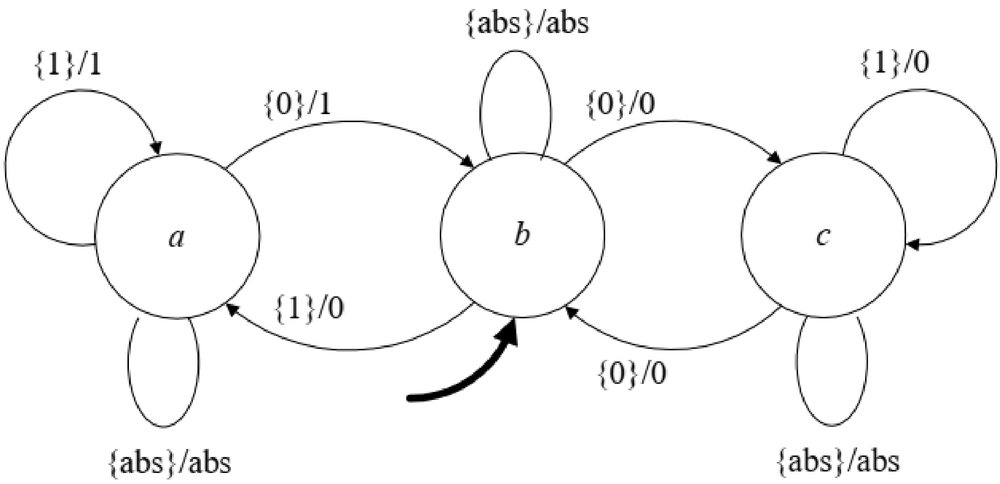
CSE314 “Systems Analysis” Final Exam Model Answer
January, 2021

Question (1):

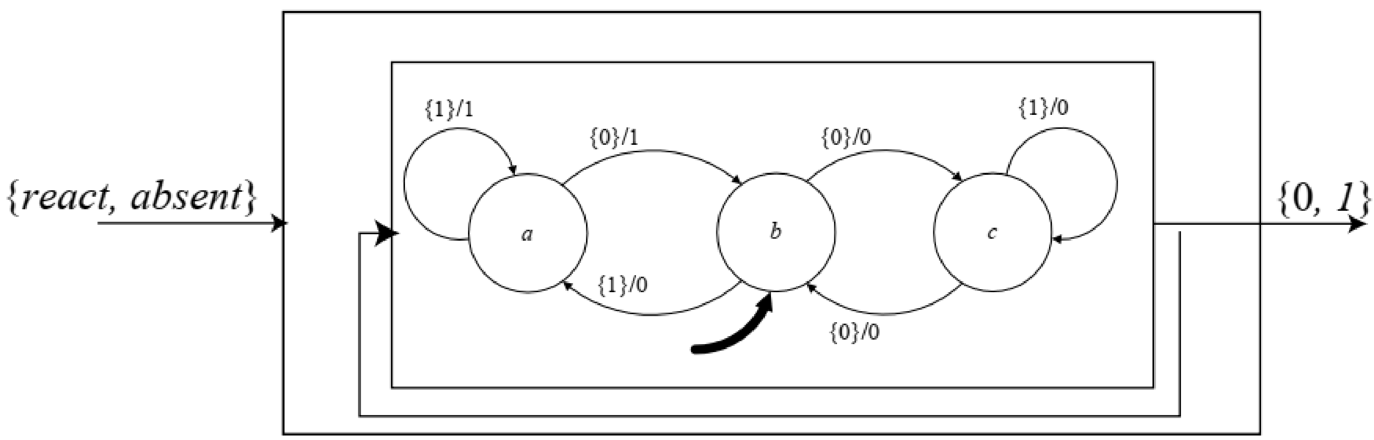


2.

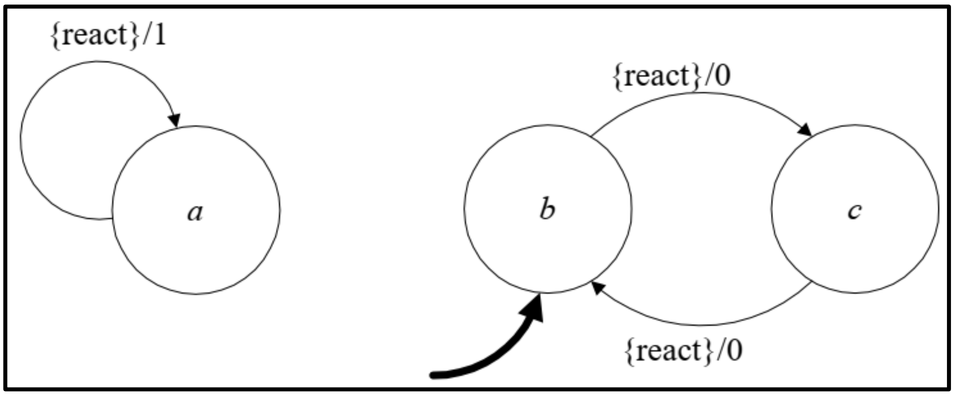
$States = \{a, b, c\}$
$Inputs = \{0,1, absent\}$
$Outputs = \{0,1, absent\}$
$initialState = b$

