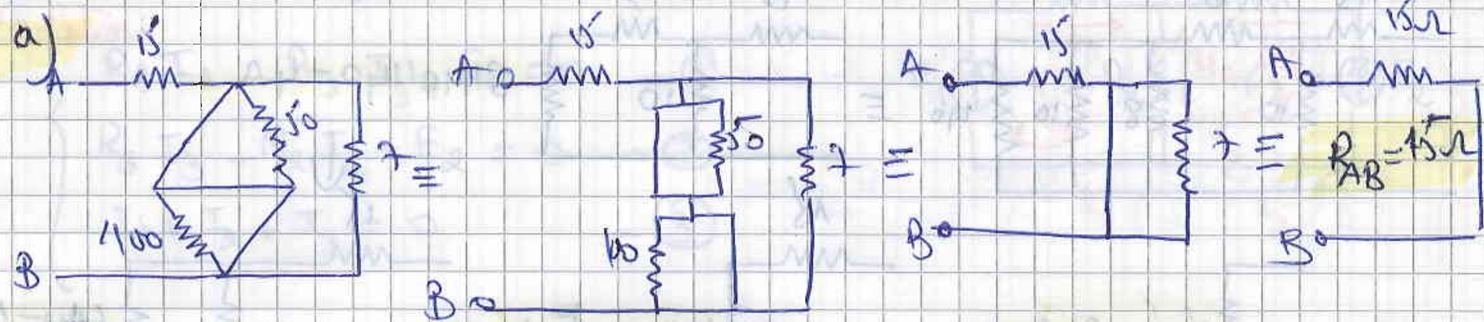
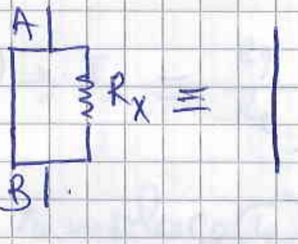
Solution TD n°1.

Ex01 :

a)

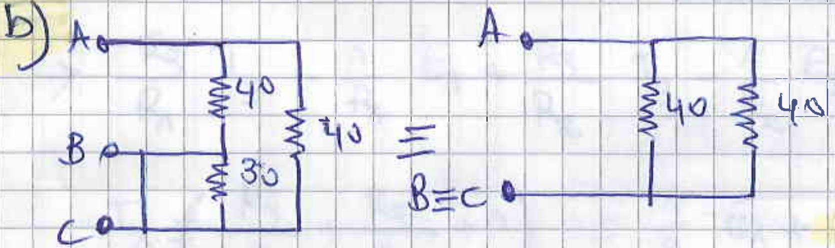


en effet :



$$R_{eq} = \frac{R_X \cdot 0}{R_X + 0} = 0 \quad \text{court-circuit.}$$

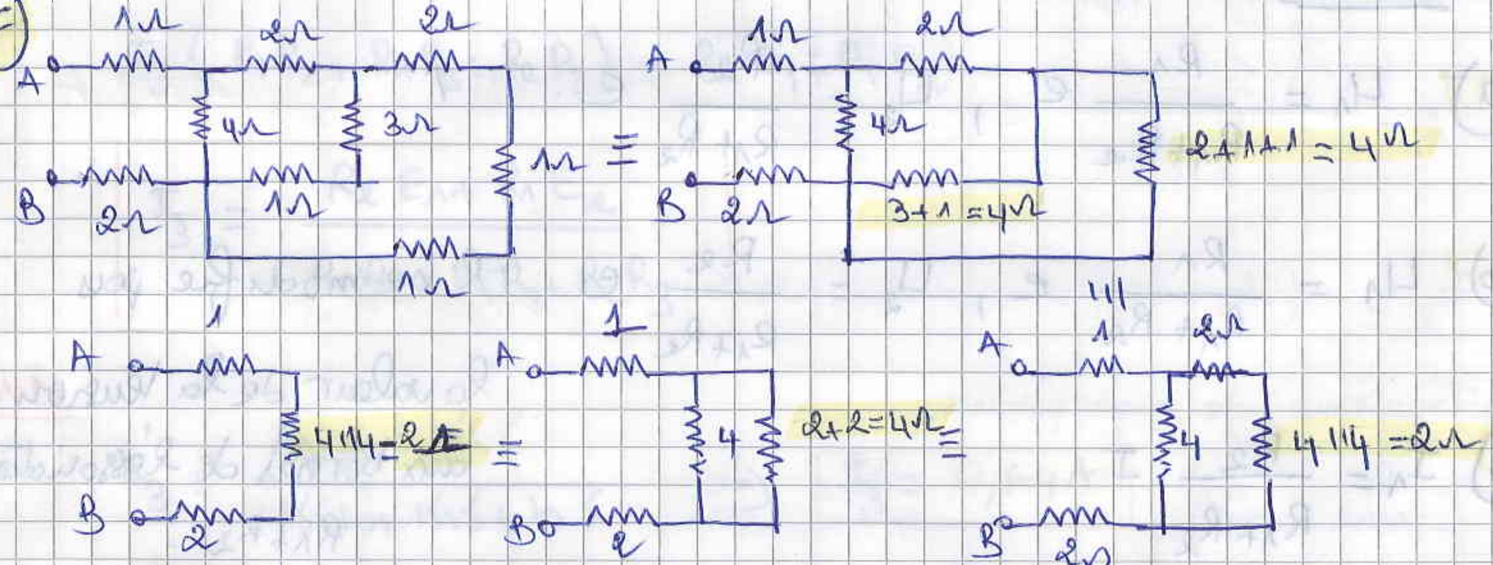
b)

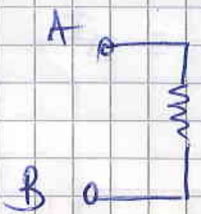


$$R_{AB} = 40 // 40 = \frac{40 \cdot 40}{40+40} = 20\,\Omega$$

$$R_{AB} = 20\,\Omega.$$

c)

