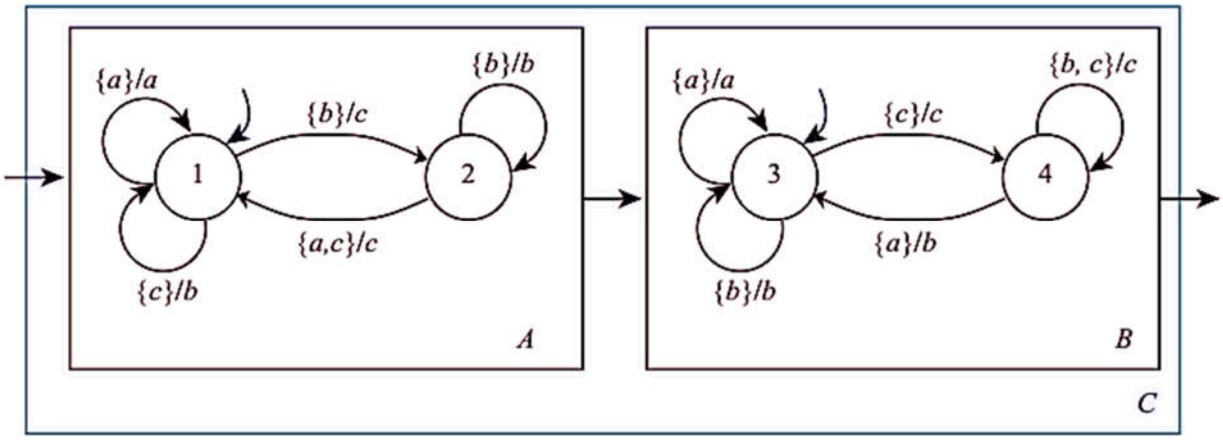
CSE314 “Systems Analysis” Final Exam Model Answer
January, 2020

Question (1):



1.

State machine A:

$States = \{1,2\}$

$Inputs = \{a, b, c\}$

$Outputs = \{a, b, c\}$

$initialState = 1$

	$(s(n+1), y(n)) = update(s(n), x(n))$		
Current state $s(n)$	Input $x(n) = a$	Input $x(n) = b$	Input $x(n) = c$
1	$(1, a)$	$(2, c)$	$(1, b)$
2	$(1, c)$	$(2, b)$	$(1, c)$

State machine B:

$States = \{3,4\}$

$Inputs = \{a, b, c\}$

$Outputs = \{a, b, c\}$

$initialState = 3$

	$(s(n+1), y(n)) = update(s(n), x(n))$		
Current state $s(n)$	Input $x(n) = a$	Input $x(n) = b$	Input $x(n) = c$
3	$(3, a)$	$(3, b)$	$(4, c)$
4	$(3, b)$	$(4, c)$	$(4, c)$

2.

Composite machine C:

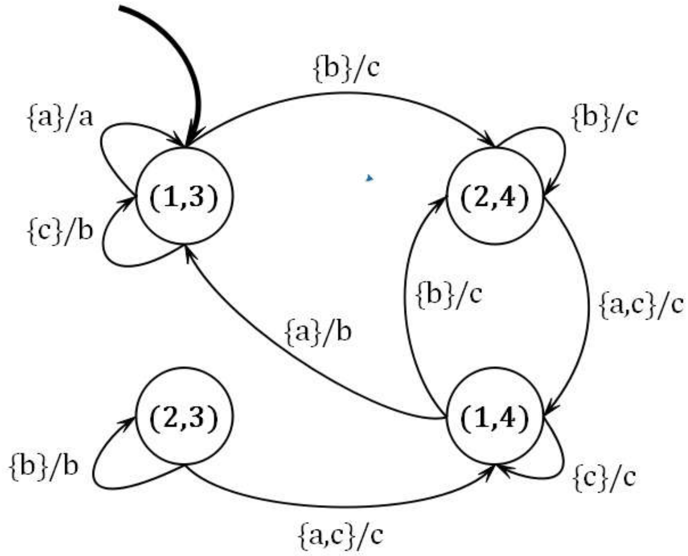
$States = \{(1,3), (1,4), (2,3), (2,4)\}$

