Multimedia
Fall 2019
Lecture 2
JPEG

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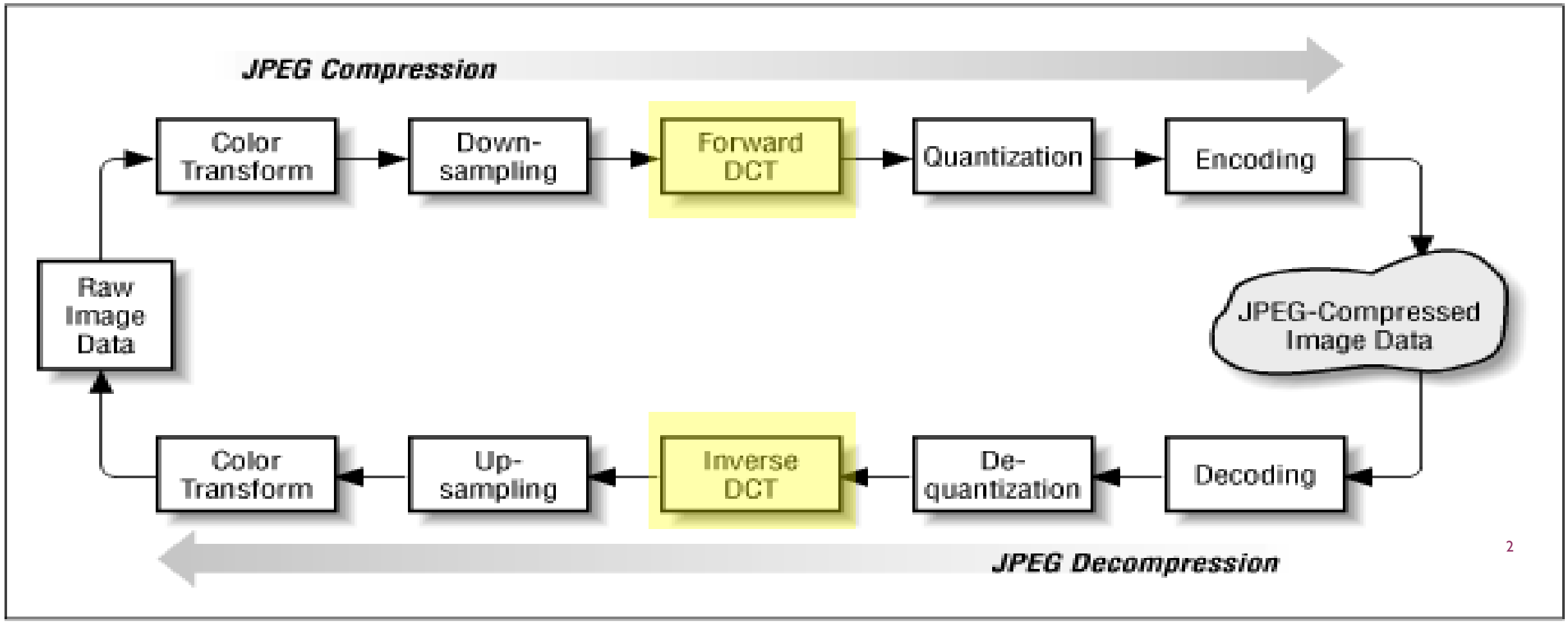
IT Dept.

JPEG


JPEG Compression
Color Transform
Down-sampling
Forward DCT
Quantization
Encoding
JPEG-Compressed Image Data
Decoding
De-quantization
Inverse DCT
Up-sampling
Color Transform
Raw Image Data
JPEG Decompression

JPEG : JOINT PHOTOGRAPHIC EXPERTS GROUP

JPEG encoder Consists of:

1. **Image/block preparation "Color Models"**
2. DCT computation
3. Quantization
4. Entropy coding [vectoring, differential encoding, run-length encoding, Huffman encoding]
5. Frame building

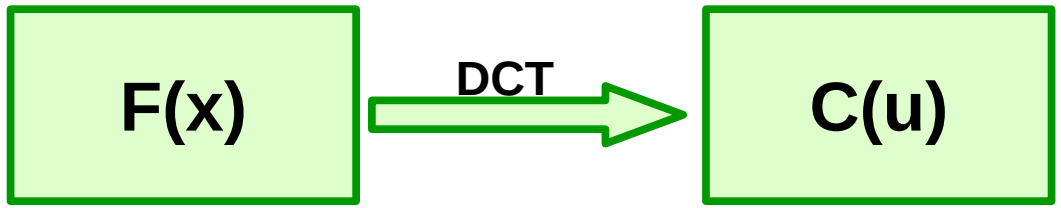
From this point on, each color component is processed independently, so a "pixel" means a single value, even in a color image

2-JPEG: DISCRETE COSINE TRANSFORM (DCT)