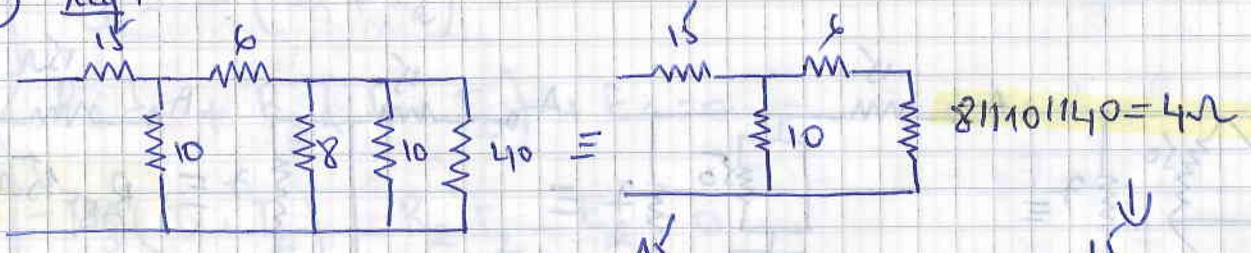


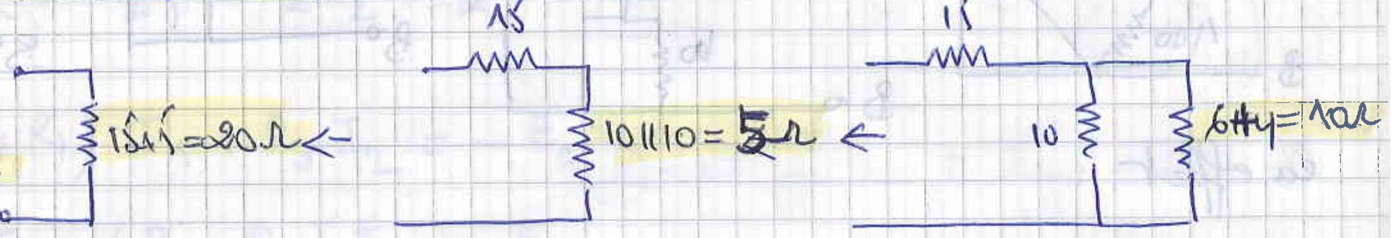


$$R_{AB} = 1+2+2 = 5\,\Omega. \qquad R_{AB} = 5\,\Omega.$$

Exo 2 :

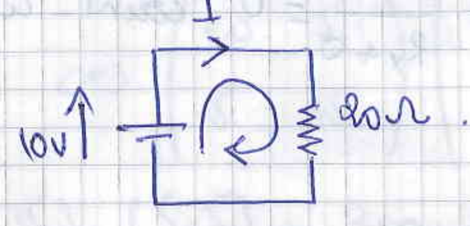
1) Req :





$R_{eq} = 20\Omega$

2) I = ?



$$10 = 20\,I$$

$$\Rightarrow \quad I = \frac{10}{20}$$

$$\boxed{I = 0.5\,A}$$

Exo 3 :

U_1 et U_2

a) $U_1 = \dfrac{R_1}{R_1+R_2}\,e \quad , \quad U_2 = \dfrac{R_2}{R_1+R_2}\,e.$

b) $U_1 = \dfrac{R_1}{R_1+R_2}\,e \quad , \quad U_2 = \dfrac{R_2}{R_1+R_2}\,e$

R ne modifie pas la valeur de la tension aux bornes de l'association R_1+R_2.

c) $I_1 = \dfrac{R_2}{R_1+R_2}\,I$

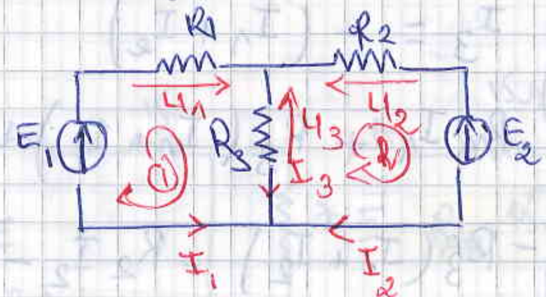
$I_2 = \dfrac{R_1}{R_1+R_2}\,I$

d) $I_1 = \dfrac{R_2 \parallel R_3}{R_1+(R_2+R_3)}\,I$

$I_2 = \dfrac{R_1 \parallel R_3}{R_2+(R_1 \parallel R_3)}\,I$

EX04 : calculer I_3 par trois méthodes (montage (a))

1) • Lois de Kirchhoff



1re solution

$$\begin{cases} R_1 I_1 - R_3 I_3 + E_1 = 0 & -(1) \\ R_3 I_3 - R_2 I_2 - E_2 = 0 & -(2) \\ I_1 + I_2 + I_3 = 0 & -(3) \end{cases}$$

$$(1) \rightarrow I_1 = \frac{R_3}{R_1} I_3 - \frac{1}{R_1} E_1 \quad -(4)$$

$$(2) \rightarrow I_2 = \frac{R_3}{R_2} I_3 - \frac{1}{R_2} E_2 \quad -(5)$$

on remplace (4) et (5) dans (3).

$$\rightarrow \frac{R_3}{R_1} I_3 - \frac{1}{R_1} E_1 + \frac{R_3}{R_2} I_3 - \frac{1}{R_2} E_2 + I_3 = 0$$

$$I_3 \left(\frac{R_3}{R_1} + \frac{R_3}{R_2} + 1 \right) = \frac{1}{R_1} E_1 + \frac{1}{R_2} E_2$$

$$I_3 \left(R_1 R_2 + R_1 R_3 + R_2 R_3 \right) = R_2 E_1 + R_1 E_2$$

$$\boxed{I_3 = \frac{R_2 E_1 + R_1 E_2}{R_1 R_2 + R_1 R_3 + R_2 R_3}}$$

A.N :

$$I_3 = \frac{10.10 + 15.5}{15.10 + 15.5 + 10.5} \Rightarrow \boxed{I_3 = 0{,}64 A}$$

$$I_1 = \frac{5}{15} \cdot (0{,}64) - \frac{1}{15} \cdot 10 \Rightarrow \boxed{I_1 = -0{,}45 A}$$