	$\big(s(n+1), y(n)\big) = update\big(s(n), x(n)\big)$		
Current state $s(n)$	Input $x(n) = 0$	Input $x(n) = 1$	Input $x(n) = abs$
a	$(b, 1)$	$(a, 1)$	$(a, absent)$
b	$(c, 0)$	$(a, 0)$	$(b, absent)$
c	$(b, 0)$	$(c, 0)$	$(c, absent)$

3.
Feedback composition:



Transition diagram for feedback composition:



4.

Reachable states: $\{b, c\}$

Question (2):

1.

$$s(n) = \left[\begin{array}{c} x(n-1) \\ x(n-2) \end{array} \right].$$

Then

$$s(n+1) = \left[\begin{array}{c} x(n) \\ x(n-1) \end{array} \right] + \left[\begin{array}{cc} 0 & 0 \\ 1 & 0 \end{array} \right] \left[\begin{array}{c} x(n-1) \\ x(n-2) \end{array} \right] + \left[\begin{array}{c} 1 \\ 0 \end{array} \right] x(n),$$

and

$$y(n) = [1 \quad 1] \left[\begin{array}{c} x(n-1) \\ x(n-2) \end{array} \right] + [1]\, x(n)$$

from which one can read off A, b, c^T, d by matching with

$$\begin{aligned} s(n+1) &= As(n) + bx(n) \\ y(n) &= c^T s(n) + dx(n) \end{aligned}$$

2.

Taking $\forall k, x(k) = \delta(k)$, and zero initial conditions, gives the impulse response:

$$h(n) = 1, n = 0, 1, 2; \quad h(n) = 0, \text{ otherwise.}$$

3.

Since the initial state is zero, the response is the convolution sum,

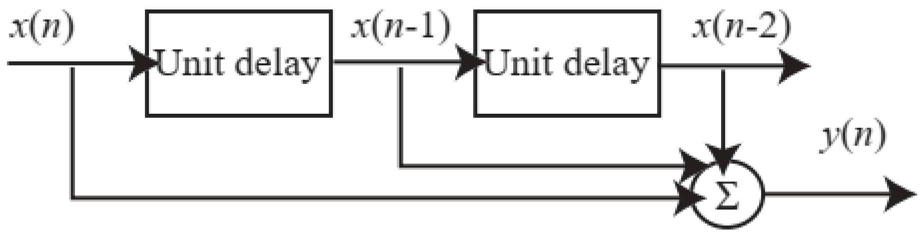
$$y(n) = \sum_{k=-\infty}^{\infty} h(k)x(n-k) = \sum_{k=0}^{2} x(n-k) = \begin{cases} 1, & n = 0 \\ 2, & n = 1 \\ 3, & n \geq 2 \end{cases}$$

4.

$$\hat{H}(z) = \frac{1}{z^{-2} + z^{-1} + 1} = \frac{z^2}{z^2 + z + 1}$$

5.

Two delay elements are needed and arranged as shown below.



Question (3):

1. a.

$\sin(\pi n)$ always equals zero and can be ignored.

$\cos\left(\frac{2\pi n}{3}\right)$ has period 3 → $x(n)$ has period 3

$p = 3,\ \omega_0 = \frac{2\pi}{p} = \frac{2\pi}{3}$

1. b.

$$x(n) = \frac{1}{2}e^{j\frac{2\pi}{3}} + \frac{1}{2}e^{j\frac{-2\pi}{3}} + 0$$

$X_1 = \frac{1}{2},\ X_{-1} = \frac{1}{2},\ X_{otherwise} = 0$

2. a.

$$y(n) = -\frac{1}{4}x(n) + \frac{1}{2}x(n-1) - \frac{1}{4}x(n-2)$$

2. b.