1-D Discrete Cosine Transform

$$C(u) = \alpha(u)\sum_{x=0}^{N-1} f(x)\cos\left(\frac{(2x+1)u\pi}{2N}\right)$$

$$\alpha(u) = \begin{cases} \sqrt{\frac{1}{N}} & \text{if } u = 0 \\ \sqrt{\frac{2}{N}} & \text{otherwise} \end{cases}.$$

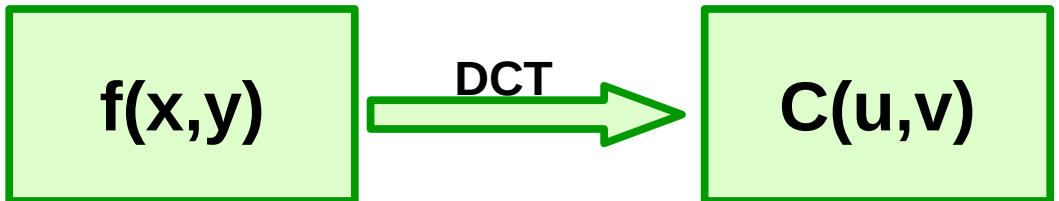


DISCRETE COSINE TRANSFORM (2D-DCT)

2-D Discrete Cosine Transform

the definition for alpha is the same as before

$$C(u,v) = \alpha(u)\alpha(v)\sum_{x=0}^{N-1}\sum_{y=0}^{N-1} f(x,y)\cos\left(\frac{(2x+1)u\pi}{2N}\right)\cos\left(\frac{(2y+1)v\pi}{2N}\right)$$



INVERSE DCT

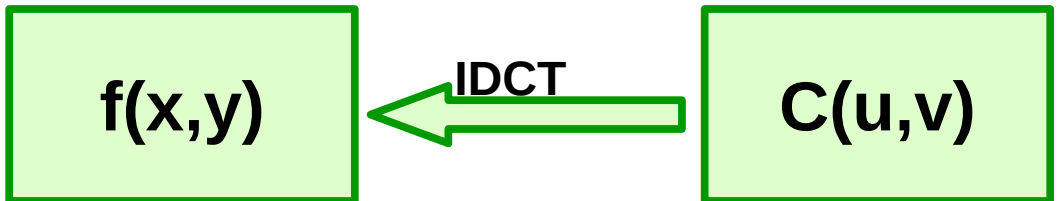
The DCT is invertible

Spatial samples can be recovered from the DCT coefficients

$$f(x) = \sum_{u=0}^{N-1} \alpha(u) C(u) \cos\left(\frac{(2x+1)u\pi}{2N}\right)$$

$$f(x,y) = \sum_{u=0}^{N-1} \sum_{v=0}^{N-1} \alpha(u)\alpha(v) C(u,v) \cos\left(\frac{(2x+1)u\pi}{2N}\right) \cos\left(\frac{(2y+1)v\pi}{2N}\right)$$

F(u,v)



SOME DCT PROPERTIES

- The DCT provides energy compaction

Low frequency coefficients have larger magnitude (typically)

High frequency coefficients have smaller magnitude (typically)

Most information is compacted into the lower frequency coefficients

(those coefficients at the 'upper-left')

- Compaction can be used for compression

Use the DCT coefficients to store image data but discard a certain percentage of the high-frequency coefficients!

JPEG does this

DCT ON A 8X8 BLOCK

139	144	149	153	155	155	155	155
144	151	153	156	159	156	156	156
150	155	160	163	158	156	156	156
159	161	162	160	160	159	159	159
159	160	161	162	162	155	155	155
161	161	161	161	160	157	157	157
162	162	161	163	162	157	157	157
162	162	161	161	163	158	158	158

$$\alpha(u) = \begin{cases} \sqrt{\frac{1}{N}} & u = 0 \\ \sqrt{\frac{2}{N}} & \text{o.w.} \end{cases} \qquad \alpha(v) = \begin{cases} \sqrt{\frac{1}{N}} & v = 0 \\ \sqrt{\frac{2}{N}} & \text{o.w.} \end{cases}$$

235.6	-1.0	-12.1	-5.2	2.1	-1.7	-2.7	1.3
-22.6	-17.5	-6.2	-3.2	-2.9	-0.1	0.4	-1.2
-10.9	-9.3	-1.6	1.5	0.2	-0.9	-0.6	-0.1
-7.1	-1.9	0.2	1.5	0.9	-0.1	-0.0	0.3
-0.6	-0.8	1.5	1.6	-0.1	-0.7	0.6	1.3
1.8	-0.2	1.6	-0.3	-0.8	1.5	1.0	-1.0
-1.3	-0.4	-0.3	-1.5	-0.5	1.7	1.1	-0.8
-2.6	1.6	-3.8	-1.8	1.9	1.2	-0.6	-0.4