$\boxed{I_2 = -0{,}18 A}$ $-0{,}18 A$

2ème solution (kramer).

$$\begin{cases} I_3 = -(I_1 + I_2) \\ R_1 I_1 + R_3(I_1 + I_2) + E_1 = 0 \\ -R_3(I_1 + I_2) - R_2 I_2 - E_2 = 0 \end{cases}$$

$$\begin{cases} (R_3 + R_1)\, I_1 + R_3 I_2 = -E_1 \\ R_3 I_1 + (R_2 + R_3)\, I_2 = -E_2 \end{cases}$$

$$\begin{cases} 20\, I_1 + 5\, I_2 = -10 \\ 5\, I_1 + 15\, I_2 = -5 \end{cases} \Leftrightarrow \begin{pmatrix} 20 & 5 \\ 5 & 15 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} -10 \\ -5 \end{pmatrix}$$

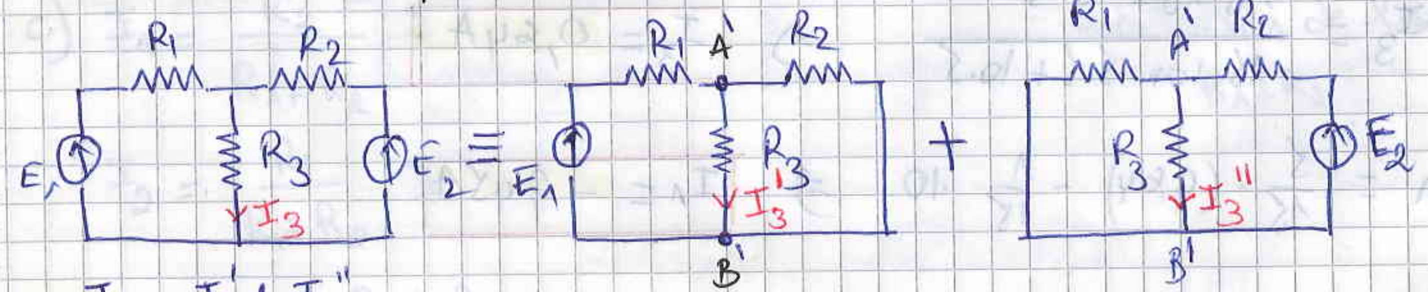
on calcule $D = (20 \cdot 15) - (5 \cdot 5) = 300 - 25 = 275$

$$I_1 = \frac{\begin{pmatrix} -10 & 5 \\ -5 & 15 \end{pmatrix}}{275} = \frac{(-150 + 25)}{275} = -0{,}45 \qquad I_1 = -0{,}45\,A$$

$$I_2 = \frac{\begin{pmatrix} 20 & -10 \\ 5 & -5 \end{pmatrix}}{275} = \frac{(-100 + 50)}{275} = -0{,}18 \qquad I_2 = -0{,}18\,A$$

$$I_3 = -(I_1 + I_2) = -(-0{,}45 - 0{,}18) \Rightarrow I_3 = 0{,}64\,A$$

* Théorème de superposition



$I_3 = I_3' + I_3''$

$I_3' = ?$

$U_{A'B'} = R_3\, I_3'$

$$U_{A'B'}\,(RDT) = \frac{(R_2 \parallel R_3)}{R_1 + (R_2 \parallel R_3)} E_1 = \frac{R_2 R_3}{R_1(R_2+R_3)+R_2R_3} E_1$$

$$I_3' = \frac{1}{R_3} U_{A'B'} = \frac{1}{\cancel{R_3}} \frac{R_2 \cancel{R_3}}{R_1(R_2+R_3)+R_2R_3} E_1$$

$$\boxed{I_3' = \frac{R_2}{R_1(R_2+R_3)+R_2R_3} E_1}$$

A.N.

$$I_3' = \frac{10}{15(10+5)+5.10}\, 10 = \frac{100}{275} = 0{,}37\,A$$

$I_3'' = ?$

$$U_{A''B''} = R_3 I_3''$$

$$U_{A''B''} = \frac{R_1 \parallel R_3}{R_2 + (R_1 \parallel R_3)} E_2 = \frac{R_1 R_3}{R_2(R_1+R_3)+R_1R_3} E_2$$

$$I_3'' = \frac{1}{R_3} U_{A''B''} = \frac{1}{\cancel{R_3}} \frac{R_1 \cancel{R_3}}{R_2(R_1+R_3)+R_1R_3} E_2$$

$$\boxed{I_3'' = \frac{R_1}{R_2(R_1+R_3)+R_1R_3} E_2}$$

A.N:

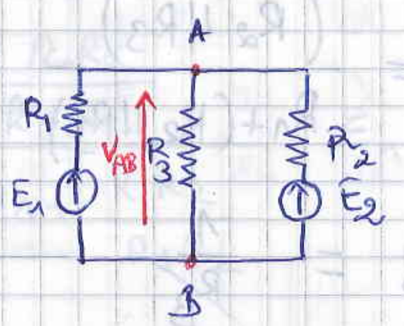
$$I_3'' = \frac{15}{10(15+5)+15.5}\, 5 = \frac{75}{275} = 0{,}27\,A$$

$$I_3 = I_3' + I_3'' = 0{,}37 + 0{,}27 = 0{,}64\,A$$

$$\boxed{I_3 = 0{,}64\,A}$$

* Théorème de Millman :

$$V_{AB} = \frac{\frac{E_1}{R_1} + \frac{E_2}{R_2} + \frac{0}{R_3}}{1/R_1 + 1/R_2 + 1/R_3}$$



A.N :

