Minia University
Faculty of Engineering
Computers and Systems Eng. Dept.
January, 2021

Course Code: CSE416
Course Title: Digital Control
Time Allowed: 3 hrs.
Total Marks: 100

<u>This exam consists of 4 questions located in 2 pages. Attempt all the questions and assume any missing data or logical assumptions.</u>

<u>Question (1):</u> (25 marks)

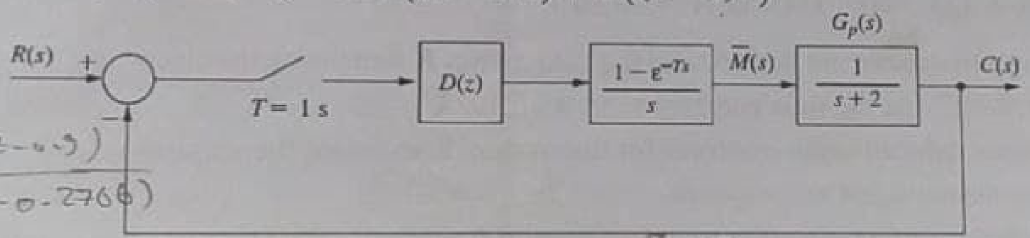
1. Consider the following continuous-time state variable model:

$$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix}=\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}u(t), \qquad y(t)=\begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}$$

   a. Check controllability and observability for the continuous-time system. Cont ✓ obsv. ✓ *(4 marks)*
   b. Convert the continuous-time state variable model into its discrete-time equivalent. Assume that the sampling time is T. *(6 marks)*
   c. Recheck controllability and observability for the obtained equivalent discrete-time state variable model. *(4 marks)*
   d. Derive the analog and digital transfer functions of the system. *(6 marks)*
2. The characteristic polynomial of a system is $Q(z)=z^2+\alpha_1 z+\alpha_2$. Use the Jury's criterion to derive a relationship between α_1 and α_2 to assure system stability. *(5 marks)*

<u>Question (2):</u> (25 marks)

1. Consider the following system. The digital filter in the block diagram has the difference equation: $m(kT)=e(kT)-0.9\,e\big((k-1)T\big)+m\big((k-1)T\big)$

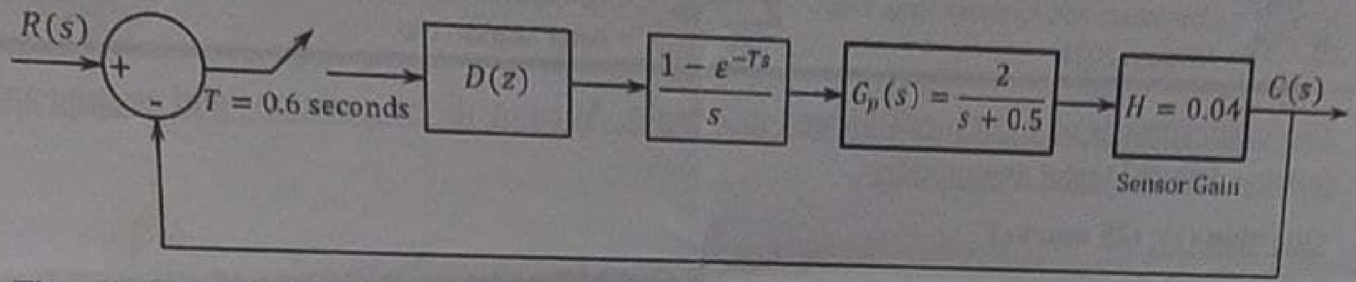


$\Rightarrow D(z)$ $\dfrac{0.86466(z-0.9)}{(z-1)(2z-0.2768)}$

   a. Obtain the digital controller transfer function. $\dfrac{z-0.9}{z-1}$ *(2 marks)*
   b. Find the system type, and steady state response for a unit step input. 1, 1 *(3 marks)*
   c. Find the time constant for the system. $\tau = 0.746$ *(3 marks)*
   d. Find the approximate time for the system to reach the steady state value. 3 sec. *(3 marks)*
   e. Obtain the unit-step response for the system. *(3 marks)*
   f. Use the unit step response to verify the time calculated (in part c) to reach the steady state value. *(3 marks)*
2. Plot the root locus ($K>0$) in the z-plane for a unity feedback system an open-loop transfer function as: $G(z)=\dfrac{Kz}{(z-0.5)^2}$ *(8 marks)*

