Learning-Based Predictive Control

Chapter 9
Predictive Safety Filters

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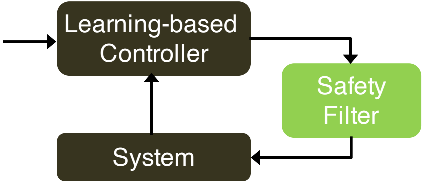
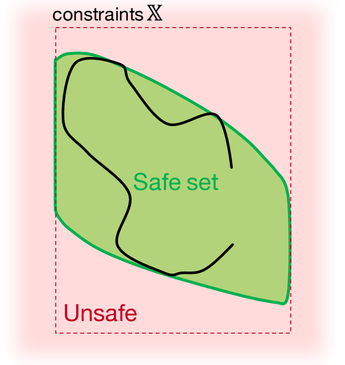
Goal of Safety Filter

Consider the controlled system $x(k+1) = f(x(k), u(k))$





Proposed (learning) input u_L which an (RL) controller intends to apply

Goal: Design safety filter $\pi_f(u_L, x) \in \mathcal{U}$ to

- Maximize safe set $\mathcal{S}$ for system $f(x, \pi_f(u_L, x))$
- Interfere as 'as little as possible': $\pi_f(u_L, x) \approx u_L$

This lecture: Implicit representation of safe set and controller via MPC

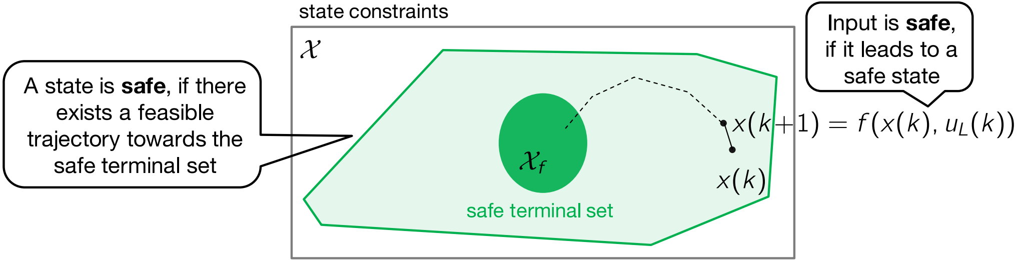
Learning Objectives: Safety Filter – Part I

- Introduce implicit safety filter formulation using predictive control
- Understand benefits of predictive safety filter
- Understand how to make predictive safety filter idea work in the context of uncertain system models
- Investigate safety filter formulation for different uncertainty assumptions
- Analyze how to successively improve terminal safe set and filter design based on data

Outline

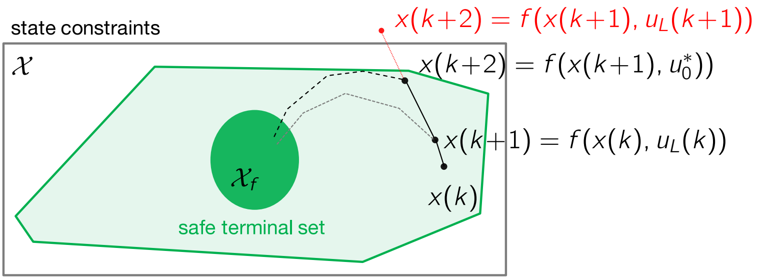
1. Model Predictive Safety Filter
2. Safety Filter based on Robust Techniques
3. Safety Filter based on Stochastic Techniques - Linear Systems
4. Successive Improvements of Safety Filter based on Data
5. Examples

Main Idea of Predictive Safety Filter



Idea: Verify safety by planning safe forward trajectory.

Main Idea of Predictive Safety Filter

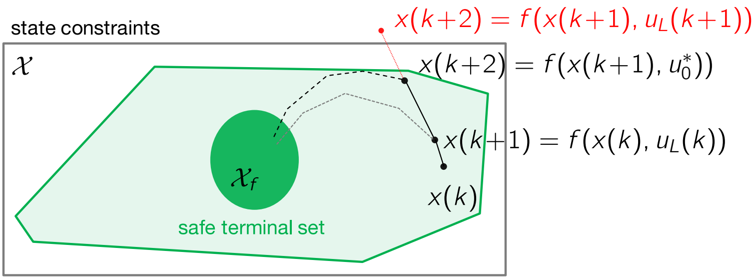


Idea: Verify safety by planning safe forward trajectory.

- Existence of safety trajectory at time step k ensures safety at time step $k+1$:
  Could follow safety trajectory computed at time k if no new safety trajectory can be found.

