

## Informed Search

Chapter 4.1 – 4.3

1

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## Informed Search

\* *Informed searches use domain knowledge to guide selection of the best path to continue searching*

- **Heuristics** are used, which are informed guesses
- Heuristic means "serving to aid discovery"

2

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## Informed Search

- Define a **heuristic** function,  $h(n)$ :
  - uses domain-specific info. in some way
  - is computable from the current state description
  - it estimates
    - the "goodness" of node  $n$
    - how close node  $n$  is to a goal
    - the cost of *minimal* cost path from node  $n$  to a goal state

3

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## Informed Search

- $h(n) \geq 0$  for all nodes  $n$
- $h(n) = 0$  implies that  $n$  is a goal node
- $h(n) = \infty$  implies that  $n$  is a dead end from which a goal cannot be reached
- All domain knowledge used in the search is encoded in the heuristic function,  $h$
- An example of a **weak method** for AI because of the limited way that domain-specific information is used to solve a problem

4

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## Best-First Search

- \* Sort nodes in the nodes list by increasing values of an evaluation function,  $f(n)$ , that incorporates domain-specific information
- This is a generic way of referring to the class of informed search methods

5

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## Greedy Best-First Search

- \* Use as an evaluation function,  $f(n) = h(n)$ , sorting nodes in the nodes list by increasing values of  $f$
- Selects the node to expand that is believed to be closest (i.e., smallest  $f$  value) to a goal node

6

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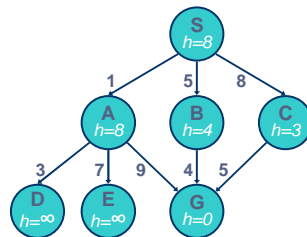
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## Greedy Best-First Search

$$f(n) = h(n)$$

# of nodes tested: 0, expanded: 0

expnd. node	OPEN list
	{S:8}



7

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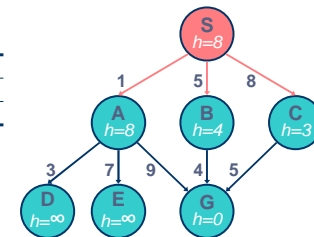
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## Greedy Best-First Search

$$f(n) = h(n)$$

# of nodes tested: 1, expanded: 1

expnd. node	OPEN list
	{S:8}
S not goal	{C:3, B:4, A:8}



8

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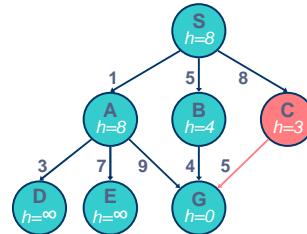
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## Greedy Best-First Search

$$f(n) = h(n)$$

# of nodes tested: 2, expanded: 2

expnd. node	OPEN list
S	{S:8}
S	{C:3,B:4,A:8}
C not goal	{G:0,B:4,A:8}



9

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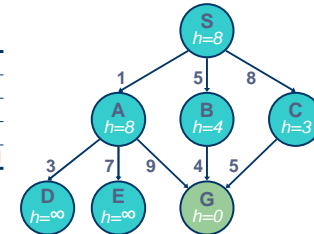
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## Greedy Best-First Search

$$f(n) = h(n)$$

# of nodes tested: 3, expanded: 2

expnd. node	OPEN list
S	{S:8}
S	{C:3,B:4,A:8}
C	{G:0,B:4,A:8}
G goal	{B:4,A:8} no expand



10

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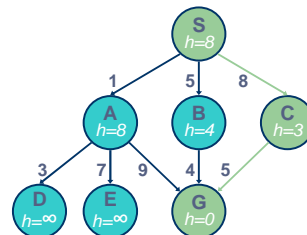
## Greedy Best-First Search

$$f(n) = h(n)$$

# of nodes tested: 3, expanded: 2

expnd. node	OPEN list
S	{S:8}
S	{C:3,B:4,A:8}
C	{G:0,B:4,A:8}
G	{B:4,A:8}

\* Fast but not optimal



path: S,C,G  
cost: 13

11

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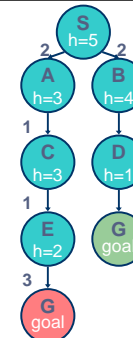
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## Greedy Best-First Search

- Not complete
- Not optimal/admissible

Greedy search finds the **left** goal (solution cost of 7)

Optimal solution is the path to the **right** goal (solution cost of 5)



12

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## Beam Search

- \* Use an evaluation function  $f(n) = h(n)$  as in greedy best-first search, but restrict the maximum size of the nodes list to a constant  $k$
- Only keep  $k$  best nodes as candidates for expansion, and throw away the rest
- More space efficient than Greedy Search, but may throw away a node on a solution path
- Not complete
- Not optimal/admissible

