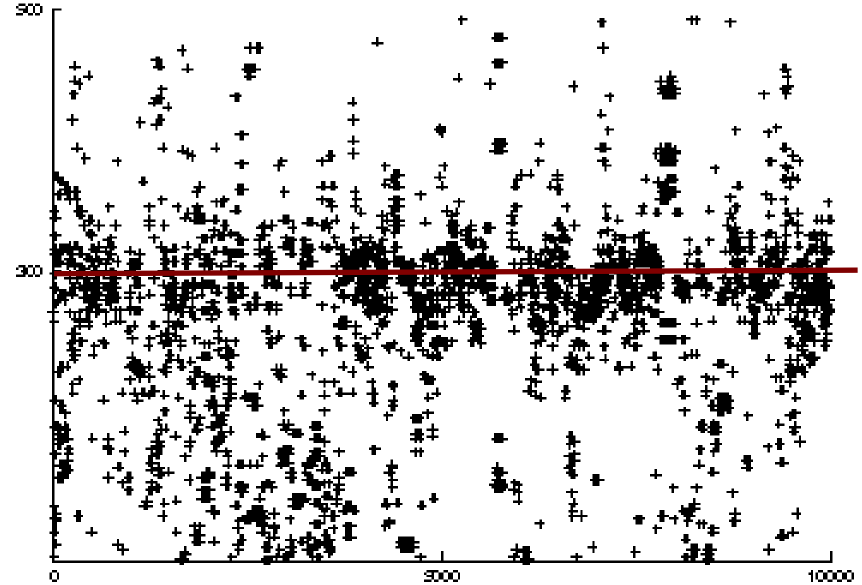
How can we determine the model parameters?

Raw Sensor Data

Measured distances for the “true” expected distance of 300 cm (maximum range 500 cm)



Sonar



Laser

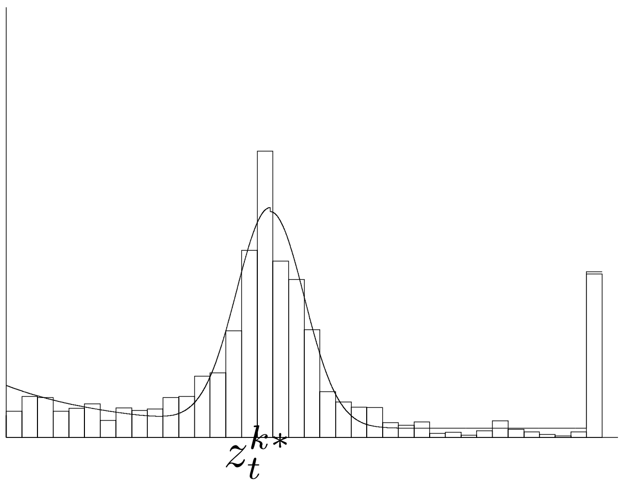
Approximation

- Maximize log likelihood of the data

$$p(z \mid z_{\exp})$$

- Search space of n-1 parameters
  - Hill climbing
  - Gradient descent
  - Genetic algorithms
  - ...
- Deterministically compute the n-th parameter to satisfy normalization constraint

Approximation Results



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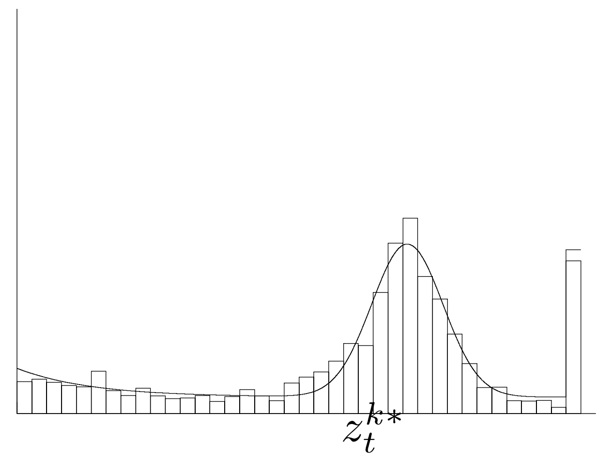
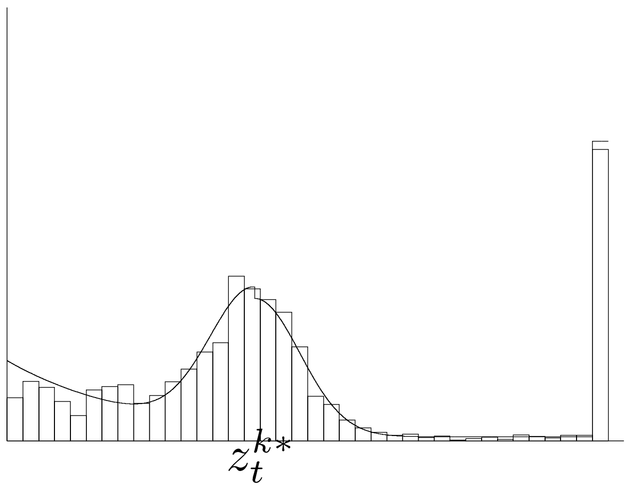
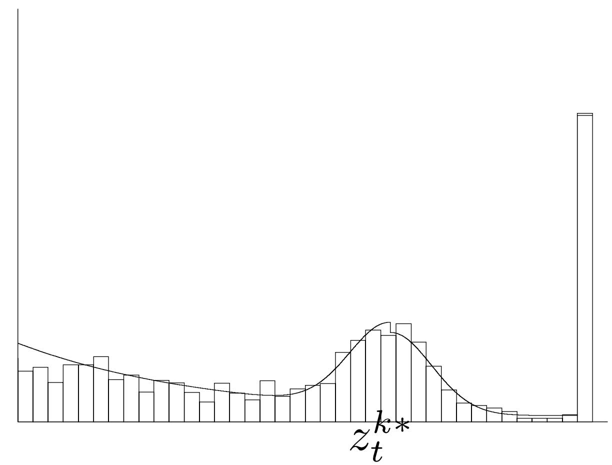




