№ 2

$$A = U \Sigma V^T$$

1° $\left(A^T A\right)^{-1} = ?$

$$A^T A = \left(U \Sigma V^T\right)^T \left(U \Sigma V^T\right) =$$

$$V \Sigma \underbrace{U^T U}_{=\mathbb{1}} \Sigma V^T = V \Sigma^2 V^T$$

$$\left(A^T A\right)^{-1} = \left(V \Sigma^2 V^T\right)^{-1} = V \Sigma^2 V^T$$

т.к V унитарная, то $V^{-1} = V^T$

2° $\left(A^T A\right)^{-1} A^T$

из п.1 $\left(A^T A\right)^{-1} = V \Sigma^2 V^T$

$$A^T = V \Sigma U^T \Rightarrow V \Sigma^2 \underbrace{V^T V}_{=\mathbb{1}} \Sigma U^T =$$

$$= V \Sigma^3 U^T$$

3° $A\left(A^T A\right)^{-1} = U \Sigma V^T V \Sigma^2 V^T = U \Sigma^3 V^T$

4° $A\left(A^T A\right)^{-1} A^T = U \Sigma^3 V^T V \Sigma U^T =$

$$= U \Sigma^4 U^T$$