6.045J/18.400J: Automata, Computability and Complexity Prof. Nancy Lynch

Quiz 1

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Please write your name in the upper corner of each page.

Problem 1:

True or False (20 points) Full credit will be given for correct answers. If you include justification for your answers, you may obtain partial credit for incorrect answers.

In all parts of this question, the alphabet Σ is $\{0, 1\}$.

1. True or False: A DFA with n states must accept at least one string of length greater than n.

False. The DFA might accept no strings at all.

2. True or False: A DFA with n states that accepts an infinite language must accept at least one string x such that $2n < |x| \leq 3n$.

True. Follows by a pumping lemma argument. The DFA has to accept at least one string longer than $3n$. Now, pump this down. In each step, we pump down by at most n, and at least 1. It follows that if one does this sufficiently many times, one gets a string with length in the range [2n. . . 3n].

3. If R is a regular language and L is some language, and $L \cup R$ is a regular language, then L must be a regular language.

False. Let $R = \Sigma^*$ and L be some non-regular language.

4. If F is a finite language and L is some language, and $L - F$ is a regular language, then L must be a regular language.

True. $L = (L - F) \cup F'$, where F' is some subset of F, and is therefore finite. The statement now follows from the closure under union of regular languages.

5. True or False: Define $FOUR(w)$, for a finite string w, to be the string consisting of the symbols of w in positions that are multiples of four. For example, $FOUR(1110011100) = 01$.
If L is a regular language, then $\{FOUR(w) : w \in L\}$ must be regular.

True. A variation of the construction used to show that TWO(L) is regular from Problem 2.7, Recitation 2, applies here.

6. True or False: For every three regular expressions R, S, and T, the languages denoted by $R(S \cup T)$ and $(RS) \cup (RT)$ are the same.

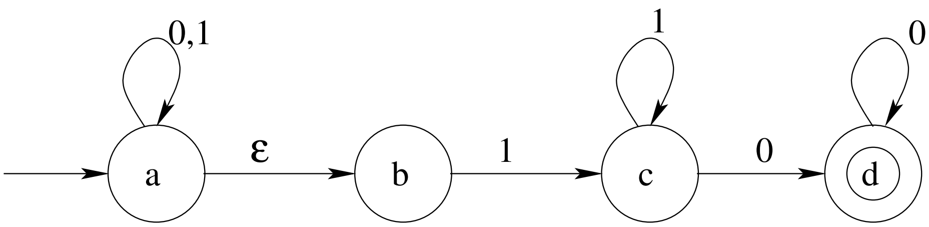
True.

7. True or False: If a language L is recognized by an n-state NFA, then it must be recognized by some DFA with no more than 2^n states.

True.

8. If a language L is recognized by an 2^n-state DFA, then it must be recognized by some NFA with no more than n states.

False. The minimal DFA and NFA could have the same size,such as the ones accepting the language $L = \phi$.



Problem 2: **(20 points)** Consider the following NFA:

1. **(12 points)** Convert this NFA into an equivalent DFA using the procedure we studied in class. **Your answer should be the state diagram of a DFA. Your diagram should include only the states that are reachable from the start state.** (Note: There are not more than 16 states in the resulting DFA). Please label your states in some meaningful way. You may explain your work, to receive partial credit for an incorrect answer.

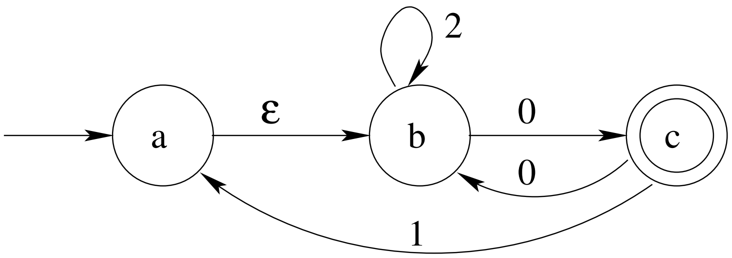
This is a standard construction, and is left as an exercise.

2. **(4 points)** Give a regular expression that defines the language that is recognized by the given NFA (and therefore, also the DFA you constructed in part 1).

$(0 \cup 1)^*11^*00^*$.

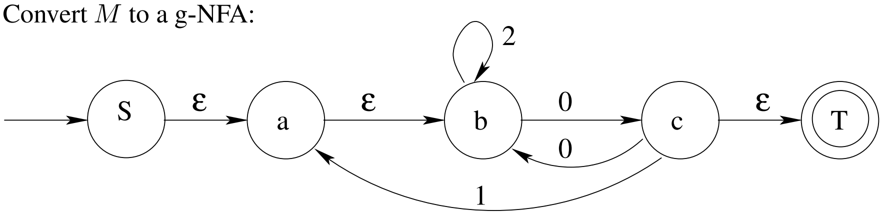
3. **(4 points)** Prove that there cannot exist a 2-state DFA for the language you defined in part (b) above. (Hint: Give three (short) strings that must lead to different states, *in any DFA that recognizes this language*.)

