CS 436
Exam 4 Solutions

0) English Translations (20 points)

a) Convert this to sentence into First Order Logic

All apartments in Belgrade have lower rent than some apartments in Bozeman

$$\forall x \left[Apt(x) \wedge In\left(x, Belgrade\right) \right] \Rightarrow \exists y \left[Apt(y) \wedge In\left(y, Bozeman\right) \wedge \left(Rent(x) < Rent(y) \right) \right]$$

b) What is wrong with the translation of this sentence?

If a house is more expensive than all houses in Belgrade, then it must be in Bozeman.

$$\forall x\, Apt(x) \wedge \left[\forall y\, Apt(y) \wedge In\left(y, Belgrade\right) \wedge \left(Rent(x) > Rent(y) \right) \right] \Rightarrow In\left(x, Bozeman\right)$$

Uses '^' with 'For All' quantifier, should use '➔'

$$\forall x\, Apt(x) \wedge \left[\forall y\, Apt(y) \wedge In\left(y, Belgrade\right) \Rightarrow \left(Rent(x) > Rent(y) \right) \right] \Rightarrow In\left(x, Bozeman\right)$$

'House' should have been 'Apartment' in English sentence, some extra credit was given for people who pointed this out.

1) CNF Conversion (20 points)

Consider these sentences

A: $\forall x \left[\exists y\, P(x,y) \right] \Rightarrow Q(x)$

B: $\forall x,y\, P(x,y) \Rightarrow Q(x)$

a) Convert to CNF

A: $\forall x \neg \left[\exists y\, P(x,y) \right] \vee Q(x)$ - Apply implication elimination, look for parentheses!

A: $\forall x \left[\forall y \neg P(x,y) \right] \vee Q(x)$ - distribute NOT operator inside parens

A: $\forall x \forall y \neg P(x,y) \vee Q(x)$ - drop parens

A: $\forall x \forall y \neg P(x,y) \vee Q(x)$ - drop universal quantifiers (optional step for test)

B: $\forall x,y \neg P(x,y) \vee Q(x)$ - Apply implication elimination

b) Prove A entails B

Simple argument: B in CNF is the same as A in CNF…. so we're done.

Pointy-headed way: Prove by resolving A against NOT B.

¬B: $\neg \left[\forall x,y \neg P(x,y) \vee Q(x) \right]$

¬B: $\exists x,y \neg \left[\neg P(x,y) \vee Q(x) \right]$

¬B: $\exists x,y \; P(x,y) \wedge \neg Q(x)$

B1: P(G, H)

B2: ¬ Q (G)

Resolve A against x=G & y=H and we get

C: Q(G)

Conflict with B2.

2) FOL vs Propositional Logic (20 points)

In comparison to Propositional Logic, First Order Logic can be more manageable in terms of the number of logical sentences in a KB. In some cases, like the game of chess, the equivalent KB in Propositional Logic can be many thousands of times larger. Which of the following are valid reasons for this? Circle Valid or Invalid and explain.

a) Chess is a complicated game with many complex rules

Valid / **Invalid**

This is the same for PL vs FOL. Rules of chess don't change when we change 'logical expression languages'

b) There are several pieces of each type {2 Bishops, 2 Knights, 8 Pawns, etc}

Valid / Invalid

FOL has the ability to generalize over multiple pieces of the same type. In PL we would have 'duplicate' rules for each individual piece.

c) There are multiple types of game pieces {Pawn, Rook, Queen, etc}

Valid / **Invalid**

This is the same for PL vs FOL. Rules of chess don't change when we change 'logical expression languages'

d) The board has 64 squares

Valid / Invalid

PL needs rules/axioms for every move on each square (start square -> end square) for every individual piece.