$Inputs = \{a, b, c\}$

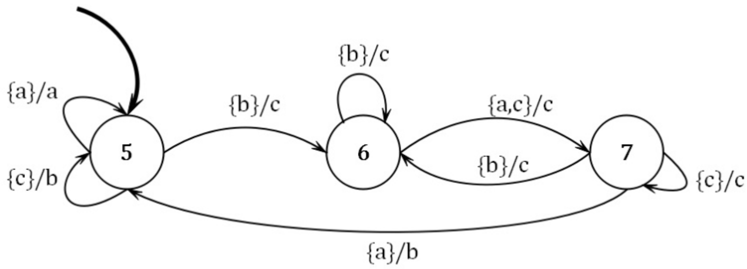
$Outputs = \{a, b, c\}$

$initialState = (1,3)$

	$\big(s(n+1), y(n)\big) = update\big(s(n), x(n)\big)$		
Current state $s(n)$	Input $x(n) = a$	Input $x(n) = b$	Input $x(n) = c$
$(1,3)$	$\big((1,3), a\big)$	$\big((2,4), c\big)$	$\big((1,3), b\big)$
$(1,4)$	$\big((1,3), b\big)$	$\big((2,4), c\big)$	$\big((1,4), c\big)$
$(2,3)$	$\big((1,4), c\big)$	$\big((2,3), b\big)$	$\big((1,4), c\big)$
$(2,4)$	$\big((1,4), c\big)$	$\big((2,4), c\big)$	$\big((1,4), c\big)$



3.



Simulation relation is $S = \{\big((1,3), 5\big), \big((2,4), 6\big), \big((1,4), 7\big)\}$

Question (2):

1. a.

$$s(n+1) = As(n) + Bx(n) \qquad s(0) = \begin{bmatrix} s_1(0) \\ s_2(0) \end{bmatrix}$$

Zero-Input State Response $\rightarrow x(n) = 0, \ \forall n$

$$
\begin{aligned}
s(n+1) &= As(n);\ A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \\
s(1) &= As(0) = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} s_1(0) \\ s_2(0) \end{bmatrix} \\
&= \begin{bmatrix} s_1(0) + s_2(0) \\ s_2(0) \end{bmatrix} \\
s(2) &= As(1) = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} s_1(0) + s_2(0) \\ s_2(0) \end{bmatrix} \\
&= \begin{bmatrix} s_1(0) + 2s_2(0) \\ s_2(0) \end{bmatrix} \\
s(n) &= \begin{bmatrix} s_1(0) + ns_2(0) \\ s_2(0) \end{bmatrix}
\end{aligned}
$$

1. b.

$$
\begin{aligned}
h(n) &= CA^{n-1}B,\ n \geq 1;\ h(0) = d = 0 \\
A^0 &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\
A^1 &= \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \\
A^2 &= \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \\
A^n &= \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}, \forall n \geq 0 \\
h(1) &= CA^0B = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = [0] \\
h(2) &= CA^1B = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = [1] \\
h(n) &= CA^{n-1}B = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & n-1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & n-1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = [n-1]
\end{aligned}
$$

1. c.

$$
s(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix};\ x(n) = \delta(n-1) = \begin{cases} 1, & n = 1 \\ 0, & \text{otherwise} \end{cases}
$$

$$
\forall n \geq 0, \qquad y(n) = C\,A^n s(0) + \sum_{k=0}^{n} h(n-k)x(k)
$$

$$
\begin{aligned}
y(0) &= C\,A^0 s(0) + h(0)x(0) = C\,A^0 s(0) \\
&= \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \\
&= \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = [1]
\end{aligned}
$$

$$
\begin{aligned}
y(1) &= C\,A^1 s(0) + \sum_{k=0}^{1} h(1-k)x(k) = C\,A^1 s(0) + h(0) \\
&= \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + [0] \\
&= \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + [0] = [2] \\
y(n) &= C\,A^n s(0) + \sum_{k=0}^{n} h(1-k)x(k) = C\,A^n s(0) + h(n-1) \\
&= \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + h[n-1] \\
&= \begin{bmatrix} 1 & n \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + h[n-1] = [1+n] + [n-2] = [2n-1]
\end{aligned}
$$

2. a.

$$\forall\, n \in \mathbb{Z}, \quad s(n+1) = (1+r)s(n) + x(n).$$

$$\forall\, n \in \mathbb{Z}, \quad y(n) = s(n).$$

2. b.

$s(0) = 100, \qquad x(0) = 1000$

$s(1) = 1.01s(0) + 1000 = 1101, \qquad x(1) = -30$

$s(2) = 1.01s(1) - 30 = 1082.01, \qquad x(2) = -30$

$s(3) = 1.01s(2) - 30 = 1062.8301$

$y(3) = s(3) = 1062.8301$ pounds

Question (3):

1.