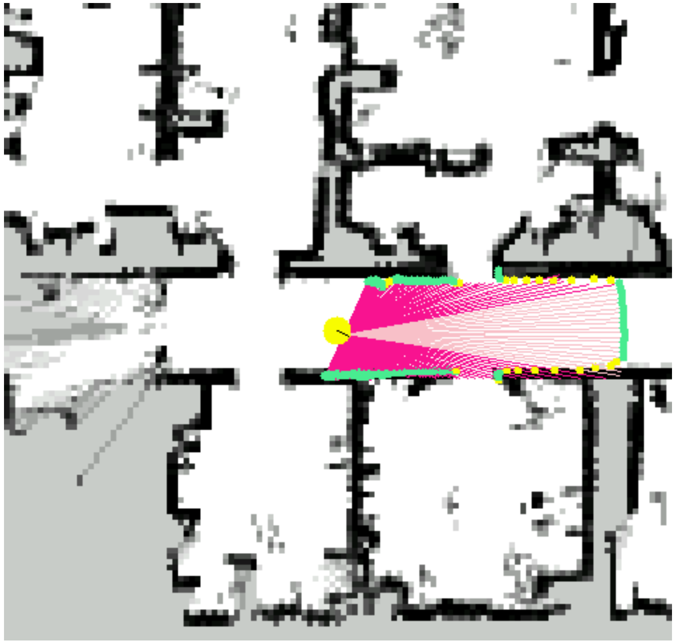
Sonar



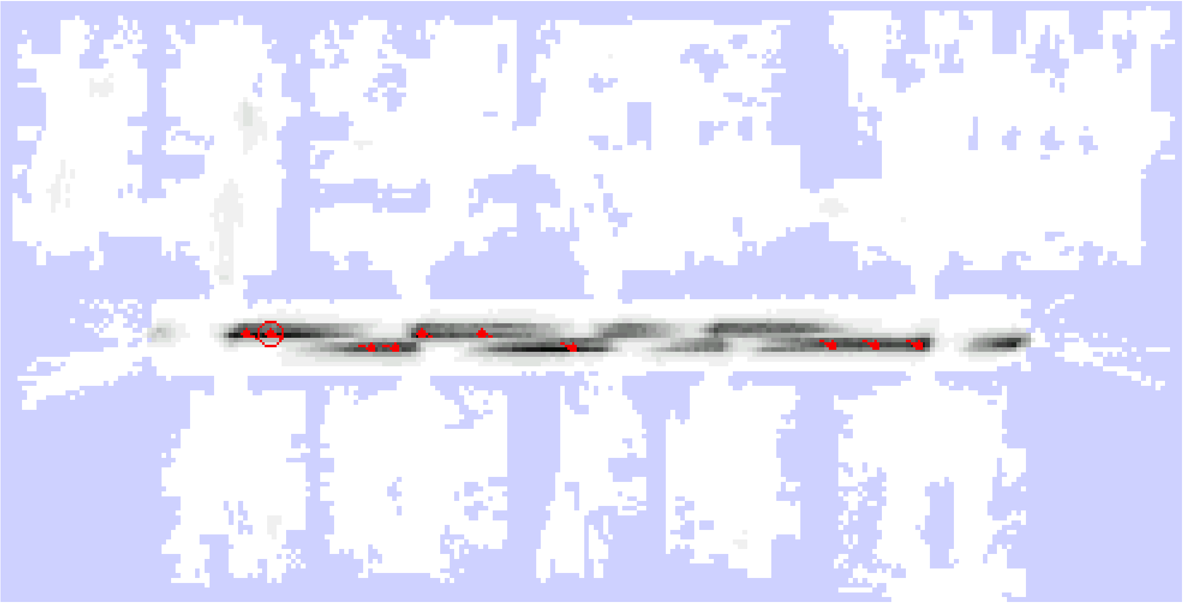
300cm

400cm

Example

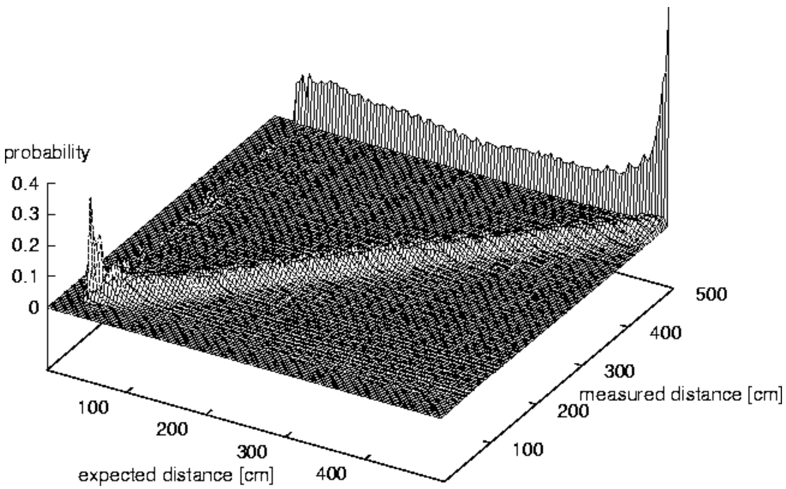


z

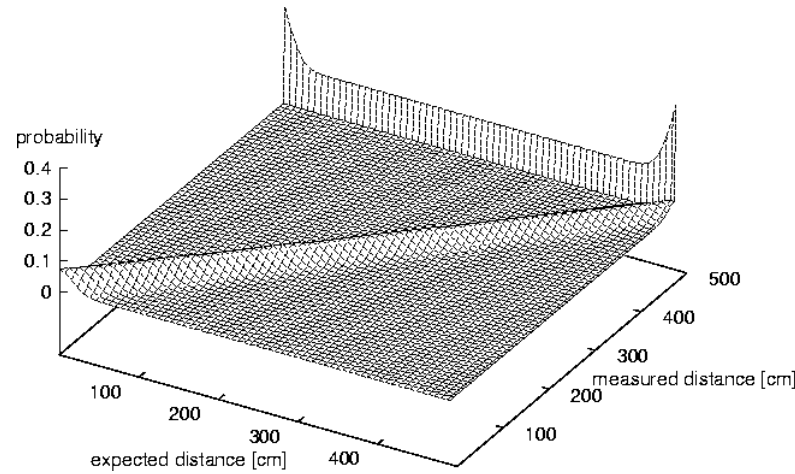


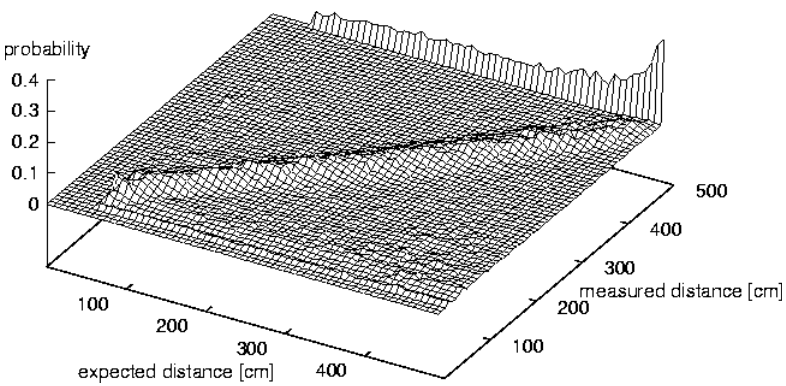