13

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## Algorithm A Search

- \* Use as an evaluation function  $f(n) = g(n) + h(n)$ , where  $g(n)$  is minimal cost path from start to current node  $n$  (as defined in UCS)
- The  $g$  term adds a breadth-first component to the evaluation function
- Nodes on the search frontier (in nodes list) are ranked by the *estimated cost of a solution*, where  $g(n)$  is the cost from the start node to node  $n$ , and  $h(n)$  is the estimated cost from node  $n$  to a goal

14

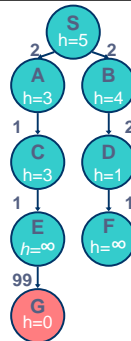
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## Algorithm A Search

- Not complete
- Not optimal/admissible

Algorithm A never expands E because  $h(E) = \infty$



15

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## Algorithm A\* Search

- \* Use the same evaluation function used by Algorithm A, except add the constraint that for all nodes  $n$  in the search space,  $h(n) \leq h^*(n)$ , where  $h^*(n)$  is the true cost of the minimal cost path from  $n$  to a goal
- The cost to the nearest goal is never overestimated
- When  $h(n) \leq h^*(n)$  holds true,  $h$  is admissible
- An admissible heuristic guarantees that a node on the optimal path cannot look so bad so that it is never considered

16

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## Algorithm A\* Search

- **Complete**  
whenever the branching factor is finite  
(as also is required for BFS, IDS, UCS),  
and every operator has a fixed positive cost
- **Optimal/Admissible**

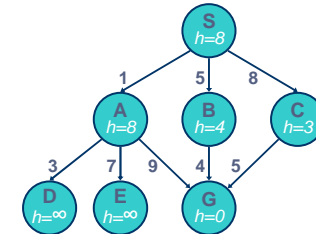
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## Example

$n$	$g(n)$	$h(n)$	$f(n)$	$h^*(n)$
S				
A				
B				
C				
D				
E				
G				



$g(n)$  = actual cost to get to node  $n$   
from start

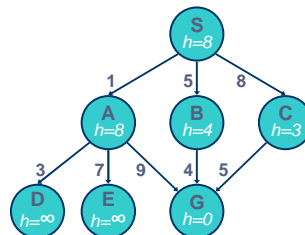
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## Example

$n$	$g(n)$	$h(n)$	$f(n)$	$h^*(n)$
S	0			
A				
B				
C				
D				
E				
G				



$g(n)$  = actual cost to get to node  $n$   
from start

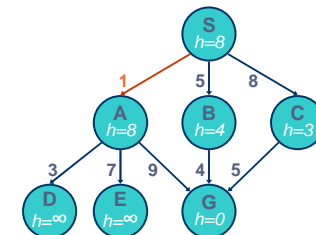
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## Example

$n$	$g(n)$	$h(n)$	$f(n)$	$h^*(n)$
S	0			
A	1			
B				
C				
D				
E				
G				



$g(n)$  = actual cost to get to node  $n$   
from start

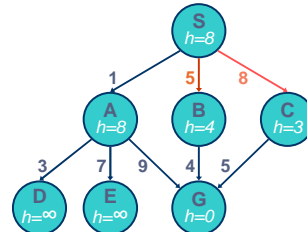
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## Example

$n$	$g(n)$	$h(n)$	$f(n)$	$h^*(n)$
S	0			
A	1			
B	5			
C	8			
D				
E				
G				



$g(n)$  = actual cost to get to node  $n$   
from start

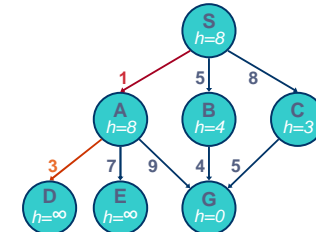
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## Example

$n$	$g(n)$	$h(n)$	$f(n)$	$h^*(n)$
S	0			
A	1			
B	5			
C	8			
D	1+3 = 4			
E				
G				



$g(n)$  = actual cost to get to node  $n$   
from start

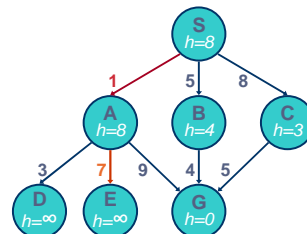
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## Example

$n$	$g(n)$	$h(n)$	$f(n)$	$h^*(n)$
S	0			
A	1			
B	5			
C	8			
D	4			
E	1+7 = 8			
G				



$g(n)$  = actual cost to get to node  $n$   
from start

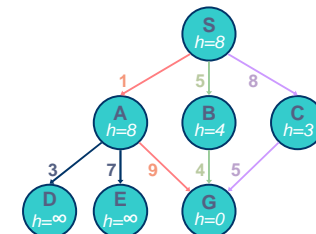
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## Example

