Introduction to Control Systems

ELECENG 3CL4

Instructor: Prof. Tim Davidson

Amr ABD EL SHAKOUR–abdelsha

2 Design of a Phase Lead-Lag Compensator for the Servomotor

We chose our zero at -0.01 and our pole location at -0.001, so our lag compensator has the following transfer function:

$$\frac{S + 0.01}{S + 0.001}$$

This lag compensator should not alter the shape of root locus significantly and it should theoretically increase the velocity error constant by a factor of 10.

- 2.1 ○ Phase lag

```
s + 0.01
---------
s + 0.001
```

  - Closed loop transfer function with lead lag compensator

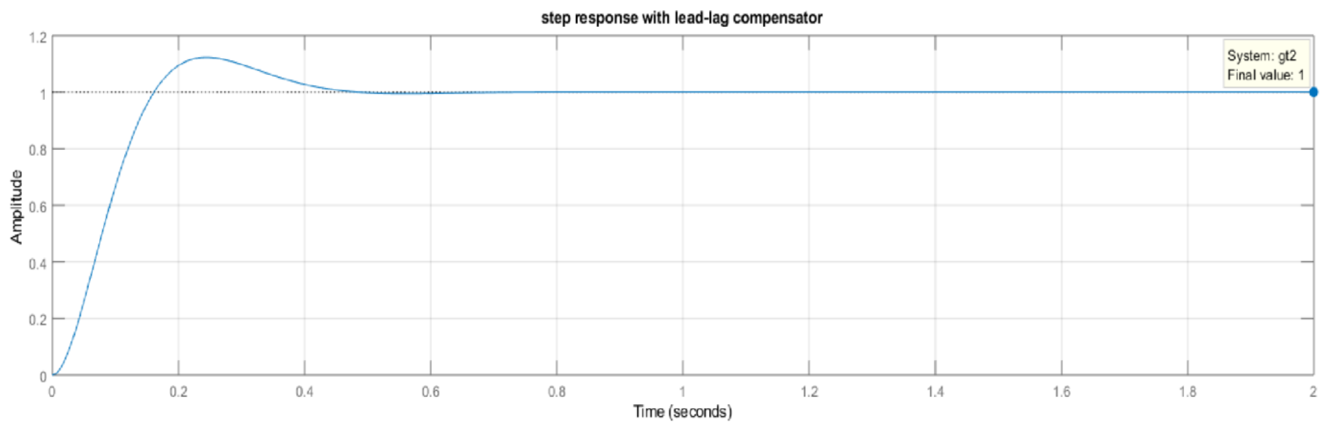
```
           320.2 s^2 + 3205 s + 32.02
---------------------------------------------
s^4 + 35.33 s^3 + 516.1 s^2 + 3205 s + 32.02
```

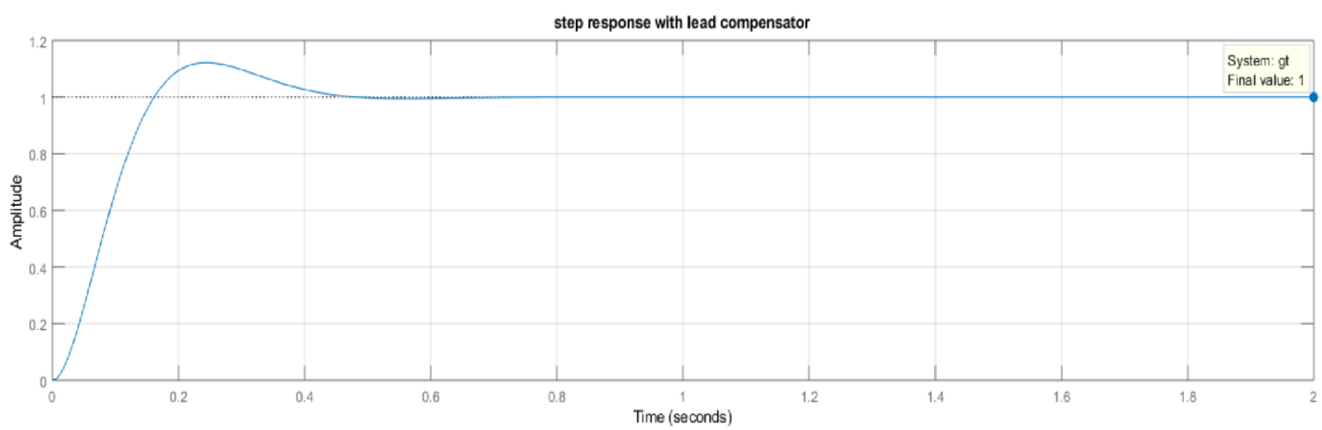
  - Closed loop transfer function with lead compensator

```
          320.2 s + 3202
--------------------------------
s^3 + 35.33 s^2 + 516.1 s + 3202
```

