MATHEMATICS

Homework 4. Combinatorics

In the questions 1, 2 and 3 you have to choose **two** correct answers from the list, in the questions 4 and 5 you have to give a **solution**.

Question 1 (2 answers)**.** How many distinct 10-letter words can be formed from the letters of EQUALITIES?

- A $10!$
- B $\frac{10!}{2}$
- C $\frac{10!}{4}$
- D $\frac{8!}{2}$ words starting with EQ.
- E $\frac{8!}{4}$ words starting with EQ.

Question 2 (2 answers)**.** In an organic market, 8 types of cakes are being sold (cakes of the same type are identical).

- A There are $\binom{27}{7}$ ways to buy 20 cakes.
- B There are $\binom{8}{6}$ ways to buy 6 cakes.
- C There are 20^8 ways to buy 20 cakes.
- D There are $\binom{20}{6}$ ways to buy 20 cakes to taste at least six types.
- E There are $\binom{19}{7}$ ways to buy 20 cakes to taste all eight types.

Question 3 (2 answers)**.** There are 20 students in the class. They are going to form two studying groups of 5 and 15 people. In how many ways can they do it?

- A If no student can be in more than one group, then $\frac{20!}{5!} \cdot 15$ ways.
- B If no student can be in more than one group, then $\binom{20}{5}$ ways.
- C If any student can be in any number of groups, then $\frac{20!}{5!15!}$ ways.
- D If any student can be in any number of groups, then $\frac{(20!)^2}{5!15!}$ ways.
- E If any student can be in any number of groups, then $\binom{20}{5}^2$ ways.

Question 4. Find with explanation the number of ways that one can choose 8 cards from a standard deck of 52 cards in such a way that all four suits are present.

Question 5. A man buys 7 donuts, each of which is a plain donut, a powdered donut or a glazed donut. How many possible selections are there if he buys at least one donut of each kind?

- A Answer the question above by determining the coefficient of x^7 in a product of polynomials and/or power series.
- B Answer the question above by computing $\binom{s+t-1}{s}$ for an appropriate choice of s and t.