Corrigé TD N°2

Exercice 1

$$\beta = \omega t = 2\pi 60 \times 11.9 \times 10^{-3} = 4.49rd$$

$$\beta = \frac{180}{3.14} \times 4.49 = 257°$$

La valeur max de la tension :

$$\beta = 257° \quad V_{\max} = 339V$$

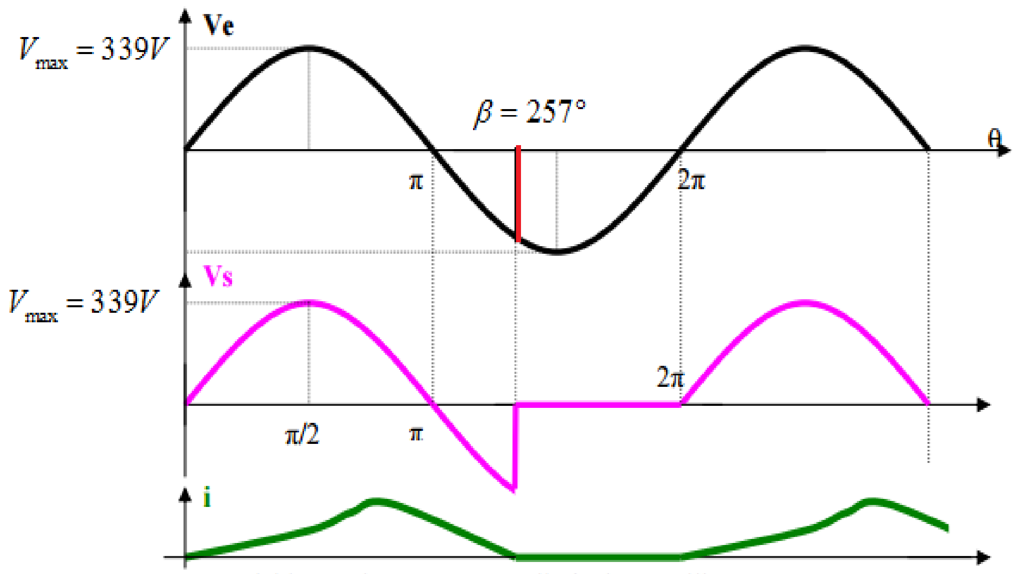
$$V_{\max} = V \times \sqrt{2} = 240 \times \sqrt{2} = 339V$$

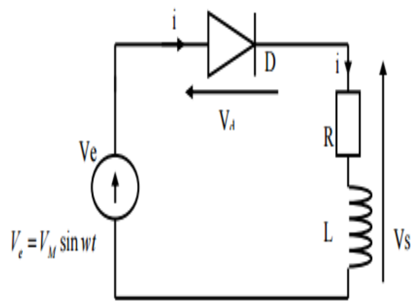
$$Z = \sqrt{R^2 + L^2\omega^2} = 40.6$$

$$\varphi = artg\left(\frac{L\omega}{R}\right) = 1.19rd = 68.3°$$

$$\begin{cases} 0 < \omega t < \beta \Rightarrow D\ passante \Rightarrow V_d \approx 0\ et\ v_s = v_e et \\ i_s = \frac{V_{e\max}}{Z}\left(\sin(\omega t - \varphi) + \sin(\varphi) e^{-\frac{\omega t}{\tan\varphi}}\right) = \frac{339}{40.6}\left(\sin(\omega t - 68.3°) + 0.93 e^{-\frac{\omega t}{25}}\right) = 7.76 e^{-\frac{\omega t}{25}} + 8.35\sin(\omega t - 68.3°) \\ \beta < \omega t < 2\pi \Rightarrow D bloquée \Rightarrow i_s = 0, V_d = v_e\ et\ v_s = 0 \end{cases}$$

Avant de représenter les formes d'onde, on doit calculer l'angle d'extinction du courant :





Charge RL

$$V_{smoy} = \frac{V_{\max}}{\pi}\left(\frac{1-\cos\beta}{2}\right) = \frac{339}{\pi}\left(\frac{1-\cos 257}{2}\right) = 66V$$

$$I_{smoy} = \frac{V_{smoy}}{R} = \frac{66}{15} = 4.4A$$

Charge R

$$V_{smoy} = \frac{V_{\max}}{\pi} = \frac{339}{\pi} = 108V$$

$$I_{smoy} = \frac{V_{smoy}}{R} = \frac{108}{15} = 7.2A$$

$$V_{seff} = \frac{U_{\max}}{2} = 169V$$

$$I_{seff} = \frac{V_{seff}}{R} = \frac{169}{15} = 11.26A$$

$$V_{seff}=\frac{U_{max}}{2}\sqrt{\frac{2\beta-\sin 2\beta}{2\pi}}=\frac{339}{2}\sqrt{\frac{2\times257-\sin 2\times257}{2\times180}}=\frac{339}{2}\sqrt{\frac{514-0.44}{360}}=202V$$

$$I_{seff}=\frac{V_{seff}}{Z},Z=\sqrt{R^2+(L\omega)^2}=\sqrt{15^2+(0.1\times2\pi60)^2}=40.6\Omega$$

