Solution abrégée du TD N°4 de Physique I.

Exo 1 :

$\vec{F} = -ky\,\vec{i} + kx\,\vec{j}$; $\vec{dl} = dx\,\vec{i} + dy\,\vec{j}$

$\delta W = -ky\,dx + kx\,dy$ (travail élémentaire)

a) le long de Ox, de $A(a,0)$ à $B(-a,0)$:

$y = 0$ et $dy = 0 \Rightarrow \delta W = 0 \Rightarrow \boxed{W_{AB} = 0}$ ($\vec{F} \perp \vec{dl}$, $\forall x$)

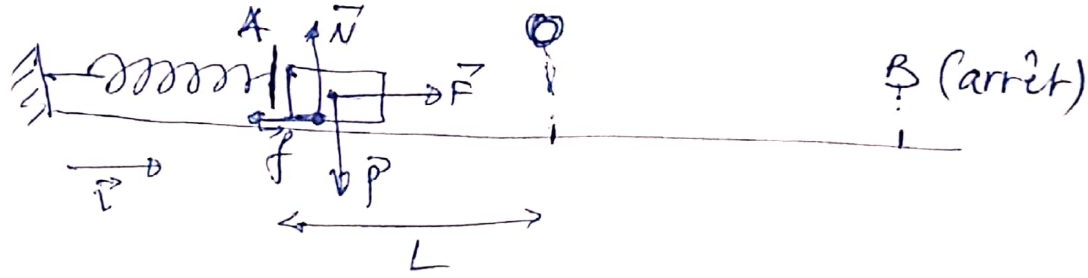
b) Demi-cercle : équations paramétriques :

$$\begin{cases} x = a\cos\theta \\ y = a\sin\theta \end{cases} \Rightarrow \begin{cases} dx = -a\sin\theta\,d\theta \\ dy = a\cos\theta\,d\theta \end{cases} \Rightarrow$$

$$\delta W = -k(a\sin\theta)(-a\sin\theta\,d\theta) + k(a\cos\theta)(a\cos\theta\,d\theta) = ka^2\,d\theta$$

$$\Rightarrow W_{AB} = \int_0^{\pi} ka^2\,d\theta \Rightarrow \boxed{W_{AB} = \pi ka^2}$$

Exo 2



a) le bloc quitte le ressort au point O :
(on néglige la masse du ressort)

Th. de l'énergie cinétique : $\Delta E_c = \sum W$

$$\frac{1}{2} m v_O^2 - \frac{1}{2} m v_A^2 = W_F + W_f \text{ ; avec } \begin{cases} v_A = 0 \\ \vec{F} = -kx\,\vec{i} \quad (x<0) \\ \vec{f} = -\mu_c mg\,\vec{i} \\ \vec{dl} = dx\,\vec{i} \end{cases}$$

$$W_F = \int_{x_A}^{x_0} -kx\,dx = \left[-\frac{1}{2}kx^2\right]_{-L}^{0} = \frac{1}{2}kL^2$$

$$W_f = \int_{x_A}^{x_0} -\mu_c mg\,dx = \left[-\mu_c mg\,x\right]_{x_A}^{x_0} = -\mu_c mg\,\overbrace{(x_0 - x_A)}^{L} = -\mu_c mg L$$

donc : $\frac{1}{2} m v_0^2 = \frac{1}{2} k L^2 - \mu_c m g L \Rightarrow \boxed{v_0 = \sqrt{\frac{k}{m} L^2 - 2\mu_c g L}}$ (2

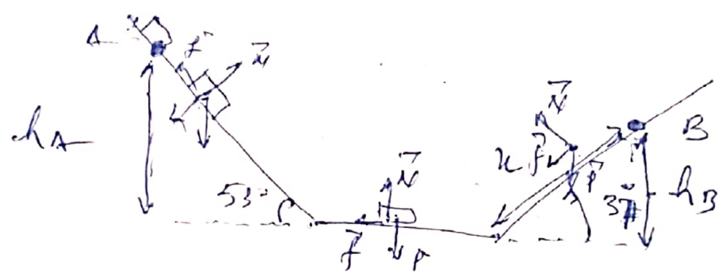
$\boxed{v_0 \simeq 2{,}2 \, m/s}$? $(g = 9{,}8 \, m/s^2)$

b) $\frac{1}{2} m v_B^2 - \frac{1}{2} m v_A^2 = W_F + W_f$ avec $\begin{cases} v_A = 0 \\ v_B = 0 \end{cases}$ on pose $AB = d$

$0 = \frac{1}{2} k L^2 - \mu_c m g d \Rightarrow \boxed{d = \frac{k L^2}{2 \mu_c m g}}$

$\boxed{d \simeq 0{,}82 \, m}$?

