$P(\neg A|B) = 1 - P(A|B)$

RB — Sheet 1

1.1

$P(\overset{\text{test}}{t}|\neg \overset{\text{Not ill}}{i}) = 0.005 \leftarrow$ false +ve قال مريض وهو سليم !

$P(\neg t| i) = 0 \leftarrow$ false -ve قال مش مريض وهو مريض !

$P(i) = 0.00002 \leftarrow$ Someone is ill

$$P(i|t) = \frac{P(t|i)P(i)}{P(t)} = \frac{(1-P(\neg t|i))P(i)}{=} = 3.984 \times 10^{-3}$$

$P(t) \rightarrow P(t|\neg i)P(\neg i) + P(t|i)P(i)$

1.2

$P(z|P) = 0.7$
$P(z|\neg P) = 0.2$
$P(P) = 0.5$

$$P(P|\neg z) = \frac{P(\neg z|P)P(P)}{P(\neg z)} = \frac{(1-0.7)(0.5)}{1-[0.7 \times 0.5 + 0.2 \times 0.5]} = 3/11$$

$P(\neg z|P) = 1 - P(z|P)$

$P(\neg z) = 1 - P(z)$

$P(z) = P(z|P)P(P) + P(z|\neg P)P(\neg P)$

1.3 $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, $\Delta t = 0.1$; Find A such that $x_{t+1} = A x_t$

$x_1 \rightarrow$ Pos, $x_2 \rightarrow$ vel

$$\begin{bmatrix} x_1)_{t+1} \\ x_2)_{t+1} \end{bmatrix} = \begin{bmatrix} 1 & 0.1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1)_t \\ x_2)_t \end{bmatrix} \Leftarrow \begin{matrix} \underset{x_1)_{t+1}}{x} = \underset{x_1)_t}{x_0} + \underset{x_2)_t}{v_0} t \\ \underset{x_2)_{t+1}}{v} = \underset{x_2)_t}{v_0} \end{matrix}$$

1.4

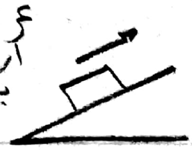
$$\begin{bmatrix} x_1)_{t+1} \\ x_2)_{t+1} \end{bmatrix} = \begin{bmatrix} 1 & 0.1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1)_t \\ x_2)_t \end{bmatrix} + \begin{bmatrix} 0.05 \\ 0.1 \end{bmatrix} u_t$$

$0.05 \rightarrow \frac{1}{2}t^2$, $0.1 \rightarrow \Delta t$, $u_t \rightarrow$ acc

1.5

$$z_t = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ v \end{bmatrix}$$

عربية بتقع من
ارتفاع وداست
بنزين تطلع!



motion Noise

$\boxed{1.6}$ $x_0 = (3, -1)$, $\Sigma_0 = \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix}$, $R = \begin{pmatrix} 0.1 & 0 \\ 0 & 0.04 \end{pmatrix}$

$\Delta t = 0.1s$

* if $u_t = 3m/s^2$, what's the Prediction of Kalman for $t=1$, Compute mean & Covariance of the state!

$$\rightarrow \bar{\mu}_t = \begin{pmatrix} 1 & 0.1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 3 \\ -1 \end{pmatrix} + \underbrace{\begin{pmatrix} 0.05 \\ 0.1 \end{pmatrix}}_{\text{Given}}(3)$$

$$= \begin{pmatrix} 2.915 \\ -0.7 \end{pmatrix}$$

$$\rightarrow \bar{\Sigma}_t = \begin{pmatrix} 1 & 0.1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0.1 & 1 \end{pmatrix} + \begin{pmatrix} 0.1 & 0 \\ 0 & 0.04 \end{pmatrix}$$

$$= \begin{pmatrix} 4.11 & 0.1 \\ 0.1 & 1.04 \end{pmatrix} \Bigg\} \text{ symmetric \& higher than } \Sigma_{t-1}$$

Prediction:

$\bar{\mu}_t = A_t \mu_{t-1} + B u_t$

$\bar{\Sigma}_t = A_t \Sigma_{t-1} A_t^T + R_t$

Correction:

$\mu_t = \bar{\mu}_t + K_t(z_t - C_t \bar{\mu}_t)$

$\Sigma_t = (I - K_t C_t)\bar{\Sigma}_t$

$\boxed{1.7}$ $z_1 = 2m$, $\sigma_z = 0.1$, what are the mean & Covariance of the Corrected State?

$\hookrightarrow C = (1 \; 0)$ $\hookrightarrow Q_t = (\sigma_z^2) = (0.01)$

$$\rightarrow \mu_t = \begin{pmatrix} 2.915 \\ -0.7 \end{pmatrix} + \begin{pmatrix} 0.998 \\ 0.024 \end{pmatrix}\overbrace{\left(2 - (1 \; 0)\begin{pmatrix} 2.915 \\ -0.7 \end{pmatrix}\right)}^{-0.915}$$

$$= \begin{pmatrix} 2.915 - 0.915 * 0.998 \\ -0.7 - 0.915 * 0.024 \end{pmatrix} = \begin{pmatrix} 2.002 \\ -0.722 \end{pmatrix}$$

$$\rightarrow \Sigma = \left[\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 0.998 & 0 \\ 0.024 & 0 \end{pmatrix}\right]\begin{pmatrix} 4.11 & 0.1 \\ 0.1 & 1.04 \end{pmatrix}$$

$$= \begin{pmatrix} 0.002 & 0 \\ -0.024 & 1 \end{pmatrix}\begin{pmatrix} 4.11 & 0.1 \\ 0.1 & 1.04 \end{pmatrix} = \begin{pmatrix} 0.008 & 0 \\ 0.001 & 1.038 \end{pmatrix}$$

$$\rightarrow S_t = (1 \; 0)\begin{pmatrix} 4.11 & 0.1 \\ 0.1 & 1.04 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + (0.01) = 4.12$$

$$\rightarrow K_t = \begin{pmatrix} 4.11 & 0.1 \\ 0.1 & 1.04 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} * \frac{1}{4.12} = \begin{pmatrix} 4.11 \\ 0.1 \end{pmatrix} * \frac{1}{4.12}$$

$$= \begin{pmatrix} 0.998 \\ 0.024 \end{pmatrix}$$

$S_t = C_t \bar{\Sigma}_t C_t^T + Q_t$

$K_t = \bar{\Sigma}_t C_t^T S_t^{-1}$