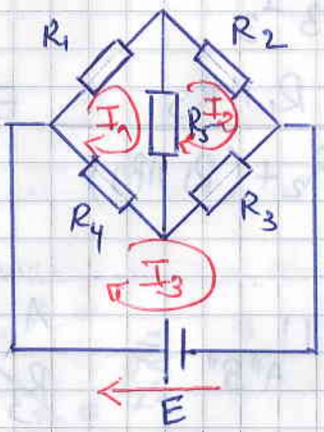
$$V_{AB} = \frac{10/15 + 5/10 + 0}{1/15 + 1/10 + 1/5} = \frac{1{,}17}{0{,}37} = 3{,}16\ V$$

$$I_3 = \frac{V_{AB}}{R_3} = 0{,}634\ A$$

2) calculer I traversant R_5 (montage (b))

* Méthode des courants fictifs :

$$\begin{cases} R_1 I_1 + R_4(I_1 - I_3) + R_5(I_1 - I_2) = 0 & (1) \\ R_5(I_2 - I_1) + R_3(I_2 - I_3) + R_2 I_2 = 0 & (2) \\ R_4(I_3 - I_1) + R_3(I_3 - I_2) = E & (3) \end{cases}$$



(1) $\rightarrow (R_1 + R_4 + R_5) I_1 - R_5 I_2 - R_4 I_3 = 0$

(2) $\rightarrow (-R_5 I_1 + (R_2 + R_3 + R_5) I_2 - R_3 I_3 = 0$

(3) $\rightarrow -R_4 I_1 - R_3 I_2 + (R_3 + R_4) I_3 = 0$

$$D = \begin{vmatrix} R_1 + R_4 + R_5 & -R_5 & -R_4 \\ -R_5 & R_2 + R_3 + R_5 & -R_3 \\ -R_4 & -R_3 & R_3 + R_4 \end{vmatrix}$$

AN :

$$\Delta = \begin{vmatrix} 10 & -5 & -4 \\ -5 & 10 & -3 \\ -4 & -3 & 7 \end{vmatrix} = 10(70-9)+5(-35-12)-4(15+40)$$

$$= 610 - 235 - 220 = 155$$

$$I_1 = \frac{\begin{vmatrix} 0 & -5 & -4 \\ 0 & 10 & -3 \\ 10 & -3 & 7 \end{vmatrix}}{155} = \frac{550}{155} = 3{,}548\,A$$

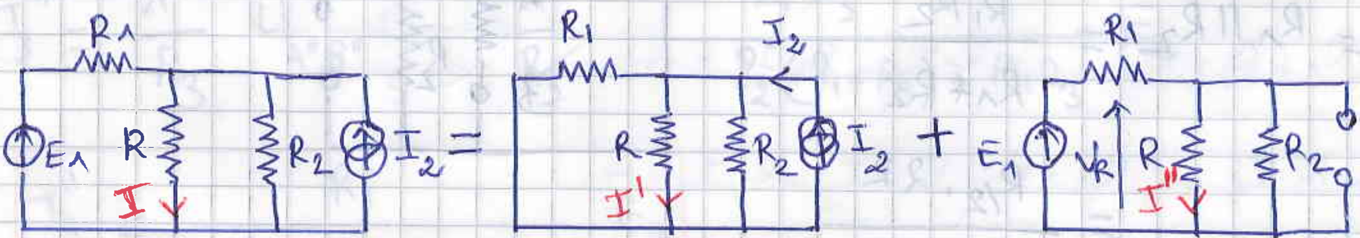
$$I_2 = \frac{\begin{vmatrix} 10 & 0 & -4 \\ -5 & 0 & -3 \\ -4 & 10 & 7 \end{vmatrix}}{155} = \frac{500}{155} = 3{,}226\,A$$

$$I_{R5} = I_1 - I_2 = 3{,}548 - 3{,}226 \Rightarrow \boxed{I_{R5} = 0{,}322\,A}$$

Exo5 : Données $R_1 = R/2$, $R_2 = 2R$.

calculer le courant I traversant la resistance R.

1) Théoreme de superposition :



$$I = I' + I''$$

$I' = ?$

$$I'(R.D.C) = \frac{G_R}{G_R + G_{R_1} + G_{R_2}} I_2 = \frac{1/R}{1/R + 1/R_1 + 1/R_2} I_2$$

$$= \frac{R_1 R_2}{R_1R_2 + RR_1 + RR_2} = \frac{R/2 \cdot 2R}{R/2 \cdot 2R + R\,R/2 + R\cdot 2R} I_2$$

$$I' = \frac{2}{7} I_2$$

$\underline{I'' = ?}$

$$V_R(R.D.T) = \frac{(R \| R_2)}{R_1 + (R \| R_2)} E_1 = \frac{\frac{R R_2}{R + R_2}}{R_1 + \frac{R R_2}{R + R_2}} E_1$$

$$= \frac{R R_2}{R_1 R + R_1 R_2 + R R_2} E_1$$

$$= \frac{R \cdot 2R}{R/2 \cdot R + R/2 \cdot 2R + R \cdot 2R} E_1$$

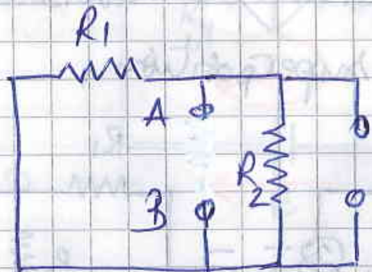
$$V_R = \frac{4}{7} E_1 = R I'' \Rightarrow I'' = \frac{4}{7R} E_1$$

$$\boxed{I = \frac{2}{7} I_2 + \frac{4}{7R} E_1}$$

2) Méthode de Thevenin :

$R_{Th} = R_{eq}$ (sources eteintes)