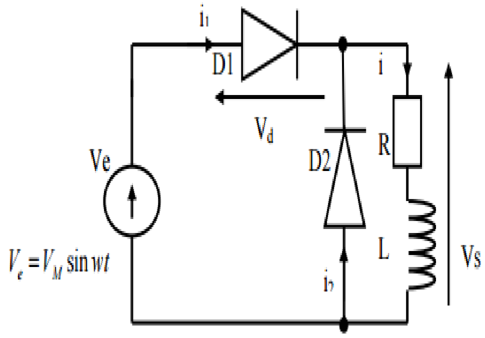
$$I_{seff}=\frac{202}{40.6}=4.97A$$

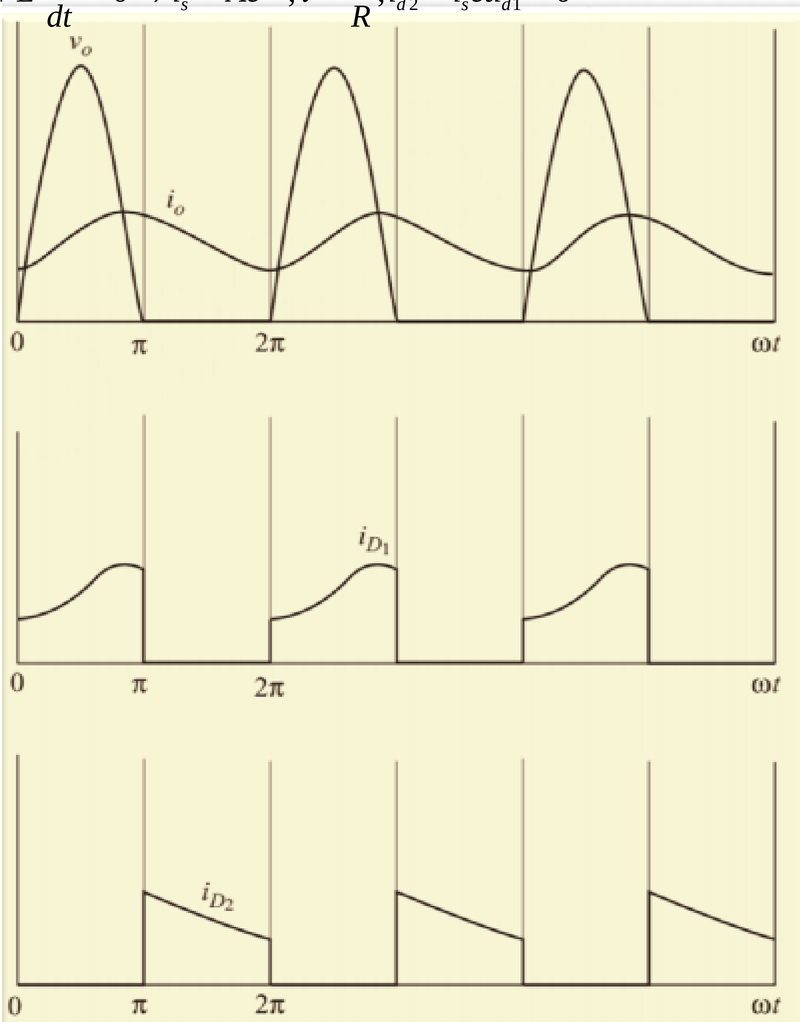
$$F_{vs}=\frac{V_{seff}}{V_{smoy}}=\frac{202}{66}=3\Rightarrow\tau=\sqrt{F^2-1}=2.82$$

$$F_{is}=\frac{I_{seff}}{I_{smoy}}=\frac{4.97}{4.4}=1.13\Rightarrow\tau=\sqrt{F^2-1}=0.56$$

Charge RL avec diode de roue lire

$$\begin{cases}0<\omega t<\pi\Rightarrow D1\,passante\Rightarrow V_{d1}\approx0, D2\,bloquée\Rightarrow V_{d2}=-v_e et\, v_s=v_e\\ i_s=\frac{V_{emax}}{Z}\left(\sin(\omega t-\varphi)+\sin(\varphi)e^{-\frac{\omega t}{\omega\tau}}\right), i_{d1}=i_s et i_{d2}=0\\ \pi<\omega t<2\pi\Rightarrow D1bloquée\Rightarrow V_{d1}=v_e, D21\,passante\Rightarrow V_{d2}=0 et\, v_s=0\\ Ri_s+L\frac{di_s}{dt}=0\Rightarrow i_s=Ae^{-\frac{t}{\tau}},\tau=\frac{L}{R}, i_{d2}=i_s et i_{d1}=0\end{cases}$$





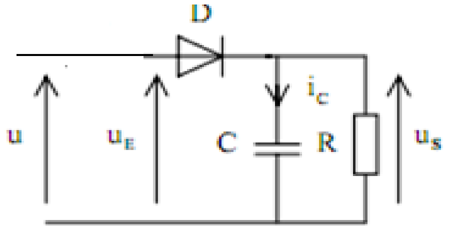
La valeur moyenne de la tension de charge :

$$V_{smoy}=\frac{V_{max}}{\pi}=\frac{339}{\pi}=108V$$

$$F_{vs}=\frac{V_{seff}}{V_{smoy}}=\frac{202}{108}=1.87\Rightarrow\tau=\sqrt{F^2-1}=1.58$$

Exercice 2

Fonctionnement : condensateur déchargé initialement :



$u_e > 0 \Rightarrow D\, passante \Rightarrow V_d \approx 0\, et u_s = U_{e\max} \sin \omega t,$

$i_s = \dfrac{U_{e\max}}{R} \sin \omega t$

$\Rightarrow i_c = C\dfrac{du_e}{dt} = C\omega U_{e\max} \cos \omega t,$

$i_D = i_c + i_s = C\omega U_{e\max} \cos \omega t + \dfrac{U_{e\max}}{R} \sin \omega t = \dfrac{U_{e\max}}{R}\left(RC\omega \cos \omega t + \sin \omega t\right)$

$a\cos \omega t + b \sin \omega t = \sqrt{a^2+b^2}\cos\left(\omega t - \tan^{-1}\left(\dfrac{b}{a}\right)\right) \Rightarrow i_D = \dfrac{U_{e\max}}{R}\sqrt{1+R^2C^2\omega^2}\cos\left(\omega t - \varphi\right)$

$,\varphi = \tan^{-1}\left(\dfrac{1}{RC\omega}\right)$

$i_D = 0 \Rightarrow \omega t_0 - \varphi = \dfrac{\pi}{2} \Rightarrow \omega t_0 = \beta = \dfrac{\pi}{2} + \varphi$

Le courant iD passe par zéro à l'instant ωt_0

$t = \omega t_0, i_D = 0 \Rightarrow D\, bloquée \Rightarrow i_D = 0, u_s = u_e\left(\omega t_0\right)$

$\dfrac{u_s}{R} + C\dfrac{du_s}{dt} = 0 \Rightarrow Ck + \dfrac{1}{R} = 0 \Rightarrow k = -\dfrac{1}{RC} \Rightarrow u_s = C_{const} e^{-\frac{t}{RC}}$

$u_s\left(\omega t_0 = \beta\right) = U_{e\max}\sin\left(\beta\right) = U_{e\max}\sin\left(\dfrac{\pi}{2}+\varphi\right) = U_{e\max}\cos\left(\varphi\right)$

$u_s\left(\omega t_0 = \beta\right) = U_{e\max}\cos\left(\varphi\right) = C_{const} e^{-\frac{\omega t_0}{\omega RC}} = C_{const} e^{-\frac{\beta}{\omega RC}} \Rightarrow C_{const} = U_{e\max}\cos\left(\varphi\right) e^{\frac{\beta}{\omega RC}}$

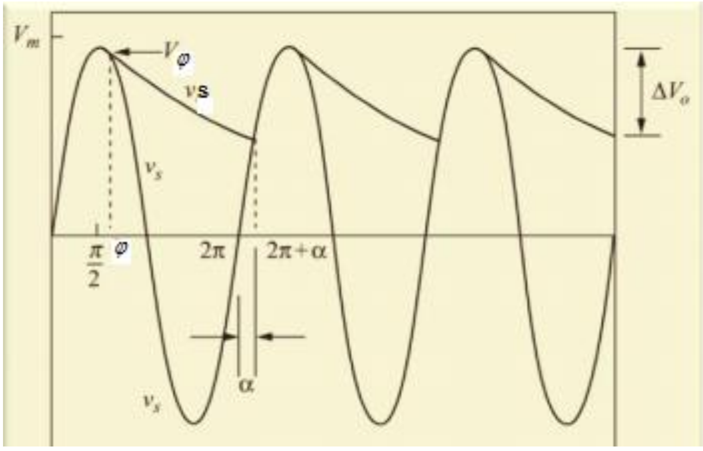
$u_s = U_{e\max}\cos\left(\varphi\right) e^{\frac{\beta}{\omega RC}} e^{-\frac{t}{RC}} = U_{e\max}\cos\left(\varphi\right) e^{\frac{\beta}{\omega RC}} e^{-\frac{\omega t}{\omega RC}} = U_{e\max}\cos\left(\varphi\right) e^{-(\omega t-\beta)tg\varphi}, tg\varphi = \dfrac{1}{\omega RC}$

