Optimization

EMINES - 2023

Africa Business School

abs.um6p.ma

Modeling

A 3 steps life-cycle

Design

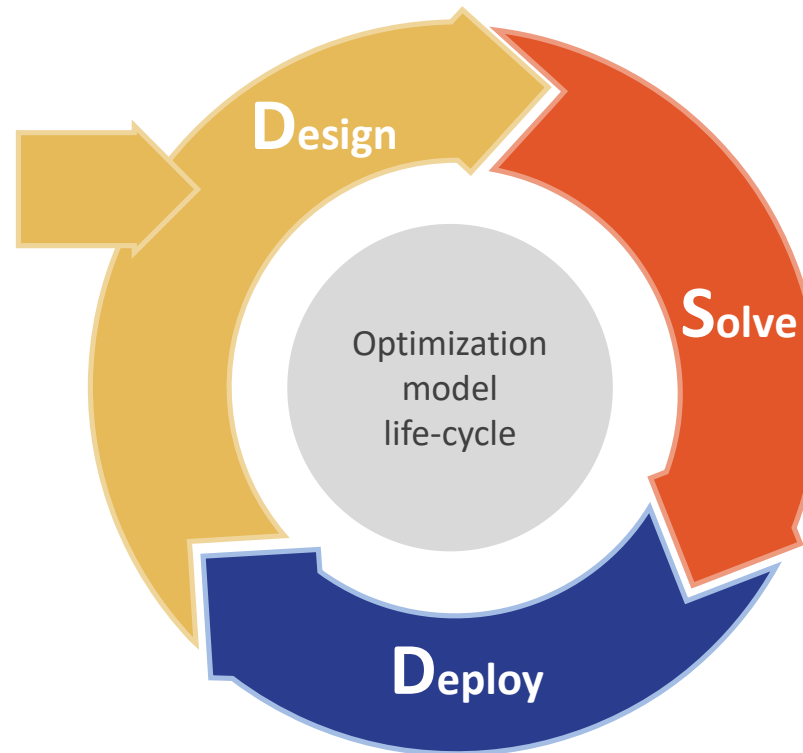**Understand the business issue**
- Discuss with decision-makers
- Identify the business decisions, constraints and criteria
- Map & analyse available data

Frame the model
- Define inputs and outputs
- Scope : break-down
- Simplification and approximation

Solve
- **Formulation** : define variables, objective and constraints
- **Choose resolution method** and write mathematics formula in adequation with this methodology
- **Implement and run the resolution algorithm**. *Example : call the linear solver*

Deploy

Validation & Adoption
- Compare the outputs to real-life decisions and adjust if needed
- Check the validity of assumptions and simplifications

Integration & productization
- Integration with other models
- Support & training
- Documentation
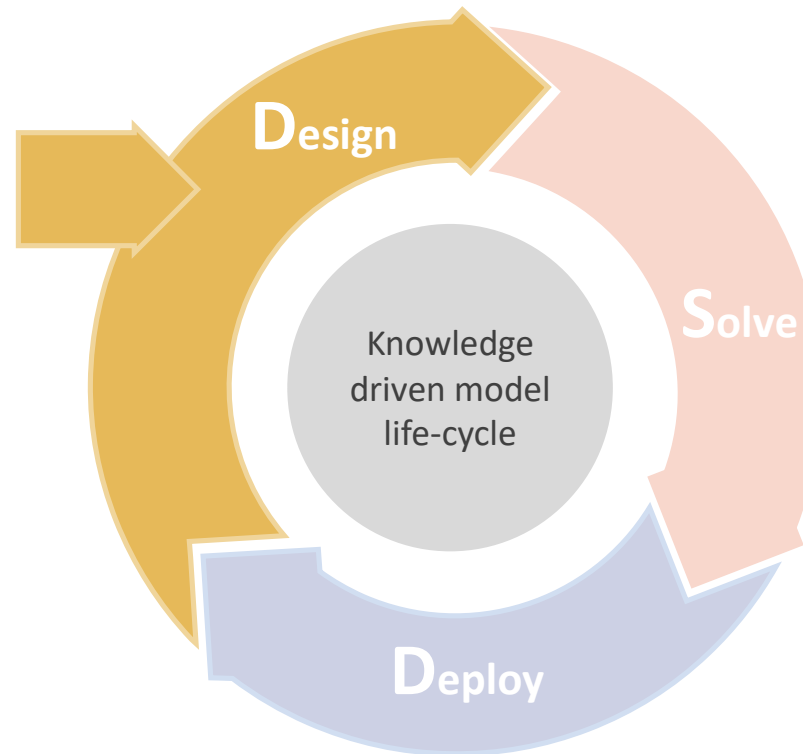
The Design phase = the Modeling phase

