Artificial Intelligence MSc
– Exam Sample Answers –

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Answers 1: Propositional Logic

(a)

A formula is

- *satisfiable* if there is a truth assignment that makes it true;

(1 Point)

- *falsifiable* if there is a truth assignment that makes it false;

(1 Point)

- *unsatisfiable* if every truth assignment makes it false;

(1 Point)

- *valid* if every truth assignment makes it true.

(1 Point)

(b)

(i) Below are the truth tables for the three formulas

1. $p \wedge \neg q$

p	q	$\neg q$	$p \wedge \neg q$
F	F	T	F
F	T	F	F
T	F	T	T
T	T	F	F

(1 Point)

2. $p \wedge \neg p$

p	$\neg p$	$p \wedge \neg p$
F	T	F
T	F	F

(1 Point)

3. $(p \wedge \neg p) \rightarrow (p \vee q)$

p	q	$p \wedge \neg p$	$p \vee q$	$(p \wedge \neg p) \rightarrow (p \vee q)$
F	F	F	F	T
F	T	F	T	T
T	F	F	T	T
T	T	F	T	T

(1 Point)

(ii) Based on the truth tables, we find that the formulas have the following properties

1. $p \wedge \neg q$: satisfiable, falsifiable, not unsatisfiable, not valid

(1 Point)

2. $p \wedge \neg p$: not satisfiable, falsifiable, unsatisfiable, not valid

(1 Point)

3. $(p \wedge \neg p) \rightarrow (p \vee q)$: satisfiable, not falsifiable, not unsatisfiable, valid

(1 Point)

(c)

(i) We let the proposition `s` stand for "the sun shines", `h` for "we go for a hike", `m` for "we go to the museum".

Then we can write the four sentences as follows:

S_1: $\mathtt{s} \rightarrow \mathtt{h}$

(1 Point)

S_2: $\neg\mathtt{h} \rightarrow \mathtt{m}$

(1 Point)

C_1: $\neg\mathtt{m} \rightarrow \mathtt{s}$

(1 Point)

C_2: $(\mathtt{s} \vee \neg\mathtt{m}) \rightarrow \mathtt{h}$

(1 Point)

(ii) One can check either one of the following

- $(S_1 \wedge S_2) \rightarrow C$ is valid
- $\neg(S_1 \wedge S_2) \wedge \neg C$ is unsatisfiable

(2 Points)

(iii) We check whether $F = (S_1 \wedge S_2) \rightarrow C_1$ is valid. To this end, we set up a truth table for

$$F = ((\mathtt{s} \rightarrow \mathtt{h}) \wedge (\neg\mathtt{h} \rightarrow \mathtt{m})) \rightarrow (\neg\mathtt{m} \rightarrow \mathtt{s})$$

s	h	m	$\mathtt{s} \rightarrow \mathtt{h}$	$\neg\mathtt{h} \rightarrow \mathtt{m}$	$(\mathtt{s} \rightarrow \mathtt{h}) \wedge (\neg\mathtt{h} \rightarrow \mathtt{m})$	$\neg\mathtt{m} \rightarrow \mathtt{s}$	F
F	F	F	T	F	F	F	T
F	F	T	T	T	T	T	T
F	T	F	T	T	T	F	F
F	T	T	T	T	T	T	T
T	F	F	F	F	F	T	T
T	F	T	F	T	F	T	T
T	T	F	T	T	T	T	T
T	T	T	T	T	T	T	T

The third line has the truth value F. Therefore, F is not valid and C_1 does not follow from S_1 and S_2.

(4 Points)

(iv) We check whether $F = (S_1 \wedge S_2) \rightarrow C_2$ is valid. That is, whether

$$F = ((\mathtt{s} \rightarrow \mathtt{h}) \wedge (\neg\mathtt{h} \rightarrow \mathtt{m})) \rightarrow ((\mathtt{s} \vee \neg\mathtt{m}) \rightarrow \mathtt{h})$$

is valid.

This can be checked using a truth table. It can also be checked using equivalence transformations if we transform F into T.

First, we apply $\neg p \rightarrow q \equiv \neg q \rightarrow p$ to $\neg\mathtt{h} \rightarrow \mathtt{m}$. This yields

$$F \equiv ((\mathtt{s} \rightarrow \mathtt{h}) \wedge (\neg\mathtt{m} \rightarrow \mathtt{h})) \rightarrow ((\mathtt{s} \vee \neg\mathtt{m}) \rightarrow \mathtt{h})$$

Then we apply $p \rightarrow q \equiv \neg p \vee q$, which yields

$$F \equiv ((\neg\mathtt{s} \vee \mathtt{h}) \wedge (\neg\neg\mathtt{m} \vee \mathtt{h})) \rightarrow (\neg(\mathtt{s} \vee \neg\mathtt{m}) \vee \mathtt{h})$$

Then we apply $\neg(p \vee q) \equiv \neg p \wedge \neg q$, which yields

$$F \equiv ((\neg\mathtt{s} \vee \mathtt{h}) \wedge (\neg\neg\mathtt{m} \vee \mathtt{h})) \rightarrow ((\neg\mathtt{s} \wedge \neg\neg\mathtt{m}) \vee \mathtt{h})$$

