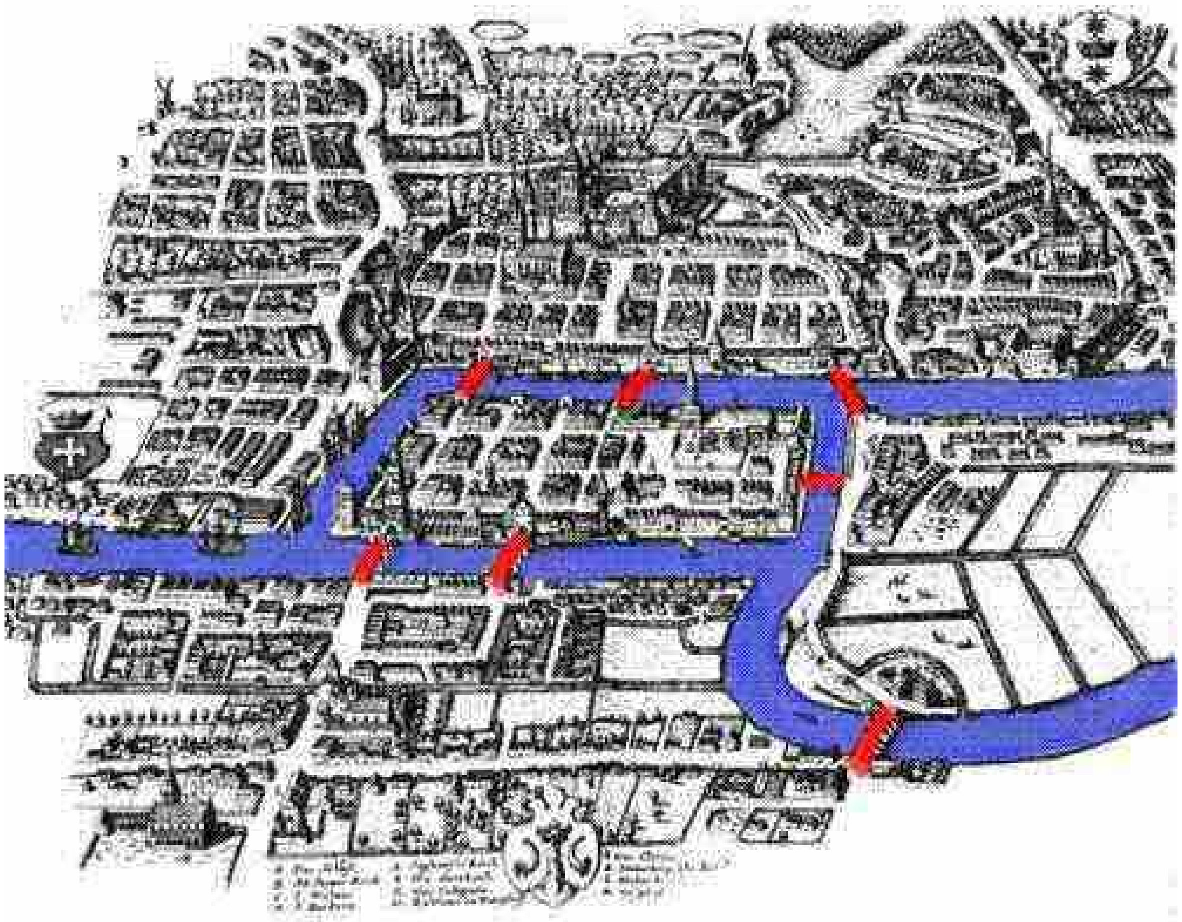
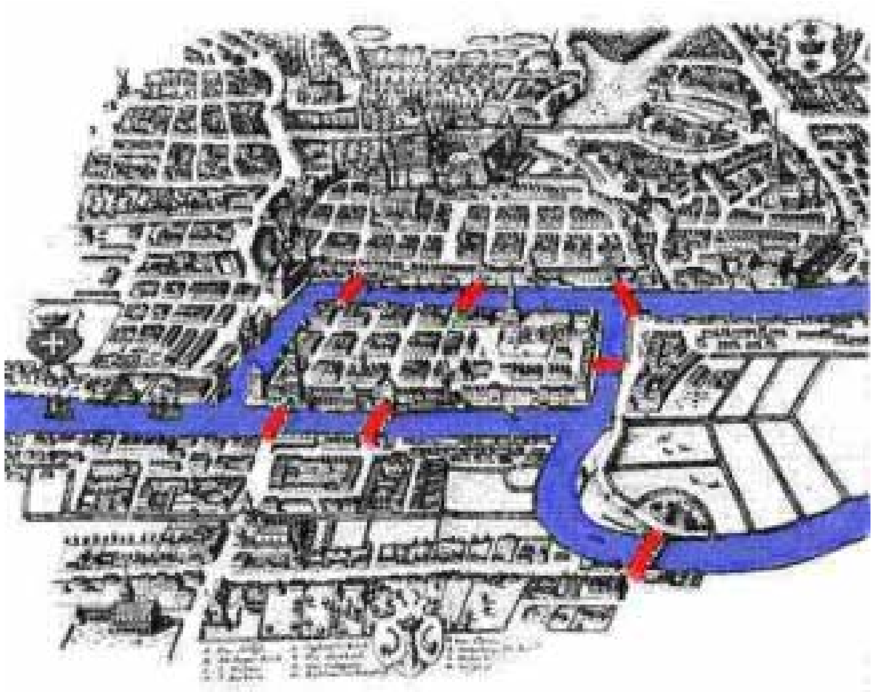
