Learning-Based Predictive Control

Chapter 2b
Performance Learning-based MPC

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Learning Objectives

- Integrate an online-learning algorithm into a control algorithm
- Understand how to decouple performance and safety
  - Recursive feasibility and constraint satisfaction are independent of the cost function
  - Performance depends on the cost function
- See how linear robust MPC can be used for mildly nonlinear systems
- Introduce the typical high-level work-flow of learning-based control

Main reference for this lecture:
[1] Anil Aswani, Humberto Gonzalez, Sastry S. Shankar. and Claire Tomlin. 2013 Provably Safe and Robust Learning-Based Model Predictive Control. Automatica.

Outline

1. Problem Formulation

2. Robust Performance Learning-Based MPC

3. Example

Problem Formulation

Consider the nonlinear system

$$x(k+1) = Ax(k) + Bu(k) + g(x(k), u(k)) + w(k)$$

The system is subject to polytopic constraints

$$\begin{aligned} x \in \mathcal{X} &= \{x | H_x x \leq h_x\}, \\ u \in \mathcal{U} &= \{u | H_u u \leq h_u\}. \end{aligned}$$

Problem Formulation

Consider the nonlinear system

$$x(k+1) = \underbrace{Ax(k) + Bu(k)}_{\text{nominal dynamics}} + \underbrace{g(x(k), u(k)) + w(k)}_{:= \bar{w}(k) \text{ model error}}$$

The linear part represents the nominal dynamics, the second term the modeling error and disturbances.

Assumption: The model error $g(x,u) + w(k)$ is bounded and lies within a polytope $\mathcal{W}$, i.e.,

$$g(x,u) + w(k) \in \mathcal{W} \quad \forall x \in \mathcal{X},\ \forall u \in \mathcal{U},\ \forall w(k) \in \mathcal{D},$$

where $\mathcal{D}$ is a bounded set.

This is a standard assumption in (linear) robust model predictive control.

Outline

Robust Performance Learning-Based MPC [1]

Consider the nonlinear system

$$x(k+1) = \underbrace{Ax(k) + Bu(k)}_{\text{nominal dynamics}} + \underbrace{g(x(k), u(k)) + w(k)}_{:= \bar{w}(k) \text{ model error}}$$

Approach: Decouple performance and safety by introducing two models in the controller:

- A nominal linear model

  $$z_{i+1} = Az_i + Bv_i$$

  with which we ensure robustness against every $\bar{w}(k) \in \mathcal{W}$.
- A learned nonlinear model

  $$x_{i+1} = Ax_i + Bv_i + \mathcal{O}_k(x_i, v_i)$$

  where $\mathcal{O}_k$ is a time-varying function called the oracle, representing an arbitrary learned model.

Oracle Examples

The oracle $\mathcal{O}_k$ is a black-box that takes in state and input and gives as answer an estimated model correction term. To allow efficient optimization, it should also return the gradient at the given input.
Examples:

- Bayesian linear regression

$$\underbrace{x(k+1) - Ax(k) - Bv(k)}_{y_i} = \phi(x(k), v(k))^T \theta$$

- Gaussian process
- Neural network
- Parametric model with parameters estimated via least-square
- etc

Note: If you make the oracle a prediction function also of time $O_k(x(k), v(k), k)$, the prediction may also include predicted disturbances, e.g. wind and weather.

Robust Performance LBMPC

The following optimization problem defines Robust Performance LBMPC

$$J_k^\star(x(k)) = \min_u \; l_f(x_N) + \sum_{i=0}^{N-1} l(x_i, v_i)$$

$$\text{s.t. } x_0 = x(k), \quad z_0 = x(k)$$

$$x_{i+1} = Ax_i + Bv_i + \mathcal{O}_k(x_i, v_i)$$

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z_{i+1} = Az_i + Bv_i& \\
z_i \in \mathcal{X} \ominus \mathcal{F}_i, \quad v_i \in \mathcal{U} \ominus K\mathcal{F}_i& \\
z_{i+N} \in \mathcal{X}_f \ominus \mathcal{F}_N&
\end{aligned}
$$

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\end{aligned}\right\} \begin{array}{c}\text{"Linear robust constraint} \\ \text{tightening MPC" constraints}\end{array}
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The applied input is $u(k) = v_0^\star(x(k))$.
Assumptions on $\mathcal{F}_i, \mathcal{X}_f$: see Lecture 1.