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Solve

Knowledge driven model life-cycle

Deploy

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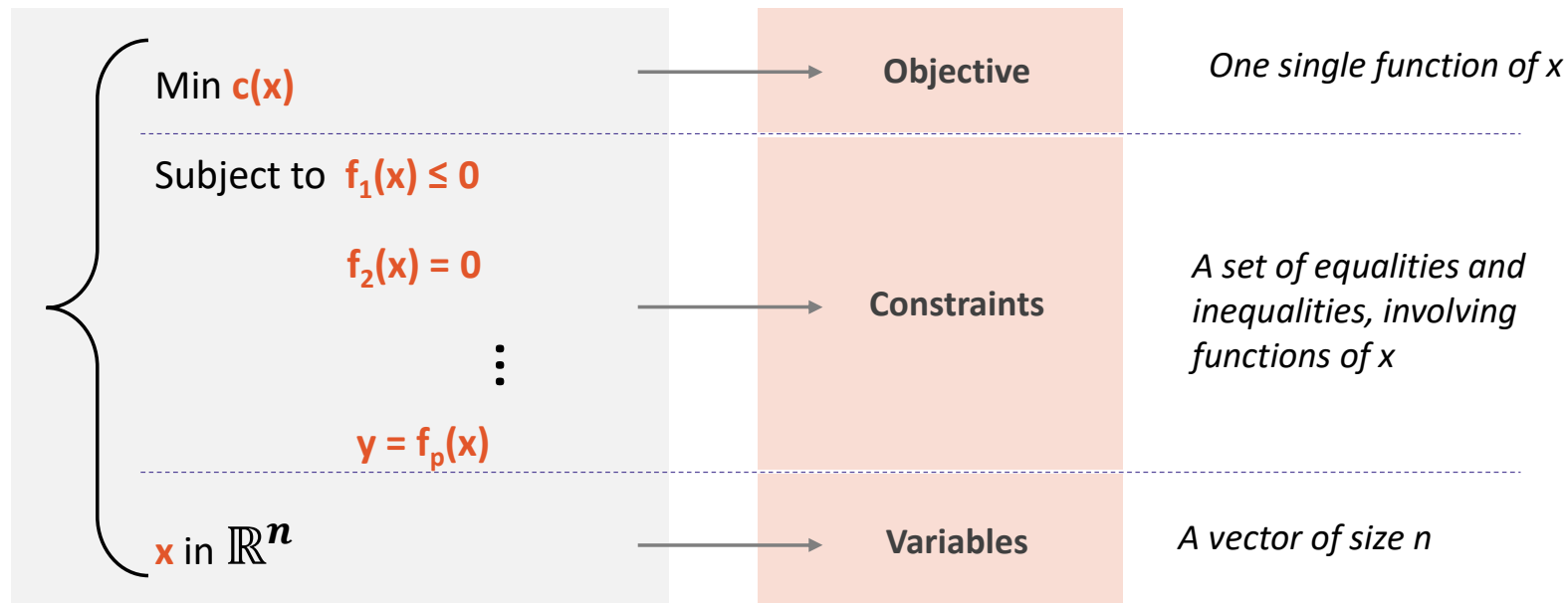
Integration & deployment
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What is a typical optimization problem ?

1. Identify the **decision variables** x
2. Identify the **set of possible choices :** $x \in \mathbf{F}$
3. Identify **decision criteria** $c(x) \in \mathbb{R}^m$

With m=1 and F defined through a set of constraints

Min **c(x)** $\longrightarrow$ **Objective** — *One single function of x*

Subject to **$f_1(x) \leq 0$**

$f_2(x) = 0$

$\vdots$ $\longrightarrow$ **Constraints** — *A set of equalities and inequalities, involving functions of x*

$y = f_p(x)$

x in $\mathbb{R}^n$ $\longrightarrow$ **Variables** — *A vector of size n*

Where to start ?

Variables

- **What are the decisions to make ?**
- How do they impact the system ?
- How do you characterise your system state ?
- What are the degree of greedom of your system ?