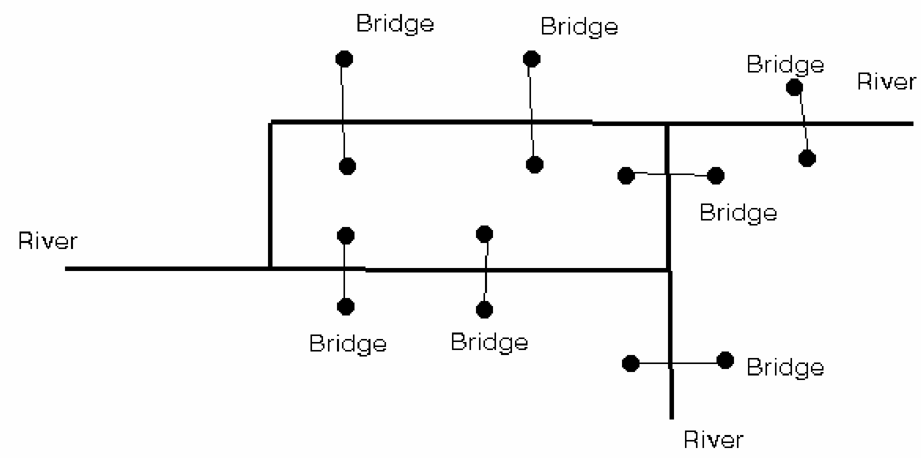
Elementary Graph Algorithms (1)

