Basic Mathematics, Test 3, 16th of May 2022.

1. Find the eigenvalues, eigenvectors, eigenspaces of the following matrix. Give the algebraic and geometric multipicities of all the eigenvalues, and decide if there is an eigenbasis in $\mathbb{R}^3$ or not. Is A diagonalizable? If it is, give the diagonalizer matrix, and the diagonal form of A. *(16 points)*

$$A = \begin{bmatrix} 4 & -2 & -2 \\ 6 & -3 & -4 \\ -3 & 2 & 3 \end{bmatrix} \in \mathbb{R}^{3\times 3}$$

2. Let $W \subset \mathbb{R}^4$ be the subspace generated by the following linearly independent vectors :

$$v_1 = (0,2,1,0),\ v_2 = (1,1,3,-1),\ v_3 = (-5,3,-1,4).$$

a) Give an orthogonal and orthonormal basis in W.

b) Decompose vector

$$x = (19,17,-19,20)$$

into a paralell and orthogonal component related to the W space. *(9+6 points)*

3. Consider the following function :

$$f(x) = 2x^2 + 4x - 2 \qquad (x \in [-1;+\infty)).$$

Prove that f is invertable, and give the sets : $D_{f^{-1}}$, $R_{f^{-1}}$ and for $y \in D_{f^{-1}}$ the value $f^{-1}(y)$. (Attention : we do not accept here the "graphical" solution.) *(9 points)*

4. Using the definition prove that :

$$\lim_{x\to+\infty} \frac{3x^4 + 7x^3 + x^2 - 11}{x^4 + 2x^3 - x^2 - 3} = 3. \ (10\ points)$$