EUROPEAN EMBEDDED CONTROL INSTITUTE (EECI) – INTERNATIONAL GRADUATE SCHOOL ON CONTROL

# Learning-Based Predictive Control

# Chapter 4
# Adaptive MPC via on-line Set Membership identification

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# Adaptive control with guaranteed constraint satisfaction

We have seen how SM estimation approaches can quantify uncertainty and deliver guaranteed uncertainty sets.

We will now see how these concepts can be used to set up a predictive control strategy that can **safely learn on-line** (i.e., during operation).

- Preserve performance when plant changes
- Avoid constraint violations during learning
- Self-tuning of control systems
- Improve performance over time

The focus is on **linear models with affine parametrization**.

# Some remarks

How restrictive are our working assumptions?

- Linear systems
- Affine parametrization

$\Rightarrow$ <u>Convex programs</u>

This is essentially a tradeoff between **generality** and **computational tractability**.

- Engineered systems are often designed to behave linearly
- Closed-loop systems are almost always designed to behave linearly
- Nonlinear systems can be often partitioned into nonlinearly-interacting linear ones
- Nonlinear dynamics can be accounted for in the uncertainty and/or in linear time-varying approaches

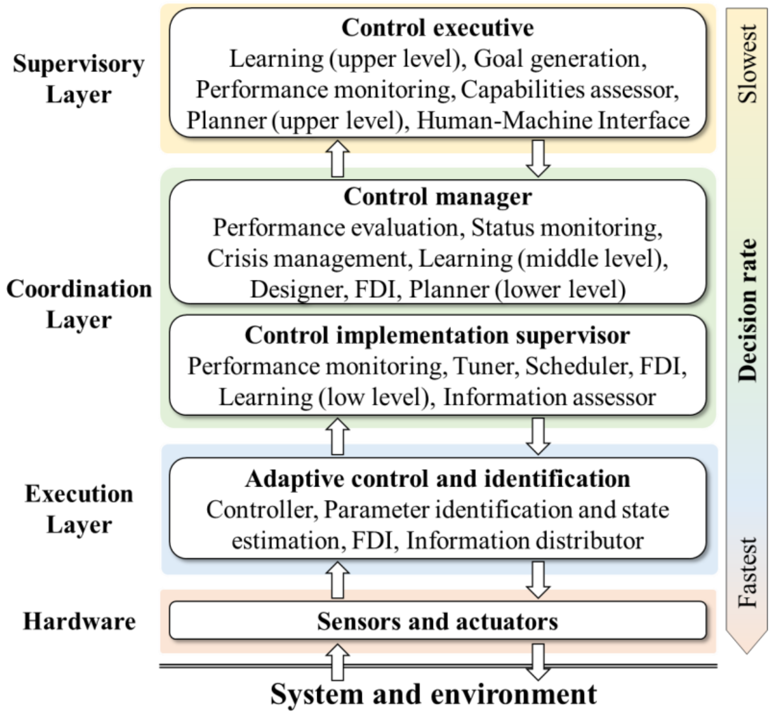
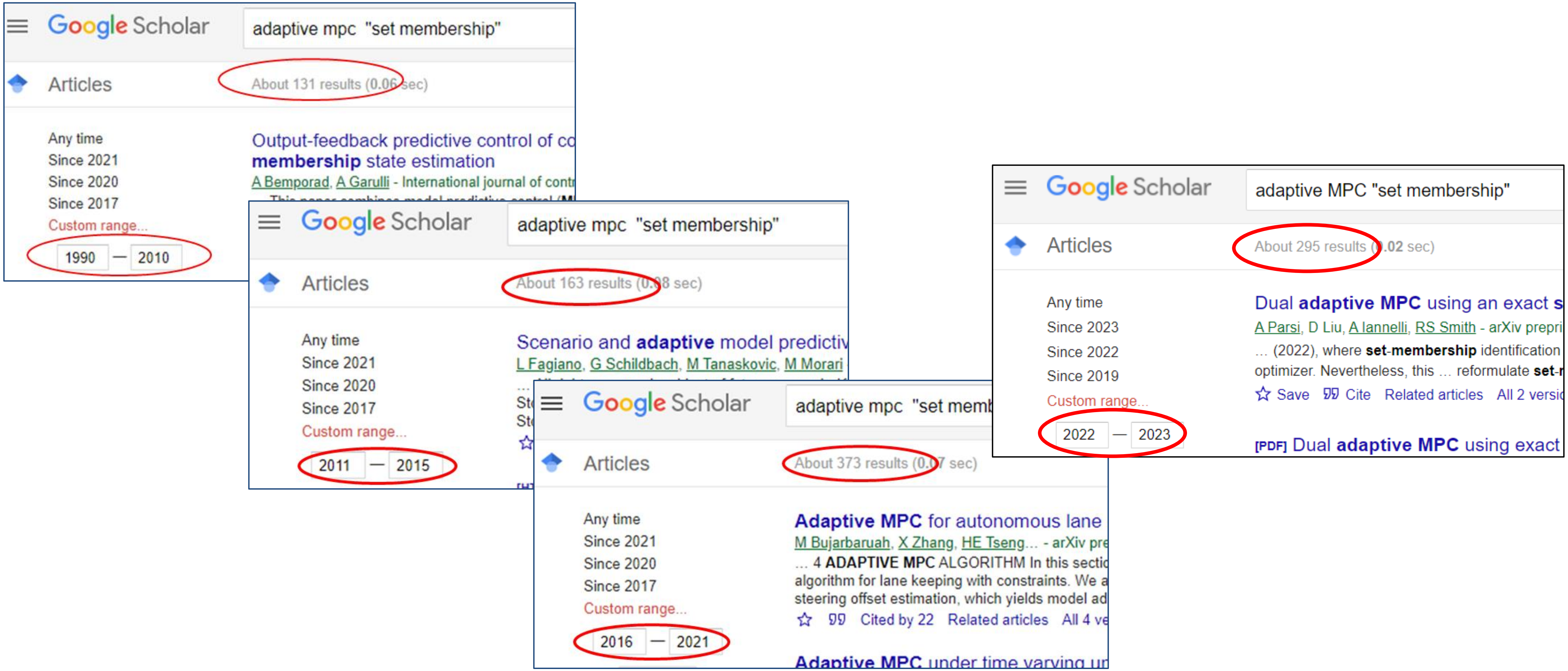



Figure adapted from [1]

# Literature

There are many contributions that employ “set-membership” concepts together with robust MPC, with (more or less) different working assumptions. Increasing interest.



# Adaptive MPC via on-line SM identification

We will review the main concepts, then provide more insights on:

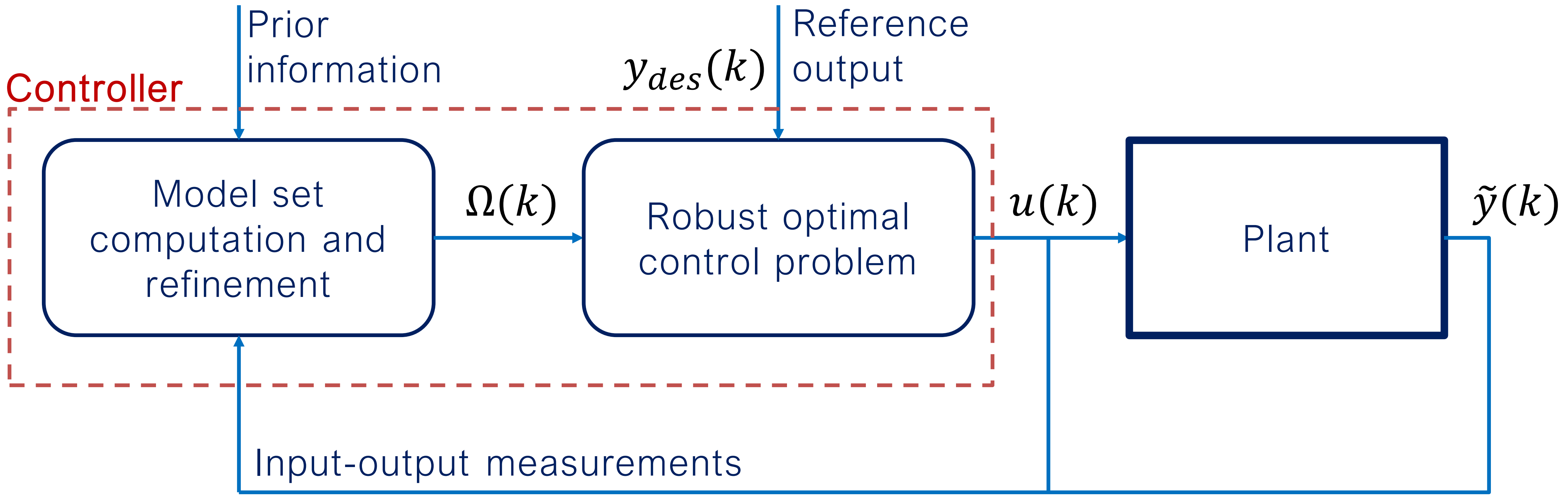
1. Output feedback with measurement noise & FIR structure
    - Time invariant systems [2]
    - Time varying systems [3]

2. State feedback w. polytopic model, time invariant (by L. Hewing in next lecture/exercise session, see refs. there)

# Adaptive MPC – prototype algorithm

1) At time step $k$, obtain the plant output measurement $\tilde{y}(k)$ and update the model set $\Omega(k)$;
2) Select a nominal model of the plant inside $\Omega(k)$;
3) Formulate and solve a Finite Horizon Optimal Control Problem (FHOCP) that:
   - Minimizes a convex cost function involving the nominal model trajectory;
   - Enforces input and output constraints for all models in $\Omega(k)$;
   - Guarantees recursive feasibility;
4) Apply the first element of the obtained optimal input sequence to the plant, set $k = k + 1$, go to 1)

