B.Sc. Engg. CSE 7th Semester

16 May 2019

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
Department of Computer Science and Engineering (CSE)

SEMESTER FINAL EXAMINATION — **WINTER SEMESTER, 2018-2019**

DURATION: 3 Hours — **FULL MARKS:150**

CSE 4703: Theory of Computing

Programmable calculators are not allowed. Do not write anything on the question paper.

There are **8 (eight)** questions. Answer any **6 (six)** of them.

Figures in the right margin indicate marks.

---

1. a) Prove that every nondeterministic Turing Machine has an equivalent deterministic Turing Machine. 8

   b) Following is the state diagram of a TM, M, give the sequence of configurations that M enters when started on the input string 00#0100#. 12

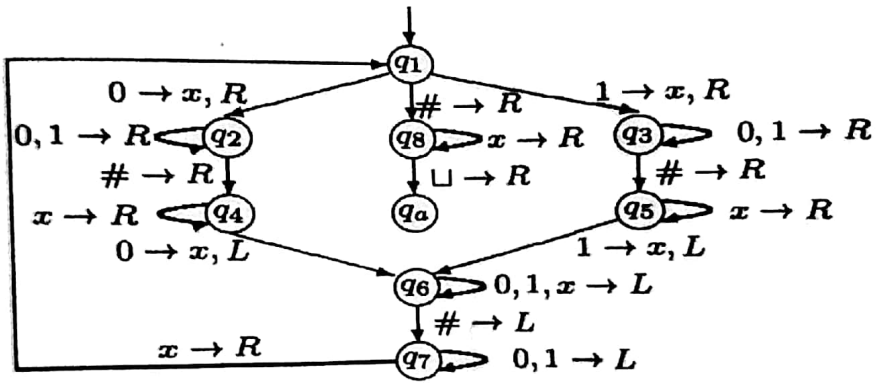

Figure 1: State diagram of a Turing Machine M

c) Consider the grammar G = {S → 0S | 0S1S | ε}, show that the grammar G is ambiguous. 5

2. a) The classic game Pac-Man requires the player to navigate through a maze, eating pellets and avoiding the ghosts who chase him through the maze. Occasionally, Pac-Man can turn the tables on his pursuers by eating a power pellet, which temporarily grants him the power to eat the ghosts. When this occurs, the ghosts' behavior changes, and instead of chasing Pac-Man they try to avoid him. 12

   The ghosts in Pac-Man have five behaviors listed below:

   1. Randomly wander the maze
   2. Chase Pac-Man, when he is within line of sight
   3. Flee Pac-Man, after Pac-Man has consumed a power pellet
   4. If dead, Return to the central base to regenerate
   5. Game over after eats Pac-Man

   Draw the state diagram of the NFA that emulates the behavior of a single ghost in Pac-Man.

b) Convert the following CFG to Chomsky Normal Form (CNF). 10

$$S \to ABA$$

$$A \to aA \mid \varepsilon$$

$$B \to bB \mid \varepsilon$$

c) What do you understand by the terms Turing recognizable and Turing decidable?

3. a) Convert the following DFA in to a regular expression. Construct GNFA as an intermediary step. 15

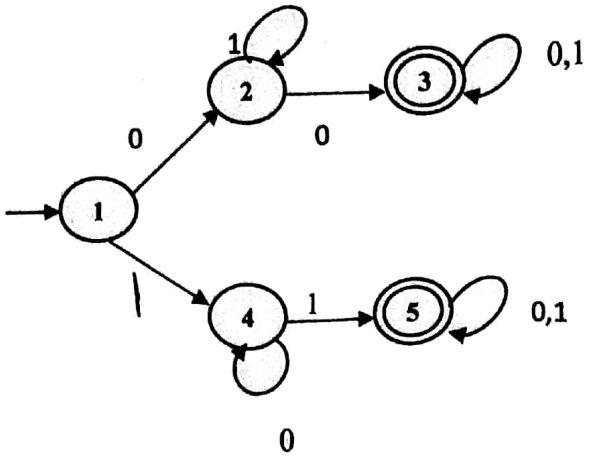

Figure 2: State diagram of a DFA

b) Suppose you are asked to design a small part of a cricket game. This game is little bit different than usual cricket. In this game for a boundary batsman gets 1 run, for an over-boundary batsman gets 2 runs otherwise batsman gets no run. When a batsman's total run is multiple of 3 (i.e. 3, 6, 9, 12…etc.) it plays a sound of cheering crowd (Hurraaaaaaaaaaaay). When batsman first enters into the crease crowd also cheers for him. Simulate this feature of the game using a DFA. 10

4. a) Prove that EQ_{DFA} is a decidable language where $EQ_{DFA} = \{\langle A,B\rangle \mid A \text{ and } B \text{ are DFAs and } L(A) = L(B)\}$. 9

b) Construct a PDA from the following Context Free Grammar (CFG). 12

$$R \rightarrow XRX \mid S$$
$$S \rightarrow aTb \mid bTa$$
$$T \rightarrow XTX \mid X \mid \varepsilon$$
$$X \rightarrow a \mid b$$

Figure 3: Definition of the CFG

c) Write a short note on Enumerators. 4

5. a) Prove that the set of real numbers R is uncountable. 11

b) Give CFGs generating the following languages: 7+7

i. Let $L = \{w \in \{0, 1\}^*$: the language L is the set of all binary integers that are even$\}$

ii. Let $L = \{w \in \{a, b\}^*$: the first, middle, and last characters of w are identical.$\}$

6. a) Prove that $B = \{a^n b^n c^n \mid n>=0\}$ is not context free. Sketch an algorithm that shows that this language is indeed Turing recognizable. 15

b) Give state diagrams of the following NFAs with the specified number of states recognizing each of the following languages. In all parts the alphabet is {0, 1}. 5+5

i. The language {w | w contains the substring 0101}, with five states.

ii. The language $0^*1^*0^+$ with three states.

7. a) Prove the pumping lemma for context free languages. 10

Convert the regular expression $((a \cup b)\ a)^*$ to an NFA in a sequence of stages, starting from the smallest sub expression to larger sub expression. 10

c) Write down the implementation level description of the Turing machine deciding the language $B = \{w\#w \mid w \in \{0, 1\}^*\}$. 5

8. a) Convert the following regular expression to NFA. 5+4

$(0 \cup 10)^* 010 (0 \cup 1)^+$

Show the computation of the NFA on input *010110*.

b) For each of the following languages, give two strings that are members and two strings that are not members of the languages. Assume the alphabet $\Sigma = \{a,b\}$ in all parts. 2+2

i. $\Sigma^* a \Sigma^* b \Sigma^* a \Sigma^*$

ii. $(a \cup ba \cup bb)\ \Sigma^*$

c) Give regular expressions that generate each of the following languages. In all cases, the alphabet is $\Sigma = \{a, b\}$. 2×6

iii. $L = \{w \in \Sigma^* : |w| \text{ is odd}\}$

iv. $L = \{w \in \Sigma^* : w \text{ has an odd number of a's}\}$

v. $L = \{w \mid w \text{ contains at least two a's, or exactly two b's}\}$.

vi. $L = \{w \in \Sigma^* : w \text{ ends in a double letter}\}$. [(A string contains a double letter if it contains aa or bb as a substring.)]

vii. $L = \{w \mid w \text{ has length at least 3 and its third symbol is a 0}\}$

viii. $L = \{w \mid w \text{ doesn't contain the substring 110}\}$