$p(z|x,m)$

Approximation Results

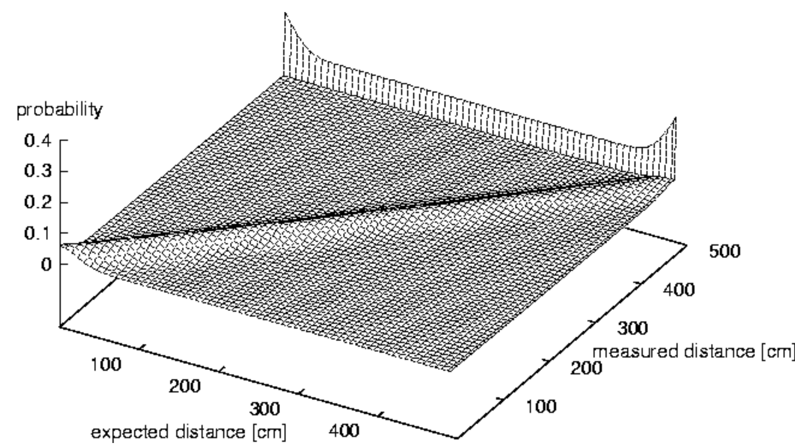


Laser

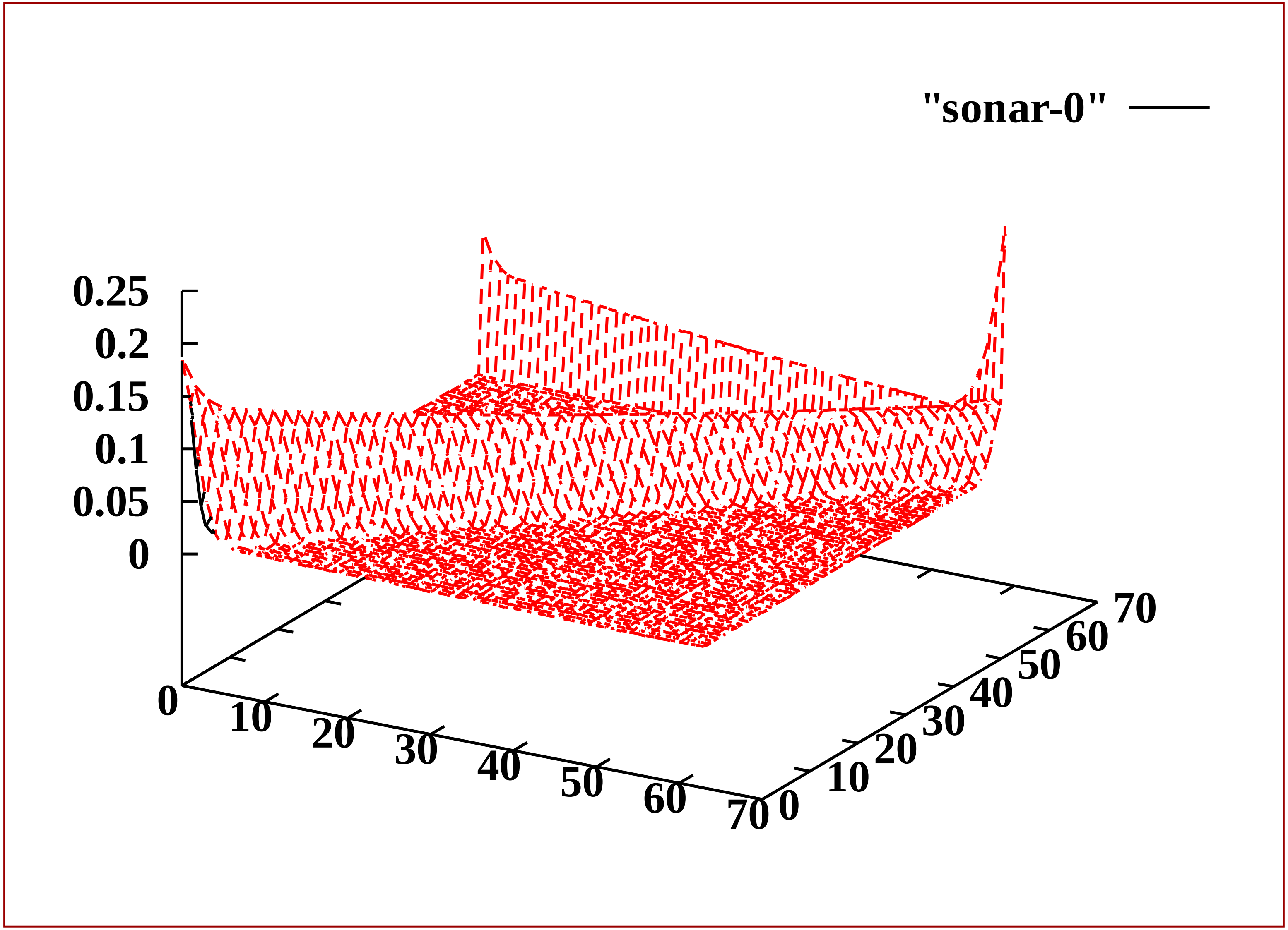




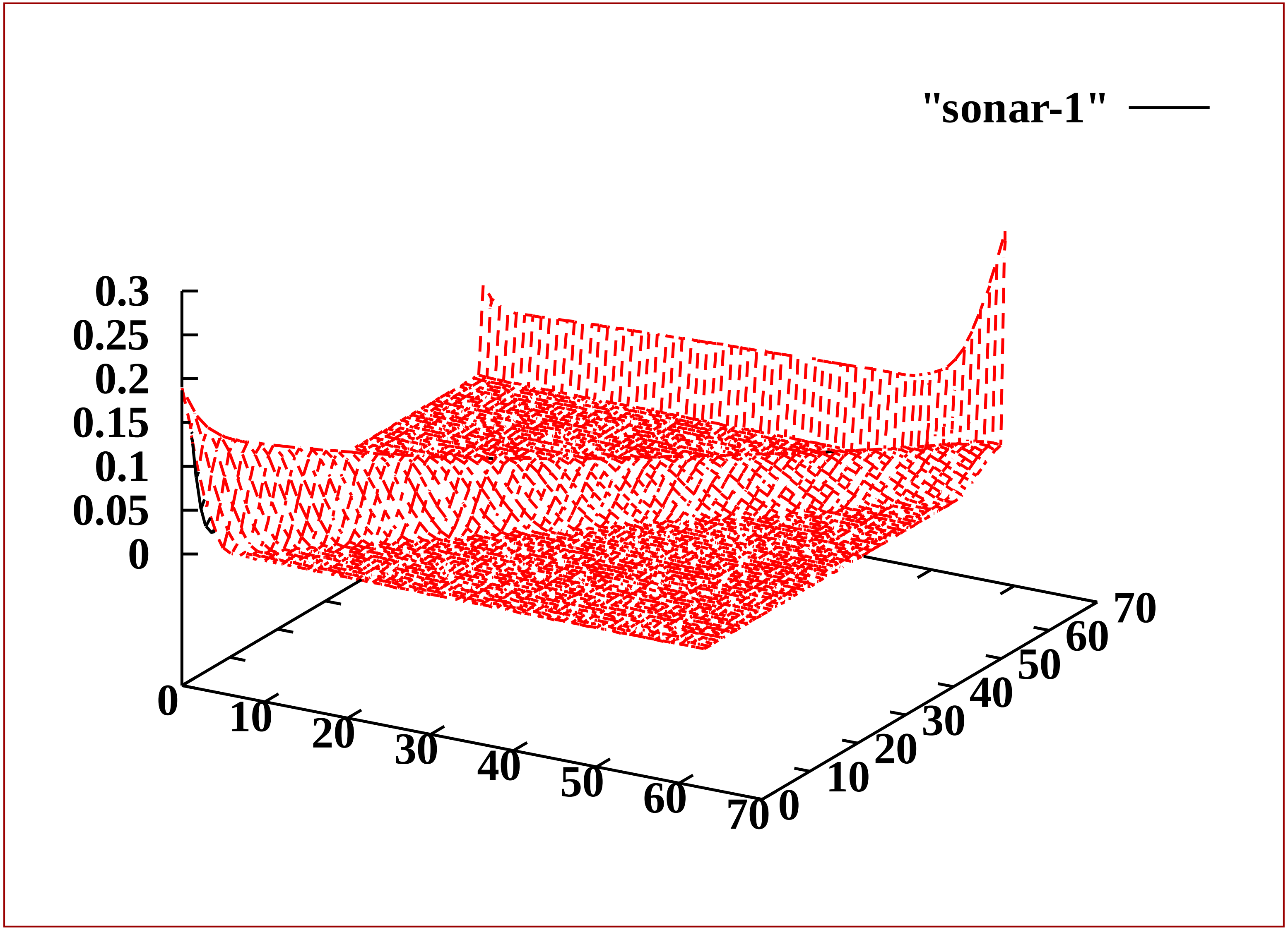
Sonar



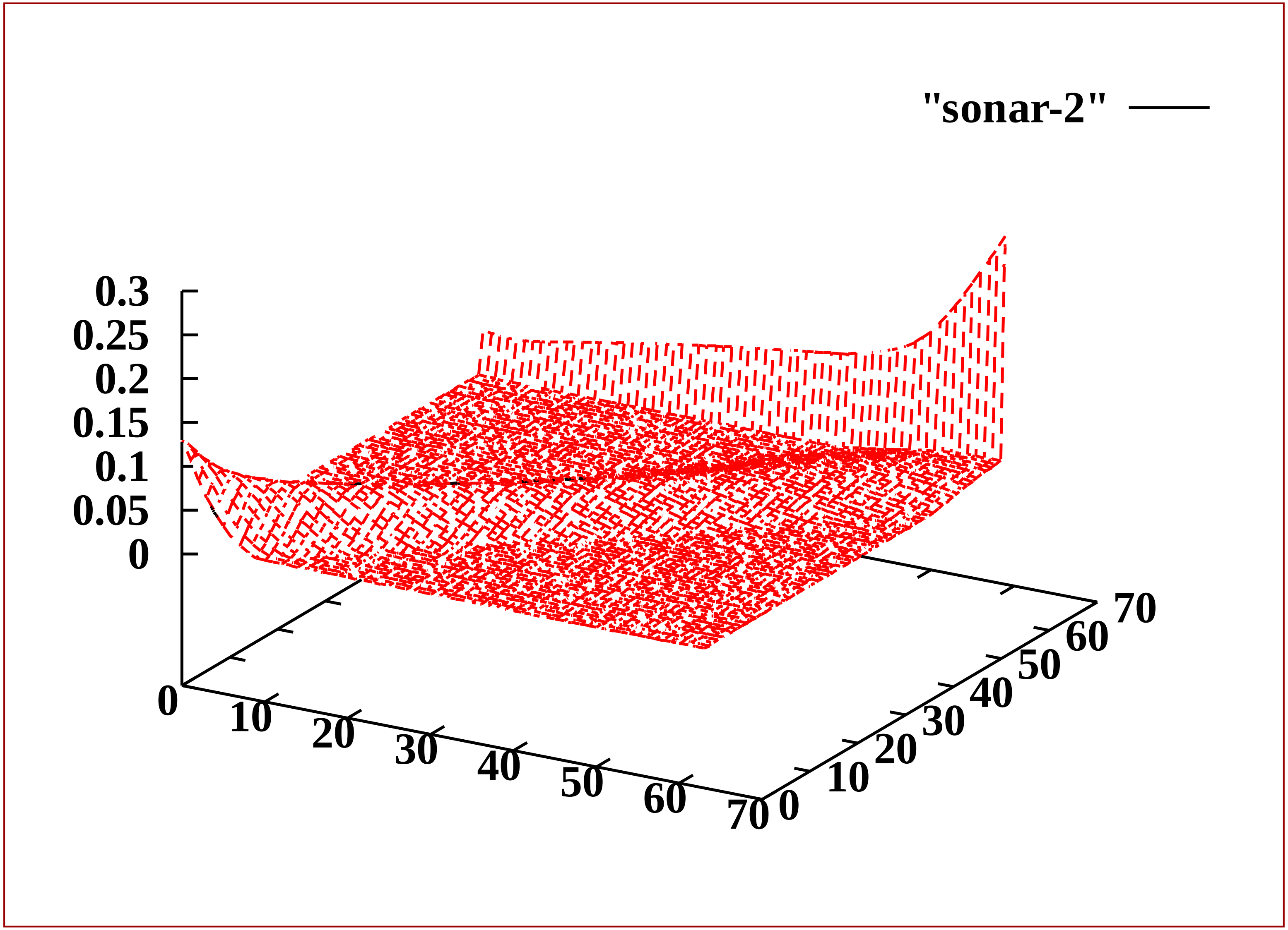
