App Physics 157

ACTIVITY 2 (Part 1) REPORT

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Background
Discrete Fourier Transform

In image processing, the Discrete Fourier Transform (DFT) is a mathematical technique used to transform a two-dimensional image into the frequency domain. The DFT of an image is a complex-valued function that represents the frequency content of the image.

The DFT is computed using a two-dimensional array of complex numbers, which represents the image. The DFT of the image is obtained by applying the one-dimensional DFT to each row of the image, and then applying the one-dimensional DFT to each column of the resulting matrix.

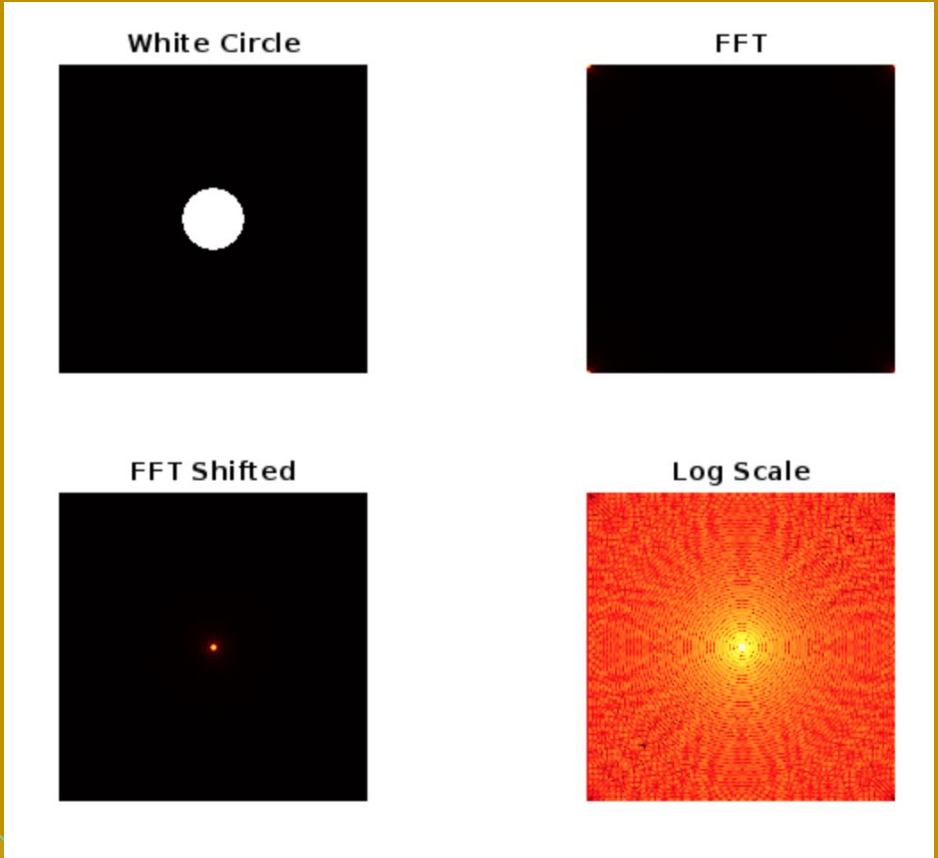
The resulting DFT image contains information about the spatial frequency content of the original image. The low-frequency components of the image are represented by the values near the center of the DFT image, while the high-frequency components are represented by the values near the edges of the DFT image.

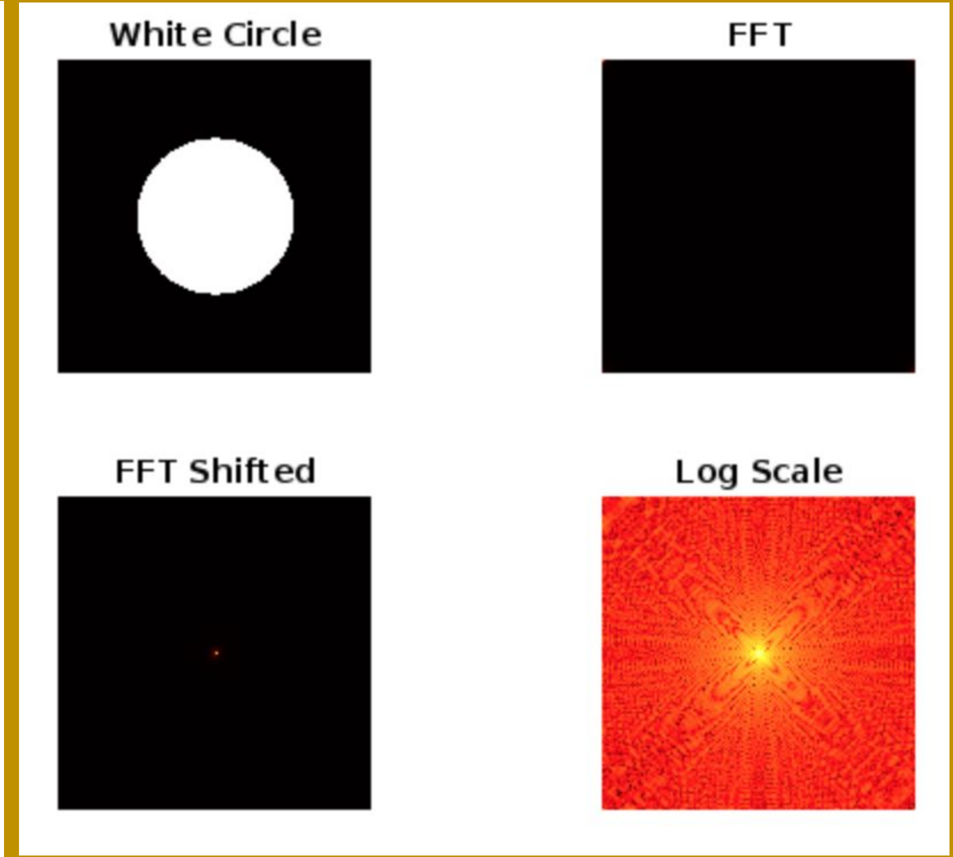
Objectives 2.1

Activity 2.1 Familiarization with Discrete FT

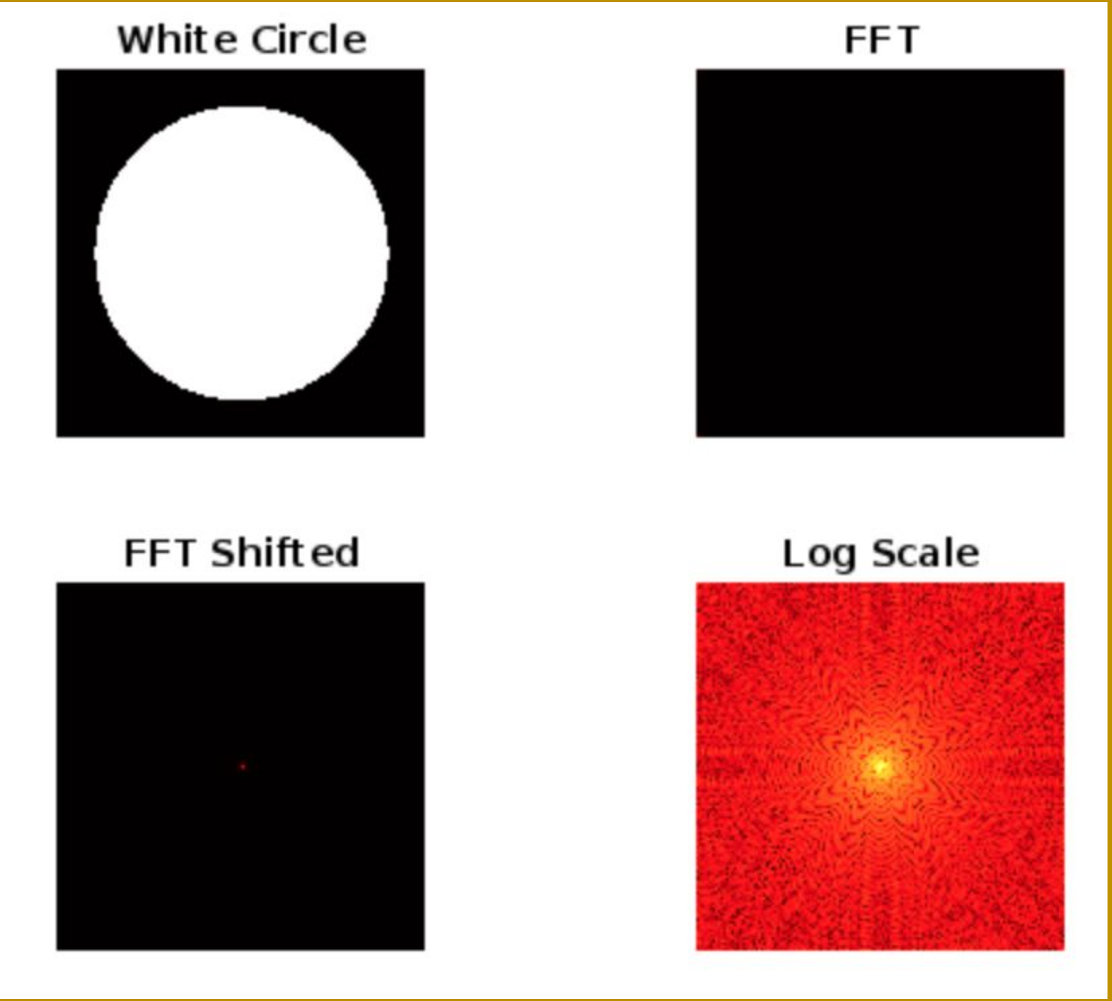
1. Create an image of a white circle against a black background centered in a 400x400 pixel matrix. Suppose the image is called A. You may use the image you used in Activity 1.
2. Apply fft2() on the image and compute the intensity values using abs(). Remember, the result of and FFT is a complex matrix. Display the FFT magnitude as an intensity image.
3. Notice that the resulting image have intensities at the corners. This is because of the property of FFT2 where the diagonal quadrants are interchanged. To make the FFT2 output appear zero-centered, use fftshift and then display. Use the “hot” colormap to make it look like a laser diffraction pattern.
4. Pick any image in your collection and crop out any square portion. Load this image in your program and convert this image into grayscale. Apply fft2 () twice on the image and display the reconstruction. Do it again, but this time apply fft2() followed by ifft2(). Comment on the appearance of the reconstructed images.