Then we apply the distributive law $(p \wedge q) \vee r \equiv (p \vee r) \wedge (p \vee r)$, which yields

$$F \equiv ((\neg\mathtt{s} \vee \mathtt{h}) \wedge (\neg\neg\mathtt{m} \vee \mathtt{h})) \rightarrow ((\neg\mathtt{s} \vee \mathtt{h}) \wedge (\neg\neg\mathtt{m} \vee \mathtt{h}))$$

Then we apply $p \rightarrow p \equiv \mathtt{T}$, which yields

$$F \equiv \mathtt{T}$$

(5 Points)

Answers 2: Resolution and Prolog

(a)

The steps are

- eliminate implications using the equivalence $p \rightarrow q \equiv \neg p \vee q$;
- push negations downward using using De Morgan's laws $\neg(p \wedge q) \equiv \neg p \vee \neg q$, $\neg(p \vee q) \equiv \neg p \wedge \neg q$ and the double negation law $\neg\neg p \equiv p$;
- push disjunctions downward using the distributive law $p \vee (q \wedge r) \equiv (p \wedge q) \vee (p \wedge r)$ and the identity laws $p \vee \neg p \equiv \mathtt{T}$ and $\mathtt{T} \vee p \equiv \mathtt{T}$.

(3 Points)

(b)

The conjunctive normal forms can be derived as follows:

(i)

$$\begin{aligned}
& \neg(a \rightarrow (b \vee c)) \wedge (b \rightarrow (a \wedge c)) \\
\equiv\ & \neg(\neg a \vee (b \vee c)) \wedge (\neg b \vee (a \wedge c)) \\
\equiv\ & (\neg\neg a \wedge \neg(b \vee c)) \wedge (\neg b \vee (a \wedge c)) \\
\equiv\ & (a \wedge (\neg b \wedge \neg c)) \wedge (\neg b \vee (a \wedge c)) \\
\equiv\ & (a \wedge \neg b \wedge \neg c \wedge (\neg b \vee a) \wedge (\neg b \vee c))
\end{aligned}$$

(3 Points)

(ii)

$$\begin{aligned}
& (a \wedge (a \rightarrow b)) \rightarrow b \\
\equiv\ & \neg(a \wedge (\neg a \vee b)) \vee b \\
\equiv\ & (\neg a \vee \neg(\neg a \vee b)) \vee b \\
\equiv\ & (\neg a \vee (\neg\neg a \wedge \neg b)) \vee b \\
\equiv\ & (\neg a \vee (a \wedge \neg b)) \vee b \\
\equiv\ & ((\neg a \vee a) \wedge (\neg a \vee \neg b)) \vee b \\
\equiv\ & ((\neg a \vee a \vee b) \wedge (\neg a \vee \neg b \vee b)) \\
\equiv\ & ((\mathtt{T} \vee b) \wedge (\neg a \vee \mathtt{T})) \\
\equiv\ & \mathtt{T} \wedge \mathtt{T} \equiv \mathtt{T}
\end{aligned}$$

(5 Points)

(c)

(i) We let the proposition $\texttt{v}$ stand for "Joe has made a vacation trip", $\texttt{t}$ for "Joe has got a tan", $\texttt{m}$ for "Joe was in the mountains", $\texttt{b}$ for "Joe was at the beach".

This information can be captured in the formula:

$$(\texttt{v} \rightarrow \texttt{b} \vee \texttt{m}) \wedge (\neg\texttt{t} \rightarrow \neg\texttt{b}) \wedge (\texttt{m} \rightarrow \texttt{t}) \wedge \neg\texttt{t}$$

(2 Points)

(ii) The conjunctive normal form of that formula is

$$(\neg\texttt{v} \vee \texttt{b} \vee \texttt{m}) \wedge (\texttt{t} \vee \neg\texttt{b}) \wedge (\neg\texttt{m} \vee \texttt{t}) \wedge \neg\texttt{t}$$

(1 Point)

(iii) We write the above formula as a collection of clauses and apply resolution:

$$\begin{array}{ll} & \{\neg\texttt{v}, \texttt{b}, \texttt{m}\}, \{\texttt{t}, \neg\texttt{b}\}, \{\neg\texttt{m}, \texttt{t}\}, \{\neg\texttt{t}\} \\ \vdash & \{\neg\texttt{b}\}, \{\neg\texttt{m}\} \\ \vdash & \{\neg\texttt{v}, \texttt{m}\} \\ \vdash & \{\neg\texttt{v}\} \end{array}$$

We have derived the clause $\neg\texttt{v}$. Hence, Joe has not made a vacation trip.

(2 Points)

(iv) Since resolution is a sound inference rule, all the clauses we have derived represent logical consequences of our original information.

(1 Point)

(d)

The predicates can be defined as follows

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mother(X) :- female(X), parent(X,Y).

brother(X,Y) :- parent(X,Z), parent(Y,Z), male(X).

uncle(X,Y) :-  brother(X,Z), parent(Z,Y).

granddaughter(X,Y) :- parent(Y,Z), parent(Z,X), female(X).
```

(8 Points)