$n$	$g(n)$	$h(n)$	$f(n)$	$h^*(n)$
S	0			
A	1			
B	5			
C	8			
D	4			
E	8			
G	10/9/13			



$g(n)$  = actual cost to get to node  $n$   
from start

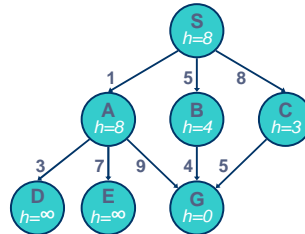
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## Example

$n$	$g(n)$	$h(n)$	$f(n)$	$h^*(n)$
S	0			
A	1			
B	5			
C	8			
D	4			
E	8			
G	10/9/13			



$h(n)$  = estimated cost to get to a goal  
from node  $n$

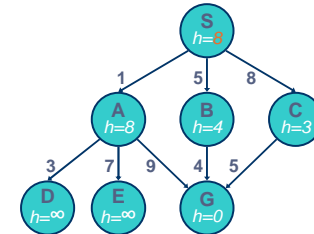
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## Example

$n$	$g(n)$	$h(n)$	$f(n)$	$h^*(n)$
S	0	8		
A	1	8		
B	5	4		
C	8	3		
D	4	∞		
E	8	∞		
G	10/9/13	0		



$h(n)$  = estimated cost to get to a goal  
from node  $n$

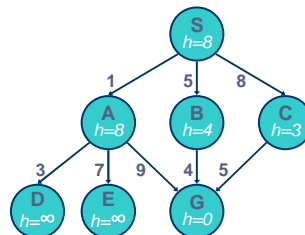
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## Example

$n$	$g(n)$	$h(n)$	$f(n)$	$h^*(n)$
S	0	8	8	
A	1	8	9	
B	5	4	9	
C	8	3	11	
D	4	∞	∞	
E	8	∞	∞	
G	10/9/13	0	10/9/13	



$f(n) = g(n) + h(n)$   
actual cost to get from start to  $n$   
plus estimated cost from  $n$  to goal

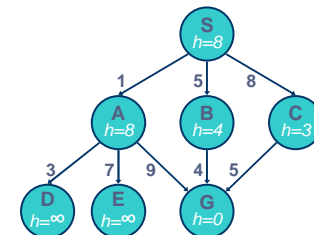
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## Example

$n$	$g(n)$	$h(n)$	$f(n)$	$h^*(n)$
S	0	8	8	
A	1	8	9	
B	5	4	9	
C	8	3	11	
D	4	∞	∞	
E	8	∞	∞	
G	10/9/13	0	10/9/13	



$h^*(n)$  = true cost of minimal path  
from  $n$  to a goal

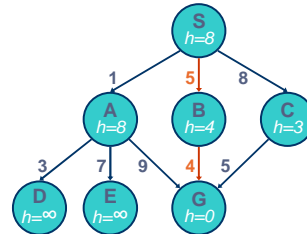
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## Example

$n$	$g(n)$	$h(n)$	$f(n)$	$h^*(n)$
S	0	8	8	9
A	1	8	9	
B	5	4	9	
C	8	3	11	
D	4	$\infty$	$\infty$	
E	8	$\infty$	$\infty$	
G	10/9/13	0	10/9/13	



$h^*(n)$  = true cost of minimal path  
from  $n$  to a goal

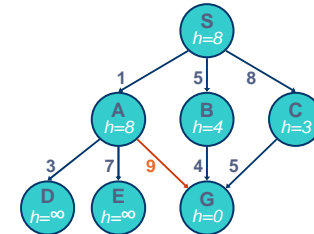
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## Example

$n$	$g(n)$	$h(n)$	$f(n)$	$h^*(n)$
S	0	8	8	9
A	1	8	9	9
B	5	4	9	
C	8	3	11	
D	4	$\infty$	$\infty$	
E	8	$\infty$	$\infty$	
G	10/9/13	0	10/9/13	



$h^*(n)$  = true cost of minimal path  
from  $n$  to a goal

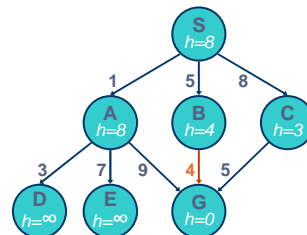
30

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## Example

$n$	$g(n)$	$h(n)$	$f(n)$	$h^*(n)$
S	0	8	8	9
A	1	8	9	9
B	5	4	9	4
C	8	3	11	
D	4	$\infty$	$\infty$	
E	8	$\infty$	$\infty$	
G	10/9/13	0	10/9/13	



