Question 1 (15 Points)

(a)

A	B	C	D	$A \wedge B$
T	T	T	T	T
T	T	T	F	T
T	T	F	T	T
T	T	F	F	T
T	F	T	T	F
T	F	T	F	F
T	F	F	T	F
T	F	F	F	F
F	T	T	T	F
F	T	T	F	F
F	T	F	T	F
F	T	F	F	F
F	F	T	T	F
F	F	T	F	F
F	F	F	T	F
F	F	F	F	F

Hence, 4 models

(b)

A	B	C	D	$(A \wedge B) \vee (B \wedge C)$
T	T	T	T	T
T	T	T	F	T
T	T	F	T	T
T	T	F	F	T
T	F	T	T	F
T	F	T	F	F
T	F	F	T	F
T	F	F	F	F
F	T	T	T	F
F	T	T	F	F
F	T	F	T	F
F	T	F	F	F
F	F	T	T	F
F	F	T	F	F
F	F	F	T	F
F	F	F	F	F

Hence, 6 models

(c) 2 models

Question 2 (15 Points)

(a)

A	B	$\neg(P \land Q)$	$\neg P \lor \neg Q$
T	T	F	F
T	F	T	T
F	T	T	T
F	F	T	T

To prove soundness, we must show that whenever the premise, $\neg(P \land Q)$, is true, then the derived sentence is true. In the truth table, $\neg(P \land Q)$ is true for line 2 – 4. In all these of these cases, $\neg P$ v $\neg Q$ is also true. Thus, the rule is sound.

(b)

A	B	. $\neg(P \lor Q)$	$\neg P \land \neg Q$
T	T	F	F
T	F	F	F
F	T	F	F
F	F	T	T

To prove soundness, we must show that whenever the premise, $\neg(P \lor Q)$, is true, then the derived sentence is true. In the truth table, $\neg(P \lor Q)$ is true for line 4. In all these of these cases, $\neg P \land \neg Q$ is also true. Thus, the rule is sound.

Question 3 (15 Points)

(a)

A	B	$(\neg P \vee \neg Q)$	$(P \wedge Q)$
T	T	F	T
T	F	T	F
F	T	T	F
F	F	T	F

Unsatisfiable

(b)

P	Q	$(Q \Rightarrow P)$	$P \Rightarrow (Q \Rightarrow P)$
T	T	T	T
T	F	T	T
F	T	F	T
F	F	T	T

Valid

(c)

P	Q	R	$P \Rightarrow R$	$(P \wedge Q) \Rightarrow R$	$Q \Rightarrow (P \Rightarrow R)$	Overall
T	T	T	T	T	T	T
T	T	F	F	F	F	T
T	F	T	T	T	T	T
T	F	F	F	T	T	T
F	T	T	T	T	T	T
F	T	F	T	T	T	T
F	F	T	T	T	T	T
F	F	F	T	T	T	T

Valid

(d)

P	Q	$(P \Rightarrow Q) \wedge \neg P$	$((P \Rightarrow Q) \wedge \neg P) \Rightarrow \neg Q$
T	T	F	T
T	F	F	T
F	T	T	F
F	F	T	T

Satisfiable

Question 4 (15 Points)
(a) Truth table

P	Q	R	$P \Rightarrow Q$	$Q \Rightarrow R$	$(P \Rightarrow Q) \wedge (Q \Rightarrow R)$	$(P \Rightarrow R)$	Overall
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

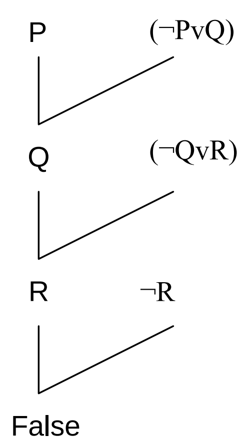
Therefore, it is valid (true for all models)

(b) Natural deduction

(1) $(P \Rightarrow Q) \wedge (Q \Rightarrow R)$, then, by implication elimination: $(P \Rightarrow Q) \equiv \neg P \vee Q$, $(Q \Rightarrow R) \equiv \neg Q \vee R$
(2) $(\neg P \vee Q) \wedge (\neg Q \vee R)$, then, by AND-Elimination, we have
(3) $(\neg P \vee Q)$, similarly, from preceding step we also have
(4) $(\neg Q \vee R)$, then, by the Resolution Rule of inference, we have from (3) and (4)
(5) $(\neg P \vee R)$ Finally, by reverse implication elimination, we have
(6) $P \Rightarrow R$

