Analysis1 Test2, 25th of May 2023

1. Evaluate the sum:

$$\sum_{n=1}^{+\infty} \frac{2}{(2n+1)\cdot(2n+3)}. \quad \textbf{(5 points)}$$

2. Decide whether the following series are convergent or divergent. Prove your statements.

$$\text{a)} \sum_{n=0}^{+\infty} \frac{n+2}{(n^2+1)^2}; \quad \text{b)} \sum_{n=0}^{+\infty} \frac{(3n)!}{8^n\cdot(n!)^3}; \quad \text{c)} \sum_{n=1}^{+\infty} \left(\frac{7n-5}{7n+2}\right)^{n^2}. \quad \textbf{(4+4+4 points)}$$

3. For what real values x is the following power series convergent, divergent, absolutely convergent:

$$\sum_{n=0}^{+\infty} \frac{(x-3)^n}{(3n+2)\cdot 2^n}? \quad \textbf{(7 points)}$$

4. Prove by definition, that:

$$\lim_{x\to 1}\left(\frac{2x^2+1}{3x^2+2}\right) = \frac{3}{5}. \quad \textbf{(6 points)}$$

5. Evaluate the following limits (do not use L'Hospital's rule here):

$$\text{a)} \lim_{x\to 1} \frac{\sqrt{x^2+x}-\sqrt{2}}{\sin(2x-2)}; \qquad \text{b)} \lim_{x\to 0} \frac{1-\cos(4x)}{e^{2x}-2e^x+1}. \quad \textbf{(5+5 points)}$$