Combined Verification and Safety Controller

$$
\begin{aligned}
\min \quad & \|u_0 - u_L\| \\
\text{s.t.} \quad & x_{i+1} = f(x_i, u_i) \\
& (x_i, u_i) \in \mathcal{X} \times \mathcal{U} \\
& x_N \in \mathcal{X}_f \\
& x_0 = x(k)
\end{aligned}
$$



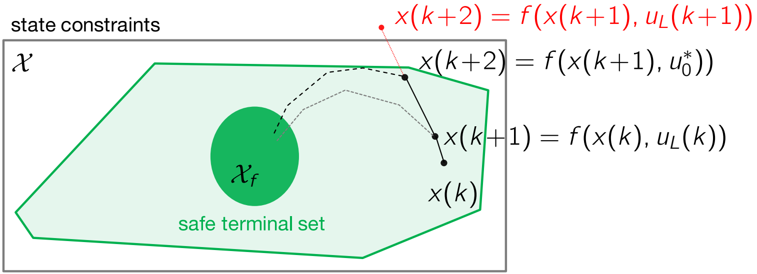
Predictive Safety Filter:

- If learning input is safe: $u_0^*(k) = u_L(k)$
- If learning input is not safe: $u_0^*(k) =$ 'closest' safe input

Assumption: $\mathcal{X}_f$ satisfies regular assumptions on MPC terminal set (invariance under local control law, state and input constraint satisfaction).

Combined Verification and Safety Controller

$$
\begin{aligned}
\min \quad & \|u_0 - u_L\| \\
\text{s.t.} \quad & x_{i+1} = f(x_i, u_i) \\
& (x_i, u_i) \in \mathcal{X} \times \mathcal{U} \\
& x_N \in \mathcal{X}_f \\
& x_0 = x(k)
\end{aligned}
$$



Predictive Safety Filter:

- If learning input is safe: $u_0^*(k) = u_L(k)$
- If learning input is not safe: $u_0^*(k) =$ 'closest' safe input

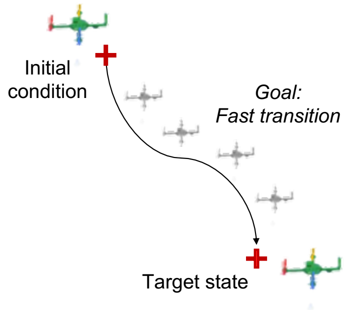
$\rightarrow$ Feasible set is a **safe set**.

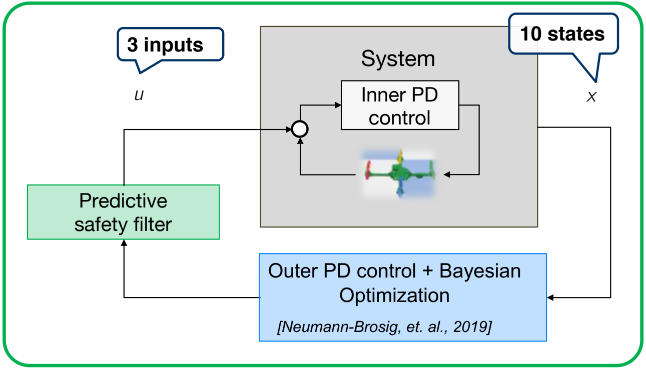
$\rightarrow$ Implicit safe set & controller

$\rightarrow$ Implicit decision module

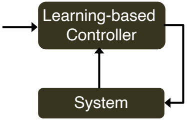
$\rightarrow$ Efficient solvers available

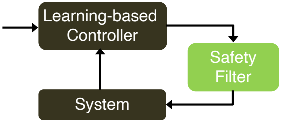
Application Example: Learning to Fly a Quadrotor

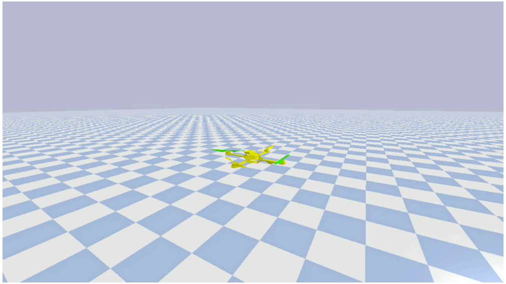




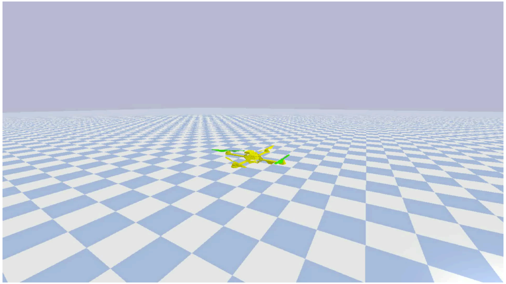
Application Example: Learning to Fly a Quadrotor