- 2.2
  - Lead lag controller
    - Zeros are -10, -0.010
    - Poles are -15.2585, -10.0292 +10.4466i, -10.0292 -10.4466i, -0.0100 ○ Lead controller
    - Zeros are-10
    - Poles are -15.2512, -10.0373 +10.4491i, -10.0373 -10.4491i ○ The impact of the phase lead controller increases the velocity error constant Kv by a factor of 10 without significantly changing the shape of the original root locus. As shown above, the newly introduced zero and pole from the lag compensator are very close to the origin, and they are not dominating compare with the rest of the poles and zeros.

- 2.3

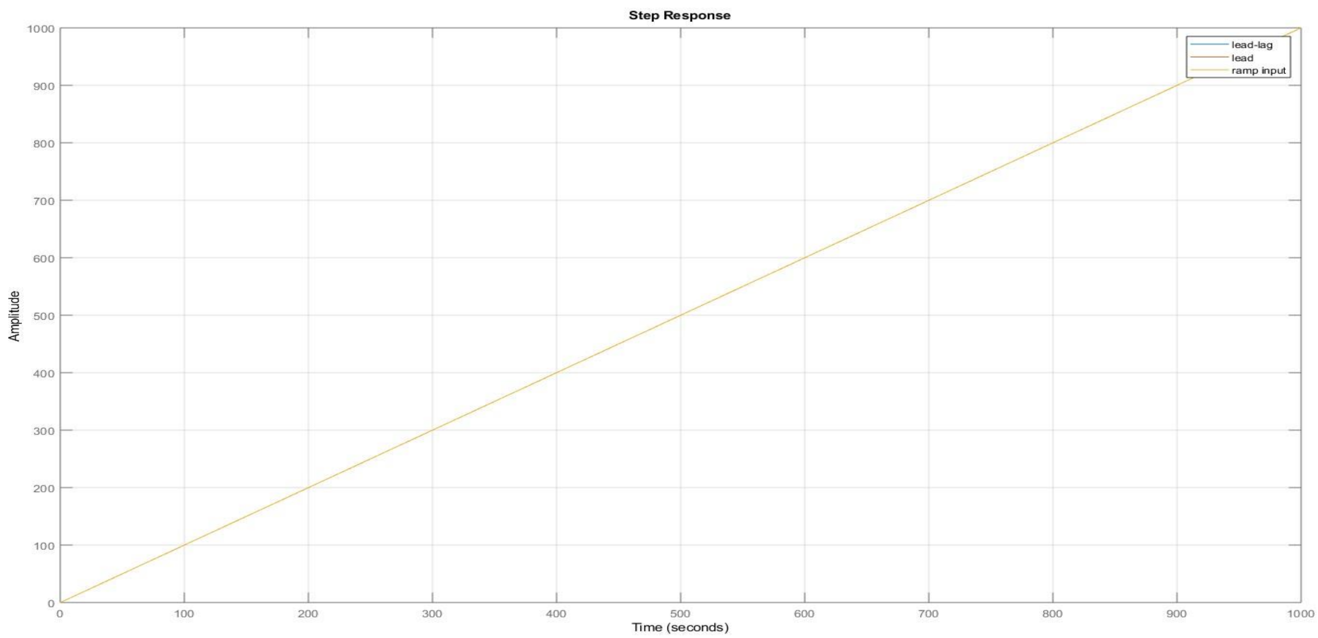


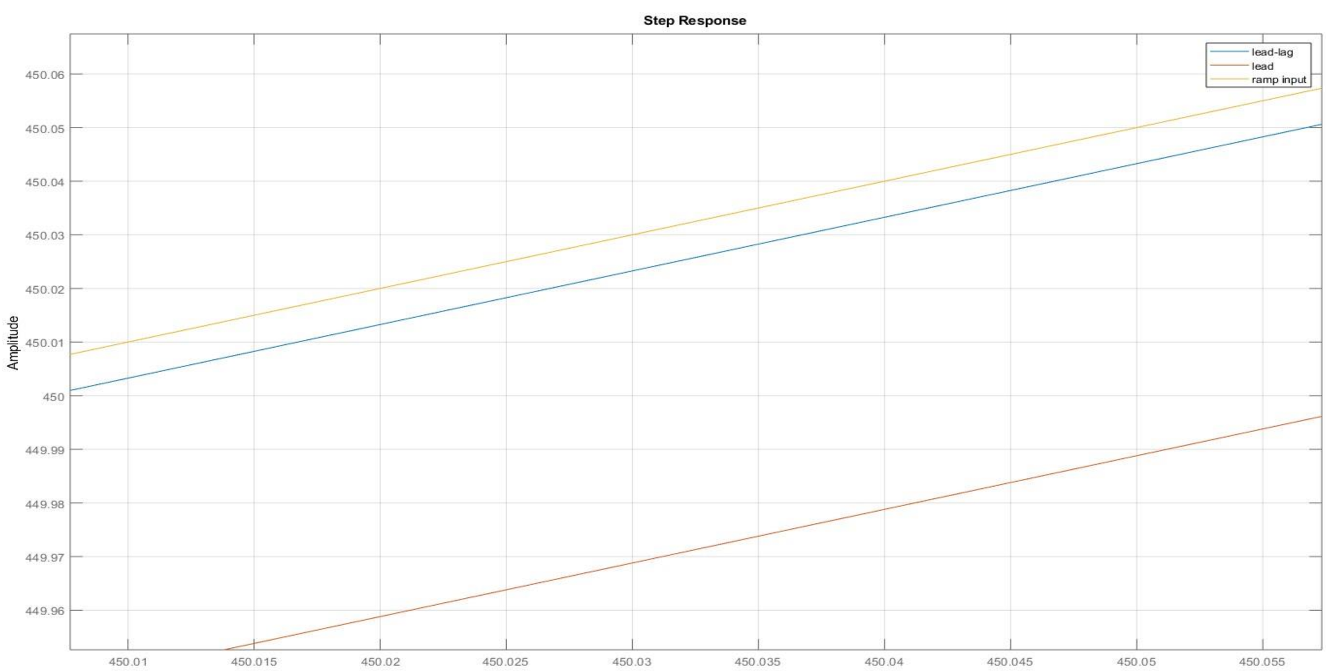


As we can observe above, the system with the lead-lag compensator has very similar step input response when compared with a system with only the lead compensator. This is our expectation because we understand the lag compensator would not reshape the root locus in any significant ways. This means for step inputs; our designed lead-lag system would have similar behaviour as the lead compensator system we developed in laboratory 4. The percentage overshoot and settling time requirements should be satisfied in theory.

- 2.4

- (Overall plot) response for a unit ramp input





- (Zoomed plot) response for a unit ramp input
- As demonstrated above, the addition of lag compensator has reduced the steady state error for a unit ramp input.

