Projet ingénierie biomédical

Modélisation de l'asthme

Réalisé par :

- Ahmed Taha LAMRANI
- Abdeljalil FARID
- Mohamed BOUHMID
- Oumar KEITA
- Anas ROUAM

Encadré par :

- Pr. Marcel FILOCHE

Plan

1) Modélisation en 2D d'une bronchiole saine

↓

2) Relation diamètre - Résistance hydraulique

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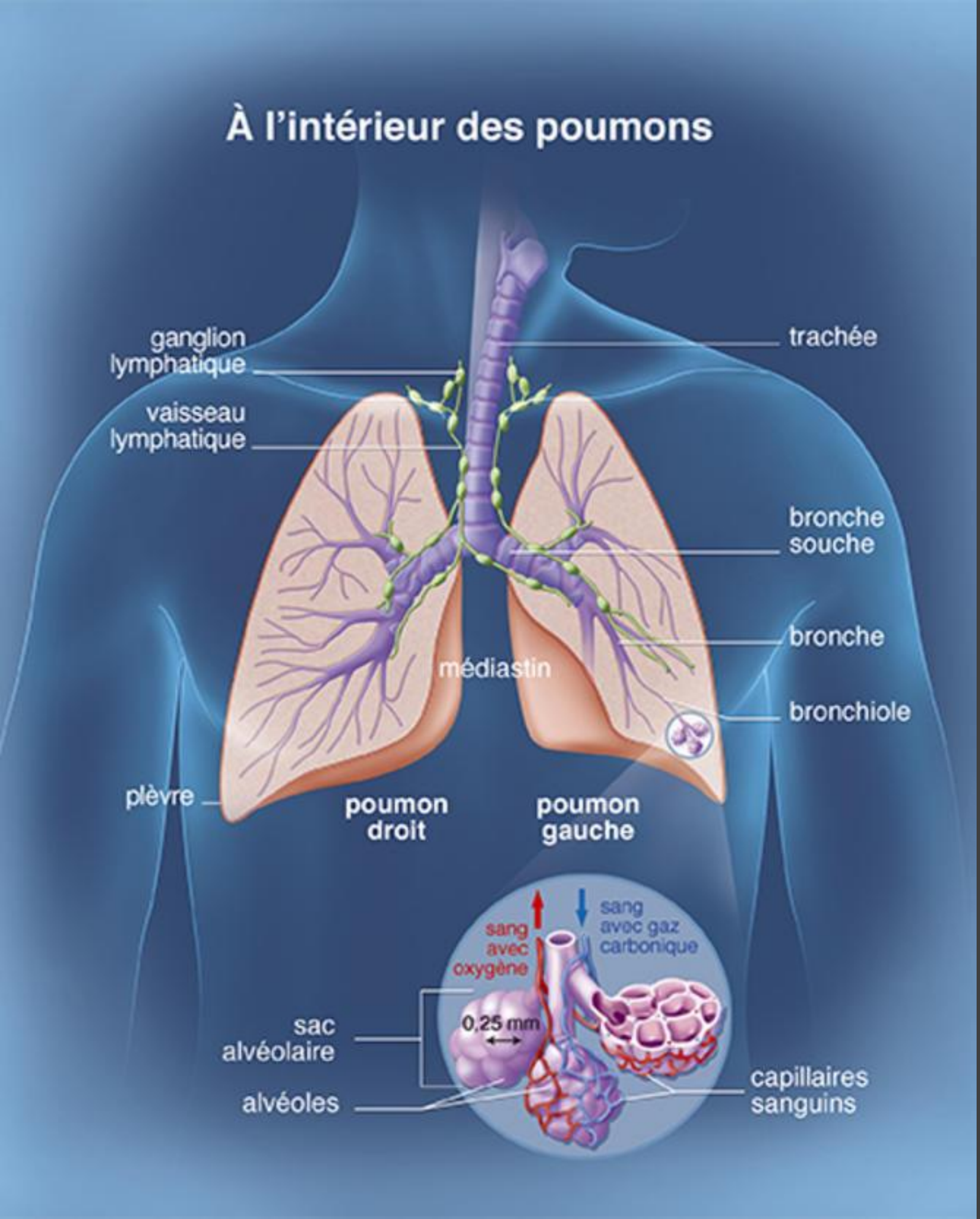
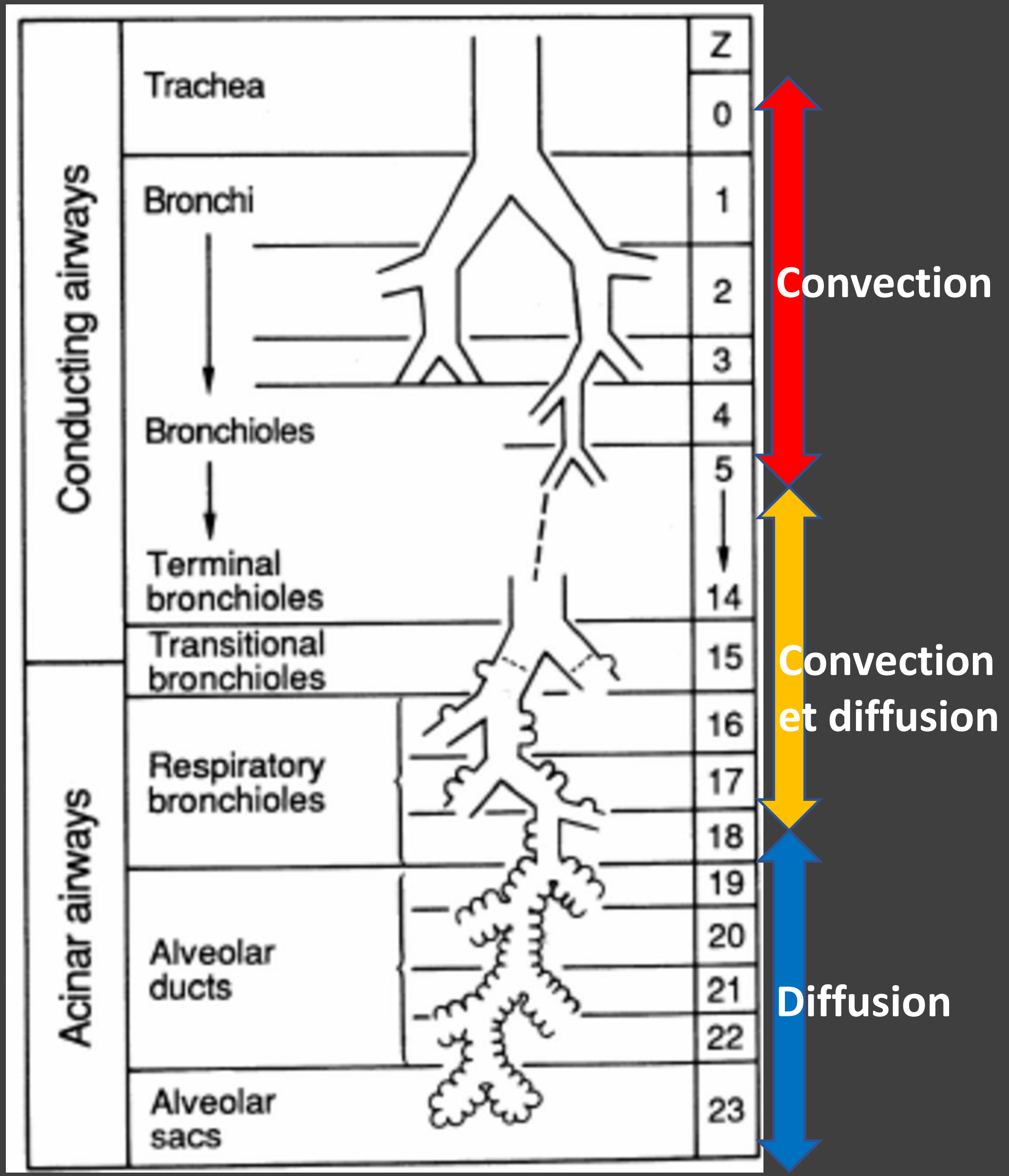
3) Modélisation de l'asthme



