Analysis I., Sample Test 2

1. Determine the sums of the following series:

$$\text{a)}\ \sum_{n=1} \frac{2^{2n+1} - 3 \cdot 2^{n+3}}{6 \cdot 5^n} \qquad \text{b)}\ \sum_{n=1} \frac{1}{n(n+3)}$$

2. Determine whether the following series are convergent or not:

$$\text{a)}\ \sum_{n=1} \left(1 + \frac{1}{n}\right)^n \qquad \text{b)}\ \sum_{n=1} \frac{n+3}{\sqrt{n^5 + n^3 + n + 2}} \qquad \text{c)}\ \sum_{n=1} \frac{\sqrt{n+1} - \sqrt{n}}{\sqrt{n}}$$

$$\text{d)}\ \sum_{n=1} \left(\frac{n+2}{2n}\right)^n \qquad \text{e)}\ \sum_{n=1} \frac{2^n \cdot n!}{n^n} \qquad \text{f)}\ \sum_{n=1} (-1)^n \cdot \frac{1}{\sqrt{n+1}}$$

3. Determine the radius of convergence and the convergence set of the following power series:

$$\sum_{n=1} \frac{1}{n^2 \cdot 3^n} \cdot (x-2)^n$$

4. Prove by the definition of the limit:

$$\lim_{x \to 1} \frac{x^2 + 4x - 5}{x^3 - 1} = 2$$

5. Determine the following limits (if they exist):

$$\text{a)}\ \lim_{x \to 1} \frac{x^3 - x^2 + x - 1}{x^2 - 1} \qquad \text{b)}\ \lim_{x \to 2} \frac{x^2 + x + 1}{x^2 - 3x + 2} \qquad \text{c)}\ \lim_{x \to 0} \frac{\sqrt{x^2 + 1} - 1}{\sqrt{x^2 + 16} - 4}$$

$$\text{d)}\ \lim_{x \to 0} \frac{1 + \sin x - \cos x}{1 - \sin x - \cos x} \qquad \text{e)}\ \lim_{x \to 0} \frac{2 - e^x - e^{-x}}{\cos x - 1}$$

6. Expand the following function into power series around the center 0:

$$f(x) = \frac{x}{x^2 - 5x + 6} \qquad (x \in \mathbb{R} \setminus \{2, 3\})$$