	Linear	Time Invariant	Causal
a. $S\big(x(t)\big) = e^{i2\pi t}x(t)$	Yes	No	Yes
b. $S\big(x(t)\big) = x(-t-2)$	Yes	No	No
c. $S\big(x(t)\big) = \big(x(t-2)\big)^2$	No	Yes	Yes
d. $S\big(x(t)\big) = x(t^2-2)$	Yes	No	No

2. a.

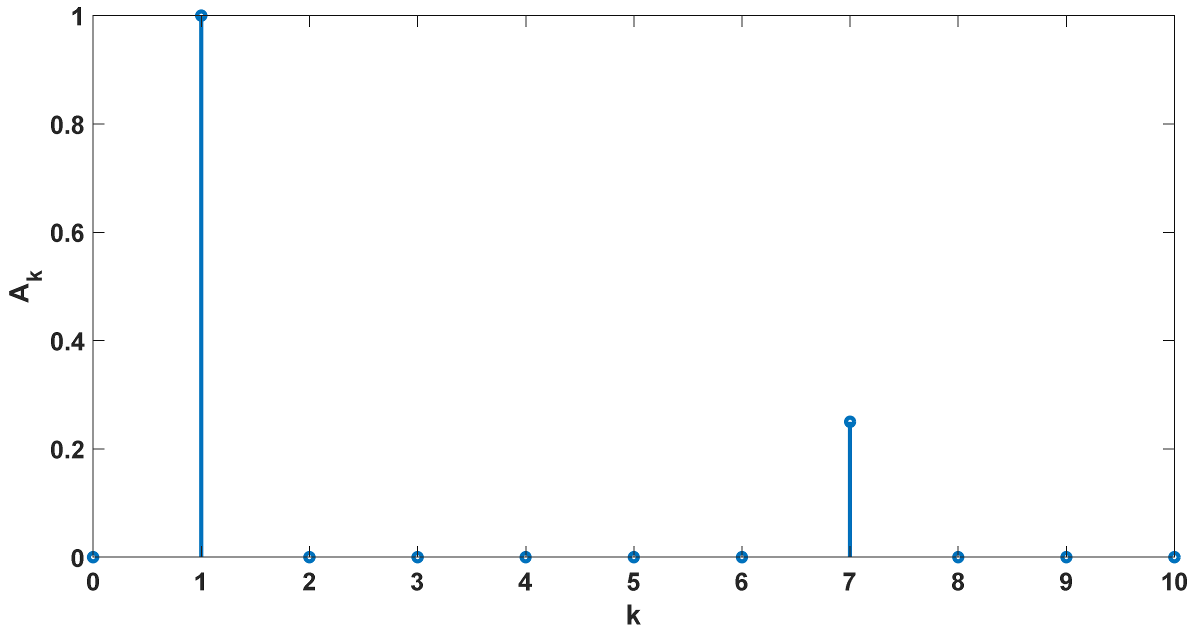
$$p = 2\pi \rightarrow \omega_0 = \frac{2\pi}{p} = 1$$

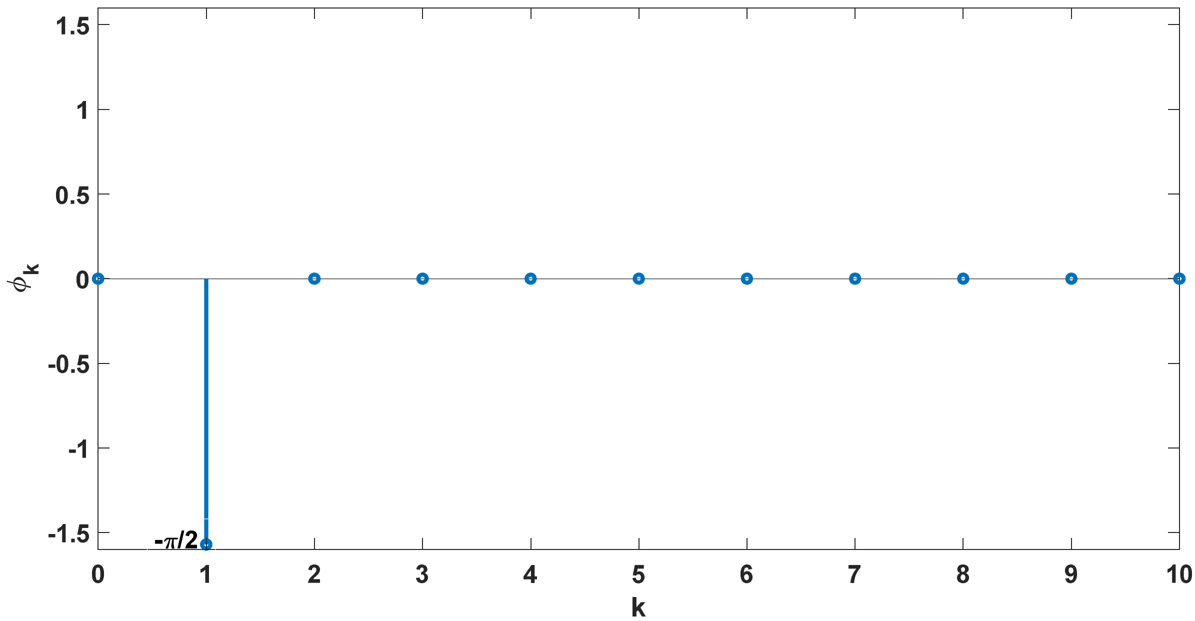
2. b. With manipulation, we can rewrite the signal as:

$$x(t) = \cos\left(t - \frac{\pi}{2}\right) + \frac{1}{4}\cos(7t)$$

$$A_1 = 1, \quad \phi_1 = -\frac{\pi}{2}, \quad A_7 = \frac{1}{4}, \quad \phi_7 = 0$$

and all other coefficients are zeros.





3. a.

$$H(\omega)e^{i\omega n} + 2H(\omega)e^{i\omega(n-2)} = e^{i\omega n}$$

$$H(\omega)e^{i\omega n}\left(1 + 2e^{-i2\omega}\right) = e^{i\omega n}$$

$$H(\omega) = \frac{1}{1 + 2e^{-i2\omega}}$$

3. b.

Choose states as: $s(n) = \begin{bmatrix} y(n-1) \\ y(n-2) \end{bmatrix}$

$$s(n+1) = \begin{bmatrix} 0 & -2 \\ 1 & 0 \end{bmatrix} s(n) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} x(n)$$

$$y(n) = \begin{bmatrix} 0 & -2 \end{bmatrix} s(n) + [1]x(n)$$

3. c.

$$y(n) = x(n) - 2y(n-2)$$

For the impulse response $x(n) = \delta(n) = \begin{cases} 1 & for \quad n = 0 \\ 0 & otherwise \end{cases}$, $\quad y(n) = h(n)$

Assuming causality:

$$\forall n < 0, h(n) = 0$$

$$h(n) = \delta(n) - 2h(n-2)$$

$$h(0) = 1$$

$$h(1) = 0$$

$$h(2) = -2$$

$$h(3) = 0$$

$$h(4) = 4$$

$$h(n) = \begin{cases} (-2)^{\frac{n}{2}} & \text{for} \quad n \text{ even and nonnegative} \\ 0 & \text{otherwise} \end{cases}$$

Question (4):

1. a.

$p = 2.$ Seconds

$\omega_0 = \pi$ radians/second.

1. b.

$A_2 = A_3 = 1, A_k = 0, \forall k \notin \{2,3\}$, and $\phi_k = 0, \forall k \in \mathbb{N}$.

2. a. Anti-causal (Left-sided)

2. b.

$$\forall z \in RoC(x), \quad \hat{X}(z) = \sum_{m=-\infty}^{\infty} a^m u(-m) z^{-m} = \sum_{m=-\infty}^{0} a^m z^{-m} = \frac{1}{1-a^{-1}z},$$

where

$$RoC(x) = \{z \in \mathbb{C} \mid \sum_{n=0}^{\infty} |a^{-1}z|^n < \infty\} = \{z \in \mathbb{C} \mid |z| < |a|\}.$$

2. c.

The signal is absolutely summable if $|a| > 1$.

2. d.

The DTFT is

$$\forall \omega \in \mathbb{R}, \quad X(\omega) = \hat{X}(e^{i\omega}) == \frac{1}{1-a^{-1}e^{i\omega}}.$$

3. a. Anti-causal (Left-sided)

3. b.

$$\begin{aligned}\forall s \in RoC(x), \quad \hat{X}(s) &= \int_{-\infty}^{\infty} x(t) e^{-st} dt \\ &= -\int_{-\infty}^{0} e^{-at} e^{-st} dt\end{aligned}$$

$$= -\int_{-\infty}^{0} e^{-(s+a)t} dt$$
$$= \frac{1}{s+a}.$$

The region of convergence is

$$RoC(x) = \{s \in \mathbb{C} \mid Re\{s\} < -Re\{a\}\}.$$

3. c.

x is absolutely integrable if $Re\{a\} < 0$.

3. d.

$$X(\omega) = \hat{X}(i\omega) = \frac{1}{i\omega + a}.$$