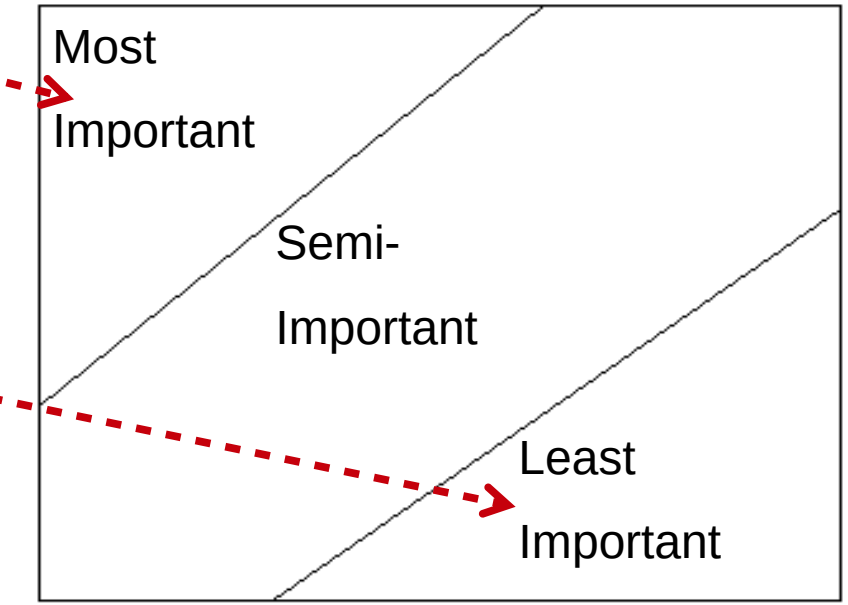
$$C(u,v) = \alpha(u)\alpha(v) \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} f(x,y) \cos\left(\frac{(2x+1)u\pi}{2N}\right) \cos\left(\frac{(2y+1)v\pi}{2N}\right)$$

IMPORTANCE OF DCT COEFFICIENTS

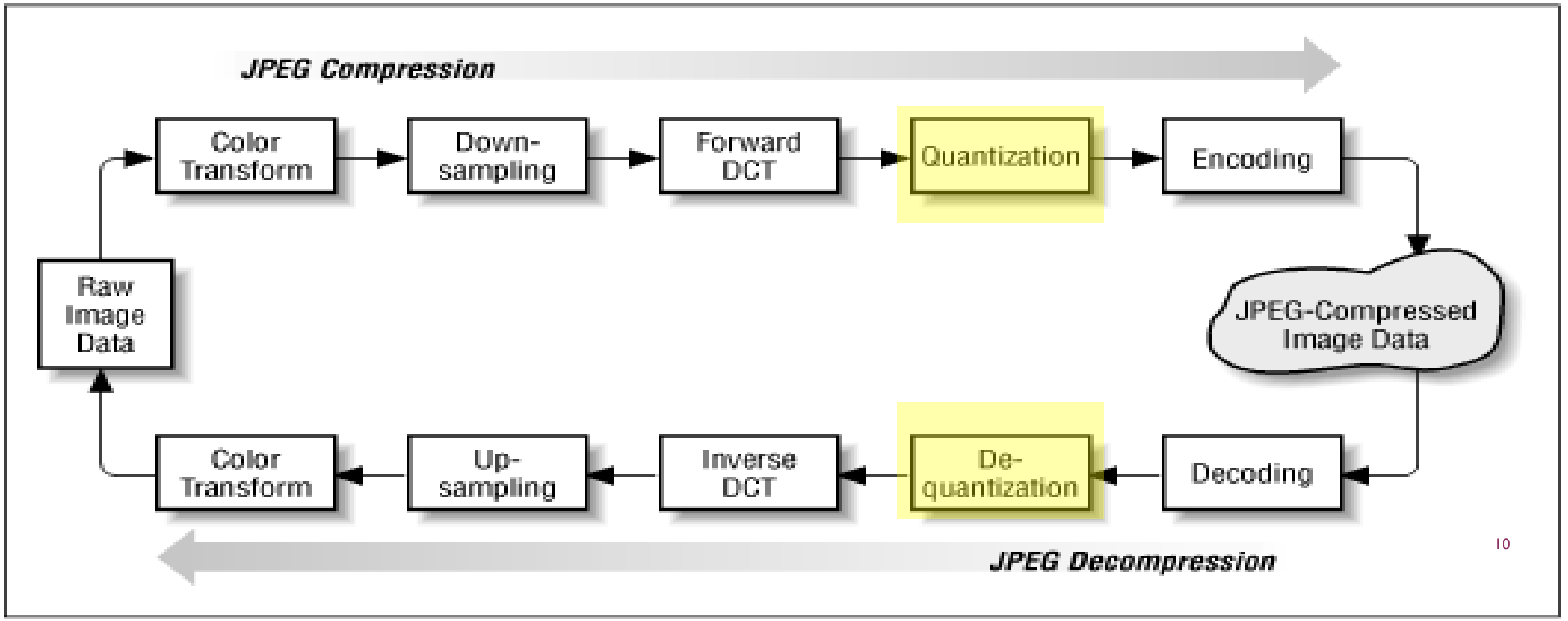
- Using the DCT, the entries will be organized based on the human visual system.

