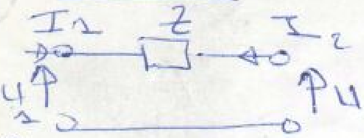
exo 1
Solution:

Serie 2 Correction



$I_2 = -I_1$

1) $Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$ $\quad U_1 = Z_{11} I_1 + Z_{12} I_2$

$U_2 = Z_{21} I_1 + Z_{22} I_2$

$Z_{11} = \frac{U_1}{I_1}\Big|_{I_2=0} \Rightarrow I_1 = 0 \Rightarrow Z_{11}$ indéfini

$\boxed{[Z] \text{ n'existe pas.}}$

2) $Y = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$ $\quad I_1 = Y_{11} U_1 + Y_{12} U_2$

$I_2 = Y_{21} U_1 + Y_{22} U_2$

$Y_{11} = \frac{I_1}{U_1}\Big|_{U_2=0} \Rightarrow U_1 = I_1 Z \Rightarrow Y_{11} = \frac{1}{Z}$

$Y_{12} = \frac{I_1}{U_2}\Big|_{U_1=0} \Rightarrow U_2 = -I_1 Z \Rightarrow Y_{12} = \frac{1}{Z}$

$Y_{21} = \frac{I_2}{U_1}\Big|_{U_2=0} \Rightarrow U_1 = -Z I_2 \Rightarrow Y_{21} = -\frac{1}{Z}$

$Y_{22} = \frac{I_2}{U_2}\Big|_{U_1=0} \Rightarrow U_2 = Z I_2 \Rightarrow Y_{22} = \frac{1}{Z}$

$\boxed{Y = \begin{bmatrix} \frac{1}{Z} & -\frac{1}{Z} \\ -\frac{1}{Z} & \frac{1}{Z} \end{bmatrix}} \Rightarrow \Delta Y = 0 \Rightarrow Z$ n'existe pas.

3) $T = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}$ $\quad U_2 = T_{11} U_1 + T_{12} I_1$

$I_2 = T_{21} U_1 - T_{22} I_1$

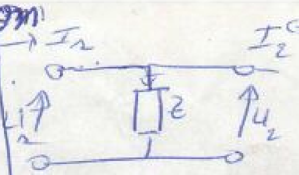
$T_{11} = \frac{U_2}{U_1}\Big|_{I_1=0} \Rightarrow I_1 = 0 \Rightarrow U_1 = U_2 \Rightarrow T_{11} = 1$

$T_{22} = \frac{-I_2}{I_1}\Big|_{U_1=0} \Rightarrow I_2 = -I_1 \Rightarrow T_{22} = 1.$

$T_{12} = \frac{-U_2}{I_1}\Big|_{U_1=0} \Rightarrow U_2 = -Z I_1 \Rightarrow T_{12} = Z$

$T_{21} = \frac{I_2}{U_1}\Big|_{I_1=0} \Rightarrow I_1 = 0 = -I_2 \Rightarrow T_{21} = 0.$

$\boxed{T = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}}$



$U_2 = U_1.$

1) $Z_{11} = \frac{U_1}{I_1}\Big|_{I_2=0} \Rightarrow U_1 = Z I_1 \Rightarrow Z_{11} = Z$

$Z_{22} = \frac{U_2}{I_2}\Big|_{I_1=0} \Rightarrow U_2 = I_2 Z \Rightarrow Z_{22} = Z$

$Z_{12} = \frac{U_1}{I_2}\Big|_{I_1=0} \Rightarrow U_1 = Z I_2 \Rightarrow Z_{12} = Z$

$Z_{21} = \frac{U_2}{I_1}\Big|_{I_2=0} \Rightarrow U_2 = Z I_1 \Rightarrow Z_{21} = Z$

$\boxed{Z = \begin{bmatrix} Z & Z \\ Z & Z \end{bmatrix}}$ $\Delta Z = 0 \Rightarrow$ $\boxed{Y \text{ n'existe pas}}$

2) $Y_{11} = \frac{I_1}{U_1}\Big|_{U_2=0} \Rightarrow U_2 = U_1 = 0 \Rightarrow$

Y_{11} indéfini $\Rightarrow [Y]$ n'existe pas.

3) $T_{11} = \frac{U_2}{U_1}\Big|_{I_1=0} \Rightarrow U_2 = U_1 \Rightarrow T_{11} = 1$

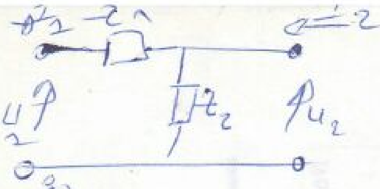
$T_{22} = \frac{-I_2}{I_1}\Big|_{U_1=0} \Rightarrow Z(I_1 + I_2) = 0$

$\Rightarrow I_1 = -I_2 \Rightarrow T_{22} = 1.$

$T_{12} = \frac{-U_2}{I_1}\Big|_{U_1=0} \Rightarrow U_2 = 0 \Rightarrow T_{12} = 0$

$T_{21} = \frac{I_2}{U_1}\Big|_{I_1=0} \Rightarrow U_1 = Z I_2 \Rightarrow T_{21} = \frac{1}{Z}$

$\boxed{T = \begin{bmatrix} 1 & 0 \\ \frac{1}{Z} & 1 \end{bmatrix}}$ ok.



