# Lecture 4. Induction

DR. YARASLAU ZADVORNY

# Mathematical Induction. The main idea

Suppose we have a sequence of statements, which we denote as $S_1$, $S_2$, $S_3$, …

The way for proving all the statements from the sequence is two prove that:

**Base Case.** $S_1$ is correct;

**Inductive Step.** For any natural $n$, if $S_n$ is correct then $S_{n+1}$ is correct.

$$S_1 \qquad S_2 \qquad S_3 \qquad S_4$$

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**Example 1.** Prove that

$$1+2+3+\ldots+n=\frac{n(n+1)}{2}$$

Another way to prove:

$$(1 + 2 + 3 + \ldots + n) + (n + (n - 1) + (n - 2) + \ldots + 1) =$$

$$= (1 + n) + (2 + n - 1) + \ldots + (n + 1) = n(n + 1)$$

$$2 \times (1 + 2 + 3 + \ldots + n) = n(n + 1)$$

$$1+2+3+\ldots+n=\frac{n(n+1)}{2}$$

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**Example 1.** Prove that

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The sequence of statements:

$$S_1:1=\frac{1\cdot 2}{2}$$

$$S_2:1+2=\frac{2\cdot 3}{2}$$

...

$$S_n:1+2+...+n=\frac{n\cdot(n+1)}{2}$$

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$$1+2+...+n+(n+1)=\frac{n\cdot(n+1)}{2}+(n+1)$$

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$$1+2+\ldots+n+(n+1)=\frac{n\cdot(n+1)}{2}+(n+1)=\frac{n\cdot(n+1)+2(n+1)}{2}=\frac{(n+1)\cdot(n+2)}{2}$$

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**Example 2.** Prove that

$$1^2 + 2^2 + \ldots n^2 = \frac{n(n+1)(2n+1)}{6}$$

The sequence of statements:

$$S_1 : 1^2 = \frac{1\cdot(1+1)(2+1)}{6}$$

$$S_2 : 1^2 + 2^2 = \frac{2\cdot(2+1)(4+1)}{6}$$

...

$$S_n = 1^2 + 2^2 + \ldots n^2 = \frac{n(n+1)(2n+1)}{6}$$

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# Mathematical Induction: Inequality

**Example 3.** Prove that for natural $n$ and $h$ > -1 there holds

$$(1+h)^n = 1+hn.$$

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**Base case:** Let *n* = 1. Then

$$(1+h) \geq (1+h).$$

# Mathematical Induction: Bernoulli‘s Inequality

**Example 3.** Prove that for natural $n$ and $h$ > -1 there holds

$$(1+h)^n = 1+hn.$$

**Base case:** Let $n$ = 1. Then

$$(1+h) \geq (1+h).$$

**Inductive Step:** Let it is known that

$$(1+h)^k \geq 1+kh.$$

Then

$$(1+h)^{k+1} = (1+h)^k (1+h) \geq (1+kh)(1+h) = 1+kh+h+kh^2 \geq 1+(k+1)h.$$

# Bernoulli‘s Inequality: Application

Suppose that a bank offers to take your money, and will give you 100% interest on it in a year's time. So if you give the bank $1000 now, you will get $2000 back in a year.

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Suppose that a bank offers to take your money, and will give you 100% interest on it in a year's time. So if you give the bank $1000 now, you will get $2000 back in a year.

Alternatively, the bank says, it will give you half that interest rate, but it will give you the interest twice as often: so it will give you 50% interest in half a year's time, and then a further 50% on that at the end of the year. Your $1000 will become $1500 after half a year, and then $2250 at the end of the year. By calculating half the interest twice as often, you end up with significantly more at the end.

# Bernoulli‘s Inequality: Application

So. this works in the following way:

$$1000\$ \times (1+1)^1 = 2000\$$$

$$1000\$ \times \left(1+\frac{1}{2}\right)^2 = 2250\$$$

$$1000\$ \times \left(1+\frac{1}{3}\right)^3 = 2370.37\$$$

$$\ldots$$

$$1000\$ \times \left(1+\frac{1}{n}\right)^n = ???$$

# Bernoulli‘s Inequality: Application

A question: Is it true that the more times we require interest rate, the more money we finally get?

