1. Colorful tree

Lemma 1.1: *Given two sets of nodes S_1 and S_2 and their diameters. Let the nodes that represent the diameter of S_1 equal to (U, V) and S_2 be equal to (X, Y). The diameter of $S_1 \cup S_2$ will be equal to the path (A, B) such that nodes A and B belong to {U, V, X, Y} and have maximum distance.*

Sub-problem 1.1: *Find the longest path between any nodes u and v such that the color of the nodes u and v are the same.*

To solve *sub-problem 1.1*, we can use *Lemma 1.1* to find the longest path for each color using a segment tree and the distance between any two nodes can be calculated in *O(1)* using LCA and a sparse table.

We can continue on the solution of *sub-problem 1.1* such that we have *C* segment trees for each color and *C* paths that represent the longest path for each color. We need to repeat the same method for these paths to get the longest path in the whole tree but for different colors. We will repeat the same approach for solving *subproblem 1.1* but for each node in the segment tree, it will hold the *MaxDiff* and *MaxSame*. *MaxDiff* will hold the longest path between two different colored nodes from the two children. *MaxSame* will hold the longest path between two same-colored nodes from the two children. *MaxSame* will take the maximum path between *X->Y* and *U->V*. *MaxDiff* will take the maximum path between the other four paths. Since there are queries you need to make a dynamic segment tree that generates a new leaf every time you need to.