1)

$Z_{11} = \frac{U_1}{I_1}\Big|_{I_2=0} \Rightarrow U_1 = (Z_1+Z_2) I_1$

$\Rightarrow Z_{11} = Z_1 + Z_2$.

$Z_{22} = \frac{U_2}{I_2}\Big|_{I_1=0} \Rightarrow U_2 = Z_2 I_2 \Rightarrow Z_{22} = Z_2$

$Z_{12} = \frac{U_1}{I_2}\Big|_{I_1=0} \Rightarrow U_1 = U_2 = Z_2 I_2 \Rightarrow Z_{12} = Z_2$

$Z_{21} = \frac{U_2}{I_1}\Big|_{I_2=0} \Rightarrow U_2 = Z_2 I_1 \Rightarrow Z_{21} = Z_2$

$$\boxed{Z = \begin{bmatrix} Z_1+Z_2 & Z_2 \\ Z_2 & Z_2 \end{bmatrix}}$$ ok.

2) $y_{11} = \frac{I_1}{U_1}\Big|_{U_2=0} \Rightarrow U_1 = Z_1 I_1 \Rightarrow y_{11} = \frac{1}{Z_1}$

$y_{12} = \frac{I_1}{U_2}\Big|_{U_1=0} \Rightarrow U_2 = Z_2(I_1+I_2) = \cancel{Z_2}\, I_1$

$\Rightarrow y_{12} = -\frac{1}{Z_1}$

$y_{21} = \frac{I_2}{U_1}\Big|_{U_2=0} \Rightarrow Z_2(I_1+I_2) = 0 \Rightarrow I_1 = -I_2$

$U_1 = I_1 Z_1 = -I_2 Z_1$

$\Rightarrow y_{21} = -\frac{1}{Z_1}$

$y_{22} = \frac{I_2}{U_2}\Big|_{U_1=0} \Rightarrow U_2 = -Z_1 I_1 = Z_2(I_2+I_1)$

$\Rightarrow U_2 = -Z_1 I_1$

$(Z_1 I_1 + Z_2 I_1) = -Z_2 I_2$

$I_1 = \frac{-Z_2}{Z_1+Z_2} I_2$

$U_2 = \frac{Z_1 Z_2}{Z_1+Z_2} I_2 \Rightarrow y_{22} = \frac{Z_1+Z_2}{Z_1 Z_2}$

ou bien $U_1 = 0 \Rightarrow Z_1 // Z_2$

$U_2 = Z_{eq} \cdot I_2 \Big| \frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2} \Rightarrow Z_{eq} = \frac{Z_1+Z_2}{Z_1 Z_2}$

$$Y = \begin{bmatrix} \frac{1}{Z_1} & -\frac{1}{Z_1} \\ -\frac{1}{Z_1} & \frac{1}{Z_1}+\frac{1}{Z_2} \end{bmatrix}$$

3)

$T_{11} = \frac{U_1}{U_2}\Big|_{I_2=0} \Rightarrow U_2 = U_1 \Rightarrow T_{11} = 1$.

$T_{12} = \frac{-U_2}{I_1}\Big|_{U_1=0} \Rightarrow U_2 \equiv I_1 Z_1 \Rightarrow T_{12} = +Z_1$.

$T_{21} = \frac{I_2}{U_2}\Big|_{I_2=0} \Rightarrow U_2 = U_1 = Z_2 I_2$

$\Rightarrow T_{21} = \frac{1}{Z_2}$

$T_{22} = \frac{-I_2}{I_1}\Big|_{U_1=0} \Rightarrow Z_1 // Z_2$

$I_1 = \frac{-Z_2}{Z_1+Z_2} I_2$ (diviseur de courant)

$\Rightarrow T_{22} = \frac{Z_1+Z_2}{Z_2} = \frac{Z_1}{Z_2} + 1$.

$$\boxed{T = \begin{bmatrix} 1 & Z_1 \\ \frac{1}{Z_2} & \frac{Z_1}{Z_2}+2 \end{bmatrix}}$$ ok.

2e méthode :

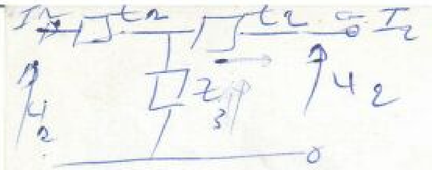
$[T] = [T_2][T_1] = \begin{bmatrix} 1 & 0 \\ \frac{1}{Z_2} & 1 \end{bmatrix} \begin{bmatrix} 1 & Z_1 \\ 0 & 1 \end{bmatrix}$

$$[T] = \begin{bmatrix} 1 & Z_1 \\ \frac{1}{Z_2} & \frac{Z_1}{Z_2}+1 \end{bmatrix}$$ ok

2e méthode

$$Y = \frac{1}{Z_1 Z_2} \begin{bmatrix} Z_2 & -Z_2 \\ -Z_2 & Z_1+Z_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{Z_1} & -\frac{1}{Z_1} \\ -\frac{1}{Z_1} & \frac{1}{Z_2}+\frac{1}{Z_1} \end{bmatrix}$$

$\Delta Z \neq 0$. ok.



