Modelling waning and boosting of COVID-19 in France with vaccination
Basic model information
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David W. Dick[1], Jane Heffernan[1]
model structure and basic assumptions developed with
Lauren Childs[2], Zhilan Feng[3a,b], John W Glasser[4],
Jing Li[5], Gergely Röst[6]

[1] Mathematics and Statistics, Centre for Disease Modelling, York University, Toronto, Canada
[2] Mathematics, Virginia Tech, Blacksburg, Virginia, USA
[3a] Mathematics, Purdue University, West Lafayette, Indiana, USA
[3b] National Science Foundation, Alexandria, Virginia, USA
[4] National Center for Immunization and Respiratory Diseases, CDC, Atlanta, Georgia, USA
[5] Mathematics, California State University, Northridge, California, USA
[6] Mathematics, University of Szeged, Szeged, Hungary

Model Equations

Our model tracks age, infection and immune status. Susceptible individuals of status i and age k are denoted by S_{ik}; similarly, infectious individuals by I_{ik}. Infected but not-yet-infectious individuals of immune status i, age k and stage j are denoted by E^j_{ik}. Vaccinated individuals of initial immune status i, age k and dose k are denoted by V^j_{ik}. Parameter descriptions are found in Table 1. The system of ODEs for age group k is given by the following set of equations:

Susceptible compartments:

$$\frac{d}{dt}S_{1k} = -\sum_{j=2}^{4} p_{1k}^{j}\Lambda_{1k}S_{1k} + \omega_{2k}S_{2k} - \sigma_{1k}^{1}(t)\rho S_{1k} + \omega_{2k}V_{1k}^{1},$$

$$\frac{d}{dt}S_{2k} = -\sum_{j=2}^{4} p_{2k}^{j}\Lambda_{2k}S_{2k} + \omega_{3k}S_{3k} - \omega_{2k}S_{2k} - \sigma_{2k}^{1}(t)\rho S_{2k} + \gamma_{2k}I_{2k} + \omega_{3k}V_{2k}^{1},$$

$$\frac{d}{dt}S_{3k} = -\sum_{j=3}^{4} p_{3k}^{j}\Lambda_{3k}S_{3k} + \omega_{4k}S_{4k} - \omega_{3k}S_{3k} - \sigma_{3k}^{1}(t)\rho S_{3k} + \gamma_{3k}I_{3k} + \omega_{4k}\left(\sum_{j=3}^{4} V_{jk}^{1} + \sum_{j=1}^{4} V_{jk}^{2}\right),$$

$$\frac{d}{dt}S_{4k} = -\omega_{4k}S_{4k} - \sigma_{4k}^{1}(t)\rho S_{4k} + \gamma_{4k}I_{4k},$$

Vaccinated compartments:

$$\frac{d}{dt}V_{1k}^{1} = \sigma_{1k}^{1}(t)\rho S_{1k} - \sigma_{1k}^{2}(t)\rho V_{1k}^{1} - \sum_{j=2}^{4} p_{2k}^{j}\Lambda_{2k}V_{1k}^{1} - \omega_{2k}V_{1k}^{1},$$

$$\frac{d}{dt}V_{2k}^{1} = \sigma_{2k}^{1}(t)\rho S_{2k} - \sigma_{2k}^{2}(t)\rho V_{2k}^{1} - \sum_{j=3}^{4} p_{3k}^{j}\Lambda_{3k}V_{2k}^{1} - \omega_{3k}V_{2k}^{1},$$

$$\frac{d}{dt}V_{3k}^{1} = \sigma_{3k}^{1}(t)\rho S_{3k} - \sigma_{3k}^{2}(t)\rho V_{3k}^{1} - \omega_{4k}V_{3k}^{1},$$

$$\frac{d}{dt}V_{4k}^{1} = \sigma_{4k}^{1}(t)\rho S_{4k} - \sigma_{4k}^{2}(t)\rho V_{4k}^{1} - \omega_{4k}V_{4k}^{1},$$

$$\frac{d}{dt}V_{1k}^{2} = \sigma_{1k}^{2}(t)\rho V_{1k}^{1} - \omega_{4k}V_{1k}^{2},$$

$$\frac{d}{dt}V_{2k}^{2} = \sigma_{2k}^{2}(t)\rho V_{2k}^{1} - \omega_{4k}V_{2k}^{2},$$

$$\frac{d}{dt}V_{3k}^{2} = \sigma_{3k}^{2}(t)\rho V_{3k}^{1} - \omega_{4k}V_{3k}^{2},$$

$$\frac{d}{dt}V_{4k}^{2} = \sigma_{4k}^{2}(t)\rho V_{4k}^{1} - \omega_{4k}V_{4k}^{2},$$