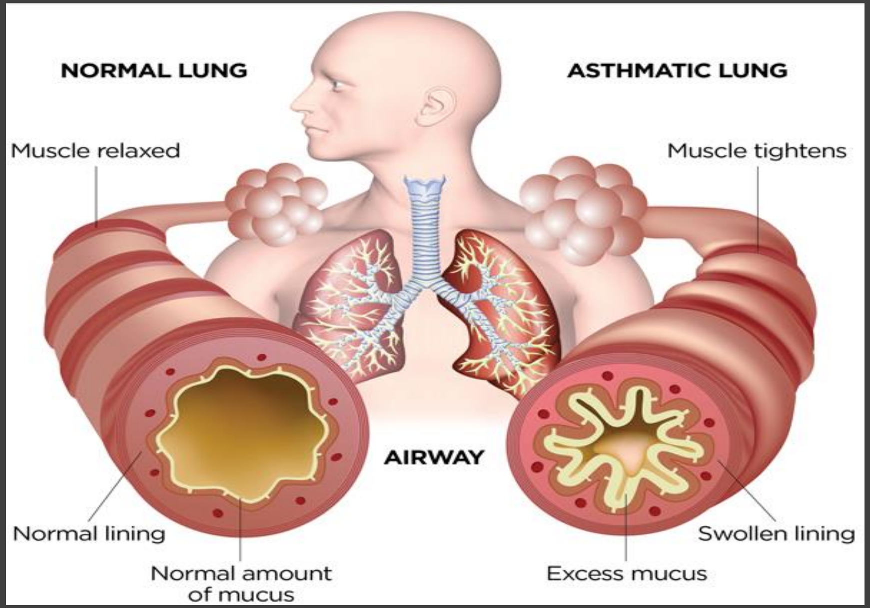
Présentation du poumon

- Chaque poumon est consitué de plusieurs bifurcations qui aboutissent à des sacs alvéolaires



Le système pulmonaire aérien

- L'asthme est une maladie chronique due à l'inflammation des voies aériennes. La contraction des muscles bronchiques et la sécrétion de mucus provoquent une obstruction bronchique



Au delà des bronchioles respiratoires

Equation de Navier Stokes

$$\rho(\frac{\partial \vec{u}}{\partial t} + (\vec{grad}.\vec{u})\vec{u}) = -\vec{grad}(p) + \eta\Delta\vec{u}$$