$z_{11} = \frac{U_1}{I_1}\Big|_{I_2=0} \Rightarrow U_1 = (z_1+z_3) I_1$

$\Rightarrow z_{11} = z_1 + z_3.$

$z_{22} = \frac{U_2}{I_2}\Big|_{I_1=0} \Rightarrow U_2 = (z_2+z_3) I_2$

$\Rightarrow z_{22} = z_2+z_3.$

$z_{12} = \frac{U_1}{I_2}\Big|_{I_1=0} \Rightarrow U_1 = z_3 I_2$

$\Rightarrow z_{12} = z_3.$

$z_{21} = \frac{U_2}{I_1}\Big|_{I_2=0} \Rightarrow U_2 = z_3 I_1.$

$\Rightarrow z_{21} = z_3$

$z = \begin{bmatrix} z_1+z_3 & z_3 \\ z_3 & z_2+z_3\end{bmatrix}$ ok.

l'admittance:

$y_{11} = \frac{I_1}{U_1}\Big|_{U_2=0}$

2e méthode:

$\Delta z = z_1z_2 + z_1z_3 + \cancel{z_3z_3} + z_3 z_2 - \cancel{z_3z_3}$

$[y] = [z]^{-1} = \frac{1}{\Delta z}\begin{bmatrix} z_2+z_3 & -z_3\\ -z_3 & z_1+z_3\end{bmatrix}$

$\Delta z \neq 0.$

$y_{11} = \frac{I_1}{U_1}\Big|_{U_2=0} \Rightarrow -z_2 I_2 = z_3 I_2 + z_3 I_1$

$\frac{U_1}{I_1} = z_1 + (z_2 // z_3) = z_1 + \frac{z_2z_3}{z_2+z_3}$

$= \frac{z_1z_2 + z_1z_3 + z_2z_3}{z_2+z_3}$

$\cancel{\Rightarrow y_{11} = \frac{z_1+z_2}{z_1z_1 + 2z_1z_2}}$

$\cancel{y_{11} = \frac{z_2+z_3}{z_1z_2 + z_1z_3 + z_2z_3}}$ si

$y_{11} = \frac{z_2+z_3}{z_1z_2+z_1z_3+z_2z_3}$ ok

$y_{22} = \frac{I_2}{U_2}\Big|_{U_1=0} \Rightarrow U_2 = I_2 \left(z_2 + (z_1 // z_3)\right)$

$\frac{U_2}{I_2} = z_2 + \frac{z_1z_3}{z_1+z_3} = \frac{z_1z_2+z_2z_3+z_1z_3}{z_1+z_3}.$

$\Rightarrow y_{22} = \frac{I_2}{U_2} = \frac{z_1+z_3}{\Delta z}$

$y_{12} = \frac{I_1}{U_2}\Big|_{U_1=0} \Rightarrow$

$I_1 z_1 + z_3(I_1+I_2) = 0$

$\Rightarrow I_1 = \frac{-z_3}{z_1+z_3} I_2$ — (1)

$U_2 = I_2 \left(z_2 + \frac{z_1z_3}{z_1+z_3}\right)$ — (2)

$\frac{I_1}{U_2} = \frac{-z_3(z_1+z_3)}{(z_1+z_3)\times \Delta z} = \frac{-z_3}{\Delta z}.$

$y_{21} = \frac{I_2}{U_1}\Big|_{U_2=0} \Rightarrow$

$I_2 z_2 + z_3(I_1+I_2) = 0 \Rightarrow I_2 = \frac{-z_3 I_1}{z_2+z_3}$

$U_1 = I_1\left(z_1 + \frac{z_2z_3}{z_2+z_3}\right).$

$\Rightarrow y_{21} = \frac{-z_3}{\Delta z}$ ok.

Calcul de $T = ?$

1re méthode :

$$T = [T_2]\cdot[T_3]\cdot[T_1] =$$

$$= \begin{bmatrix} 1 & z_2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1}{z_3} & 1 \end{bmatrix} \begin{bmatrix} 1 & z_1 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & z_2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & z_1 \\ \frac{1}{z_3} & \frac{z_1}{z_3}+1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+\frac{z_2}{z_3} & z_1 + \frac{z_2 z_1}{z_3} + z_2 \\ \frac{1}{z_3} & \frac{z_1}{z_3}+1 \end{bmatrix} \quad \text{ok}$$

