Course: Computer Controlled Systems	Mid-Term Exam (1) Model Answer
Date: 18 / 5 / 2022	Time: 60 min.

1. a.

No.

$\sin(a \cdot x) \neq a \cdot \sin(x)$ or $\sin(x + y) \neq \sin(x) + \sin(y)$

1. b.

Yes.

Let $\ddot{\theta}(t) = f(x(t)) = -\frac{g}{L}\sin(x(t))$ and $x(t) = \theta(t)$.

Delay the output: $\ddot{\theta}(t - \tau) = f(x)(t - \tau) = -\frac{g}{L}\sin(\theta(t - \tau))$.

Delay the input: let $x(t - \tau) = \theta(t - \tau)$ and pass it through the function as $f(x(t - \tau)) = -\frac{g}{L}\sin(\theta(t - \tau))$.

Delaying the input is the same as delaying the output.

1. c.

Yes. Current output depends on current input.

1. d.

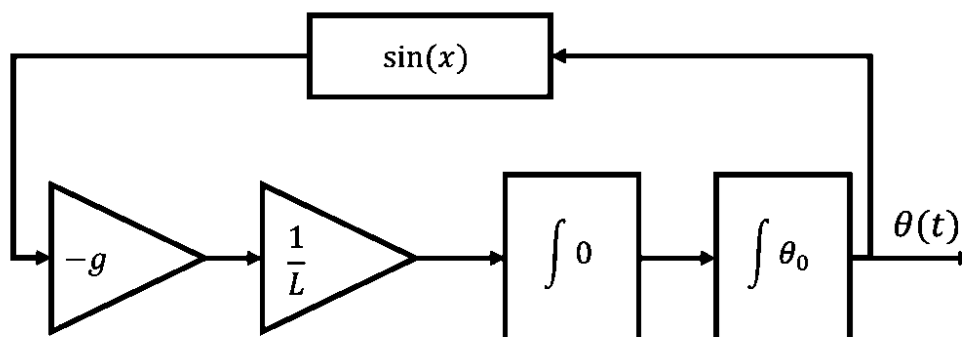No. Current output depends on current input. Strictly causal systems only depend on past input for current output.

1. e.

Yes. Current output depends **only on** current input.

1. f.

Yes, $sin(x)$ is bounded for bounded x.

2.

This actor model has memory because it contains integrators, which have memory.
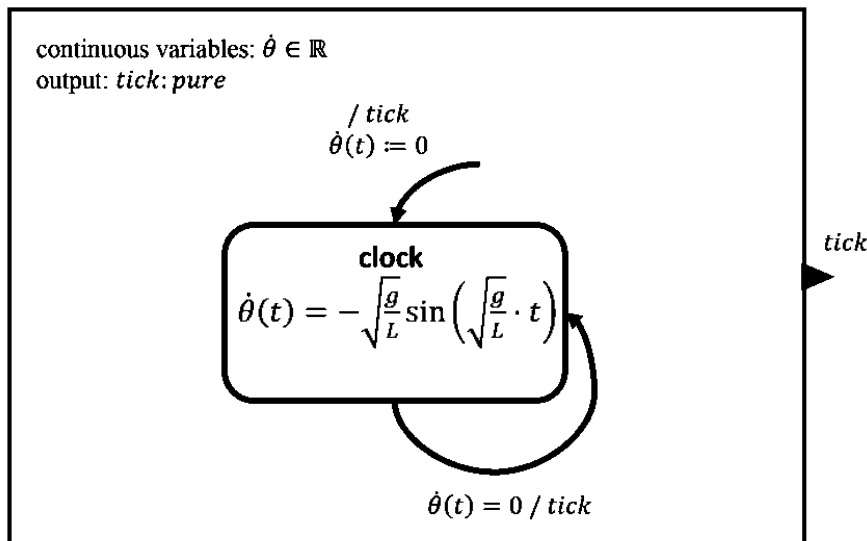
3. a.

$$\ddot{\theta}(t) = -\frac{g}{L} \cdot \theta(t)$$

3. b.

Yes, satisfies superposition.

4.

Period is given by:

$$T = 2\pi \cdot \sqrt{\frac{L}{g}}$$

We want **half the period** to be equal to 1sec, so the string length can be obtained as:

$$\frac{1}{2}T = 1 = \pi \cdot \sqrt{\frac{L}{g}} \Rightarrow L = \frac{g}{\pi^2}$$