Artificial Intelligence MSc

– Exam Questions –

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3 Propositional Logic

(a)

A propositional formula can be

- satisfiable
- falsifiable
- unsatisfiable
- valid.

Explain what is meant by each of the four properties.

(4 Points)

(b)

Consider the following propositional formulas

1. $p \wedge \neg q$
2. $p \wedge \neg p$
3. $(p \wedge \neg p) \rightarrow (p \vee q)$

(i) For each formula, set up a truth table.

(3 Points)

(ii) For each formula, state which of the four properties satisfiable, falsifiable, unsatisfiable, and valid apply to it and which not.

(3 Points)

(c)

In this part of the question, we want to analyse whether some sentences about our weekend plans are logical consequences of other sentences.

Consider the following two statements:

S_1: If the sun shines, then we go for a hike

S_2: If we do not go for a hike, then we go to the museum

and the two claims

C_1: If we do not go to the museum, then the sun shines.

C_2: If the sun shines or we do not go to the museum, then we go for a hike.

(i) Translate the statements and the claims into propositional logic, using appropriate atomic propositions.

(4 Points)

(ii) Explain how one can check whether a claim C is a logical consequence of the two sentences S_1 and S_2.

(2 Points)

(iii) Use an approach of your choice to check whether C_1 is a logical consequence of S_1 and S_2.

(4 Points)

(iv) Use an approach of your choice to check whether C_2 is a logical consequence of S_1 and S_2.

(5 Points)

4 Resolution and Prolog

(a)

Every propositional formula can be transformed into an equivalent formula in conjunctive normal form (CNF).
Suppose, formulas are built up using the operators "$\rightarrow$", "$\neg$", "$\vee$", and "$\wedge$". Which are the main steps in the algorithm that transforms such a formula into an equivalent CNF formula?
(3 Points)

(b)

Transform the following formulas into conjunctive normal form:

(i) $\neg(a \rightarrow (b \vee c)) \wedge (b \rightarrow (a \wedge c))$

(3 Points)

(ii) $(a \wedge (a \rightarrow b)) \rightarrow b$

(5 Points)

(c)

Consider the following statements:

- If Joe has made a vacation trip, he was either at the beach or in the mountains.
- If Joe has not got a tan, he certainly was not at the beach.
- If Joe was in the mountains, he has got a tan.
- Joe has not got a tan.

(i) Translate the statements into a propositional logic formula, using appropriate atomic propositions. (2 Points)

(ii) Transform your formula into conjunctive normal form. (1 Points)

(iii) Use resolution to find out whether Joe has made a vacation trip or not.

(2 Points)

(iv) Explain why resolution was suitable to answer the quesiton under (iii), using the soundness of the resolution inference rule.

(1 Point)

(d)

This part of the question is about defining predicates in terms of some given predicates, using Prolog rules. Assume the predicate `parent(X,Y)`, `male(X)`, and `female(X)` are given, where

- `parent(X,Y)` means "`X` is a parent of `Y`",
- `male(X)` means "`X` is male", and
- `female(X)` means "`X` is female".

Based on the above relations, use Prolog rules to define the relations `mother(X)`, `brother(X,Y)`, `uncle(X,Y)`, and `grandmother(X,Y)`.

- `mother(X)` means "`X` is a mother"
- `brother(X,Y)` means "`X` is a brother of `Y`"
- `uncle(X,Y)` means "`X` is an uncle of `Y`"
- `granddaughter(X,Y)` means "`X` is a granddaughter of `Y`"

(8 Points)