CS 540: Introduction to Artificial Intelligence

HW #3: CSP, Logic and Inference

Assigned: March 5
Due: March 12

Late Policy:

Homework must be handed in by noon on the due date and electronically turned in by this same time.

- If it is 0 – 24 hours late, including weekend days, a deduction of 10% of the maximum score will be taken off in addition to any points taken off for incorrect answers.
- If it is 24 – 48 hours late, including weekend days, a deduction of 25% of the maximum score will be taken off in addition to any points taken off for incorrect answers.
- If it is 48 – 72 hours late, including weekend days, a deduction of 50% of the maximum score will be taken off in addition to any points taken off for incorrect answers.
- If it is more than 72 hours late, you will receive a '0' on the assignment.
- In addition, there are 2 'late days' you may use any time throughout the semester. Each late day has to be used as a whole – you can't use only 3 hours of it and "save" 21 hours for later use.

Collaboration Policy:

You are to complete this assignment individually. However, you are encouraged to discuss the general algorithms and ideas with classmates, TA, and instructor in order to help you answer the questions. You are also welcome to give each other examples that are not on the assignment in order to demonstrate how to solve problems. But we require you to:

- not explicitly tell each other the answers
- not to copy answers or code fragments from anyone or anywhere
- not to allow your answers to be copied
- not to get any code or help on the Web

In those cases where you work with one or more other people on the general discussion of the assignment and surrounding topics, we suggest that you specifically record on the assignment the names of the people you were in discussion with.

Question 1: Models

Consider a vocabulary with only four propositions, A, B, C, and D. How many models are there for the following sentences:

(a) $A \wedge B$
(b) $(A \wedge B) \vee (B \wedge C)$
(c) $A \Leftrightarrow B \Leftrightarrow C \Leftrightarrow D$

Question 2: Soundness

DeMorgan's Laws can be stated as

(a) $\neg(P \wedge Q) \vdash \neg P \vee \neg Q$
(b) $\neg(P \vee Q) \vdash \neg P \wedge \neg Q$

Prove or disprove the soundness of each of these laws using a truth table. Be explicit, stating why your truth table proves or disproves the soundness of each law.

Question 3: Valid and Unsatisfiable Sentences in Propositional Logic

Is each of the following sentences in Propositional Logic valid, unsatisfiable, or neither? Justify your answer.

(a) $(\neg P \vee \neg Q) \Leftrightarrow (P \wedge Q)$
(b) $P \Rightarrow (Q \Rightarrow P)$
(c) $((P \wedge Q) \Rightarrow R) \Rightarrow (Q \Rightarrow (P \Rightarrow R))$
(d) $((P \Rightarrow Q) \wedge \neg P) \Rightarrow \neg Q)$

Question 4: Deductive Inference in Propositional Logic

Prove that implication is transitive in Propositional Logic by showing that $((P \Rightarrow Q) \wedge (Q \Rightarrow R)) \Rightarrow (P \Rightarrow R)$ using

(a) The truth table method.
(b) The natural deduction method using any of the rules of inference described in Section 7.5 of the textbook. For this method only, assume you are given $((P \Rightarrow Q) \wedge (Q \Rightarrow R))$ and you are to prove $(P \Rightarrow R)$.
(c) The resolution refutation method.

Question 5: Translation to First-Order Logic

Problem 8.6, parts (a), (b), (c), and (d), and Problem 8.7 in the textbook. Use only the predicates *Student*(x), *Takes*(x,y,z) meaning student x takes course y in semester z, *Passes*(x,y,z) meaning student x passes course y in semester z, $>(x,y)$ (or $x > y$) means x is greater than y, $=(x,y)$ (or $x = y$) means x is equal to y, *German*(x), *Language*(x), and *Speaks*(x,l) meaning person x speaks language l. The function *Score*(x,y,z) means the score obtained by student x in course y in semester z.

Question 6: Constraint Satisfaction

Consider the following constraint satisfaction problem: There are four variables, A,B,C, and D, and each variable can take any of three possible values, 1, 2, and 3. The following constraints are given: A≠B, A≠C, B>C, and B<D.

(a) Draw the constraint graph for this problem.

(b) Assuming an alphabetical/numerical ordering on both variables and values, list the variable assignments after each step of **Backtracking DFS with Forward Checking**. That is, after each step, give the remaining domains for all variables. For example, the steps of DFS are (1) A=1; B,C,D={1,2,3}; (2) A=1, B=1; C,D={1,2,3}; (3) A=1, B=1, C=1; D={1,2,3}; (4) A=1, B=1, C=1, D=1; (5) A=1, B=1, C=1, D=2; (6) A=1, B=1, C=1, D=3; (7) A=1, B=1, C=2; D={1,2,3}; etc.

(c) Give the variable assignments after each step when using **Backtracking DFS with Forward Checking along with the two heuristics, Minimum Remaining Values (MRV) and Least-Constraining-Value (LCV)**. Break MRV ties first using the degree heuristic (explained on page 144) and second alphabetically. Break LCV ties numerically, using smaller values first.

(d) Give the variable assignments when using the **Arc Consistency** algorithm, AC-3, first as a preprocessing step with the given initial domains for each variable, and then after each assignment (use the same ordering process as in (c)). After each use of AC-3 (repeatedly propagating until no inconsistencies remain), show the remaining domains of the variables.