$\Rightarrow$ Recursive feasibility follows from linear constraint tightening MPC results.
$\Rightarrow$ Robust constraint satisfaction: $x(k+1) = Ax(k) + Bv_0^*(x(k)) + \bar{w}(k) = z_1^*(x(k)) + \bar{w}(k) \in \mathcal{X}$.

Performance and Safety

1. Note that the cost function depends on the states of the learned model, which uses the oracle $\mathcal{O}_k(x, v)$ for predictions.
2. Recursive feasibility and therefore robust safety does not depend on the terms in the cost function. This is one of the reasons why Robust Performance LBMPC decouples safety from performance.
3. It is important that the oracle $\mathcal{O}_k(x, v)$ can be time-varying, as we want to update the learned model from online collected data.

Updating uncertainty bounds: While the performance model can be updated online, the uncertainty set $\mathcal{W}$ can not. One major problem is that most techniques, e.g. Bayesian linear regression, would not ensure $\mathcal{W}_{k+1} \subseteq \mathcal{W}_k$. In the next lecture, we will address this with a specific estimation technique.

Stability

Lemma

Let $\mathcal{X}_N$ be the feasible set of the Robust Performance LBMPC. If the functions l, l_F, and $\mathcal{O}_k$ are continuous, then $J_k^\star(x(k))$ is continuous on $\text{int}(\mathcal{X}_N)$.

Input to state stability (ISS) can be proven by assuming the oracle to be a disturbance on the MPC solution.

Summary:

- practical effect of oracle typically "good"
- without additional assumptions: "bad" for theoretical stability analysis (the larger the oracle, the worse the stability property)

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Robust Performance Learning-Based MPC on a Quadrotor [2]

Goal
Real-time control of a quadcopter using Robust Performance LBMPC.

Figure: Credits IEEE Spectrum.

Experiment - Step Response

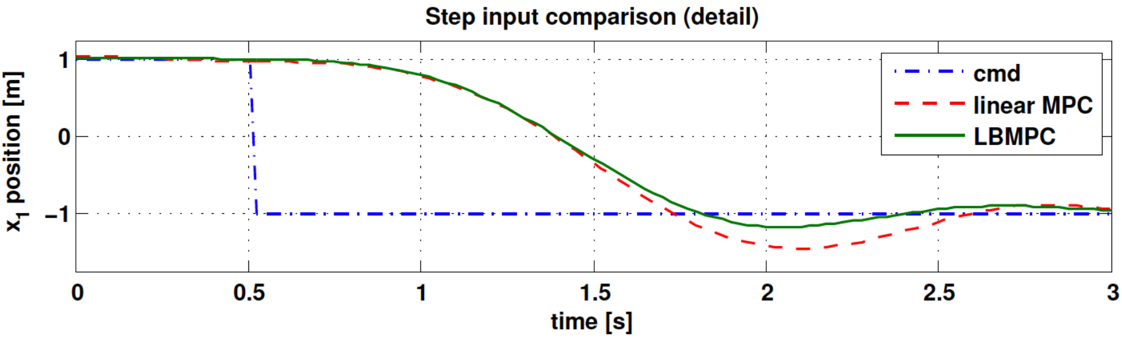
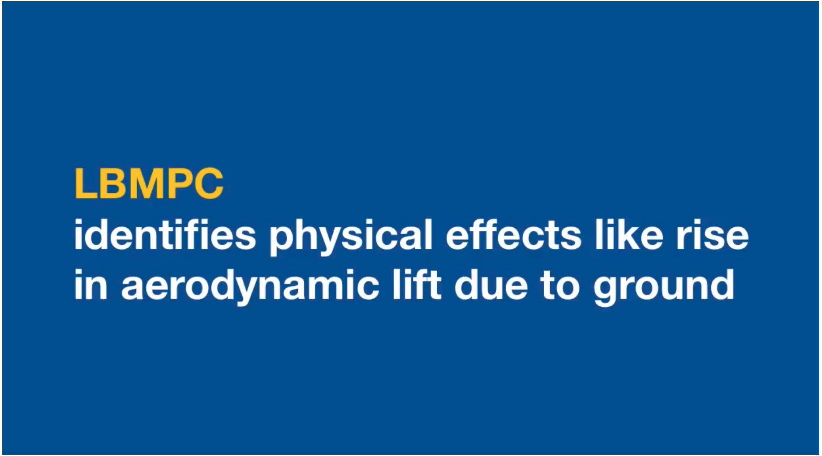


Figure: Variation of thrust input mapping [2].