Constraints

- **What limits the decisions ?**
- How does the system work ?

Criteria

- **What makes you think this is a good decision or not ?**

Inputs → **f** → Outputs = f(inputs)

Outputs = f(inputs) : example
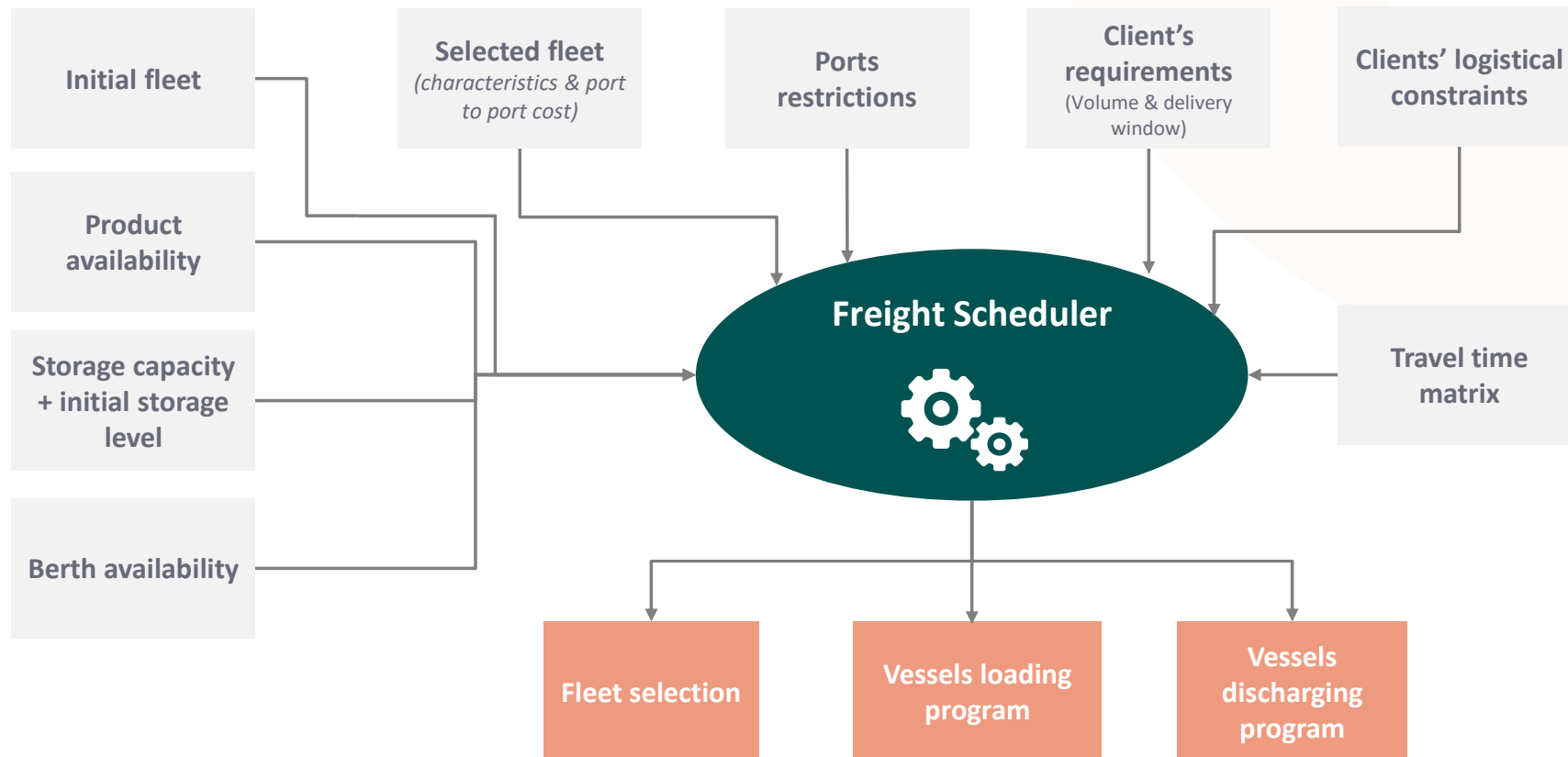
<u>Variables</u> : not all of them correspond to actual decisions