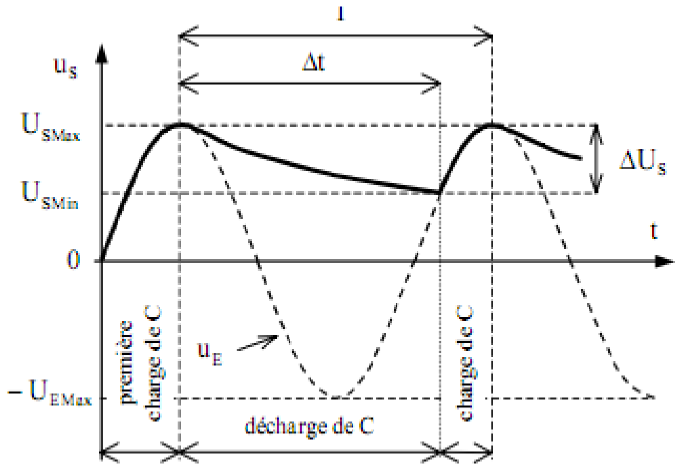
$i_C = i_R = \dfrac{u_s}{R} = \dfrac{U_{e\max}\cos\left(\varphi\right)}{R} e^{-(\omega t-\beta)tg\varphi}$

Donc les équations de la tension et le courant :

$$u_s = \begin{cases} U_{e\max}\sin\omega t & 0<\omega t<\beta \\ U_{e\max}\cos(\varphi)e^{-(\omega t-\beta)tg\varphi} & \beta<\omega t<2\pi+\alpha \end{cases}$$

$$i_s = \begin{cases} \dfrac{U_{e\max}}{R}\sin\omega t & 0<\omega t<\beta \\ \dfrac{U_{e\max}\cos(\varphi)}{R}e^{-(\omega t-\beta)tg\varphi} & \beta<\omega t<2\pi+\alpha \end{cases}$$



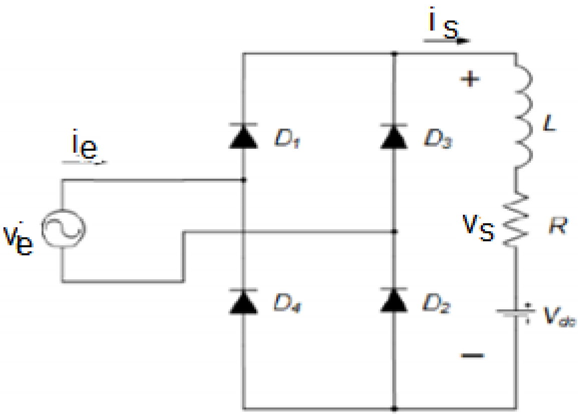


L'angle α est donné par l'équation suivante :

$$\Delta u_s = U_{e\max} - U_{e\max}\sin\alpha = U_{e\max}(1-\sin\alpha) \Rightarrow \alpha = \sin^{-1}\left(1-\frac{\Delta u_s}{U_{e\max}}\right)$$

Exercice 3

$$u_s = \begin{cases} U_{e\max}\sin\omega t & 0<\omega t<\pi \\ -U_{e\max}\sin\omega t & \pi<\omega t<2\pi \end{cases}$$



$$U_{smoy} = \frac{1}{T}\int_0^T i(t)\,dt = \frac{1}{\pi}\int_0^{\pi} V_{\max}\sin\theta\, d\theta = \frac{V_{\max}}{\pi}\left|-\cos\theta\right|_0^{\pi}$$

$$= -\frac{V_{\max}}{\pi}\left[\cos\pi - \cos 0\right] = \frac{2V_{\max}}{\pi} = 216V$$

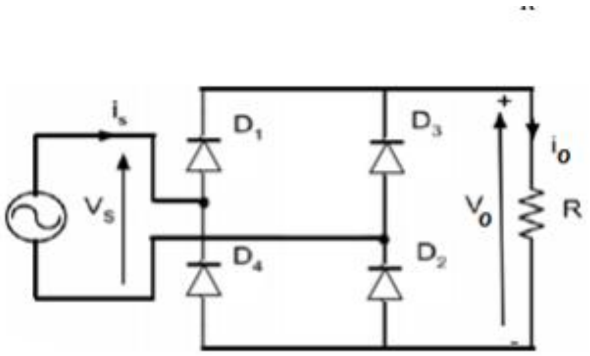
$$U_{seff} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt} = \sqrt{\frac{1}{\pi}\int_0^{\pi} V_{\max}^2\sin^2\theta\, d\theta} = \sqrt{\frac{V_{\max}^2}{\pi}\int_0^{\pi}\frac{1-\cos 2\theta}{2}\, d\theta}$$

$$= \sqrt{\frac{V_{\max}^2}{2\pi}\int_0^{\pi}(1-\cos 2\theta)\, d\theta} = \sqrt{\frac{V_{\max}^2}{2\pi}\left|\theta + \frac{1}{2}\sin 2\theta\right|_0^{\pi}}$$

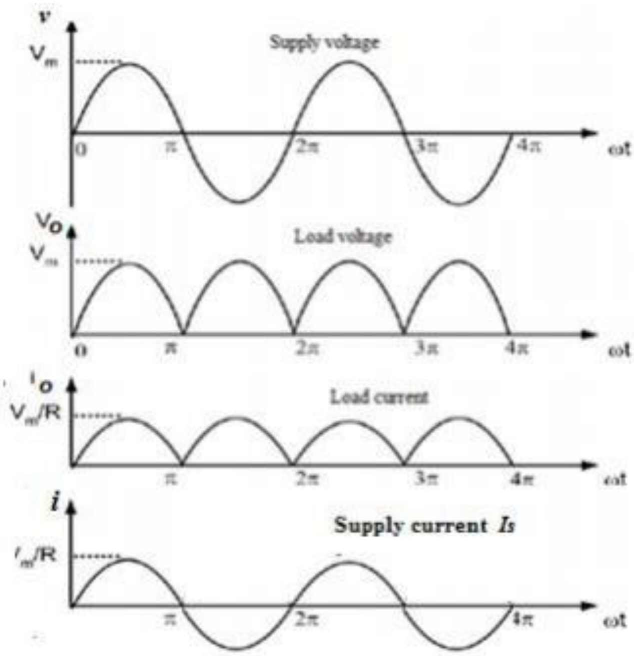
$$= \sqrt{\frac{V_{\max}^2}{2\pi}\left(\pi + \frac{1}{2}\sin 2\pi\right) - \left(0 + \frac{1}{2}\sin 0\right)}$$

$$= \sqrt{\frac{V_{\max}^2}{2\pi}(\pi)} = \frac{V_{\max}}{\sqrt{2}} = 240V$$

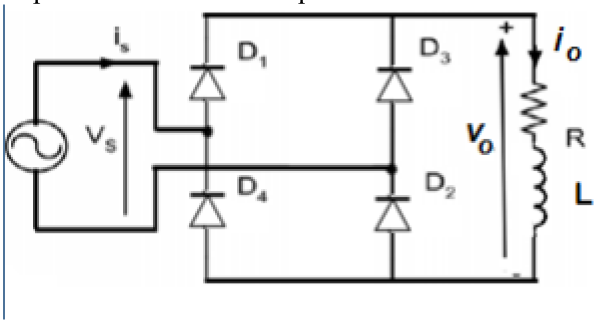
$$F_{vs} = \frac{V_{seff}}{V_{smoy}} = \frac{240}{216} = 1.11 \Rightarrow \tau = \sqrt{F^2 - 1} = 0.48$$

