Minia University
CSE Dept.
Electric circuits analysis

Midterm Exam 3, Time allowed: 1.5 Hrs
Name:____________________________
Date:____________________________

- *This is a closed book exam.*
- *The exam has 5 questions in two pages, answer all of them.*
- *Good Luck!*

**1. a)** For the network in Figure 1, determine the level of capacitance that will ensure maximum power to the load if the range of capacitance is limited to 1 nF to 10 nF.

**b)** Using the results of part (a), determine the value of $R_L$ that will ensure maximum power to the load.

**c)** Using the results of parts (a) and (b), determine the maximum power to the load. ***[10 marks]***

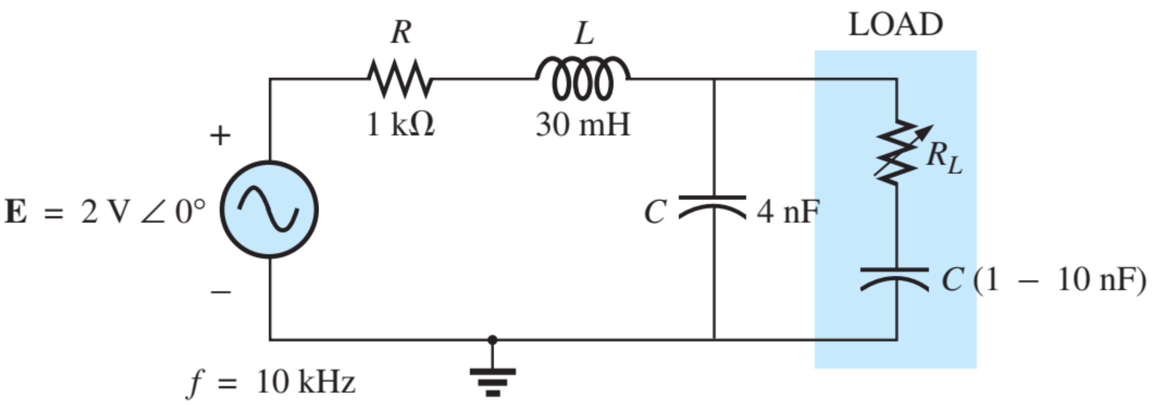
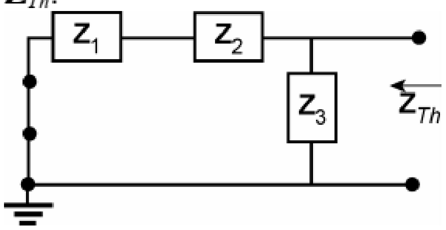



Figure 1

**a.** $\mathbf{Z}_{Th}$:




$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(10\text{ kHz})(4\text{ nF})}$$
$$\cong 3978.87\ \Omega$$
$$X_L = 2\pi fL = 2\pi(10\text{ kHz})(30\text{ mH})$$
$$\cong 1884.96\ \Omega$$
$$\mathbf{Z}_1 = 1\text{ k}\Omega\ \angle 0°,\ \mathbf{Z}_2 = 1884.96\ \Omega\ \angle 90°$$
$$\mathbf{Z}_3 = 3978.87\ \Omega\ \angle{-90°}$$

$$\mathbf{Z}_{Th} = (\mathbf{Z}_1 + \mathbf{Z}_2) \parallel \mathbf{Z}_3 = (1\text{ k}\Omega + j1884.96\ \Omega) \parallel 3978.87\ \Omega\ \angle{-90°})$$
$$= 2133.79\ \Omega\ \angle 62.05° \parallel 3978.87\ \Omega\ \angle{-90°})$$
$$= 3658.65\ \Omega\ \angle 36.52°$$

$$\therefore\quad \mathbf{Z}_L = \mathbf{3658.65\ \Omega\ \angle{-36.52°} = 2940.27\ \Omega - j2177.27\ \Omega}$$

