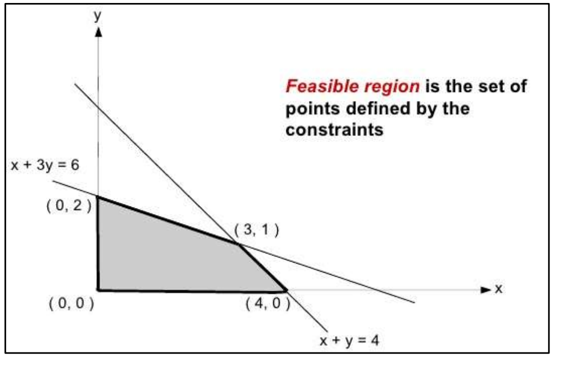
Blending problem

What is a Linear Program ?

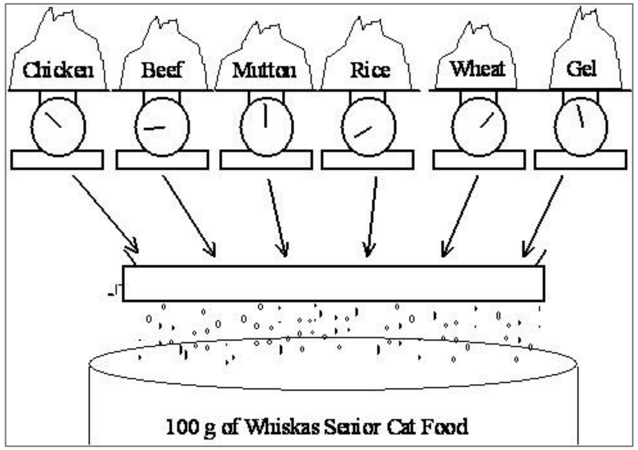
A linear program is an optimization problem where the objective and constraints are all linear (or affine)

Reminder : an optimization problem	Example of a linear program
$$\begin{cases} \text{Min} \quad c(x) \\ \text{Subject to} \quad f_1(x) \leq 0 \\ \qquad f_2(x) = 0 \\ \qquad \vdots \\ \qquad f_p(x) = 0 \\ \boldsymbol{x} \in \mathbb{R}^n \end{cases}$$	$$\begin{cases} \text{Min} \quad 2x + y \\ \text{Subject to} \quad x + 3y \leq 6 \\ \qquad x + y \leq 4 \\ \qquad x, y \geq 0 \\ \qquad (\boldsymbol{x}, \boldsymbol{y}) \in \mathbb{R}^2 \end{cases}$$

Then the feasible set of the problem is a polyhedron :

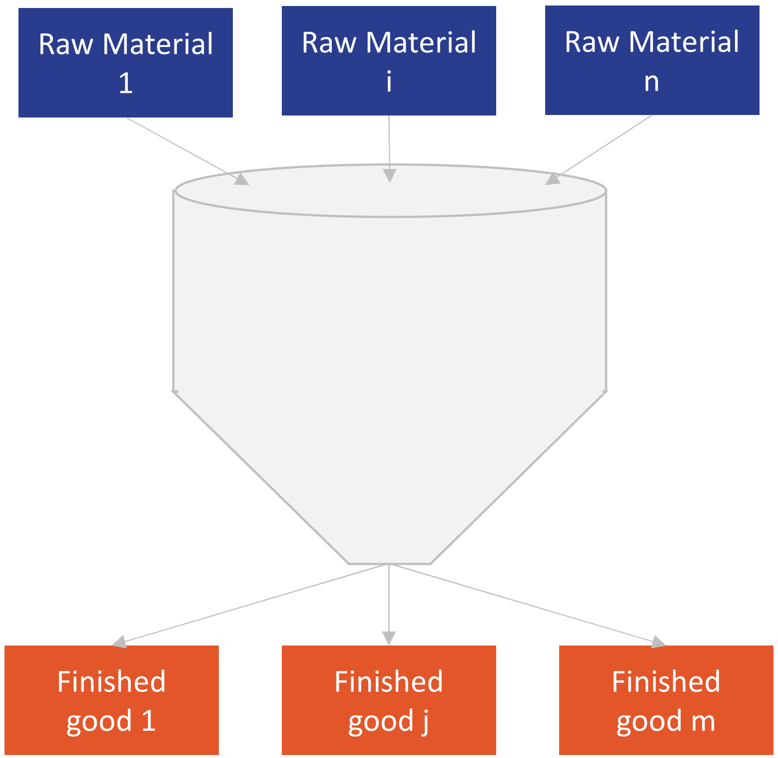


The blending problem : example of applications



- Blending various types of crude oils to produce different types of gasoline and other outputs.
- Blending various chemicals to produce other chemicals
- Blending various types of metal alloys to produce various types of steels

Example of Linear Problem : The Blending Problem



Various raw materials are blended so as to produce finished goods that satisfy requirements on their chemical composition

Objective : satisfy the demand in finished goods while minimizing the raw materials cost

Each finished good shall comply with specifications relative to certains components, taking the form of an admissible range.