Influence of Angle to Obstacle



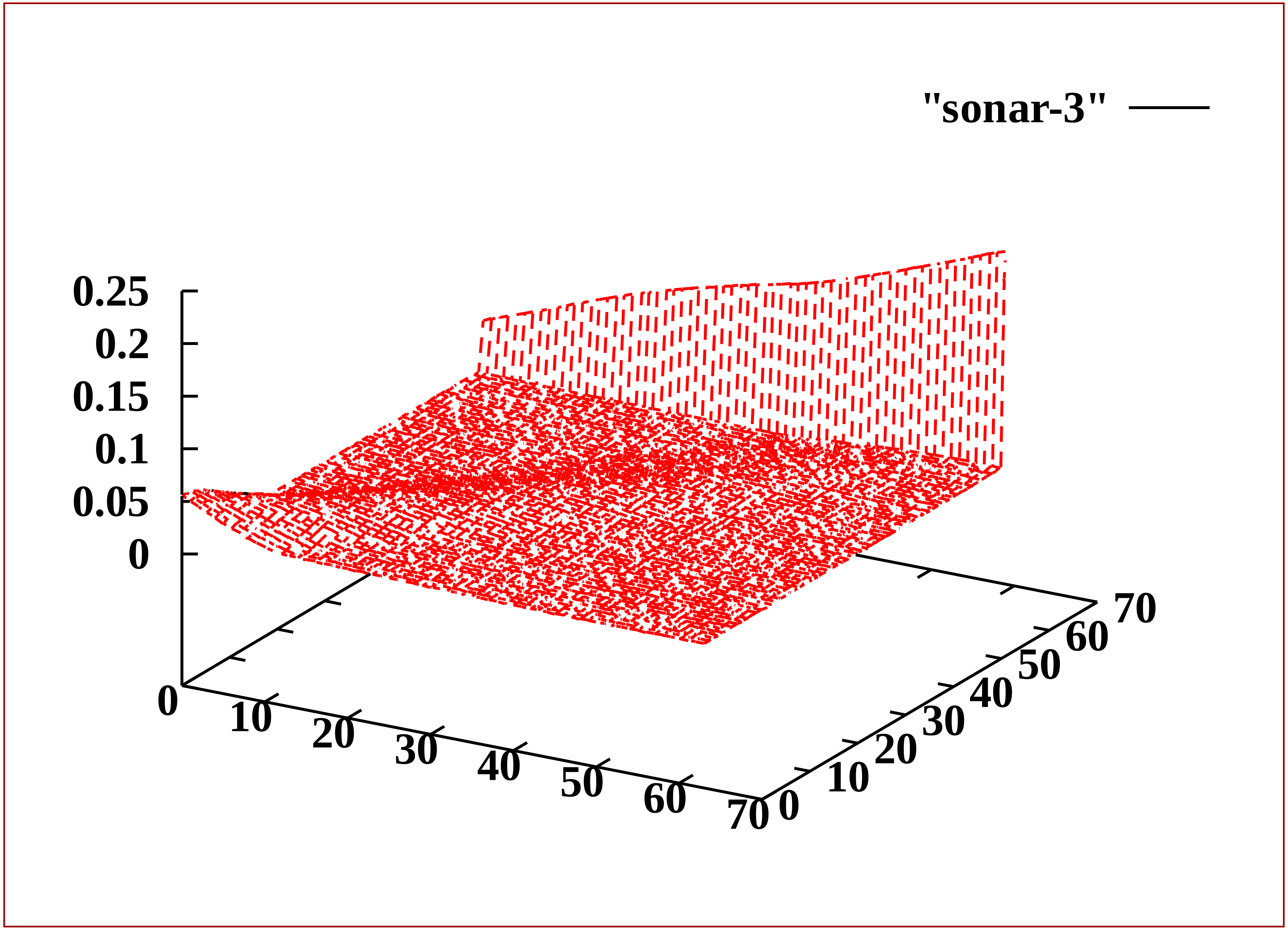
Influence of Angle to Obstacle



Influence of Angle to Obstacle



Influence of Angle to Obstacle



Summary Beam-Based Model (1)