Ecoulement permanent + diffusive

$$\eta\Delta\vec{u} - \vec{grad}(p) = 0 \quad (1)$$

Equation de Conservation de la masse

$$\frac{\partial \rho}{\partial t} + div(\vec{u}) = 0$$

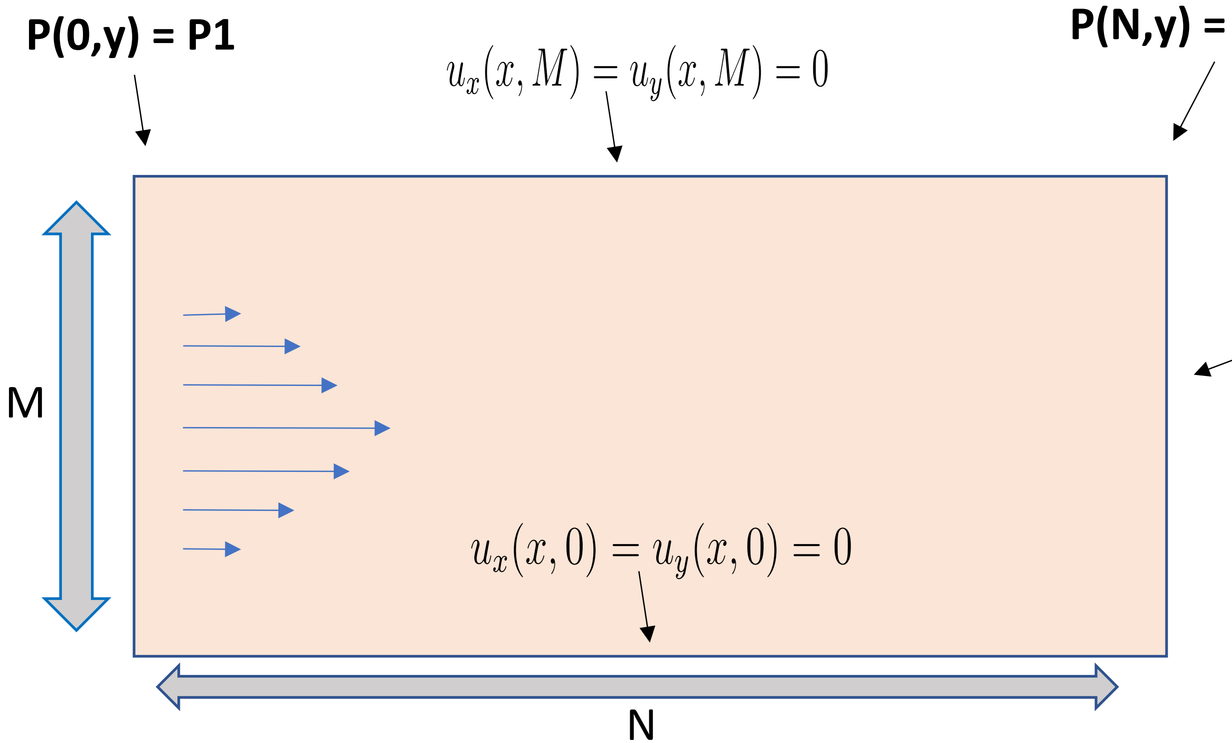
Ecoulement incompressible et homogène

$$div(\vec{u}) = 0 \quad (2)$$

Modèle des branches saines en 2D

$$(1) \quad \Rightarrow \quad \begin{cases} \dfrac{\partial^2 u_x}{\partial x^2} + \dfrac{\partial^2 u_x}{\partial y^2} - \dfrac{\partial P}{\partial x} = 0 \\ \dfrac{\partial^2 u_y}{\partial x^2} + \dfrac{\partial^2 u_y}{\partial y^2} - \dfrac{\partial P}{\partial y} = 0 \end{cases}$$

$$(2) \quad \Rightarrow \quad \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} = 0$$