$h^*(n)$  = true cost of minimal path  
from  $n$  to a goal

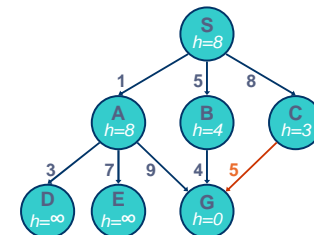
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## Example

$n$	$g(n)$	$h(n)$	$f(n)$	$h^*(n)$
S	0	8	8	9
A	1	8	9	9
B	5	4	9	4
C	8	3	11	5
D	4	$\infty$	$\infty$	
E	8	$\infty$	$\infty$	
G	10/9/13	0	10/9/13	



$h^*(n)$  = true cost of minimal path  
from  $n$  to a goal

32

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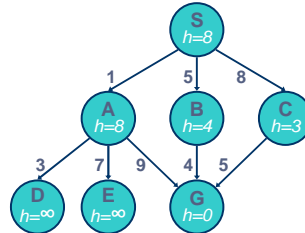
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## Example

$n$	$g(n)$	$h(n)$	$f(n)$	$h^*(n)$
S	0	8	8	9
A	1	8	9	9
B	5	4	9	4
C	8	3	11	5
D	4	$\infty$	$\infty$	$\infty$
E	8	$\infty$	$\infty$	$\infty$
G	10/9/13	0	10/9/13	0

$h^*(n)$  = true cost of minimal path  
from  $n$  to a goal



33

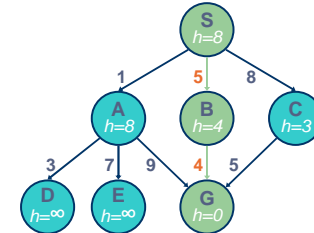
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## Example

$n$	$g(n)$	$h(n)$	$f(n)$	$h^*(n)$
S	0	8	8	9
A	1	8	9	9
B	5	4	9	4
C	8	3	11	5
D	4	$\infty$	$\infty$	$\infty$
E	8	$\infty$	$\infty$	$\infty$
G	10/9/13	0	10/9/13	0

optimal path = S,B,G  
cost = 9



34

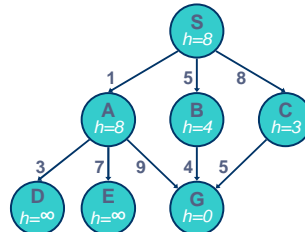
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## Example

$n$	$g(n)$	$h(n)$	$f(n)$	$h^*(n)$
S	0	8	8	9
A	1	8	9	9
B	5	4	9	4
C	8	3	11	5
D	4	$\infty$	$\infty$	$\infty$
E	8	$\infty$	$\infty$	$\infty$
G	10/9/13	0	10/9/13	0

Since  $h(n) \leq h^*(n)$  for all  $n$ ,  
 $h$  is admissible



35

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## A\* Algorithm

- Put the start node  $S$  in the nodes list, called *OPEN*
- If *OPEN* is empty, exit with failure
- Remove from *OPEN* the node  $n$  for which  $f(n)$  is minimum and place it in *CLOSED*, the list of nodes that have been expanded
- If  $n$  is a goal node, exit (for solution path trace back links from  $n$  to  $S$ )
- Expand  $n$ , generating all its successors and store in them a link back to  $n$   
For each successor  $n'$  of  $n$ 
  - If  $n'$  is not already in either *OPEN* or *CLOSED*, calculate  $h(n')$ ,  $g(n')=g(n)+c(n,n')$ ,  $f(n')=g(n')+h(n')$ , and place  $n'$  in *OPEN*
  - If  $n'$  is already in either *OPEN* or *CLOSED*, Check if  $g(n')$  is lower for the newly generated  $n'$ 
    - If it is not lower, do nothing
    - If it is lower replace existing  $n'$  with new  $g(n')$ ,  $h(n')$ , link back to this  $n$  and if in *CLOSED* move node from *CLOSED* to *OPEN*
- Go to 2**

36

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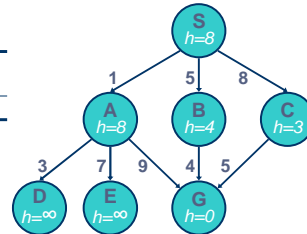
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## A\* Search

$$f(n) = g(n) + h(n)$$

# of nodes tested: 0, expanded: 0

expnd. node	OPEN list
	{S:0+8}



37

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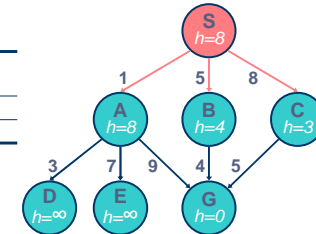
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## A\* Search

$$f(n) = g(n) + h(n)$$

# of nodes tested: 1, expanded: 1

expnd. node	OPEN list
	{S:8}
S not goal	{A:1+8, B:5+4, C:8+3}



38

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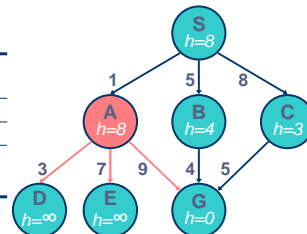
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## A\* Search

$$f(n) = g(n) + h(n)$$

# of nodes tested: 2, expanded: 2

expnd. node	OPEN list
	{S:8}
S	{A:9, B:9, C:11}
A not goal	{B:9, G:1+9+0, C:11, D:1+3+∞, E:1+7+∞}



