Name : Neptun : Teacher :

Points :

Basic Mathematics, Test 3, 16th of May 2019.

1. Consider the following matrix :

$$A := \begin{pmatrix} 4 & -3 & -3 \\ 3 & -2 & -3 \\ -1 & 1 & 2 \end{pmatrix} \in \mathbb{K}^{3\times 3}.$$

a) Find all the eigenvalues, eigenvectors and eigenspaces of A on $\mathbb{R}$ and $\mathbb{C}$ as well. What are the algebraic and geometric multiplicities of the eigenvalues? **(8 points)**

b) Give an eigenbasis in $\mathbb{R}^3$ if there is one. Is A diagonalizable in $\mathbb{R}$ and in $\mathbb{C}$? Give the diagonal form of A, the matrix C with the transfomation which gives the diagonal form. **(4 points)**

2. a) Consider the following linearly independent vectors :

$$b_1 = (1,0,1,-1);\ \ b_2 = (0,1,-1,1);\ \ b_3 = (1,-1,0,1) \in \mathbb{R}^4.$$

Give an orthogonal system u_1, u_2, u_3 which is equivalent to the b_1, b_2, b_3 vector system. If possible choose the components of u_1, u_2, u_3 to be integers. (5 **points)**

b) Give the decomposition of the following vector

$$x = (3,0,5,20)$$

into paralel and orthogonal components to the subspace $W = Span(u_1, u_2, u_3)$. **(8 points)**

3. Consider the functions :

$$f(x) := \frac{2x+5}{x-1} \quad (x \in (1;+\infty)) \quad \text{and} \quad g(x) := x^2 - 6x + 1 \quad (x \in \mathbb{R}) \quad \text{and} \quad F(x) := \frac{x-1}{\sqrt{1-x}} \quad (x \in (-\infty;1)).$$

a) Prove that f is invertable and give $D_{f^{-1}}; R_{f^{-1}}$ and for all $y \in D_{f^{-1}}$ the value $f^{-1}(y)$. **(6 points)**

b) Give the function $F \circ g$. **(4 points)**

4. Find the following limit, and then prove by definition your result :

$$\lim_{x\to+\infty} \left(\frac{2x^2 - 7x + 3}{5x^2 + x + 4} \right). \quad (8 \text{ points})$$

THEORY :

5. Write down and prove the *Projection theorem.* **(7 points)**