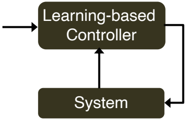


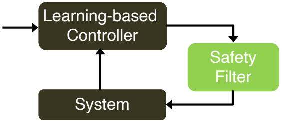
Video: `02_QUADROTOR_UNSAFE`

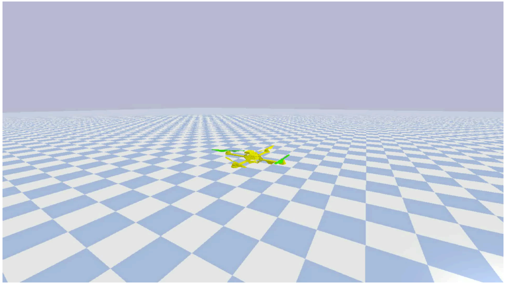


Video: `03_QUADROTOR_SAFE`

Application Example: Learning to Fly a Quadrotor







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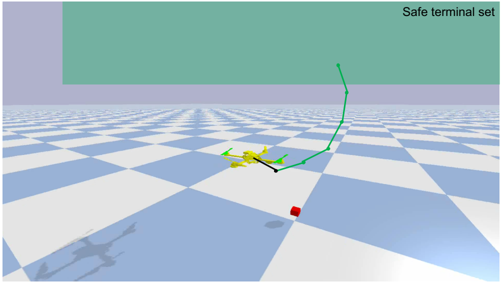

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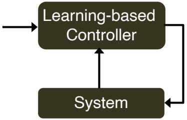
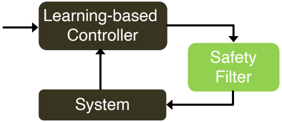
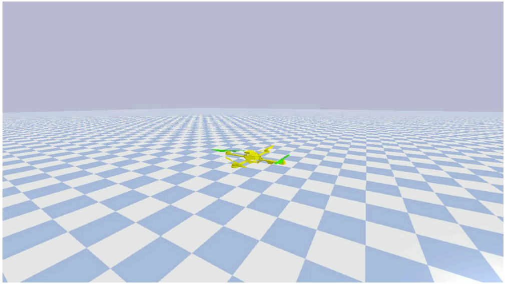
Application Example: Learning to Fly a Quadrotor







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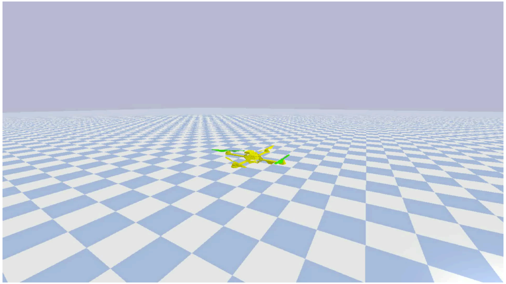

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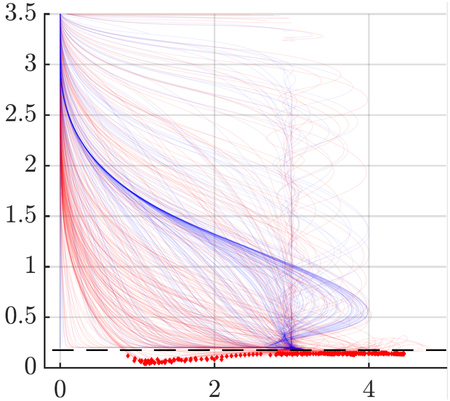
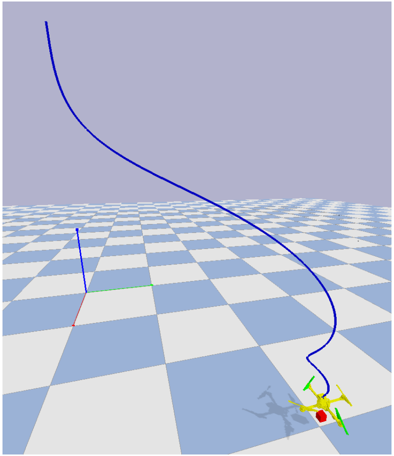
Application Example: Learning to Fly a Quadrotor



Blue: With safety filter

Red: Without safety filter



Remark: Effects of safety filter on performance

- Filter interventions can negatively affect performance of learning (or other) algorithm
- Common approach: Inclusion of safety metric in learning controller
  - Based on safety value function
    Example: Add barrier term to cost used in learning algorithm

    $$C_s(x,u) = C(x,u) - \gamma log(1 - V(x))$$

    (barrier goes to infinity at boundary of safe set)
  - Based on distance to learning input (e.g. for predictive filter)

    $$C_s(x,u) = C(x,u) - \gamma \|u(k) - u_L(k))\|^2$$

Comparison Predictive vs. Invariance-based Safety Filter

Invariance-based safety filter

+ Scales to high-dimensional systems
+ Cheap online verification of safety and computation of safety controller
- Conservative safe set

Predictive safety filter

+ Scales to high-dimensional systems
+ Reduced conservatism w.r.t. maximum control invariant set