39

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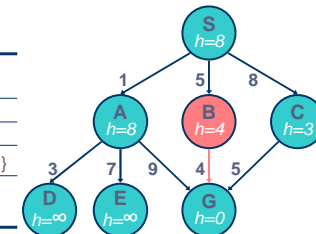
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## A\* Search

$$f(n) = g(n) + h(n)$$

# of nodes tested: 3, expanded: 3

expnd. node	OPEN list
	{S:8}
S	{A:9, B:9, C:11}
A	{B:9, G:10, C:11, D:∞, E:∞}
B not goal	{G:5+4+0, C:11, D:∞, E:∞} replace



40

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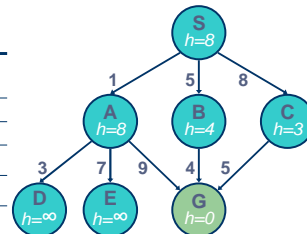
## A\* Search

$$f(n) = g(n) + h(n)$$

# of nodes tested: 4, expanded: 3

expnd. node	OPEN list
S	{S:8}
A	{B:9,B:9,C:11}
B	{G:9,C:11,D:∞,E:∞}
G goal	{C:11,D:∞,E:∞}

not expanded



41

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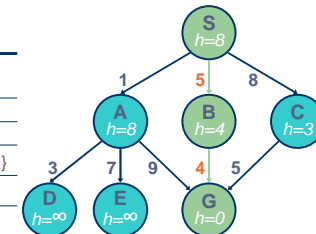
## A\* Search

$$f(n) = g(n) + h(n)$$

# of nodes tested: 4, expanded: 3

expnd. node	OPEN list
S	{S:8}
A	{B:9,B:9,C:11}
B	{G:9,C:11,D:∞,E:∞}
G	{C:10,C:11,D:∞,E:∞}

\* Pretty fast and optimal



path: S,B,G  
cost: 9

42

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## Proof by Contradiction of A\* Optimality

- Let:
  - G be the optimal goal
  - G2 be a sub-optimal goal
  - $f^*$  be the cost of the optimal path from start to G
- $g(G2) > f^*$ 
  - assume G2 is found using A\*
  - where  $f(n) = g(n) + h(n)$  and  $h(n)$  is admissible
- \* That is, A\* found a sub-optimal path (which it shouldn't)

43

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## Proof by Contradiction of A\* Optimality

- Let  $n$  be some node on the optimal path but not on the path to G2
- $f(n) \leq f^*$ 
  - by admissibility, since  $f(n)$  never overestimates the cost to the goal it must be  $\leq$  the cost of the optimal path
- $f(G2) \leq f(n)$ 
  - G2 was chosen over  $n$  for the sub-optimal goal to be found
- $f(G2) \leq f^*$ 
  - combining equations

44

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## Proof by Contradiction of A\* Optimality

- $f(G2) \leq f^*$
- $g(G2) + h(G2) \leq f^*$   
substituting the definition of  $f$
- $g(G2) \leq f^*$   
 $h(G2) = 0$  since  $G2$  is a goal node
- This contradicts the assumption that  $G2$  was sub-optimal,  $g(G2) > f^*$
- \* Therefore,  $A^*$  is optimal with respect to path cost;  $A^*$  search never finds a sub-optimal goal

45

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## Devising Heuristics

- Are often devised by **relaxing the problem**  
compute exact cost of a solution to a *simplified* version
  - remove constraints: 8-puzzle movement
  - simplify problem: straight line distance for actual
  - see text

46

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## Devising Heuristics

- \* *Goal of an admissible heuristic is to get as close to the actual cost without going over*
- \* *Must also be relatively fast to compute*
- Trade off:  
using time to compute complex heuristic versus  
using time expand more nodes with a simpler heuristic

47

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## Devising Heuristics

- If  $h(n) = h^*(n)$  for all  $n$ ,
  - only nodes on optimal solution path are expanded
  - no unnecessary work is performed
- If  $h(n) = 0$  for all  $n$ ,
  - the heuristic is admissible
  - $A^*$  performs exactly as Uniform-Cost Search (UCS)
- \* *The closer  $h$  is to  $h^*$ , the fewer extra nodes that will be expanded*