$$\begin{cases} u_x(0,y) = u_x(N,y) \\ u_y(0,y) = u_y(N,y) \\ \frac{\partial u_x}{\partial x}(0,y) = \frac{\partial u_x}{\partial x}(N,y) \\ \frac{\partial u_y}{\partial x}(x,y) = 0 \end{cases}$$

Discrétisation des équations

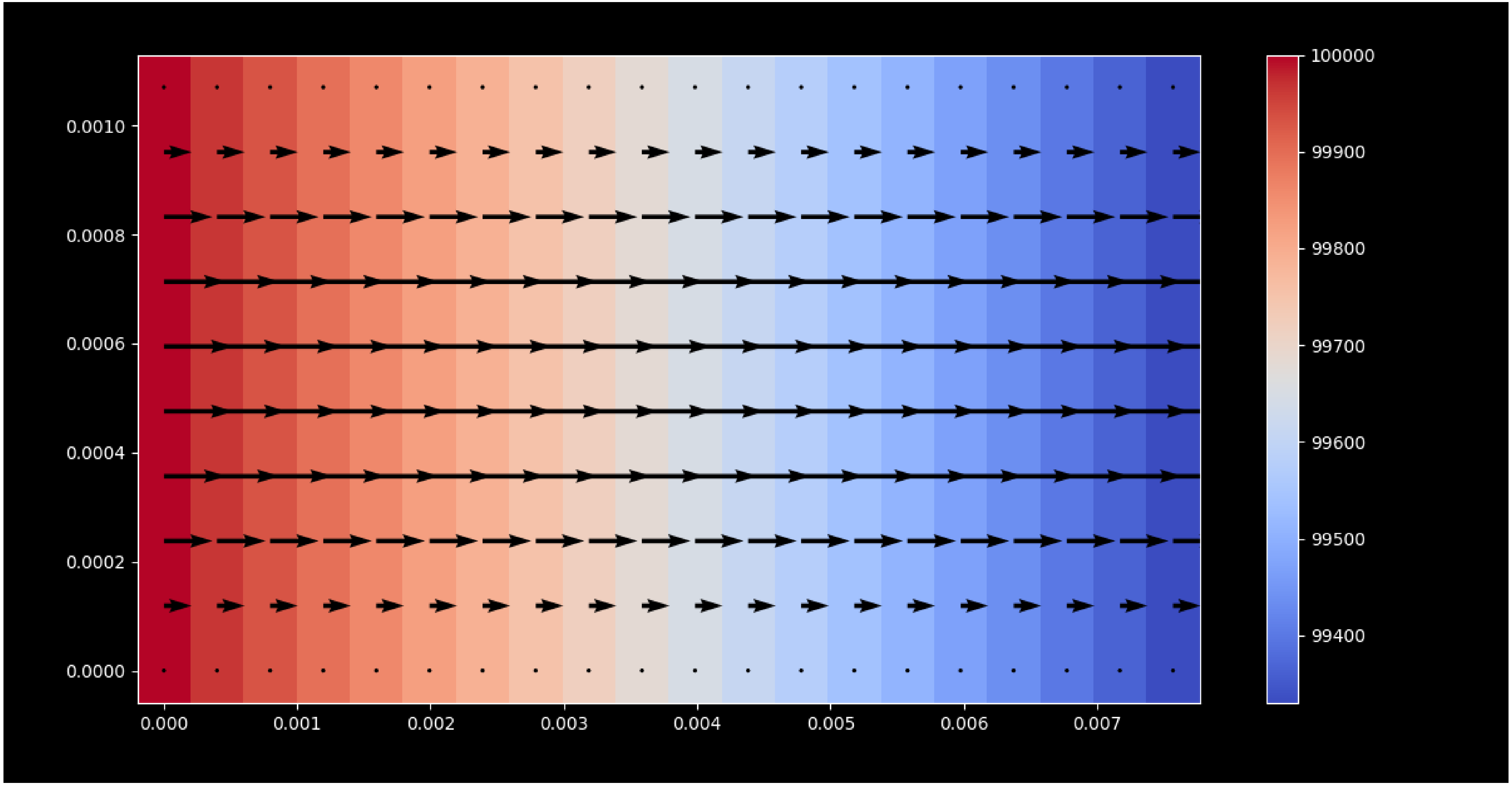
$$\begin{cases} \frac{u_{i+1,j}-2u_{i,j}+u_{i-1,j}}{(\Delta x)^2} + \frac{u_{i,j+1}-2u_{i,j}+u_{i,j-1}}{(\Delta y)^2} - \frac{p_{i+1,j}-p_{i,j}}{(\Delta x)} = 0 \\ \frac{v_{i+1,j}-2v_{i,j}+v_{i-1,j}}{(\Delta x)^2} + \frac{v_{i,j+1}-2v_{i,j}+v_{i,j-1}}{(\Delta y)^2} - \frac{p_{i,j+1}-p_{i,j}}{(\Delta y)} = 0 \\ \frac{u_{i+1,j}-u_{i,j}}{(\Delta x)} + \frac{v_{i,j+1}-v_{i,j}}{(\Delta y)} = 0 \end{cases}$$

