QuantLib Project : Monte Carlo

MOHMOH Abdellah
OLIVIER Julien
TRAORE Idrissa
NALIATO Anthony

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1. Introduction

1.1 Overview of Instructions

In Monte Carlo engines, repeatedly calling process methods can lead to performance issues, especially when the process is an instance of the `GeneralizedBlackScholesProcess` class. This is because its methods make expensive calls to the underlying term structures.
To improve performance (at the cost of some accuracy), the task is to:

1. Create a new class that models a **Black-Scholes process with constant parameters** (underlying value, risk-free rate, dividend yield, and volatility).
2. Modify the Monte Carlo engines in the repository to accept:
   - A generic Black-Scholes process.
   - An additional boolean parameter.
     - If `false`, the engine runs as usual.
     - If `true`, the engine extracts constant parameters from the original process (e.g., the risk-free rate is the zero-rate of the full risk-free curve at the option's exercise date) and runs the simulation using the constant process.

1.2 Description of Project Architecture

The QuantLib project consists of three main files:

- `mc_discr_arith_av_strike.hpp`
- `mcbarrierengine.hpp`
- `mceuropeanengine.hpp`

Each file uses the Monte Carlo process for different types of options: European, Asian, and Barrier.
The Monte Carlo process involves the following steps:

1. **Random number generation**:
   - Two types: `MersenneTwister` (fully random) and `SobolRsg` (random based on dimensionality and vector size).
2. **Stochastic processes**:
   - Modeled as $dX = \mu(t,X)\ dt + \sigma(t,X)\ dW$
     i. $\mu(t,X)$: *Drift component.*
     ii. $\sigma(t,X)$: *Diffusion component.*
     iii. dW : *Random component*
3. **Path generation**:
   - Combines random numbers and stochastic processes to define the time grid.
4. **Path pricing**:
   - Calculates the option price based on the generated paths.
5. **Simulations**:
   - Aggregates results from path generation and pricing.

To improve pricing performance, the method reuses components of the Black-Scholes process. The next section introduces two additional files:

- `constantblackscholesprocess.cpp`
- `constantblackscholesprocess.hpp`

These files implement a **Black-Scholes process with constant parameters**.

2. Implementation

2.1 Explanation of the Black-Scholes Process

The `ConstantBlackScholesProcess` class models a **Black-Scholes process with constant parameters**, such as:

- Underlying value (`underlyingValue`)
- Risk-free rate (`riskFreeRate`)
- Dividend yield (`dividend`)
- Volatility (`volatility`)

This class inherits from the `StochasticProcess1D` class and implements four key methods:

1. **`x0()`**: Returns the initial value of the underlying asset.
2. **`drift(Time t, Real x)`**: Computes the drift component of the process.
3. **`diffusion(Time t, Real x)`**: Computes the diffusion (volatility) component.
4. **`apply(Real x0, Real dx)`**: Applies a change `dx` to the initial value `x0`.

These methods are used by the **`evolve`** function in the `StochasticProcess1D` class, which plays a central role in generating paths for the Monte Carlo simulation. The `evolve` function relies on three key methods:

- **`apply`**: Applies the change to the underlying value.
- **`expectation`**: Computes the expected value of the process.
- **`stdDeviation`**: Computes the standard deviation of the process.

Here's a snippet of the `ConstantBlackScholesProcess` class in C++:

```cpp
class ConstantBlackScholesProcess : public StochasticProcess1D {

public:
    ConstantBlackScholesProcess(double underlyingValue_, double riskFreeRate_, double volatility_, double dividend_);

    Real x0() const;
    Real drift(Time t, Real x) const;
    Real diffusion(Time t, Real x) const;
    Real apply(Real x0, Real dx) const;

private:
    double underlyingValue;
    double riskFreeRate;
    double volatility;
    double dividend;

};
```

- **`x0()`**: Returns the initial underlying value.
- **`drift()`** : Computes the drift as $r - q - 0.5\,\sigma ** 2$ where r is the risk-free rate, q is the dividend yield, and σ is the volatility.
- **`diffusion()`**: Returns the constant volatility.
- **`apply()`**: Applies the change using $x0 \cdot e ** dx$

2.2 Role of the Boolean Parameter

The goal here is to introduce a **boolean parameter** (`constantParameters`) in each Monte Carlo engine. This parameter determines whether the engine uses:

- The **original process** (if `false`).
- A **constant Black-Scholes process** (if `true`).

The boolean parameter is added to the constructors of three engine classes:

1. `MCEuropeanEngine_2`
2. `MCDiscreteArithmeticASEngine_2`
3. `MCBarrierEngine_2`

Each engine overrides the **`pathGenerator`** method to handle the boolean parameter:

- If `constantParameters` is `true`, the engine extracts constant parameters (underlying value, risk-free rate, dividend yield, and volatility) from the original process and creates an instance of `ConstantBlackScholesProcess`.
- If `constantParameters` is `false`, the engine uses the original process.