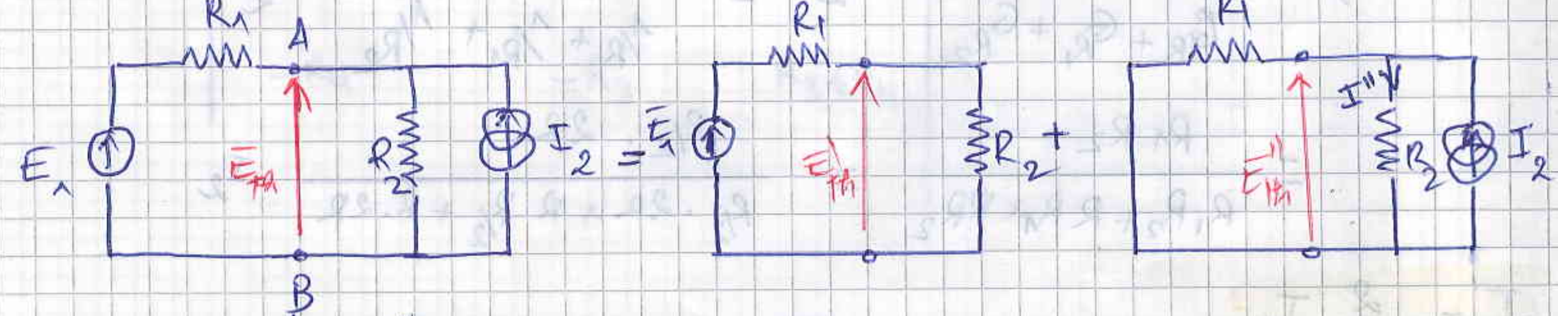
$$R_{Th} = R_1 \| R_2 = \frac{R_1 R_2}{R_1 + R_2}$$



$$= \frac{R/2 \cdot 2R}{R/2 + 2R}$$

$$R_{Th} = \frac{2}{5} R$$

$\underline{E_{Th}?}$



$$E_{Th} = E'_{Th} + E''_{Th}$$

$$E_{Th}' = V_{R_2}(RDT) = \frac{R_2}{R_1+R_2} E_1$$

$$= \frac{2R}{R/2 + 2R} E_1$$

$$E_{Th}' = \frac{4}{5} E_1$$

$$E_{Th}'' = V_{R_2} = R_2 I''$$

$$I''(RDC) = \frac{R_1}{R_1+R_2} I_2$$

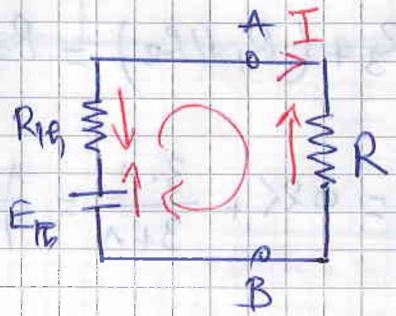
$$E_{Th}'' = \frac{R_1 R_2}{R_1+R_2} I_2$$

$$E_{Th}'' = \frac{R/2 \cdot 2R}{R/2 + 2R} I_2$$

$$E_{Th}'' = \frac{2}{5} R I_2$$

$$\boxed{E_{Th} = \frac{4}{5} E_1 + \frac{2}{5} R I_2}$$

$$E_{Th} = (R + R_{Th}) I \Rightarrow I = \frac{E_{Th}}{R + R_{Th}}$$

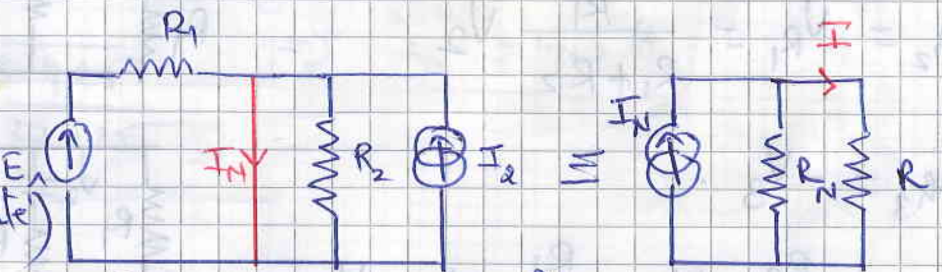


$$I = \frac{4/5 E_1 + 2/5 R I_2}{2/5 R + R}$$

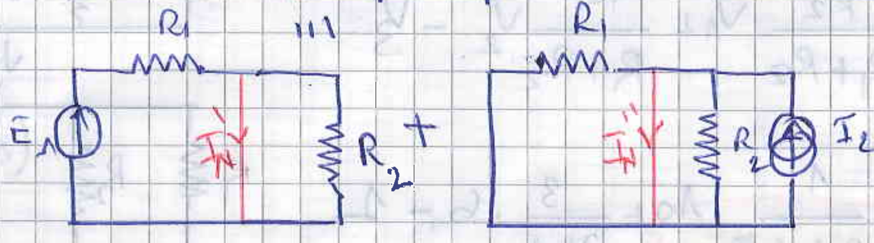
$$I = \frac{4E_1 + 2R I_2}{7R} \Rightarrow \boxed{I = \frac{4}{7R} E_1 + \frac{2}{7} I_2}$$

3) Méthode de Norton :

* $R_N = R_{Th} = R_{eq} = \frac{2}{5} R$



$$I_N' = \frac{E_1}{R_1} \quad (R_2 \text{ court-circuité})$$



$$I_N'' = I_2.$$

$$I = \frac{R_N}{R_N + R} I_N$$

$$I = \frac{\frac{2}{5}R}{R + \frac{2}{5}R}\left(\frac{E_1}{R_1} + I_2\right)$$

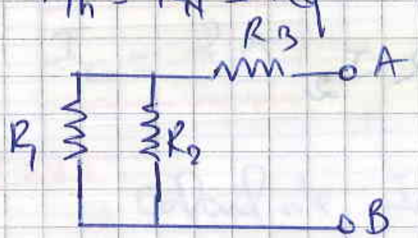
$$= \frac{2}{7}\left(\frac{2E_1}{R} + I_2\right) \Rightarrow \boxed{I = \frac{4}{7R}E_1 + \frac{2}{7}I_2}$$

Exo 6 :

1) Methode thevenin :

* R_{th} : ?