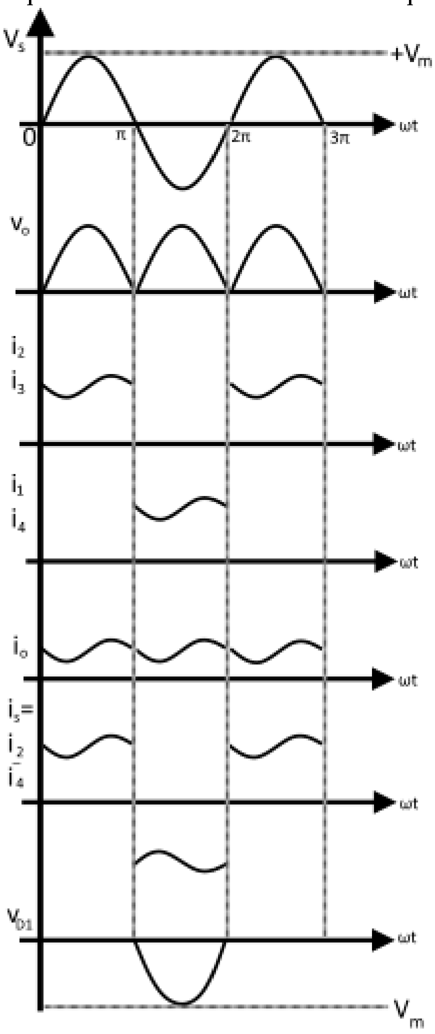


(a) Circuit.



(b) Waveforms





Le courant de charge :

$$i_s = \begin{cases} i_e & 0 < \omega t < \pi \\ -i_e & \pi < \omega t < 2\pi \end{cases}$$

$0 < \omega t < \pi$

$$V_{e\max}\sin\omega t = Ri_s + L\frac{di_s}{dt} + E \Rightarrow V_{e\max}\sin\omega t - E = Ri_s + L\frac{di_s}{dt}$$

Le courant dans la charge est la somme d'une composante libre i_{sl} caractérisant le régime transitoire caractérisant le régime transitoire et d'une composante forcée i_{sf} .

$$i_s = i_{sl} + i_{sf}$$

La composante i_{cl} est la solution de l'équation sans second membre :

$$Ri_s + L\frac{di_s}{dt} = 0 \Rightarrow i_s = Ae^{-\frac{t}{\tau}}, \tau = \frac{L}{R}$$

La composante i_{cf} est la solution de l'équation avec second membre :

On pose : $i_{sf} = A\cos\omega t + B\sin\omega t + C \Rightarrow i'_{sf} = -A\omega\sin\omega t + B\omega\cos\omega t$

$$V_{e\max}\sin\omega t - E = R\left(A\cos\omega t + B\sin\omega t + C\right) + L\left(-A\omega\sin\omega t + B\omega\cos\omega t\right) = \left(RA + LB\omega\right)\cos\omega t + \left(B - AL\omega\right)\sin\omega t$$

$$\Rightarrow \begin{cases} RA + LB\omega = 0 \\ RB - AL\omega = \widehat{V}_e \\ RC = -E \end{cases} \Rightarrow \begin{cases} A = -\dfrac{LB\omega}{R} \\ RB + \dfrac{L^2B\omega 2}{R} = \widehat{V}_e \\ C = \dfrac{-E}{R} \end{cases} \Rightarrow \begin{cases} A = -\dfrac{L\omega}{R^2 + L^2\omega^2}\widehat{V}_e \\ B = \dfrac{R}{R^2 + L^2\omega^2}\widehat{V}_e \\ C = \dfrac{-E}{R} \end{cases}$$

$$i_{sf} = -\frac{L\omega}{R^2 + L^2\omega^2}\widehat{V}_e\cos\omega t + \frac{R}{R^2 + L^2\omega^2}\widehat{V}_e\sin\omega t - \frac{E}{R}$$

$$i_{sf} = A\cos\omega t + B\sin\omega t - \frac{E}{R} = \sqrt{A^2 + B^2}\sin\left(\omega t - \varphi\right) - \frac{E}{R}, \varphi = artg\left(\frac{-A}{B}\right)$$

$$\widehat{V}_e\sin\omega t - E = Ri_s + L\frac{di_s}{dt} \Rightarrow i_{sf} = \frac{\widehat{V}_e}{Z}\sin\left(\omega t - \varphi\right) - \frac{E}{R}$$

$$\varphi = artg\left(\frac{L\omega}{R}\right), Z = \sqrt{R^2 + L^2\omega^2}$$

La solution générale est alors :

$$i_s = Ae^{-\frac{R}{L}t} + \frac{\widehat{V}_e}{\sqrt{R^2 + L^2\omega^2}}\sin\left(\omega t - artg\left(\frac{L\omega}{R}\right)\right) - \frac{E}{R}$$

La constante est déterminée à partie des conditions initiales. En effet à $t=0$ le courant dans la charge est nul

$$i_s(\omega t) = Ae^{-\frac{\omega t}{\tan\varphi}} + \frac{\widehat{V}_e}{\sqrt{R^2+L^2\omega^2}}\sin\left(\omega t - artg\left(\frac{L\omega}{R}\right)\right) - \frac{E}{R}$$

$$i_s\big|_{\omega t=0} = i_s\big|_{\omega t=\pi} \Rightarrow \begin{cases} i_s\big|_{\omega t=0} = A + \frac{\widehat{V}_e}{\sqrt{R^2+L^2\omega^2}}\sin\left(-artg\left(\frac{L\omega}{R}\right)\right) - \frac{E}{R} \\ i_s\big|_{\omega t=\pi} = Ae^{-\frac{\pi}{\tan\varphi}} + \frac{\widehat{V}_e}{\sqrt{R^2+L^2\omega^2}}\sin\left(\pi - artg\left(\frac{L\omega}{R}\right)\right) - \frac{E}{R} \end{cases}$$

$$A + \frac{\widehat{V}_e}{\sqrt{R^2+L^2\omega^2}}\sin\left(-artg\left(\frac{L\omega}{R}\right)\right) - \frac{E}{R} = Ae^{-\frac{\pi}{\tan\varphi}} + \frac{\widehat{V}_e}{\sqrt{R^2+L^2\omega^2}}\sin\left(\pi - artg\left(\frac{L\omega}{R}\right)\right) - \frac{E}{R}$$

$$\Rightarrow A\left(1 - e^{-\frac{\pi}{\tan\varphi}}\right) = \frac{\widehat{V}_e}{\sqrt{R^2+L^2\omega^2}}\left[\sin(\varphi) + \sin(\varphi)\right]$$

$$A = \frac{\frac{\widehat{V}_e}{\sqrt{R^2+L^2\omega^2}}\left[2\sin(\varphi)\right]}{\left(1 - e^{-\frac{\pi}{\tan\varphi}}\right)}$$