Infected compartments:

$$\frac{d}{dt}E^1_{2k} = p^2_{1k}\Lambda_{1k}S_{1k} + p^2_{2k}\Lambda_{2k}S_{2k} + p^2_{2k}\Lambda_{2k}V^1_{1k} - \kappa^1_{2k}E^1_{2k},$$
$$\frac{d}{dt}E^1_{3k} = p^3_{1k}\Lambda_{1k}S_{1k} + p^3_{2k}\Lambda_{2k}S_{2k} + p^3_{2k}\Lambda_{2k}V^1_{1k} + p^3_{3k}\Lambda_{3k}S_{3k} + p^3_{3k}\Lambda_{3k}V^1_{2k} - \kappa^1_{3k}E^1_{3k},$$
$$\frac{d}{dt}E^1_{4k} = p^4_{1k}\Lambda_{1k}S_{1k} + p^4_{2k}\Lambda_{2k}S_{2k} + p^4_{2k}\Lambda_{2k}V^1_{1k} + p^4_{3k}\Lambda_{3k}S_{3k} + p^4_{3k}\Lambda_{3k}V^1_{2k} - \kappa^1_{4k}E^1_{4k},$$
$$\frac{d}{dt}E^2_{2k} = \kappa^1_{2k}E^1_{2k} - \kappa^2_{2k}E^2_{2k},$$
$$\frac{d}{dt}E^2_{3k} = \kappa^1_{3k}E^1_{3k} - \kappa^2_{3k}E^2_{3k},$$
$$\frac{d}{dt}E^2_{4k} = \kappa^1_{4k}E^1_{4k} - \kappa^2_{4k}E^2_{4k},$$
$$\frac{d}{dt}E^3_{2k} = \kappa^2_{2k}E^2_{2k} - \kappa^3_{2k}E^3_{2k},$$
$$\frac{d}{dt}E^3_{3k} = \kappa^2_{3k}E^2_{3k} - \kappa^3_{3k}E^3_{3k},$$
$$\frac{d}{dt}E^3_{4k} = \kappa^2_{4k}E^2_{4k} - \kappa^3_{4k}E^3_{4k},$$
$$\frac{d}{dt}I_{2k} = \kappa^3_{2k}E^3_{2k} - \gamma_{2k}I_{2k} - \delta_{2k}I_{2k},$$
$$\frac{d}{dt}I_{3k} = \kappa^3_{3k}E^3_{3k} - \gamma_{3k}I_{3k} - \delta_{3k}I_{3k},$$
$$\frac{d}{dt}I_{4k} = \kappa^3_{4k}E^3_{4k} - \gamma_{4k}I_{4k} - \delta_{4k}I_{41},$$

where for $1 \leq k \leq N$,

$$\Lambda_{ik}(t) = \alpha_{ik}A_k\lambda_{ik}(t),$$
$$\lambda_{ik}(t) = \sum_{m=1}^{N} c_{km}\frac{\sum_{j=2}^{4}\beta_{jm}I_{jm}(t)}{\sum_{j=1}^{4}T_{jm}(t)},$$
$$T_{jm}(t) = S_{jm}(t) + V^1_{jm}(t) + V^2_{jm}(t) + E^1_{jm}(t) + E^2_{jm}(t) + E^3_{jm}(t) + I_{jm}(t).$$

Parameters

Disease Parameters

Susceptibility is assumed to decrease with increasing immunity status, but does not depend on age. Thus, $\alpha_{1n} = 1$, $\alpha_{2n} = \frac{2}{3}$, and $\alpha_{3n} = \frac{1}{3}$ for any age group n. Infectivity is assumed to vary by severity of infection and severity of disease. Our infectivity is chosen to produce a basic reproductive number of $R_0 = 2.6$ (**?**). By immunity status, we assume $\beta_{3n} = \beta = 0.08$, $\beta_{2n} = 0.5\beta$, $\beta_{4n} = 0.1\beta$ for any age group n. Milder infections are expected to have lower viral loads and, thus, lower infectivity. Simultaneously, more severe disease outcomes are expected to induce behavioral changes, such as being too sick to go out, that lower infectivity.

