

Reasoning and justification are needed in the solutions.

1. (7 points) Determine the inverse function (domain and formula) of the the following function, if it is invertible:

$$f(x) = x^2 - 2x - 3 \quad (x \geq 2)$$

2. (8 points) Determine (without using the concept of limit) $\sup H$, $\inf H$, $\max H$, $\min H$ if

$$H = \left\{ \frac{6n-2}{2n+1} \mid n \in \mathbb{N} = \{1, 2, 3, \dots\} \right\}$$

3. (6 points) Prove by definition that

$$\lim_{n \rightarrow \infty} \frac{n^3 + 2n^2 + 1}{3n^3 + n + 2} = \frac{1}{3}$$

4. (5 points) $\lim_{n \rightarrow \infty} (\sqrt{n^2 + n + 5} - \sqrt{n^2 - 7n + 7}) = ?$

5. (5 points) $\lim_{n \rightarrow \infty} \left(\frac{2n+7}{2n+3} \right)^{4n-1} = ?$

6. (9 points) Prove that the following sequence is convergent, and compute its limit.

$$a_0 = 0, \quad a_{n+1} = \sqrt{6 + 5a_n} \quad (n \in \mathbb{N}_0)$$