# General structure of the control system




NOTE: this is a **dynamic controller** (vs. classical and robust MPC, which are typically static control laws)

# Adaptive MPC – key aspects

1) At time step $k$, obtain the plant output measurement $\tilde{y}(k)$ and update the model set $\Omega(k)$ → recursive (on-line) Set Membership identification;
2) Select a nominal model of the plant inside $\Omega(k)$;
3) Formulate and solve a Finite Horizon Optimal Control Problem (FHOCP) that:
   - Minimizes a convex cost function involving the nominal model trajectory;
   - Enforces input and output constraints for all models in $\Omega(k)$ → robust constraint satisfaction
   - Guarantees recursive feasibility → terminal ingredients
4) Apply the first element of the obtained optimal input sequence to the plant, set $k = k + 1$, go to 1)

# Adaptive MPC – challenges

- Recursive on-line SM identification
  - Deal with generally growing memory requirement;
  - Deal with time-varying case
- Robust constraint satisfaction
  - Deal with polytopic parameter set & time-varying case
- Recursive feasibility
  - Employ suitable terminal ingredients

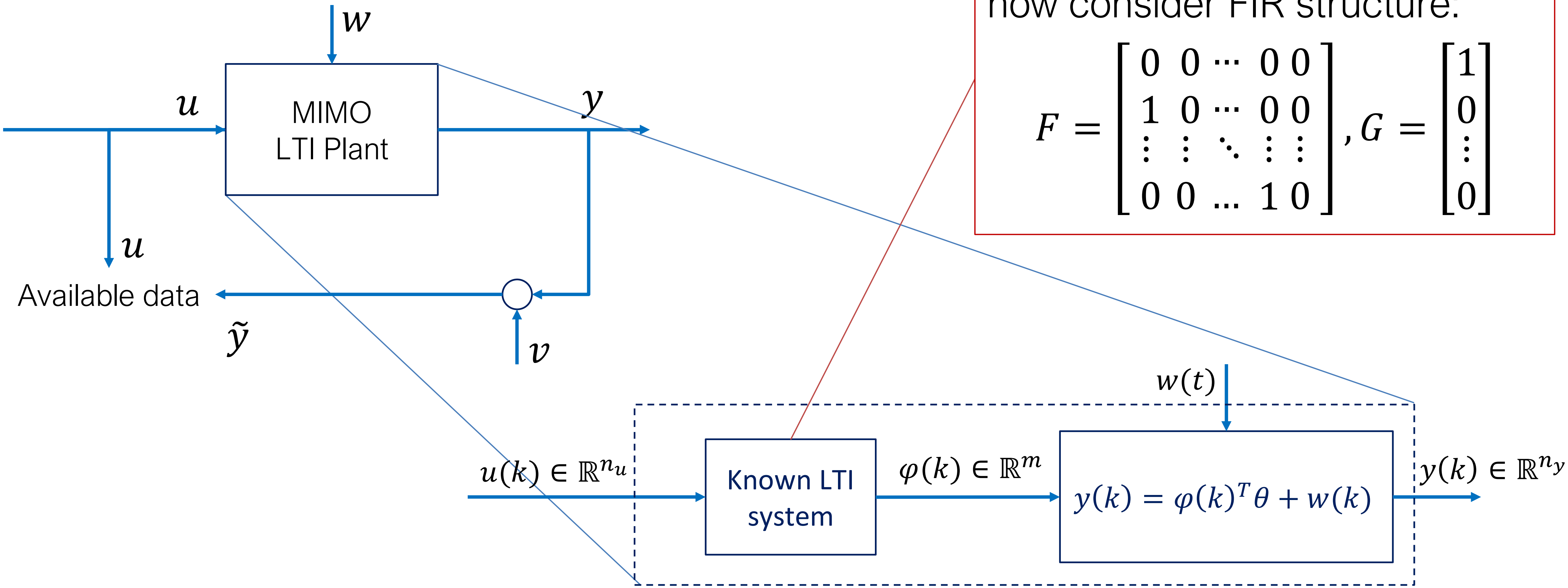
We will see how these are addressed through the description of three specific approaches. Other techniques in the literature follow similar strategies.

# Adaptive output-feedback MPC for LTI MIMO systems

Considered plant structure

$\varphi(k+1) = F\varphi(k) + Gu(k)$, for now consider FIR structure:

$$F = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & 0 \end{bmatrix}, G = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$



# Assumptions on system and signals

- $y_j(k) = \varphi(k)^T \theta_j + w_j(k), j =, \dots, n_y$, order $m$
  (for FIR model: $\varphi(k) = [u(k-1)^T, \dots, u(t-m)^T]^T \in \mathbb{R}^{n_u m}$)
- $\tilde{y}_j(k) = y_j(k) + v_j(k), j =, \dots, n_y$

Prior assumptions on $w$ and $v$:

- $|w_j| \leq \bar{w}_j$, $j =, \dots, n_y$
- $|v_j| \leq \bar{v}_j$, $j =, \dots, n_y$

From now on, we consider $j = 1$ and drop the index for notational simplicity. Then $\theta$ is a vector in $\mathbb{R}^{n_u m}$ and $\theta_i$ will denote its $i$-th component.

Prior assumptions on system

- Asymptotically stable
- $\theta = \left[\theta_1^T, \dots, \theta_{n_y}^T\right]^T \in \Omega_0 = \{\theta \in \mathbb{R}^{n_u m \times n_y}: H_{\Omega_0,j}\theta_j \leq h_{\Omega_0,j}, \; j =, \dots, n_y\}$ compact

# Assumptions on system and signals (2)

Asymptotic stability of the system is usually considered in identification and in adaptive control problems

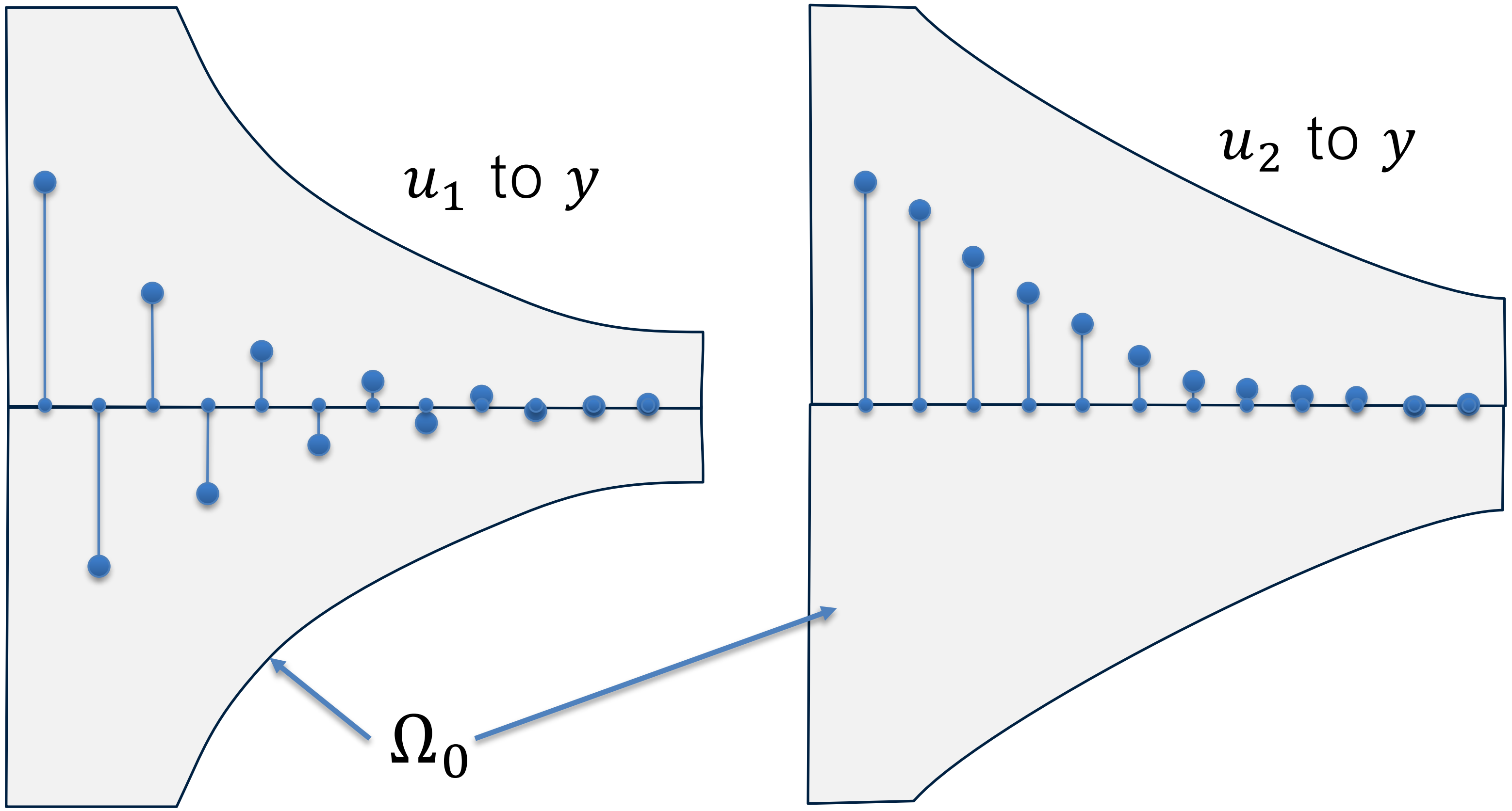
- Either explicitly
- Or implicitly (e.g., assuming the knowledge of a pre-stabilizing controller in technical proofs)

Under this assumption, the Infinite Impulse Response (IIR) coefficients of the system are exponentially decaying. Thus, sensible constraints $\Omega_0$ are:

$$\begin{aligned} |\theta_i| &\leq L && \text{if } i \in [1, \mu] \\ |\theta_i| &\leq L\rho^{i-\mu} && \text{if } i \in [\mu + 1, m] \end{aligned} \qquad \text{(for each input)}$$

# Assumptions on system and signals (3)

$$\begin{aligned} |\theta_i| &\leq L && \text{if } i \in [1, \mu] \\ |\theta_i| &\leq L\rho^{i-\mu} && \text{if } i \in [\mu+1, m] \end{aligned}$$ (for each input)




If additional prior information is available (e.g., the sign of the input-output gains), it can be included as further affine constraints in $\Omega_0$.