$$C = \frac{1}{2\pi fX_C} = \frac{1}{2\pi(10\text{ kHz})(2177.27\ \Omega)} = \mathbf{7.31\ nF}$$

b. $R_L = R_{Th} = \mathbf{2940.27\ \Omega}$

c. $$\mathbf{E}_{Th} = \frac{\mathbf{Z}_3(\mathbf{E})}{\mathbf{Z}_3 + \mathbf{Z}_1 + \mathbf{Z}_2} = \frac{(3978.87\ \Omega\ \angle{-90°})(2\text{ V}\angle 0°)}{1\text{ k}\Omega + j1884.96\ \Omega - j3978.87\Omega} = 3.43\text{ V}\angle{-25.53°})$$

$$P_{\max} = E_{Th}^2/4R_{Th} = (3.43\text{ V})^2/4(2940.27\ \Omega) = \mathbf{1\ mW}$$

**2.** For the circuit shown in Figure 2, Find the Norton's equivalent as "seen" by the load resistor $R_L$. ***[10 marks]***

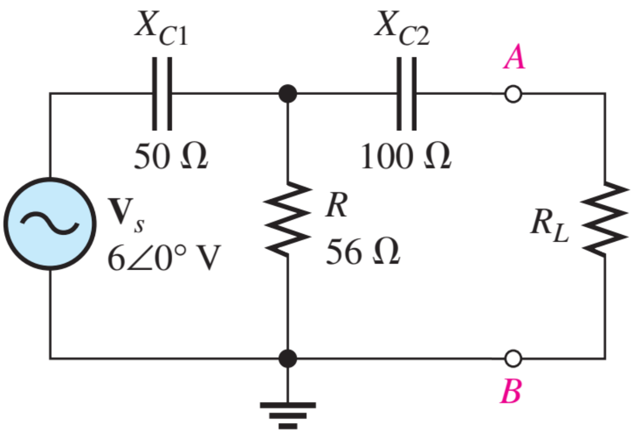



Figure 3

$\mathbf{I}_n$ is the current through the short and is calculated as follows. First, the total impedance viewed from the source is

$$\mathbf{Z} = \mathbf{X}_{C1} + \frac{\mathbf{R}\mathbf{X}_{C2}}{\mathbf{R} + \mathbf{X}_{C2}} = 50\angle -90°\ \Omega + \frac{(56\angle 0°\ \Omega)(100\angle -90°\ \Omega)}{56\ \Omega - j100\ \Omega}$$
$$= 50\angle -90°\ \Omega + 48.9\angle -29.3°\ \Omega$$
$$= -j50\ \Omega + 42.6\ \Omega - j23.9\ \Omega = 42.6\ \Omega - j73.9\ \Omega$$

Converting to polar form yields

$$\mathbf{Z} = 85.3\angle -60.0°\ \Omega$$

Next, the total current from the source is

$$\mathbf{I}_s = \frac{\mathbf{V}_s}{\mathbf{Z}} = \frac{6\angle 0°\ \text{V}}{85.3\angle -60.0°\ \Omega} = 70.3\angle 60.0°\ \text{mA}$$

Finally, apply the current-divider formula to get $\mathbf{I}_n$ (the current through the short between terminals $A$ and $B$).

$$\mathbf{I}_n = \left(\frac{\mathbf{R}}{\mathbf{R} + \mathbf{X}_{C2}}\right)\mathbf{I}_s = \left(\frac{56\angle 0°\ \Omega}{56\ \Omega - j100\ \Omega}\right)70.3\angle 60.0°\ \text{mA} = \mathbf{34.4\angle 121°\ mA}$$