Exo 3 :



E^T énergie totale : $\Delta E^T = \sum W_{nmc}$, W_{nmc} travail des forces non-conservatives.

au point A : $v_A = 2 \, m/s$, $m g h_A$
au point B : $v_B = 0$, $m g h_B = m g x \sin 37°$ $(x = ?)$

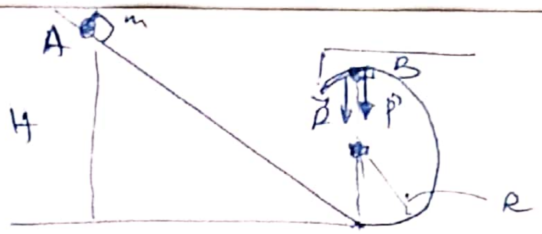
$(m g h_B - \frac{1}{2} m v_A^2 - m g h_A) = -\mu_c m g \cos 53° \frac{h_A}{\sin 53°} - \mu_c m g d$
$- \mu_c m g \cos 37° \cdot x$

$\Rightarrow x (\sin 37° + \mu_c \cos 37°) = \frac{v_A^2}{2g} + h_A - \mu_c h_A \, tg \, 37° - \mu_c d$; $\begin{cases} \cos 53 = \sin 37 \\ \sin 53 = \cos 37 \end{cases}$

$x = \dfrac{\frac{v_A^2}{2g} + h_A - \mu_c h_A \, tg \, 37° - \mu_c d}{\sin 37° + \mu_c \cos 37°} \Rightarrow \boxed{x = 1{,}95 \, m}$?

$(g = 9{,}8 \, m/s^2)$

Exo 4 :



a) $\vec{R}+\vec{P} = \frac{m\vartheta_B^2}{R}\vec{u_N}$ ou $R+mg = m\frac{\vartheta_B^2}{R(\text{rayon})}$

le bloc ne quitte pas le cercle au point B

$\Rightarrow$ la vitesse ϑ_B est assez grande pour que

$R \geq 0$ ou $\boxed{\frac{m\vartheta_B^2}{R(\text{rayon})} - mg \geq 0}$

pas de frottement $\Rightarrow$ énergie totale conservée.

au point A : $\vartheta_A = 0$ $\quad mgH$
au point B : ϑ_B $\quad mg(2R)$

$\Rightarrow mgH = \frac{1}{2}m\vartheta_B^2 + mg\,2R$

$\Rightarrow \boxed{\frac{m\vartheta_B^2}{R} = 2mg\frac{(H-2R)}{R}}$ $\quad 2mg\frac{(H-2R)}{R}$

$\frac{m\vartheta_B^2}{R} \geq mg \Rightarrow 2\cancel{mg}\frac{(H-2R)}{R} \geq \cancel{mg} \Rightarrow H \geq 2R + \frac{R}{2}$

$\Rightarrow H \geq \frac{5}{2}R \Rightarrow \boxed{H_{min} = 5/2\,R}$

b) $H = 2.H_{min} = 5R$.

$R + mg = m\frac{\vartheta_B^2}{R}$, avec $\frac{m\vartheta_B^2}{R} = 2mg\frac{H-2R}{R} = 6mg$

réaction

$\Rightarrow R + mg = 6mg \Rightarrow \boxed{R = 5mg}$

Exo 5 : $\vec{F} = (y^2 - x^2)\vec{i} + 2xy\vec{j}$; $\vec{dl} = dx\,\vec{i} + dy\,\vec{j}$

a) $\text{rot}\,\vec{F} = \vec{\nabla} \wedge \vec{F} = 0 \Rightarrow \exists\, E_p \;/\; \vec{F} = -\vec{\nabla} E_p$

$$\Rightarrow \begin{cases} y^2 - x^2 = -\dfrac{\partial E_p}{\partial x} & (1) \\ 2xy = -\dfrac{\partial E_p}{\partial y} & (2) \end{cases}$$

(1) $\Rightarrow E_p = \frac{1}{3}x^3 - xy^2 + f(y)$; $f(y)$ fonction de y seulement, à déterminer

E_p vérifie (2) $\Rightarrow -2xy + \dfrac{df}{dy} = -2xy \Rightarrow \dfrac{df}{dy} = 0$

$\Rightarrow f(y) = C$ (constante numérique)

pour $\begin{cases} x = 0 \\ y = 0 \end{cases}$ $E_p = 0 \Rightarrow C = 0$

$$\boxed{E_p = \frac{1}{3}x^3 - xy^2}$$

b)

$B(2,1)$; $E_p^B = \frac{8}{3} - 2 = \frac{2}{3}$ J ?

$A(1,2)$; $E_p^A = \frac{1}{3} - 4 = -\frac{11}{3}$ J ?

donc $W_{AB} = -\Delta E_p = E_p^A - E_p^B = -\frac{11}{3} - \frac{2}{3} = -\frac{13}{3}$ J