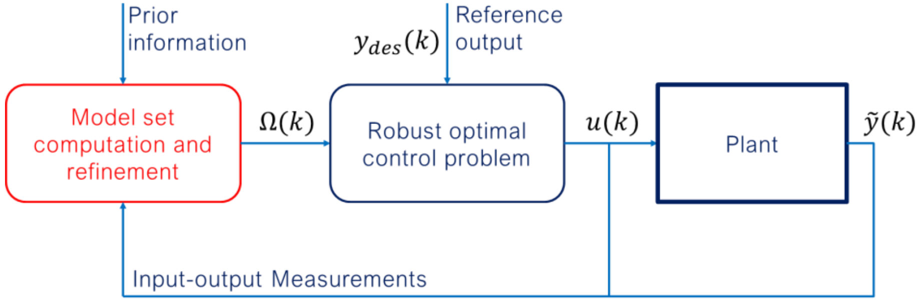
# Constraints & control objective

$$Cu(k) \leq g \quad \text{(input constraints)}$$
$$L\Delta u(k) \leq f \quad \text{(input rate constraints)}$$
$$Ey(k) \leq p \quad \text{(output constraints)}$$

Bounded input + exponentially decaying rate of IIR can be used to estimate a bound on the error due to IIR truncation, which can be included in the disturbance bound $\bar{w}$.

The control objective is to track a desired output signal $y_{des}$ (assumed constant for notational simplicity, no loss of generality).

# On-line set Membership identification



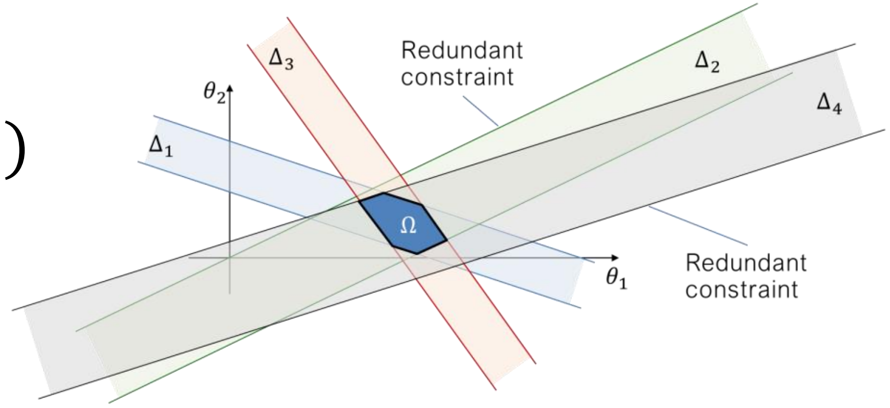

Recall what we have seen for affine parametrizations

- At time $k$, consider $\Omega(k-1)$ (without loss of generality $\Omega(0)=\Omega_0$ if there are no data points available)
- Consider the available data pair $(\tilde{y}(k), \varphi(k))$. Compute the set $\Delta(k)$ as:

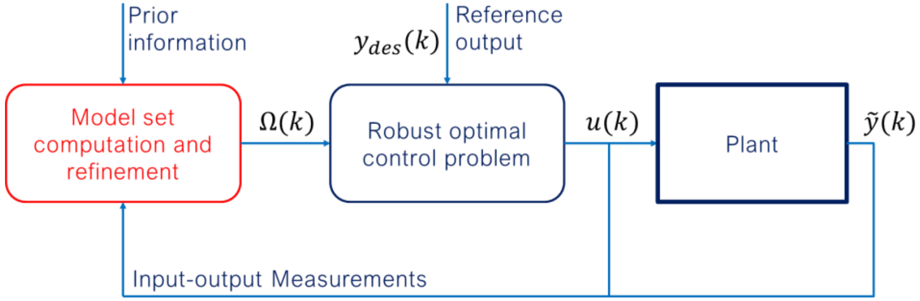
$$\Delta(k) = \{\theta \,|\, |\tilde{y}(k) - \varphi(k)^T\theta| \leq \bar{w} + \bar{v}\} = \left\{\theta \,\middle|\, \begin{matrix} \varphi(k)^T\theta \leq \tilde{y}(k) + \bar{w} + \bar{v} \\ \varphi(k)^T\theta \geq \tilde{y}(k) - \bar{w} - \bar{v} \end{matrix}\right\}$$ (slab in $m$ dimensions)

- Then we can update the model set as:

$$\Omega(k) = \Omega(k-1) \cap \Delta(k)$$



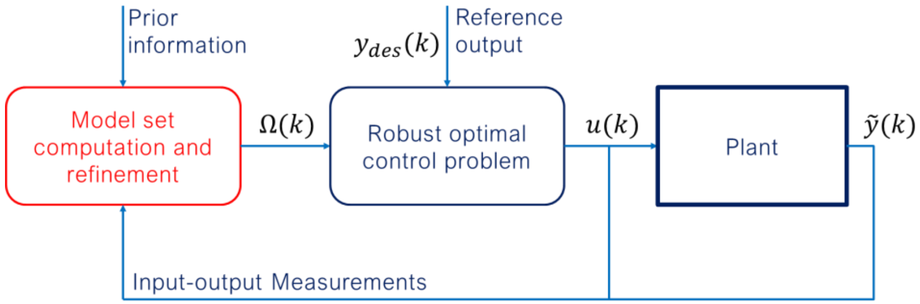
# Limiting the complexity of $\Omega(k)$




In general, the number of inequalities defining $\Omega(k)$ grows linearly with $k$.

- Redundant constraints can be removed to limit such a growth, by solving auxiliary LPs. However, this does not solve the problem;
- A way to keep the number of constraints below a user-defined maximum value $M = M_1 + M_2$ is the following:
  - Apply the normal update rule as long as the number of inequalities is smaller than $M_1$ (separating hyperplanes are determined by data);
  - Once the limit $M_1$ has been reached, add new separating hyperplanes that are parallel to one of $M_2$ pre-defined directions inside a user-defined set $D$.

# Limited-complexity model set update




$$\Omega(k) = \Omega(k-1) \cap \left\{ \theta \;\middle|\; \begin{array}{l} \varphi^+(k)^T\theta \le \tilde{y}(k) + \delta^+(k) \\ \varphi^-(k)^T\theta \le -\tilde{y}(k) + \delta^-(k) \end{array} \right\}$$

Where:

$$\varphi^+(k) = \arg\max_{f\in D} \varphi(k)^T f$$

$$\varphi^-(k) = \arg\max_{f\in D} -\varphi(k)^T f$$

Directions inside $D$ that are "closest" to the regressor $\varphi(k)$

$$\delta^+(k) = \max_{\theta} \varphi^+(k)^T\theta - \tilde{y}(k)$$

$$\delta^+(k) = \max_{\theta} \varphi^-(k)^T\theta - \tilde{y}(k)$$

subject to

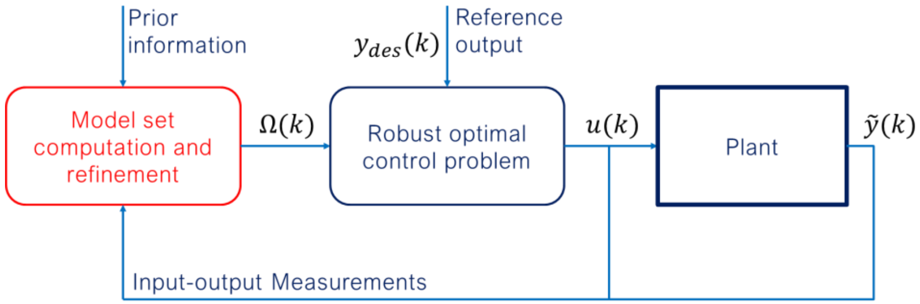
$$\theta \in \Omega(k-1) \cap \Delta(k)$$

