Homework-1

Machine Learning 1

Instructions: All questions are mandatory. Do submit the answers in PDF format, with the file name being your name. The deadline for submission is December 06, 2022, at 11:59 p.m.

Problem 1. This symmetric matrix is called the Hadamard matrix and it has orthogonal columns:

$$H = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

also, $H^2 = 4I$

(a) What is the determinant of H ?

(b) What are the eigenvalues of H ?

(c) What are the singular values of H ? **(3+3+4 = 10 points)**

Problem 2. Calculate the Singular Value Decomposition for the following matrices.

$$A = \begin{bmatrix} 1 & -1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix}$$

If $A = U\Sigma V^T$ is an SVD for A, What can you say about the SVD for A^T ? **(4+4+2 = 10 points)**

Problem 3. In image processing, consider this situation: A satellite takes a picture and wants to send it to Earth. In order to do that, a compression is used.

Take an image from the web of Einstein. Apply SVD on the 2D image and compress it to 30% singular values (make the 70% lower singular values as zero). Depict the original image and the compressed image and comment. **(10 points)**

Problem 4. Calculate the Gradient of the following functions

(a) $f(x, y) = x + 3y^2$, (b) $f(x, y) = \sqrt{x^2 + y^2}$ **(5+5 = 10 points)**

Problem 5. Consider the optimization problem:

$$\begin{array}{ll} \text{minimize} & (1/2)x^T Px + q^T x + r \\ \text{subject to} & -1 \leq x_i \leq 1, \quad i = 1, 2, 3, \end{array}$$

where

$$P = \begin{bmatrix} 13 & 12 & -2 \\ 12 & 17 & 6 \\ -2 & 6 & 12 \end{bmatrix}, \quad q = \begin{bmatrix} -22.0 \\ -14.5 \\ 13.0 \end{bmatrix}, \quad r = 1$$

(a) Show that the problem is *convex* (Hint : $x^T Ax$ is *convex* if A is positive semi-definite)

(b) Solve the optimization problem using Projected Gradient Descent method.

(c) Attempt to solve the problem mathematically and find the optimal solution. (BONUS)

(5+5 = 10 points, BONUS = 10 points)