3 types of variables
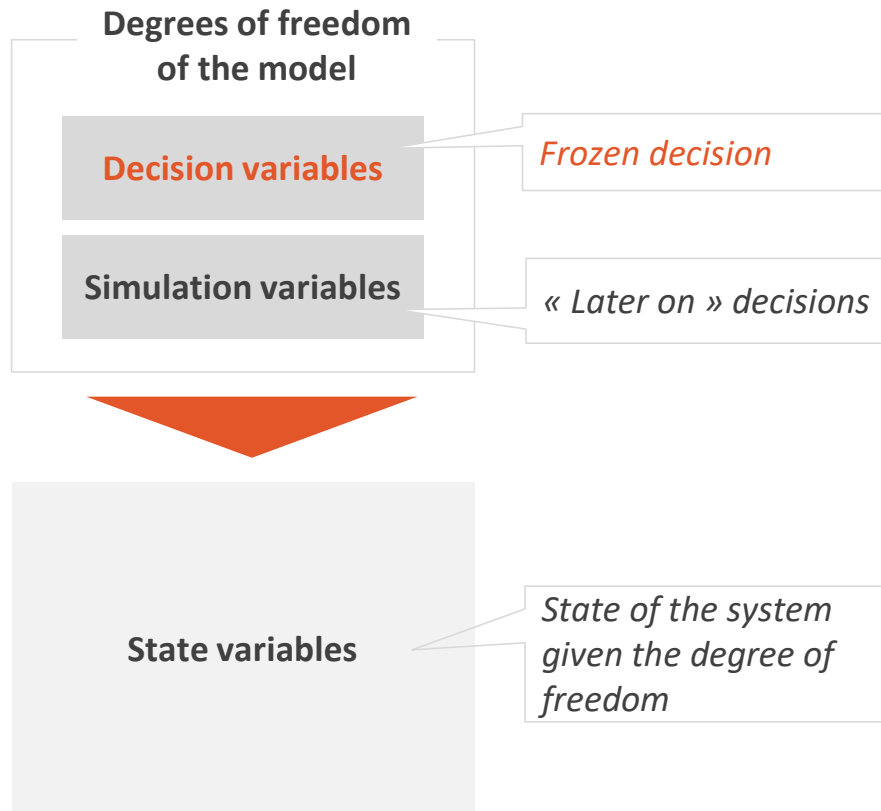
Example : *freight scheduling*

Degrees of freedom of the model

| Decision variables | *Frozen decision* |

| Simulation variables | *« Later on » decisions* |

State variables — *State of the system given the degree of freedom*

| **Vessel selection** |

| **Loading and discharging plan** |

Inventory levels
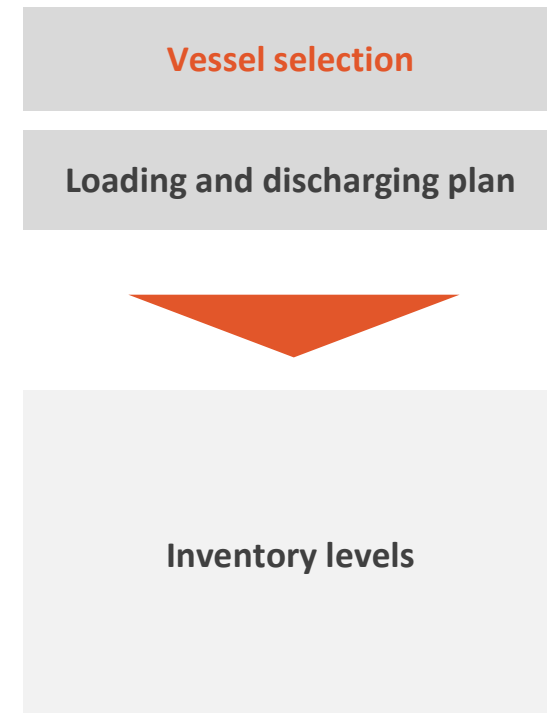
Constraints reflect the way your system works

Structural Vs. additional constraints

Structural

Enforce system's dynamics

Equality constraints :
- *Inventory[t] = Inventory[t-1] + FlowIn[t] – FlowOut[t]*

Additional

Enforce management practices and limits of ressources
Ex :
- *Capacity constraints*

