Analysis II

Computer Science BSc

October 27, 2023

Test 1

1. Discuss the continuity of the following function. *(10 points)*

$$f(x) = \begin{cases} \dfrac{x^2-2x-3}{x^2-x-2}, & x \in (-4,2) \setminus \{-1\} \\ \dfrac{\sin(x-2)}{x^2-x-2}, & x > 2 \\ \dfrac{3-x}{3}, & x \in (-\infty,-4] \cup \{-1,2\} \end{cases}$$

2. Consider the function: *(6 points)*

$$f(x) = \log_2(3+\cos x) - x \qquad (x \in \mathbb{R})$$

Prove that the equation $f(x) = 0$ has at least one real solution. Determine the equation of the tangent line to the graph of f at the point $a = 0$.

3. Discuss the following function and sketch its graph. *(11 points)*

$$f(x) = \frac{3x}{4+x^2} \qquad (x \in \mathbb{R})$$

4. Determine the below limit. *(5 points)*

$$\lim_{x \to 0} \left(1 - 3x^2\right)^{7/x^2}$$

5. Find the second degree Taylor polynomial of f centered at $a = 0$ and estimate the error of approximation on the interval I. *(8 points)*

$$f(x) = \frac{1}{\sqrt{1+2x}} \quad \left(x > -\tfrac{1}{2}\right), \qquad I = \left(0,\ \tfrac{1}{4}\right)$$