MPC Exercise 3 - MPC Optimization problem

Robust MPC optimal control problem

At time t, given $x(t)$, solve

$$\underset{z(t|t),v(\cdot|t)}{\text{minimize}} \quad J(x(t), v(\cdot|t))$$

$$= \underset{z(t|t),v(\cdot|t)}{\text{minimize}} \quad \sum_{k=t}^{t+N-1} \delta L(z(k|t), v(k|t)) + F(z(t+N|t))$$

such that
$$z(k+1|t) = Az(k|t) + Bv(k|t) \qquad t \leq k \leq t+N-1$$
$$x(t) \in z(t|t) \oplus S$$
$$z(k|t) \in \mathbb{Z} := \mathcal{X} \ominus S \qquad\qquad t \leq k \leq t+N$$
$$v(k|t) \in \mathbb{V} := \mathcal{U} \ominus KS \qquad\qquad t \leq k \leq t+N-1$$
$$z(t+N|t) \in \mathbb{Z}^f \subseteq \mathbb{Z}$$

$u(t) = v(t|t) + Kx(t)$
$\rightarrow$ Need to compute K, S, $\mathbb{Z}^f$, $F(\cdot)$!

What is the MPT...

- free Matlab Toolbox
  - extensive library for polytope operations and computational geometry
  - various solvers for convex optimization and multi-parametric programming ($\rightarrow$ explicit MPC)
  - modeling and controller design for constrained systems (PWA, hybrid)
  - post-processing, analysis, and graphical evaluation

- user-friendly interface, "unpack-and-use" toolbox; but usage and modification of lower level functions also possible

- interfaces to external optimization software (SeDuMi, YALMIP, CDD,...)

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MPT for Polyhedron computations

- Define Polyhedron $S = \{x \mid Ax \le b, A_{eq}x = b_{eq}\}$
  ```
  S = Polyhedron('A',A,'b',b'Ae',Aeq,'be',beq)
  ```
  or (Vertex-Representation)
  ```
  A = Polyhedron('V',V)
  ```
  (V containts vertices row-wise)
- Minkowski sum $S_3 = S_1 \oplus S_2$
  ```
  S3 = S1+S2
  ```
- Pontryagin difference $S_3 = S_1 \ominus S_2$
  ```
  S3 = S1-S2
  ```
- Extract matrices $A, b, \dots$ from Polyhedron $S = \{x \mid Ax \le b, A_{eq}x = b_{eq}\}$
  ```
  A = S.A, ...
  ```
- Extract Vertices from Polyhedron $S = \{x \mid Ax \le b, A_{eq}x = b_{eq}\}$
  ```
  v = S.V
  ```
  ($v = [v_1, \dots, v_n]$)
- Plot Polyhedron S
  ```
  plot(S) or S.plot
  ```
- Set inclusion $S_1 \subseteq S_2$
  ```
  S1 <= S2
  ```
- And many more operations:
  projection, extreme point, chebyball, vertex represenation, convex hull, check equivalence, union, ...