Results & Analysis 2.1



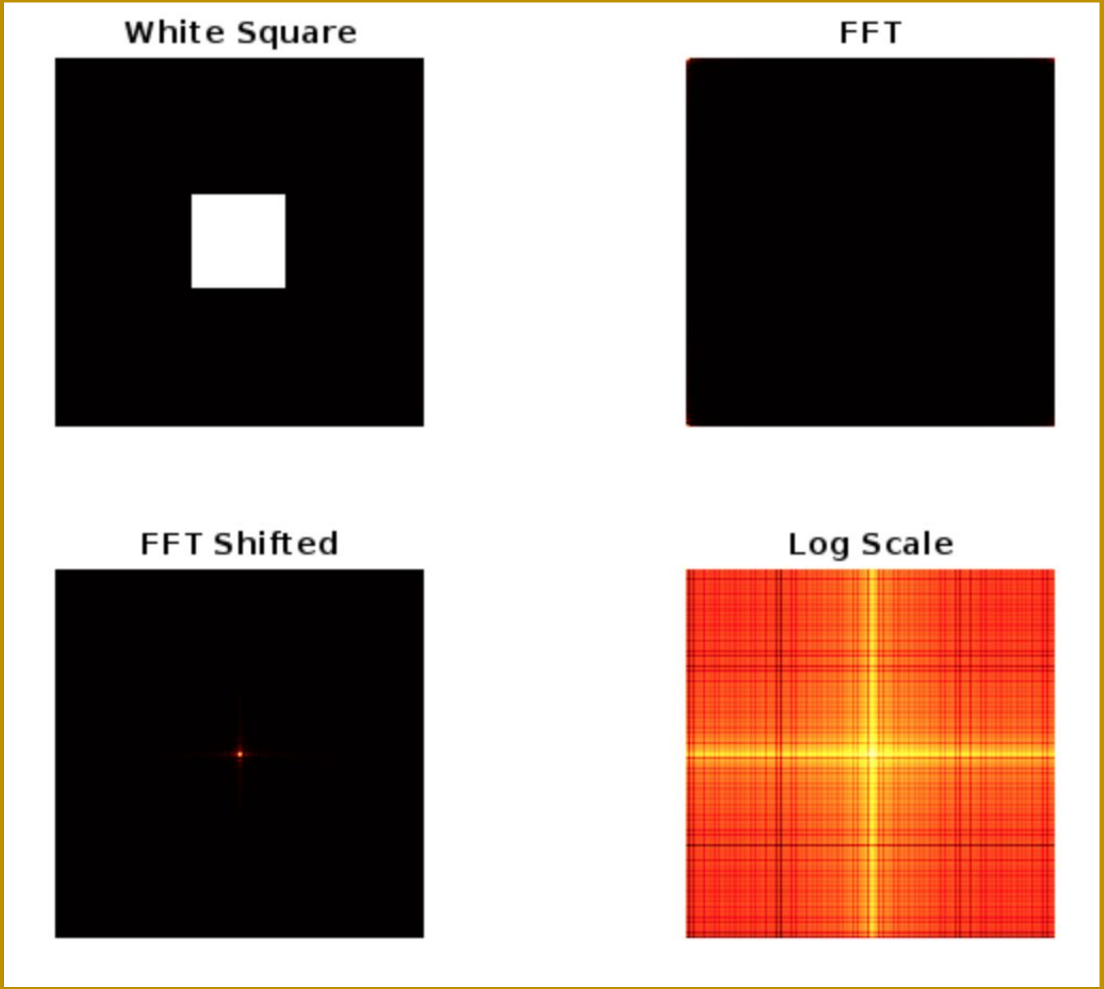


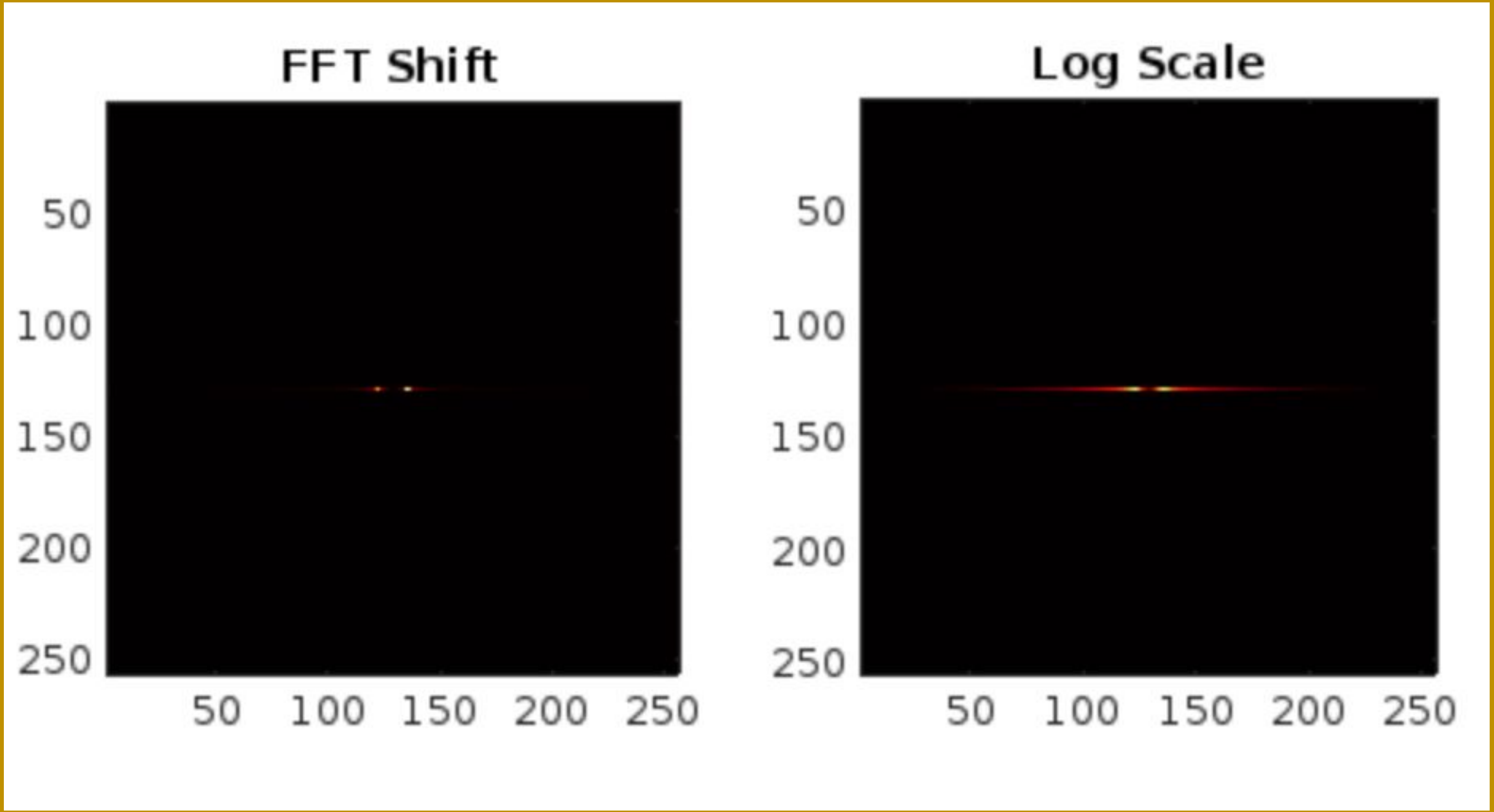
CIRCLE IMAGE



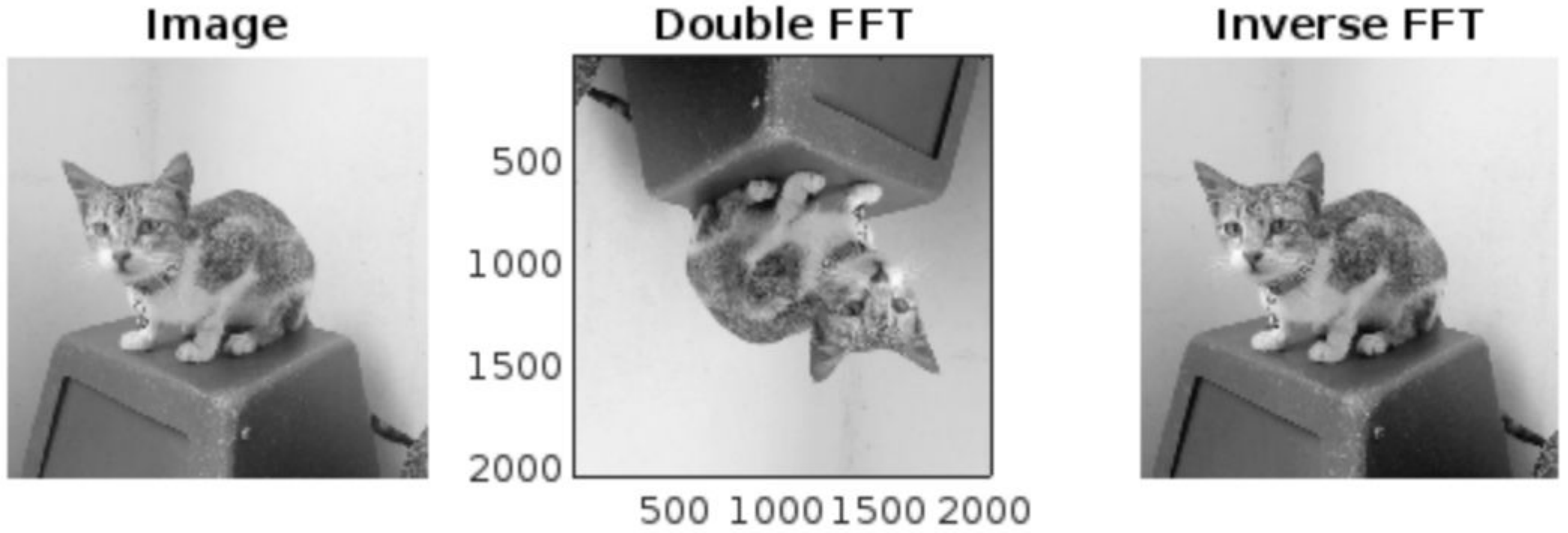
CIRCLE IMAGE

SQUARE IMAGE





SINUSOID



GENERAL IMAGE

DFT gives us a novel way of viewing data, by transforming information from the spatiotemporal domain to the complex frequency domain.

When applying FFT twice to an image, it inverts the image 180 degrees. When applying inverse FFT to an image, it increases contrast in the image.

ANALYSIS

Background
Convolution

With two functions h(t) and g(t), and their corresponding Fourier transforms H(f) and G(f), the convolution, denoted f = g * h, is defined by

$$f(t) = g * h = \int_{-\infty}^{\infty} g(\tau)h(t-\tau)d\tau$$

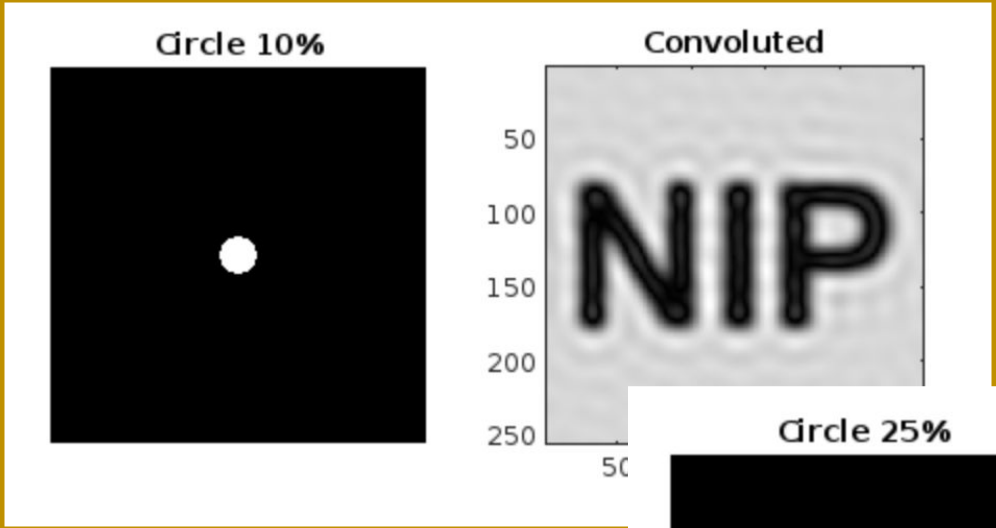
The Fourier Transform Convolution theorem states that convolution in the time domain is equivalent to multiplication in the frequency domain. Specifically, the Fourier Transform of the convolution of two signals is equal to the product of the Fourier Transform of each signal

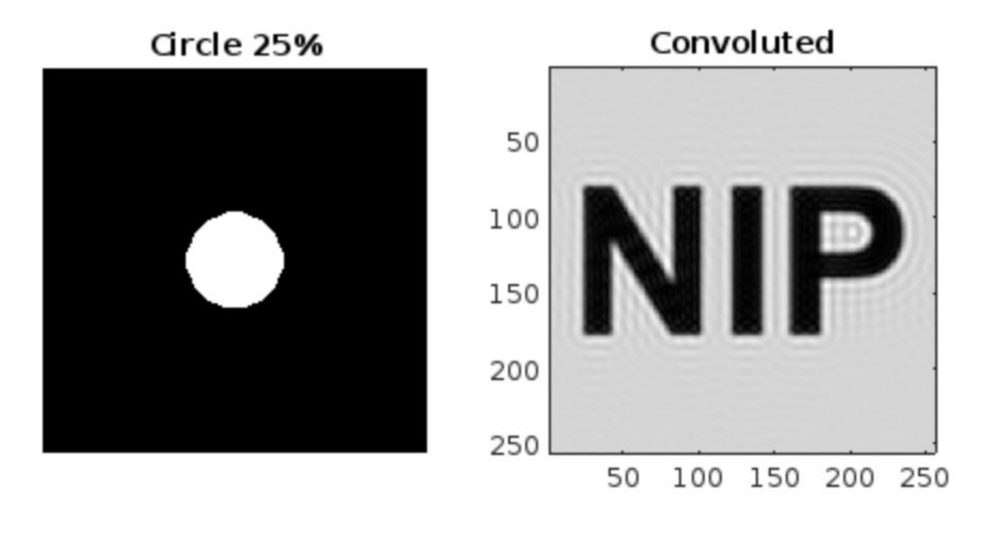
Objectives 2.2

Activity 2.2 Simulation of an imaging system

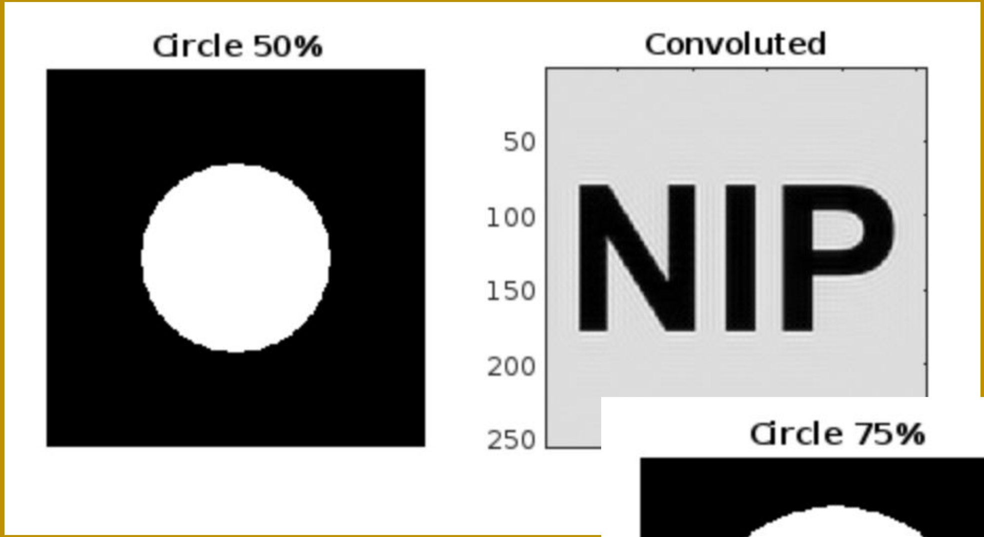
