Solution de fiche TD4

Exercice 4 :

1)

$\vec{V} = a\overrightarrow{u_\theta} + b\,\overrightarrow{u_y}$ où a et b sont deux constantes. On a :

$\overrightarrow{u_y} = (\sin(\theta)).\overrightarrow{u_\rho} + (\cos(\theta)).\overrightarrow{u_\theta}$

$$\Rightarrow \vec{V} = b\sin(\theta)\,.\overrightarrow{u_\rho} + (a + b\cos(\theta)\ \overrightarrow{u_\theta} = \dot{\rho}.\overrightarrow{u_\rho} + \rho.\dot{\theta}.\overrightarrow{u_\theta} \Rightarrow \frac{d\rho}{dt}\sin(\theta)\ \ et\ \rho\frac{d\theta}{dt}$$

$$= a + b\cos(\theta)\ \Rightarrow\ \frac{d\rho}{\rho} = \frac{b\sin(\theta)}{a + b\cos(\theta)}d\theta$$

Soit $C=\ln(\rho_0)$

$$\ln(\rho) + C = \int\frac{b\sin(\theta)}{a + b\cos(\theta)}d\theta = \ln(\rho) - \ln(\rho_0) = \ln\left(\frac{\rho}{\rho_0}\right) \Rightarrow \rho(\theta) = \frac{\rho_0}{a + b\cos(\theta)}$$

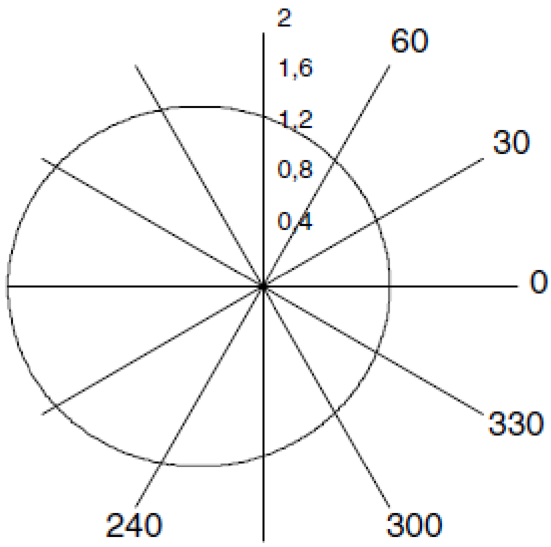
2)

a=3b et θ=0

$\rho(0)=1=\frac{\rho_0}{3b+b} \Rightarrow \rho(0) = 4b$

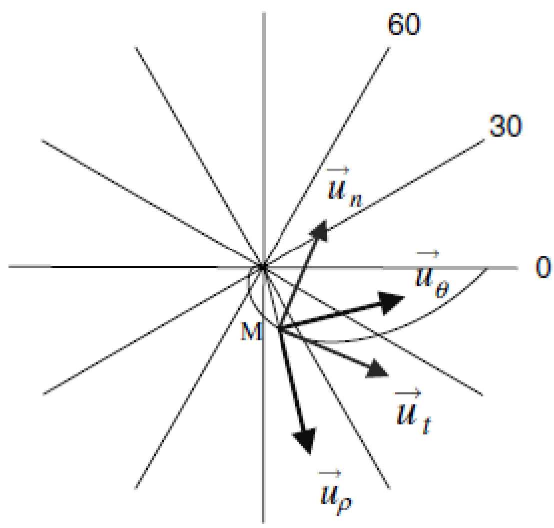
$$\rho(\theta) = \frac{4}{3 + b\cos(\theta)} = \frac{4/3}{1 + \frac{1}{3}\cos(\theta)}$$

Equation d’une ellipse en coordonnées polaires.



Exercice 5 :

1)



2) : $\rho = \rho_0 e^{\theta}$, θ = ωt avec ω constant.