- **The <u>most</u> important values** to
- our eyes will be placed in the
- upper left corner of the matrix.

- **The <u>least</u> important values**
- will be mostly in the lower
- right corner of the matrix.



JPEG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3-JPEG: QUANTIZATION

- After DCT transform we have fraction, +ve and –ve numbers, which mean more bits is needed to store each block, we have to quantize DCT blocks But any change in any number of DCT will propagate to overall image ,so HOW TO QUANTIZE??
- We formulate quntizer for each block with 64 different steps , but HOW TO DETERMINE STEP SIZE??

135.6	-1.0	-12.1	-5.2	2.1	-1.7	-2.7	1.3
-22.6	-17.5	-6.2	-3.2	-2.9	-0.1	0.4	-1.2
-10.9	-9.3	-1.6	1.5	0.2	-0.9	-0.6	-0.1
-7.1	-1.9	0.2	1.5	0.9	-0.1	-0.0	0.3
-0.6	-0.8	1.5	1.6	-0.1	-0.7	0.6	1.3
1.8	-0.2	1.6	-0.3	-0.8	1.5	1.0	-1.0
-1.3	-0.4	-0.3	-1.5	-0.5	1.7	1.1	-0.8
-2.6	1.6	-3.8	-1.8	1.9	1.2	-0.6	-0.4

3-JPEG: QUANTIZATION

- Each of the *64 positions* of the DCT output block has its *own quantization* coefficient, with the higher-order terms with low magnitude being quantized more heavily than the low-order terms (that is, the *higher-order terms have larger quantization coefficients*).
- *Separate* quantization tables are employed for *luminance and chrominance* data, with the chrominance data being quantized more heavily than the luminance data. This allows JPEG to exploit further the eye's differing sensitivity to luminance and chrominance.

Quantization

16	11	10	16	24	40	51	61
12	12	14	19	26	58	60	55
14	13	16	24	40	57	69	56
14	17	22	29	51	87	80	62
18	22	37	56	68	109	103	77
24	35	55	64	81	104	113	92
49	64	78	87	103	121	120	101
72	92	95	98	112	100	103	99

The Luminance Quantization Table

17	18	24	47	99	99	99	99
18	21	26	66	99	99	99	99
24	26	56	99	99	99	99	99
47	66	99	99	99	99	99	99
99	99	99	99	99	99	99	99
99	99	99	99	99	99	99	99
99	99	99	99	99	99	99	99
99	99	99	99	99	99	99	99

The Chrominance Quantization Table

- The entries of Q(u,v) tend to have larger values towards the lower right corner. This aims to introduce more loss at the higher spatial frequencies.
- The tables above show the default Q(u,v) values obtained from psychophysical studies with the goal of maximizing the compression ratio while minimizing perceptual losses in JPEG images.

HOW TO FORMULATE MY QUANTIZATION MATRIX ??

$$Q = \left\lfloor \frac{x}{\Delta} + 0.5 \right\rfloor$$

If x is =+ve

$$Q = \left\lfloor \frac{x}{\Delta} - 0.5 \right\rfloor$$

If x is =-ve

Quantization

x^q=QXΔ

De-Quantization

EXAMPLE:

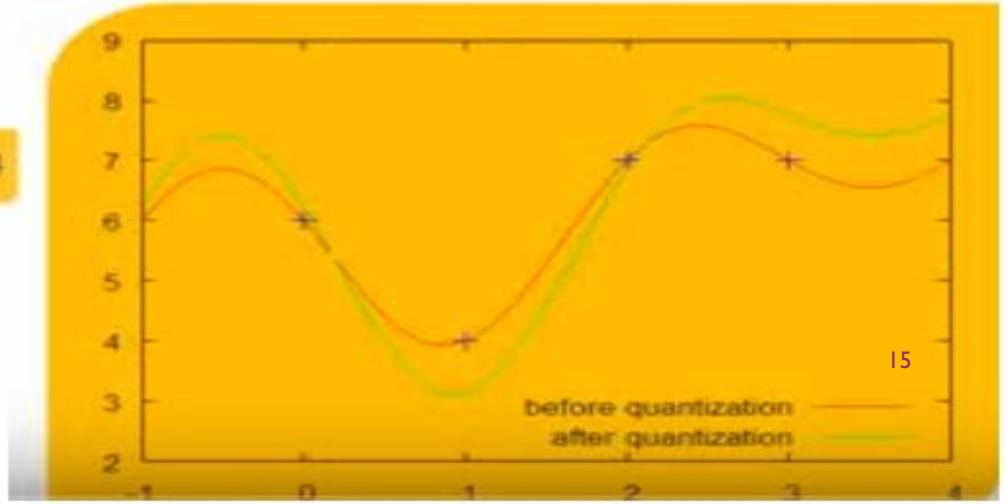
Example: Uniform quantization with $\Delta = 2$ of x = {6.00, -1.47, 1.00, 1.69}

$$q = \left\lfloor \frac{x}{\Delta} + 0.5 \right\rfloor \Rightarrow q = \{3, -1, 1, 1\}$$

$$\hat{x} = q \cdot \Delta \Rightarrow \hat{x} = \{6, -2, 2, 2\}$$

Quantization error:

$$MSE = \frac{1}{4}\left((6-6)^2 + ((-2)-(-1.47))^2 + (2-1)^2 + (2-1.69)^2\right) = 0.34$$