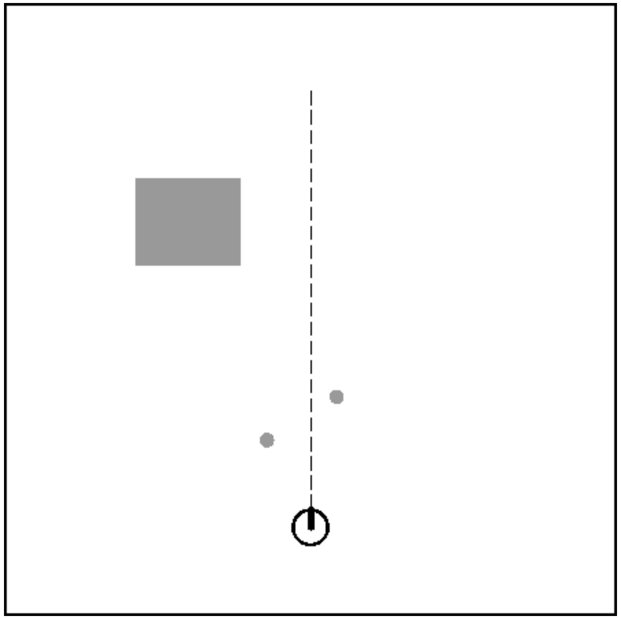
- **Assumes independence between beams**
  - Justification?
  - Problem: Overconfident estimates
- **Models physical causes for measurements**
  - Mixture of densities for these causes
  - Assumes independence between causes

Summary Beam-Based Model (2)

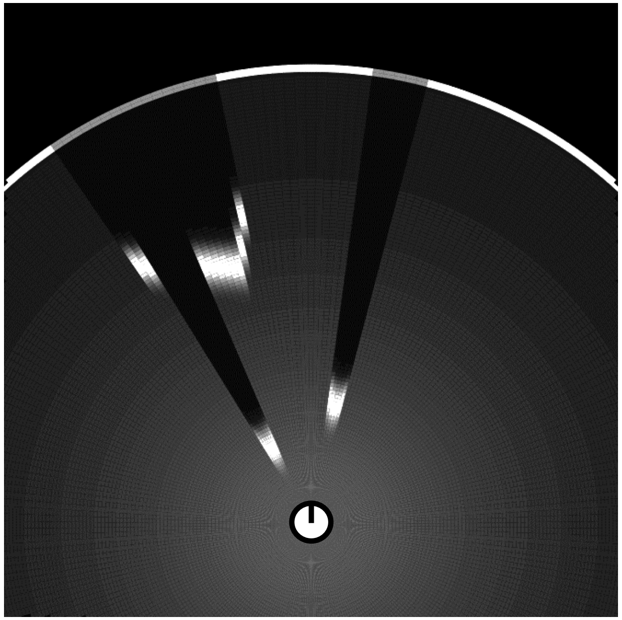
- **Implementation**
  - Learn parameters based on real data
  - Different models should be learned for different angles at which the sensor beam hits the obstacle (sonar)
  - Determine expected distances by ray casting
  - Expected distances can be pre-computed

Summary Beam-Based Model (3)

- **Disadvantages:** The beam-based model is
  - not smooth at edges
  - not very efficient (ray casting or precomputed lookup tables)



Map *m*



Likelihood field

End-Point Model

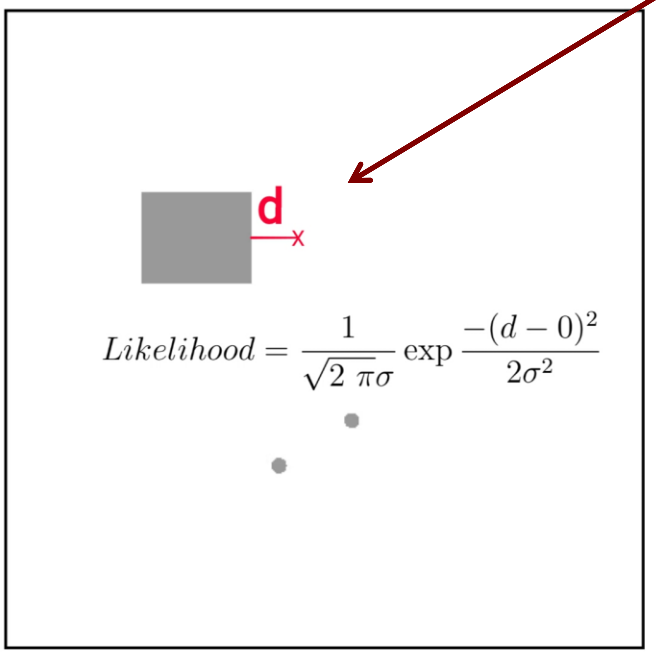
- Idea of the end-point model: Instead of following along the beam, just check the end point of the beam
- Precompute a so-called likelihood field (distance grid)

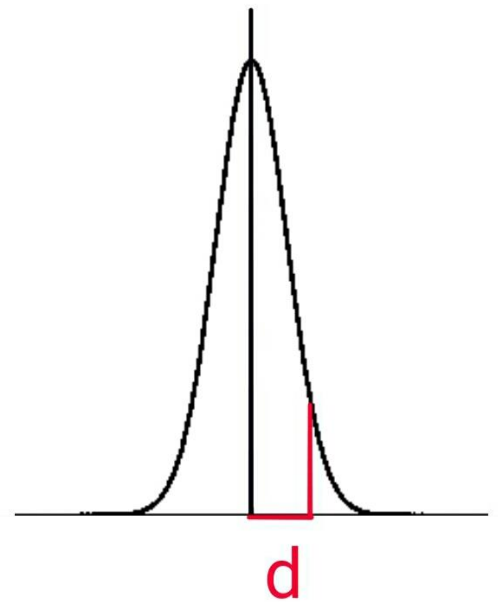
End-Point Model

- Probability is a mixture of:
  - a Gaussian distribution evaluating the **distance to the closest obstacle**,
  - a uniform distribution for random measurements, and
  - a small uniform distribution for max range measurements
- Again, independence between different components is assumed