$\Rightarrow \dot{\rho} = \omega\rho \quad \ddot{\rho} = \omega^2\,\rho$

La vitesse angulaire constante ω = cste

$\Rightarrow \dot{\theta} = \omega \quad \ddot{\theta} = 0$

$\vec{\mathrm{V}} = \dot{\rho}.\overrightarrow{u_\rho} + \rho.\dot{\theta}.\overrightarrow{\mathrm{u}_\theta} = \omega\rho(\overrightarrow{u_\rho} + \overrightarrow{\mathrm{u}_\theta})$

$\vec{\gamma} = (\ddot{\rho} - \rho.\dot{\theta}^2)\overrightarrow{u_\rho} + (2\dot{\rho}\dot{\theta} + \rho\ddot{\theta})\overrightarrow{\mathrm{u}_\theta} = (\omega^2\,\rho\text{-}\rho.\,\omega^2)\,\overrightarrow{u_\rho} + 2\omega^2\,\rho\overrightarrow{\mathrm{u}_\theta} = 2\omega^2\,\rho\overrightarrow{\mathrm{u}_\theta}$

$\|\vec{\mathrm{V}}\| = \sqrt{2}\,\omega\rho$

-$\|\vec{\gamma}\| = 2\omega^2\,\rho$

$$\vec{\mathrm{V}}\,\overrightarrow{\mathrm{u}_\theta} = \omega\rho = \sqrt{2}\,\omega\rho\cos(\alpha) \Rightarrow \cos(\alpha) = \frac{\sqrt{2}}{2} \Rightarrow \alpha = \frac{\pi}{4}$$

3)

$\vec{\gamma} = \gamma_t\overrightarrow{u_t} + \gamma_n\overrightarrow{\mathrm{u_n}} \quad = \frac{\mathrm{dV}}{\mathrm{dt}}\overrightarrow{\mathrm{u_t}} + \frac{\mathrm{V}^2}{\mathrm{R}}\overrightarrow{\mathrm{u_n}} = \sqrt{2}\,\omega^2\rho\,\overrightarrow{\mathrm{u_t}} + \frac{2\omega^2\rho^2}{\mathrm{R}}\overrightarrow{\mathrm{u_n}}$

$\|\vec{\gamma}\| = 2\omega^2\,\rho = \sqrt{(\sqrt{2}\,\omega^2\rho)^2 + (\frac{2\omega^2\rho^2}{\mathrm{R}})^2} \quad =$

$$\gamma^2 = {\gamma_t}^2 + {\gamma_n}^2 \Rightarrow \gamma_n = \sqrt{\gamma^2 - {\gamma_t}^2} \Rightarrow \frac{\mathrm{V}^2}{\mathrm{R}} = \sqrt{\gamma^2 - {\gamma_t}^2} \Rightarrow R = \frac{\mathrm{V}^2}{\sqrt{\gamma^2 - {\gamma_t}^2}}$$

$$\Rightarrow R = \frac{(\sqrt{2}\,\omega\rho)^2}{\sqrt{(2\omega^2\,\rho)^2 - (\sqrt{2}\,\omega^2\rho)^2}} = \frac{(\sqrt{2}\,\omega\rho)^2}{\sqrt{2\omega^2(\sqrt{2}\,\omega\,\rho)^2 - \omega^2(\sqrt{2}\,\rho\omega)^2}}$$

$$= \frac{(\sqrt{2}\,\omega\rho)^2}{\sqrt{2}\,\omega\,\rho\sqrt{2\omega^2 - \omega^2}} = \sqrt{2}\,\rho$$

4)

$$\|\vec{\mathrm{V}}\| = \sqrt{2}\,\omega\rho = \sqrt{2}\,\frac{d\theta}{dt}\rho = cste \Rightarrow \frac{d\theta}{dt} = \frac{V}{\sqrt{2}\,\rho_0 e^{\theta}} \Rightarrow e^{\theta}d\theta = \frac{V}{\sqrt{2}\,\rho_0}dt$$

$$\Rightarrow e^{\theta} = \frac{V}{\sqrt{2}\,\rho_0}t + C$$

Si t=0 θ=0 $\Rightarrow$ C=1

Donc

$$e^{\theta} = \frac{V}{\sqrt{2}\,\rho_0}t + 1 \Rightarrow \theta = \ln\left(\frac{V}{\sqrt{2}\,\rho_0}t + 1\right) \Rightarrow \frac{d\theta}{dt} = \frac{V}{\sqrt{2}\,\rho_0}.\left(\frac{V}{\sqrt{2}\,\rho_0}t + 1\right)^{-1}$$

Exercice 1:

$$\vec{\mathrm{r}} = (1{,}5 - 1t^2)\vec{\imath} + (-3{,}2t + 0{,}5t^2)\,\vec{\jmath}$$