Soft Vs. Hard constraints

Hard

- **Technical**
- **Reglementary**
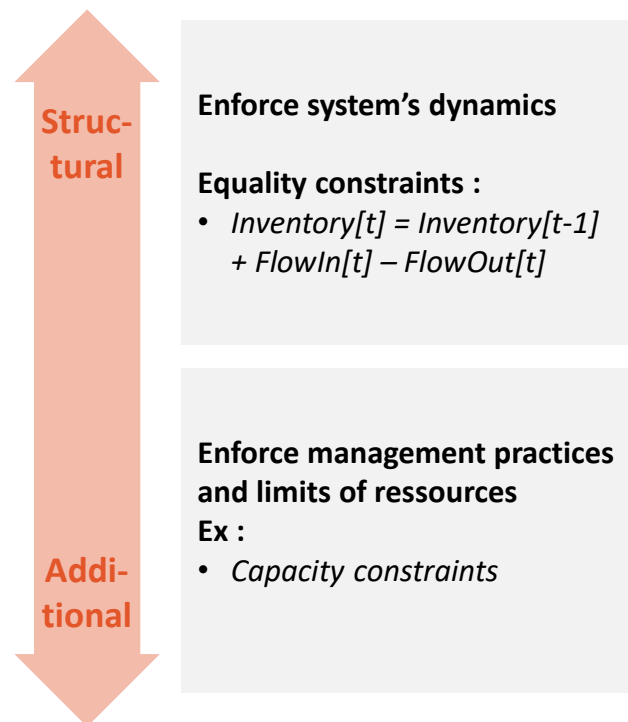
Soft

- **Preference**
- **Policies**
- **Rules**
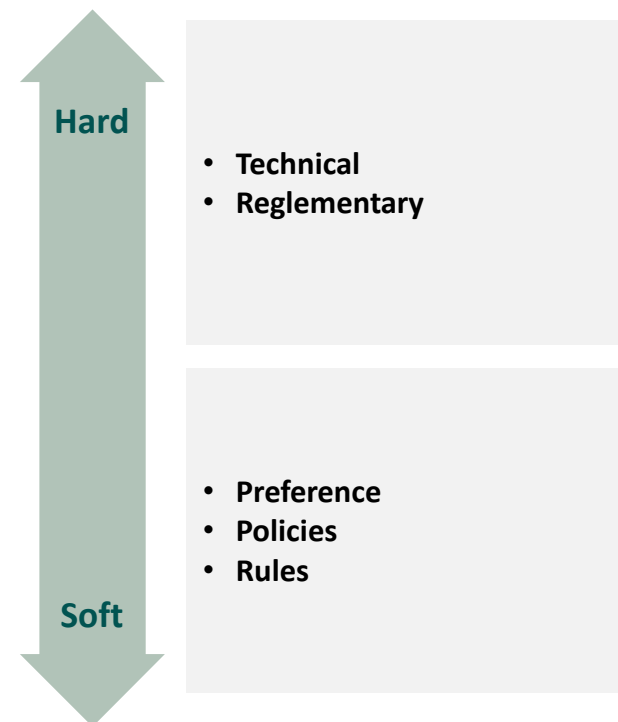
Optimization criteria : there might be several

No criteria	→ *Constraints satisfaction problems*

One single criteria	→ *Classical optimization problem*

Multiple criteria *Look for the best trade-off*	**Causes :** • Multiple criteria (cost, security, environnement,…) • Multiple decision-makers • Multiple scenarios • Mutliple time-horizons