2e méthode :

$$T_{11} = \frac{U_1}{U_2}\Big|_{I_2=0} \Rightarrow U_2 = z_3 I_2$$

$$U_1 = (z_3 + z_2) I_2$$

$$\Rightarrow \frac{U_1}{U_2} = \frac{z_3+z_2}{z_3} \quad \text{ok}$$

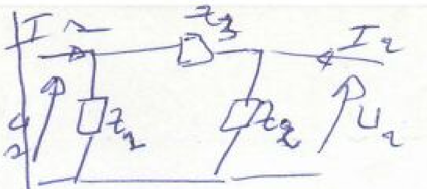
$$T_{12} = -\frac{U_1}{I_2}\Big|_{U_2=0} = -\frac{1}{y_{12}} = +\frac{\Delta z}{z_3} \quad \text{ok}$$

$$T_{21} = \frac{I_1}{U_2}\Big|_{I_2=0} = \frac{1}{z_{12}} = \frac{1}{z_3} \quad \text{ok}$$

$$T_{22} = \frac{-I_1}{I_2}\Big|_{U_2=0} \Rightarrow I_2 z_1 = -z_3 (I_1 + I_2)$$

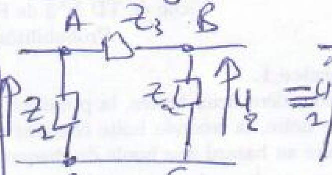
$$\Rightarrow I_1 = -I_2 \frac{z_3}{z_1+z_3} \Rightarrow \frac{-I_1}{I_2} = \frac{z_1+z_3}{z_3}$$

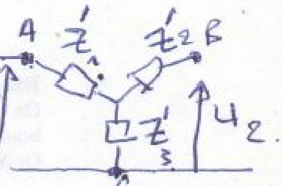
ok



1re méthode :

On transforme le circuit en étoile en utilisant la méthode de Kennelly :





avec

$$z'_1 = \frac{z_1 z_3}{z_1+z_2+z_3}$$

$$z'_2 = \frac{z_2 \cdot z_3}{z_1+z_2+z_3}$$

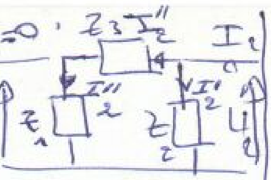
$$z'_3 = \frac{z_1 \cdot z_2}{z_1+z_2+z_3}$$

Une fois transformée on peut utiliser des résultats d'avant (précédent)

à titre d'exemple :

$$Z = \begin{bmatrix} \frac{z_1(z_2+z_3)}{z_1+z_2+z_3} & \frac{z_1 \cdot z_2}{z_1+z_2+z_3} \\ \frac{z_1 z_2}{\Sigma} & \frac{z_2(z_1+z_3)}{\Sigma} \end{bmatrix}$$

vérification :



$Z_{12} = \frac{U_1}{I_2}\Big|_{I_1=0} \Rightarrow$

En utilisant le diviseur de courant

$I_2'' = \frac{Z_2}{Z_2+Z_3+Z_1} . I_2$ — (1)

$U_1 = Z_1 I_2''$ — (2)

$\Rightarrow \frac{U_1}{I_2} = \frac{Z_1 Z_2}{Z_2+Z_3+Z_1} = Z_{12}$ ok

$Z_{11} = \frac{U_1}{I_1}\Big|_{I_2=0} =$

$U_1 = Z_1 . I_1''$.

$I_1'' = \frac{Z_1}{Z_1+Z_2+Z_3} . I_1$.

$U_1 = Z_1 . I_1'$.

$I_1' = \frac{Z_2+Z_3}{Z_1+Z_2+Z_3} I_1$ — (3)

$\frac{U_1}{Z_1} = \frac{Z_2+Z_3}{Z_1+Z_2+Z_3} I_1$.

$\Rightarrow \frac{U_1}{I_1} = \frac{Z_1(Z_2+Z_3)}{\Sigma}$. ok.

pour Y :

$$Y = \frac{1}{\Delta Z}\begin{bmatrix} \frac{Z_2(Z_1+Z_3)}{\Sigma} & -\frac{Z_1Z_2}{\Sigma} \\ -\frac{Z_1Z_2}{\Sigma} & \frac{Z_1(Z_2+Z_3)}{\Sigma}\end{bmatrix}$$

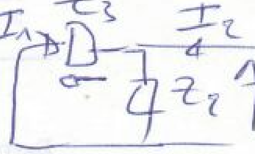
avec $\Delta Z = (Z_1'+Z_3')(Z_2'+Z_3') - Z_3'^2$

$\Delta Z = Z_1'Z_2' + Z_1'Z_3' + Z_2'Z_3'$.