Bijection entre les coordonnées

$$k = f(i, j, N) = i + Nj$$

$$(i, j) = f^{-1}(k) = (k\%N,\ k//N)$$

Graphique du profil du vitesse et pression



Calcul de la résistance hydraulique

$$\frac{\partial u_y}{\partial y} = 0 \Rightarrow \Delta u_y = 0$$

$$(3) \Rightarrow \frac{\partial u_x}{\partial x} = 0 \Rightarrow u_x = u_x(y)$$

$$(2) \quad \Rightarrow \quad \frac{\partial P}{\partial y} = 0 \quad \Rightarrow \quad P = P(x) \quad \Rightarrow \quad \frac{\partial P}{\partial x} = \frac{P2 - P1}{L}$$

$$\frac{\partial^2 u_x}{\partial x^2} = 0 \Rightarrow (1) \ : \ \frac{\partial^2 u_x}{\partial y^2} + \frac{\partial P}{\partial x} = 0$$

$$u_x(y) = \frac{P2 - P1}{2\eta L} y^2 + by + c$$

$$u_x(0) = u_x(L) = 0 \Rightarrow \begin{cases} b = -\frac{P2-P1}{2\eta L} D \\ c = 0 \end{cases}$$

$$u_x = \frac{P2 - P1}{2\eta L}(y^2 - Dy)$$

$$Q_s = \int_0^D u_x(y) dy = -\frac{P2 - P1}{12\eta L} D^3$$

$$\boldsymbol{R_H = \frac{\Delta P}{Q_s} = \frac{12\eta L}{D^3}}$$

Pour une branche de 17ème niveau

Analytiquement :

$\eta = 1,81 * 10^{-5} \; Pa.s$

$L = 7,57 \; mm$

$D = 1,07 \; mm$

$\Longrightarrow$ $R_H = 1342 \; Pa.s.m^{-2}$

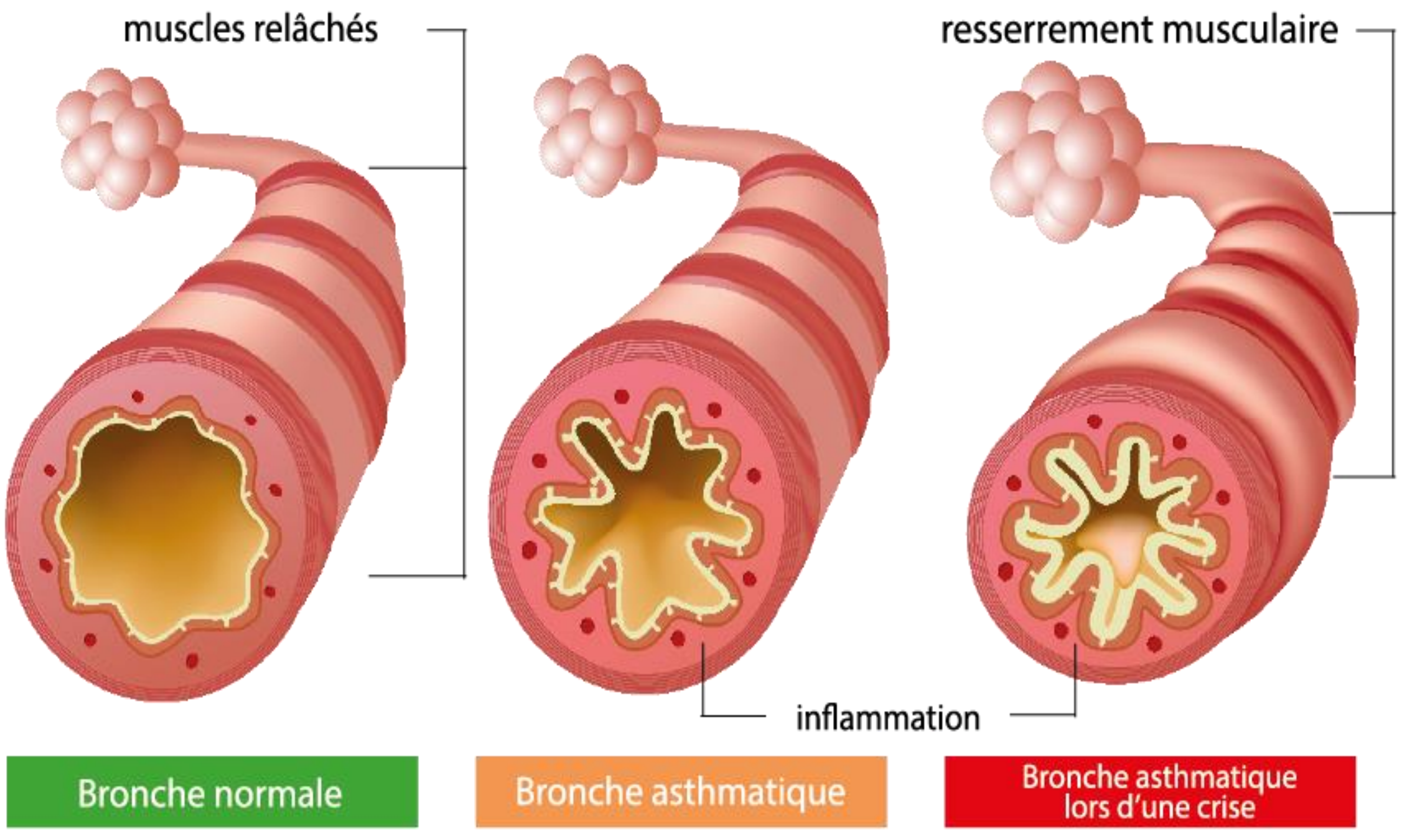
Numériquement :