QUANTIZED DCT COEFFICIENTS

16	11	10	16	24	40	51	61
12	12	14	19	26	58	60	55
14	13	16	24	40	57	69	56
14	17	22	29	51	87	80	62
18	22	37	56	68	109	103	77
24	35	55	64	81	104	113	92
49	64	78	87	103	121	120	101
72	92	95	98	112	100	103	99

Quantization Matrix

235.6	-1.0	-12.1	-5.2	2.1	-1.7	-2.7	1.3
-22.6	-17.5	-6.2	-3.2	-2.9	-0.1	0.4	-1.2
-10.9	-9.3	-1.6	1.5	0.2	-0.9	-0.6	-0.1
-7.1	-1.9	0.2	1.5	0.9	-0.1	-0.0	0.3
-0.6	-0.8	1.5	1.6	-0.1	-0.7	0.6	1.3
1.8	-0.2	1.6	-0.3	-0.8	1.5	1.0	-1.0
-1.3	-0.4	-0.3	-1.5	-0.5	1.7	1.1	-0.8
-2.6	1.6	-3.8	-1.8	1.9	1.2	-0.6	-0.4

DCT Block

$$\left\lfloor \frac{235.6}{16} + 0.5 \right\rfloor = 15$$

15	0	-1	0	0	0	0	0
-2	-1	0	0	0	0	0	0
-1	-1	0	0	0	0	0	0
-1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

Quantizatied DCT B

JPEG Example

Quantization Table Used

8	5	5	8	12	20	25	30
6	6	7	9	13	29	30	27
7	6	8	12	20	28	34	28
7	8	11	14	25	43	40	31
9	11	18	28	34	54	51	38
12	17	27	32	40	52	56	46
24	32	39	43	51	60	60	50
36	46	47	49	56	50	51	49

16	11	10	16	24	40	51	61
12	12	14	19	26	58	60	55
14	13	16	24	40	57	69	56
14	17	22	29	51	87	80	62
18	22	37	56	68	109	103	77
24	35	55	64	81	104	113	92
49	64	78	87	103	121	120	101
72	92	95	98	112	100	103	99

64	44	40	64	96	160	204	244
48	48	56	76	104	232	240	220
56	52	64	96	160	228	276	224
56	68	88	116	204	300	300	248
72	88	148	224	272	300	300	300
96	140	220	256	300	300	300	300
196	256	300	300	300	300	300	300
288	300	300	300	300	300	300	300

128	128	128	128	128	128	128	128
128	128	128	128	128	128	128	128
128	128	128	128	128	128	128	128
128	128	128	128	128	128	128	128
128	128	128	128	128	128	128	128
128	128	128	128	128	128	128	128
128	128	128	128	128	128	128	128
128	128	128	128	128	128	128	128