$= \frac{Z_1Z_2Z_3^2 + Z_1^2 Z_2 Z_3 + Z_2^2 Z_1 Z_3}{(Z_1+Z_2+Z_3)^2}$

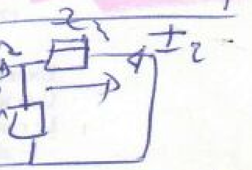
$\Delta Z = \frac{Z_1Z_2Z_3 \cancel{\Sigma}}{\Sigma^{\cancel{2}}}$.

vérification :



$y_{12} = \frac{I_1}{U_2}\Big|_{U_1=0}$

$U_2 = -Z_3 I_1 \Rightarrow \boxed{y_{12} = -\frac{1}{Z_3}}$ ok



$y_{21} = \frac{I_2}{U_1}\Big|_{U_2=0}$

$U_1 = -Z_3 I_2 \Rightarrow \frac{I_2}{U_1} = -\frac{1}{Z_3}$. ok

$y_{11} = \frac{I_1}{U_1}\Big|_{U_2=0} \Rightarrow U_1 = I_1 (Z_1 // Z_3)$.

$\boxed{\frac{I_1}{U_1} = \frac{Z_1+Z_3}{Z_1Z_3} = y_{11}}$ ok.

$$Y = \frac{\Sigma}{Z_1Z_2Z_3}\begin{bmatrix}\frac{Z_2(Z_1+Z_3)}{\Sigma} & -\frac{Z_1Z_2}{\Sigma}\\ -\frac{Z_1Z_2}{\Sigma} & \frac{Z_1(Z_2+Z_3)}{\Sigma}\end{bmatrix}$$

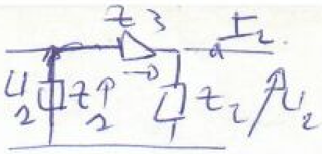
$y_{11} = \frac{Z_1+Z_3}{Z_1 Z_3} = \frac{1}{Z_3}+\frac{1}{Z_1}$. ok.

$y_{12} = \frac{-Z_1Z_2}{Z_1Z_2Z_3} = -\frac{1}{Z_3}$. ok.

$\boxed{\Delta Z = \frac{Z_1Z_2Z_3}{Z_1+Z_2+Z_3}}$ $y_{22} = \frac{Z_3+Z_2}{Z_3 Z_2}$

$$T = \begin{bmatrix} 1 + \frac{Z_2 Z_3}{Z_1 Z_2} & Z_3 \\ \frac{Z_1 + Z_2 + Z_3}{Z_1 Z_2} & 1 + \frac{Z_3}{Z_2} \end{bmatrix}$$

$$T_{11} = \frac{U_2}{U_1}\Big|_{I_2=0}$$



$U_2 = Z_2 I_2'$. $U_2 = U_1 + Z_3 I_2''$

$U_1 = Z_1 I_2''$.

$U_2 = U_1 + Z_3 \frac{U_1}{Z_1} \Rightarrow \frac{U_2}{U_1} = \boxed{1 + \frac{Z_3}{Z_2}}$ ok

vérification :

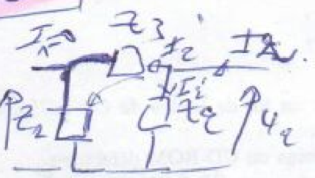
$$T_{12} = \frac{-U_1}{I_2}\Big|_{U_2=0} = -y_{12} = \boxed{+Z_3}.\ \text{ok}$$

$$T_{22} = \frac{-I_2}{I_1}\Big|_{U_1=0}.$$ Diviseur de courant

$$I_1 = \frac{-Z_2}{Z_2+Z_3} I_2 \Rightarrow \frac{-I_2}{I_1} = \frac{Z_2+Z_3}{Z_2}$$

$$T_{22} = \boxed{1 + \frac{Z_3}{Z_2}}.\ \text{ok}$$

$$T_{21} = \frac{I_2}{U_2}\Big|_{I_1=0}$$



$$U_2 = + I_2 \left((Z_3 + Z_1) // Z_2 \right)$$

$$= + I_2 \left(\frac{(Z_3+Z_1) Z_2}{Z_1 + Z_2 + Z_3} \right)$$

$$\frac{I_2}{U_2} = \frac{Z_1 + Z_2 + Z_3}{Z_1 (Z_3 + Z_2)}$$

$$T_{21} = \frac{I_2}{U_2}\Big|_{I_1=0} = \frac{1}{Z_{12}} = \boxed{\frac{Z_1 + Z_2 + Z_3}{Z_1 . Z_2}}$$

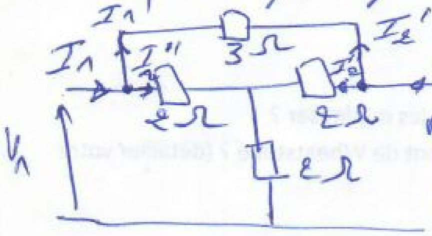
$$I_2'' = \frac{Z_2}{Z_2 + Z_3 + Z_1} I_2 \ ; \ U_2 = Z_1 I_2''$$

$$\frac{U_2}{Z_1} = \frac{Z_2}{Z_2 + Z_3 + Z_1} I_2 \Rightarrow \frac{I_2}{U_2} = \boxed{\frac{Z_3 + Z_2 + Z_1}{Z_1 Z_2}} = T_{21}$$

exo2 :

Calcul de $[Y] = ?$

1- En utilisant le principe d'association de quadripôle.