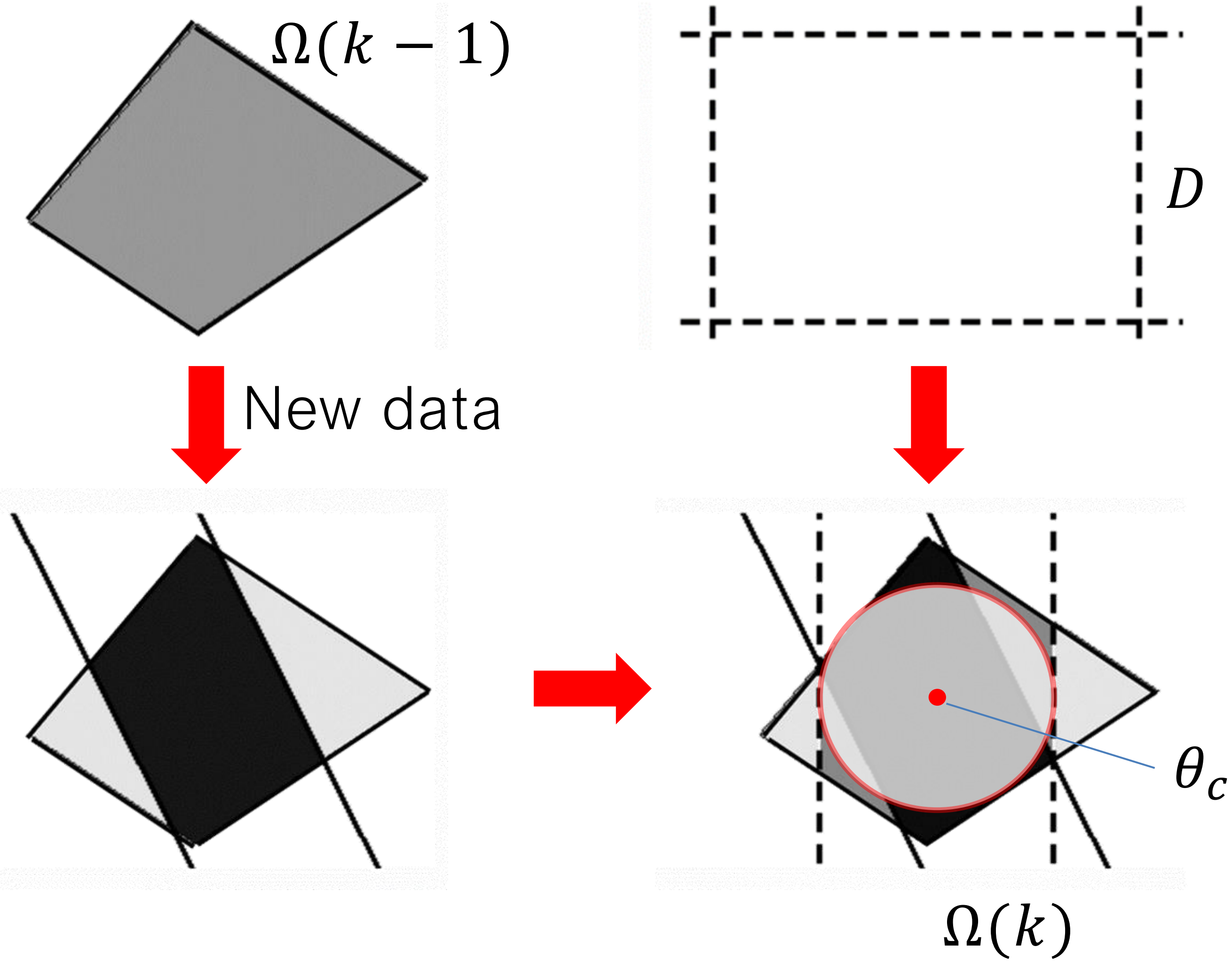
Offset values are computed to guarantee that

$$\Omega(k-1) \cap \Delta(k) \subseteq \Omega(k)$$

Needs 2 LPs per output

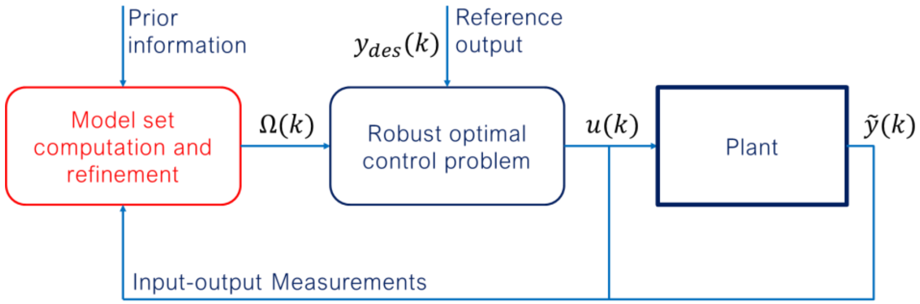
# Limited-complexity model set update (2)




The value of $M_1 + M_2$ can be tuned to trade-off conservativeness and accuracy (i.e. size of $\Omega(k)$)

# Guaranteed properties of the model set

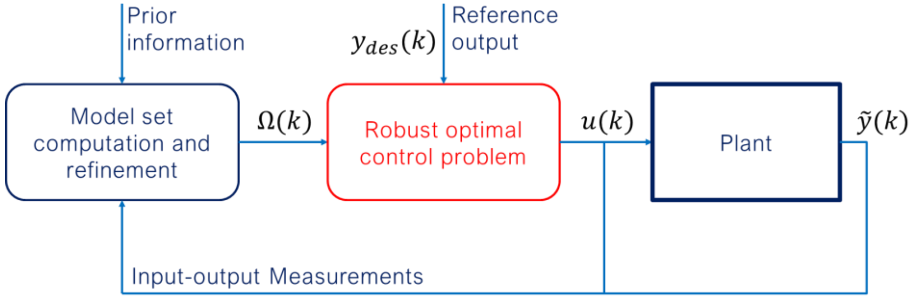



Under the considered prior assumptions, with the presented algorithm:

- $\Omega(k) \subseteq \Omega(k-1), \forall k \geq 0$
- $\Omega(k) \neq \emptyset, \forall k \geq 0$
- $\Omega(k)$ contains the true systems parameters, $\forall k \geq 0$

Proof by induction, see [2]

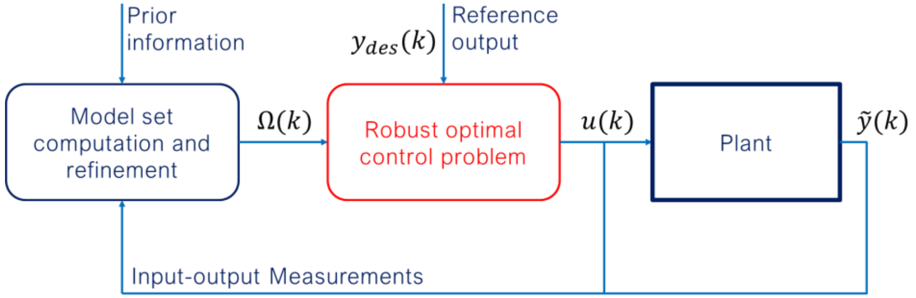
# Robust optimal control problem




Denote with

- $N \geq m$ prediction horizon (minimum value needed to ensure recursive feasibility)
- $\varphi_i$ predicted regressor (affine in the predicted input values $U = [u_0^T, \dots, u_{N-1}^T]^T$)
- $\Delta u_i$ predicted input variations $(u_{i+1} - u_i)$, with $\Delta u_0 = u_0 - u(k-1)$
- $\hat{w}(k)$ estimate of the disturbance $w(k)$: $\hat{w}(k) = \tilde{y}(k) - \varphi(k)^T \theta_c(k)$
- $\hat{y}_i$ predicted output with the nominal model, accounting for $\hat{w}(k)$: $\hat{y}_i = \varphi_i^T \theta_c(k) + \hat{w}(k)$

# Robust optimal control problem (2)




Cost function:

$$J\left(U, \tilde{y}(k), \varphi(k)\right) = \sum_{i=0}^{N-1} (\hat{y}_{i+1} - y_{des})^T Q \, (\hat{y}_{i+1} - y_{des}) + \Delta u_i^T R \Delta u_i \qquad (1)$$

Constraints:

$$C u_i \leq g, i = 0, \dots, N-1 \qquad (2a)$$

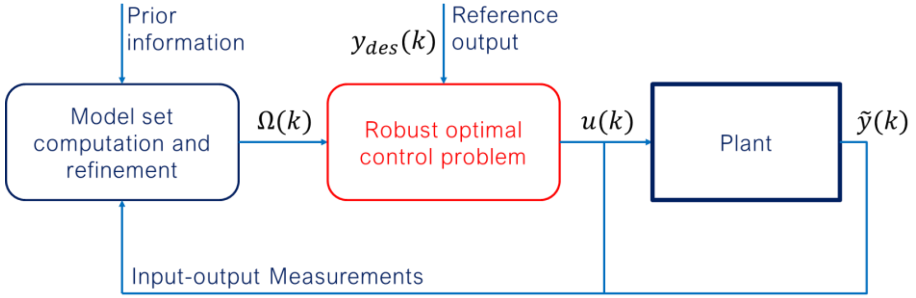
$$L \Delta u_i \leq f, i = 0, \dots, N-1 \qquad (2b)$$

$$E \varphi_i^T \theta + \overline{\overline{w}} \leq p, \forall \theta \in \Omega(k), i = 1, \dots, N \qquad (3)$$

Robust constraints

Computed on the basis of matrix $E$ and bounds $\overline{w}$

# Robust constraints - reformulation

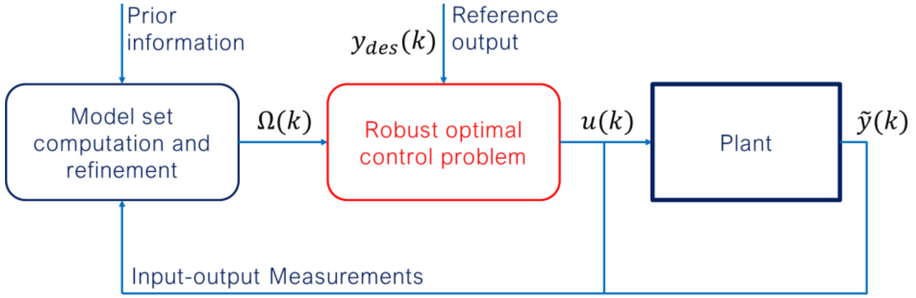



Let $E \in \mathbb{R}^{n_o}$ ($n_o$ output constraints, recall that we consider a scalar output for notational simplicity). Let $\Omega(k) = \{\theta: H_{\Omega(k)}\theta \leq h_{\Omega(k)}\}$ with $H_{\Omega(k)} \in \mathbb{R}^{n_{\Omega(k)} \times n_u m}$. Finally let $\Lambda = \left[\lambda_{1,1}^T, \dots, \lambda_{1,n_o}^T, \dots, \lambda_{N,n_o}^T\right]^{\mathrm{T}} \in \mathbb{R}^{n_{\Omega(k)} n_o N}$.