Recovery rate (from infectiousness) is assumed to depend on on disease severity, with milder disease associated with shorter infectious periods. Thus, $\gamma_{1n} < \gamma_{2n} < \gamma_{3n}$, with $\gamma_{1n} = \frac{1}{5}$, $\gamma_{2n} = \frac{1}{10}$, $\gamma_{3n} = \frac{1}{15}$ for any age group n. We assume that disease-induced mortality only occurs from the most severe infectious class, I_4. Thus, $\delta_{2n} = 0$, $\delta_{3n} = 0$, $\delta_{4n} = \delta = 0.0001$ for any age group n.

Every susceptible class can become infected and result in equal or greater immunity. The proportion of susceptible class going to higher severity classes increases with age, i.e. $p^{(j)}_{in} <= p^{(j)}_{im}$ for $n < m$. Here, we assume that age is a proxy for co-morbidity. Precise values are determined from **?**.

Parameter	Definition
α_{in}	susceptibility of individuals from S_{in} (i immunity status, n age group)
β_{jm}	infectivity of infected individuals from I_{jm} (j immunity status, m age group)
γ_{jm}	recovery rate of infected individuals from I_{jm} (j immunity status, m age group)
κ^k_{jm}	rates of progress through the pre-infectious period of infeciton (i immunity status, n age group, k stage)
δ_{jm}	disease-induced mortality rate of infected individuals from I_{jm} (j immunity status, m age group)
ω_{in}	waning rate of immunity of individuals from S_{in} (i immunity status, n age group)
p^j_{in}	proportion of S_i going to I_j upon infection with $i = 1, 2, 3$ and $j = 2, 3, 4$ (i, j immunity status, n age group)
ρ_{in}	vaccine efficacy (i immunity status, n age group)
σ_k	vaccination rate for first $k = 1$ and second $k = 2$ dose
A_n	per capita activity counts of individuals in age group n
c_{an}	mixing matrix between individuals in age group a and age group n, modified given mitigation strategy and PPE, social distancing, and hand washing compliance (k-value)

Table 1: Parameter definitions for the ODE model.

We assume immunity lasts on average one year between successive immunity stages and is independent of age and immunity status. Thus, the waning of immunity occurs at rate $\omega_{in} = \omega = \frac{1}{365}$ for $i = 1, 2, 3$ and any age group n. For vaccinated classes, we assume that classes V^1_{1n} wane in immunity to S_{1n}, V^1_{2n} wane to S_{2n}, and all other vaccinated classes wane to S_{3n}.

Vaccination Parameters

Vaccination rate of the first dose is determined by the coverage data from France as supplied by Carole - Melanie, and Linda. Weekly coverage, y_{in}, is scaled to a daily rate, σ^1_{in}, using a translation equation.

Demographic Parameters

Given the short period of examination, we currently assume the absence of birth, natural mortality and aging but this can be added in if wanted. We focus on the importance of contact structure among age groups. We assume that the standard mixing matrix and associated activities only depend on age and are taken from **?**. The base matrix provides the number of contacts between ages at school (S), work (W), home (H) and other (O). We also determine matrices for different stages of mitigation and relaxation: strict mitigation (phase 0); moderate mitigation (phase 1); moderate relaxation (phase 2); and increased relaxation (phase 3). In the Canadian context, these contact matrices were used and scaled by a compliance index, k, as follows: January 19 to March 14 baseline contacts with $k = 1$; March 15 to March 18 baseline contacts with $k = 0.87$; March 19 to March 24 phase 2 with $k = 1$; March 25 to March 31 phase 2 with $k = 0.9$; April 1 to April 25 phase 1 with $k = 0.8$; April 26 to May 15 phase 0 with $k = 0.8$; May 16 to June 19 phase 0 with $k = 0.5$; June 20 to July 19 phase 1 with $k = 0.7$; July 20 to September 1 phase 1 with $k = 0.7$; September 2 to December 31 phase 2 with $k = 0.5$. Our simulations of the vaccination period and post vaccination period continue to use phase 2 with $k = 0.5$. These contact matrices and compliance values were chosen in accordance with France policy changes.