3. Image Transformation

Fourier Transformation

$$F(U,V) = F(f(x,y)) = \frac{1}{NM}\sum_{Y=0}^{M-1}\sum_{X=0}^{N-1} f(x,y)e^{-2\pi j(\frac{UX}{N}+\frac{VY}{M})}$$

$$e^{-2\pi jX} = cos(\pi X) - jsin(\pi X) \;\; Where\ \pi = 180$$

Inverse

$$F(X,Y) = F^{-1}(f(u,v)) = \sum_{V=0}^{M-1}\sum_{U=0}^{N-1} f(u,v)e^{2\pi j(\frac{UX}{N}+\frac{VY}{M})}$$

$$e^{2\pi jX} = cos(\pi X) + jsin(\pi X) \;\; Where\ \pi = 180$$

$Magnitude = ||e|| = \sqrt{R^2 + I^2}$
$Power\ element = R^2 = I^2$
$Phase\ element = tan^{-1}\frac{1}{R}$

Discrete cosine transform

$$C(U,V) = \alpha(u)\alpha(v)\sum_{Y=0}^{M-1}\sum_{X=0}^{N-1} f(x,y)cos(\frac{2X+1}{2N} * \pi u)cos(\frac{2Y+1}{2M} * \pi v)$$

$$Inverse\ F(X,Y) = \sum_{v=0}^{M-1}\sum_{u=0}^{N-1} \alpha(u)\alpha(v)C(x,y)cos(\frac{2X+1}{2N} * \pi u)cos(\frac{2Y+1}{2M} * \pi v)$$

$\alpha(u) = \frac{1}{\sqrt{n}}\ if\ u = 0\ else\ \alpha(u) = \sqrt{\frac{2}{n}}$
$\alpha(v) = \frac{1}{\sqrt{m}}\ if\ v = 0\ else\ \alpha(v) = \sqrt{\frac{2}{m}}$

Ideal low pass filter

- Filter that passes signals with a frequency lower than a certain cut of frequency and weaken the signals with frequencies higher than the cut off frequency
- Frequency of index = $\sqrt{u^2 + V^2}$
- F(x,y) --> F(u,v) then if F > FC --> 0

Ideal high pass filter

- Filter that passes signals with a frequency greater than a certain cut of frequency and waken the signals with frequencies higher than the cut off frequency.
- Frequency of index = $\sqrt{u^2 + V^2}$
- F(x,y) --> F(u,v) then if F < FC --> 0

Power and cut off frequency

1. Matrix of Power --> p = $R^2 + I^2$
2. Total power
3. Matrix of Frequency --> F = $\sqrt{u^2 + v^2}$
4. Calculate percentage --> perc = $(\frac{Totalpower - (power + prev\ steps\ powers)}{Totalpower}) * 100$
    1. when asking for high pass starting from (0,0)
    2. when asking for low pass starting from (N,M)

Butter worth filter

- low pass filter --> H(f)=($\frac{1}{1+[\frac{f}{f_0}]^{2n}}$)
- High pass filter --> H(f)=($\frac{1}{1+[\frac{f_0}{f}]^{2n}}$)
- n --> filter order
- f_0 --> mid frequency

Gaussian filters

- low pass filter -> $H(u,v) = e^{-D^2(u,v)/2s^2}$
- high pass filter -> $H(u,v) = 1 - e^{-D^2(u,v)/2s^2}$
- $D(u,v) = \sqrt{u^2 + v^2}$
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