Finalement :

$$i_s = \frac{\frac{\widehat{V}_e}{\sqrt{R^2+L^2\omega^2}}\left[2\sin(\varphi)\right]}{\left(1-e^{-\frac{\pi}{\tan\varphi}}\right)} e^{-\frac{\omega t}{\tan\varphi}} + \frac{\widehat{V}_e}{\sqrt{R^2+L^2\omega^2}}\sin\left(\omega t - artg\left(\frac{L\omega}{R}\right)\right) - \frac{E}{R}$$

$$i_s = \frac{\widehat{V}_e}{\sqrt{R^2+L^2\omega^2}}\left(\sin\left(\omega t - artg\left(\frac{L\omega}{R}\right)\right) - \sin\left(-artg\left(\frac{L\omega}{R}\right)\right)e^{-\frac{R}{L}t}\right) - \frac{E}{R}\left(1-e^{-\frac{R}{L}t}\right)$$

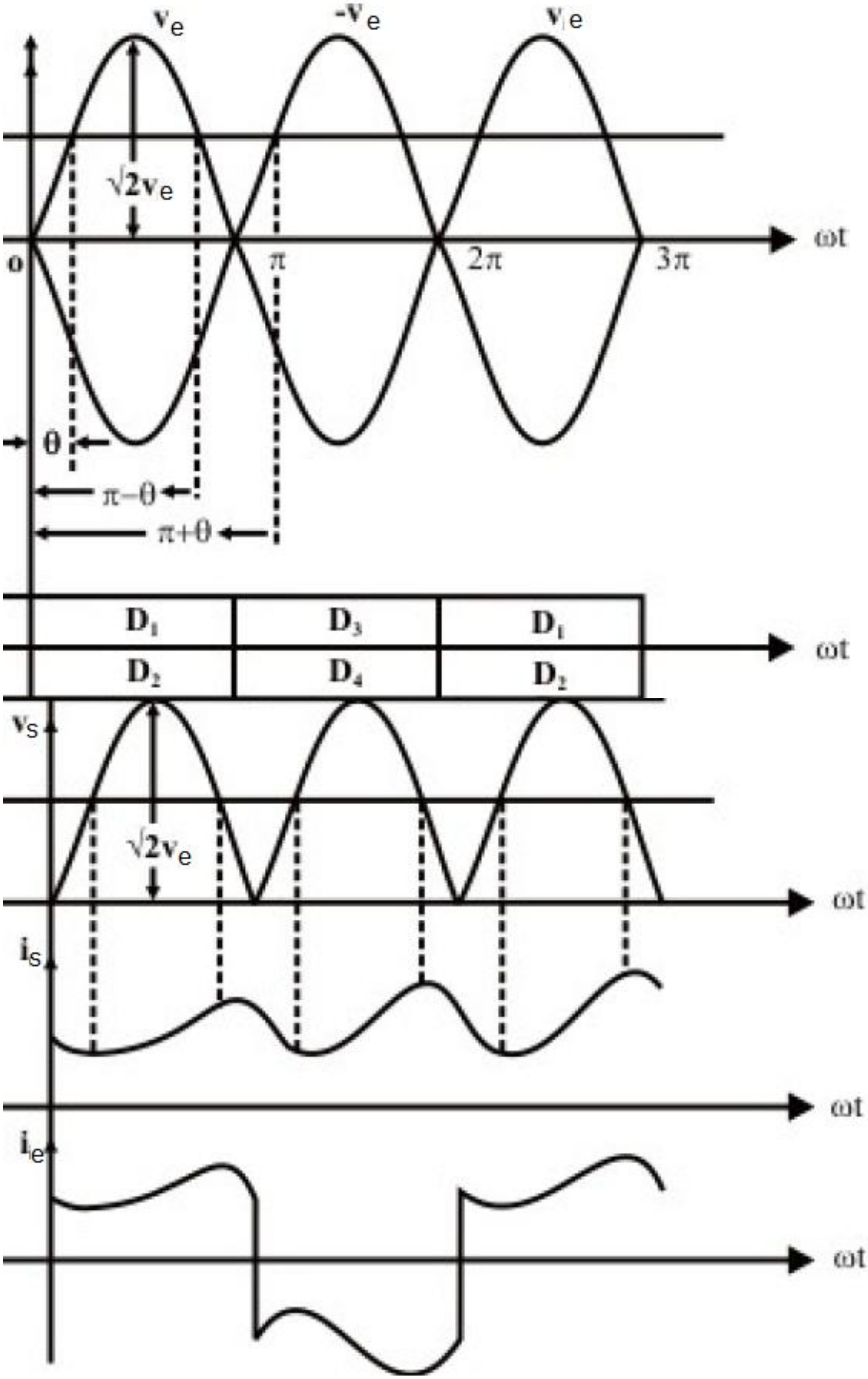
$$i_s(\omega t) = \frac{\widehat{V}_e}{Z}\left(\frac{2\sin(\varphi)}{\left(1-e^{-\frac{\pi}{\tan\varphi}}\right)} e^{-\frac{\omega t}{\tan\varphi}} + \sin(\omega t - \varphi) - \frac{\sin\theta}{\frac{R}{Z}}\right)$$

$$E = \widehat{V}_e \sin\theta \Rightarrow \sin\theta = \frac{E}{\widehat{V}_e}$$

$$Z = \sqrt{R^2+L^2\omega^2} \Rightarrow Z = R\sqrt{1+\frac{L^2\omega^2}{R^2}} = R\sqrt{1+\tan^2\varphi} \Rightarrow \frac{R}{Z} = \frac{1}{\sqrt{1+\tan^2\varphi}} = \cos\varphi$$

$$\cos^2\varphi = \frac{1}{1+\tan^2\varphi}$$

$$i_s(\omega t) = \frac{\widehat{V}_e}{Z}\left(\frac{2\sin(\varphi)}{\left(1-e^{-\frac{\pi}{\tan\varphi}}\right)} e^{-\frac{\omega t}{\tan\varphi}} + \sin(\omega t - \varphi) - \frac{\sin\theta}{\cos\varphi}\right)$$



v e
-v e
v e
√2v e
o
π
2π
3π
ωt
θ
π−θ
π+θ
D1
D3
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D4
D2
ωt
vS
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iS
ωt
ie
ωt