The blending action mixes linearly the components of the raw materials

Ex : mix 1t of RM1 + 1t of RM2, with component 1 = 20 for RM1, = 30 for RM2 → we get 2 t of a finished good with 25 of component 1

Each raw material has a limited availability (Capacity)

The blending problem : the data

Indices

- **i :** raw material in {Chicken, Beef, Mutton, Rice, Wheat, Gel}
- **j :** finished good in {Senior Cat food, Junior Cat food}
- **c :** component in {Vitamin, Protein }

Per raw material i

- Cost[i]
- Capacity[i]
- Content[c][i]

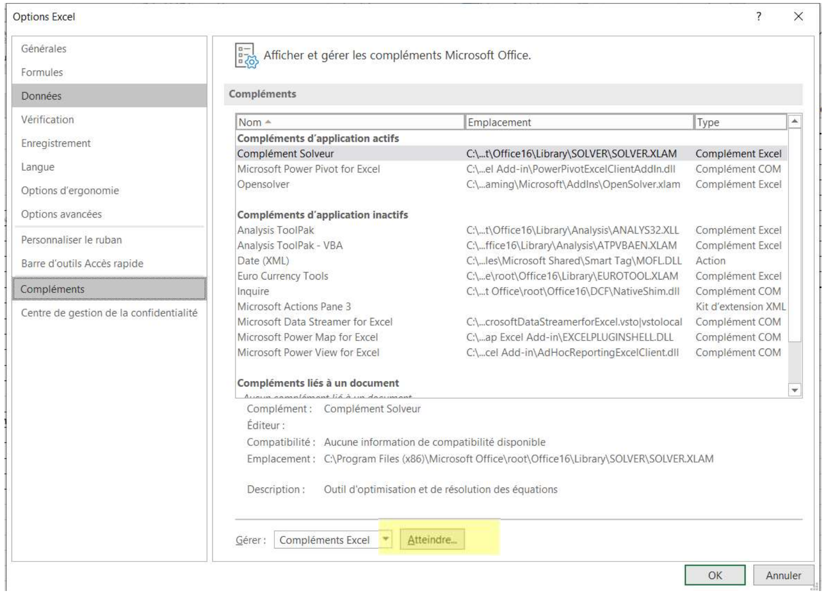
Raw materials	Cost (/kg)	Capacity (kg)	Protein (/kg)	Vitamin (/kg)
Chicken	2,86	1000	1,65	4,62
Beef	2,73	1000	0,36	3,60
Mutton	2,28	600	1,74	3,84
Rice	2,11	500	1,26	1,74
Wheat	2,40	500	2,31	0,84
Gel	2,01	800	1,95	3,84

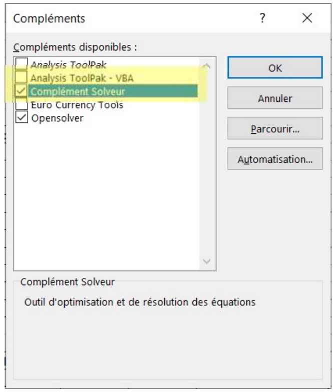
Per finished good j

- Demand[j]
- MinContent[c][j]
- MaxContent[c][j]