1. Create a 256x256 image of the letters “**NIP**” in large bold fonts (Arial or Helvetica font recommended) using PAINT or similar apps. Let the letters fill 50% to 75% of the space. Save this image as TIF to preserve the crisp edges. Note the edges of the sans serif font are sharp.
2. Create another 256 x 256 image of a white circle (centered) against a black background. This image represents the “aperture” of a circular lens. Let the diameter of this circle be at 10% of the image width.
3. Convolve the two images using the following steps:
   3.1. FFTshift the circular aperture. For a lens, this aperture function is already in the FT space so no need to FFT2 the aperture.
   3.2. Take the FFT2 of the NIP image. Do not use abs. We need both real and imaginary parts.
   3.3. Multiply the FFT2(NIP) with the fftshifted aperture.
   3.4. Inverse FFT2 the product in 3.3 using ifft2();
   3.5. Plot the modulus (abs ()) of the result in 3.4.
4. Repeat steps 2 and 3 for apertures with diameters 25%, 50%, 75% and 100% of the array width. Comment on the appearance of the reconstruction.
5. 5. Simulate the image of a star produced by the James Webb Space Telescope. Stars may be considered point objects. Simply get the FT of the JWST honeycomb mirror configuration you created in Activity 1. The resulting FT image is how a single star would look like as imaged by JWST.

