EECI Course on

# Nonlinear and Data-based

# Model Predictive Control


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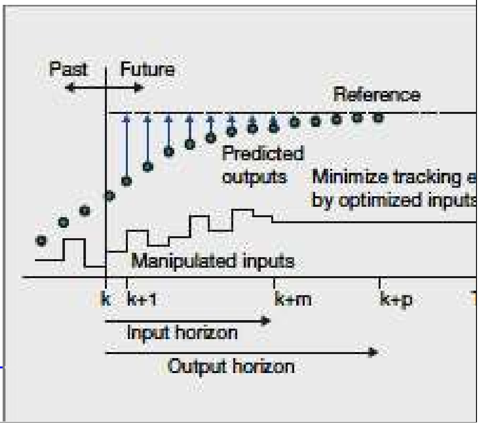


## The Lecturers of this Module


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## Main Goals of the Course

- Give an introduction to *Model Predictive Control (MPC)* and in particular to *Nonlinear MPC (NMPC)*
- Focus of course is on systems theoretic properties and issues
- Show how to achieve a stable closed loop system
- Give an introduction to data-based MPC
- Give an introduction to economic MPC
- Give an introduction to robust MPC
- Give a feeling and some insight into selected current trends in the MPC field (Explicit MPC, Moving Horizon Estimation, Output feedback MPC, Distributed MPC)

# An Introduction to Model Predictive Control

# Analytical vs. Numerics-based Control Methods

**Analytical controllers**

- Control law is synthesized off-line
- Control action is analytical function of present and past measurements

**Numerics-based controllers**

- Controller not implemented as an analytical function of past and present measurements
- On-line numerical computations take, at least, partly role of off-line controller design

A numerics-based control technique:

Model Predictive Control (MPC)

## A VERY GENERAL Control Problem !!!

$$\dot{x} = f(x, u), \quad x(0) = x_0$$

General nonlinear system

Find **stabilizing** control strategy that

- minimizes **objective functional**

$$J = \int_t^{\infty} F(x(\tau), u(\tau)) d\tau$$

General nonlinear objective functional

- satisfies **constraints**

$$u(\tau) \in \mathcal{U}$$
$$x(\tau) \in \mathcal{X}$$

Additional constraints on inputs and states

## A Possible Solution - Model Predictive Control

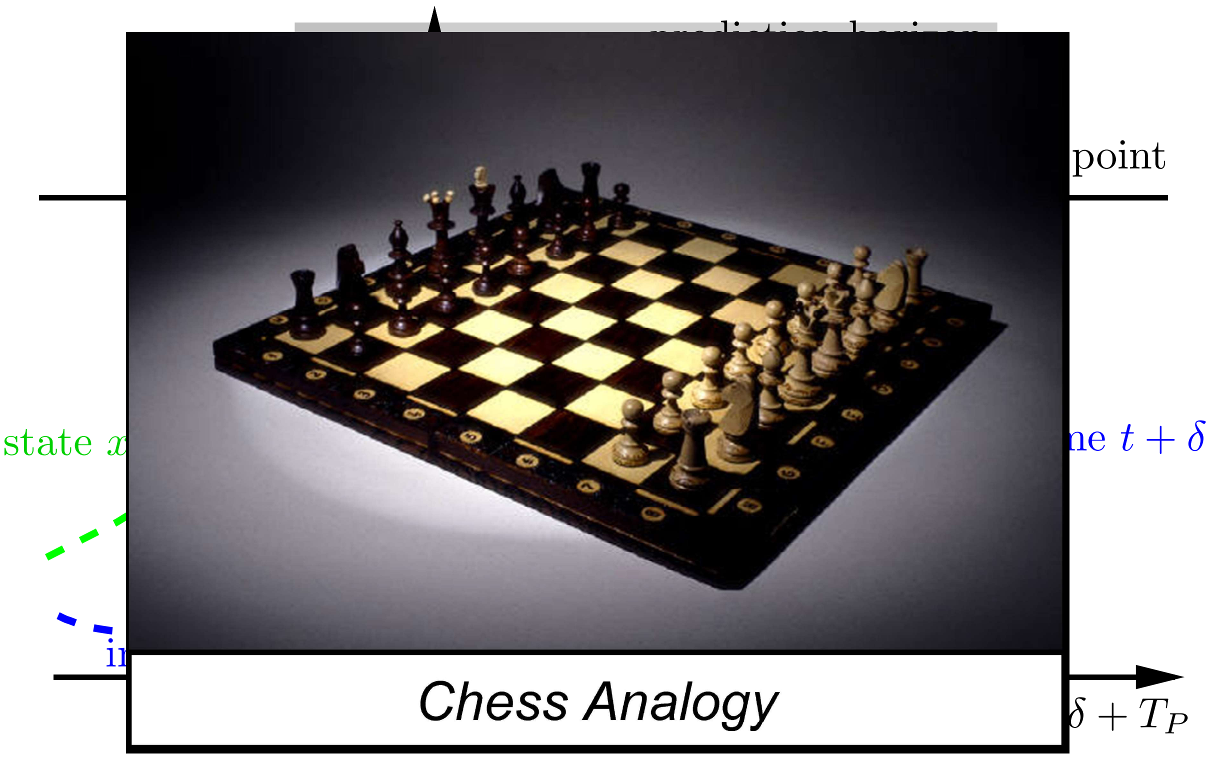
MPC= repeated **open-loop optimal control**