48

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## Devising Heuristics

- If  $h1(n) \leq h2(n) \leq h^*(n)$  for all  $n$  that aren't goals then  $h2$  **dominates**  $h1$ 
  - $h2$  is a *better heuristic* than  $h1$
  - $A^*$  using  $h1$  (i.e.,  $A1^*$ ) expands *at least as many* if not more nodes than using  $A^*$  with  $h2$  (i.e.,  $A2^*$ )
  - $A2^*$  is said to be **better informed** than  $A1^*$

49

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## Devising Heuristics

- For an admissible heuristic
  - $h$  is frequently very simple
  - therefore search resorts to (almost) UCS through parts of the search space

50

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## Devising Heuristics

- If optimality is *not* required, i.e., **satisficing solution** okay, then
  - \* *Goal of heuristic is then to get as close as possible, either under or over, to the actual cost*
- It results in many fewer nodes being expanded than using a poor but provably admissible heuristic

51

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## Devising Heuristics

- Unfortunately,  $A^*$  often suffers because it cannot venture down a single path unless it is almost continuously having success (i.e.,  $h$  is decreasing); any failure to decrease  $h$  will almost immediately cause the search to switch to another path

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## Local Searching

- **Systematic searching:** search for a path from start state to a goal state, then “execute” solution path’s sequence of operators
  - BFS, IDS, UCS, Greedy Best-First, A, A\* etc.
  - **ok** for small search spaces that are often “toy world” problems
  - **not ok** for NP-Hard problems requiring exponential time to find the optimal solution

53

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## Traveling Salesperson Problem (TSP)

- **Classic NP-Hard problem:**
  - A salesman wants to visit a list of cities
    - stopping in each city only once
    - returning to the first city
    - traveling the shortest distance

54

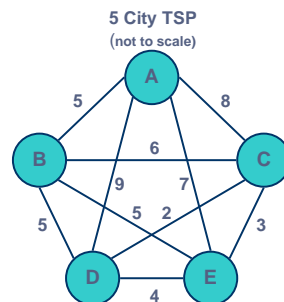
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## Traveling Salesperson Problem (TSP)

Nodes are cities  
Arcs are labeled with distances between cities  
Adjacency matrix (notice the graph is fully connected):

	A	B	C	D	E
A	0	5	8	9	7
B	5	0	6	5	5
C	8	6	0	2	3
D	9	5	2	0	4
E	7	5	3	4	0



55

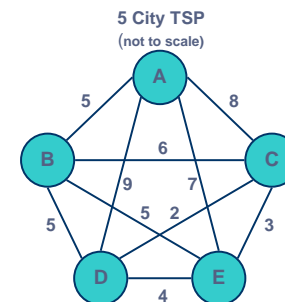
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## Traveling Salesperson Problem (TSP)

a solution is a permutation of cities, called a **tour**

	A	B	C	D	E
A	0	5	8	9	7
B	5	0	6	5	5
C	8	6	0	2	3
D	9	5	2	0	4
E	7	5	3	4	0



56

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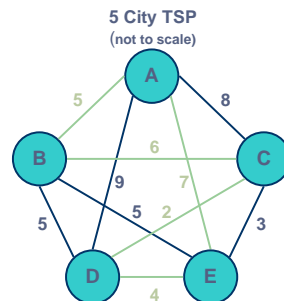
## Traveling Salesperson Problem (TSP)

a solution is a permutation of cities, called a **tour**

e.g. A – B – C – D – E

assume tours return home

	A	B	C	D	E
A	0	5	8	9	7
B	5	0	6	5	5
C	8	6	0	2	3
D	9	5	2	0	4
E	7	5	3	4	0



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## Traveling Salesperson Problem (TSP)

How many solutions exist?

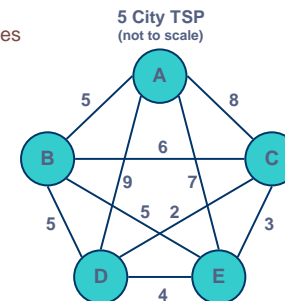
$(n-1)! / 2$  where  $n = \#$  of cities

$n = 5$  results in 12 tours

$n = 10$  results in 181440 tours

$n = 20$  results in  $\sim 6 \cdot 10^{16}$  tours

	A	B	C	D	E
A	0	5	8	9	7
B	5	0	6	5	5
C	8	6	0	2	3
D	9	5	2	0	4
E	7	5	3	4	0



58

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## Other Example Problems

- **N-Queens**

- Place  $N$  queens on  $N \times N$  checkerboard so that no one can capture another

