Computer Science BSc
Basic Mathematics TEST-3
14-th of December 2020

Reasoning and justification are needed in the solutions

1. *(12 points)* Determine the eigenvalues and eigenvectors of the following matrix. Determine the algebraic and geometric multiplicities of the eigenvalues. Discuss the digonalizability of A (determine C and $C^{-1}AC$

$$A = \begin{bmatrix} 0 & 1 & -1 \\ 3 & -2 & 3 \\ 1 & -1 & 2 \end{bmatrix} \in \mathbb{R}^{3\times 3}$$

2. Denote by W the subspace in $\mathbb{R}^4$ generated by the linearly independent system:

$$b_1 = (-1,\, 0,\, 1,\, 2), \quad b_2 = (0,\, 1,\, 0,\, 1), \quad b_3 = (1,\, 1,\, 1,\, 1),$$

a) *(8 points)* Determine an orthogonal and an orthonormal basis in W.

b) *(6 points)* Decompose the vector $x = (-1,\, 2,\, 2,\, 0)$ by the subspace W into parallel and orthogonal components.

3. *(8 points)* Consider the following $\mathbb{R} \to \mathbb{R}$ type function f:

$$f(x) = x^2 - 4x + 5 \qquad (x \in [3\,,\, +\infty)\,)$$

Prove that f is invertible, and determine the sets $D_{f^{-1}}$, $R_{f^{-1}}$ and the for $y \in D_{f^{-1}}$ the function value $f^{-1}(y)$.

4. *(8 points)* Prove by definition that

$$\lim_{x\to+\infty} \frac{3x^4 - x^3 + 2x^2 + 5}{x^4 - 2x^2 + 3} = 3$$

5. *(8 points)* Using the EBT-method (=Gauss-Jordan method) determine the inverse of the following matrix:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix} \in \mathbb{R}^{3\times 3}$$