B.Sc. Engg. CSE 7th Semester

18 May 2019

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
Department of Computer Science and Engineering (CSE)

SEMESTER FINAL EXAMINATION **WINTER SEMESTER, 2018-2019**
DURATION: 3 Hours **FULL MARKS: 150**

MATH 4741: Mathematical Analysis

Programmable calculators are not allowed. Do not write anything on the question paper.
There are **8 (eight)** questions. Answer any **6 (six)** of them.
Figures in the right margin indicate marks.

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1. a) An urn always contains 2 balls. Ball colors are red and blue. At each stage a ball is randomly chosen and then replaced by a new ball, which with probability 0.8 is the same color, and with probability 0.2 is the opposite color, as the ball it replaces. If initially both balls are red, find the probability that the fifth ball selected is red. 8

b) Suppose that balls are successively distributed among 8 urns, with each ball being equally likely to be put in any of these urns. What is the probability that there will be exactly 3 nonempty urns after 9 balls have been distributed? 8

c) On any given day Gary is either cheerful (C), so-so (S), or glum (G). If he is cheerful today, then he will be C, S, or G tomorrow with respective probabilities 0.5, 0.4, 0.1. If he is feeling so-so today, then he will be C, S, or G tomorrow with probabilities 0.3, 0.4, 0.3. If he is glum today, then he will be C, S, or G tomorrow with probabilities 0.2, 0.3, 0.5. In the long run, what proportion of time is the process in each of the three states? 9

2. a) Consider a gambler who at each play of the game has probability p of winning one unit and probability $q = 1 - p$ of losing one unit. Assuming that successive plays of the game are independent, what is the probability that, starting with i units, the gambler's fortune will reach N before reaching 0? 13

b) "In a city, on average most of the buses are empty but according to passengers everyone found their bus to be crowded" – Are the statements contradictory? Explain with justification. 6

c) Suppose Max and Patty decide to flip pennies; the one coming closest to the wall wins. Patty, being the better player, has a probability 0.6 of winning on each flip. (a) If Patty starts with five pennies and Max with ten, what is the probability that Patty will wipe Max out? (b) What if Patty starts with 10 and Max with 20? 3+3

3. Consider a shoe shine shop consisting of two chairs. Suppose that an entering customer first will go to chair 1. When his work is completed in chair 1, he will go either to chair 2 if that chair is empty or else wait in chair 1 until chair 2 becomes empty. Suppose that a potential customer will enter this shop as long as chair 1 is empty. (Thus, for instance, a potential customer might enter even if there is a customer in chair 2.) If we suppose that potential customers arrive in accordance with a Poisson process at rate λ, and that the service times for the two chairs are independent and have respective exponential rates of $\mu 1$ and $\mu 2$, then

a) What proportion of potential customers enters the system? 7

b) What is the mean number of customers in the system? 6

c) What is the average amount of time that an entering customer spends in the system? 6

d) Find π_b, equal to the fraction of entering customers that are blockers? That is, find the fraction of entering customers that will have to wait after completing service with server 1 before they can enter chair 2. 6

4. a) Explain with examples about three different behaviors of arrival for a queueing model. 6

b) Explain birth and death queueing models. Show and explain the conditions that we impose on the following queueing model to model them as birth and death model: 2+6

i. The M/M/1 queueing system

ii. The M/M/1 queueing system with balking

c) Consider a single-server queue where customers arrive according to a Poisson process with rate λ and where the service distribution is exponential with rate μ, but now suppose that each customer will only spend an exponential time with rate α in queue before quitting the system. Assume that the impatient times are independent of all else, and that a customer who enters service always remains until its service is completed. 11

i. Model it as birth and death model (impose condition on arrival rate and departure rate).

ii. determine the proportion of arrivals that receive service, π_s and verify it.

iii. Let, a_n equal to the proportion of arrivals that find n in the system. Derive equation for a_n.

5. a) Explain: M/M/1 and M/M/k queueing systems. 5

b) For an M/M/1 queueing system with infinite capacity derive equations for L and W. Make any necessary assumptions for the derivation and show step by step approach. 15

c) What is the general condition that must be satisfied for limiting probabilities to exist in most single-server queueing systems? Explain mathematically/logically. 5

6. a) Suppose that customers arrive at a Poisson rate of one per every 12 minutes, and that the service time is exponential at a rate of one service per 8 minutes. What are L and W for M/M/1 with infinite queue? Increase the arrival rate by 20% and recalculate L and W. Compare the L & W of both scenarios and explain why we see such changes in the values. 10

b) For an M/M/1 queueing system with finite capacity N derive equations for L and W. Make any necessary assumptions for the derivation and show step by step approach. 15

7. a) Suppose that it costs **c$\mu$$/hour** to provide service at a rate μ. Suppose also that we incur a gross profit of **A$** for each customer served. If the system has a capacity N, what service rate μ maximizes our total profit? Let, $N = 2, \lambda = 1, A = 10, c = 1$. 10

b) Consider yourself to be a very devoted researcher working under a professor. Your devotion is so high that you don't get the chance to go outside of the room and perceive the weather by yourself. But a thing that you have noticed that your professor sometimes brings umbrella and sometimes he doesn't. After asking your professor each day you learn that when it's sunny outside 20% of time he brings umbrella and if it is rainy then 40% of time he forgets the umbrella. Another fact that you have noticed is that if today is sunny tomorrow is sunny with 80% probability and if today is rainy tomorrow is rainy with 60% probability. You should be aware that in your environment weather can be of only two kinds, sunny and rainy. Given that a random day is sunny with probability 2/3 and rainy with probability 1/3, answer the following questions with respect to the scenario described above: 5+10

i. What kind of model the above scenario fits into? Draw the transition diagram of that model.

ii. For four consecutive days, you observe that the professor didn't bring umbrella in the first two days and brought umbrella in the last two days. What was the weather like in those days? (Show calculations with proper reasoning for your answer)

8. In the tv-show called "Game of Thrones" characters die with a rate $\lambda = .6$/episode. Considering you haven't watched the show before and you start watching it. You will watch the series for at least two episodes but if any of the characters die within those episodes you will stop watching after the second episode. Otherwise you will continue to watch the series past the second episode until any character dies. Answer the following question based on given scenario:

a) P (watch for more than two episodes) 5

b) P (watch for more than two episodes but less than five episodes) 5

c) P (watching at least two characters die) 5

d) E [number of deaths] 5

e) E [total watch time] 5