In other words, is it true that

$$\left(1+\frac{1}{n+1}\right)^{n+1} > \left(1+\frac{1}{n}\right)^{n} ?$$

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$$n^3+3n^2+3n+2 > n^3+3n^2+3n+1$$

# Bernoulli‘s Inequality: Application

In fact, the greatest sum you can get is

$$1000\$ \times e = 2.7182...;$$

here

$$e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$$

# Mathematical Induction: One More Trick

**Example 4. Prove that**

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**Inductive Step:**

$$1+\frac{1}{2^2}+\frac{1}{3^2}+\ldots+\frac{1}{k^2}<2$$

$$1+\frac{1}{2^2}+\frac{1}{3^2}+\ldots+\frac{1}{(k+1)^2}<2$$

It looks like we must find another way.

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**Let us prove a stronger inequality:**

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**Inductive Step:**

$$1+\frac{1}{2^2}+\frac{1}{3^2}+\ldots+\frac{1}{k^2}\le 2-\frac{1}{k}$$

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# Mathematical Induction: One More Trick

**Example 4. Prove that**

$$1+\frac{1}{2^2}+\frac{1}{3^2}+\ldots+\frac{1}{n^2}<2$$

**Let us prove a stronger inequality:**

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$$1\le\frac{k^2+k+1}{k(k+1)}$$

# Another Schemes of Induction

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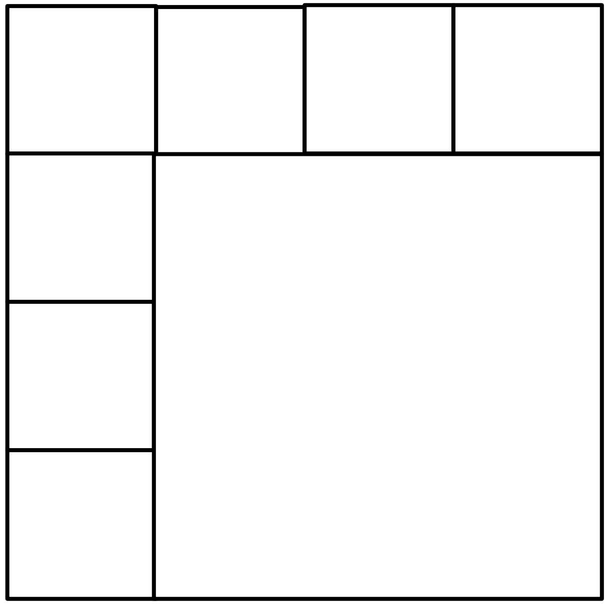

An example for 8 squares

# Another Schemes of Induction

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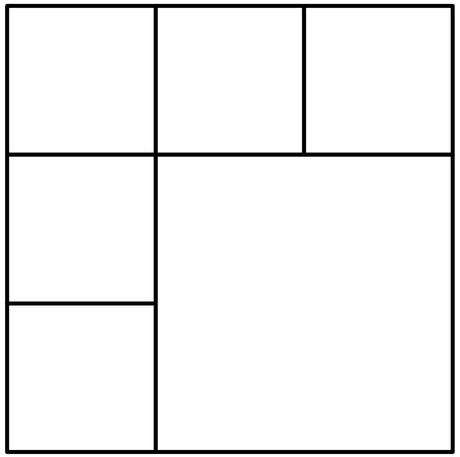
**Let's try! Base case: *n* = 6.**