**THM** The constraints (3) are satisfied if and only if there exist $\varphi_i, i = 1, \dots, N$ and $\Lambda$ such that:

$$\left.\begin{array}{c} H_{\Omega(k)}^T \lambda_{i,l} = E_l \varphi_i \\ h_{\Omega(k)}^T \lambda_{i,l} \leq p_l - \overline{\overline{w}} \\ \lambda_{i,l} \geq 0 \end{array}\right\} l = 1, \dots, n_o, i = 1, \dots, N \qquad (4)$$

# Robust constraints - proof




$$E\varphi_i^T\theta + \bar{\bar{w}} \leq p, \forall\theta \in \Omega(k), i = 1, \dots, N$$

1. Note that (3) holds $\Leftrightarrow$ $\gamma_{i,l} \leq p_l - \bar{\bar{w}}$, $l = 1, \dots, n_o, i = 1, \dots, N$, where

$$\gamma_{i,l} = \max_{\theta\in\Omega(k)} E_l\,\varphi_i^T\theta \qquad (5)$$

2. The dual of the LP (5) reads:

$$\tilde{\gamma}_{i,l} = \min_{\lambda_{i,l}} h_{\Omega(k)}^T\lambda_{i,l}$$

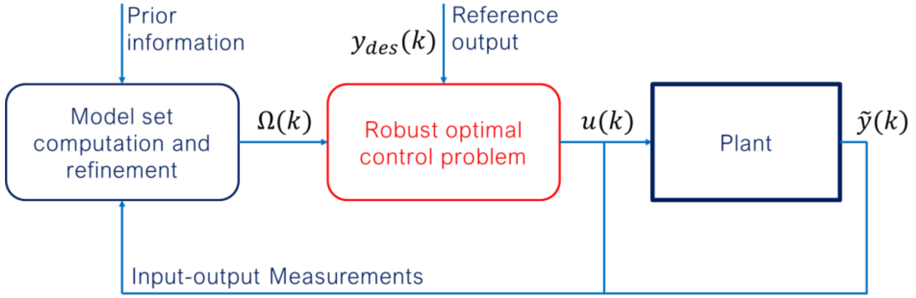
subject to

$$H_{\Omega(k)}^T\lambda_{i,l} = E_l\varphi_i \qquad (6)$$

$$\lambda_{i,l} \geq 0 \qquad (7)$$

3. Since $\Omega(k)$ is compact, by strong duality we have $\gamma_{i,l} = \tilde{\gamma}_{i,l}$
4. Thus, for any $\lambda_{i,l}$ satisfying (6)-(7) it holds that $\gamma_{i,l} \leq h_{\Omega(k)}^T\lambda_{i,l}$

# Robust constraints – proof (2)



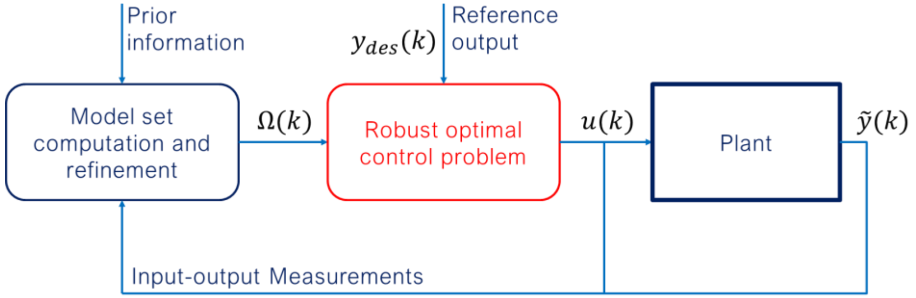

$$\left.\begin{array}{c} H_{\Omega(k)}^T\lambda_{i,l} = E_l\varphi_i \\ h_{\Omega(k)}^T\lambda_{i,l} \le p_l - \bar{\bar{w}} \\ \lambda_{i,l} \ge 0 \end{array}\right\} l = 1, \dots, n_o, i = 1, \dots, N$$

5. Thus, satisfaction of (4) implies $\gamma_{i,l} \le h_{\Omega(k)}^T\lambda_{i,l} \le p_l - \bar{\bar{w}}$, which in turn implies satisfaction of (3).

$$E\varphi_i^T\theta + \bar{\bar{w}} \le p, \forall\theta \in \Omega(k), i = 1, \dots, N$$

6. Necessity can be proven in a similar way.

# Robust optimal control problem (3)




$$J(U, \tilde{y}(k), \varphi(k)) = \sum_{i=0}^{N-1} (\hat{y}_{i+1} - y_{des})^T Q\, (\hat{y}_{i+1} - y_{des}) + \Delta u_i^T R \Delta u_i$$

Minimize (1)

subject to

$$Cu_i \leq g, i = 0, \dots, N-1$$
$$L\Delta u_i \leq f, i = 0, \dots, N-1$$

(2)

(4)

$$\left.\begin{array}{c} H^T_{\Omega(k)} \lambda_{i,l} = E_l \varphi_i \\ h^T_{\Omega(k)} \lambda_{i,l} \leq p_l - \bar{\bar{w}} \\ \lambda_{i,l} \geq 0 \end{array}\right\} l = 1, \dots, n_o, i = 1, \dots, N$$

$$u_i = u_{i-1}, i = N-m, \dots, N-1$$

Terminal steady-state (with FIR parametrization; it can be extended to other structures)

$\boxed{\text{This is a QP}}$

# Close loop guarantees & other good features

- Recursive feasibility, i.e. guaranteed constraint satisfaction
- Integral action (under further assumptions)
- Only convex programs (LPs and QPs), many can be parallelized
- Can trade-off intuitively computational complexity and model set accuracy

# Drawbacks & extensions

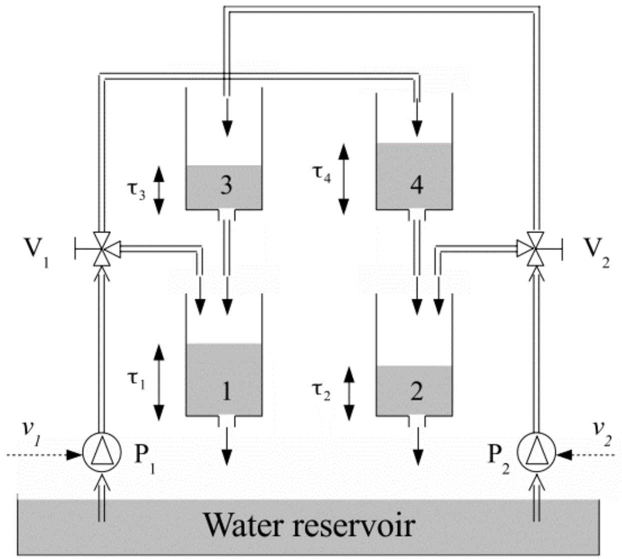
<u>Main drawback</u>: FIR parametrization can lead to large number of parameters

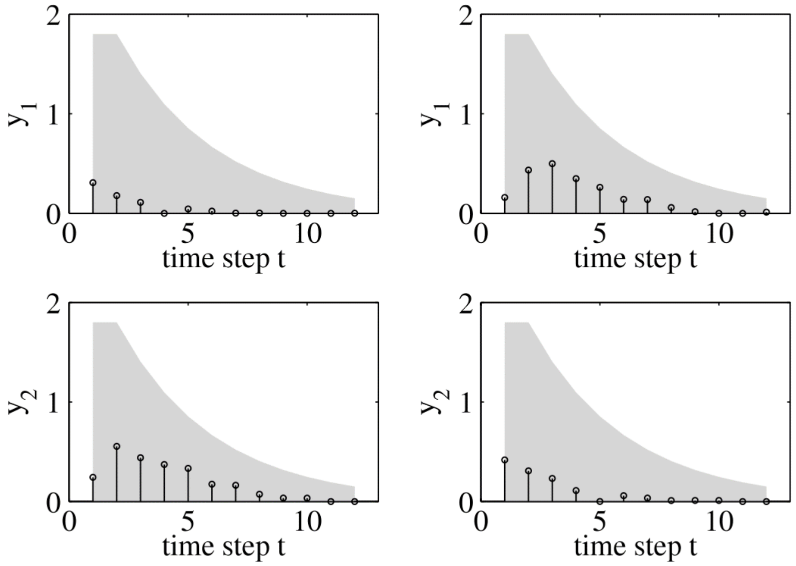
<u>Extensions</u>:

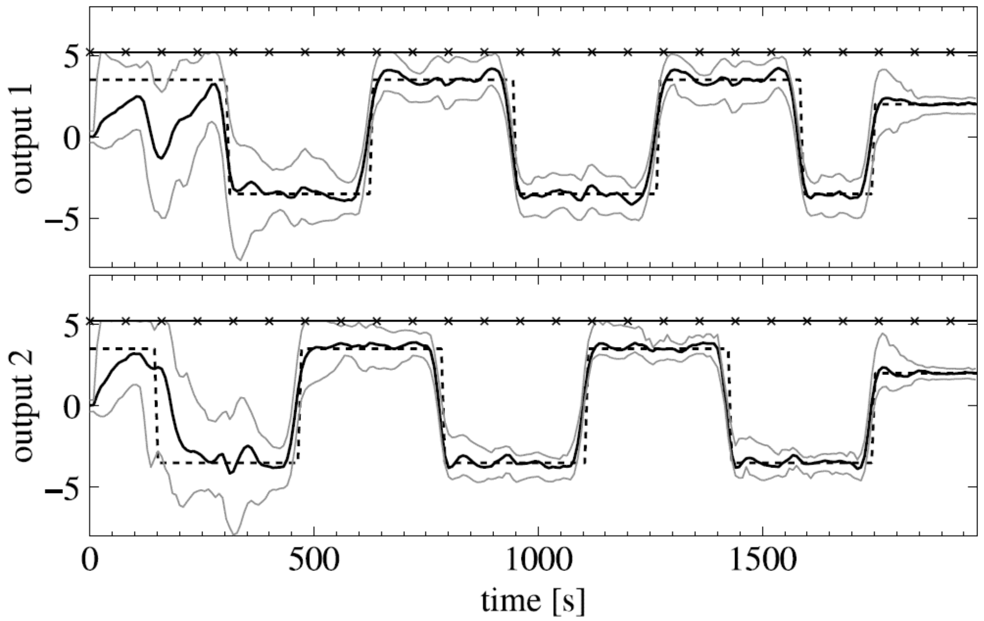
- Parametrization with basis functions


- Active exploration (albeit to a limited extent, still subject of research)

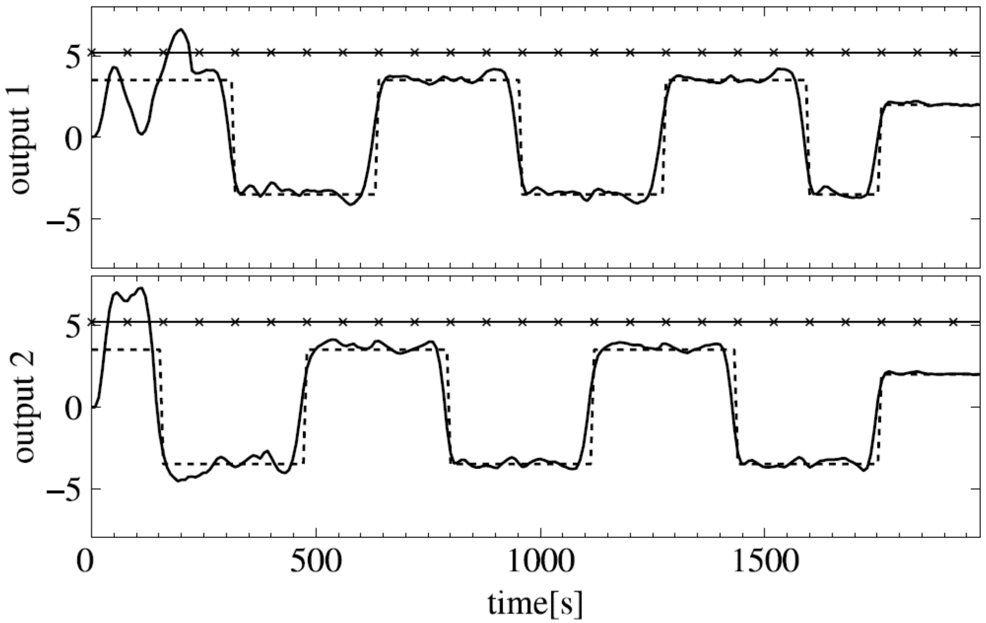
# Experimental results (lab)




Adaptive MPC with SM model learning




Adaptive MPC with certainty equivalence & soft constraints

# Adaptive output-feedback MPC for LTV MIMO systems

Almost all concepts and features carry over to the LTV case.

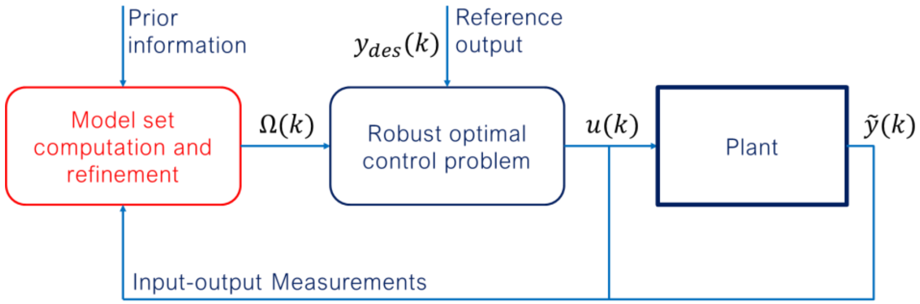
The main difference is in how the model set is updated, and in **additional prior assumptions to retain the theoretical guarantees**:

- Assume bounded rate of parameter variation:

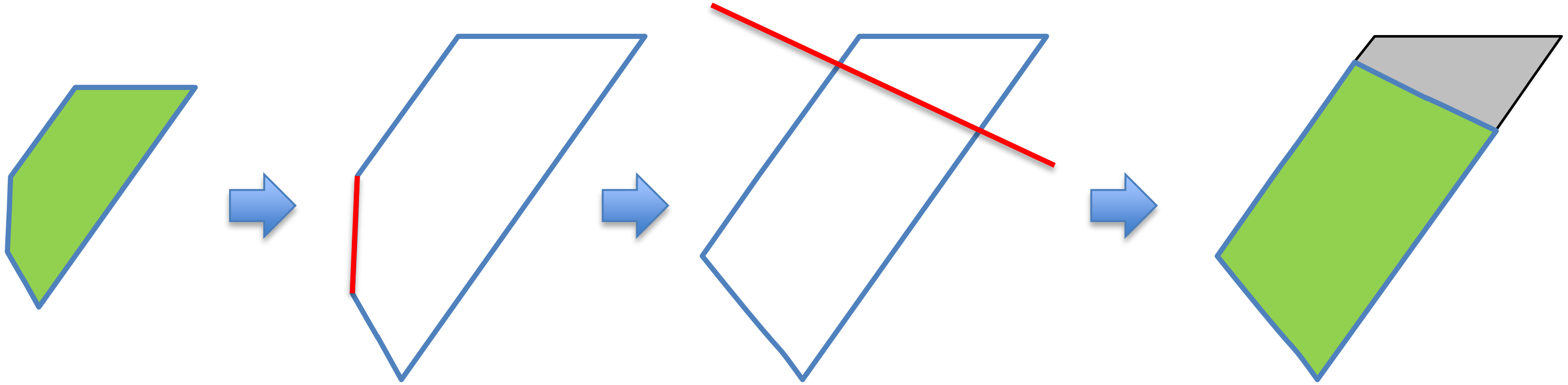
$$\Delta\theta(k) \doteq \theta(k) - \theta(k-1) \in \mathcal{D}, \mathcal{D} \text{ polytope}$$

- Consequently, predict an expanding model set until the maximal one, $\Omega_0$, is reached.