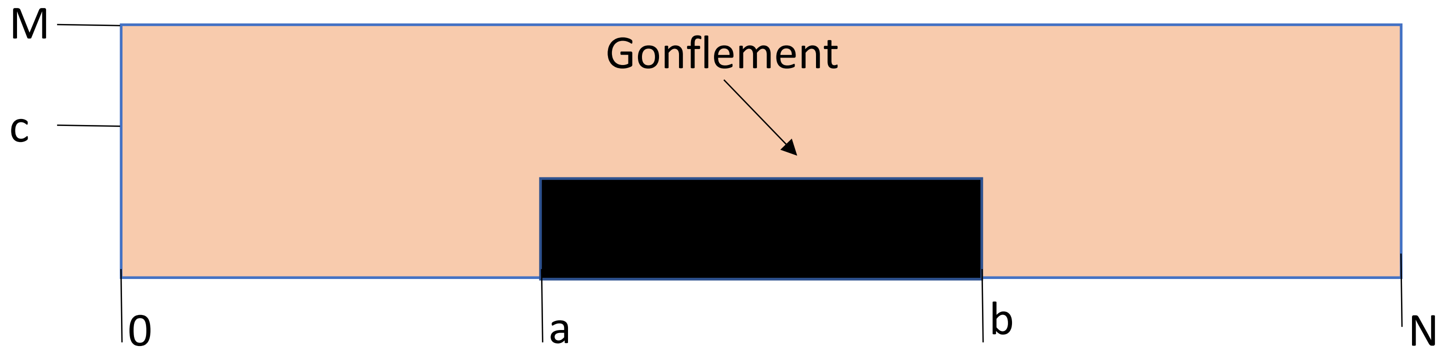
$\Delta P = P1 - P2 = 30 \, Pa$

$$Q_s = \int_0^D u dy \; \simeq \; \Delta y * \sum_i^M u_i \; = \; 0.019 \, m^2.s^{-1}$$

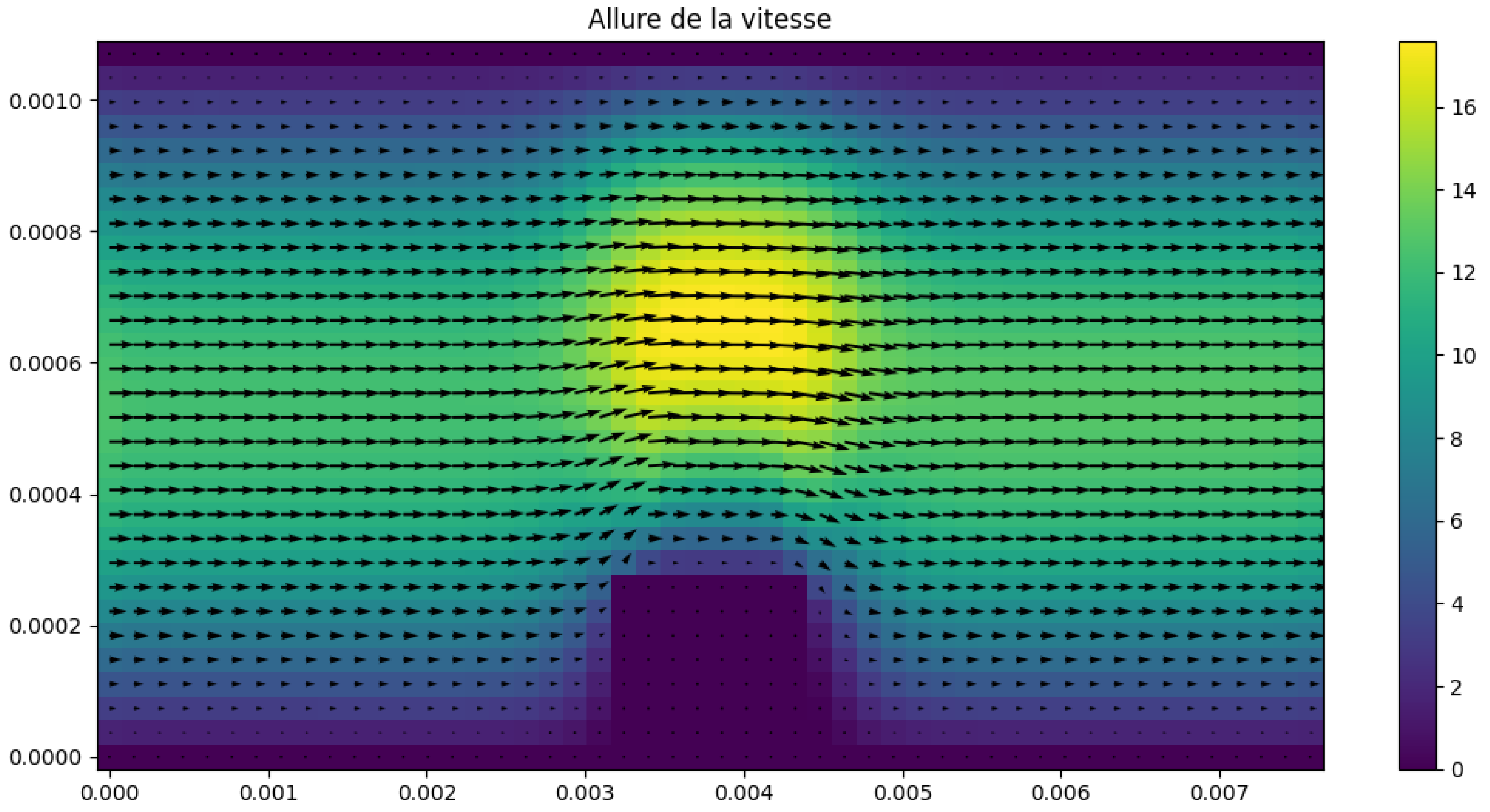
$\Longrightarrow$ $R_H = 1578 \; Pa.s.m^{-2}$