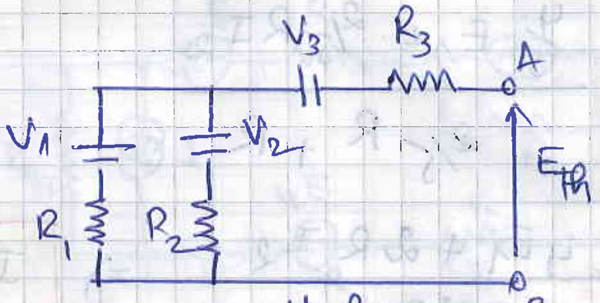
$R_{th} = R_N = R_{eq}$



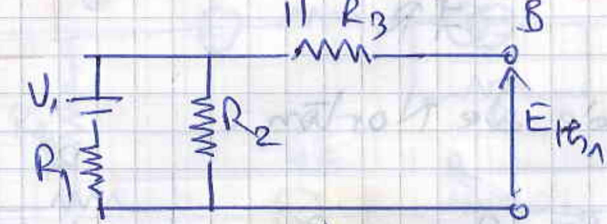
$$R_{eq} = R_3 + (R_1 // R_2) = R_3 + \frac{R_1 R_2}{R_1 + R_2}$$

A.N :
$$R_{eq} = 0{,}25 + \frac{3 \cdot 1}{3+1} = \frac{1}{4} + \frac{3}{4} \Rightarrow \boxed{R_{th} = R_N = 1\,\Omega}$$

* E_{th} : ?

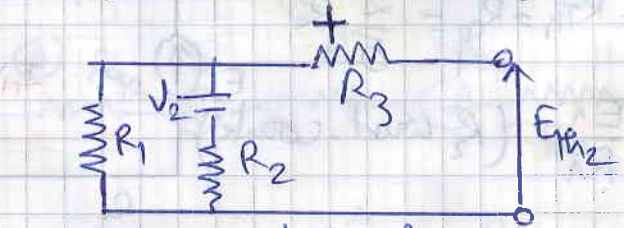


$E_{th} = E_{th_1} + E_{th_2} + E_{th_3}$.



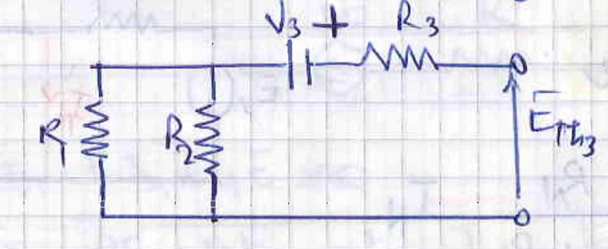
$$E_{th_1} = V_{R_2} = \frac{R_2}{R_1 + R_2} V_1$$

$$E_{th_2} = V_{R_1} = \frac{R_1}{R_1 + R_2} V_2$$



$$E_{th_3} = -V_3.$$

$$E_{th} = \frac{R_2}{R_1 + R_2} V_1 + \frac{R_1}{R_1 + R_2} V_2 - V_3$$



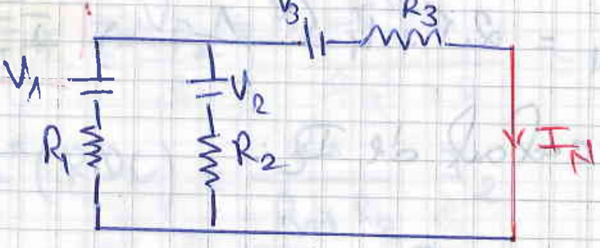
A.N :

$$E_{th} = \frac{1}{3+1} \cdot 10 + \frac{3}{3+1} \cdot 6 - 1$$

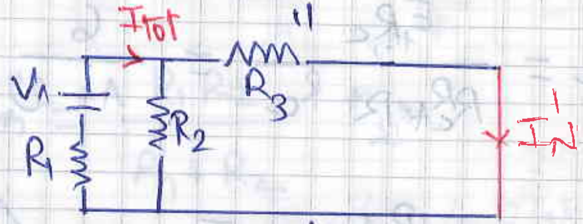
$$\boxed{E_{th} = 6V}.$$

a) Norton:

* $I_N = ?$

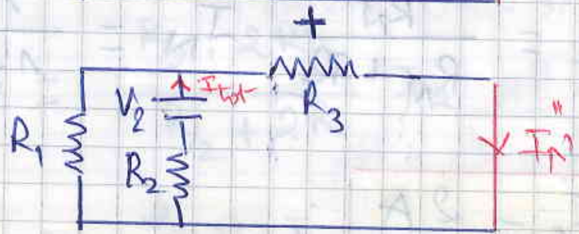


$$I_N' = \frac{R_2}{R_2+R_3} I_{tot}$$



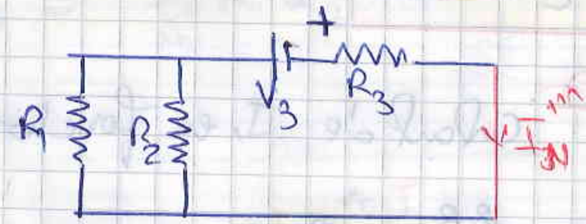
$$I_{tot} = \frac{V_1}{R_1 + (R_2 \parallel R_3)}$$

$$I_N' = \frac{R_2}{R_2+R_3} \quad \frac{V_1}{R_1 + (R_2 \parallel R_3)}$$



A.N :

$$I_N' = \frac{1}{1+0{,}25} \cdot \frac{10}{3 + \frac{1 \cdot 0.25}{1+0.25}}$$



$$\boxed{I_N' = 2.5\,A}$$

* $I_N'' = ?$

$$I_N'' = \frac{R_1}{R_1+R_3} I_{tot} \quad , \quad I_{tot} = \frac{V_2}{R_2 + (R_1 \parallel R_3)}$$

$$I_N'' = \frac{R_1}{R_1+R_3} \quad \frac{V_2}{R_2 + (R_1 \parallel R_3)}$$

A.N :

$$I_N'' = \frac{3}{3+0.25} \quad \frac{6}{1 + \frac{3 \cdot 0.25}{3+0{,}25}} \Rightarrow \boxed{I_N'' = 4{,}5\,A}$$

