CS 436
Exam 3 Solutions

Wednesday November 15, 2006

DUE FRIDAY NOVEMBER 17, 2006 at 12:00 Noon.

Take Home Exam

You may NOT work with anyone on this exam. You may not use the Internet.
Use your mind, the book and a pencil.

Name **Solutions**

Problem	Points	Max Points
0	______	**20**
1	______	**20**
2	______	**20**
3	______	**20**
4	______	**20**
5	______	**20**
TOTAL	______	**120**

0) Model Counting (20 points)

Consider a propositional language with four symbols, A, B, C, and D. How many models are there for each of the following sentences?

(a) B ∨ C

12 <= 3 for B or C times 4 for all values of A, D

(b) ¬A ∨ ¬B ∨ ¬C ∨ ¬D

15 <= Only invalid assignment is A=B=C=D=True

(c) (A => B) ^ A ^ ¬B ^ C ^ D

0 <= First three conjuncts are invalid as a unit.

(d) A ^ B

4 <= A=B=True. Four total models with values of T/F for C, D.

(e) A <=> B <=> C

4 <= 2 A=B=C= True or False, times 2 possbilities for C T/F

(f) (A ^ B) ∨ (B ^ C)

6 <= 3 for above times 2 for D=T/F

1) Working With the Rules of Logic (20 points)

Decide whether each of the following sentences is valid, unsatisfiable, or neither. Verify your decisions using truth tables or the equivalence rules of Figure 7.11.

(a) Rain => Rain

Rain	**Rain => Rain**
False	True
True	True

Valid, last column all true

(b) Rain => Wet

Rain	**Wet**	**Rain => Wet**
TRUE	TRUE	TRUE
TRUE	FALSE	FALSE
FALSE	TRUE	TRUE
FALSE	FALSE	TRUE

Last column contains both true and false, so it is not valid. It is satisfiable however, thus it is neither valid or unsatisfiable.

(c) (Rain ∨ Wet) => (¬Rain => ¬Wet)

Rain	**Wet**	**~Rain**	**~Wet**	**Rain V Wet**	**~Rain => ~Wet**	**(Rain V Wet) => (¬Rain => ¬Wet)**
TRUE	TRUE	FALSE	FALSE	FALSE	TRUE	TRUE
TRUE	FALSE	FALSE	TRUE	TRUE	TRUE	TRUE
FALSE	TRUE	TRUE	FALSE	TRUE	FALSE	FALSE
FALSE	FALSE	TRUE	TRUE	TRUE	TRUE	TRUE

Last column contains both true and false, so it is not valid. It is satisfiable however, thus it is neither valid or unsatisfiable.

(d) Rain ∨ Wet ∨ ¬Wet

Rain	**Wet**	**~Wet**	**Rain V Wet V ~Wet**
TRUE	TRUE	FALSE	TRUE
TRUE	FALSE	TRUE	TRUE
FALSE	TRUE	FALSE	TRUE
FALSE	FALSE	TRUE	TRUE

Valid, last column all true

(e) ((Rain ^ Clouds) => Wet) <=> ((Rain => Wet) V (Clouds => Wet))

Rain	**Clouds**	**Wet**	**Rain ^ Clouds**	**(Rain ^ Clouds) => Wet**	**Rain => Wet**	**Clouds => Wet**	**(Rain => Wet) V (Clouds => Wet)**	**((Rain ^ Clouds) => Wet) <=> ((Rain => Wet) V (Clouds => Wet))**
TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
TRUE	TRUE	FALSE	TRUE	FALSE	FALSE	FALSE	FALSE	TRUE
TRUE	FALSE	TRUE	FALSE	TRUE	TRUE	TRUE	TRUE	TRUE
TRUE	FALSE	FALSE	FALSE	TRUE	FALSE	TRUE	TRUE	TRUE
FALSE	TRUE	TRUE	FALSE	TRUE	TRUE	TRUE	TRUE	TRUE
FALSE	TRUE	FALSE	FALSE	TRUE	TRUE	FALSE	TRUE	TRUE
FALSE	FALSE	TRUE	FALSE	TRUE	TRUE	TRUE	TRUE	TRUE
FALSE	FALSE	FALSE	FALSE	TRUE	TRUE	TRUE	TRUE	TRUE