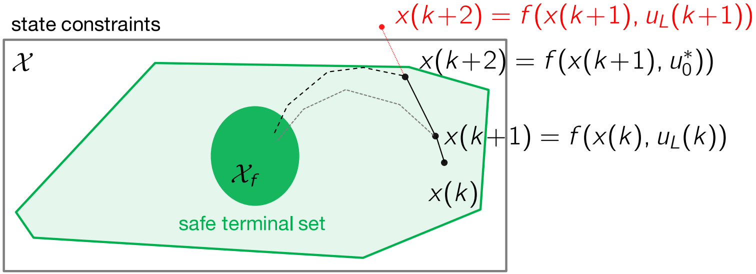
(+) (Conceptually) easily extended to nonlinear systems

- Requires solution of an optimization problem

Model Predictive Safety Filter

Safety filter

$$\begin{aligned} \min \quad & \|u_0 - u_L\| \\ \text{s.t.} \quad & x_{i+1} = f(x_i, u_i) \\ & (x_i, u_i) \in \mathcal{X} \times \mathcal{U} \\ & x_N \in \mathcal{X}_f \\ & x_0 = x(k) \end{aligned}$$



- If learning input is safe: $u_0^*(k) = u_L(k)$
- If learning input is not safe: $u_0^*(k) =$ 'closest' safe input

Main challenge: Model is required for planning a safety trajectory.

Model Uncertainty & Disturbances

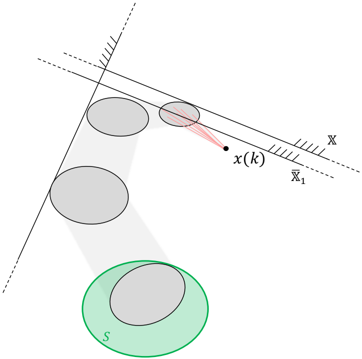
Real world systems are subject to **disturbances** and **model mismatch**

Real system: $x(k+1) = f(x(k), u(k), \theta) + w(k)$

Nominal system: $z(k+1) = f(x(k), u(k), \hat{\theta})$

Tracking controller: $u(k) = v(k) + \pi(x(k), z(k))$

$$
\begin{aligned}
\min \quad & \|v_0 - u_L\| \\
s.t. \quad & z_{i+1} = f(z_i, v_i) \\
& z_i \in \bar{\mathcal{X}}_i, v_i \in \bar{\mathcal{U}}_i \\
& z_N \in \bar{\mathcal{X}}_f \\
& z_0 = x(k)
\end{aligned}
$$



- Robust MPC: Assuming $\theta \in \Omega$ (or $\theta \sim \mathcal{Q}^{\theta}$) and $w(k) \in \mathcal{W}$
- Stochastic MPC: Assuming $\theta \sim \mathcal{Q}^{\theta}$ and $w(k) \sim \mathcal{Q}^{w}$ i.i.d.

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Robust Model

System: $x(k+1) = f(x(k), u(k), \theta_t) + \tilde{w}(k),\ \tilde{w} \in \tilde{\mathcal{W}}$

θ_t constant but unknown

$\rightarrow$ Assume parametric uncertainty with prior distribution $\theta \sim \mathcal{Q}_\theta$

$\rightarrow$ Confidence set: $\Pr(\theta \in \mathcal{C}_\theta) \geq p_s$.

Model: $x(k+1) = f(x(k), u(k), \hat{\theta}) + w(k)$

Assumption: $\Pr(w(k) \in \mathcal{W}\ \forall k \geq 0) \geq p_s$

$\Rightarrow \mathcal{W} = \{w \in \mathbb{R}^n | w = f(x, u, \theta) + \tilde{w} - f(x, u, \hat{\theta}), \theta \in \mathcal{C}_\theta,\ x \in \mathcal{X},\ u \in \mathcal{U},\ \tilde{w} \in \tilde{\mathcal{W}}\}$.

Goal: $\Pr(\forall k \geq 0 : x(k) \in \mathcal{X}, u(k) \in \mathcal{U}) \geq p_s$ (robust in probability)

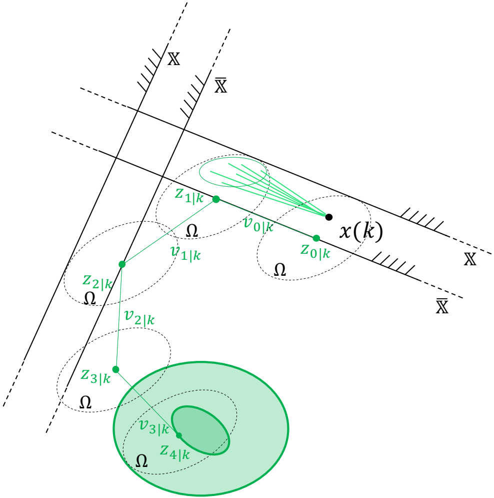
Safety Filter based on Robust MPC

- Uncertain model with uniform error bound:

$$x(k+1) = f(x(k), u(k)) + w(k), w(k) \in \mathcal{W}$$

- Plan nominal trajectory and apply tracking controller
- Tighten constraints on nominal system