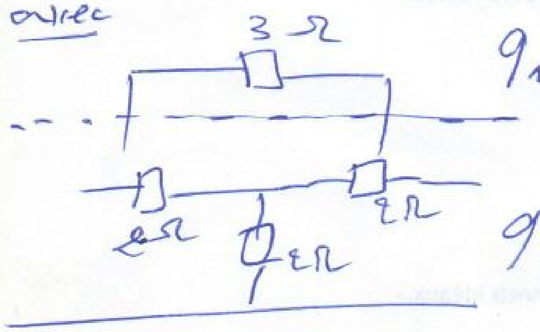
on remarque que :

$I_1 = I_1' + I_1''$; $I_2 = I_2' + I_2''$

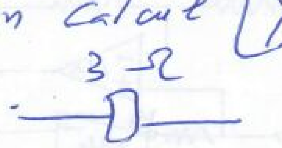
donc on a bien une additivité des courant donc :

$[Y_{eq}] = [Y_1] + [Y_2]$.

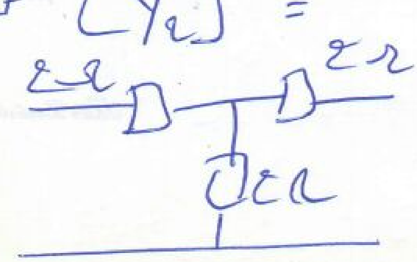
avec



donc on calcul $[Y_1]$ de



et $[Y_2] =$



avec : $Y_1 = \begin{bmatrix} \frac{1}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{1}{3} \end{bmatrix}$

On l'a déjà Calculé dans le 1er exercice :

$$[Y_2] = \begin{bmatrix} \frac{Z_2+Z_3}{\Delta Z} & \frac{-Z_3}{\Delta Z} \\ \frac{-Z_3}{\Delta Z} & \frac{Z_1+Z_3}{\Delta Z} \end{bmatrix}$$

avec $\Delta Z = Z_1Z_2 + Z_1Z_3 + Z_2Z_3$

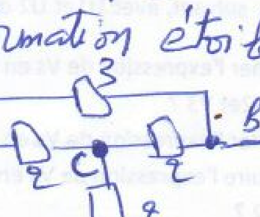
AN : $\Delta Z = 4 + 4 + 4 = 12$.

$$[Y_2] = \begin{bmatrix} \frac{4}{12} & -\frac{2}{12} \\ -\frac{2}{12} & \frac{4}{12} \end{bmatrix}$$

$[Y_{eq}] = [Y_1] + [Y_2]$

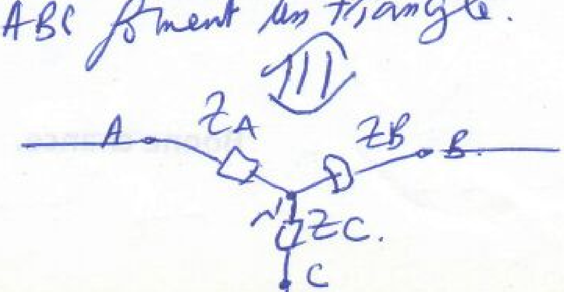
association parallèle.

2- calcul de $[Y]$ en utilisant la transformation étoile-triangle

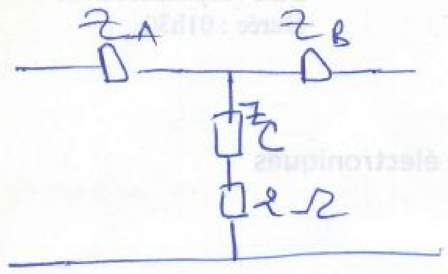


ABC forment un triangle.

$\Updownarrow$



Suite de l'exo 2.



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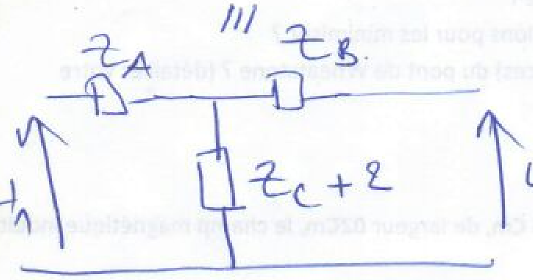
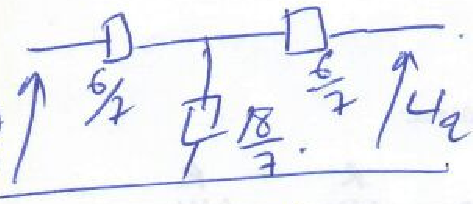


avec $Z_A = \dfrac{3\times 2}{3+2+2} = \dfrac{6}{7}$

$Z_B = \dfrac{3\cdot 2}{3+2+2} = \dfrac{6}{7}$

$Z_C = \dfrac{2\cdot 2}{3+2+2} = \dfrac{4}{7}$

donc le circuit devient.



le premier circuit est devenu sous forme T.

et on a déjà calculé dans l'ex 1 [Y] d'un T.