Valid, last column all true

(f) (Rain => Wet) => ((Rain ^ Clouds) => Wet)

Rain	**Wet**	**Clouds**	**Rain => Clouds**	**Rain ^ Wet**	**(Rain ^ Wet) => Clouds**	**(Rain => Clouds) => ((Rain ^ Wet) => Clouds)**
TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
TRUE	TRUE	FALSE	FALSE	TRUE	FALSE	TRUE
TRUE	FALSE	TRUE	TRUE	FALSE	TRUE	TRUE
TRUE	FALSE	FALSE	FALSE	FALSE	TRUE	TRUE
FALSE	TRUE	TRUE	TRUE	FALSE	TRUE	TRUE
FALSE	TRUE	FALSE	TRUE	FALSE	TRUE	TRUE
FALSE	FALSE	TRUE	TRUE	FALSE	TRUE	TRUE
FALSE	FALSE	FALSE	TRUE	FALSE	TRUE	TRUE

Valid, last column all true

(g) Elephant V Smart V (Elephant => Smart)

Elephant	**Smart**	**Elephant => Smart**	**Elephant V Smart V (Elephant => Smart)**
TRUE	TRUE	FALSE	TRUE
TRUE	FALSE	FALSE	TRUE
FALSE	TRUE	TRUE	TRUE
FALSE	FALSE	TRUE	TRUE

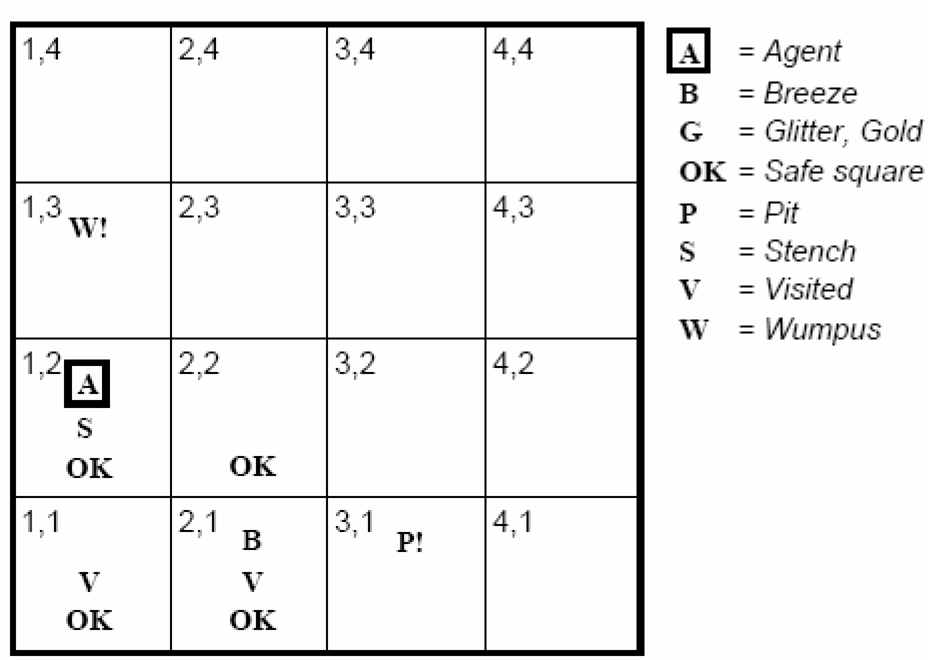
Valid, last column all true

(h) (Elephant ^ Smart) ^ ¬Smart

Elephant	Smart	Elephant ^ Smart	~ Smart	(Elephant ^ Smart) V ~ Smart
TRUE	TRUE	TRUE	FALSE	TRUE
TRUE	FALSE	FALSE	TRUE	TRUE
FALSE	TRUE	FALSE	FALSE	FALSE
FALSE	FALSE	FALSE	TRUE	TRUE

Last column contains both true and false, so it is not valid. It is satisfiable however, thus it is neither valid or unsatisfiable.

