Institut für Regelungstechnik
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MPC – Exercise 1

The main goals of this exercise are:

- Learn how to implement an MPC algorithm with zero-terminal-constraint.
- Explore the influence of the prediction horizon and the constraints on the closed-loop behavior.

Consider the continuous-time constrained optimal control problem (compare [1])

$$
\begin{aligned}
&\underset{\bar{u}(\cdot;t)}{\text{minimize}} && \int_t^{t+T} \|\bar{x}(\tau;t)\|_Q^2 + \|\bar{u}(\tau;t)\|_R^2 \,\mathrm{d}\tau && \text{(1a)}\\
&\text{subject to} && \dot{\bar{x}} = f(\bar{x},\bar{u}) && \text{(1b)}\\
& && \bar{x}(t;t) = x(t) && \text{(1c)}\\
& && \bar{u}(\tau;t) \in \mathcal{U} && \text{(1d)}\\
& && \bar{x}(t+T;t) = 0, && \text{(1e)}
\end{aligned}
$$

where $\|x\|_Q^2 = x^\top Qx$, and with the nonlinear system dynamics $\dot{x} = f(x,u)$, where $f(x)$ is given by

$$
\begin{aligned}
\dot{x}_1 &= x_2 + u\,(\mu + (1-\mu)\,x_1)\\
\dot{x}_2 &= x_1 + u\,(\mu - 4\,(1-\mu)\,x_2).
\end{aligned} \tag{2}
$$

The system dynamics $f(x,u)$ are already implemented as a function `dynamics(x,u)` in the file `MPC_Exercise_1.m`. The constraint set is

$$\mathcal{U} := \{u \in \mathbb{R} : |u| \leq 2\} \ .$$

In the following, we set $\mu = 0.5$. Moreover, the weighting matrices are

$$Q = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix} \text{ and } R = 1.0 \ .$$

The above optimal control problem (1) is discretized using the RK4 method, which is given by

$$
\begin{aligned}
k_1 &= f(x(t),u(t)) & k_2 &= f(x(t) + \frac{\delta}{2}k_1, u(t))\\
k_3 &= f(x(t) + \frac{\delta}{2}k_2, u(t)) & k_4 &= f(x_t + \delta k_3, u(t))\\
x(t+1) &\approx x(t) + \frac{\delta}{6}\,(k_1 + 2k_2 + 2k_3 + k_4)\,.
\end{aligned}
$$

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Problem 1 Solve the optimal control problem using the Casadi Toolbox (available for download from `https://web.casadi.org/get/`). For the discretization of the nonlinear continuous-time system, the classical Runge-Kutta method (RK4) with zero-order hold and a sampling time $\delta = 0.1$ should be used. The prediction horizon T is chosen to be 5 time-units. The initial state is $x(0) = \begin{bmatrix} 0.4 & -0.5 \end{bmatrix}^\top$.

a) Define the optimization variables $\bar{u}(\cdot; t)$ and $\bar{x}(\cdot; t)$.

b) Implement the constraints of the optimal control problem:

- Implement the dynamics constraint (1b). In order to do so, discretize the continuous-time system dynamics (2) using the RK4 method above.
- Implement the initial constraint (1c).
  Hint: Define $x(t)$ as a parameter of the optimization problem and set $x(t) = x(0)$.
- Implement the terminal constraint (1e).
- Implement the input constraint (1d).

c) Define the cost function of the optimization problem (1a).

d) Provide an initial guess for the optimization variables.

e) Solve the optimal control problem and plot the results.

Problem 2 We want to implement an MPC algorithm for this system, i.e., not only solve the optimal control problem once, but repeatedly in a closed-loop manner. For this, we set the simulation time to 5 and solve the optimal control problem within a `for`-loop and store the closed-loop data. Implement the following additional changes in order to get a closed-loop simulation.
At each iteration,

a) update the initial constraint, i.e., the parameter $x(t)$,

b) and update the initial guess.

c) Simulate the closed loop. What do you expect? What happens?

Problem 3 Try different initial states, prediction horizon lengths, weights Q and R, different constraint sets $\mathcal{U}$, and add state constraints $\mathcal{X}$, and analyze the changes.

Literatur

[1] Hong Chen and Frank Allgöwer. A Quasi-Infinite Horizon Nonlinear Model Predictive Control Scheme with Guaranteed Stability. *Automatica*, 34(10):1205–1217, 1998.