Question (3): (25 marks)

Consider the following feedback system:



The plant transfer function can be discretized with zero-order hold as:

$$G(z) = Z\left\{\frac{1-\varepsilon^{-Ts}}{s}\frac{2}{s+0.5}\right\} = \frac{1.037}{z-0.7408}$$

1. Draw the bode diagram for $G(z)$. $12\,dB \rightarrow -4.6\,dB$, $0 \rightarrow -180^\circ$ *(8 marks)*
2. Assuming $D(z) = 1$, obtain the steady state error for a unit step input. 0.862 *(5 marks)*
3. Design a phase lag compensator that makes the system have a steady state error of 0.05 and a phase margin of 45°. $\omega_{\omega 1} = 3.65$, $a_0 = 118.75$, $\omega_{\omega p} = 0.0073$ *(8 marks)*
4. Realize the designed phase lag compensator as a difference equation. $k_d = 1.406$ *(4 marks)*

$D(z) = \frac{1.046(z-0.8024)}{z-0.9977}$ $\frac{1.406z - 0.8393}{z-0.9977}$

Question (4): (25 marks)

Consider the following continuous-time state variable model for a satellite attitude system. x_1 is the angular position while x_2 is the angular velocity. The sampling time is 1 second.

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 0.125 \\ 0.25 \end{bmatrix} u(t), \qquad y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

1. Using pole-placement design, find the gain matrix $\boldsymbol{K}$ that yields the closed-loop damping ratio $\zeta = 0.707$ and the time constant $\tau = 4\,s$. $K = [0.39 \quad 1.763]$ *(8 marks)*
2. Design a reduced-order observer for this system to estimate the angular velocity x_2, with the time constant equal to 2 seconds. $G = [0.3935]$ *(8 marks)*
3. Find the controller-observer transfer function $D_{ce}(z)$. $D_{ce}(z) = \frac{1.09z - 0.9326}{z - 0.2513}$ *(4 marks)*
4. Convert the designed controller-observer into a pseudo-code for implementation. *(5 marks)*

Some useful design laws for Question (4): a: measured, b: estimated.

$$\mathbf{q}_b(k+1) = (\mathbf{A}_{bb} - \mathbf{G}\mathbf{A}_{ab})\mathbf{q}_b(k) + \mathbf{G}\mathbf{y}(k+1) + (\mathbf{A}_{ba} - \mathbf{G}\mathbf{A}_{aa})\mathbf{y}(k) + (\mathbf{B}_b - \mathbf{G}\mathbf{B}_a)\mathbf{u}(k)$$

$$\mathbf{K} = [0\,0 \,\cdots\, 0\,1][\mathbf{B} \;\; \mathbf{AB} \;\cdots\; \mathbf{A}^{n-2}\mathbf{B} \;\; \mathbf{A}^{n-1}\mathbf{B}]^{-1}\alpha_c(\mathbf{A})$$

$$D_{ce}(z) = \frac{-U(z)}{Y(z)} = K_1 + \mathbf{K}_b[z\mathbf{I} - \mathbf{A}_{bb} + \mathbf{G}\mathbf{A}_{ab} + (\mathbf{B}_b - \mathbf{G}\mathbf{B}_a)\mathbf{K}_b]^{-1} \cdot [\mathbf{G}z + \{\mathbf{A}_{ba} - \mathbf{G}\mathbf{A}_{aa} - K_1(\mathbf{B}_b - \mathbf{G}\mathbf{B}_a)\}]$$

$$\mathbf{G} = \alpha_e(\mathbf{A}_{bb})\begin{bmatrix} \mathbf{A}_{ab} \\ \mathbf{A}_{ab}\mathbf{A}_{bb} \\ \vdots \\ \mathbf{A}_{ab}\mathbf{A}_{bb}^{n-2} \end{bmatrix}^{-1}\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}$$

$\frac{M(z)}{E(z)}$

With my best wishes, Dr. Ahmed Mahmou

$u(k-1) + 0.7925\,y(k) - 0.4715\,y(k-1)$