Konigsberg City



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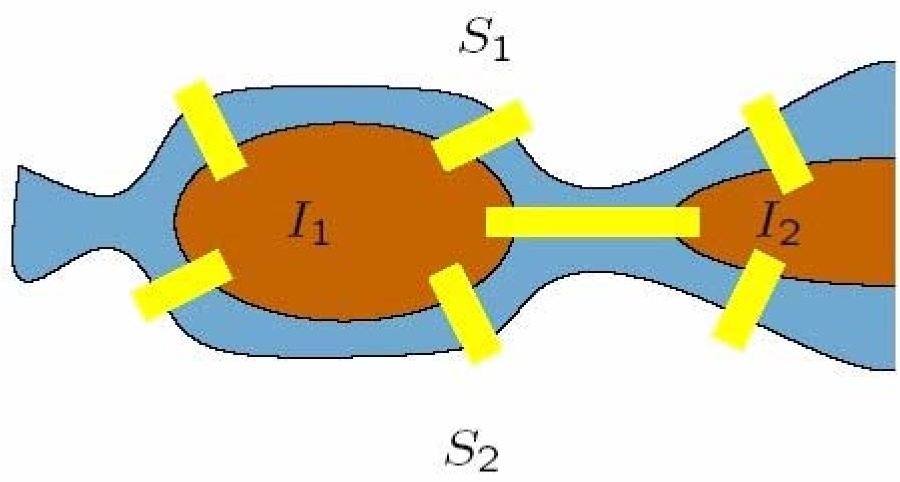
S_1
I_1
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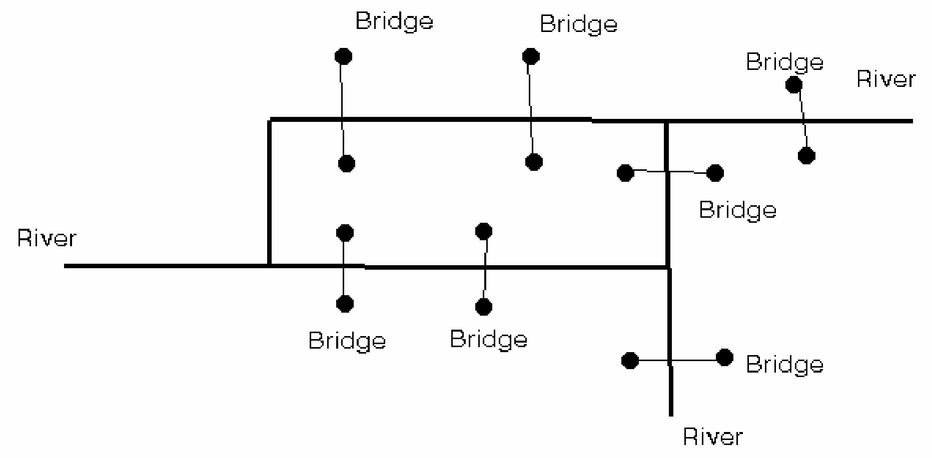
Definitions

- A **graph** G = (V, E)
  - V $\Rightarrow$ set of vertices (singular: vertex) {node}
  - E $\Rightarrow$ set of edges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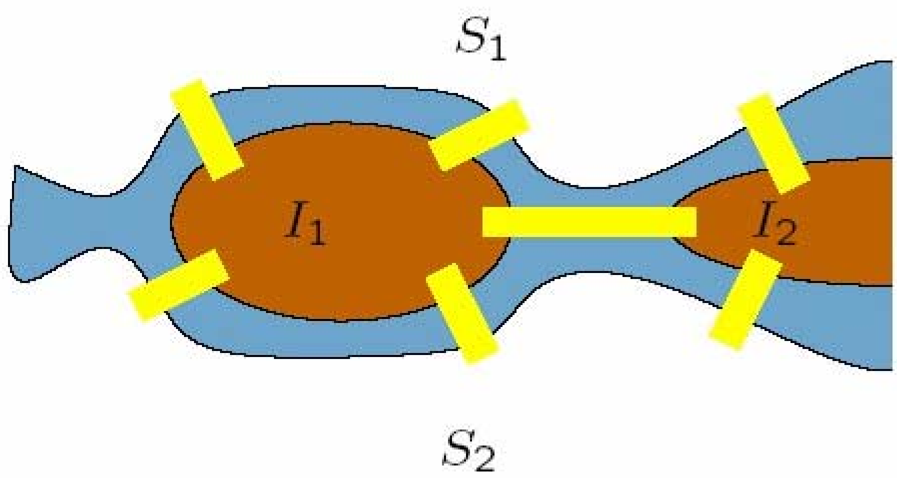
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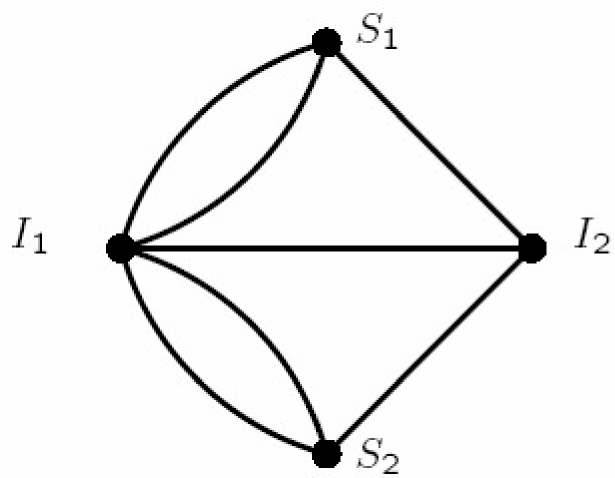
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- **Directed Graph** (digraph)
  - Edge (*u*,*v*) incident from (leaves) vertex *u* and is incident to (enter) vertex *v*
  - Self-loops—edges from a vertex to itself— are possible
  - If (*u*,*v*) is an edge in G, *v* is adjacent to vertex *u* ($u \rightarrow v$)
    - ✓ Adjacency relation is not necessarily symmetric
  - A vertex ***out-degree*** → number of edges leaving it
  - A vertex ***in-degree*** → number of edges entering it
  - A vertex ***degree*** ← in-degree + out-degree