Gaussian Used within the Likelihood of a Measurement

end point and
distance to the
closest obstacle

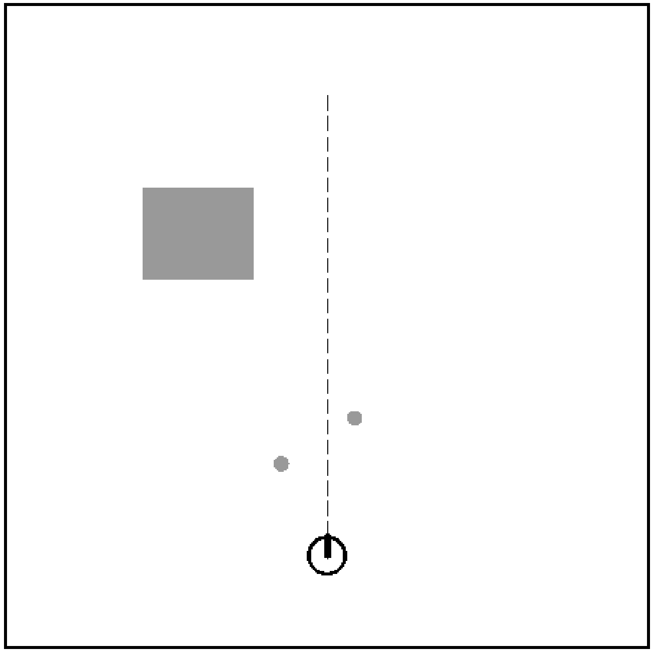




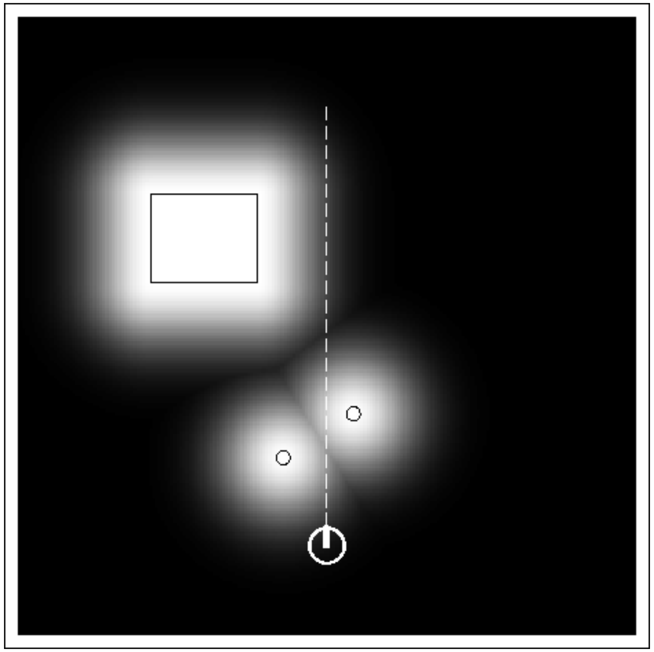
Map with
three obstacles

Gaussian to evaluate
the distance

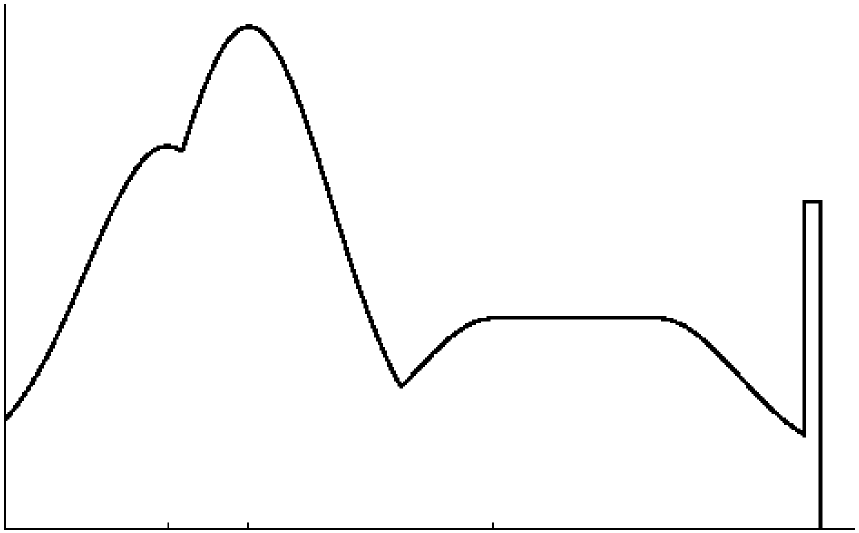
Example



Map m



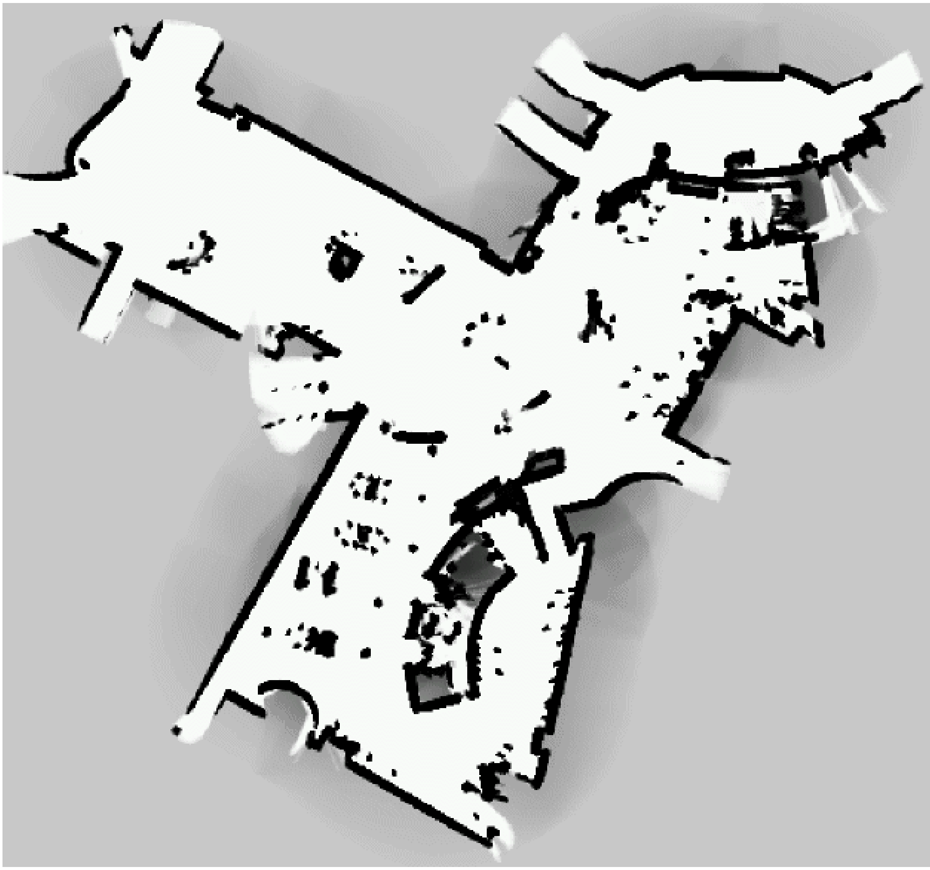
Likelihood field



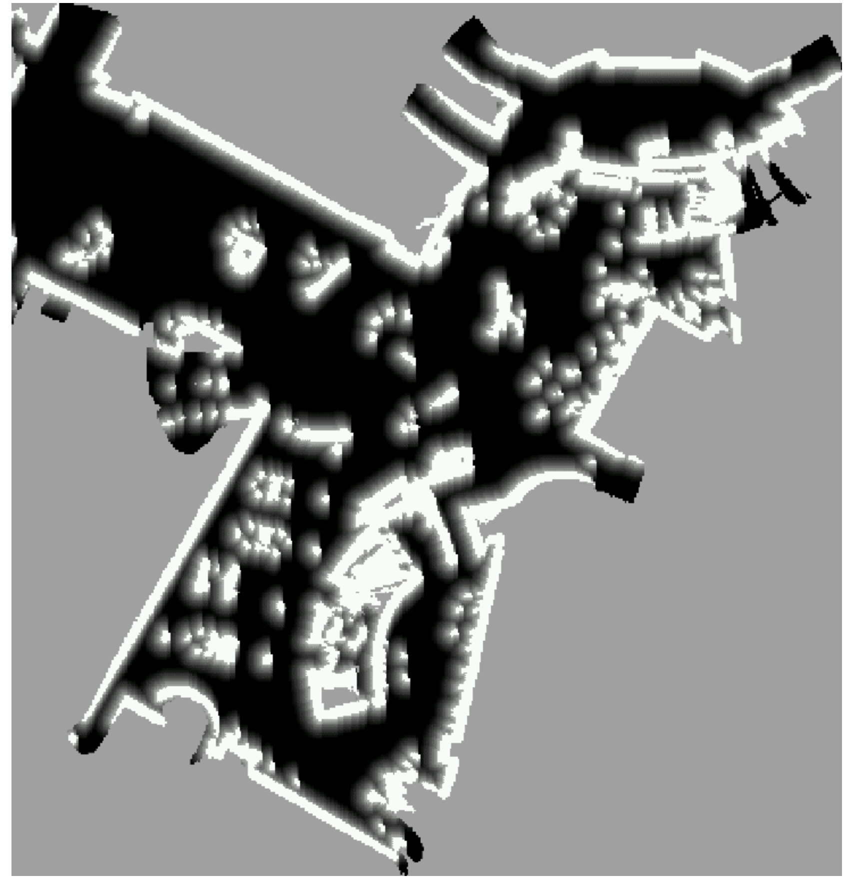
$p(z|x,m)$

measured length z

San Jose Tech Museum



Occupancy grid map



Likelihood field

Note: Precomputed independently of robot pose

Properties End-Point Model

- Highly efficient, uses 2D tables only
- Distance grid is smooth w.r.t. to small changes in robot position
- Ignores physical properties of beams
- Treats sensor as if it can see through walls