1) Supposant M_0 position à t= 0, M position à t= 2,4 s on a $\overrightarrow{\mathrm{M_0M}} = (1{,}5 - 1t^2 - 1{,}5)\vec{\imath} + (-3{,}2t + 0{,}5t^2)\vec{\jmath} \Rightarrow \|\overrightarrow{\mathrm{M_0M}}\| = \sqrt{t^4 + (-3{,}2t + 0{,}5t^2)^2} = \sqrt{2{,}4^4 + (-3{,}2.2{,}4 + 0{,}5.\,2{,}4^4)^2}$ =7.49m
2) $\mathrm{V_m} = \frac{\|\overrightarrow{\mathrm{M_0M}}\|}{\Delta\mathrm{t}} = 3{,}08\ m/s$
3)

$$\vec{\mathrm{V}}(t) = \frac{d\overrightarrow{OM}}{dt} = -2\mathrm{t}\,\vec{\imath} + (-3{,}2 + t)\vec{\jmath} \Rightarrow V = \sqrt{4t^2 + (-3{,}2 + t)^2}$$=1,76 m/s

4) $\vec{\gamma} = \frac{d\vec{V}}{dt} == -2\,\vec{\imath} + \vec{\jmath} \Rightarrow \gamma = \sqrt{5}$ =2,23 m/s²

Exercice 2:

$\overrightarrow{V_0} = 4\vec{\imath} + 2{,}8\,\vec{\jmath}$

$\vec{\gamma} = -3\vec{\imath} + 1{,}1\,\vec{\jmath} = \overrightarrow{cste}$

1) Trouver t pour x=0 , il faut trouver les équations horaires du mouvement :

$$\vec{V} = \int \vec{\gamma}\, dt = \int (-3\vec{\imath} + 1{,}1\,\vec{\jmath})dt = -3t\,\vec{\imath} + 1{,}1t\ \vec{\jmath} + \vec{V_0} = (-3t + 4)\,\vec{\imath} + (1{,}1t + 2.8)\,\vec{\jmath}$$

$$\overrightarrow{OM} = \int \vec{V}\, dt = \int (-3t + 4)\vec{\imath} + (1{,}1t + 2.8)\, dt$$

$$= \left(-\left(\frac{3}{2}\right) t^2 + 4t\right)\vec{\imath} + \left(\left(\frac{1{,}1}{2}\right) t^2 + 2.8t\right)\vec{\jmath} + \overrightarrow{OM_0}$$

t= 0 $\overrightarrow{OM_0} = \vec{0}$

donc

$x = -\left(\frac{3}{2}\right) t^2 + 4t$

Pour x= 0 $\Rightarrow = -\left(\frac{3}{2}\right) t + 4 = 0 \Rightarrow t = \frac{8}{3} = 2{,}66\ s$

2) $y = \left(\frac{1{,}1}{2}\right) t^2 + 2.8t = \left(\frac{1{,}1}{2}\right) 2{,}66^2 + 2.8.\,(2{,}66) = 11{,}4\ m$
3) $V = \sqrt{(-3t + 4)^{\,2} + (1{,}1t + 2.8)^2}$ =5,72 m/s

Exercice 3

$\gamma = 2{,}45\ \mathrm{m/s^2}$

R= 5m

1) $\vec{\mathrm{a}} = a_t\overrightarrow{e_t} + a_n\overrightarrow{\mathrm{e_n}}$
   $a_n = \overrightarrow{\mathrm{a}}.\overrightarrow{\mathrm{e_n}} = \mathrm{a}\cos(\alpha)$
   Avec α l'angle entre $\overrightarrow{\mathrm{a}}$. et $\overrightarrow{\mathrm{e_n}}$.
   α= 90°-35°=55°
   $a_n = 2{,}45\cos(55)$=1,4m/s²
2) $a^2 = {a_t}^2 + {a_n}^2 \Rightarrow a_t = \sqrt{a^2 - {a_n}^2} \Rightarrow a_t = 2{,}01\mathrm{m/s^2}$

3) $a_n = \frac{\mathrm{V}^2}{\mathrm{R}} \Rightarrow V = \sqrt{a_n R} = \sqrt{1{,}4.5} = 2{,}64\ m/s$

Exercice 6

$\vec{v}_1 = 2\vec{\imath} + 5\,\vec{\jmath}$, $\vec{v}_2 = 2\vec{\imath} + 4\,\vec{\jmath}$ et $\vec{v}_3 = 2\vec{\imath} + 3\,\vec{\jmath}$

1. Temps de vol $t = d/v_{0x}$ donc $t_1 = t_2 = t_3$
2. $h = \frac{V_0^{\,2}}{2.g}\sin(\alpha)^2 = \frac{V_{0z}^{\,2}}{2.g}$ donc $h_3 < h_2 < h_1$