

ICS 171, Summer 2000: Lecture 6 Solutions

(1) Use truth tables to show the following sentences are valid:

- $P \Rightarrow Q \Leftrightarrow \neg P \vee Q$
- $\neg(P \wedge Q) \Leftrightarrow \neg P \vee \neg Q$
- $\neg(P \vee Q) \Leftrightarrow \neg P \wedge \neg Q$

(a)

| P | Q | $P \Rightarrow Q$ | $\neg P$ | $\neg P \vee Q$ | $P \Rightarrow Q \Leftrightarrow \neg P \vee Q$ |
|-----|-----|-------------------|----------|-----------------|---|
| F | F | T | T | T | T |
| F | T | T | T | T | T |
| T | F | F | F | F | T |
| T | T | T | F | T | T |

(b)

| P | Q | $P \wedge Q$ | $\neg(P \wedge Q)$ | $\neg P$ | $\neg Q$ | $\neg P \vee \neg Q$ | $\neg(P \wedge Q) \Leftrightarrow \neg P \vee \neg Q$ |
|-----|-----|--------------|--------------------|----------|----------|----------------------|---|
| F | F | F | T | T | T | T | T |
| F | T | F | T | T | F | T | T |
| T | F | F | T | F | T | T | T |
| T | T | T | F | F | F | F | T |

(c)

| P | Q | $(P \vee Q)$ | $\neg(P \vee Q)$ | $\neg P$ | $\neg Q$ | $\neg P \wedge \neg Q$ | $\neg(P \vee Q) \Leftrightarrow \neg P \wedge \neg Q$ |
|-----|-----|--------------|------------------|----------|----------|------------------------|---|
| F | F | F | T | T | T | T | T |
| F | T | T | F | T | F | F | T |
| T | F | T | F | F | T | F | T |
| T | T | T | F | F | F | F | T |

(2) Read Question 6.2 in the course text (Russell & Norvig, page 180). Be familiar with these equivalence relations.

(3) Question 6.3 in the course text (Russell & Norvig, page 180).

(a) valid

(b) satisfiable

(c) satisfiable

(d) valid

(e) valid

(f) valid

(g) valid

(h) satisfiable

(4) Question 6.7 in the course text (Russell & Norvig, page 181)

(a) 4

(b) 12

(c) 2

(5) Consider the knowledge base:

If it is hot and humid, then it is raining. If it is humid, then it is hot. It is humid.

(a) Describe a set of propositional letters which can be used to represent the knowledge base.

Let H represent it is hot. Let M represent it is humid and let R represent it is raining.

(b) Translate the KB into propositional logic using your propositional letters from part a.

1. $(H \wedge M) \Rightarrow R$

2. $M \Rightarrow H$

3. M

(c) Is it raining? Answer this question by using logical inference rules with the KB.

Use Modus Ponens to combine 3 and 2

4. H

And introduction to combine 3 and 4

5. $H \wedge M$

Modus Ponens to combine 5 and 1

6. R

Therefore it is raining.

(6) Modus Tollens is an inference rule which states

$$\frac{P \Rightarrow Q, \quad \neg Q}{\neg P}$$

Prove that Modus Tollens is sound. Use either a truth table or sound logical inference rules.

Using a truth table:

| P | Q | $P \Rightarrow Q$ | $\neg Q$ | KB $(P \Rightarrow Q) \wedge \neg Q$ | α $\neg P$ |
|-----|-----|-------------------|----------|---|----------------------|
| F | F | T | T | T | T |
| F | T | T | F | F | T |
| T | F | F | T | F | F |
| T | T | T | F | F | F |

Since $\alpha (\neg P)$ is true everywhere the KB is true, then Modus Tollens is sound.

Using logical inference rules:

Convert to CNF form

1. $\neg P \vee Q$

2. $\neg Q$

Combine 1 and 2 using unit resolution

3. $\neg P$

(7) Consider the knowledge base:

If it is raining out then Ann puts the top up on her convertible. Ann did not put the top up on her convertible.

(a) Describe a set of propositional letters which can be used to represent the knowledge base.

Let R represent it is raining out. Let T represent Ann puts the top up on her convertible.

(b) Translate the KB into propositional logic using your propositional letters from part a.

1. $R \Rightarrow T$

2. $\neg T$

(c) Is it raining? Answer this question by using logical inference rules with the KB.

Combine 1 and 2 using Modus Tollens (see question 6)

3. $\neg R$

Therefore it is not raining out.