Landmarks

- Active beacons (e.g., radio, GPS)
- Passive markers (e.g., visual, retro-reflective)
- Standard approach: triangulation

- Sensor provides
  - distance, or
  - bearing, or
  - distance and bearing

Distance and Bearing

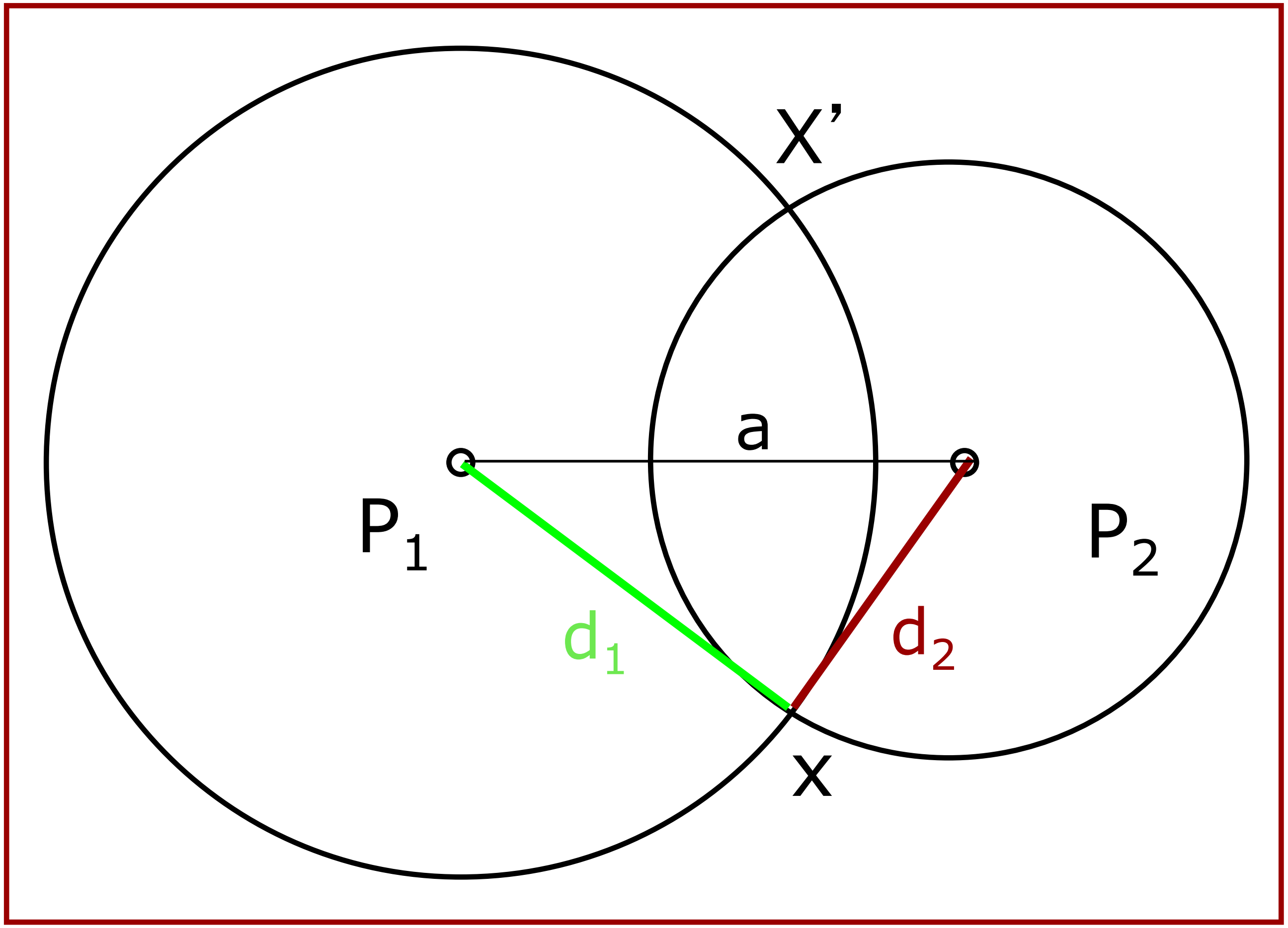
Probabilistic Model

1. Algorithm **landmark_detection_model**(z,x,m):
   $z = \langle i, d, \alpha \rangle, x = \langle x, y, \theta \rangle$
2. $\hat{d} = \sqrt{(m_x(i) - x)^2 + (m_y(i) - y)^2}$ — Expected distance
3. $\hat{a} = \mathrm{atan2}(m_y(i) - y, m_x(i) - x) - q$ — Expected angle
4. $p_{\mathrm{det}} = \mathrm{prob}(\hat{d} - d, e_d) \times \mathrm{prob}(\hat{a} - a, e_a)$ — Independence assumption, two Gaussians
5. Return p_{det}

Assumption: Correspondences are known

Distances Only
No Uncertainty

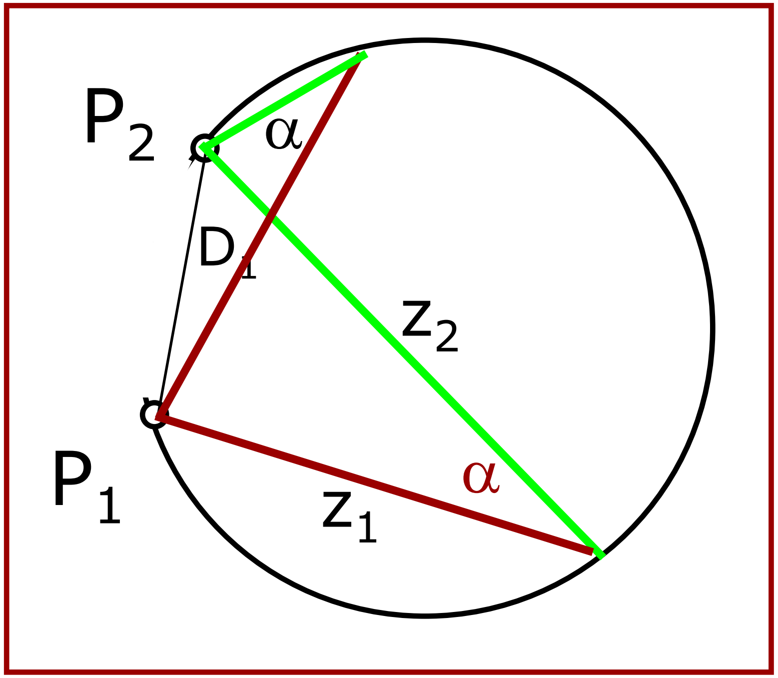
$$x = (a^2 + d_1^2 - d_2^2)/2a$$
$$y = \pm\sqrt{(d_1^2 - x^2)}$$