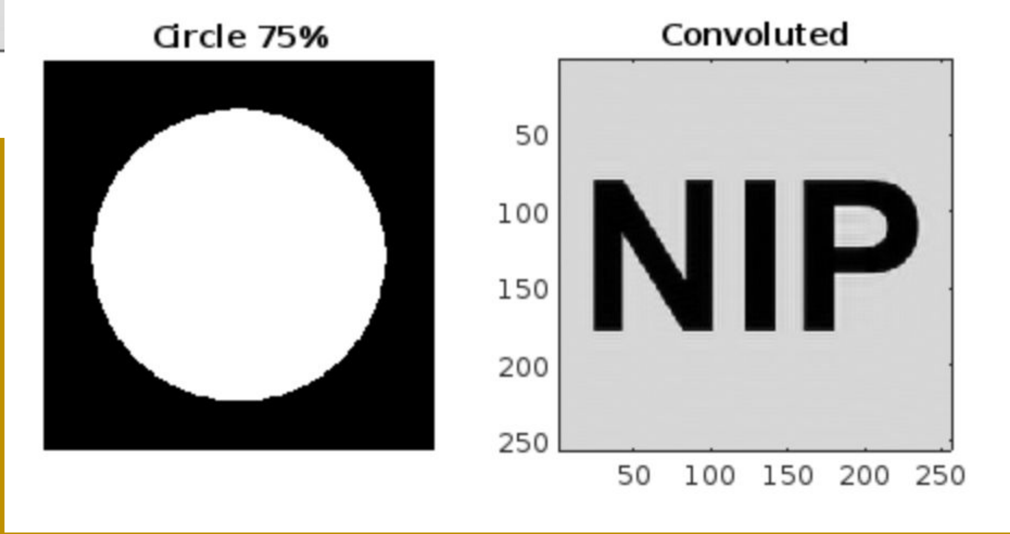
Results & Analysis 2.2



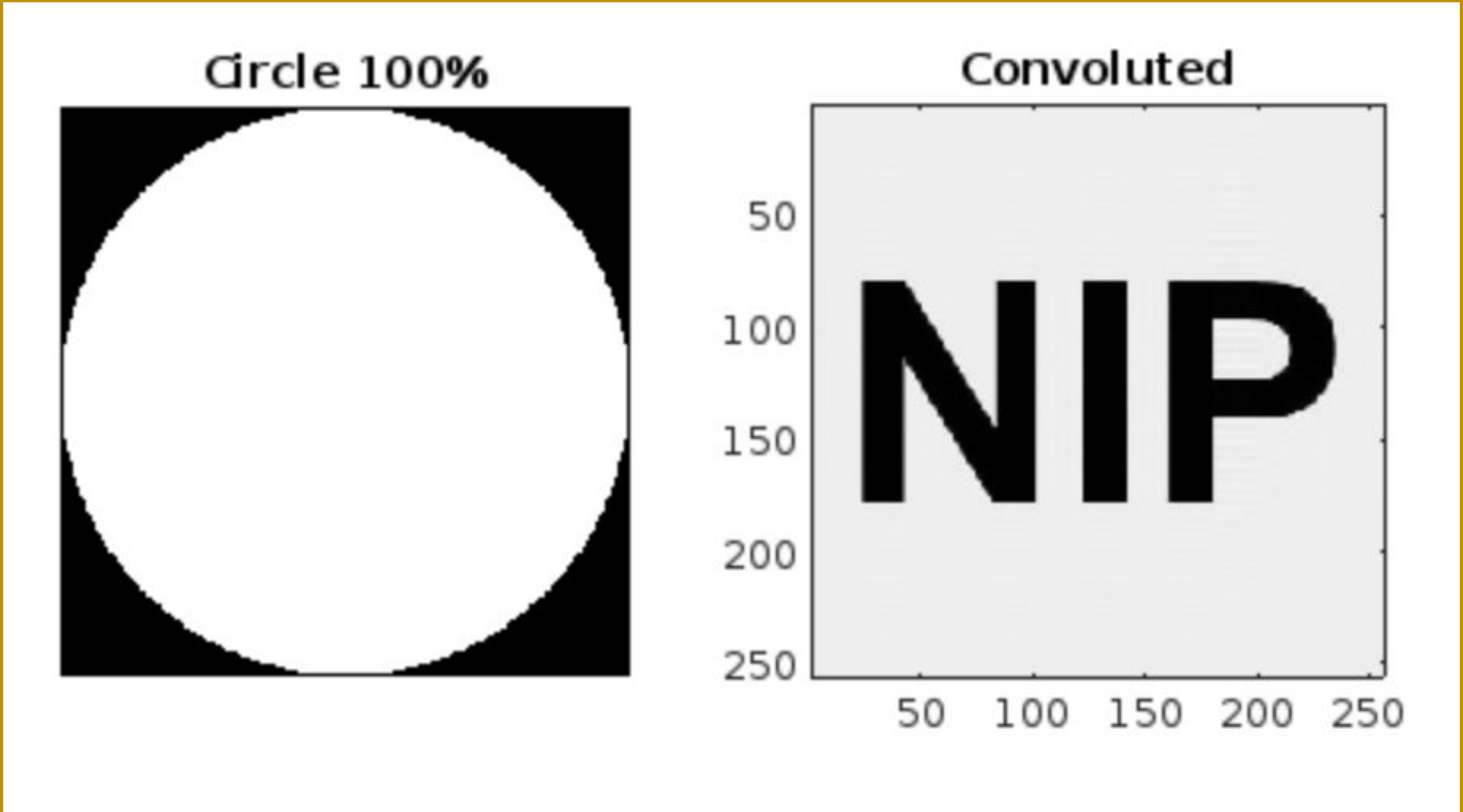


TEXT CONVOLUTION

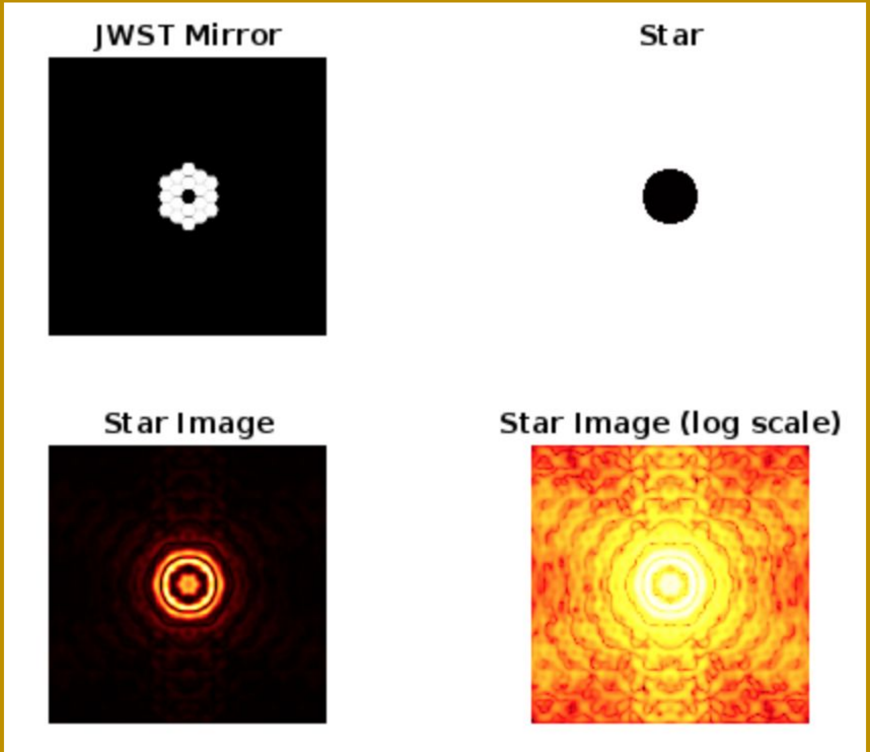


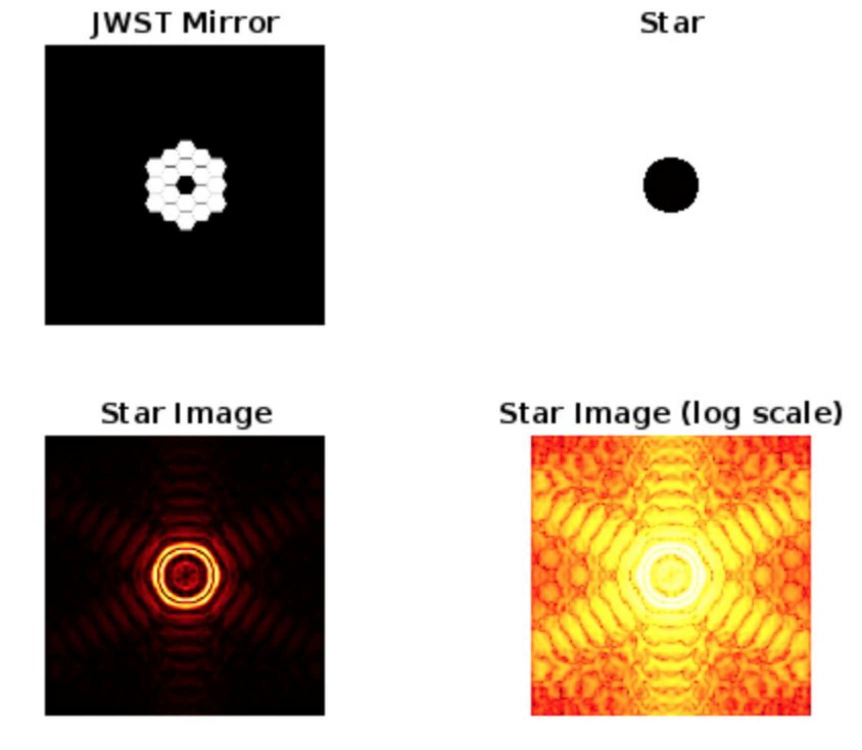