A.N

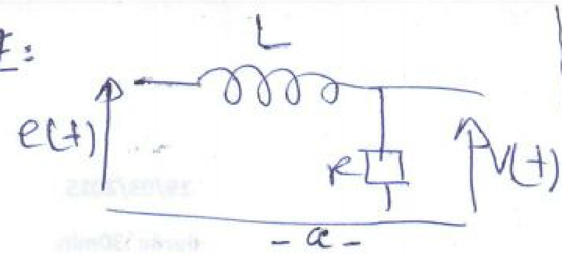
$$[Y] = \begin{bmatrix} \dfrac{\frac{6}{7}+\frac{18}{7}}{\Delta z} & \dfrac{-\frac{18}{7}}{\Delta z} \\ \dfrac{-\frac{18}{7}}{\Delta z} & \dfrac{\frac{6}{7}+\frac{18}{7}}{\Delta z} \end{bmatrix}$$

avec

$$\Delta z = \frac{6}{7}\cdot\frac{18}{7} + \frac{6}{7}\cdot\frac{18}{7} + \frac{6}{7}\cdot\frac{6}{7}$$

il reste de faire les calculs

Exo II :



1) Diviseur de tension :

$$V(t) = \frac{R}{R + jL\omega}\, e(t)$$

$$V(t) = \frac{1}{1 + j\frac{L}{R}\omega}\, e(t)$$

$$\boxed{\frac{V(t)}{e(t)} = \frac{1}{1 + j\frac{L}{R}\omega}}$$

2) $$\left|\frac{V}{e}\right| = \frac{\sqrt{1^2 + 0^2}}{\sqrt{1^2 + \left(\frac{L}{R}\right)^2 \omega^2}} = \frac{1}{\sqrt{1 + \left(\frac{L\omega}{R}\right)^2}}$$

$$\varphi(\omega) = Arg(1) - Arg\left(1 + j\frac{L}{R}\omega\right)$$

$$= Arctg\,\frac{0}{1} - Arctg\left(\frac{L\omega}{R}\right)$$

$$\varphi(\omega) = 0 - Arctg\left(\frac{L\omega}{R}\right)$$

3) Le diagramme de Bode :

$$A_{dB} = 20 \log A = 20 \log \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_0}\right)^2}}$$

$$\boxed{\text{avec } \omega_0 = \frac{R}{L}}$$

on peut poser $\frac{\omega}{\omega_0} = X$.

$$A_{dB} = 20 \log \frac{1}{\sqrt{1 + X^2}}$$

Les asymptotes.

qd :

$X \to 0 \Rightarrow A_{dB} = 0$.

$X \to \infty \Rightarrow A_{dB} = 20 \log \frac{1}{X}$

$\Rightarrow A_{dB} = -20 \log X$.

Soit une chute de -6 dB/octave.

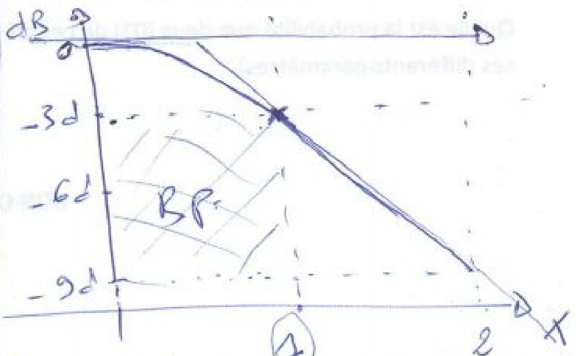
Calculons la fréquence de coupure :

$$A_{dB} = -3\,dB = 20 \log \frac{1}{\sqrt{1 + X^2}}$$

$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1 + X^2}}$$

$$\Leftrightarrow 2 = 1 + X^2 \Rightarrow X^2 = 1 \Rightarrow \boxed{X = 1}$$

$\Rightarrow A_{dB} = -3\,dB$.



$f_c \Rightarrow X = \frac{\omega}{\omega_0} = 1 \Leftrightarrow 2\pi f_c = \frac{R}{L}$.

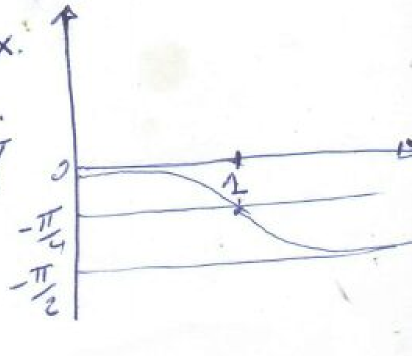
$\Rightarrow \boxed{f_c = \frac{R}{2\pi L}}$ si $f \gg f_c$ l'amplitude est fortement atténuée donc il laisse passer les tensions à petite fréquence $\Rightarrow$ filtre $\Rightarrow$ passe bas.