Video

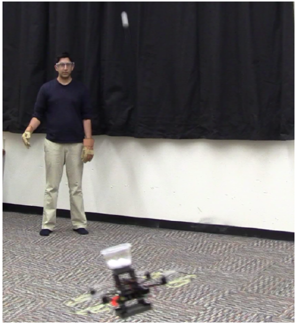


[*Robust Performance LBMPC* `https://www.youtube.com/watch?v=dL_ZFSvLX1U`]

Ball Catching Experiment

Example

Ball catching experiment: catch a ball thrown by a human with a quadcopter before it hits the ground.



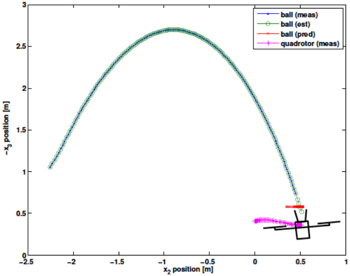
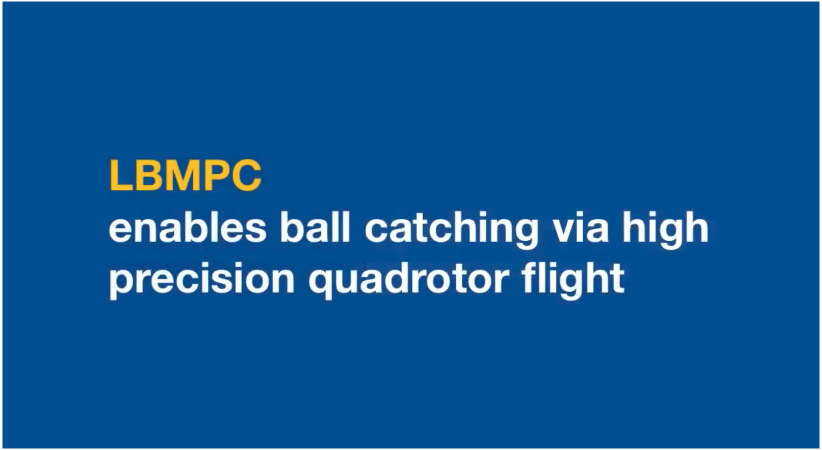


Figure: Ball catching experiment [2].

Video



[*Ball Catching Experiment* `https://www.youtube.com/watch?v=dL_ZFSvLXlU`]

Summary: Robust Performance Learning-Based MPC

- Main idea: Safety and performance is decoupled by using two models
  - Constraint satisfaction is guaranteed by the nominal linear model
  - Performance is handled by the performance model
- Benefits:
  - Online learning can significantly improve the performance of the system
  - Guarantees on recursive feasibility and constraint satisfaction
- Limitations:
  - Uncertainty set cannot be updated
  - Learned model considered as "disturbance" (the more freedom given to the oracle, the more restrictive the controller)

References and further reading

[1] Anil Aswani, Humberto Gonzalez, Sastry S. Shankar. and Claire Tomlin. 2013 Provably Safe and Robust Learning-Based Model Predictive Control. Automatica.

[2] Patrick Bouffard, Anil Aswani, and Claire Tomlin. 2012. Learning-Based Model Predictive Control on a Quadrotor: On-board Implementation and Experimental Results. 2012 IEEE International Conference on Robotics and Automation.

[3] Anil Aswani, Patrick Bouffard, and Claire Tomlin. 2012. Extensions of Learning-Based Model Predictive Control for Real-Time Application to a Quadrotor Helicopter. In Proceedings American Control Conference (ACC), (Montreal, Canada).