$I_N''' = ?$

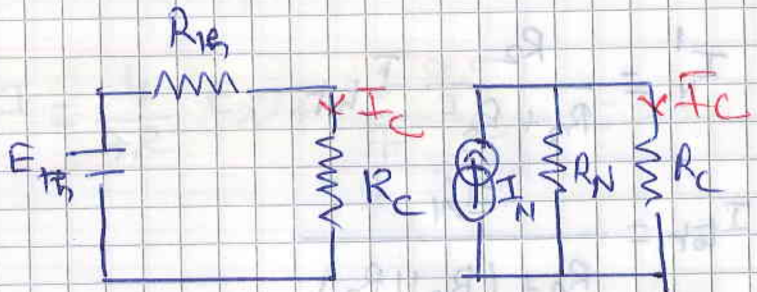
$$I_N''' = -\frac{V_3}{R_3 + (R_1 \parallel R_3)}$$

A.N :

$$I_N''' = -\frac{1}{0{,}25 + \frac{3 \cdot 1}{3+1}} = -\frac{1}{\frac{1}{4}+\frac{3}{4}} \Rightarrow \boxed{I_N''' = -1}$$

$I_N = 2.5 + 4.5 - 1 \Rightarrow \boxed{I_N = 6A}$

3) calcul de I_C :

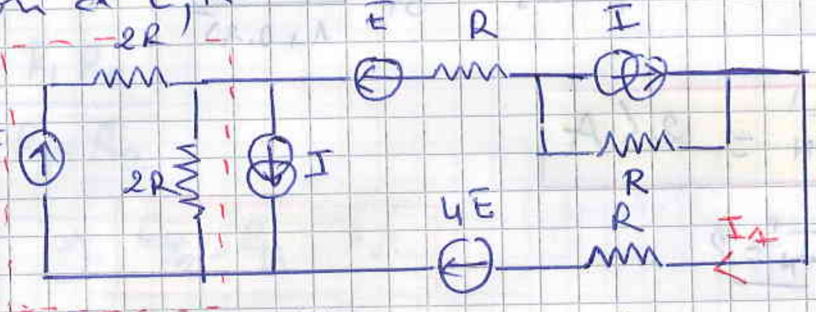


$$I_C = \frac{E_{th}}{R_{th}+R_C} = \frac{6}{1+2}$$

$$I_C = \frac{R_N}{R_N+R_C} I_N = \frac{1}{1+2} 6$$

$$\boxed{I_C = 2A}$$

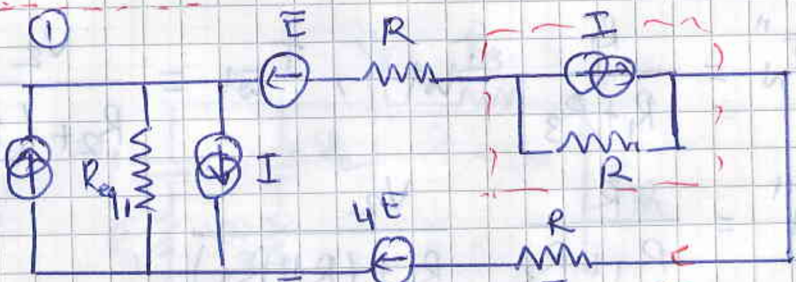
Exo7. Calcul de I_1 en fonction de E, R



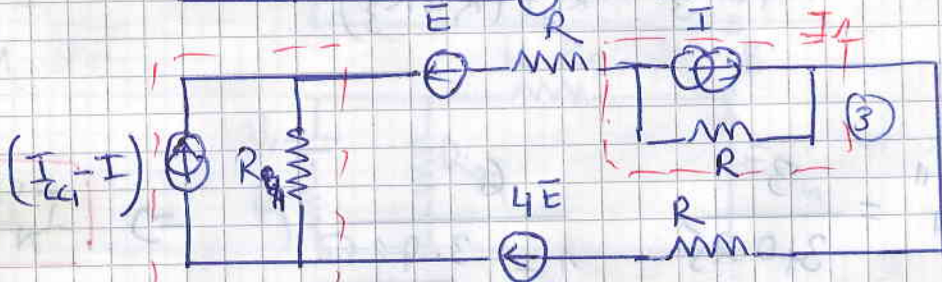
$$R_{eq_1} = \frac{2R}{2} = R$$

$$E_{th_1} = \frac{2R}{2R+2R} \cdot 2E = E$$

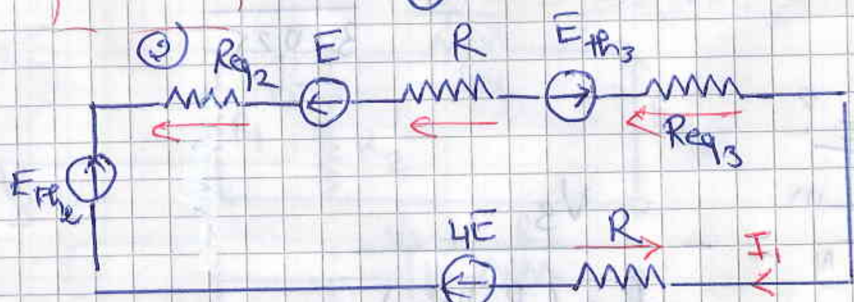
$$I_{cc1} = \frac{E}{R}$$



$$\begin{cases} E_{th_2} = (I_{cc_1} - I) R_{eq_1} \\ R_{eq_2} = R_{eq_1} = R \end{cases}$$



$$\Rightarrow \begin{cases} E_{th_2} = E - RI \\ R_{eq_2} = R. \end{cases}$$



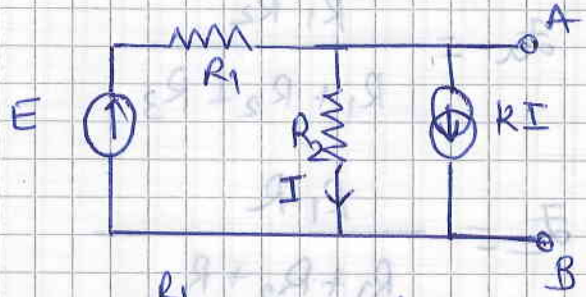
$$\begin{cases} E_{th_3} = R \cdot I \\ R_{eq_3} = R \end{cases}$$

$E_{th_2} - R_{eq_2} I_1 - E - R I_1 + E_{th_3} - R_{eq_3} I_1 - R I_1 + 4E = 0$.