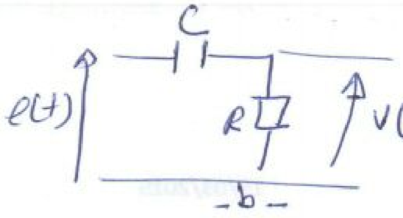
$\varphi(\omega) = -Arctg\,X$.

$X \to 0 \Rightarrow \varphi(\omega) = 0$.

$X \to \infty \Rightarrow \varphi(\omega) = -\frac{\pi}{2}$

$X = 1 \Rightarrow \varphi(\omega) = -\frac{\pi}{4}$





1) Diviseur de tension.

$$v(t) = \frac{R}{R + \frac{1}{j\omega C}} e(t) = \frac{jRC\omega}{jRC\omega + 1} e(t)$$

$$\frac{v(t)}{e(t)} = \frac{jRC\omega}{jRC\omega + 1}.$$

2) $$\left|\frac{V}{e}\right| = \frac{\sqrt{(RC\omega)^2}}{\sqrt{1 + (RC\omega)^2}}.$$

$$\left|\frac{V}{e}\right| = \frac{RC\omega}{\sqrt{1 + (RC\omega)^2}}$$

$$\varphi(\omega) = Arg(jRC\omega) - Arg(1 + jRC\omega)$$

$$= Artg\,\frac{RC\omega}{0} - Artg\left(\frac{RC\omega}{1}\right)$$

$$\varphi(\omega) = \frac{\pi}{2} - Artg(RC\omega)$$

3) Les asymptotes.

$$A_{dB} = 20 \log A = 20 \log \frac{RC\omega}{\sqrt{1 + (RC\omega)^2}}$$

on pose $\omega_0 = \frac{1}{RC}$ et $\frac{\omega}{\omega_0} = x$.

$$A_{dB} = 20 \log \frac{x}{\sqrt{1 + x^2}} = 20 \log \frac{\frac{x}{x}}{\sqrt{\frac{1}{x^2} + 1}}$$

$x \to \infty \Rightarrow A_{dB} = 0.$

$x \to 0 \Rightarrow A_{dB} = 20 \log x.$

Soit un gain de $+6dB$/octave.

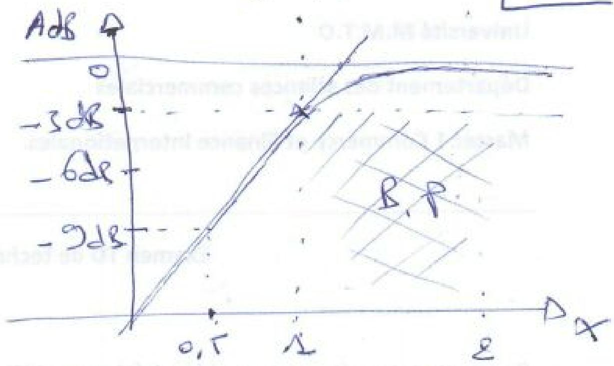
⇒ Calcul de la fréquence de coupure.

$$A_{dB} = -3dB = 20 \log \frac{1}{\sqrt{2}}$$

$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{x}{\sqrt{1 + x^2}}$$

$$\sqrt{2} = \frac{\sqrt{1 + x^2}}{x} \Leftrightarrow 2 = \frac{1 + x^2}{x^2}.$$

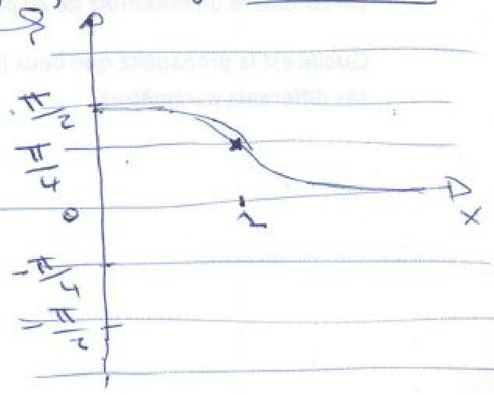
$$\Rightarrow 2x^2 - x^2 = 1 \Leftrightarrow x^2 = 1 \Rightarrow \boxed{x = 1}$$



$$f_c \Rightarrow x = 1 \Leftrightarrow \frac{\omega}{\omega_0} = 1 \Rightarrow 2\pi f_c = \omega_0 = \frac{1}{RC}$$

$$\Rightarrow f_c = \frac{1}{2\pi RC}$$

Si $f \ll f_c \Rightarrow$ l'atténuation de l'amplitude est fortement atténuée ⇒ il laisse passer les grandes fréquences ⇒ filtre passe haut.



$$\varphi(\omega) = \frac{\pi}{2} - Arctg(x) \quad \text{avec } x = RC\omega.$$

Si $x \to 0 \Rightarrow \varphi(\omega) = \frac{\pi}{2} - 0 = \frac{\pi}{2}.$

Si $\to \infty \Rightarrow \varphi(\omega) = \frac{\pi}{2} - \frac{\pi}{2} = 0.$

Si $x = 1 \Rightarrow \varphi(\omega) = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}.$