Basic Mathematics, Test 4/Retake Test from Part 3, 7th of June 2022.

1. Evaluate the eigenvalues, eigenvectors, eigenspaces of the following matrix. Is there an eigenbasis in $\mathbb{R}^3$? Is A diagonalizable? Give its diagonal form, and the diagonalizer matrix C and transformation which gives the diagonal form of A. **(16 points)**

$$A = \begin{bmatrix} 1 & 2 & -2 \\ -3 & 4 & 0 \\ -3 & 1 & 3 \end{bmatrix} \in \mathbb{R}^{3\times 3}$$

2. Let $W \subset \mathbb{R}^4$ denote the span generated by the following linearly independent vectors :

$$v_1 = (0, 2, 1, 0), v_2 = (-1, -3, 1, 0), v_3 = (6, 2, 1, 3).$$

a) Give an orthogonal and orthonormed basis in W.
b) Decompose vector $x = (-8, 8, -6, 7)$ into a paralell and orthogonal component related to W. **(9+6 points)**

3. Consider the function :

$$f(x) = 1 - 2x - x^2 \qquad (x \in (-\infty; -1]).$$

Prove that f is invertable, and give the sets $D_{f^{-1}}$, $R_{f^{-1}}$ and for $y \in D_{f^{-1}}$ values $f^{-1}(y)$.

(ATTENTION : prove your statements, we do not accept graphical solution here) **(9 points)**

4. Using the definition of the limit prove that :

$$\lim_{x \to +\infty} \frac{2x^4 + 7x^3 - 7x^2 + 2}{x^4 + 3x^3 - 2x^2 - 3} = 2. \textbf{(10 points)}$$