

Analysis II

Computer Science BSc

December 15, 2023

Name: _____

Code: _____

Group/Teacher: _____

Test 2

1. Evaluate the following integrals:

(5+6 points)

(a) $\int \left(\frac{1}{x^2} - \sqrt{1 + \frac{1}{x}} \right)^2 dx \quad (x \in (0, +\infty));$

(b) $\int (1 - 2x) \cdot \arctan x \, dx \quad (x \in \mathbb{R}).$

2. Find the area of the bounded and closed region enclosed by the following curves:

(6 points)

$$y = x^2 + 2x, \quad y = 4 - x^2.$$

3. Find the volume of the solid obtained by rotating the graph of the following function around x axes:

(8 points)

$$f(x) := \frac{1}{x} \cdot \sqrt{\frac{x+1}{x-1}} \quad (x \in [2, 3]).$$

4. Compute the local extrema of the following function:

(7 points)

$$f(x, y) = \frac{x^3}{3} - 3x^2 - 4xy + y^2 \quad ((x, y) \in \mathbb{R}^2).$$

5. Consider the function:

$$f(x, y) = \frac{\sin^2 x}{y^2} \quad ((x, y) \in \mathbb{R} \times (0, +\infty)).$$

Evaluate the double integral of f over the following domain:

(8 points)

$$D := \{(x, y) \in \mathbb{R}^2 : \pi/4 \leq x \leq \pi/2, \sin x \leq y \leq 1\}$$