TEXT CONVOLUTION

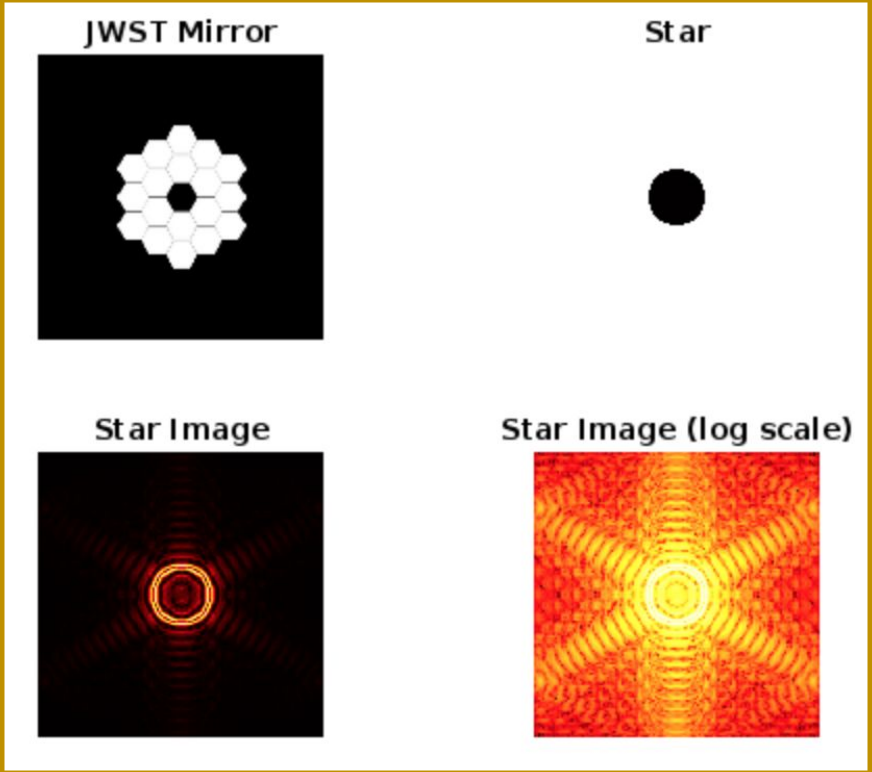


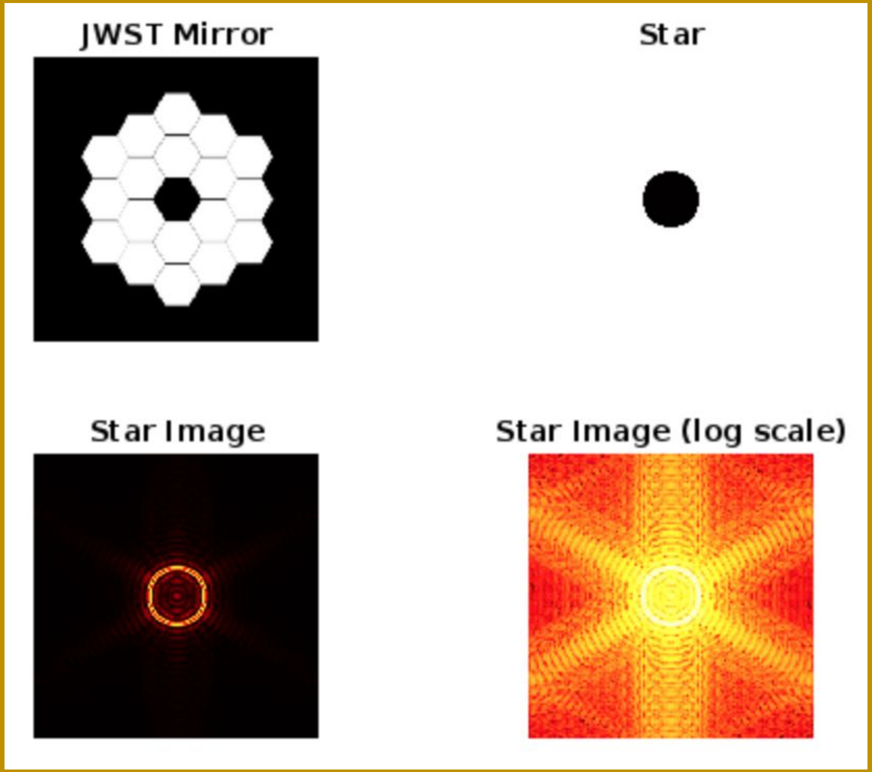
TEXT CONVOLUTION



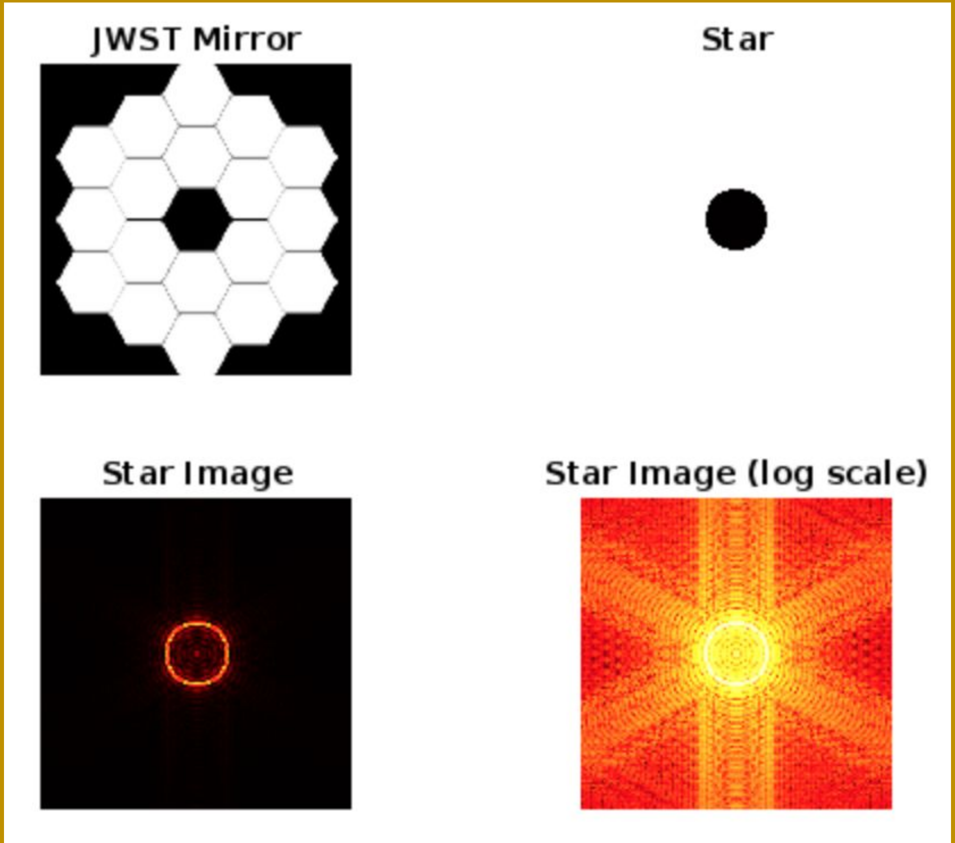


CONVOLUTION AS CAMERA (Camera Size Variation)