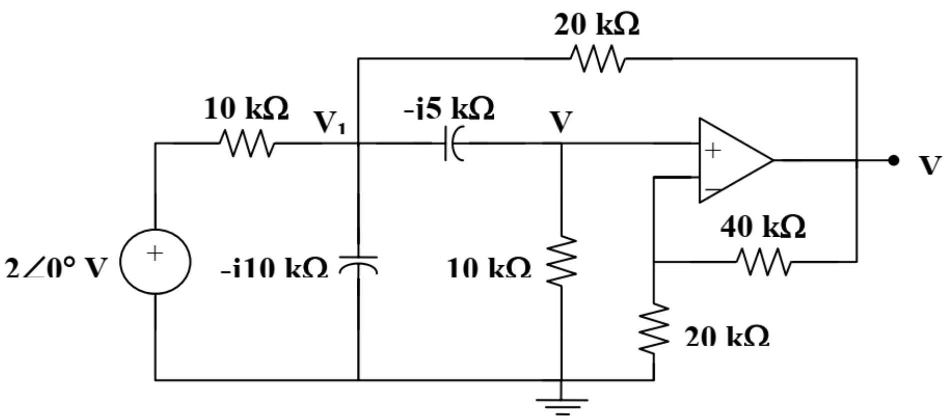
**3.** Determine $v_o(t)$ for the op amp circuit in Figure 3. ***[10 marks]***

$$2\sin(400t) \longrightarrow 2\angle 0^\circ, \quad \omega = 400$$

$$0.5\ \mu F \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(400)(0.5\times 10^{-6})} = -j5\ k\Omega$$

$$0.25\ \mu F \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(400)(0.25\times 10^{-6})} = -j10\ k\Omega$$

Consider the circuit as shown below.




At node 1,

$$\frac{2-\mathbf{V}_1}{10} = \frac{\mathbf{V}_1}{-j10} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{-j5} + \frac{\mathbf{V}_1 - \mathbf{V}_o}{20}$$

$$4 = (3 + j6)\,\mathbf{V}_1 - j4\,\mathbf{V}_2 - \mathbf{V}_o \qquad (1)$$

At node 2,

$$\frac{\mathbf{V}_1 - \mathbf{V}_2}{-j5} = \frac{\mathbf{V}_2}{10}$$

$$\mathbf{V}_1 = (1 - j0.5)\,\mathbf{V}_2 \qquad (2)$$

But

$$\mathbf{V}_2 = \frac{20}{20+40}\mathbf{V}_o = \frac{1}{3}\mathbf{V}_o \qquad (3)$$

From (2) and (3),

$$\mathbf{V}_1 = \frac{1}{3}\cdot(1 - j0.5)\,\mathbf{V}_o \qquad (4)$$

Substituting (3) and (4) into (1) gives

$$4 = (3+j6)\cdot\frac{1}{3}\cdot(1-j0.5)\,\mathbf{V}_o - j\frac{4}{3}\mathbf{V}_o - \mathbf{V}_o = \left(1 + j\frac{1}{6}\right)\mathbf{V}_o$$

$$\mathbf{V}_o = \frac{24}{6+j} = 3.945\angle -9.46^\circ$$

Therefore,

$v_o(t) =$ **<u>3.945 sin(400t – 9.46°) V</u>**

**4. Complete the following sentences:** ***[10 marks]***

i. For a certain load, the true power is 10 W and the reactive power is 10 VAR. The apparent power is **14.14 VA.**

ii. A low pass filter is a circuit that **Blocks high-frequency signals from passing through.**

iii. An ideal operational amplifier has **zero output impedance and infinite input impedance.**

iv. The Q factor of a coil is given by **$X_L/R$**.

v. Series resonance occurs when **$X_L = X_C$.**

vi. Increasing the number of magnetic poles in a basic generator will **increase the generator's output frequency** .

vii. A transformer is plugged into a 120 V rms source and has a primary current of 300 mA rms. The secondary is providing 18 V rms across a 10 Ω load, the efficiency of the transformer is **90%**

viii. The maximum output voltage of a certain low-pass filter is 15 V. The output voltage at the critical frequency is **10.60 V**

ix. The impedance at the resonant frequency of a series RLC circuit with L = 20 mH, C = 0.02 F, and $R_W$ = 90Ω is **90Ω**

x. Another name for a unity gain amplifier is **voltage follower**