$\rightarrow$ Apply robust MPC methods, such as linear or nonlinear tube-based MPC or constraint tightening methods (Lecture 1/2)



Safety Filter based on Linear Constraint Tightening MPC

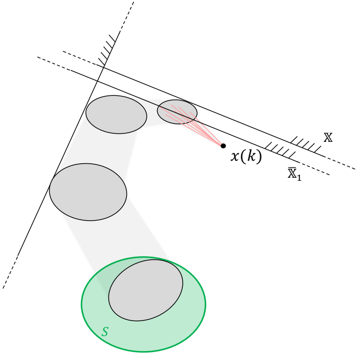
Uncertain system: $x(k+1) = Ax(k) + Bu(k) + w(k),\ \Pr(w(k) \in \mathcal{W}\ \forall k \geq 0) \geq p_s$

Nominal system: $z(k+1) = Az(k) + Bv(k)$

Tracking controller: $u(k) = v(k) + K(x(k) - z(k))$

Error system: $e(k+1) = (A + BK)e(k) + w(k),\ \Pr(w(k) \in \mathcal{W}\ \forall k \geq 0) \geq p_s$

$$
\begin{aligned}
\min \quad & \|v_0 - u_L\| \\
s.t. \quad & z_{i+1} = f(z_i, v_i) \\
& z_i \in \mathcal{X} \ominus \mathcal{F}_i,\ v_i \in \mathcal{U} \ominus K\mathcal{F}_i \\
& z_N \in \mathcal{Z}_f \\
& z_0 = x(k)
\end{aligned}
$$



The closed-loop system with $u(k) = v_0^*$ satisfies constraints robustly with probability p_s.

Outline

Stochastic Linear Model

System: $x(k+1) = f(x(k), u(k), \theta_t) + w_s(k)$, $w_s \sim \mathcal{Q}_w$, i.i.d. (e.g. Gaussian noise)

θ_t constant but unknown

$\rightarrow$ assume parametric uncertainty with prior distribution $\theta \sim \mathcal{Q}_\theta$

Model: $x(k+1) = Ax(k) + Bu(k) + w_\theta(k) + w_s(k)$, $w_s \sim \mathcal{Q}_w$, i.i.d.

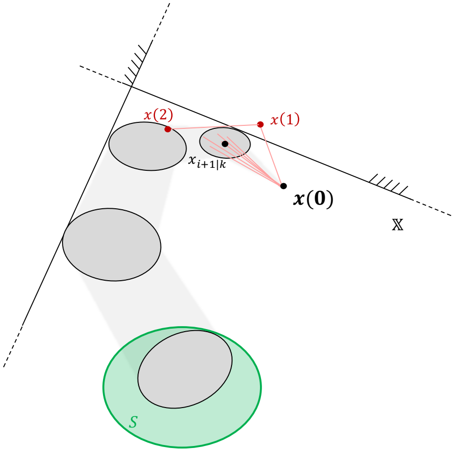
Assumption: $\Pr(w_\theta(k) \in \mathcal{W}_\theta \ \forall k \geq 0) \geq p_\theta$

Goal: $\Pr(x(k) \in \mathcal{X}) \geq p_s, \Pr(u(k) \in \mathcal{U}) \geq p_s$ for all $k \geq 0$

Stochastic Predictive Safety Filter

General predictive safety filter formulation

$$\begin{aligned}
\min \quad & \|u_0 - u_L(k)\| \\
s.t. \quad & x_{i+1} = Ax_i + Bu_i + w_\theta + w_s \\
& \Pr(x_i \in \mathcal{X} \mid x(0)) \geq p_s \\
& \Pr(u_i \in \mathcal{U} \mid x(0)) \geq p_s \\
& x_0 = x(k)
\end{aligned}$$



Address with

- "Constraint tightening" SMPC for distributions with bounded support (Lecture 5)
- SMPC for unbounded disturbances (Lecture 5)
  - Recovery Initialization
  - Indirect Feedback

Constraint Tightening using Probabilistic Reachable Sets

Nominal system: $z(k+1) = Az(k) + Bv(k)$

Tracking controller: $u(k) = v(k) + K(x(k) - z(k))$

Error system: $e(k+1) = (A+BK)e(k) + w_\theta(k) + w_s(k)$, $\Pr(w_\theta \in \mathcal{W}_\theta) \geq p_\theta$, $w_s \sim \mathcal{Q}_w$

$\rightarrow$ Error can be split in parametric error and stochastic error $e(k) = e^\theta(k) + e^s(k)$:

$$e^\theta(k+1) = (A+BK)e^\theta(k) + w_\theta(k), \quad e^s(k+1) = (A+BK)e^s(k) + w_s(k)$$

Robust treatment of e^θ:

Compute robust positively invariant set (RPI) $\mathcal{R}^\theta$

$$e^\theta(0) \in \mathcal{R}^\theta \Rightarrow e^\theta(k) \in \mathcal{R}^\theta \ \forall k \geq 0$$

$\rightarrow \mathcal{R}^\theta$ is RPI set for e^θ with probability p_θ

Constraint Tightening using Probabilistic Reachable Sets

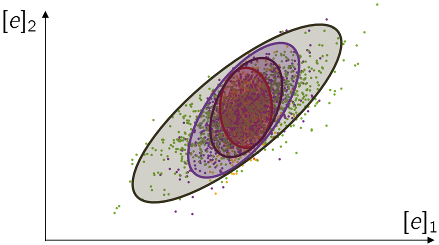
Nominal system: $z(k+1) = Az(k) + Bv(k)$

Tracking controller: $u(k) = v(k) + K(x(k) - z(k))$

Error system: $e(k+1) = (A+BK)e(k) + w_\theta(k) + w_s(k)$, $\Pr(w_\theta \in \mathcal{W}_\theta) = p_\theta$, $w_s \sim \mathcal{Q}_w$

$\rightarrow$ Error can be split in parametric error and stochastic error $e(k) = e^\theta(k) + e^s(k)$:

$$e^\theta(k+1) = (A+BK)e^\theta(k) + w_\theta(k), \quad e^s(k+1) = (A+BK)e^s(k) + w_s(k)$$



Stochastic treatment of e^s:

Compute probabilistic reachable set (PRS) $\mathcal{R}^s$

$$e^s(0) = 0 \Rightarrow \Pr(e^s(k) \in \mathcal{R}^s) \geq p \ \forall k \geq 0$$

Constraint Tightening using Probabilistic Reachable Sets

Nominal system: $z(k+1) = Az(k) + Bv(k)$

Tracking controller: $u(k) = v(k) + K(x(k) - z(k))$

Error system: $e(k+1) = (A + BK)e(k) + w_\theta(k) + w_s(k)$, $\Pr(w_\theta \in \mathcal{W}_\theta) = p_\theta$, $w_s \sim \mathcal{Q}_w$

$\rightarrow$ Error can be split in parametric error and stochastic error $e(k) = e^\theta(k) + e^s(k)$:

$$e^\theta(k+1) = (A + BK)e^\theta(k) + w_\theta(k), \quad e^s(k+1) = (A + BK)e^s(k) + w_s(k)$$

Robust treatment of e^θ:

Compute robust positively invariant set (RPI) $\mathcal{R}^\theta$

$$e^\theta(0) \in \mathcal{R}^\theta \Rightarrow e^\theta(k) \in \mathcal{R}^\theta \; \forall k \geq 0$$

$\rightarrow$ $\mathcal{R}^\theta$ is RPI set for e^θ with probability p_θ

Stochastic treatment of e^s:

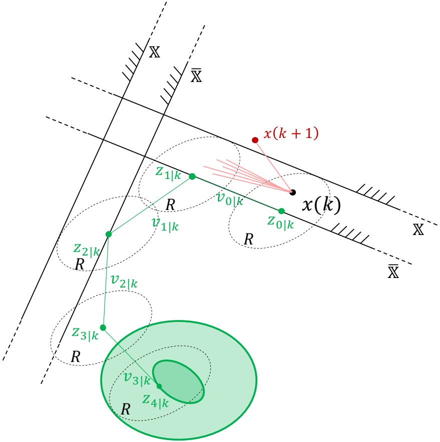
Compute probabilistic reachable set (PRS) $\mathcal{R}^s$

$$e^s(0) = 0 \Rightarrow \Pr(e^s(k) \in \mathcal{R}^s) \geq p \; \forall k \geq 0$$

$$\mathcal{R} = \mathcal{R}^\theta \oplus \mathcal{R}^s \text{ is a PRS for } e(k) \text{ at probability level } p_\theta \cdot p$$

Stochastic Predictive Safety Filter with Probabilistic Reachable Sets

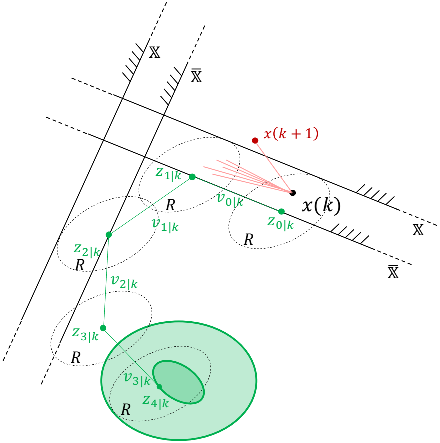
$$\begin{aligned}
\min \quad & \|u_0 - u_L(k)\| \\
s.t. \quad & u_0 = v_0 + K(x(k) - z_0) \\
& z_{i+1} = Az_i + Bv_i \\
& z_i \in \mathcal{X} \ominus \mathcal{R} \\
& u_i \in \mathcal{U} \ominus K\mathcal{R} \\
& z_N \in \mathcal{Z}_f \\
& {\color{red} z_0 = x(k)}
\end{aligned}$$