2) Entailment (20 points)



(a)

Suppose the agent has progressed to the point shown in above [Figure 7.4(a) in the book], having perceived nothing in [1,1], a breeze in [2,1], and a stench in [1,2], and is now concerned with the contents of [1,3], [2,2], and [3,1]. Each of these can contain a pit and at most one can contain a wumpus.

List the 'Worlds' in which the KB is true and those in which each of the following sentences is true:

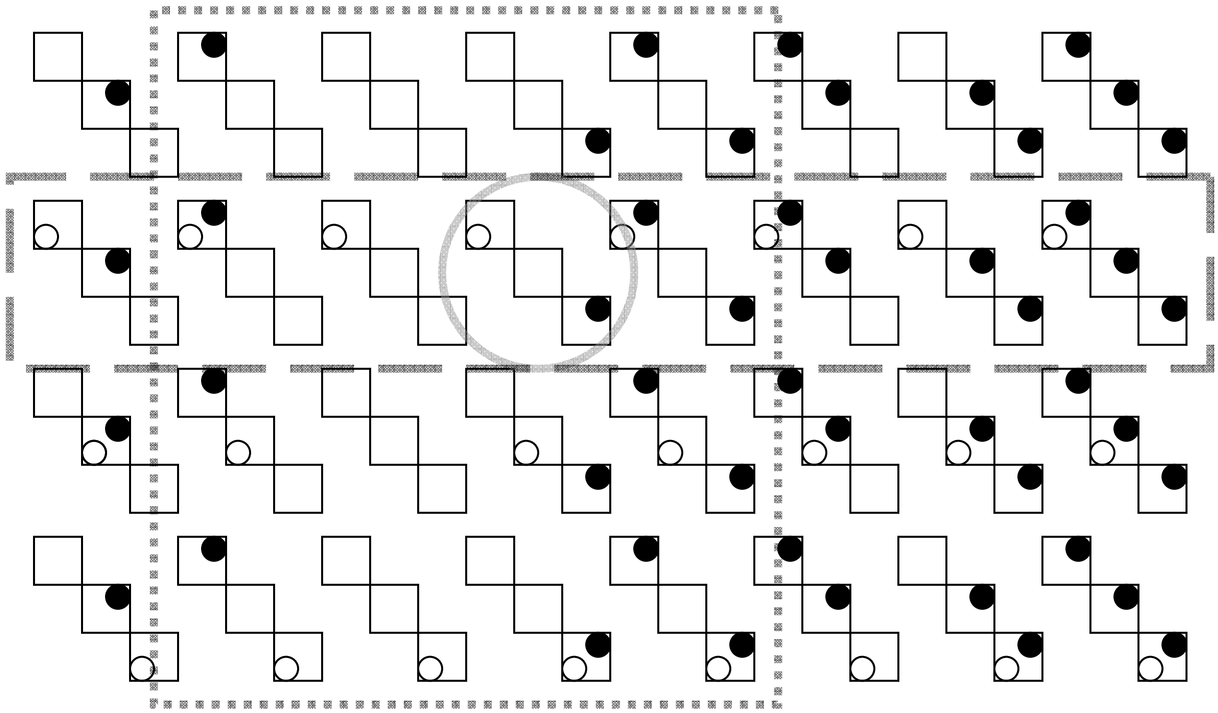
$\alpha 2$ = "There is no pit in [2,2]."
$\alpha 3$ = "There is a wumpus in [1,3]."

Hence show that $KB \models \alpha 2$ and $KB \models \alpha 3$.

Given the KB, there is a stench at [1,2] and no stench in [2,1], implying that the wumpus is in [1,3]. There is a breeze at [2,1] and none at [1,2], implying there is a pit in [3,1] and no pits in [2,2] or [1,3]. The model of the KB contains only one configuration meeting this criteria.

$\alpha 2$ = "There is no pit in [2,2]". Dotted boundary on next page
$\alpha 3$ = "There is a wumpus in [1,3]". Dashed boundary on next page
The KB entails both of these statements.

The wumpus a circle and pits are black dots.