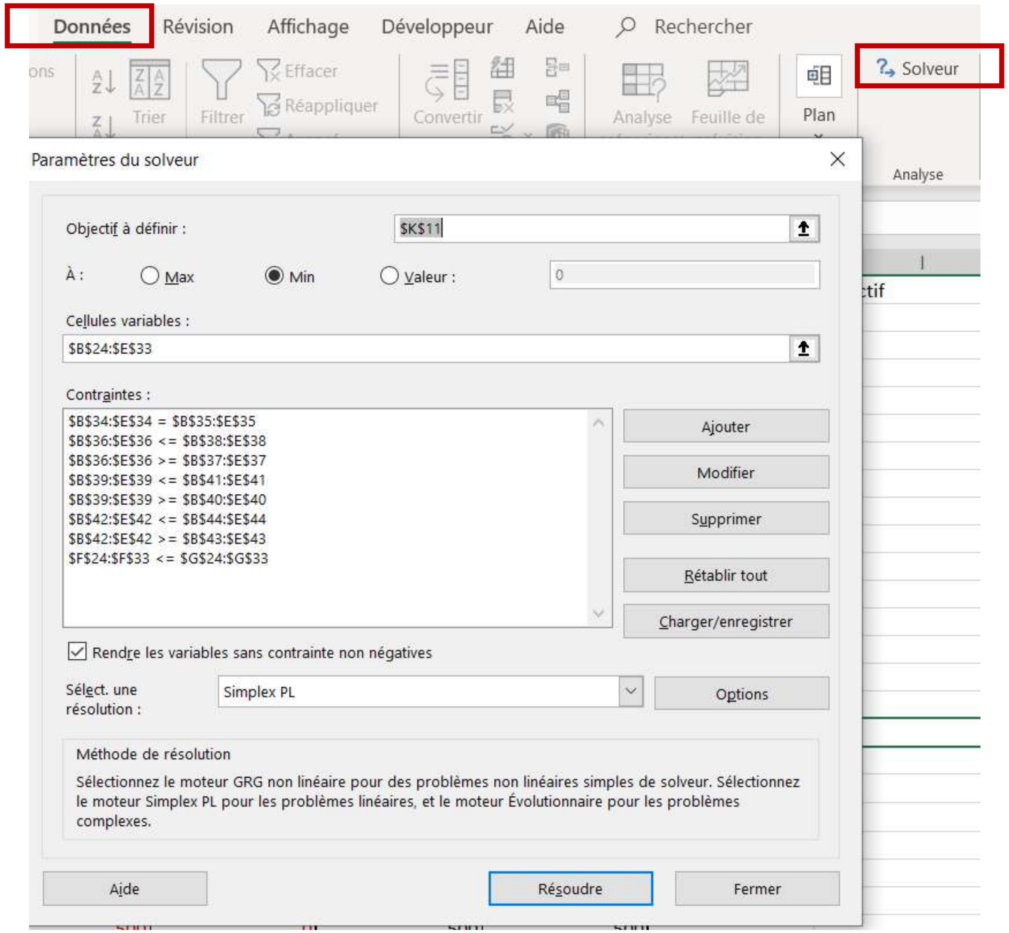
Finished goods	Demand (kg)	MinProtein (/kg)	MaxProtein (/kg)	MinVitamin (/kg)	MaxVitamin (/kg)
Senior cat food	1000	1,44	1,77	3,42	4,32
Junior cat food	1500	1,98	2,40	2,34	3,12

What does it look like in practice ? Install Excel Solver





What does it look like in practice ? Solveur Excel



+ Equality constraints definining states variables can be populated as a classical Excel formula

Let’s practice

Blending problem v1 : OK
- 6 raw materials
- 2 finished goods

Blending problem v2 : to be done
- 7 raw materials
- 3 finished goods

1. Solve the problem

Remark : constraints 1, 2, 5 are already implemented. Only constraints 3 & 4 are to implement

2. Observe what happen to the objective function when tightening a constraint that is already binding

3. For the demand, try to replace the equality by an inequality. Which one and why does still it work ?

The blending problem : Resolution with Excel Solver

Data
Decision variables
State variables

Objectif 441

Raw materials	Cost (/kg)	Capacity (kg)	Protein (/kg)	Vitamin (/kg)
Chicken	2,86	1000	1,65	4,62
Beef	2,73	1000	2,36	3,60
Mutton	2,28	600	1,74	3,84
Rice	2,11	500	1,26	1,74
Wheat	2,40	500	2,31	1,84
Gel	2,31	800	1,95	3,84
Barley	2,51	1000	2,15	3,27

Finished goods	Demand (kg)	MinProtein (/kg)	MaxProtein (/kg)	MinVitamin (/kg)	MaxVitamin (/kg)
Senior cat food	1000	1,44	1,77	3,42	4,32
Junior cat food	1500	1,98	2,20	2,34	3,12
HIgh protein cat food	1000	2,20	2,40	2,34	3,12

Recipe	Senior cat food	Junior cat food	High protein cat	RM_Volume	Capacity
Chicken	10	10	10	30	1000
Beef	10	10	10	30	1000
Mutton	10	10	10	30	600
Rice	10	10	10	30	500
Wheat	10	10	10	30	500
Gel	10	10	10	30	800
Barley	10	10	10	30	1000
FG_Volume	70	70	70		
Demande	1000	1500	1000		
Protein	0,13	0,09	0,13		
MinProtein	1,44	1,98	2,20		
MaxProtein	1,77	2,20	2,40		
Vitamin	0,23	0,15	0,23		
MinVitamin	3,42	2,34	2,34		
MaxVitamin	4,32	3,12	3,12		

Formulation : the variables

Recipe

Recipe	Senior cat food	Junior cat food	High protein cat food	RM_Volume
Chicken	10	10	10	30
Beef	10	10	10	30
Mutton	10	10	10	30
Rice	10	10	10	30
Wheat	10	10	10	30
Gel	10	10	10	30
Barley	10	10	10	30
FG_Volume	70	70	70	

Total raw materials that are used

Total Finished Goods that is produced