$\rightarrow$ Initialization can lead to feasibility issues and loss of closed-loop chance constraint satisfaction

Stochastic Predictive Safety Filter with Indirect Feedback

$$
\begin{aligned}
\min \quad & \|u_0 - u_L(k)\| \\
s.t. \quad & u_0 = v_0 + K(x(k) - z_0) \\
& z_{i+1} = Az_i + Bv_i \\
& z_i \in \mathcal{X} \ominus \mathcal{R} \\
& u_i \in \mathcal{U} \ominus K\mathcal{R} \\
& z_N \in \mathcal{Z}_f \\
& \color{red}{z_0 = z_1^*(k-1)}
\end{aligned}
$$



→ Known closed-loop error distribution

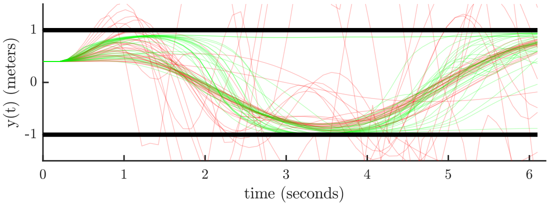
→ Indirect feedback through tracking controller

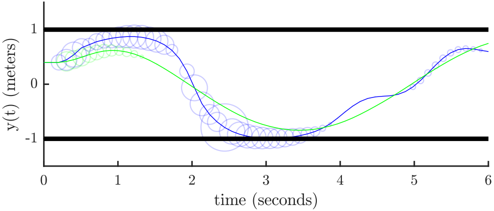
If the problem is initially feasible, then the closed-loop system under the control law $u(k) = v_0^* + K(x(k) - z_0^*)$ satisfies chance constraints for all $u_L(k)$ and for all k.

Safely Learning to Track a Trajectory with a Car

Goal: Learn to drive a simulated autonomous car along a desired trajectory without leaving a road.

Learning controller: Learn linear control law (via Bayesian optimization)





- First 30 learning episodes without (red) and with (green) the stochastic safety filter.
- Safety constraints shown in black.

- Safe closed-loop trajectories during learning with initial policy parameters (blue) and final policy parameters (green).
- Circle radii indicate the relative magnitude of safety ensuring modifications.

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4. Successive Improvements of Safety Filter based on Data

    Improvement of Terminal Safe Set

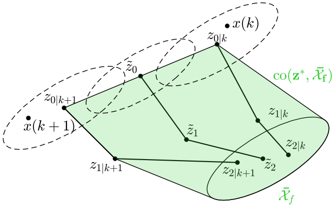
    Improvement of Model and Uncertainty Bounds

Online Enlargement based on Safe States

A larger terminal safe set leads to a larger implicit safe set.

Idea: Leverage previously encountered safe states to enlarge terminal safe set

Consider the set of nominal predicted states obtained from successfully solved instances of the safety filter problem: $\mathbf{z}^*(k) = \{z^*_{j|i}, i \in [1, \dots, k], j \in [0, \dots, N]\}$



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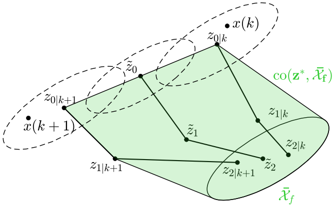


If the predictive safety filter problem is convex and $\bar{\mathcal{X}}_f$ is a safe terminal set, then the set

$$\bar{\mathcal{X}}_f^k = \mathrm{co}(\mathbf{z}^*(k), \bar{\mathcal{X}}_f) \quad \text{(where co}(\cdot)\text{ denotes the convex hull)}$$

is again a safe terminal set, i.e. it is invariant for the nominal system under a terminal safe control law and all tightened state and input constraints are satisfied.

Outline

4. Successive Improvements of Safety Filter based on Data

    Improvement of Terminal Safe Set

    Improvement of Model and Uncertainty Bounds

Learning-based Model and Uncertainty Bounds

Safety filter for learning-based controllers (e.g. RL):