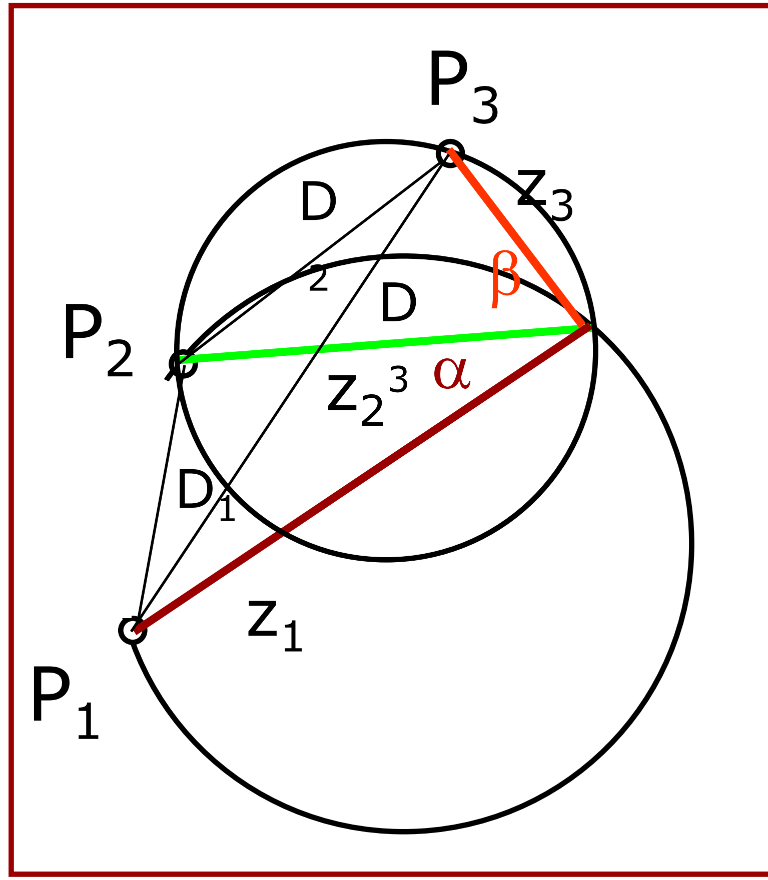


$P_1=(0,0)$

$P_2=(a,0)$

Bearings Only
No Uncertainty





Law of cosine

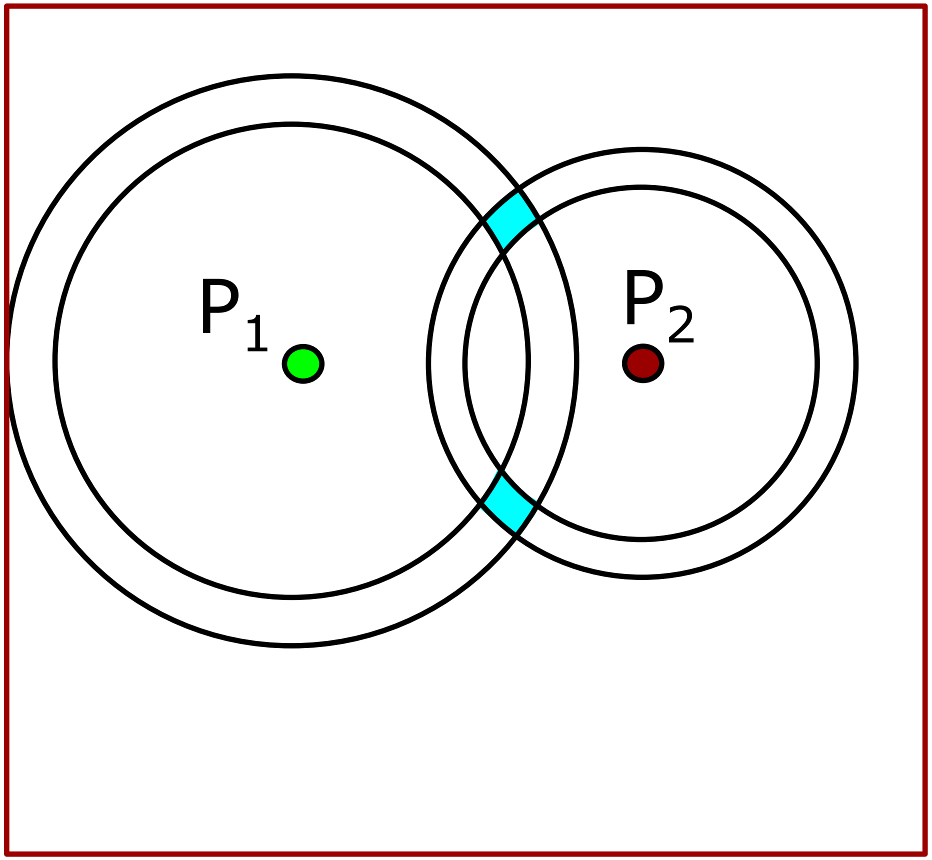
$$D_1^2 = z_1^2 + z_2^2 - 2\, z_1 z_2 \cos\alpha$$

$$D_1^2 = z_1^2 + z_2^2 - 2\, z_1 z_2 \cos(\alpha)$$

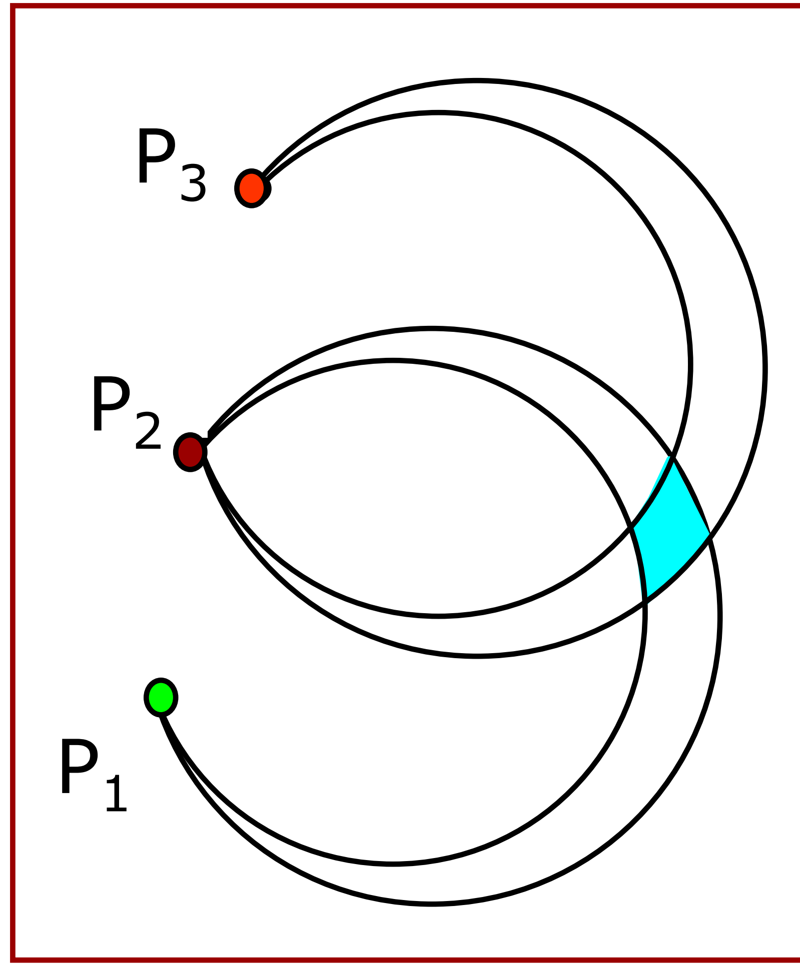
$$D_2^2 = z_2^2 + z_3^2 - 2\, z_1 z_2 \cos(\beta)$$

$$D_3^2 = z_1^2 + z_3^2 - 2\, z_1 z_2 \cos(\alpha + \beta)$$

Landmark Measurements with Uncertainty



Distance only



Bearings only

Summary (1)

- **Explicitly modeling uncertainty in sensing is key to robustness**
- In many cases, good models can be found by the following approach:
  - Determine the parametric model of a noise-free measurement
  - Analyze the individual sources of noise
  - Add adequate noise to parameters (add densities for noise)
  - Learn parameters by fitting a model to the data

Summary (2)

- **The likelihood of a measurement is given by “probabilistically comparing” the actual with the expected measurement**
- It is extremely important to be aware of the underlying assumptions!

Midterm

- This lecture is included in the Midterm

Acknowledgment

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