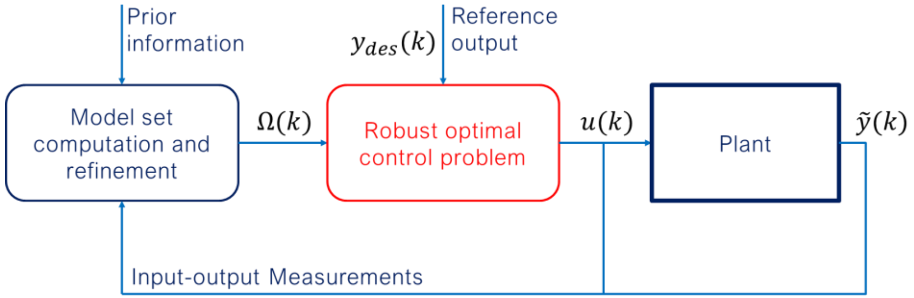
# LTV systems – parameter set update

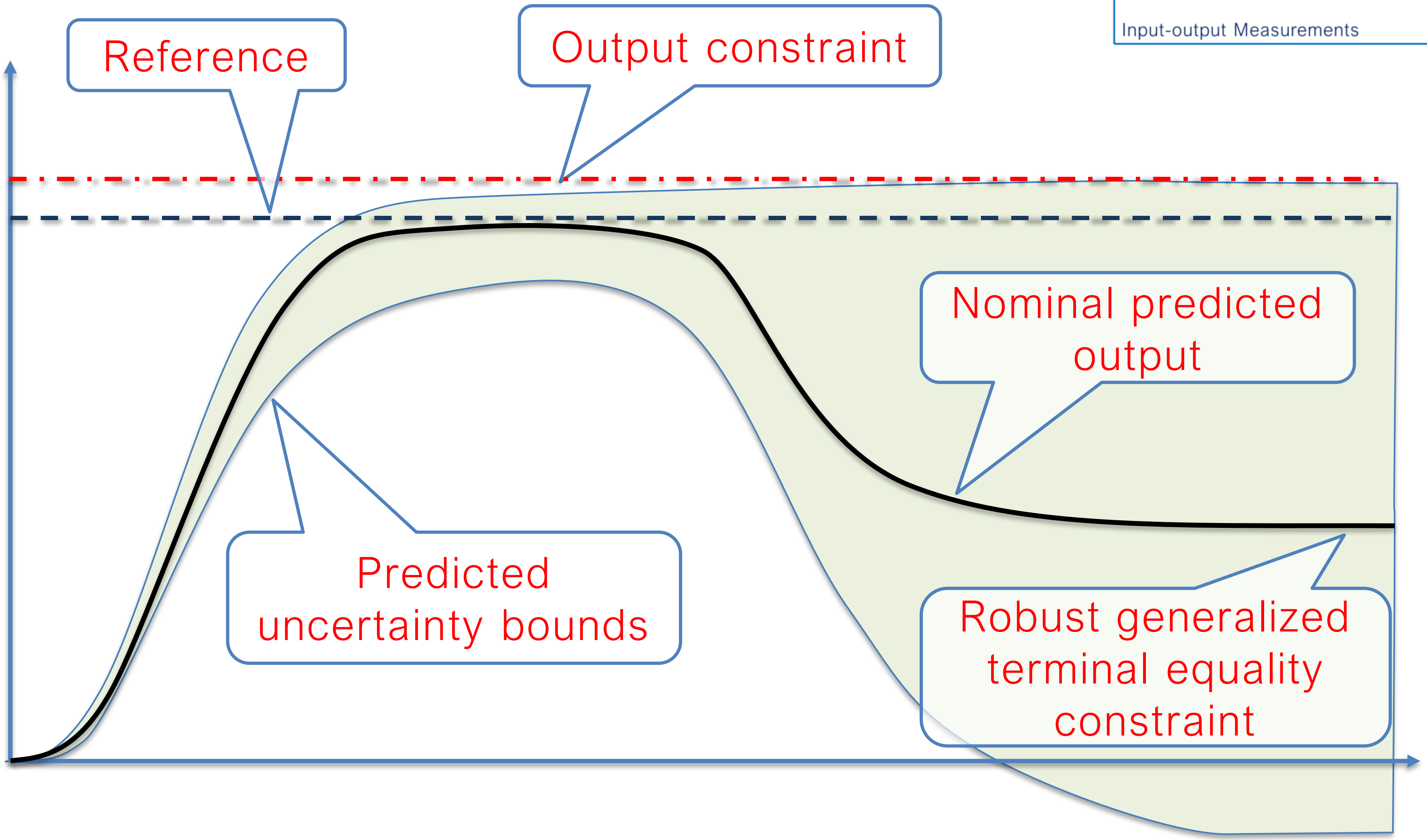



- Consider the assumed bounded rate of variation of the plant dynamics
- Inflate accordingly the Feasible Parameter set at each time step
- Apply a forgetting factor for older data (to avoid memory saturation)
- Refine the model-set set with new data



# LTV systems – optimal control problem



# Final considerations

Adaptive MPC with on-line set membership identification has been actively researched in the last years

Probably one of the best trade-offs between theoretical guarantees, computational tractability, and practical performance

Good results obtained so far, yet research questions still remain:

- How to incorporate active learning?
- How to deal with faults and/or system reconfiguration?

# References

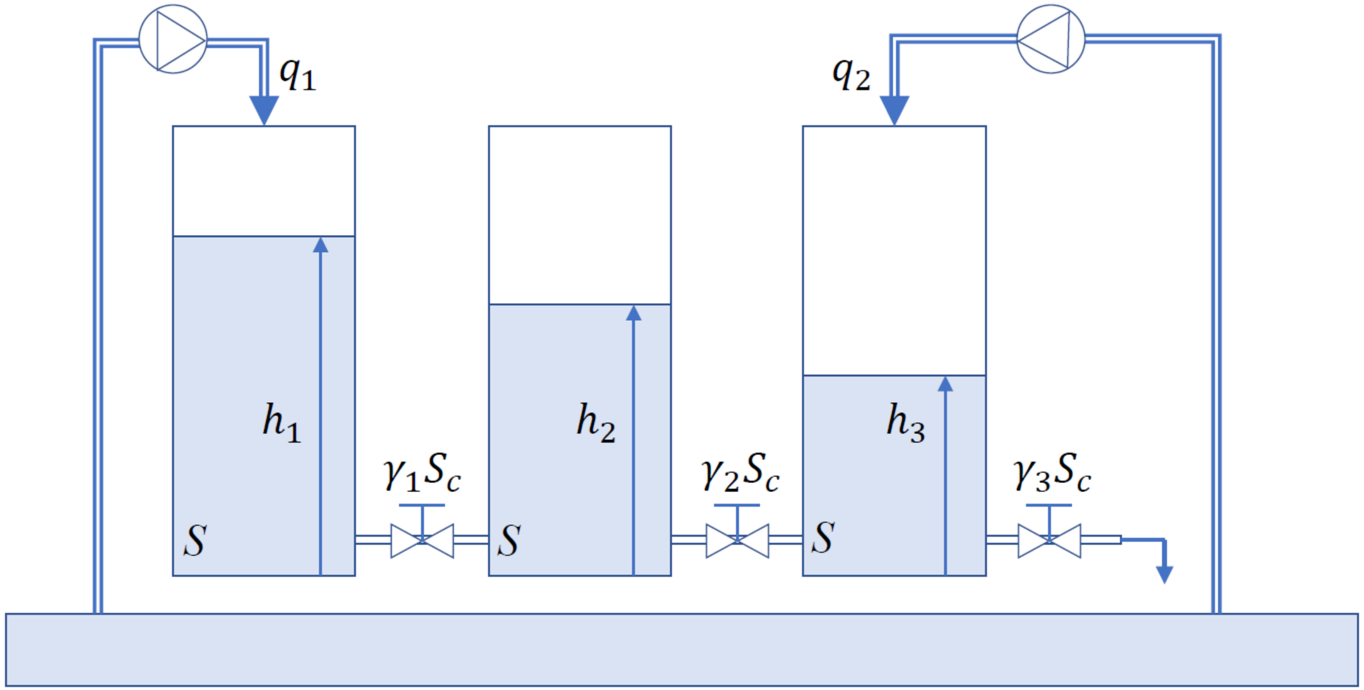

[1] P. J. Antsaklis, K. M. Passino, and S. J. Wang, "An introduction to autonomous control systems," *IEEE Control Systems Magazine*, vol. 11, no. 4, pp. 5–13, June 1991.

[2] M. Tanaskovic, L. Fagiano, R. Smith, and M. Morari, "Adaptive receding horizon control for constrained MIMO systems," *Automatica*, vol. 50, no. 12, pp. 3019–3029, 2014.

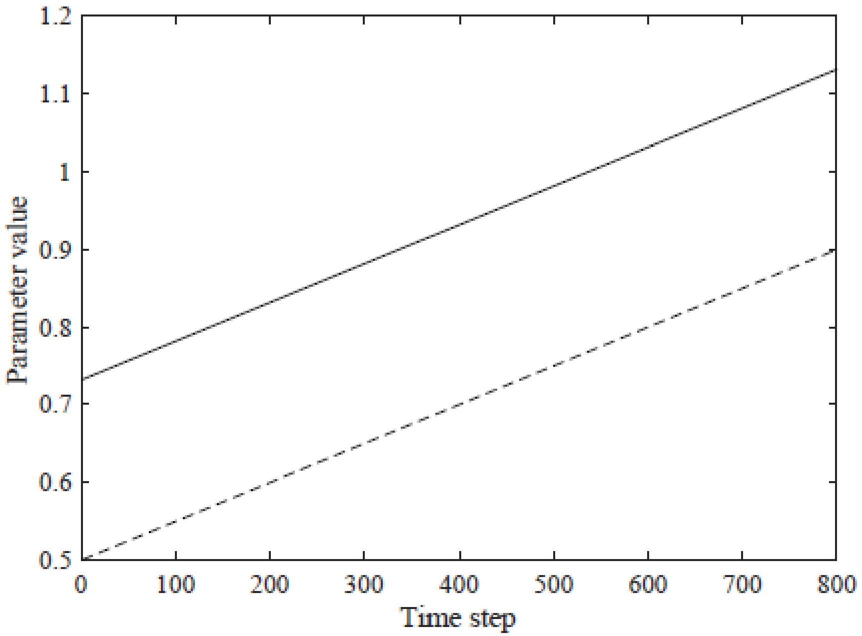
[3] M. Tanaskovic, L. Fagiano, and V. Gligorovski, "Adaptive model predictive control for linear time varying MIMO systems," *Automatica*, vol. 105, pp. 237 – 245, 2019

