Artificial Intelligence MSc
– Exam Questions –

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2004

3 Propositional Logic

(a)

Consider the following propositional formulas

1. $p \lor \neg p$
2. $(p \to \neg p) \leftrightarrow (\neg p \to p)$
3. $(p \to \neg q) \leftrightarrow (\neg q \to p)$

(i) For each formula, set up a truth table.

(4 Points)

(ii) For each formula, state which of the four properties satisfiable, falsifiable, unsatisfiable, and valid apply to it and which not.

(3 Points)

(b)

Suppose F and G are two arbitrary propositional formulas. What can you say about F and G if you know that

(i) $F \lor G$ is satisfiable?

(ii) $F \lor G$ is unsatisfiable?

(iii) $F \lor G$ is falsifiable?

(iv) $F \lor G$ is valid?

(8 Points)

(c)

In this part of the question, we want to analyse whether some sentences about the election of class representatives are logical consequences of other sentences.

Consider the following two statements:

S_1: If David is elected president, then Eric is not elected vice-president or Fred is not elected treasurer.

S_2: David is elected president and Fred is elected treasurer.

and the claim

C: Eric is not elected vice-president.

(i) Translate the statements and the claim into propositional logic, using appropriate atomic propositions.

(3 Points)

(ii) Explain how one can check whether a claim C is a logical consequence of the two sentences S_1 and S_2.

(2 Points)

(iii) Use an approach of your choice to check whether C is a logical consequence of S_1 and S_2.

(5 Points)

4 Resolution

(a)

In the lectures, we have said that a formula is a *literal* if it is a propositional atom or a negated propositional atom. For instance,

$$p, \ \neg p, \ q, \ \neg q$$

are literals. Moreover, we have said that a formula is in *disjunctive normal form* if it is a disjunction of conjunctions of literals. For instance, the formula

$$(p \wedge \neg q \wedge r) \vee (\neg p \wedge \neg r \wedge \neg s) \vee (r \wedge \neg s)$$

is in disjunctive normal form.

Answer each of the following questions either by giving a method and explaining why it is correct, or by explaining why such a method is unlikely to exist.

(i) Suppose formula F is a *disjunction* of literals. Is there an efficient method, not using truth tables, to check whether F is *valid*?

(2 Points)

(ii) Suppose formula F is a *conjunction* of literals. Is there an efficient method, not using truth tables, to check whether F is *unsatisfiable*?

(2 Points)

(iii) Suppose formula F is in *disjunctive normal form.*. Is there an efficient method, not using truth tables, to check whether F is *unsatisfiable*?

(3 Points)

(b)

Resolution is an inference rule that allows one to derive new clauses from a given set of clauses.

(i) Describe precisely how one applies the resolution rule, i.e., under which conditions it is applicable and how one can derives a new clause.

(3 Points)

(ii) Is the resolution rule sound? Explain your answer!

(3 Points)

(c)

You have arrived on the island of knights and knaves. Everything a knight says is true. Everything a knave says is false.

You meet two inhabitants, Bob and Peggy. Bob claims that Peggy is a knave. Peggy tells you, 'I am a knight or Bob is a knight.'

Your task is to find out who is a knight and who is a knave.

(i) Formalise the puzzle as a formula in propositional logic. Explain what the atoms in your formula stand for.

(3 Points)

(ii) Transform the puzzle formula into an equivalent formula in conjunctive normal form. For each step, explain what you are doing.

(5 Points)

(iii) Apply resolution to find out who is what.

(4 Points)