- Solve open-loop optimization problem all $\delta$ sampling instances

$$\min_{u(\cdot)} J(u(\cdot); x(t)) = \int_t^{T_P} F(x(\tau), u(\tau)) d\tau$$

- Apply optimal open-loop input for $\tau \in [t, t + \delta]$
- Use finite Prediction horizon $T_p$

## MPC




*Chess Analogy*

## Characteristics of MPC

- Moving horizon implementation
- Performance oriented **time domain formulation**
- Incorporation of **constraints**
- **Explicit system model used** to predict future plant dynamics

## MPC: Open-Loop Optimization Problem

**(to be solved at each time step)**

**Finite horizon optimal control problem**

$$\min_{u(t)} \sum_{k=0}^{N-1} \ell\big(x(k|t), u(k|t)\big)$$

$$\text{s.t. } x(k+1|t) = f\big(x(k|t), u(k|t)\big),\ x(0|t) = x(t)$$

$$\big(x(k|t), u(k|t)\big) \in \mathbb{Z}, \quad \forall k \in \mathbb{I}_{[0,N-1]},$$

- Dynamics/model of the system
- Stage cost function
- Hard constraints on states and inputs
-

ist

# Linear MPC - Nonlinear MPC

### Linear MPC

- Uses **linear model**: $\dot{x} = Ax + Bu$
- Quadratic cost function $F = x^T Rx + u^T Ru$
- Linear constraints $Hx + Gu < 0$
- $\Rightarrow$ Quadratic program

### Nonlinear MPC (NMPC)

- **Nonlinear model:** $\dot{x} = f(x, u)$
- Cost function can be nonquadratic $F(x, u)$
- Nonlinear constraints $h(x, u) < 0$
- => Nonlinear program

# A Brief History of MPC

**Idea is rather old:**

> "One technique for obtaining a **feedback controller synthesis from knowledge of open-loop controllers** is to measure the current control process state and then compute vary rapidly for this the open-loop control function. The first portion of this function is then used during a short time interval, after which a new value of the function is computed for this measurement, The process is then repeated."

(Lee & Marcus, 1967)

# A Brief History of early MPC

**Early industrial MPC applications:**

- Model Predictive Heuristic Control (IDCOM)
  *Richalet et al. 1976 ...*
  Adersa
- Dynamic Matrix Control (DMC)
  *Cutler & Ramaker 1979 ...*
  Shell Oil

**Academic research:**

- Few early theoretical investigations:
  *Kleinmann 1970, Thomas 1975, Chen & Shaw 1982, ...*
- Predictive control theory:
  *Keerthi & Gilbert 1988, Mayne & Michalska 1990, ...*

## Issues in Nonlinear MPC

- **System theoretic formulation and investigation:**
  *stability, performance, robustness, ...*
- **Implementation issues:**
  *efficient and reliable real-time optimization, modeling, ...*



## Systems Theory for Predictive Controllers

Predictive controllers differ from most other control schemes in that

- no explicit control law is given
  => no standard system theoretic analysis is possible

## Are MPC Controllers Generically Stabilizing the Closed Loop?

No !!!!!

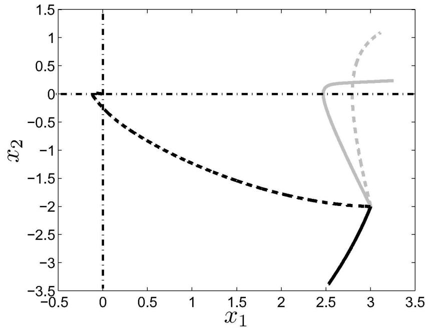
## Unstable closed loop for academic example

