Analysis I., Sample Test 1

1. Determine the inverse function (domain and formula) of the following function, if it is invertible:

$$f(x) = \frac{5x+3}{2x-4} \qquad (x > 2)$$

2. Determine the composition $f \circ g$ (domain and formula):

$$f(x) = \sqrt{3-x} \ \ (x \leq 3), \qquad g(x) = \sqrt{x^2-16} \ \ (x \in (-\infty, -4] \cup [4, +\infty))$$

3. Determine (without using the concept of limit) $\sup H$, $\inf H$, $\min H$ and $\max H$:

$$H = \left\{ \frac{7n-2}{2n+5} \in \mathbb{R} \,:\, n \in \mathbb{N} \right\}$$

4. Prove by the definition of the limit:

$$\lim \left(\frac{n^3 - 3n^2 + n - 1}{1 - 2n^3 + n} \right) = -\frac{1}{2}$$

5. Prove by the definition of the limit:

$$\lim \left(\frac{n^3 - 3n^2 + n - 1}{5n^2 + n - 3} \right) = +\infty$$

6. Determine the following limits:

a) $\lim \dfrac{(n+2)^6 - (n+3)^6}{(n^3 - 2n - 5)(2n^3 + n^2 + 3)}$ b) $\lim \dfrac{\sqrt{n+1} - \sqrt{n}}{\sqrt{n} - \sqrt{n-1}}$ c) $\lim \dfrac{n^3 \cdot 2^n + 5^{n+1}}{5^{n-1} - n \cdot 3^n}$

d) $\lim \sqrt[n]{\dfrac{n+1}{2^n+3}}$ e) $\lim \left(\dfrac{6n-7}{6n+4} \right)^{3n+2}$ f) $\lim \left(3n - \sqrt{7n^2+2} \right)$

7. Determine the limit of the following recursive sequence, if it exists:

$$a_0 = 0, \qquad a_{n+1} = \sqrt{4 + 3a_n} \quad (n \in \mathbb{N})$$