Combine criteria or convert them as constraints
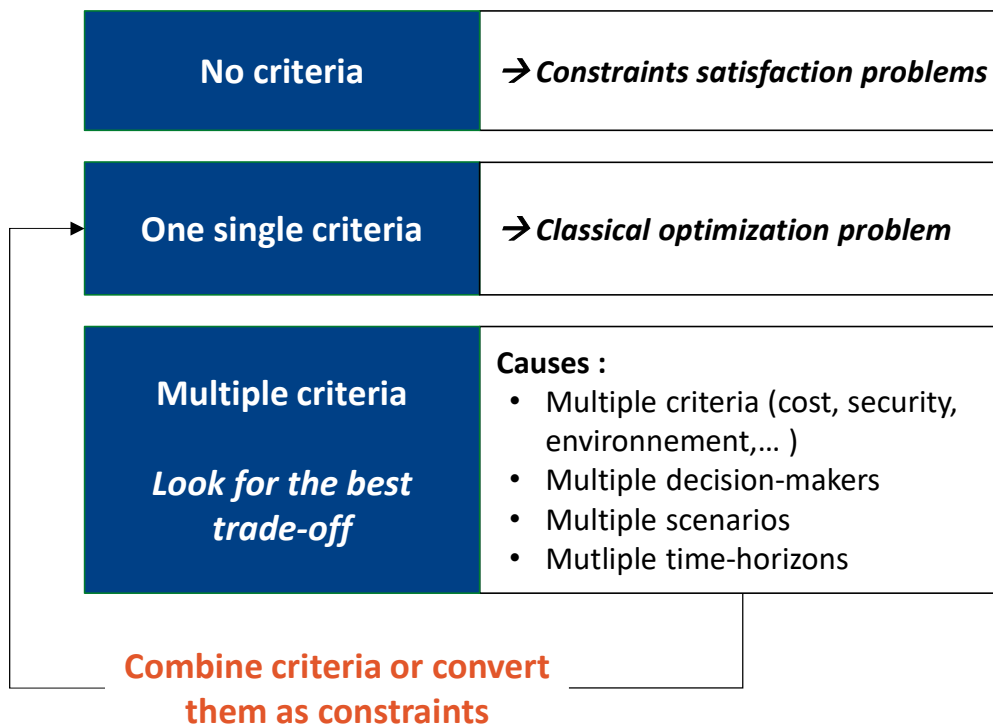
Example

► Depends on multiple simultaneous objectives

► Finding a trade-offs between different aspects

► Several conflicting viewpoints that need to be taken into account

► Example: criteria for the potential location of a factory :

   – Investment cost;

   – Cost of operations;

   – Use ;

   – Transportation;

   – Environmental impact ;

   – Social impact
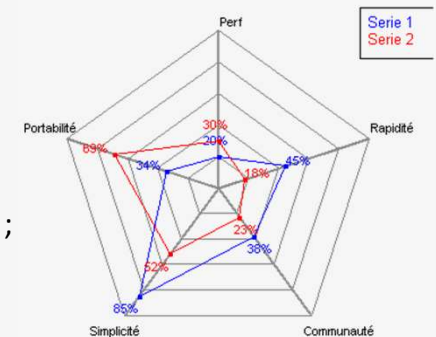
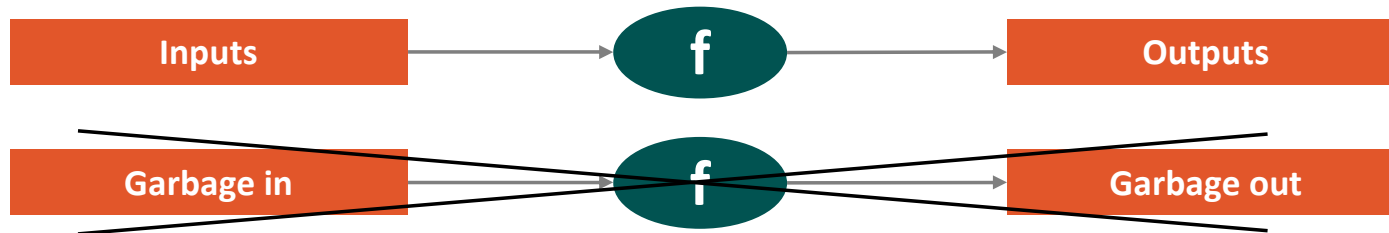# A quantitative model is nothing without suitable data

Inputs	→ f →	Outputs
Garbage in	→ f →	Garbage out

Data determines models as much as models determines data !

Inputs data management is crucial

→ Structure / dimension / granularity / format ?

→ Consistency and error handling ?

→ **Which process & governance for populating inputs data ?**

⚠ • Do not over-estimate data availability, accuracy and reliability

Outputs are the key for well interpreting the results

→ **Which KPIs** are the most significant ?

→ How will the user be able to understand and interpret the results ?

→ Leverage the « WaW » effect with a user-friendly dashboard !!

Optimization provides a framework to build collective model for decisions-making, raising key questions

Decision variables
- What are our decisions variables ? What are their granularity ?
- What is the scope impacted by these decisions ?
- Which ones are frozen / not frozen ?

System dynamics
- How to represent the state of my system ?
- What are my state variables ?
- How to compute them as function of the decision variables

Constraints
- What are my constraints ?
- What happen if they are not satisfied ?
- Soft Vs. Hard constraints

Criteria
- What criteria(s) do we want to minimize/maximize ?
- If multiple :
    - Which ones shall we prioritize ?
    - How to combine them ?
    - Which one could we consider as a constraint ?