# Adaptive MPC for LTV system - example

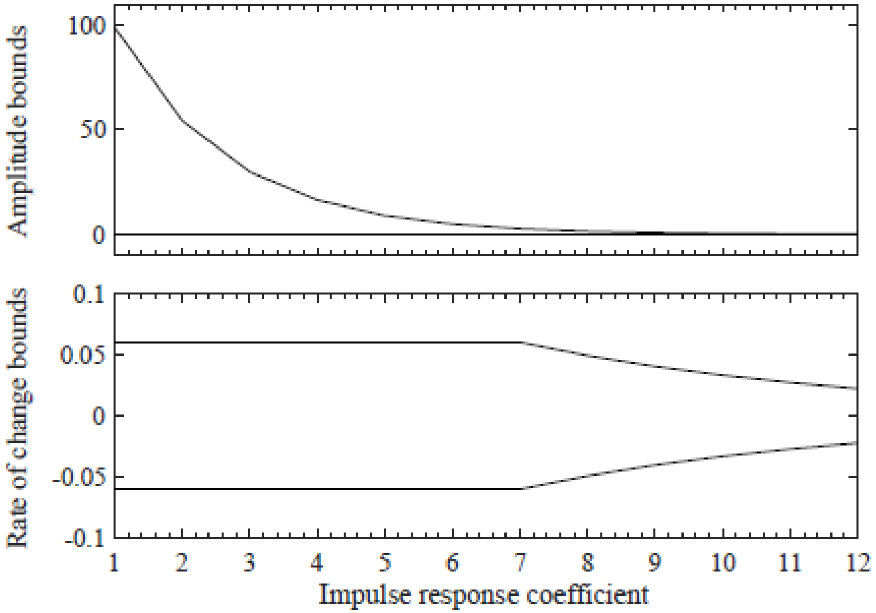
- $q_1, q_2$ inputs, $h_1, h_2, h_3$ outputs
- Valve positions $\gamma_1, \gamma_2$ change over time
- Want to robustly keep $h_1 > h_2 > h_3$ and track reference for $h_2$



# Adaptive MPC for LTV system - example

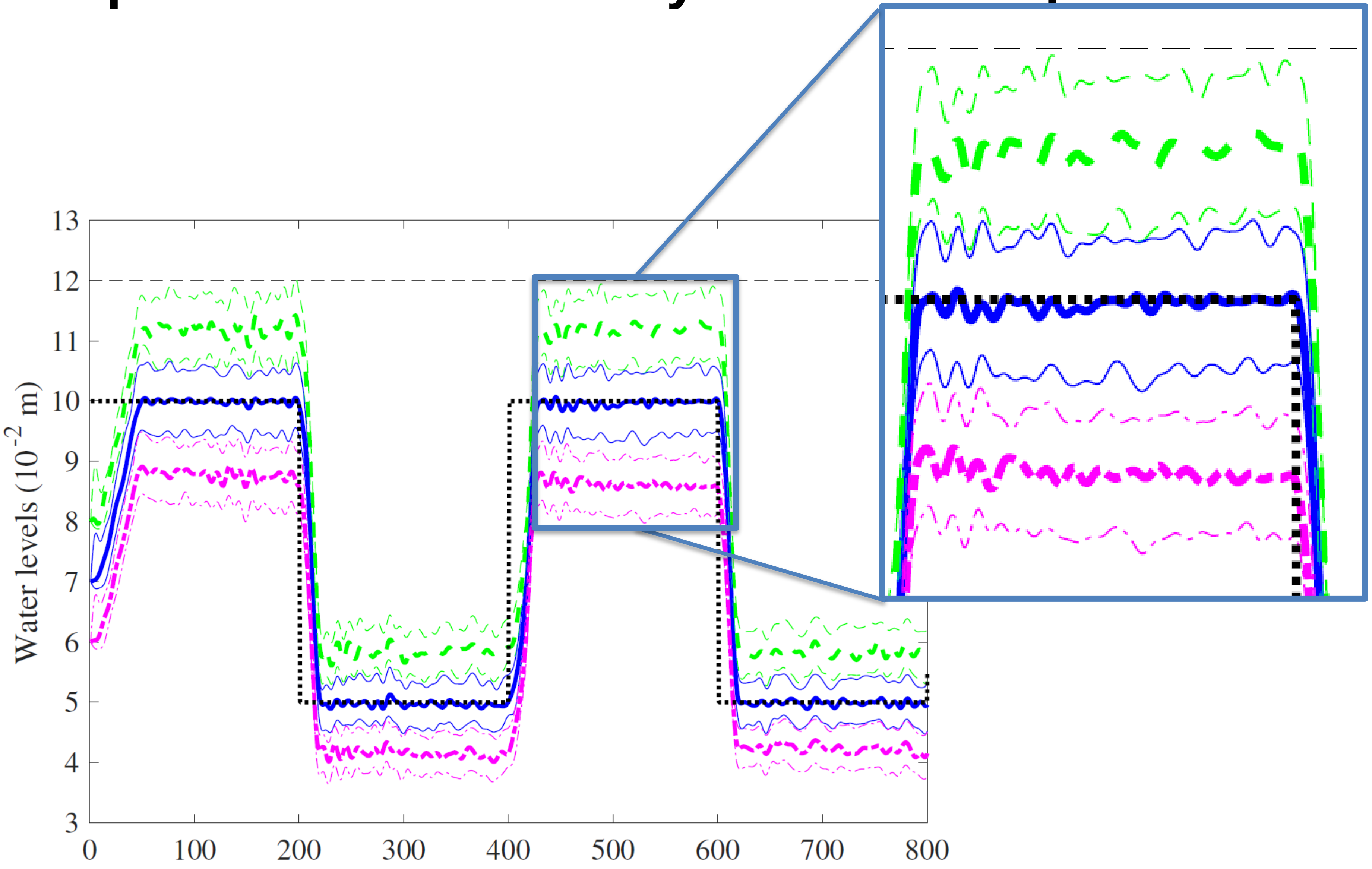



Time variation of the parameters $\gamma_1$ (dashed) and $\gamma_3$ (solid)



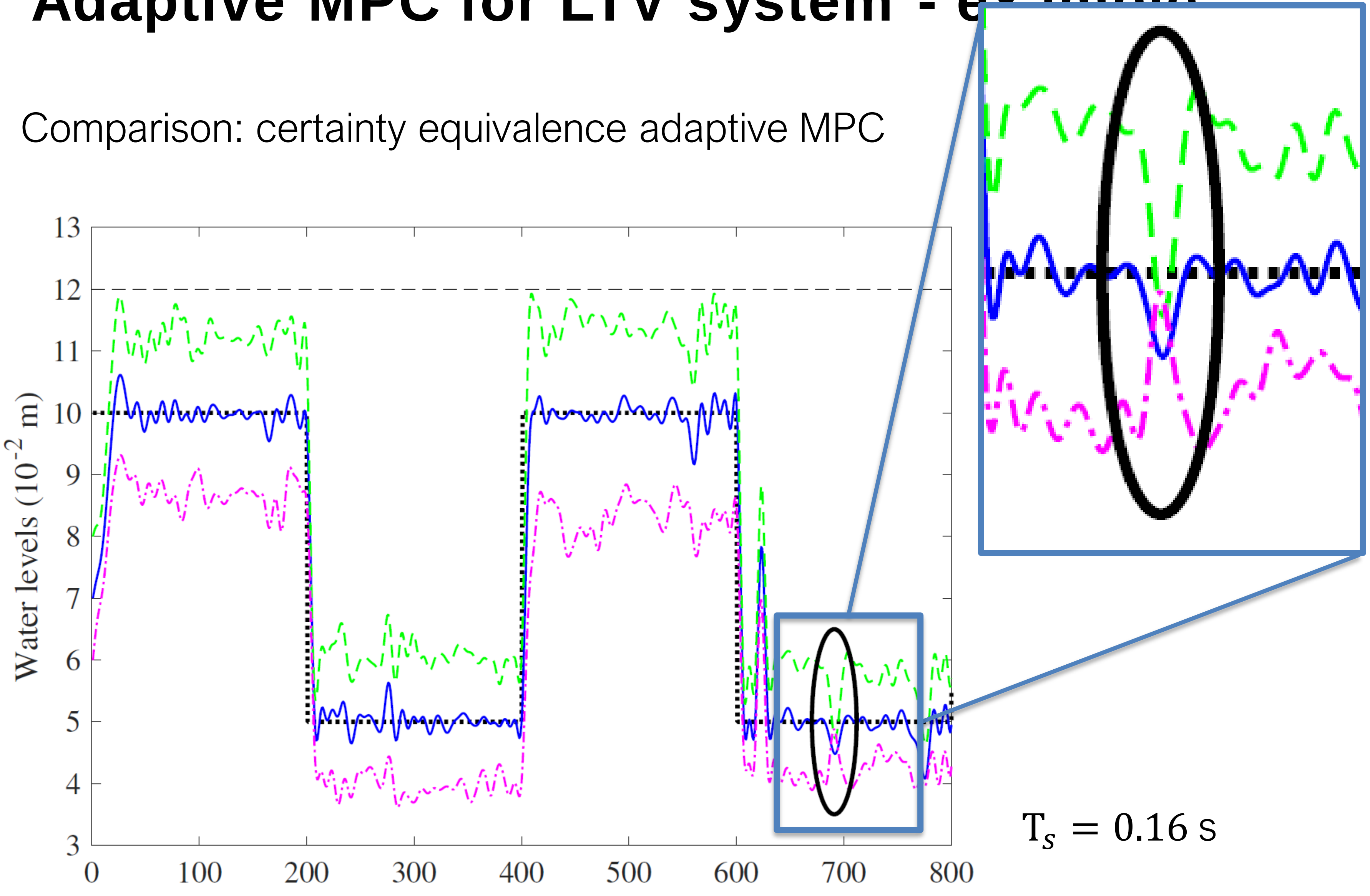

Initial assumed bounds on FIR parameters (top) and on their rate of change (bottom)

# Adaptive MPC for LTV system - example



# Adaptive MPC for LTV system - example

Comparison: certainty equivalence adaptive MPC




$T_s = 0.16$ s