- **Undirected Graph**
  - Edge (u,v) and (v,u) are considered to be the same edge and is called incident on vertices u and v
  - No self-loops
  - If (u,v) is an edge in G, v is adjacent to vertex u
    - ✓ Adjacency relation is symmetric
  - A vertex ***degree*** → number of edges incident on it

- A ***path*** of ***length*** k from vertex u to a vertex u' in G(V,E) is a sequence $\langle v_0, v_1, v_2, \ldots, v_k \rangle$ such that $u = v_0$, $u' = v_k$ and $(v_{i-1}, v_i) \in E$ for $i=0,1,2,\ldots,k$.
- A path ***length*** is number of edges in a path.
- u' is ***reachable*** from u if there is a path from u to u'
- A path is ***simple*** if all vertices are distinct

- A ***path*** $\langle v_0, v_1, v_2, \ldots., v_k \rangle$ form a ***cycle*** if $v_0 = v_k$ and the path contains at least one edge.
- A ***cycle*** is simple if $\langle v_0, v_1, v_2, \ldots., v_k \rangle$ are distinct.
- A ***self-loop*** is a cycle of length 1
- A digraph is ***simple*** if it doesn't contains any self-loop

Trees

- A ***tree*** is a connected, acyclic, undirected graph .
- A ***forest*** is a disconnected, acyclic, undirected graph

Connectivity

- Undirected Graph
  - Every pairs of vertices is connected by a path.
  - ***Connected components***: classes of vertices under the "*is reachable from*" relation
- Directed graph
  - Strongly connected if every two vertices are reachable from each other.
  - ***Strongly connected components***: classes of vertices under the "are mutually reachable" relation

Isomorphism

- For any two graphs $G=(V,E)$ and $G'=(V', E')$, if there exists a bijection $f: V \rightarrow V'$ such that $(u,v) \in E$ iff $(f(u), f(v)) \in E'$

Special Names Graphs

- A ***Complete Graph***: is an undirected graph in which every pairs of vertices is adjacent.
- A ***Bipartite Graph***: is an undirected graph G=(V,E) in which V can be partitioned into two sets V1 and V2 such that $(u,v) \in E$ implies either $u \in V_1$ and $v \in V_2$ or $v \in V_1$ and $u \in V_2$
- A ***Weighted Graph***: associates weights with either the edges or the vertices

Representation of Graphs

Adjacency List
Sparse Graph
$E << |V|^2$

Adjacency Matrix
Dense Graph
$E \approx |V|^2$