

Analysis1 Test3 from part 2, 24th of May 2022

1. (6 points) Evaluate the following sum :

$$\sum_{n=2}^{+\infty} \frac{2}{n^2 + 4n + 3}.$$

2. (4 + 4 + 3 points) Decide if the following series are convergent or divergent (prove your statements) :

$$\text{a) } \sum_{n=1} \frac{1}{3^n} \cdot \left(\frac{n}{n+1}\right)^{-n^2}; \quad \text{b) } \sum_{n=1} \frac{(n!)^2}{(2n+1)!}; \quad \text{c) } \sum_{n=1} \frac{n+2}{\sqrt{n^5 + 3n^4 + 10}}.$$

3. (8 points) Determine the convergence set of the following power series :

$$\sum_{n=0} \frac{n}{4^n \cdot (n^2 + 1)} \cdot (x - 5)^n.$$

4. (7 points) Evaluate the following limit, and then prove it by *definition* :

$$\lim_{x \rightarrow 2} \frac{x^3 - 8}{x^2 - x - 2}.$$

5. (4 + 4 points) Evaluate the following limits (do not use L'Hospital's rule here) :

$$\text{a) } \lim_{x \rightarrow 1} \left(\frac{1}{x-1} - \frac{2x}{x^2-1} \right); \quad \text{b) } \lim_{x \rightarrow 0} \frac{1 - \sqrt{\cos x}}{\sin x \cdot \sin(3x)}.$$