The velocity error constant for a system with only lead compensator can be calculated as follow:

$$Kv = \frac{1}{424.2698 - 424.2087} \approx 16.367$$

$$Ess = \frac{1}{Kv} = 0.0611$$

The velocity error constant for a system with lead-lag compensator can be calculated as follow:

$$Kv = \frac{1}{424.2698 - 424.263} \approx 147.05$$

$$Ess = \frac{1}{Kv} = 6.8 * 10^{-3}$$

We noticed that the new Kv is not exactly ten times greater than the original Kv. We suspect this is due to reading issues. To accurately observe the differences of step responses from different systems, we zoomed in to the figure significantly. MATLAB cursor tool was not able to function properly at such magnification; thus accurate readings are challenging to obtain. • 2.5

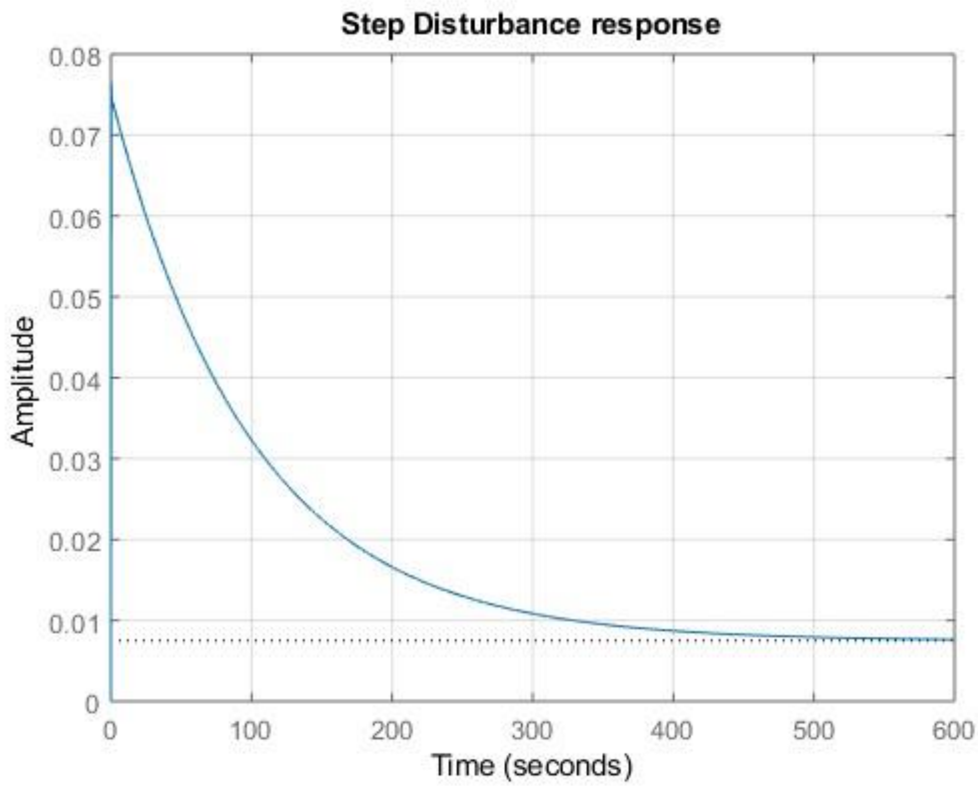
The transfer function is calculated as followed:

$$\frac{Y(s)}{Td(S)} \quad \frac{G(s)}{1 + G(s)Gc(s)} =$$

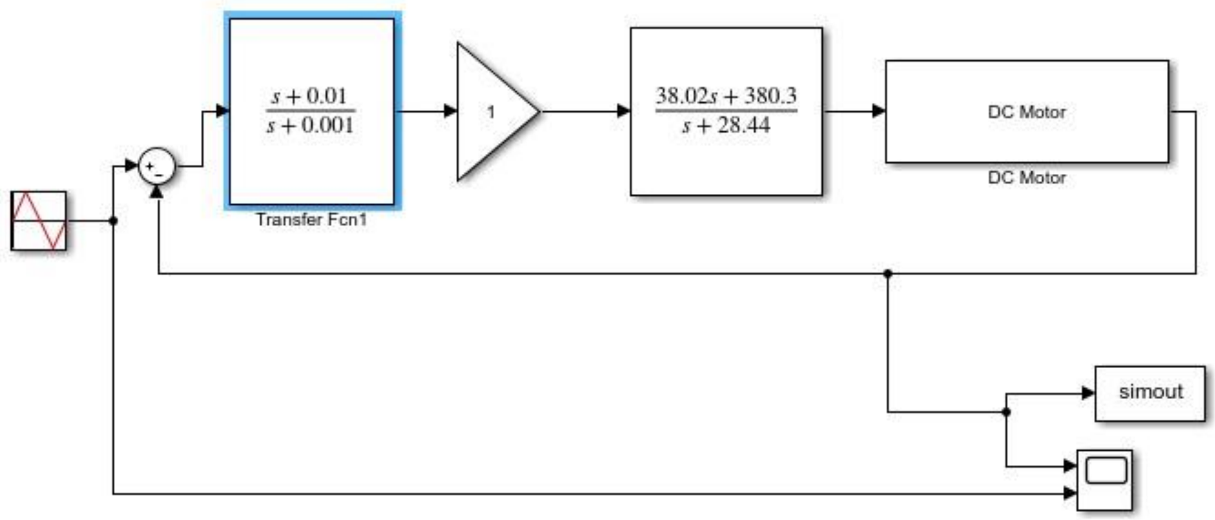
Thus,

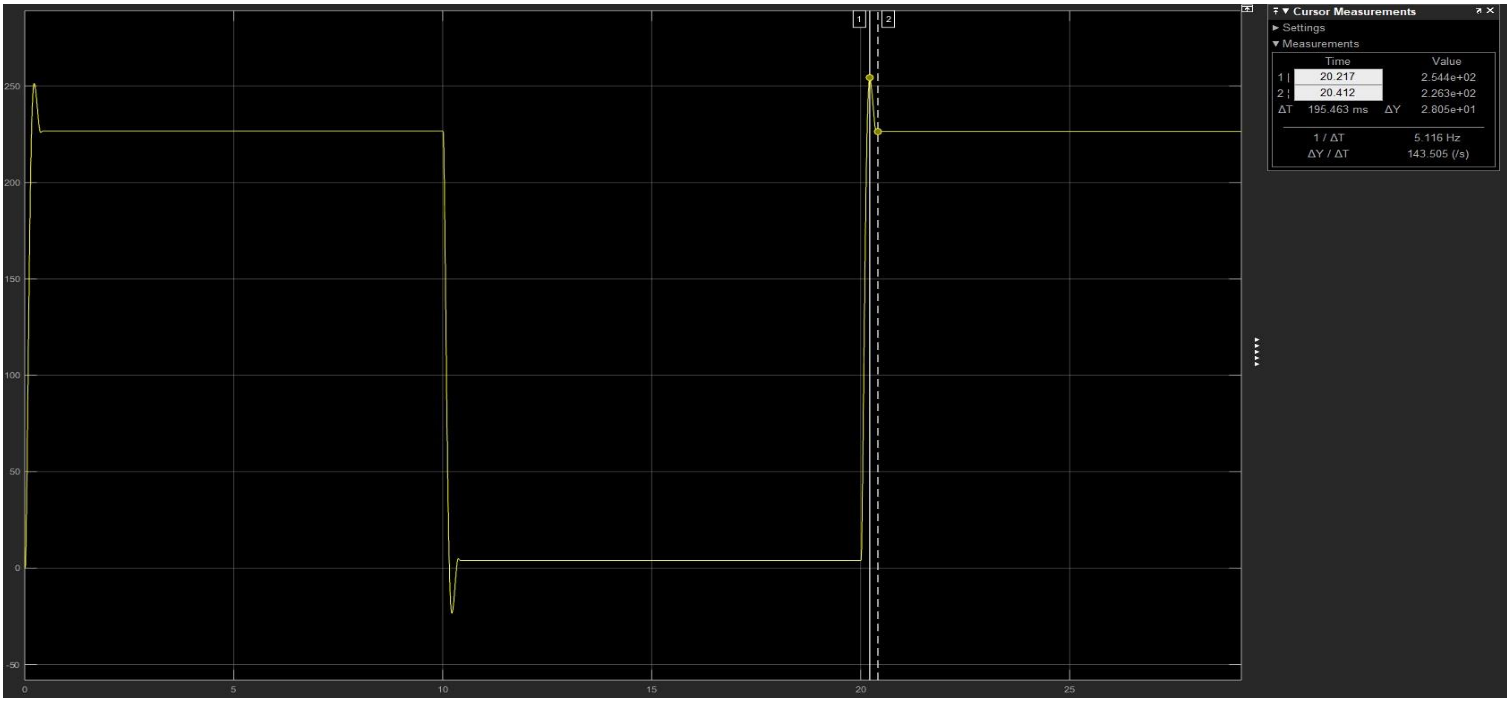
```
           320.2 s^2 + 3205 s + 32.02
-------------------------------------------
s^4 + 35.33 s^3 + 516.1 s^2 + 3205 s + 32.02
```