Modélisation numérique de l'asthme

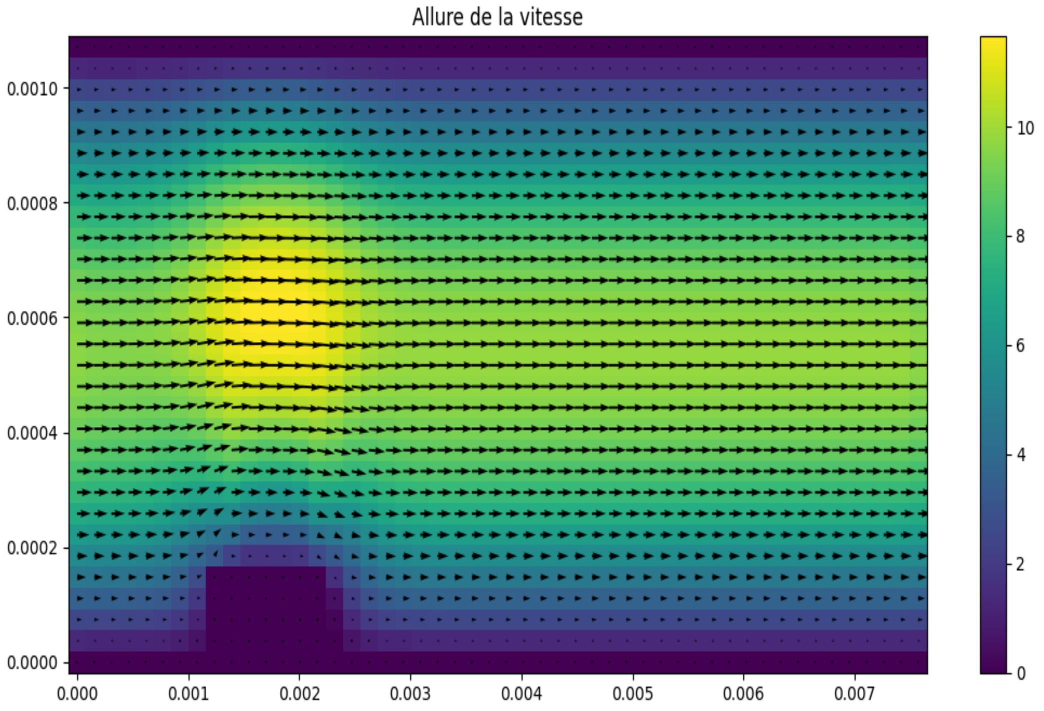


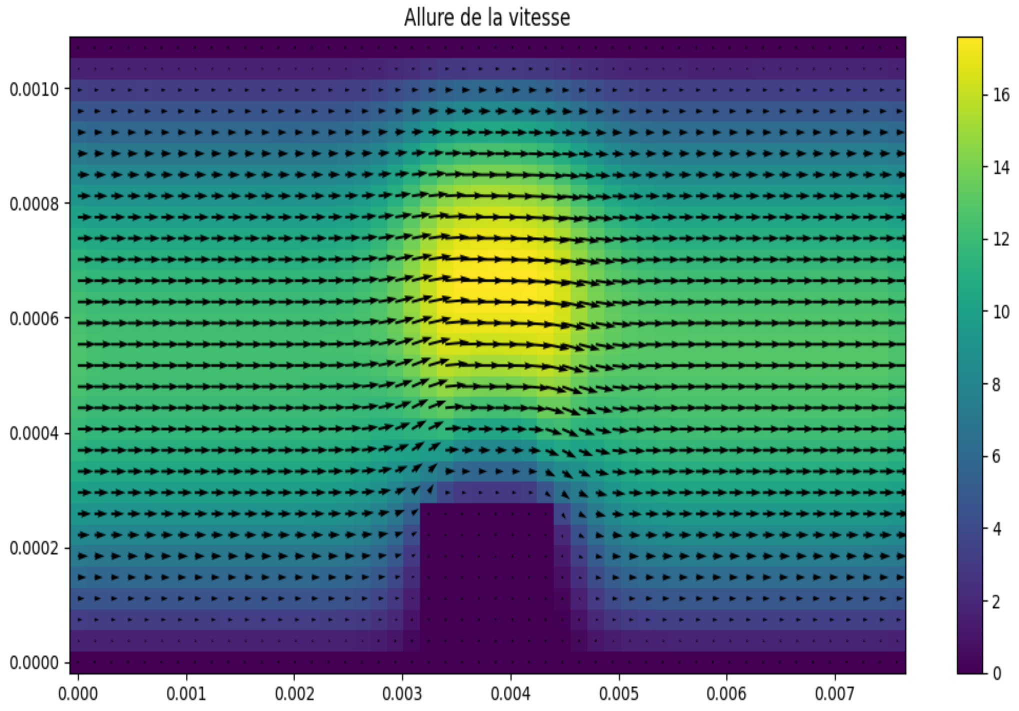


Simulation de premiere deformation

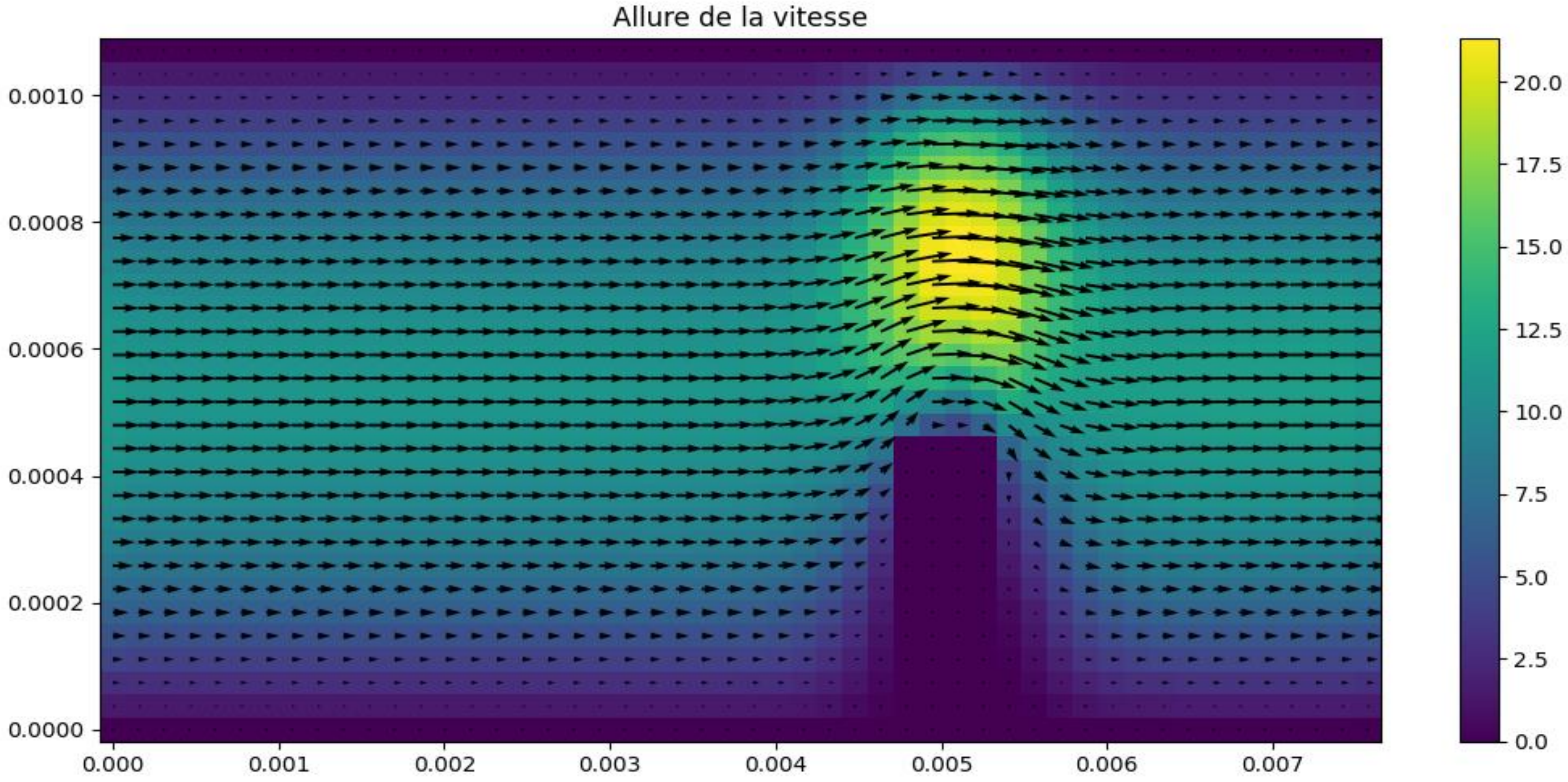


Variation de la deformation en fonction de la taille et la position de la deformation