Compressed Image

Compression Ratio: 7.7

Compression Ratio: 12.3

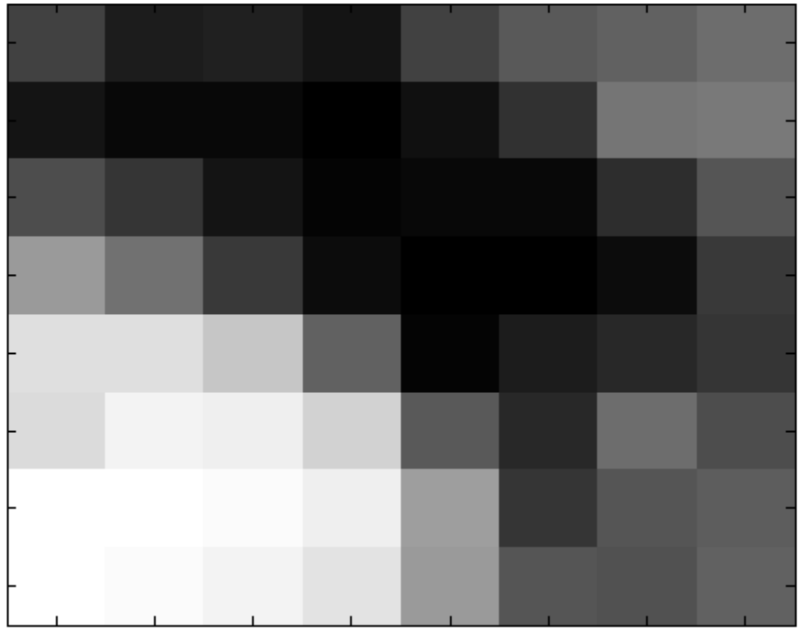
Compression Ratio: 33.9

Compression Ratio: 60.1

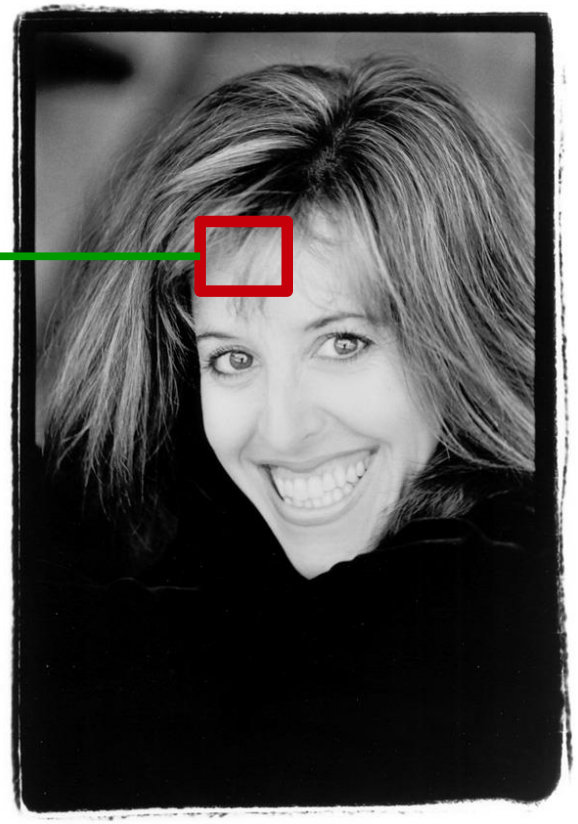
Original Image

EXAMPLE: DCT WITH QUANTIZATION

- 8 x 8 Pixels



Original Image

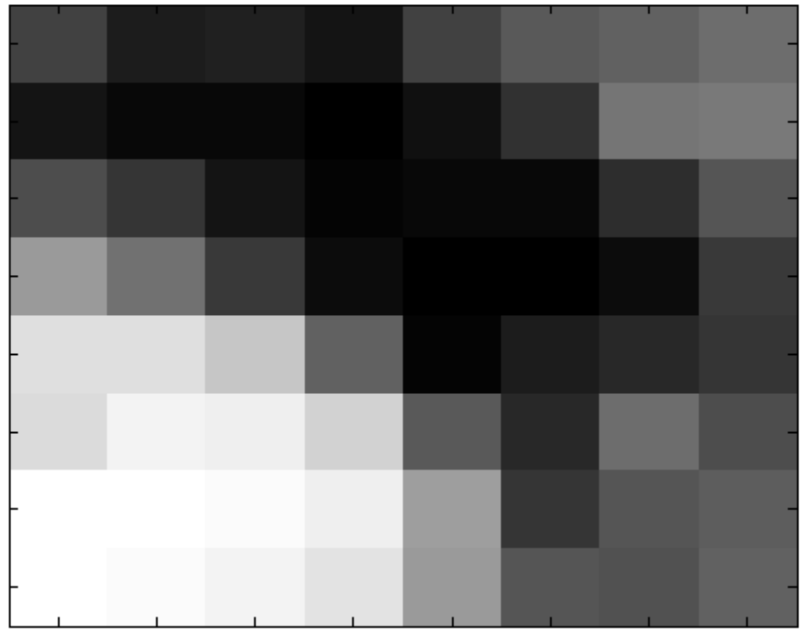


EXAMPLE: DCT WITH QUANTIZATION

- **Gray-Scale Example:** Value Range 0 (black) --- 255 (white)

63	33	36	28	63	81	86	98
27	18	17	11	22	48	104	108
72	52	28	15	17	16	47	77
132	100	56	19	10	9	21	55
187	186	166	88	13	34	43	51
184	203	199	177	82	44	97	73
211	214	208	198	134	52	78	83
211	210	203	191	133	79	74	86

Y (8x8 Block)



EXAMPLE: DCT WITH QUANTIZATION

63	33	36	28	63	81	86	98
27	18	17	11	22	48	104	108
72	52	28	15	17	16	47	77
132	100	56	19	10	9	21	55
187	186	166	88	13	34	43	51
184	203	199	177	82	44	97	73
211	214	208	198	134	52	78	83
211	210	203	191	133	79	74	86

Y (8x8 Block)

