B.Sc. Engg. CSE 6th Semester

25 October 2019

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
Department of Computer Science and Engineering (CSE)

SEMESTER FINAL EXAMINATION

SUMMER SEMESTER, 2018-2019

DURATION: 3 Hours

FULL MARKS: 150

MATH 4641: Numerical Analysis

Programmable calculators are not allowed. Do not write anything on the question paper.

There are 8 (eight) questions. Answer any 6 (six) of them.

Figures in the right margin indicate marks.

1. a) Define different types of errors in mathematical calculations with appropriate examples. Don't forget to mention specific cases when you calculate those different types of errors. 5

   b) Find out the number of terms required to take in consideration from McLaurin Series for calculating the value of $e^{1.5}$ up to 3 significant digit correctness. 8

   c) Given that $f(4)=6$, $f'(4)=8$, $f''(4)+f(4)=17$, $f'''(4)=f''(4)+5$ and that all other higher order derivatives of $f(x)$ are zero at $x=4$, and assuming the function and all its derivatives exist and are continuous between $x=4.0$ and $x=4.5$. Find the value of $f(4.3)$. 12

2. a) Define *Inflection Point* with appropriate example. Mention the shortcomings of bisection method of finding roots of non-linear equations with appropriate examples. 7

   b) Derive Newton-Raphson method of finding roots of non-linear equations from Taylor Series. 6

   c) Use the Newton-Raphson method to estimate the root of $f(x)=e^{-x}-x$, employing an initial guess of $x_0=0$ and taking at least two iterations. 12

3. a) Solve the following nonlinear equation with false position method: 10

$$e^{-2x}+4x^2-36=0$$

   Consider two initial guesses, $x_l=1$ and $x_u=4$

   b) Why do we calculate the sum of square of the residuals in case of regression? Explain with appropriate diagram. 8

   c) Mention the problems associated with using higher order polynomial equations for interpolation. How could you solve those problems? 7

4. a) The upward velocity of a rocket is given as a function of time in Table 1. Determine the value of the velocity at t =16 seconds using the direct method of interpolation with third order polynomial. 12

Table 1: Velocity as a function of time

t (sec)	V(t) (m/s)
0	0
10	227.04
15	362.78
20	517.35
22.5	602.97
30	901.67

   b) From the answer of the Question 4(a) calculate the absolute relative approximate error for the third order polynomial approximation. 5

   c) Calculate the velocity and acceleration of the rocket from t=11 to t=16 using the third order polynomial interpolant obtained in 4(a) 8

5. a) Why is it useful to decompose matrix A into LU format when a simulation system is given by the format $\boldsymbol{Ax}=\boldsymbol{b}$ with different values of b? Justify your answer with appropriate scenario. 7

b) Solve the following system with the help of LU *Decomposition*. 10

$$x_1 + x_2 - x_3 = 4$$
$$x_1 - 2x_2 + 3x_3 = -6$$
$$2x_1 + 3x_2 + x_3 = 7$$

c) Show that in case of calculating numerical differentiation on a discrete dataset, the error in central difference method is always smaller than that both forward and backward difference method. 8

6. a) With the help of necessary figures, derive the formula for Euler's method of solving ordinary differential equations. 7

b) Human vision has the remarkable ability to infer 3D shapes from 2D images. The intriguing question is: can we replicate some of these abilities on a computer? Yes, it can be done and to do this, integration of vector fields is required. The following integral needs to be integrated. 10

$$I = \int_0^{100} f(x)dx$$

where

$$f(x) = 0,\ 0 < x < 40$$
$$= -9.1688\times10^{-6}x^3 + 2.7961\times10^{-3}x^2 - 2.8487\times10^{-1}x + 9.6778,\ 40 \le x \le 172$$
$$= 0,\ 172 < x < 200$$

Use Simpson's 1/3 Rule to find the integral I.

c) The following equation represents the approximation using Runge-Kutta 4^{th} order of solving ordinary differential equations. 8

$$w_{i+1} = w_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)h$$

Describe how the constants are approximated using appropriate diagram

7. a) How can you derive Runge Kutta 2^{nd} order and 4^{th} order method of solving ordinary differential equations from Taylor Series. 8

b) A rectifier-based power supply requires a capacitor to temporarily store power when the rectified waveform from the AC source drops below the target voltage. To properly size this capacitor a first-order ordinary differential equation must be solved. For a particular power supply, with a capacitor of 150 μF, the ordinary differential equation to be solved is 17

$$\frac{dv(t)}{dt} = \frac{1}{150\times10^{-6}}\left\{-0.1 + \max\left(\frac{|18\cos(120\pi(t))| - 2 - v(t)}{0.04}, 0\right)\right\}$$

$v(0) = 0$

Using the Runge-Kutta 2^{nd} order method, find the voltage across the capacitor at t = 0.00004s Use step size h = 0.00002s and follow Heun's method.

8. a) How can you check the existence of a root of any nonlinear equation between two given points using bisection method? Explain with appropriate diagrams. 6

b) Mention at least three applications of Taylor Series in Numerical Approximation methods with appropriate examples. 9

c) Find an approximate value of the following using Euler's method of solving an ordinary differential equation. Use a step size of $h=1.5$ 10

$$I = \int_5^8 6x^3 dx$$