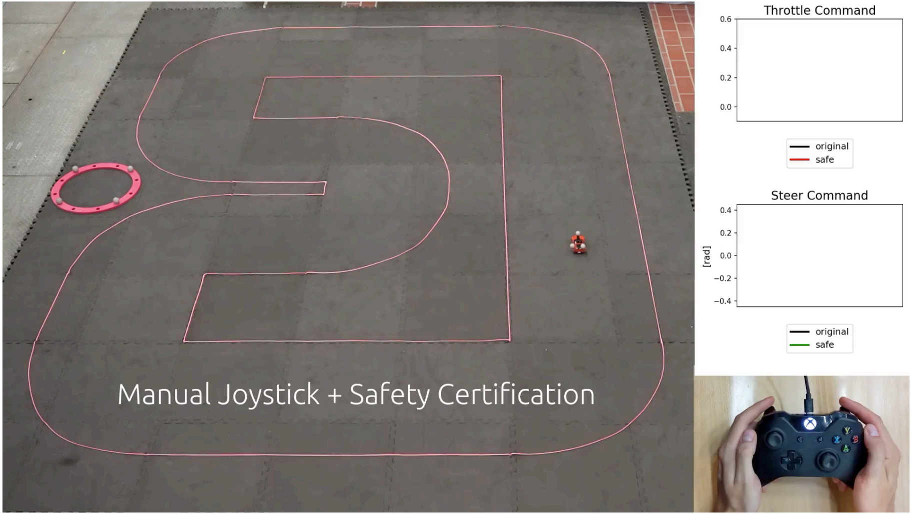
- If learning happens in episodes
  - Improve filter design after every episode to reduce conservativeness
  - Safety filters for uncertain systems usually rely on nominal model

    $\rightarrow$ improve nominal model based on collected data, update uncertainty bound

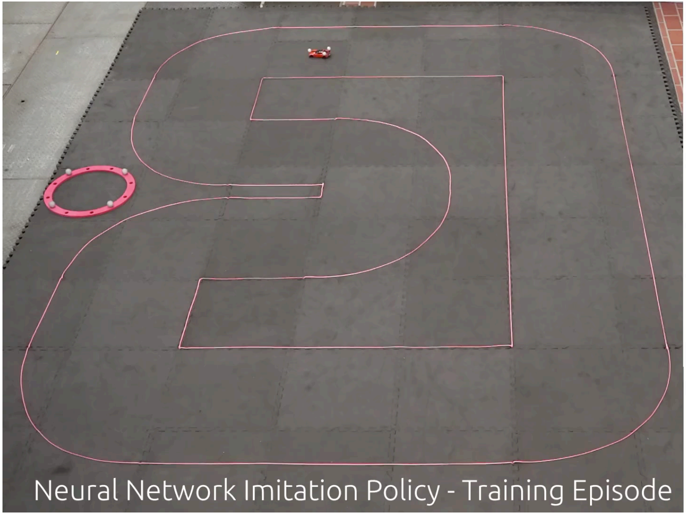
- If learning happens online
  Any learning-based MPC method can be employed that ensures recursive feasibility and constraint satisfaction (see e.g. [5])

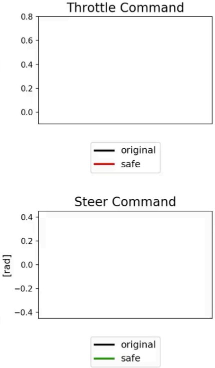
Outline

Safety filter for a Human Learner



Safety filter for Imitation Learning





Summary: Predictive Safety Filter

Idea: Approximate maximum control invariant set with a predictive optimal control problem

- Implicit optimization-based formulation
- Controller verification and backup controller computation in one problem
- Scalable to high-dimensional systems
- Leverage stochastic and robust MPC formulations to deal with model uncertainty
- Leverage efficient MPC solvers

$\Rightarrow$ Provide safety for any controller by passing proposed input through safety filter

Note: Extensions to reduce conservativeness of robust MPC in [3] and to distributed systems in [4].

References

[1] K.P. Wabersich and M.N. Zeilinger, “Linear model predictive safety certification for learning-based control”, CDC 2018.

[2] K.P. Wabersich, L. Hewing, A. Carron and M.N. Zeilinger, “Probabilistic model predictive safety certification for learning-based control”, IEEE Transactions on Automatic Control, 2021.

[3] K.P. Wabersich and M. N. Zeilinger, “A predictive safety filter for learning-based control of constrained nonlinear dynamical systems,” Automatica, vol. 129, p. 109 597, 2021.

[4] S. Muntwiler, K.P. Wabersich, A. Carron and M.N. Zeilinger, “Distributed Model Predictive Safety Certification for Learning-based Control”, IFAC World Congress 2020.

[5] A. Didier, K.P. Wabersich and M.N. Zeilinger,“Adaptive Model Predictive Safety Certification for Learning-based Control” , IEEE Conf. on Decision and Control, 2021.

[6] Brunner et al., “Stabilizing linear model predictive control: On the enlargement of the terminal set,” in European Control Conference (ECC), 2013.

[7] U. Rosolia and F. Borrelli, “Learning model predictive control for iterative tasks: a computationally efficient approach for linear system,” IFACPapersOnLine, vol. 50, no. 1, 2017.

[8] B. Tearle, K.P. Wabersich, A. Carron and M.N. Zeilinger, “A predictive safety filter for learning-based racing control”, IEEE Robotics and Automation Letters 2021.