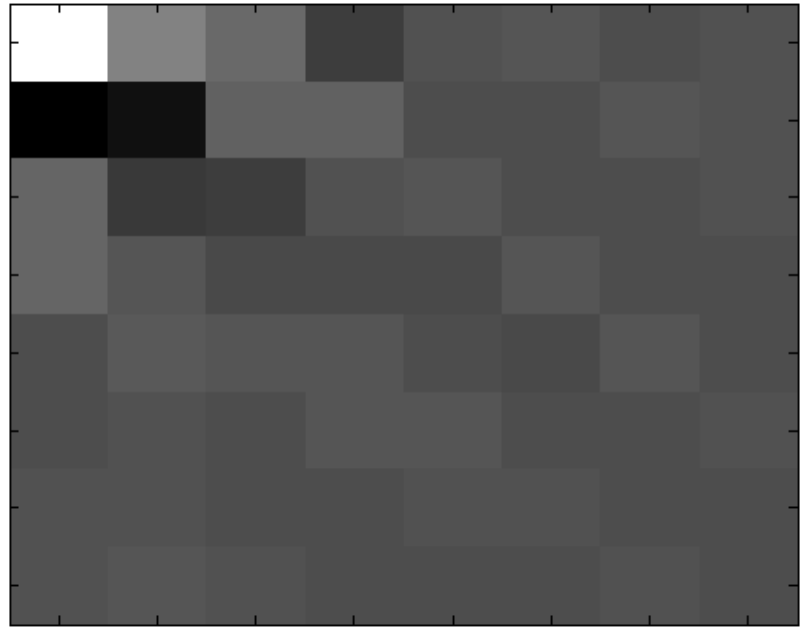
340	210	104	-69	10	20	-12	7
-327	-260	67	70	-10	-15	21	8
93	-84	-66	16	24	-2	-5	9
89	33	-19	-20	-26	21	-3	0
-9	42	18	27	-7	-17	29	-7
-5	15	-10	17	32	-15	-4	7
10	3	-12	-1	2	3	-2	-3
12	30	0	-3	-3	-6	12	-1

EXAMPLE: DCT WITH QUANTIZATION

- 2D-DCT of matrix

Numbers are coefficients of polynomial

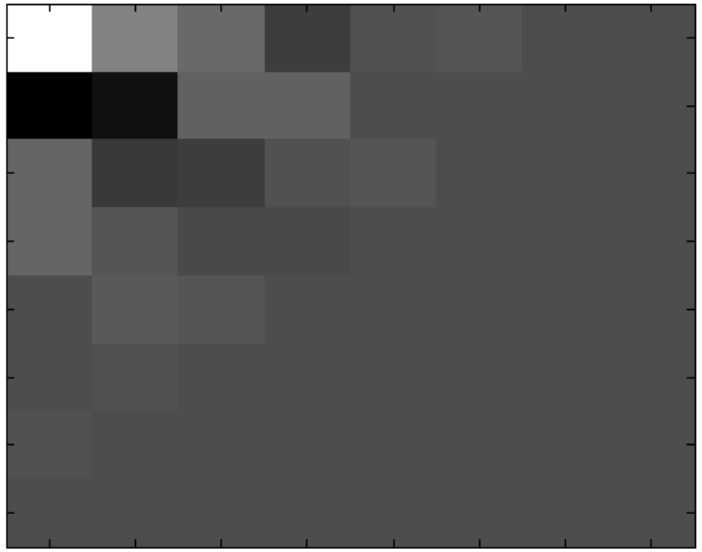
- 340 210 104 -69 10 20 -12 7
- -327 -260 67 70 -10 -15 21 8
- 93 -84 -66 16 24 -2 -5 9
- 89 33 -19 -20 -26 21 -3 0
- -9 42 18 27 -7 -17 29
- -5 15 -10 17 32 -15 -4 7
- 10 3 -12 -1 2 3 -2 -3
- 12 30 0 -3 -3 -6 12 -1



EXAMPLE: DCT WITH QUANTIZATION

- Cut the least significant components (High Frequency Components) after quantization

340	210	104	-69	10	20	-12	0
-327	-260	67	70	-10	-15	0	0
93	-84	-66	16	24	0	0	0
89	33	-19	-20	0	0	0	0
-9	42	18	0	0	0	0	0
-5	15	0	0	0	0	0	0
10	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0



EXAMPLE: DCT WITH QUANTIZATION

340	**210**	**104**	**-69**	**10**	**20**	**-12**	**0**
-327	**-260**	**67**	**70**	**-10**	**-15**	**0**	**0**
93	**-84**	**-66**	**16**	**24**	**0**	**0**	**0**
89	**33**	**-19**	**-20**	**0**	**0**	**0**	**0**
-9	**42**	**18**	**0**	**0**	**0**	**0**	**0**
-5	**15**	**0**	**0**	**0**	**0**	**0**	**0**
10	**0**	**0**	**0**	**0**	**0**	**0**	**0**
0	**0**	**0**	**0**	**0**	**0**	**0**	**0**