Example *[Primbs '00]*: Stabilize the nonlinear system

$$\dot{x}_1 = x_2$$

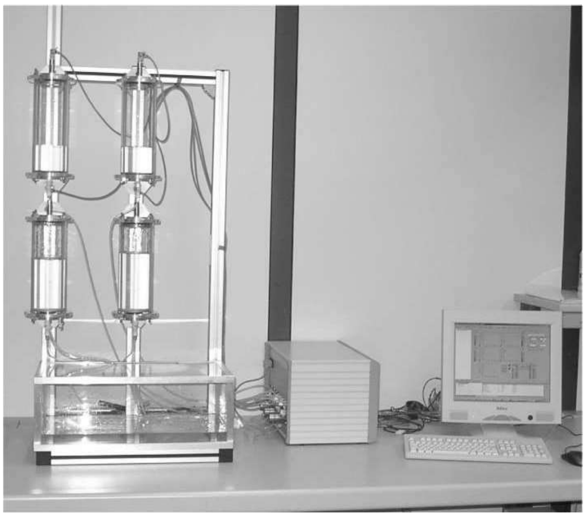
$$\dot{x}_2 = -x_1\left(\frac{\pi}{2} + \arctan(5x_1)\right) - \frac{5x_1^2}{2(1+25x_1^2)} + 4x_2 + 3u$$

via NMPC **without stability constraints** and the cost function $\int_{t_i}^{t_i+T} x_2^2 + u^2 d\tau$.




- — $T = 0.2$: unstable trajectory
- - - $T = 0.3$: stable trajectory
- — $T = 0.4$: unstable trajectory
- - - $T = 1.0$: unstable trajectory

# Experimental stability study of NMPC



Four tank system at the IST laboratory

Four tank system:

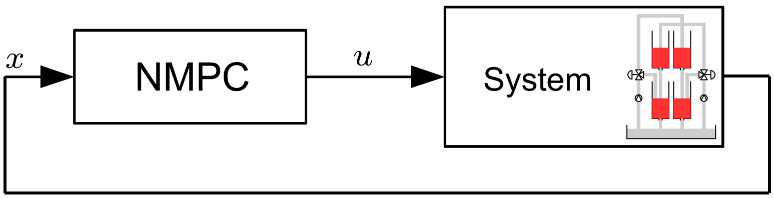
- Nonlinear system
- Input constraints
- State constraints
- “Slow” dynamics

Four tank system is suited for an experimental stability study of NMPC

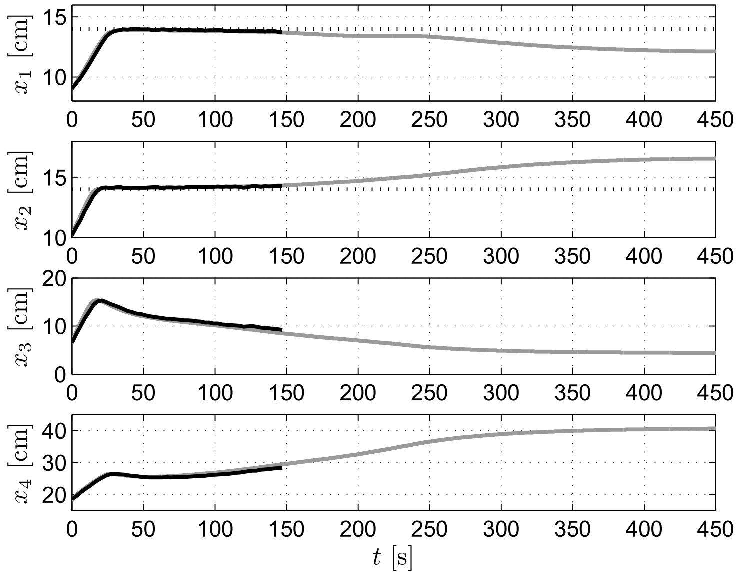
# NMPC of a Four Tank System

NMPC setup:

- $F(x,u) = (x_1 - x_{1s})^2 + (x_2 - x_{2s})^2 + 0.001((u_1 - u_{1s})^2 + (u_2 - u_{2s})^2)$
- State constraints:
  - $x_{min} = [7.5cm\ 7.5cm\ 7.5cm\ 4.5cm]^T$
  - $x_{max} = [28cm\ 28cm\ 28cm\ 28cm]^T$
- Input constraints:
  - $u_{min} = [0ml/s\ 0ml/s]^T$
  - $u_{max} = [60ml/s\ 60ml/s]^T$
- Control input is piecewise constant
- Sampling time is $\Delta = t_{i+1} - t_i = 3s$
- NMPC implementation based on *OptCon toolbox [Nagy’05]*