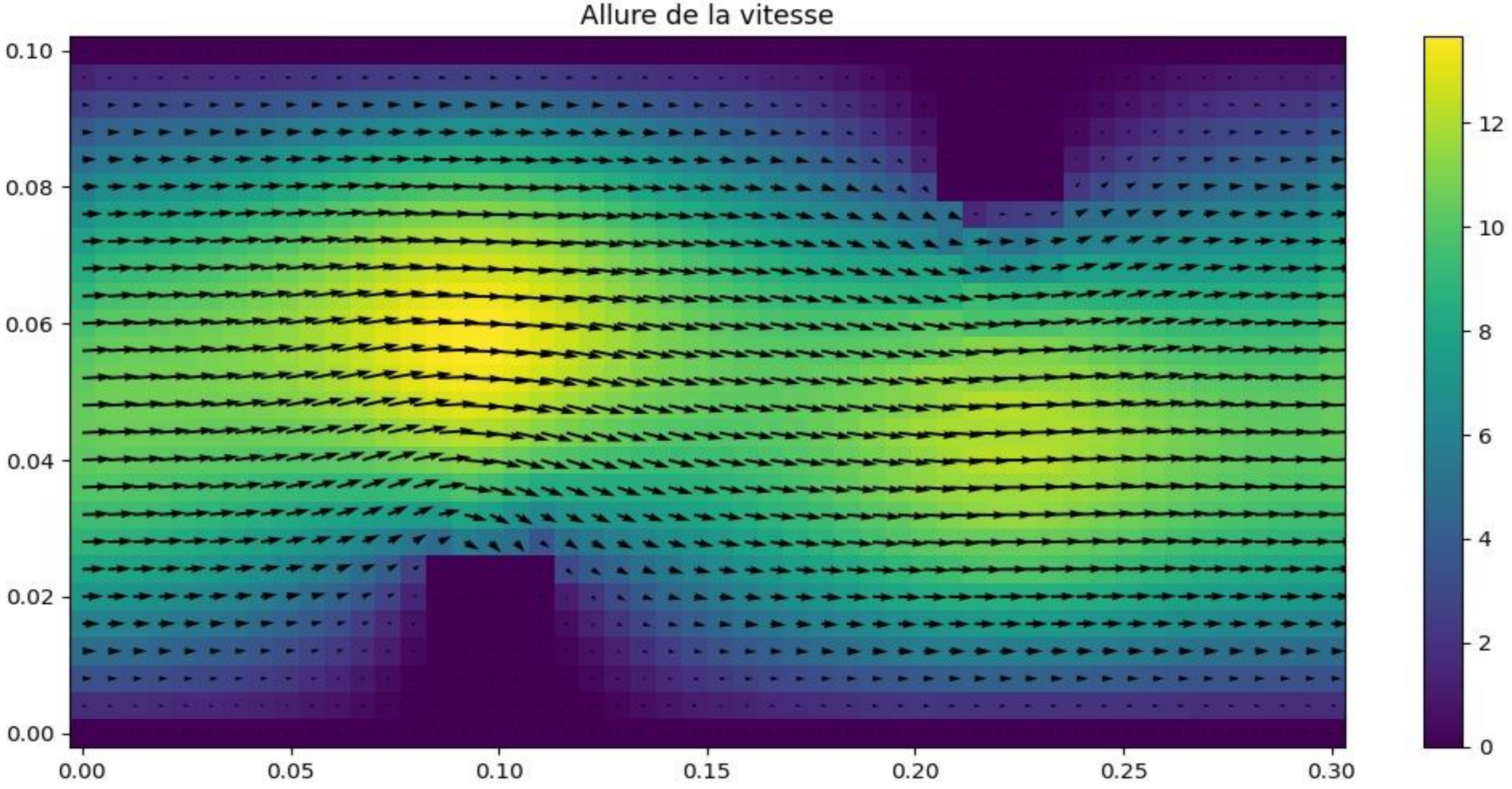




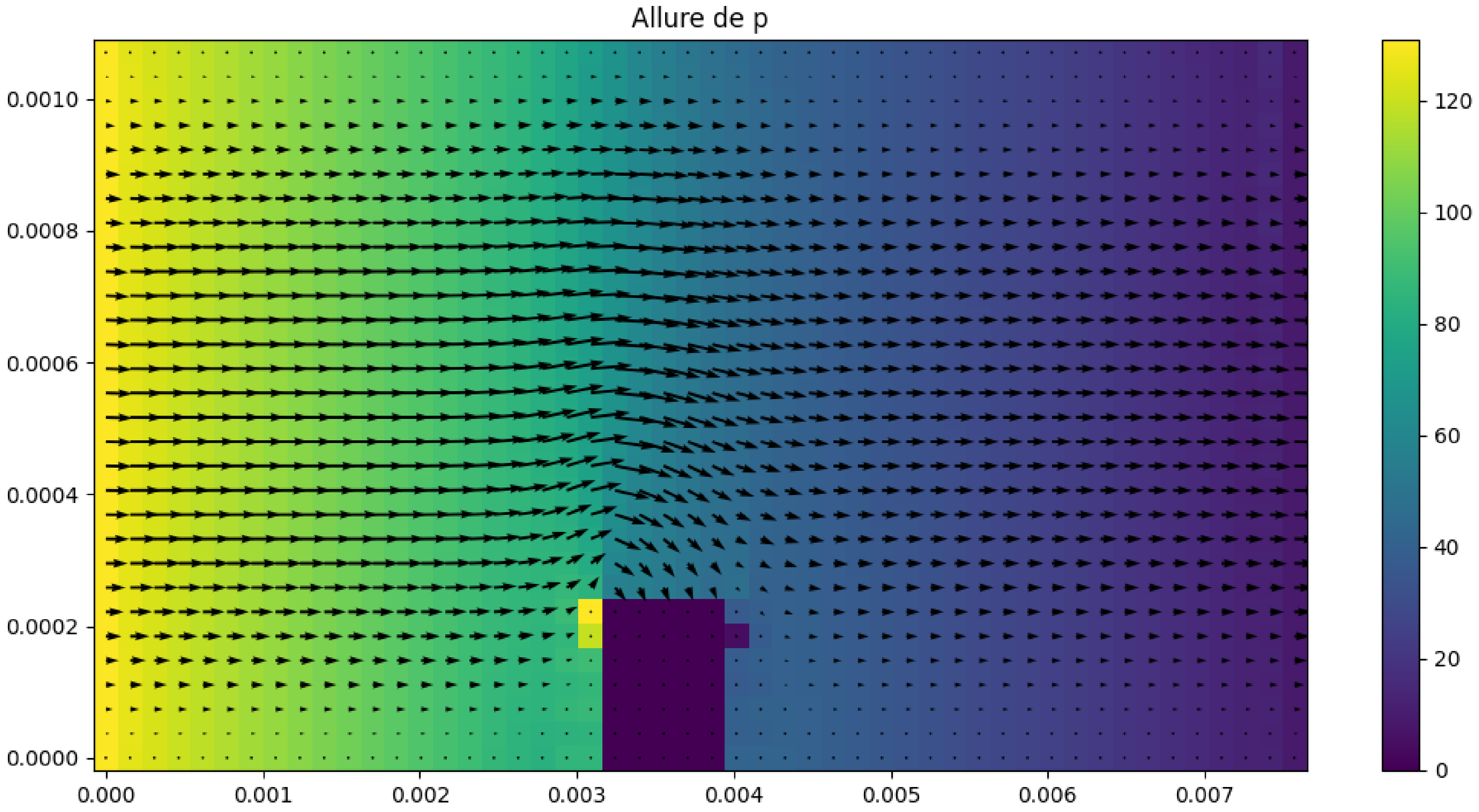
Conservation du debit



Cas de deux obstacles



Impact sur la pression



CONCLUSION

$$R_H = \frac{\Delta P}{Q_s} = \frac{12\eta L}{D^3}$$