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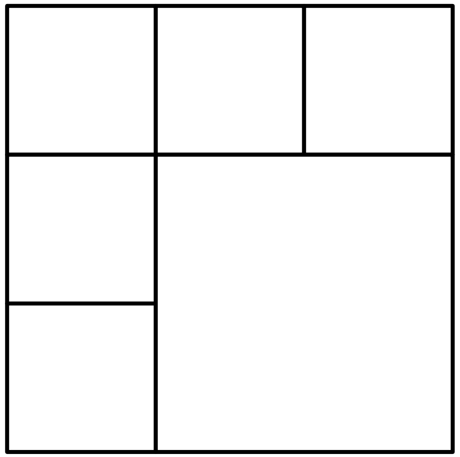

**Inductive Step: …???**

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**Let's try another way.**

**Base Case:** the statement holds for $n = 6, 7, 8$.

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**Why does it work?**

$$S_6 \to S_9 \to S_{12} \to \dots$$
$$S_7 \to S_{10} \to S_{13} \to \dots$$
$$S_8 \to S_{11} \to S_{14} \to \dots$$

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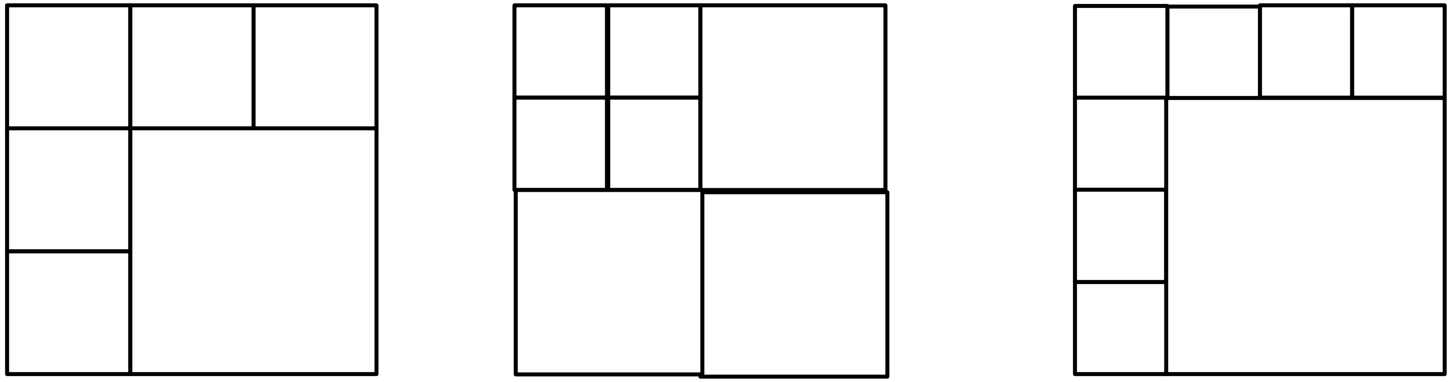

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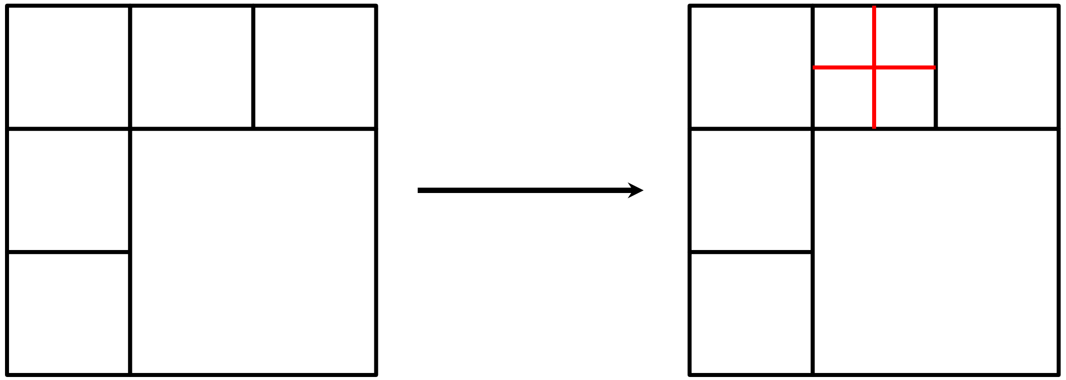

Thank you!