Name : Neptun : Teacher :

Points :

Basic Mathematics, Test 3, 14th December 2018.

1. Consider the following matrix :

$$A := \begin{pmatrix} 3 & 6 & 1 \\ 1 & 8 & 1 \\ 1 & 6 & 3 \end{pmatrix} \in \mathbb{R}^{3\times 3}.$$

a) Find all the eigenvalues, eigenvectors and eigenspaces of A. What are the algebraic and geometric multiplicities of the eigenvalues ?

b) Is there an eigenbasis in $\mathbb{R}^3$? If yes, give one.

c) Is A diagonalizable ? Give the diagonal form of A, the matrix C with the transformation which gives the diagonal form.

2. Consider the following generated subspace :

$$W := \text{Span}\,((1,-1,0,-1);(-1,0,1,-1);(3,2,4,1)) \subset \mathbb{R}^4.$$

Give the decomposition of vector $x = (5, 13, 0, -11)$ into paralell and orthogonal components to the subspace W.

3. Give an orthogonal basis in the following subspace of $\mathbb{R}^3$:

$$W := \{(x,y,z) \in \mathbb{R}^3 \mid x + 2y - z = 0\}.$$

4. Consider the functions :

$$f(x) := \frac{x-5}{2x+1} \quad (x \in (-1/2; +\infty)) \quad \text{and} \quad g(x) := x^2 + \cos^2 x - \pi x \quad (x \in \mathbb{R}).$$

a) Are f and g invertable ?

b) Give $D_{f^{-1}}; R_{f^{-1}}$ and for all $x \in D_{f^{-1}}$ the value $f^{-1}(x)$.

5. Proove by definition that :

$$\lim_{x\to+\infty} \left(\frac{x^2-1}{2x^2+5}\right) = \frac{1}{2}.$$

THEORY :

6. Write down and proove the *projection (or decomposition)* theorem.