- 2.6

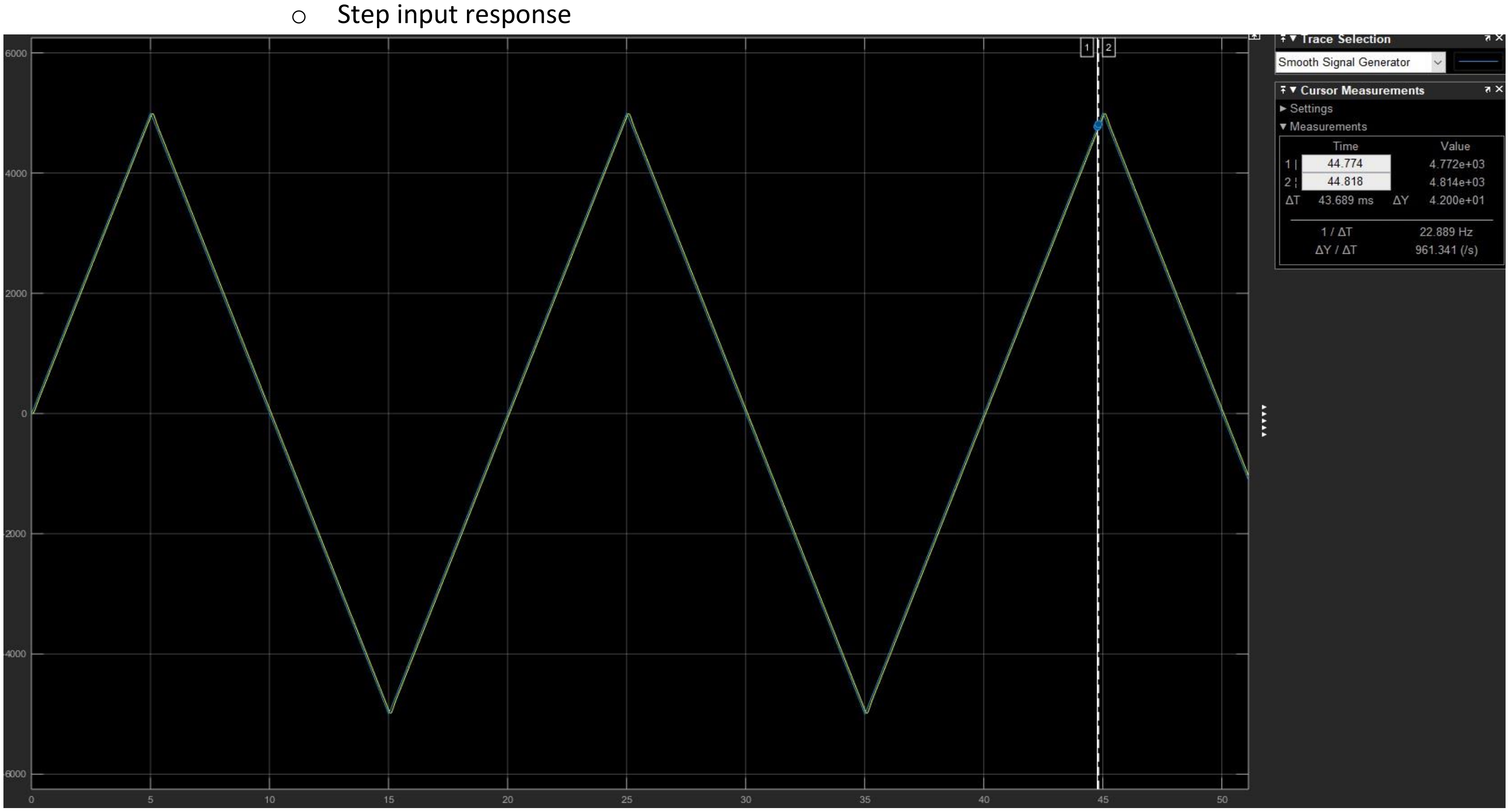

Step Disturbance response
Amplitude
Time (seconds)
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0
100
200
300
400
500
600

