Computer Science BSc
Basic Mathematics TEST-3
19-th of December 2022

Reasoning and justification and calculations are needed in the solutions

1. *(7 points)* Using Gauss-Jordan method determine the inverse of the following matrix (only the Gauss-Jordan method is acceptable):

$$A = \begin{bmatrix} 1 & 4 & 4 \\ 1 & 2 & 4 \\ 1 & 3 & 2 \end{bmatrix} \in \mathbb{R}^{3\times 3}$$

2. *(7 points)* Determine the eigenvalues and eigenvectors of the following (complex) matrix.

$$A = \begin{bmatrix} 6 & -1 \\ 8 & 2 \end{bmatrix} \in \mathbb{C}^{2\times 2}$$

3. *(10 points)* Determine the eigenvalues and eigenvectors of the following matrix. Discuss the diagonalizability of A (determine C and $C^{-1}AC$)

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 3 & 3 & -3 \\ 2 & 2 & -2 \end{bmatrix} \in \mathbb{R}^{3\times 3}$$

4. Consider the following subspace in $\mathbb{R}^4$:

$$W = \big\{(x,\, y,\, z,\, u) \in \mathbb{R}^4 \;\mid\; x + y + z - u = 0\big\}$$

   a) *(7 points)* Determine an orthogonal basis in W.

   b) *(5 points)* Decompose the vector $x = (4,\, -4,\, 4,\, 0) \in \mathbb{R}^4$ into parallel and orthogonal components by the subspace W.

5. *(7 points)* Consider the following $\mathbb{R} \to \mathbb{R}$ type function f:

$$f(x) = x^2 + 8x - 9 \qquad (x \in [\,0\,,\, +\infty)\,)$$

   Prove that f is invertible, and determine the sets $D_{f^{-1}}$, $R_{f^{-1}}$ and for $y \in D_{f^{-1}}$ the function value $f^{-1}(y)$.

6. *(7 points)* Prove by definition that

$$\lim_{x\to+\infty} \frac{x^4 - x^3 - 2x^2 - 3x - 4}{2x^3 + 3x + 1} = +\infty$$