

Homework 8. Growth of Functions and Probability of Events

In the questions 1 and 2 you have to choose 1 correct answer from the list, in the questions 3, 4 and 5 you have to give a solution.

Question 1 (1 answer). Let f be the function defined by $f(n) = n^4 + 2$ for each $n \in \mathbb{N}$. For which of the following functions g is $f(n) = \Theta(g(n))$?

- ☐ A $g(n) = 4n^2 + 1$
- ☐ B $g(n) = 4^n$
- ☐ C $g(n) = 2n^4 + \log n$
- ☐ D $g(n) = n^3 \log n + 2$
- ☐ E $g(n) = n^4 \log n$

Question 2 (1 answer). A card is drawn at random from an ordinary deck of 52 playing cards.

- ☐ A The probability that it is a queen is $1/52$.
- ☐ B The probability that it is a hearts or a diamonds is $1/3$.
- ☐ C The probability that it is a two of clubs or a five of diamonds is $1/13$.
- ☐ D The probability that it is any suit except hearts is $1/3$.
- ☐ E The probability that it is a ten or a spade is $4/13$.

Question 3. Let f and g be the functions defined by $f(n) = 3^n$ and $g(n) = 5^n$ for each $n \in \mathbb{N}$. Show that $f(n) = O(g(n))$ but $f(n) \neq \Theta(g(n))$.

Question 4. Write an algorithm (in Python) that finds the first term in a sequence of integers that is equal to some previous term in the sequence. Then show that your algorithm has time complexity $O(f(n))$ for some common function f .

Question 5. Find the probability that at least three people in a group of 10 people have the same birthday (assuming that a year has 365 days).