(c) Resolution refutation method
First, negate, then put into CNF form
$\neg ((P \Rightarrow Q) \wedge (Q \Rightarrow R)) \Rightarrow (P \Rightarrow R)$
$= \neg((\neg P \vee Q) \wedge (\neg Q \vee R) \Rightarrow (\neg P \vee R))$
$= \neg(\neg(\neg P \vee Q) \wedge (\neg Q \vee R) \vee (\neg P \vee R))$
$= ((\neg P \vee Q) \wedge (\neg Q \vee R)) \wedge \neg (\neg P \vee R)$
$= (\neg P \vee Q) \wedge (\neg Q \vee R) \wedge P \wedge \neg R$

The refutation proof graph:



Question 5 (15 Points)

(a) $\exists x\left[student(x) \wedge Takes(x, French, Spring2001)\right]$

(b) $\forall x, z\left[(student(x) \wedge Takes(x, French, z)) \Rightarrow Passes(x, French, z)\right]$

(c)

$\exists x Student(x) \wedge Takes(x, Greek, Spring2001) \wedge \forall y\left[\left(Student(y) \wedge Takes(y, Greek, Spring2001)\right) \Rightarrow (x = y)\right]$

(d)

$(\forall a, b, s)(\exists c, d)(< \left(Score(a, greek, s), Score(c, greek, s)\right) \wedge (a \neq c) \wedge$
$< \left(Score(b, french, s), Score(d, greek, s)\right) \wedge (b \neq d) \Rightarrow > \left(Score(c, greek, s), Score(d, greek, s)\right)$

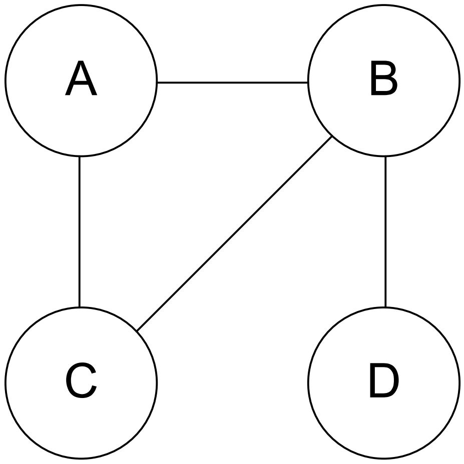
or

$\forall s \exists x \forall y (Student(x) \wedge Student(y) \wedge Takes(x, Greek, s) \wedge Takes(y, French, s)) \Rightarrow$
$> (Score(x, Greek, s), Score(y, French, s))$

(e) $\left[\forall x, y, l\left(German(x) \wedge German(y) \wedge Language(l) \wedge Speaks(x, l)\right) \Rightarrow Speaks(y, l)\right]$

Question 6 (20 Points)

(a)



(b)

Step	A	B	C	D
0	{1, 2, 3}	{1, 2, 3}	{1, 2, 3}	{1, 2, 3}
1	(1)	{2, 3}	{2, 3}	{1, 2, 3}
2	1	(2)	{}	{1}
3	1	(3)	{2}	{}
4	(2)	{1, 3}	{1, 3}	{1, 2, 3}
5	2	(1)	{}	{2,3}
6	2	(3)	{1}	{}
7	(3)	{1, 2}	{1, 2}	{1, 2, 3}
8	3	(1)	{}	{2,3}
9	3	(2)	{1}	{3}
10	3	2	(1)	{3}
11	3	2	1	(3)

(c)

Step	A	B	C	D
0	{1, 2, 3}	{1, 2, 3}	{1, 2, 3}	{1, 2, 3}
1	{1, 2}	(1)	{}	{2, 3}
2	{1, 3}	(2)	{1}	{3}
3	{3}	2	(1)	{3}
4	(3)	2	1	{3}
5	3	2	1	(3)

(d)

Step	A	B	C	D	Arcs Queue	Arc
0	{1, 2, 3}	{1, 2, 3}	{1, 2, 3}	{1, 2, 3}	AB, AC, BC, BD	
1	{1, 2, 3}	{2, 3}	{1, 2}	{1, 2, 3}	AB, AC, BD	BC
2	{1, 2, 3}	2	{1, 2}	{3}	AB, BC, AC	BD
3	{1, 2, 3}	2	1	3	AB, AC	BC
4	{1, 3}	2	1	3	AC	AB