3 Experiment: Implementation of the Lead-Lag Compensator



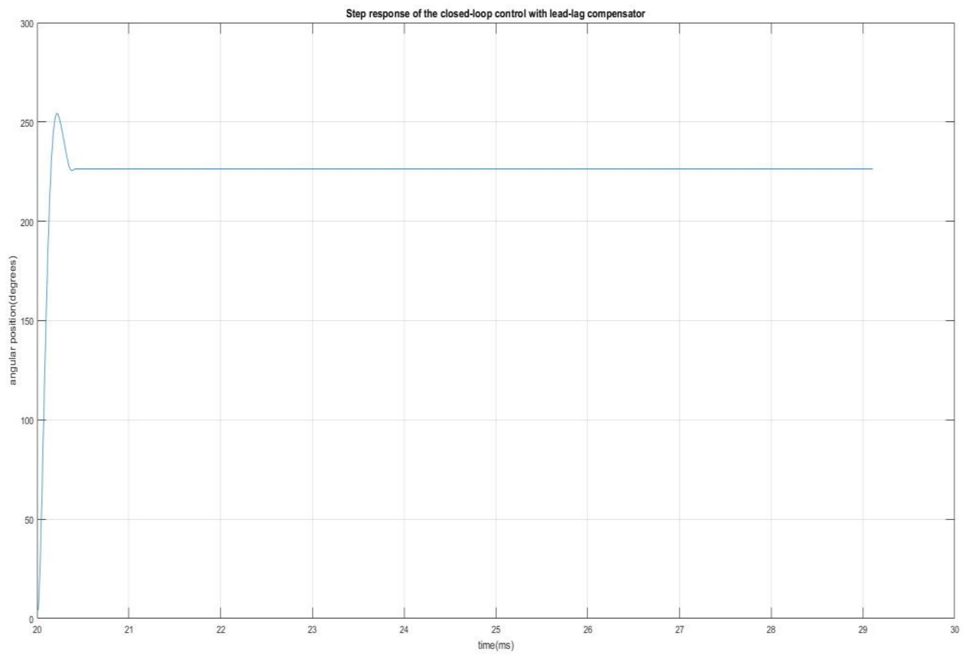


- Step input response

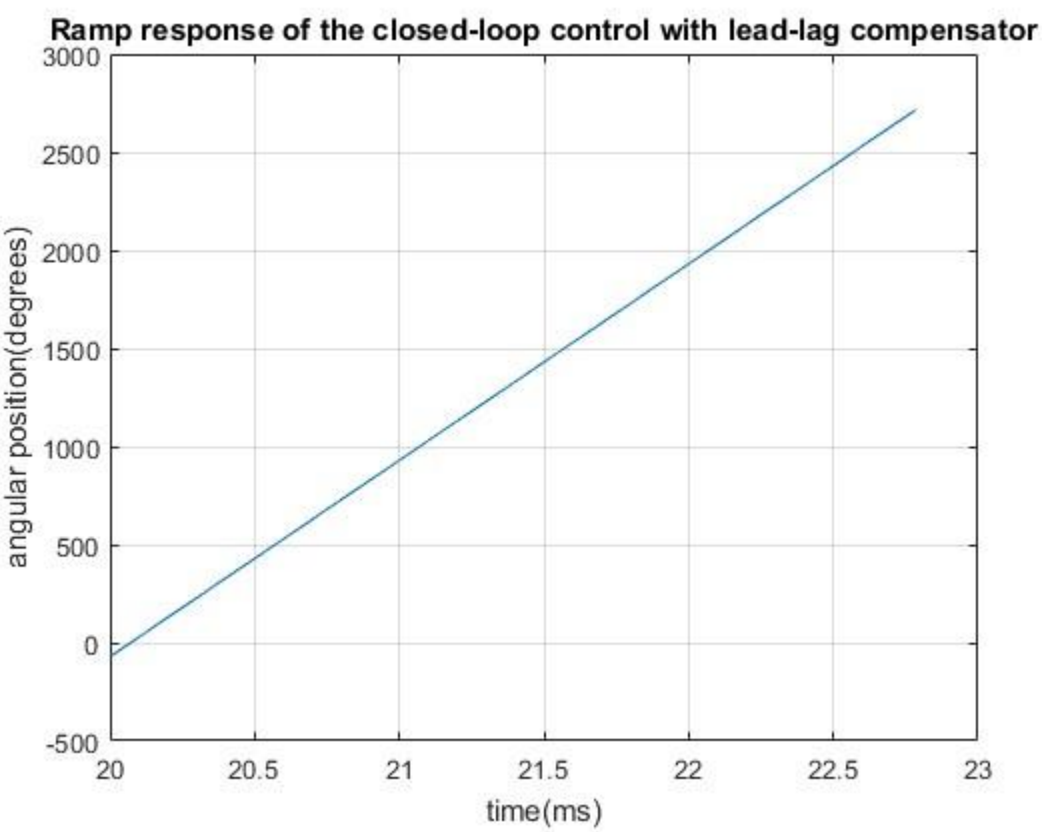


- Ramp input response

MATLAB plot for step input response:



MATLAB plot for ramp input response:



(Please note these plots are partial plots due to errors of the "to workspace" block in Simulink. For full plots please refer to the measurement plots)

The percentage overshoot of the actual system:

$$P.O = \left(\frac{254.4 - 226.3}{226.3}\right) * 100 \approx 12.4\%$$

The velocity error constant of the actual system:

$$Kv = \frac{400}{(1205 - 1179)} \approx 15$$

We graphically determined that the 2% settling time due to a step input is approximately 0.4s.

Unfortunately, our system only satisfies the 2% settling time due to a step input requirement.

Conclusion

In this laboratory, we observed the lag compensator's ability to increase velocity error constant without changing the shape of root locus in a significant manner. The lag compensator zero and pole are placed very close to each other so that in the phase criterion, the these zero and pole will cancel each other(approximately). Also, the new zero and pole are placed very close to the origin so that their effect on the magnitude criterion of the design is minor. These zero and pole, however, can yield high ratio so they can effectively alter the velocity error constant.

Our calculations and simulations in the previous sections have demonstrated that the designed system would satisfy all requirements. Unfortunately, our actual systems only meet the 2% settling time due to a step input criterion. We suspect the friction of the motor is the primary cause of this observation, and the laboratory teaching assistant informed us the old service age of our motor system. This information verified our suspicion; the motor system experiences a non-negligible amount of friction due to long time usage; this is a significant factor that influences our design.

To combat this, we purpose to intentionally design the lag controller to increase the velocity error constant by a factor much higher than 10 to compensate for the effect of friction. We also need to readjust our lead controller with experiments to fully satisfy the requirement of percentage overshoot.