Here's how the `pathGenerator` method is implemented in the `MCEuropeanEngine_2` class:

```
//bool constantParameters=true;
if (this->constantParameters)
{
    ext::shared_ptr<GeneralizedBlackScholesProcess> BS_process =
        ext::dynamic_pointer_cast<GeneralizedBlackScholesProcess>(this->process_);

    // Get the parameters from the ConstantBlackScholesProcess class
    Time time_of_extraction = grid.back();
    double strike = ext::dynamic_pointer_cast<StrikedTypePayoff>(MCVanillaEngine<SingleVariate, RNG, S>::arguments_.payoff
    double riskFreeRate_ = BS_process->riskFreeRate()->zeroRate(time_of_extraction, Continuous);
    double dividend_ = BS_process->dividendYield()->zeroRate(time_of_extraction, Continuous);
    double volatility_ = BS_process->blackVolatility()->blackVol(time_of_extraction, strike);
    double underlyingValue_ = BS_process->x0();

    // We instanciate a constantBSProcess named cst_BS_process with the parameters
    ext::shared_ptr<ConstantBlackScholesProcess> cst_BS_process(new ConstantBlackScholesProcess(underlyingValue_, riskFree

    // We return a new path generator with constantBSProcess
    return ext::shared_ptr<path_generator_type>(
        new path_generator_type(cst_BS_process, grid,
            generator, MCVanillaEngine<SingleVariate, RNG, S>::brownianBridge_));
}
```

Key Points

- **Performance Improvement**: Using the constant process reduces computational costs by avoiding repeated calls to expensive methods in the original process.
- **Flexibility**: The boolean parameter allows users to switch between the original and constant processes easily.
- **Inheritance**: Each engine class inherits its `pathGenerator` method from a parent class (`MCVanillaEngine`, `MCDiscreteAveragingAsianEngineBase`, or `McSimulation`), ensuring consistency across different option types.

In the last section, the results of simulations are presented. They determine whether the constantBlackScholesProcess class makes computational costs lower and the performance is increased.

3. Results

3.1 Execution of the Main Program

The results showcase the performance and Net Present Values (NPV) for three types of options (European, Asian, and Barrier) using three different configurations:

- **Old Engine**: The original Monte Carlo engine without optimization.
- **Non Constant**: The new Monte Carlo engine without using constant parameters.
- **Constant**: The new Monte Carlo engine with constant parameters enabled.

Here's a detailed analysis of the results:

1. European Options

NPV: The NPV remains exactly the same across all configurations. This indicates that using constant parameters does not affect the accuracy of the calculation for European options.
Time: The execution time is significantly reduced with the constant parameters (1.41447 s) compared to the old engine (9.88391 s) and the non-constant engine (15.3154 s). This demonstrates that the optimization works well for European options.

Configuration	NPV	Time (s)
Old Engine	4.17073	9.88391
Non Constant	4.17073	15.3154
Constant	4.17073	1.41447

2. Asian Options

NPV: The NPV is slightly different for the constant configuration (0.731168) compared to the others (0.729431). This minor difference may be due to the approximation introduced by using constant parameters.
Time: The execution time is drastically reduced with the constant parameters (0.884244 s) compared to the old engine (11.0943 s) and the non-constant engine (8.32357 s). This shows that the optimization is highly effective for Asian options.

Configuration	NPV	Time (s)
Old Engine	0.729431	11.0943
Non Constant	0.729431	8.32357
Constant	0.731168	0.884244

3. Barrier Options

NPV: The NPV is slightly different for the constant configuration (0.274946) compared to the others (0.273727). This small difference may be due to the approximation introduced by using constant parameters.
Time: The execution time is reduced with the constant parameters (2.91737 s) compared to the old engine (11.1662 s) and the non-constant engine (11.267 s). This demonstrates that the optimization works well for Barrier options.

Configuration	NPV	Time (s)
Old Engine	0.273727	11.1662
Non Constant	0.273727	11.267
Constant	0.274946	2.91737

3.2 Global analysis

1. Accuracy (NPV):

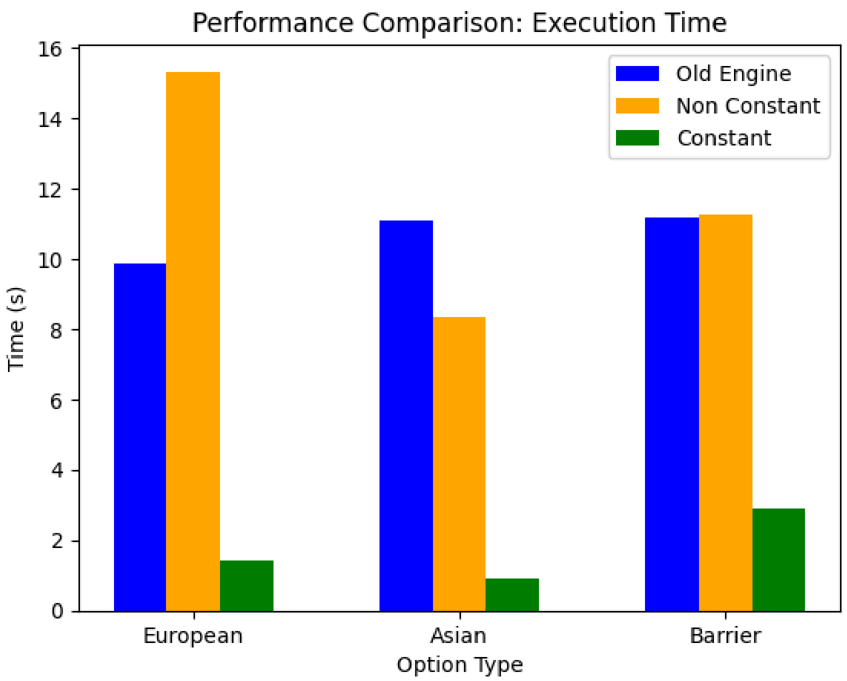
For European options, the NPV remains exactly the same, showing that using constant parameters does not affect precision.

For Asian and Barrier options, the NPV is slightly different with constant parameters. This is likely due to the approximation introduced by using constant parameters, but the difference is minimal.

2. Performance (Execution Time):

Using constant parameters significantly reduces execution time for all three types of options.

The performance gain is particularly notable for European options (85% reduction) and Asian options (92% reduction).



3. Conclusion:

The optimization using constant parameters is highly effective in reducing execution time while maintaining acceptable accuracy.

For Asian and Barrier options, there is a slight loss in precision, but this is outweighed by the significant performance improvement.