# Simulation and Experiment (1)

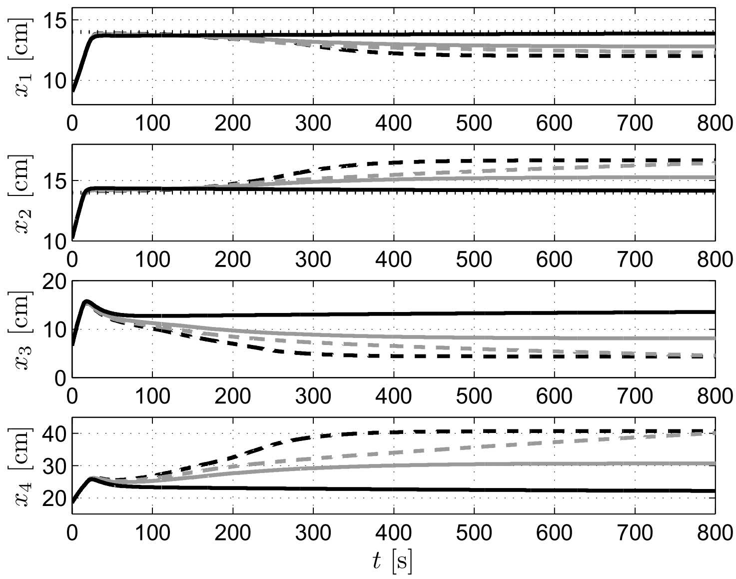



Closed-loop is unstable

- Sim.—, Exp. —
- Setpoint:
  - $x_{1,2s} = 14cm$
  - $\gamma_{1,2} = 0.4$
- Prediction horizon:
  - $T = 60s$
- Input constraints



# Simulation and Experiment (2)




- Only simulations
- Setpoint:
  - $x_{1,2s} = 14cm$
  - $\gamma_{1,2} = 0.4$
- Prediction horizon:
  - - -$T = 60s$: unstab.
  - - -$T = 75s$: unstab.
  - —$T = 90s$: unstab.
  - —$T = 135s$: stable
- Input constraints

## Systems Theory for Predictive Controllers

Another idea:

- **use predicted trajectories to assess closed loop properties**



## An Important Fact in (N)MPC Theory

Even in nominal case:

- no model plant mismatch
- no disturbances

Predicted open-loop trajectories
$\neq$
Closed-loop trajectories

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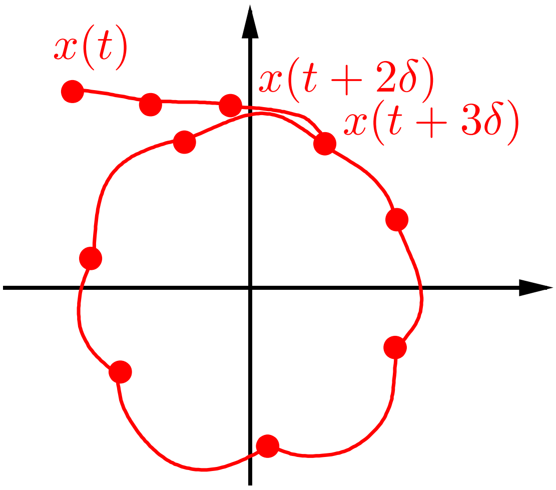


- **Performance?** Goal: $\min \int_0^\infty F(x(\tau), u(\tau))d\tau$. What is achieved by repeatedly minimizing $\int_t^{t+T_p} F(\mathbf{x}(\tau), u(\tau))d\tau$ ?
- **Stability?** Why should the closed-loop be stable?

## Approaches to Achieve Stability

Two possible approaches:

a) **Choose** $T_p$, $F$, ... such that closed loop is stable

b) **Alter problem setup** (cost functional, constraints) such that closed loop is stable independent of the choice of $T_p$, ...

$\Rightarrow$ **"guaranteed stability" NMPC scheme**

Many schemes that guarantee stability exist:

- Infinite horizon NMPC
- Zero terminal constraint NMPC
- Expanding the horizon quasi to infinity
- Unconstraint NMPC
- ...

## Learning Outcomes of *Introduction to MPC*

- MPC solves very **general Optimal Control Problem.**
- In particular: **Constraints** on inputs and states can be considered.
- MPC requires **repeated online solution** of **finite-horizon open-loop optimal control problem** in real time.
- Finite horizon is not a feature, but an (unwanted) computational necessity.
- **“Plain vanilla” MPC is not a stabilizing control approach !!!**
- Relationship between MPC solution and infinite horizon optimal control solution is not clear for “plain vanilla” MPC.
- Stability analysis of MPC cannot be done using standard methods, because we have no control law $u(x)$.
- **“Plain vanilla” MPC is not a good control approach: We need better MPC formulations -> goal of this course**

## Structure of Course: Focus is on MPC Theory

- The principle of model predictive control (MPC)
- Systems theoretic background material
- Stability of the nominal closed loop
  - Zero-terminal-constraint NMPC
  - Quasi-infinite horizon NMPC
  - Unconstrained NMPC
- Data-based MPC
- Economic MPC
- Robust MPC
- Current trends in MPC
  - Explicit MPC
  - Receding Horizon Estimation
  - Output feedback MPC
  - Distributed MPC
- Exercises