Computer Science BSc Analysis-1 Retake Test of TEST-1
05-th of June, 2023

Reasoning and justification are needed in the solutions.

1. *(9 points)* Determine (without using the concept of limit) $\sup H$, $\inf H$, $\max H$, $\min H$ if

$$H = \left\{ \frac{2n+1}{4n-3} \mid n \in \mathbb{N} = \{1, 2, 3, \dots\} \right\}$$

2. *(7 points)* Prove by definition that

$$\lim_{n\to\infty} \frac{n^3+3n^2+2}{2n^3+n^2+n+1} = \frac{1}{2}$$

3. *(7 points)* $\lim_{n\to\infty} \sqrt{n} \cdot (\sqrt{n^3+n} - \sqrt{n^3-n}) = ?$

4. *(7 points)* $\lim_{n\to\infty} \left(\frac{3n+1}{3n-2}\right)^{2n+1} = ?$

5. *(10 points)* Prove that the following sequence is convergent, and compute its limit.

$$a_0 = \frac{3}{4}, \qquad a_{n+1} = \frac{2}{1+\frac{1}{a_n}} \qquad (n \in \mathbb{N}_0)$$