Inputs data
- What inputs data do I need to represent my system ? With which granularity ?

Problème du péage: écriture sous forme d'un problème d'optimisation

Problème

Données du problème :

- **M** postes de traitement (= cabines) en parallèle, indexés par k=1,…, M

- **N** pas de temps, indexés par t=1,..,N=*240*

- **DebitEntrant[t]** : nombre de véhicule qui arrive à chaque pas de temps t

- **Durée** : durée du traitement d'un véhicule = 1/6 = 0,166666 min

- ➢ **Combien de poste doit-on activer pour garantir que les véhicules attendent au max « AttenteMax » pas de temps ?**

- ➢ Mettre sous forme d'un problème d'optimisation

- ➢ Le problème est-il linéaire ?

How to solve an optimization problem

The Resolution phase

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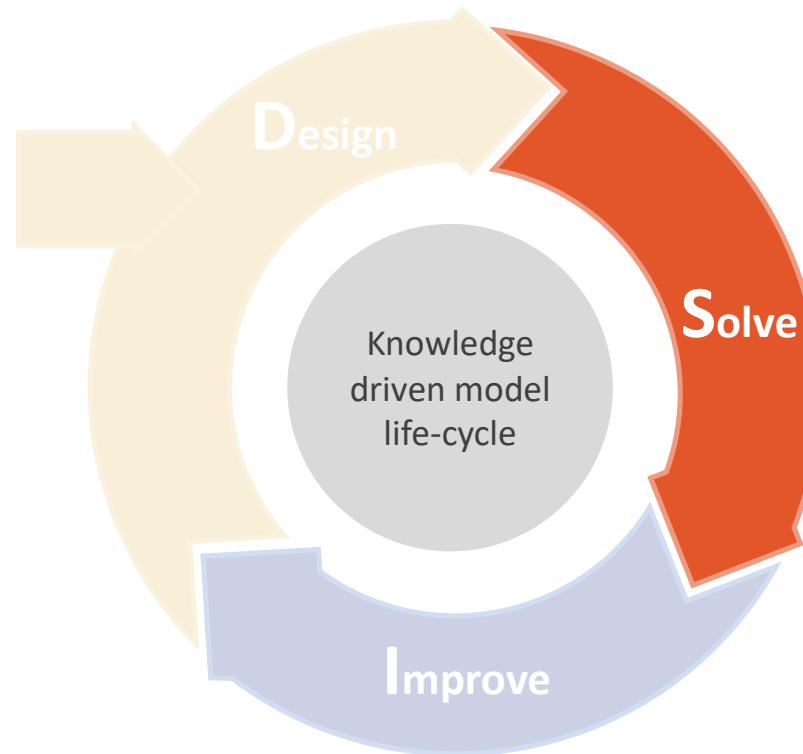
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Improve

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What kind of deliveries are associated to each phase ?

Design · **Solve** · **Deploy**

Understand the business issue	Frame the model	Mathematical formulation	Resolution	Validation & adoption	Integration & productization
Description of the business problem, and available data, in all complexity	Description of the model : scope, inputs and outputs, simplification, approximation, assumptions	**Mathematics formulas for objective and constraints, in adequation with the resolution method**	**A code that :** - **Read inputs data** - **Implement or call the resolution algorithm** - **Write outputs data**	Run different tests : - Unit tests - Functionnal tests - Back-tests	Integration with other models Support & training Documentation

...

The choice of the resolution algorithm depends on the mathematical structure of the problem

Optimization problem

$Max\ \mathbf{c}(x)\ \text{for}\ x \in \mathbf{F}$

Is there an objective function ? — *No* → **Constraints programming** $\mathbf{c}(x) = \mathbf{K}$

↓ *Yes*

Are there multiple optimization criteria ? — *Yes* → **Multi-criteria optimization** $\mathbf{c}(x) \in \mathbb{R}^m$

↓ *No*

Are there any constraints over x ? — *No* → **Unconstrained optimization** $\mathbf{F} = \mathbb{R}^n$

↓ *Yes*

Constraints and objective function are linear — *Yes* → **Linear optimization**

Yes except integrity constraints → **Integer Linear Optimization**

↓ *No*

Constraints and/or objective function are quadratic — *Yes* → **Quadratic optimization**

↓

Constraints and objective function are convex — *Yes* → **Convex optimization**

… and can be solved with various methods