$E - RI - RI_1 - E - RI_1 + RI - RI_1 - RI_1 - RI_1 + 4E \Rightarrow \boxed{I_1 = \frac{E}{R}}$

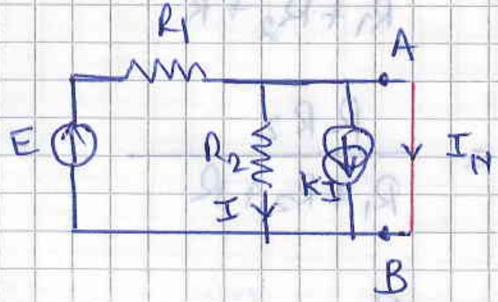
Ex 8 :

1) $R_{eq} = \dfrac{E_{Th}}{I_N}$



* Théorème de Norton :

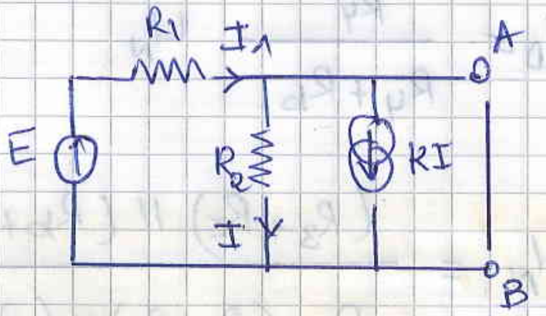
$I_1 = I + KI + I_N$



le dipôle étant en court circuit on a : $U_{AB} = 0 \Rightarrow I = 0$

$$\Rightarrow \boxed{I_N = I_1}$$

* Théorème de Thevenin :



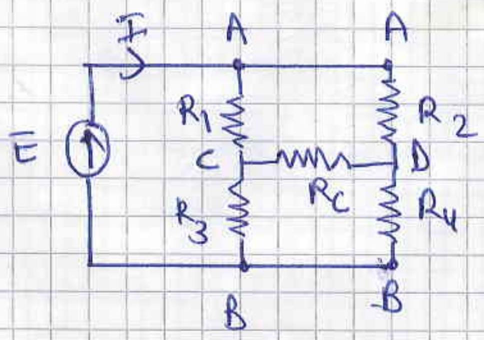
$E_{Th} = U_{AB}\,(\text{à vide}) = R_2 I$

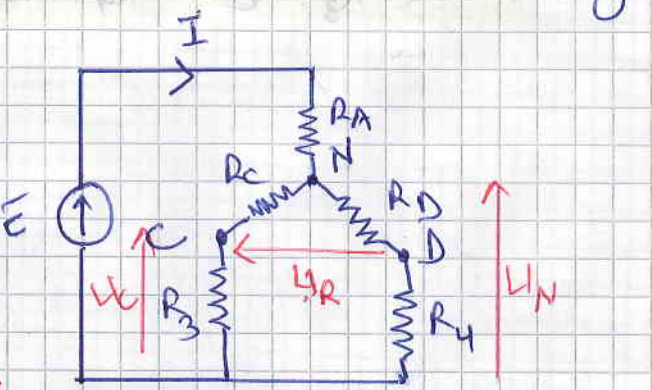
$I_1 = I + KI = (K+1)\, I$

$$\Rightarrow I = \frac{I_1}{K+1}$$

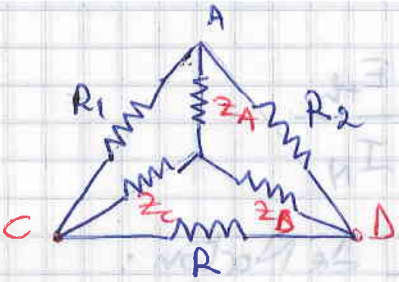
$$\boxed{E_{Th} = \frac{R_2 I_1}{K+1}} \qquad \boxed{R_{eq} = \frac{R_2}{K+1}}$$

Ex 9 : calcul de U_R en appliquant le théorème de Kennely.

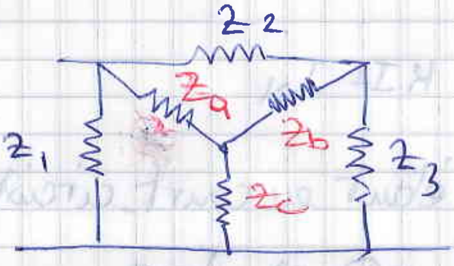




$$Z_a = \frac{R_1 R_2}{R_1 + R_2 + R}$$



$$Z_C = \frac{R_1 R}{R_1 + R_2 + R}$$



$$Z_D = \frac{R R_2}{R_1 + R_2 + R}$$

$\Delta \to Y$

$$U_C = \frac{R_3}{R_3 + R_C} U_N$$

$$Z_a = \frac{Z_1 Z_2}{Z_1 + Z_2 + Z_3}$$

$$U_D = \frac{R_4}{R_4 + R_b} U_N$$

$$Z_b = \frac{Z_2 Z_3}{Z_1 + Z_2 + Z_3}$$

$$U_N = \frac{(R_3 + R_C) \,//\, (R_b + R_4)}{R + (R_3 + R_C) \,//\, (R_4 + R_b)} E$$

$$Z_C = \frac{Z_1 Z_3}{Z_1 + Z_2 + Z_3}$$

$$U_R = U_C - U_D = \frac{R_3}{R_3 + R_C} U_N - \frac{R_4}{R_4 + R_b} U_N$$

$$U_R = \left(\frac{R_3}{R_3 + R_C} - \frac{R_4}{R_4 + R_b} \right) U_N$$