The strings $\epsilon, 1$ and 10 must go to different states. 10 must go to a state different from both 1 and ϵ, since $10 \in L$ (and therefore should go to a final state), whereas $\epsilon, 1 \notin L$. Reading a 0 after 1, we should get to an accept state, since $10 \in L$. Reading 0 after ϵ should not get us to an accept state, since $0 \notin L$. Thus, 1 and ϵ should go to different states too. Thus, we need at least 3 states in the minimal DFA recognizing L.



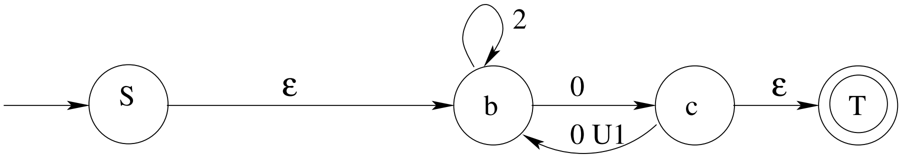
Problem 3: **(20 points)** Find a regular expression for the language recognized by this NFA M, using the procedure we studied in class. Remove the states in the order a, then b, then c.

Convert M to a g-NFA:

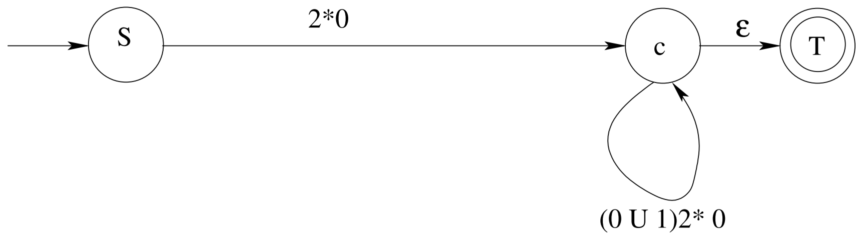


We choose to omit transitions labeled with ϕ from the diagram.

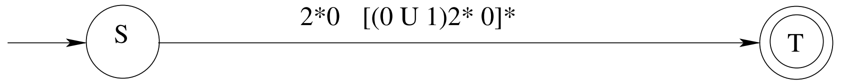
Remove state a:



Remove state *b*:



Remove state *c*:



Problem 4: **(20 points)** Describe a procedure (something that can be implemented using a program), that takes any two regular expressions and outputs a correct answer to the following questions:

1. (10 points) "Is $L(R_1) = L(R_2)$?"

Convert the regular expressions R_1 and R_2 into NFAs, and then to DFAs D_1 and D_2.
Now, construct the DFAs D_3 and D_4 that recognize $L(R_1) - L(R_2)$ and $L(R_2) - L(R_1)$ respectively. Check if both D_3 and D_4 recognize the empty language, by determining if the accept state(s) can be reached from the start state. If $L(D_3)$ and $L(D_4)$ are both empty, accept. Else reject.

2. (10 points) "Does $L(R_2)$ contain all the words in $L(R_1)$, plus exactly one additional word?"

Convert the regular expressions R_1 and R_2 into NFAs, and then to DFAs D_1 and D_2.
Now, construct the DFAs D_3 and D_4 that recognize $L(R_1) - L(R_2)$ and $L(R_2) - L(R_1)$ respectively. Check if both of the following conditions hold:

1. $L(D_3) = \phi$. This can be done as above.
2. $|L(D_4)| = 1$. For all strings x upto length $2|D_4|$, test if D_4 accepts x. Accept, if for exactly one such string x, D_4 accepts, and reject otherwise.

Problem 5: **(20 points)**

If w is a string over an alphabet Σ and $\Sigma' \subseteq \Sigma$ is a (possibly) smaller alphabet, then we write $w|_{\Sigma'}$, the projection of w on Σ', for the string obtained from w by including just the symbols in Σ', that is, by removing all the symbols in $\Sigma - \Sigma'$.

For example, $012012012|_{\{0,1\}} = 010101$.

1. (14 points) Use the Pumping Lemma to prove that the following language L over the alphabet $\{0, 1\}$ is not regular:

$$L = \{wx : w, x \in \{0, 1, 2\}^* \text{ and } w|_{\{0,1\}} = (x|_{\{0,1\}})^R\}$$

That is, the restriction of x to $\{0, 1\}$ is the reverse of the restriction of w to $\{0, 1\}$.

For example, 0120122212010 is in L.

For a pumping length of p, use the string 0^p110^p.

2. (6 points) Give an alternative proof that L is not regular based on a non-regularity result already proved in class or homework and one or more closure properties for regular languages.

Consider $L \cap \{0, 1\}^*$. If L were regular, this would be regular too. But this is exactly $\{ww^R \mid w \in \{0, 1\}^*\}$, which we know is non-regular.