3) CNF Conversion & Satisfiability (20 points)
Consider the following set of sentences on propositional calculus:

(a) Convert them to CNF

1) A => (B <=> C)
A => (B => C) ^ (C => B)
A => (¬B v C) ^ (¬C v B)
¬A v [(¬B v C) ^ (¬C v B)]
(¬A v ¬B v C) ^ (¬A v ¬C v B)

2) C ^ D => ¬B
¬ (C ^ D) v ¬B
¬C v ¬D v ¬B

3) E => (B v C)
¬E v (B v C)
¬E v B v C

4) A ^ ¬D => E
¬(A ^ ¬D) v E
¬A v D v E

5) E => D
¬E v D

6) ¬A => C
A v C

(b) Which two algorithms in chapter can be used to determine if there is a satisfying assignment of T/F for the 5 symbols A, B, C, D, E? Explain how one of them works.

DPLL and WalkSAT. See text for explaination of how they function.

4) Resolution (20 points)

Consider the following sentence:

[(Hamburger => Dinner) V (Soda => Dinner)] => [(Hamburger ^ Soda) => Dinner]

(a) Determine whether this sentence is valid, satisfiable (but not valid), or unsatisfiable, using enumeration. (You may abbreviate the proposition symbols in your answer.)

H	S	D	H => D	S => D	(H=>D) V (S=>D)	(H ^ S)	(H ^ S) => D	Total
TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
TRUE	TRUE	FALSE	FALSE	FALSE	FALSE	TRUE	FALSE	TRUE
TRUE	FALSE	TRUE	TRUE	TRUE	TRUE	FALSE	TRUE	TRUE
TRUE	FALSE	FALSE	FALSE	TRUE	TRUE	FALSE	TRUE	TRUE
FALSE	TRUE	TRUE	TRUE	TRUE	TRUE	FALSE	TRUE	TRUE
FALSE	TRUE	FALSE	TRUE	FALSE	TRUE	FALSE	TRUE	TRUE
FALSE	FALSE	TRUE	TRUE	TRUE	TRUE	FALSE	TRUE	TRUE
FALSE	FALSE	FALSE	TRUE	TRUE	TRUE	FALSE	TRUE	TRUE

Valid

(b) Convert the left-hand and right-hand sides of the main implication into CNF, showing each step, and explain how the results confirm your answer to (a).

[(Hamburger => Dinner) V (Soda => Dinner)] => [(Hamburger ^ Soda) => Dinner]

LHS converts to
[(¬Hamburger V Dinner) V (¬Soda V Dinner)]
[¬Hamburger V Dinner V ¬Soda V Dinner]
[¬Hamburger V Dinner V ¬Soda]

RHS coverts to
[(Hamburger ^ Soda) => Dinner]
[¬ (Hamburger ^ Soda) V Dinner]
[(¬Hamburger V ¬Soda) V Dinner]
[¬Hamburger V ¬Soda V Dinner]

Two sides are identical.. and P => P is always valid for any P.

(c) Prove your answer to (a) using resolution
Convert negation to CNF. Prove the negation of the sentence is unsatisfiable and your prove the sentence is valid.

¬ [[(Hamburger => Dinner) V (Soda => Dinner)] => [(Hamburger ^ Soda) => Dinner]]
[(Hamburger => Dinner) V (Soda => Dinner)] ^ ¬ [(Hamburger ^ Soda) => Dinner]
(¬Hamburger V ¬Soda V Dinner) ^ Hamburger ^ Soda ^ ¬Dinner
Resolve further and we get the empty clause.. meaning the orginal sentence is valid.

5) English to Logic Conversion (20 points)

Adapted from Barwise and Etchemendy (1993)

If the jackalope is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the jackalope is either immortal or a mammal, then it is horned. The jackalope is magical if it is horned.

(a) Convert this into Propositional Logic sentences

Mythical => ¬Mortal
¬Mythical => (Mortal ^ Mammal)
(¬Mortal v Mammal) => Horned
Horned => Magical

(b) Can you prove that the jackalope is mythical? How about magical? Horned?

We cannot prove the jackalope is either Mythical or ¬Mythical. No contradiction results from adding either of these statements to the KB.. meaning we can not prove either.

We can prove the jackalope is Magical and Horned. Negating either and adding to the KB we can find a contradiction.