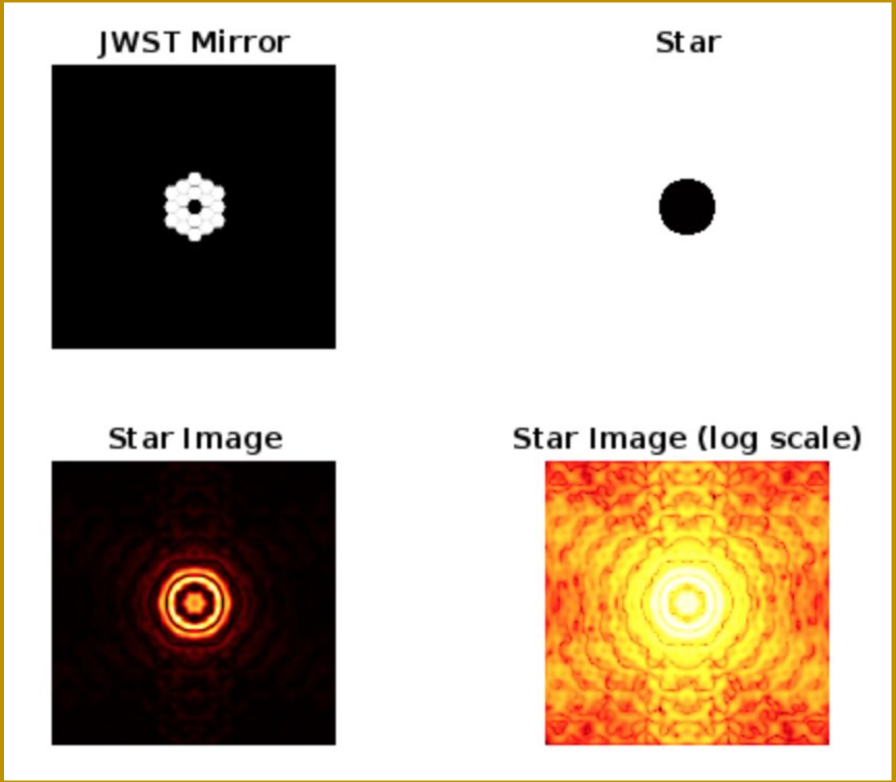


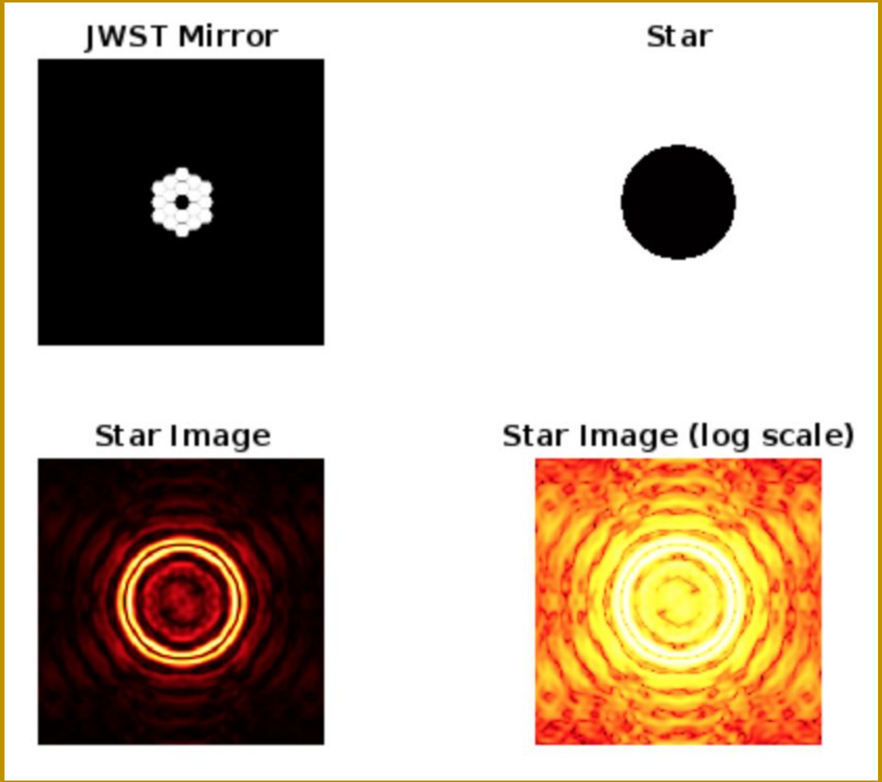


CONVOLUTION AS CAMERA (Camera Size Variation)

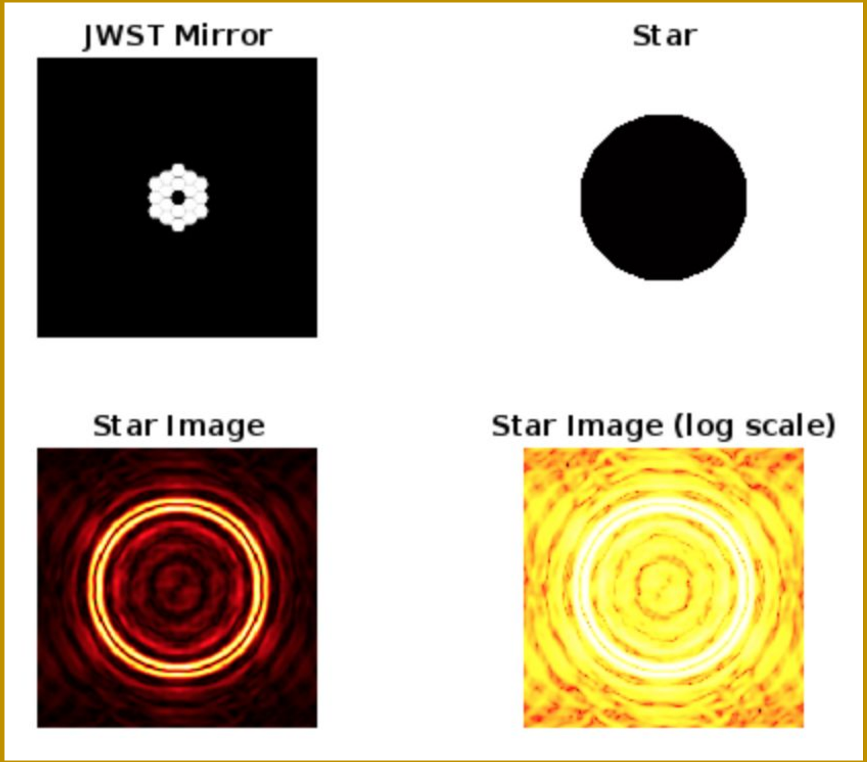


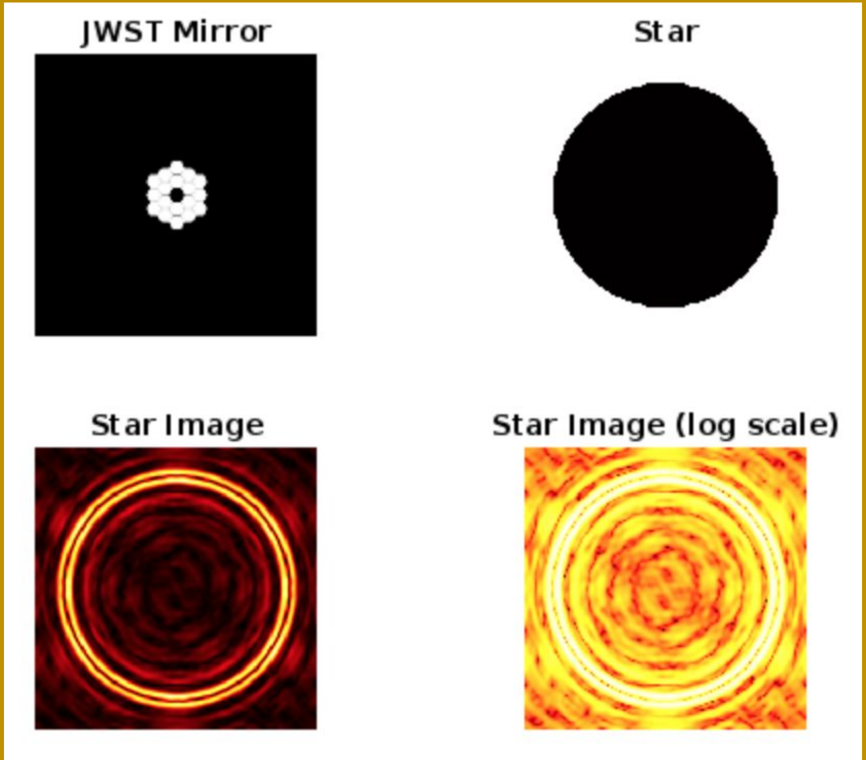
CONVOLUTION AS CAMERA (Camera Size Variation)