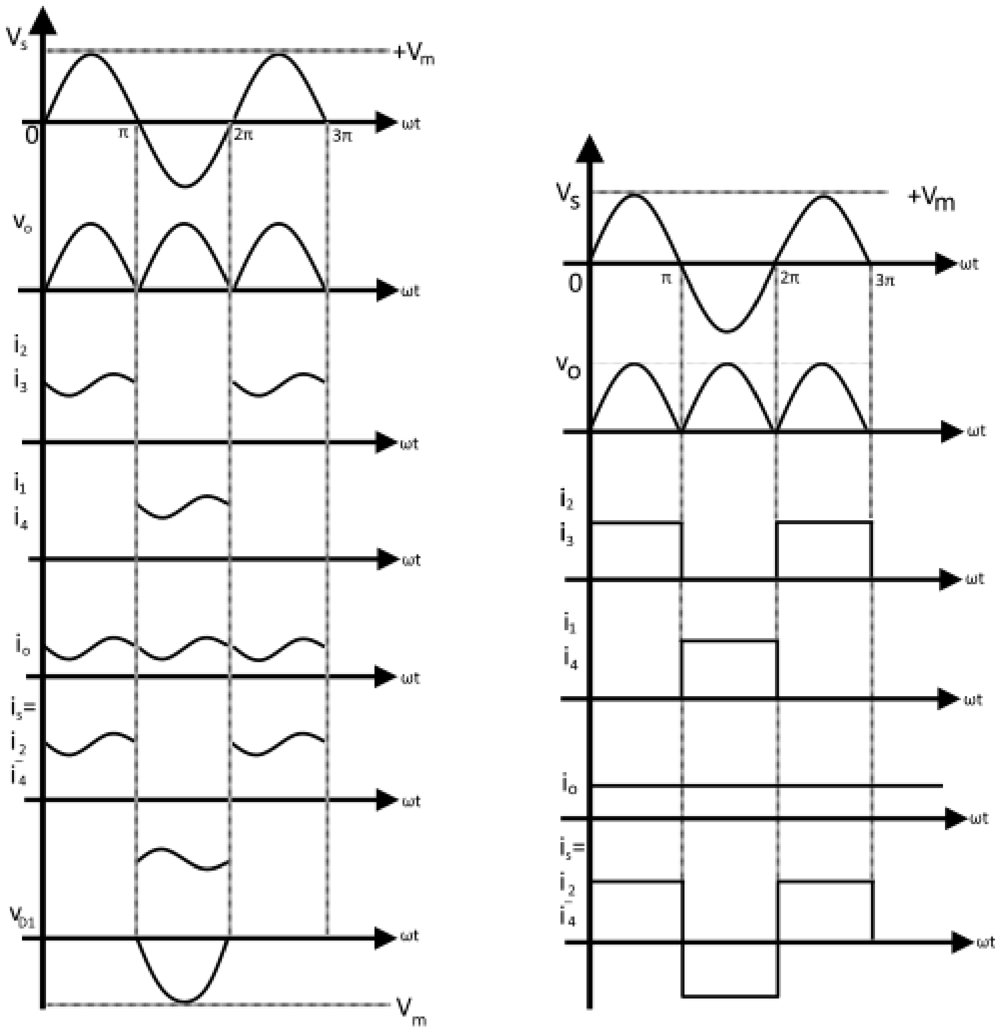
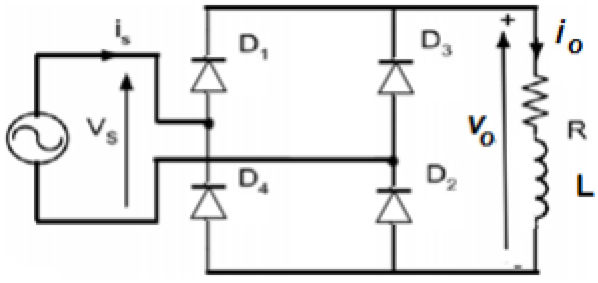
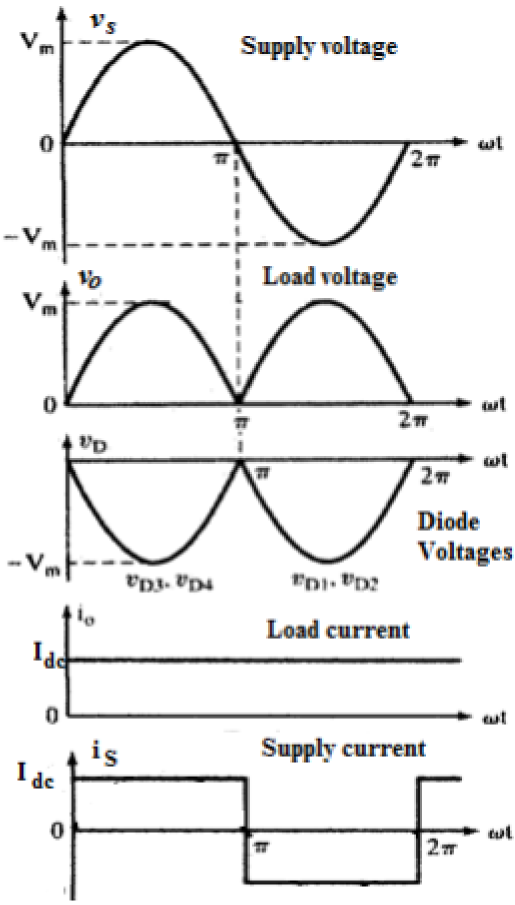


Figure 4: Full-wave Bridge Rectifier with Inductive Load (a) Waveforms for (L = R) (b) Waveform for (L >> R)

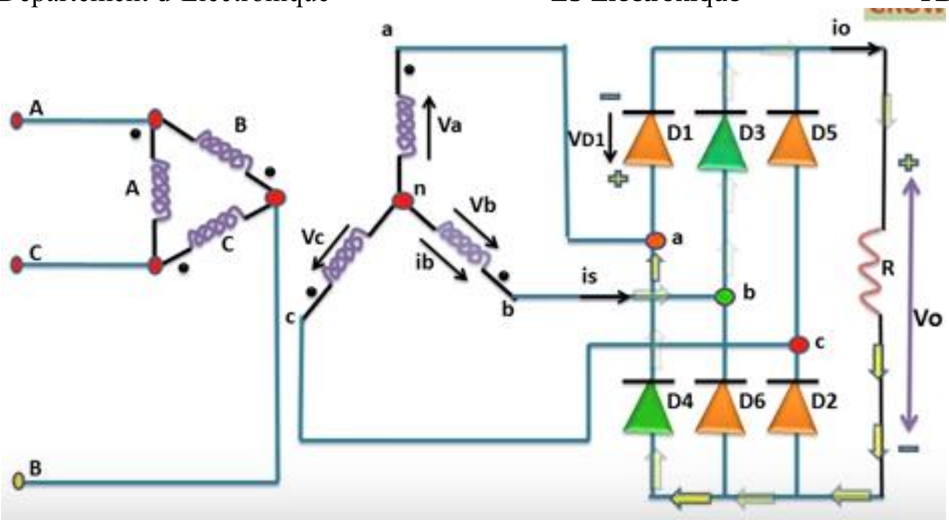


(a) Circuit.