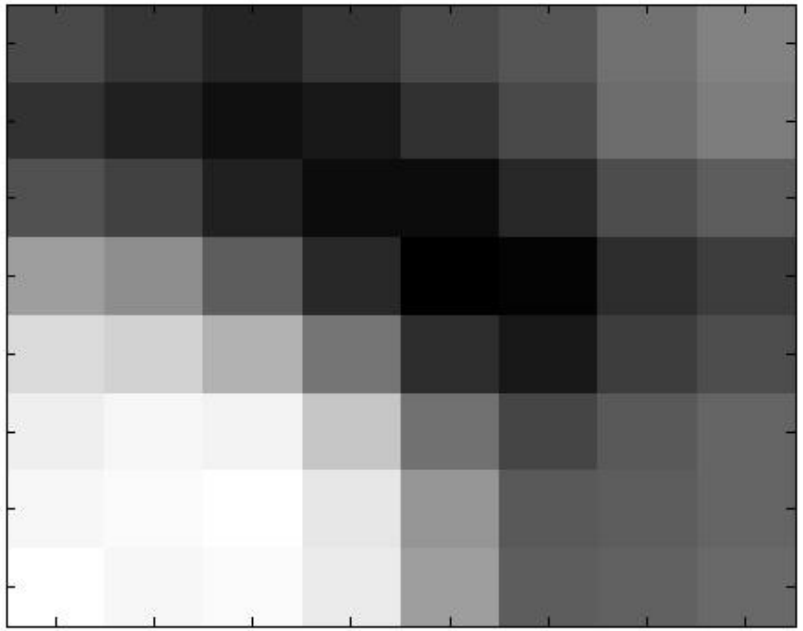
55	41	27	39	56	69	92	106
35	22	7	16	35	59	88	101
65	49	21	5	6	28	62	73
130	114	75	28	-7	-1	33	46
180	175	148	95	33	16	45	59
200	206	203	165	92	55	71	82
205	207	214	193	121	70	75	83
214	205	209	196	129	75	78	85

EXAMPLE: DCT WITH QUANTIZATION

- Apply Inverse DCT in the Image

55	41	27	39	56	69	92	106
35	22	7	16	35	59	88	101
65	49	21	5	6	28	62	73
130	114	75	28	-7	-1	33	46
180	175	148	95	33	16	45	58
200	206	203	165	92	55	71	82
205	207	214	193	121	70	75	83
214	205	209	196	129	75	78	85

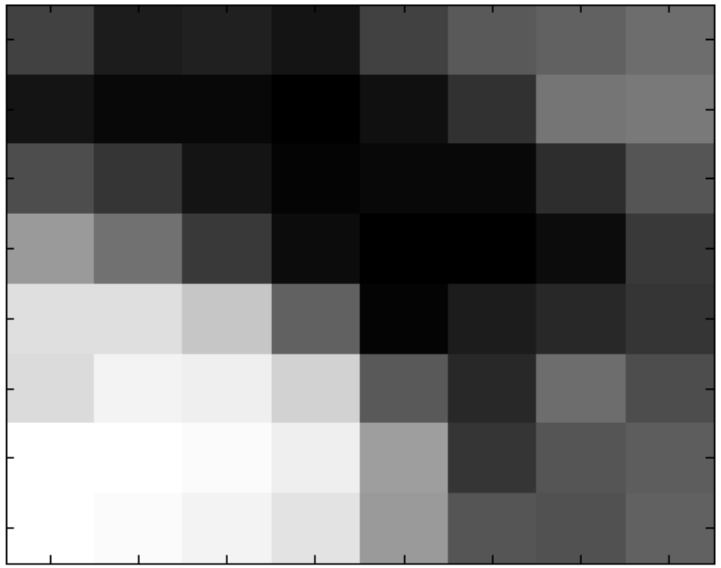
IDCT

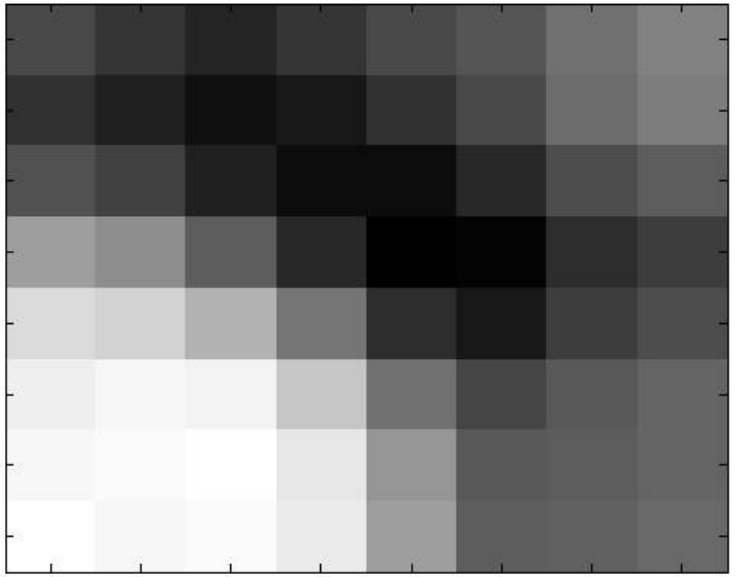


EXAMPLE: DCT WITH QUANTIZATION

- Original

Compressed-Decompressed





EXAMPLE: DCT WITH QUANTIZATION

- Original

Compressed-Decompressed