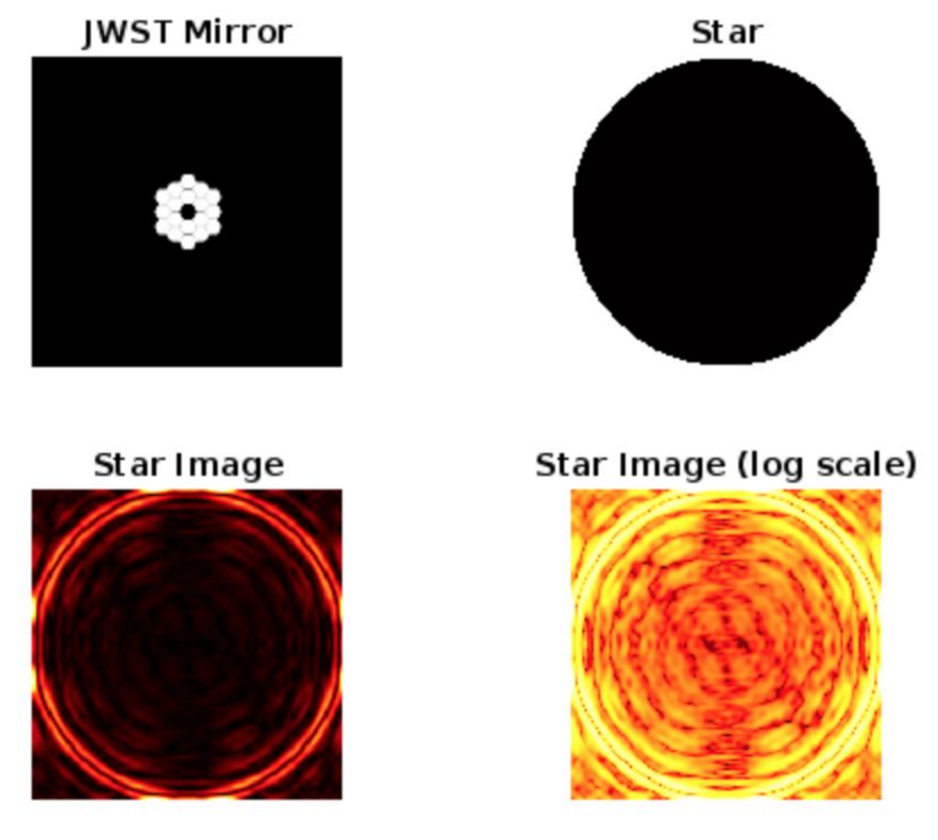


CONVOLUTION AS CAMERA (Star Size Variation)

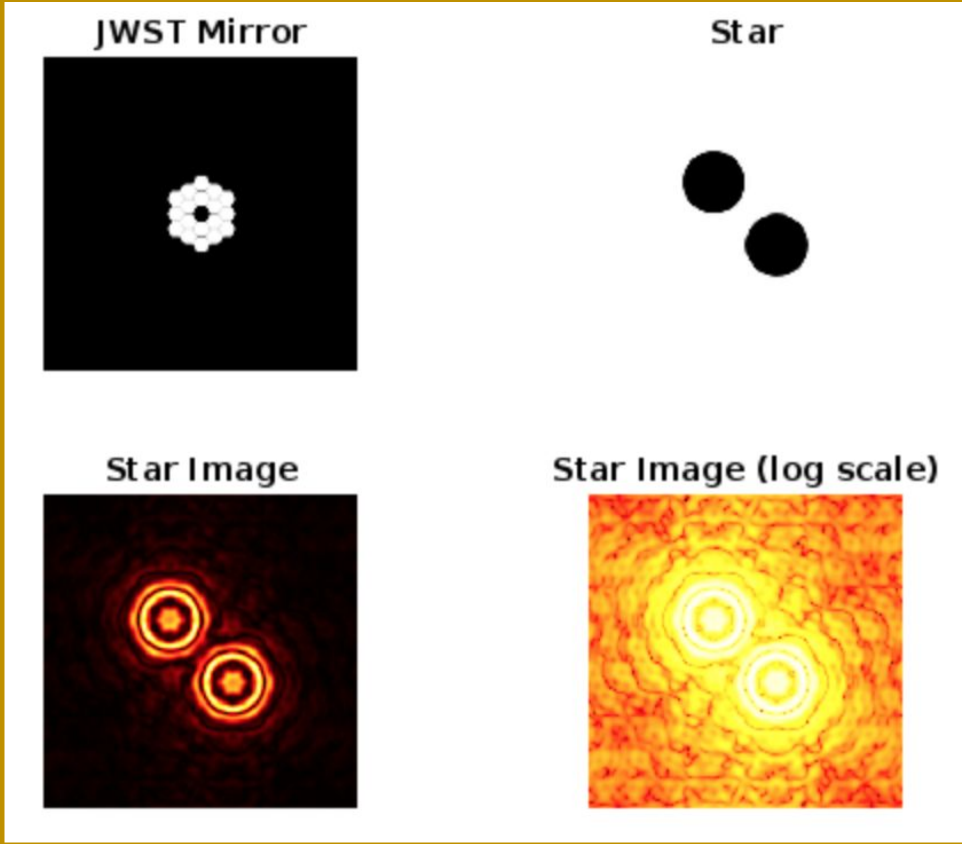




CONVOLUTION AS CAMERA (Star Size Variation)



CONVOLUTION AS CAMERA (Star Size Variation)



CONVOLUTION AS CAMERA (Binary Star)

Convolution can be used as a ‘filter’ that produces an output image that is based on the characteristics of that filter. In the case of this activity, the image of the enlarging circle acts as the filter that enhances the sharpness of the text image.

Convolution can also be used as a camera simulation, as can be seen with the JWST mirror and point star images.

ANALYSIS

Background Correlation

The correlation ρ measures the degree of similarity between two functions h and g. The more identical they are at a certain position (x,y) the higher their correlation value.

$$Corr(g,h) = \int_{-\infty}^{\infty} g(\tau + t)h(t)d\tau$$

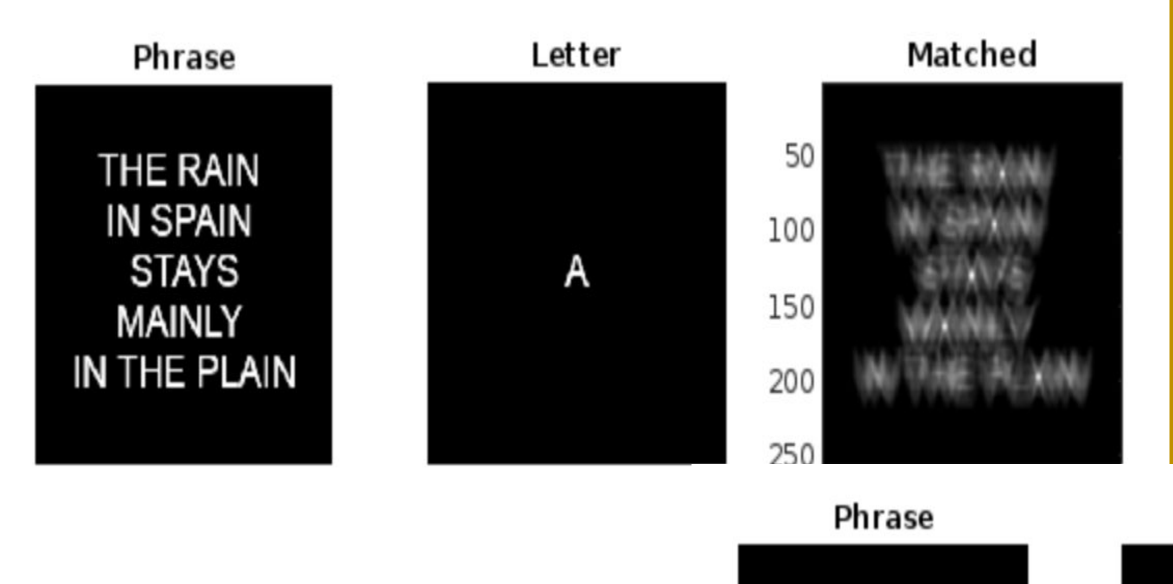
The Fourier Transform Correlation theorem states that correlation in the time domain is equivalent to multiplication in the frequency domain. Specifically, the Fourier Transform of the correlation of two signals is equal to the product of the Fourier Transform of one signal and the complex conjugate of the Fourier Transform of the other signal.