- **Boolean Satisfiability**

- Given a Boolean expression containing  $N$  Boolean variables, find an assignment of  $\{T, F\}$  to each variable so that the expression evaluates to  $T$
- $(A \vee \neg B \vee C) \wedge (\neg A \vee C \vee D)$

59

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## Local Searching

- Hard problems can be solved in a reasonable time (i.e., polynomial) by using either:
  - **approximate model**: find an exact solution to a simpler version of the problem
  - **approximate solution**: find a non-optimal solution of the original hard problem
- Next we'll explore means to search through a **solution space** by **iteratively improving** solutions until one is found that is optimal or near optimal

60

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## Local Searching

- **Local searching:** every node is a solution
  - operators go from one solution to another
  - can stop any time and have a valid solution
  - search is now finding a *better* solution
- \* **No longer searching state space for a solution path and then executing the steps of the solution path**
- A\* isn't local searching since it searches different partial solutions by looking at estimated cost of a solution path

61

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## Local Searching

- \* **An operator is needed to transform one solution to another!**
- **TSP: two-swap (common)**
  - take two cities and swap their location in the tour
  - e.g. A-B-C-D-E swap(A,D) yields D-B-C-A-E
  - possible since graph is fully connected
- **TSP: two-interchange**
  - reverse the path between two cities
  - e.g. A-B-C-D-E interchange(A,D) yields D-C-B-A-E

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## Local Searching

- Those solutions that can be reached with one application of an operator are in the current solution's **neighborhood**
- **Local search** considers only those solutions in the neighborhood
- The neighborhood should be much smaller than the size of the search space (otherwise the search degenerates)

63

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## Local Searching

- \* **An evaluation function  $f(n)$  is used to map each solution to a number corresponding to the quality of that solution**
- **TSP:** Use the distance of the tour path; A better solution has a shorter tour path
- **Maximize  $f(n)$ :**  
called **hill climbing** (**gradient ascent** if continuous)
- **Minimize  $f(n)$ :**  
called **valley finding** (**gradient descent** if continuous)
- **Can be used to maximize/minimize some cost**

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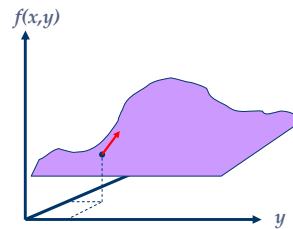
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## Local Searching

Visualized as a 2D surface

- Height is quality of solution  
 $f = f(x, y)$
- Solution space is a 2D surface
- Initial solution is a point
- Goal is to find a higher point on the surface of solution space
- **Hill-Climbing** follows the direction of the steepest ascent, i.e., where  $f$  increases the most



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## Local Searching

```
Node hillClimbing (Problem problem) {
    Node currentNode = new Node(problem.getStartPoint());
    while (true) {
        Node newNode = best neighbor of currentNode;
        //neighbor with highest evaluation (lowest for VF)
        if (problem.eval(newNode) >= problem.eval(currentNode))
            //(<= for VF: valley finding)
            //to avoid cycles a CLOSED list could be used
            currentNode = newNode;
        else
            return currentNode;
    }
}
```

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## Hill-Climbing (HC)

- **HC exploits the neighborhood**
  - like Greedy Best-First search it chooses what looks best *locally*
  - but **doesn't allow backtracking or jumping to an alternative path** since there is no OPEN list
- **HC is very space efficient**
  - Like Beam search with a beam width of 1 recall beam width is size of OPEN list
  - but if used, CLOSED nodes list could become large
- **HC is very fast and effective in practice**

67

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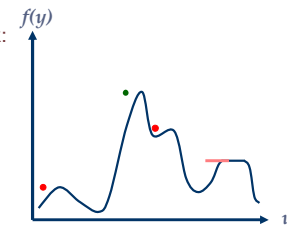
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## Hill-Climbing (HC)

Solution found by HC is totally determined by the starting point; fundamental weakness is getting stuck:

- **at local maximum**
- **at plateaus and ridges**
- **global maximum may not be found**

**Trade off:**  
greedily exploiting locality as with HC vs. exploring state space as with BFS



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## Hill-Climbing with Restarts

- Run HC multiple times (independently) with randomly generated start solutions
- Use best solution found
- Fast, easy to implement, works well for many applications where the solution space surface is not too “bumpy” (i.e., not too many local maxima)

69

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## Escaping Local Maxima

- \* *HC gets stuck at a local maximum, limiting the quality of the solution found*
- Two ways to modify HC:
  1. choice of neighbor
  2. criteria for deciding to move to neighbor
- For example:
  1. choose neighbor randomly
  2. move to neighbor if it is better or if it isn't, accept with some fixed probability  $p$