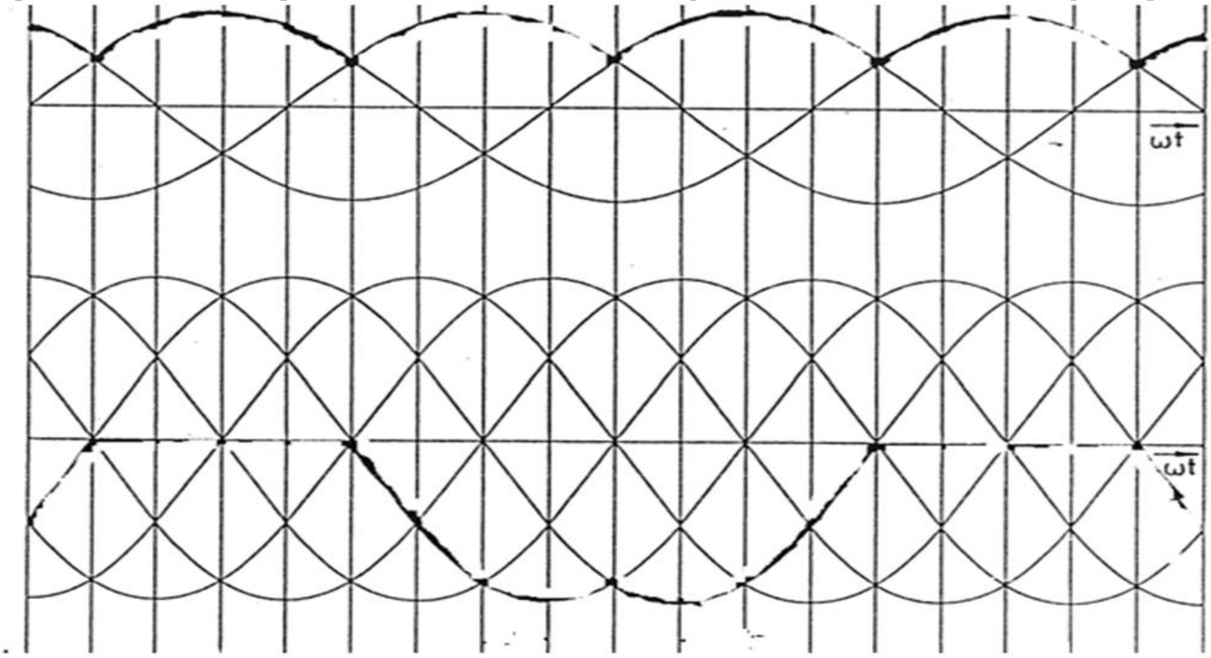
(b) Waveforms.

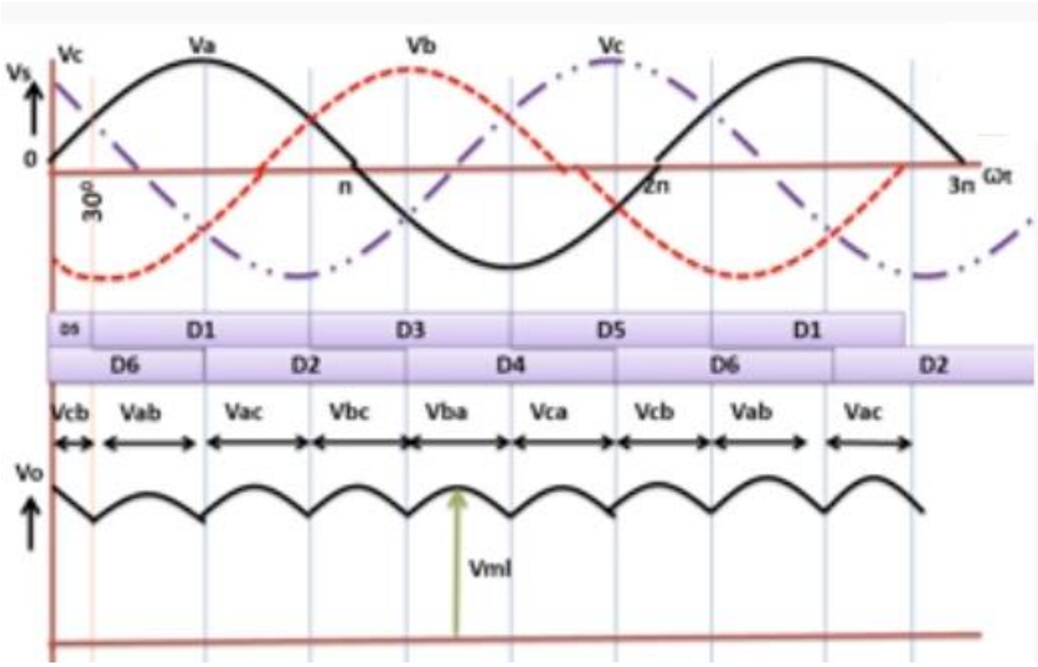
Exercice 3



Intervalles	$\left[\frac{\pi}{6},\frac{\pi}{2}\right]$	$\left[\frac{\pi}{2},\frac{5\pi}{6}\right]$	$\left[\frac{5\pi}{6},\frac{7\pi}{6}\right]$	$\left[\frac{7\pi}{6},\frac{9\pi}{6}\right]$	$\left[\frac{9\pi}{6},\frac{11\pi}{6}\right]$	$\left[\frac{11\pi}{6},\frac{13\pi}{6}\right]$
Diodes	**D16**	**D12**	**D32**	**D34**	**D54**	**D56**
Tension v_0	U_{ab}	U_{ac}	U_{bc}	U_{ba}	U_{ca}	U_{cb}
Courant i_0	$\frac{U_{ab}}{R}$	$\frac{U_{ac}}{R}$	$\frac{U_{bc}}{R}$	$\frac{U_{ba}}{R}$	$\frac{U_{ca}}{R}$	$\frac{U_{cb}}{R}$
Tension v_{D1}	**0**	**0**	U_{ab}	U_{ab}	U_{ac}	U_{ac}
Courant i_{D1}	$\frac{U_{ab}}{R}$	$\frac{U_{ac}}{R}$	**0**	**0**	**0**	**0**
Courant i_a	$\frac{U_{ab}}{R}$	$\frac{U_{ac}}{R}$	**0**	$-\frac{U_{ba}}{R}$	$-\frac{U_{ca}}{R}$	**0**

Les formes d'ondes :