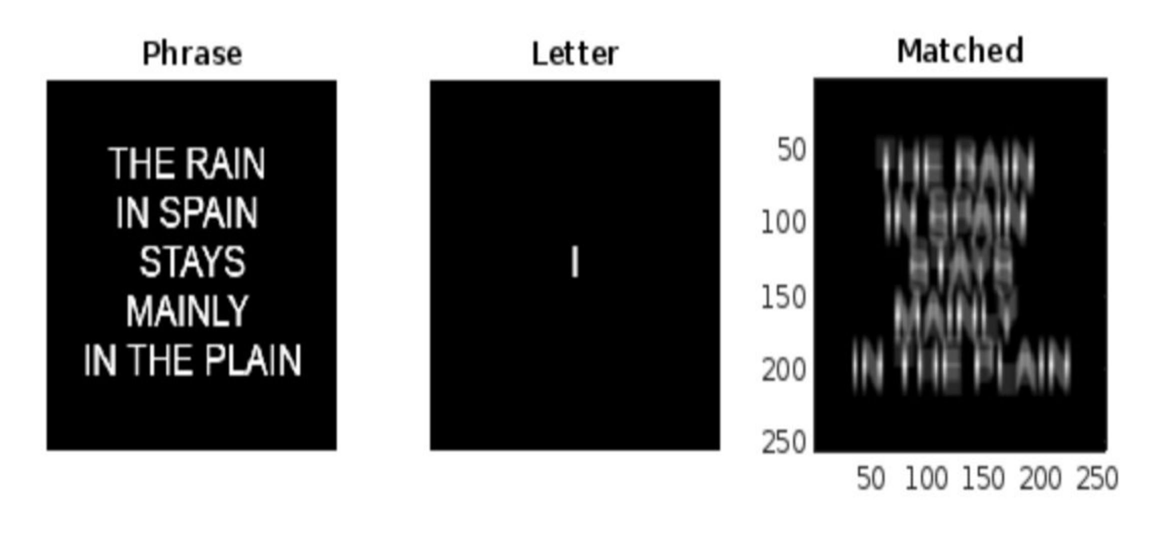
Objectives 2.3

Activity 2.3 Template matching using correlation.

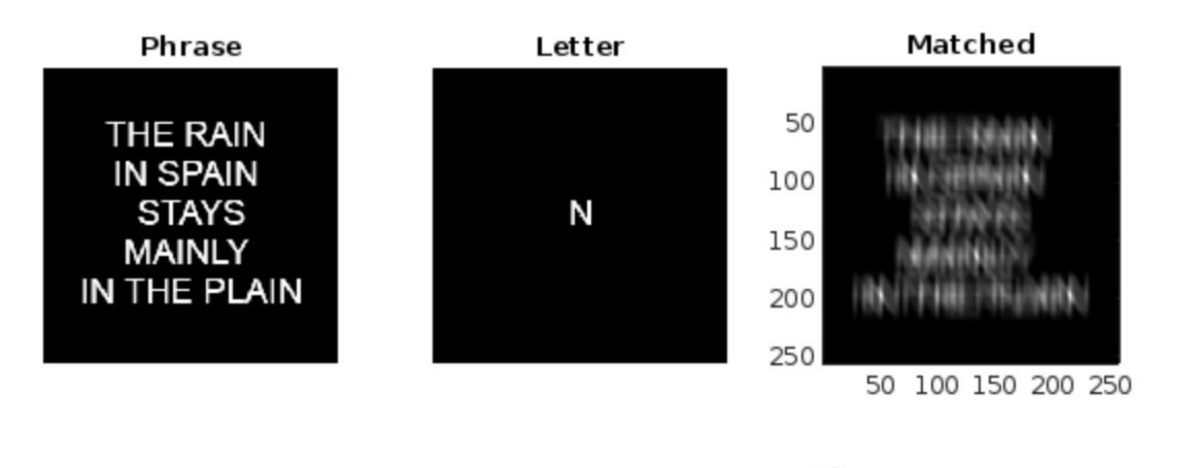
1. Create a 256x 256 image in PAINT with the phrase "THE RAIN IN SPAIN STAYS MAINLY IN THE PLAIN"). All caps please and use sans serif fonts such as Arial or Helvetica. Let the letters be white and the background black.
2. Create a 256x256 image in PAINT of the letter "A" using the same font and font size as in step 1. Make sure "A" is in the geometric center of the image. Again, letter is white, background black. A is the "template" and we want to find matches of "A" in the target phrase image.
3. Get the FFT2 of both images.
4. Multiply the complex conjugate of FFT2(A) with the FFT2 of the phrase image.
5. Get the inverse FFT of the result in 4 and display as abs(). Comment on the result.

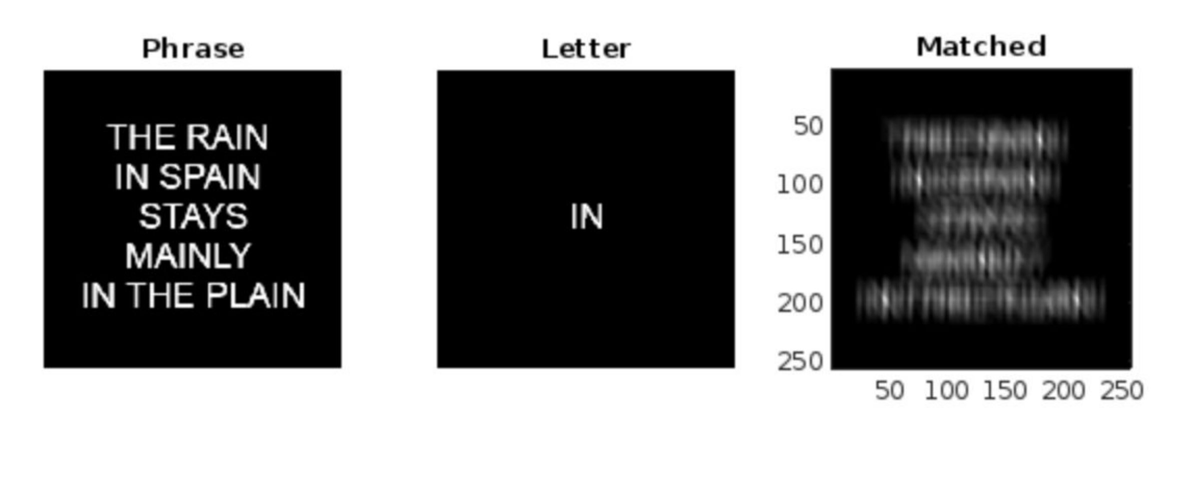
Results & Analysis 2.3





TEMPLATE MATCHING





TEMPLATE MATCHING

Correlation compares 2 images, one acts as a template and the other is a general image, then detects patterns, computes their similarities, and outputs another image. The resulting image visualizes the degree of similarity of the two. Highlighted values of the resultant image shows where the two images matched coherently (template matching) white darker areas show contrasts of the two. Correlation can be used for pattern detection.

ANALYSIS

Reflection

Having done Activity 2 Part 1 shows how Fourier transform is a very powerful mathematical and virtual tool for image processing and analysis, and other practical uses. It gives us a new point of view on images, as it transforms them from one domain to another.

My only setback was the mathematical theory behind which hindered my initial understanding, but upon performing the code and seeing the image outputs, it enlightened me. The fact that I can also tinker with the activity beyond the instructions made the activity much more appreciable.

Overall, it was a very fun activity, mostly because I used a picture of a cat again!

Self Grade

CRITERIA	QUALIFICATIONS	SCORE
Technical Correctness	❑ Met all objectives ❑ Results are complete ❑ Results are verifiably correct ❑ Understood the lesson	33
Presentation Quality	❑ All text and images are good quality ❑ Code has sufficient comments/guides ❑ Plots are properly labeled and visually understandable ❑ Report is clear	35
Self Reflection	❑ Explained validity of results ❑ Discussed what went wrong/right in activity ❑ Justified self score ❑ Acknowledged sources	27
Initiative	❑ Experimented beyond what was required ❑ Made significant improvements to existing code	10

References

[1] Graham, J. (March 25, 2005). Convolution, correlation, and Fourier transforms. *UC Berkeley.* Retrieved from http://ugastro.berkeley.edu/infrared/ir_clusters/convolution.pdf

[2] Adongo, P. (June 19, 2022). Implementation of 2-D discrete Fourier transforms of square functions and natural images in matlab. *Section.* Retrieved from https://www.section.io/engineering-education/implementation-of-2d-discrete-fourier-transform-of-square-functions-and-natural-images-in-matlab/#:~:text=The%20discrete%2Dtime%20Fourier%20transform,representation%20with%20rich%20local%20covariance.

Warm thanks also to Johnenn Manalang & Richmond Jumawan for helping me with the codes for Activity 2.2.