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## Escaping Local Maxima

- Modified HC can escape from local maxima but
  - chance of making a bad move is the same at the beginning of the search as at the end
  - magnitude of improvement or lack of is ignored
- Fix by replacing fixed probability  $p$  that a bad move is accepted with a probability that decreases as the search proceeds
- Now as the search progresses, the chances of taking a bad move reduces

71

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## Escaping Local Maxima

- Lets change to Valley Finding (VF):
  - want to find the global minima (lowest solution)
  - search for a new node with a lower value of  $f(n)$
  - replace fixed probability  $p$  with a temperature  $T$  that decreases as the search proceeds:

$$e^{(eval(currentNode) - eval(newNode)) / T}$$

72

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## Escaping Local Maxima

Let  $\Delta E = \text{eval}(\text{currentNode}) - \text{eval}(\text{newNode})$

$p = e^{\Delta E / T}$  (Boltzman's equation)

- $\Delta E \rightarrow -\infty, p \rightarrow 0$   
as badness (VF) of the move *increases*  
probability of taking it *decreases* exponentially
- $T \rightarrow 0, p \rightarrow 0$   
as temperature *decreases*  
probability of taking bad move *decreases*

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## Escaping Local Maxima

Let  $\Delta E = \text{eval}(\text{currentNode}) - \text{eval}(\text{newNode})$

$p = e^{\Delta E / T}$  (Boltzman's equation)

- $\Delta E \ll T$   
if badness of move (VF) small compare to T  
move likely to be accepted
- $\Delta E \gg T$   
if badness of move (VF) large compare to T  
move unlikely to be accepted

74

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## Simulated Annealing (SA)

```
//finds minima, VF valley finding
Node simulatedAnnealing (Problem problem) {
    int temperature = problem.getInitialTemperature();
    Node currentNode = new Node(problem.getStartPoint());
    Node bestNode = currentNode;
    while (temperature > problem.getStoppingTemperature()) {
        Node newNode = random neighbor of currentNode;
        check if newNode is accepted
        check if newNode is better than bestNode
        temperature = problem.schedule(temperature);
    }
    return bestNode;
}
```

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## Simulated Annealing (SA)

```
//check if newNode is accepted
int deltaE = problem.eval(currentNode) - problem.eval(newNode);
if (deltaE > 0) //VF: newNode is better
    currentNode = newNode;
else //allow for backtracking
    currentNode = newNode with probability  $p = e^{(\text{deltaE}/T)}$ ;

//check if newNode is better than bestNode
//needed since current node may not be best
if (problem.eval(currentNode) < problem.eval(bestNode))
    bestNode = currentNode;
```

76

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## Simulated Annealing (SA)

- Can perform multiple backward steps in a row to escape local optimum
- Chance of finding a global optima increased
- Fast
  - only one neighbor generated each iteration
  - whole neighborhood isn't checked to find best neighbor as in HC
- Usually finds a good quality solution in a very short amount of time

77

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## Simulated Annealing (SA)

- Requires parameters to be set
  - starting temperature
    - must be high enough to escape local optima but not too high to be random exploration of space
  - cooling schedule
    - typically exponential
  - halting temperature
- Domain knowledge helps set values: size of search space, bounds of maximum and minimum solutions

78

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## Summary

- **Systematic Searching**
  - \* look through state space for a goal from which solution path can be determined
  - **nodes**: state descriptions, partial solution path
  - **arcs**: operator changes state for some cost
  - **solution**: sequence of operators that change from start to goal state

79

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## Summary

- **Systematic Searching**
  - **Uninformed**: domain info not used to guide search
    - complete/optimal if arc costs uniform: BFS, IDS
    - complete/optimal: UCS (uses arc costs)
    - not complete/optimal: DFS
  - **Informed**: domain info weakly used to guide search
    - $g(n)$ : cost from start to  $n$
    - $h(n)$ : estimated cost from  $n$  to goal, **heuristic**
    - $f(n) = g(n) + h(n)$ : estimated cost of solution through  $n$
    - complete/optimal: A\* when  $h(n)$  admissible
    - not complete/optimal: Greedy Best-First, Beam, A

80

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## Summary

- **Local Searching**

- \* *Iteratively improve solution*

- **nodes**: complete solution
  - **arcs**: operator changes to another solution
  - can stop at any time
  - technique suited for:
    - hard problems, e.g., TSP
    - optimization problems

81

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## Summary

- **Local Searching**

- $f(n)$  evaluates quality of solution by weakly using domain knowledge
  - HC: maximizes  $f(n)$ , VF: minimizes  $f(n)$ 
    - solution found determined by starting point
    - can get stuck, which prevents finding global optimum
  - SA: explores, then settles down
    - bad moves accepted with probability that decreases as the search progress (T decreases) with the badness of move ( $\Delta E$  worsens)
    - requires parameters to be set

82

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