ωt
ωt


Vs
Vc
Va
Vb
Vc
0
30°
π
2π
3π
ωt
D5
D1
D3
D5
D1
D6
D2
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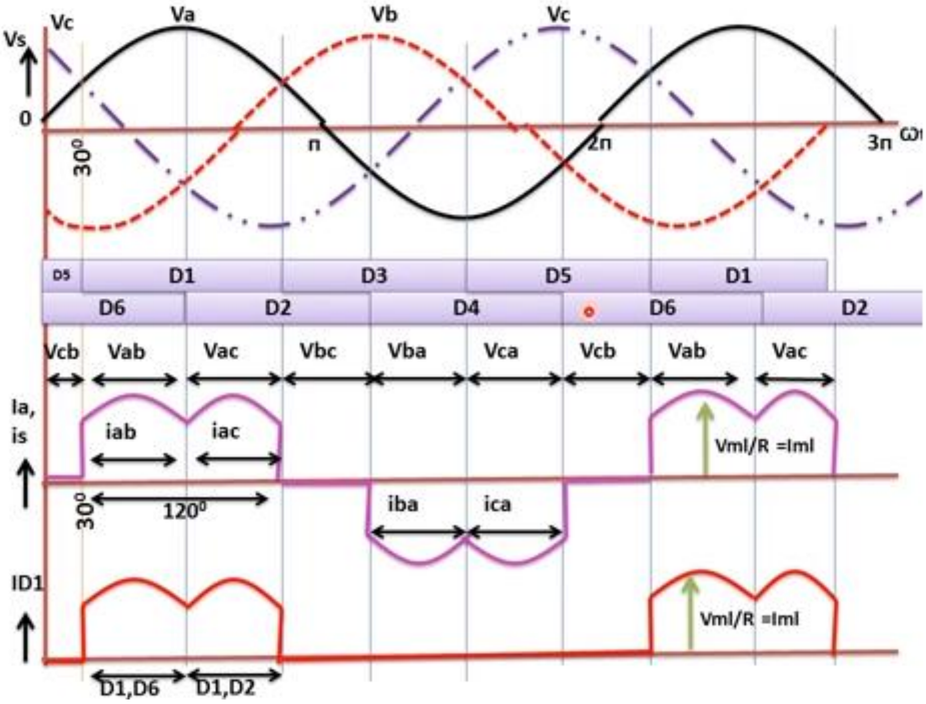
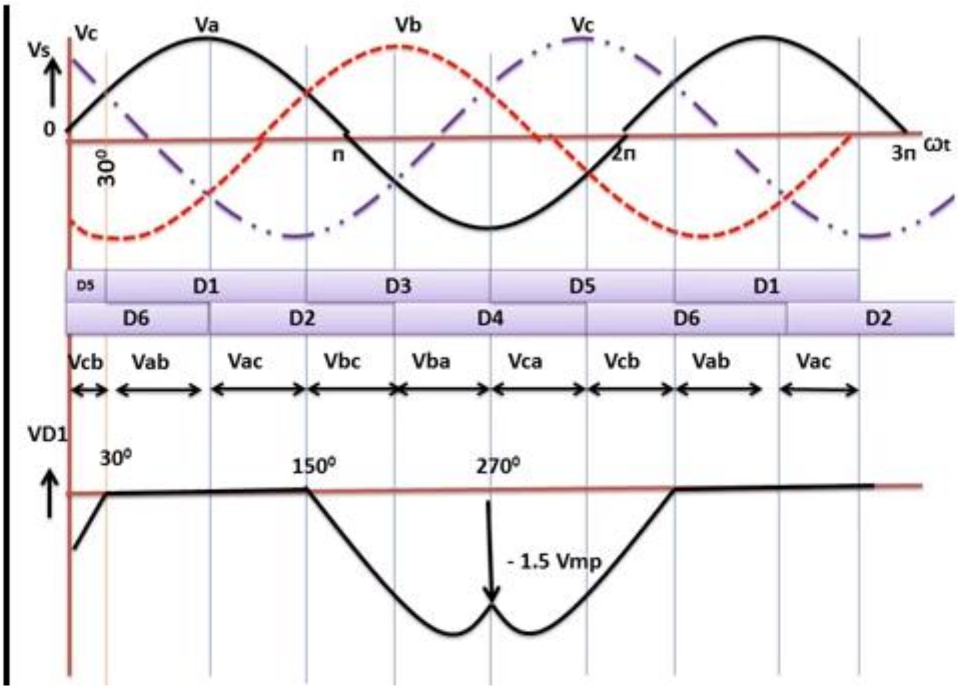
Vab
Vac
Vo
Vml


Vs
Vc
Va
Vb
Vc
0
30⁰
π
2π
3π
ωt
D5
D1
D3
D5
D1
D6
D2
D4
D6
D2
Vcb
Vab
Vac
Vbc
Vba
Vca
Vcb
Vab
Vac
Ia,
is
iab
iac
Vml/R =Iml
30⁰
120⁰
iba
ica
ID1
Vml/R =Iml
D1,D6
D1,D2


Vs
Vc
Va
Vb
Vc
0
30⁰
π
2π
3π
ωt
D5
D1
D3
D5
D1
D6
D2
D4
D6
D2
Vcb
Vab
Vac
Vbc
Vba
Vca
Vcb
Vab
Vac
VD1
30⁰
150⁰
270⁰
- 1.5 Vmp

$$V_{smoy}=\frac{1}{\frac{2\pi}{6}}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\sqrt{3}V_{Max}\sin\left(\theta+\frac{\pi}{6}\right)d\theta=\frac{3\sqrt{3}V_{Max}}{\pi}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\left(\sin\theta\cos\frac{\pi}{6}+\sin\frac{\pi}{6}\cos\theta\right)d\theta$$

$$=\frac{3\sqrt{3}V_{Max}}{\pi}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\left(\frac{\sqrt{3}}{2}\sin\theta+\frac{1}{2}\cos\theta\right)d\theta=\frac{3\sqrt{3}V_{Max}}{\pi}\left[-\frac{\sqrt{3}}{2}\left(\cos\theta\right)\Big|_{\frac{\pi}{6}}^{\frac{\pi}{2}}+\frac{1}{2}\left(\sin\theta\right)\Big|_{\frac{\pi}{6}}^{\frac{\pi}{2}}\right]$$

$$=\frac{3\sqrt{3}V_{Max}}{\pi}\left[-\frac{\sqrt{3}}{2}\left(\cos\frac{\pi}{2}-\cos\frac{\pi}{6}\right)+\frac{1}{2}\left(\sin\frac{\pi}{2}-\sin\frac{\pi}{6}\right)\right]$$

$$=\frac{3\sqrt{3}V_{Max}}{\pi}\left[-\frac{\sqrt{3}}{2}\left(-\cos\frac{\pi}{6}\right)+\frac{1}{2}\right]\left(1-\sin\frac{\pi}{6}\right)$$

$$=\frac{3\sqrt{3}V_{Max}}{\pi}\left[\frac{\sqrt{3}}{2}\frac{\sqrt{3}}{2}+\frac{1}{2}\frac{1}{2}\right]=\frac{3\sqrt{3}V_{Max}}{\pi}\left[\frac{3}{4}+\frac{1}{4}\right]=\frac{3\sqrt{3}V_{Max}}{\pi}$$