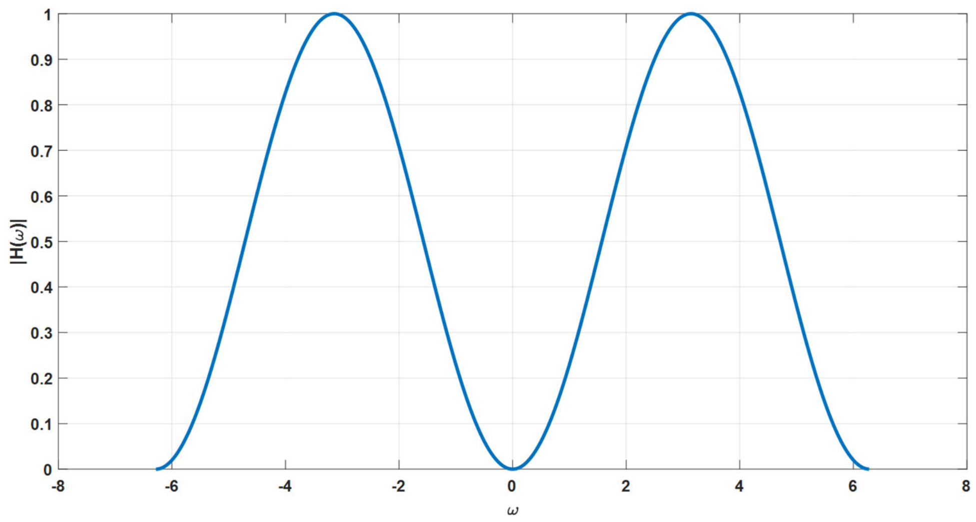
$$H(\omega) = -\frac{1}{4} + \frac{1}{2}e^{-j\omega} - \frac{1}{4}e^{-j2\omega}$$

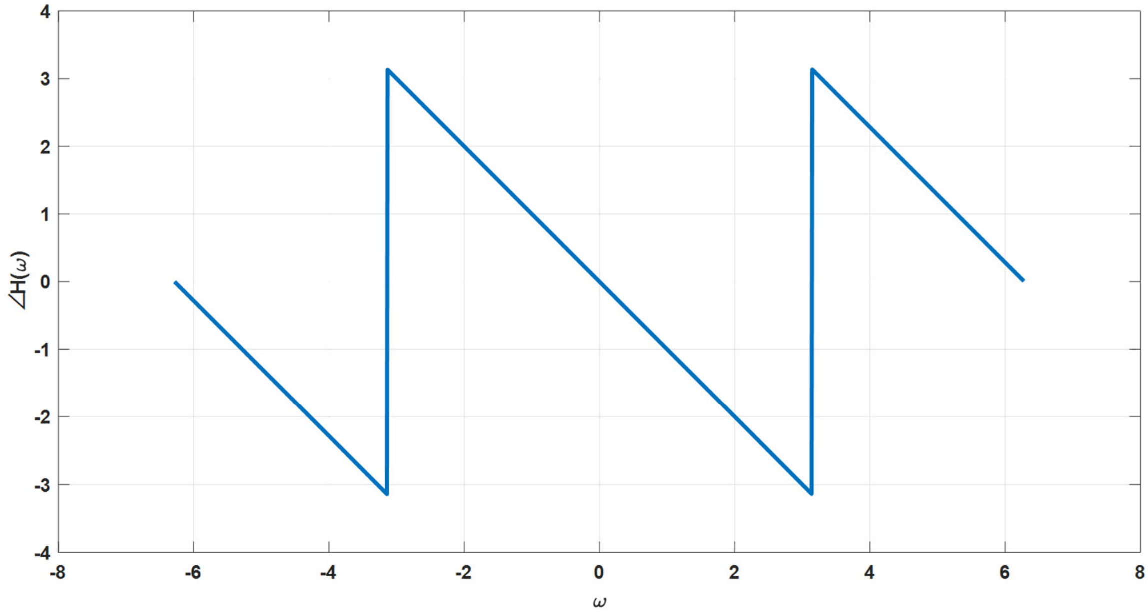
$$H(\omega) = -\frac{1}{4}e^{-j\omega}\left(e^{j\omega} + e^{-j\omega}\right) + \frac{1}{2}e^{-j\omega}$$

$$H(\omega) = -\frac{1}{2}\cos\omega\, e^{-j\omega} + \frac{1}{2}e^{-j\omega} = \frac{1}{2}(1 - \cos\omega)e^{-j\omega}$$

$$|H(\omega)| = \frac{1}{2}(1 - \cos\omega), \qquad \angle H(\omega) = -\omega$$

2. c.





2. d. This filter is a high pass filter.

<u>Question (4):</u>

1. a.

$$5(s\hat{Y}(s)-\bar{y}(0))+10\hat{Y}(s)=2\hat{X}(s),$$

so

$$\hat{Y}(s)=\frac{\bar{y}(0)}{s+2}+\frac{2}{5s+10}\hat{X}(s),$$

1. b. zero-input response:

$$\hat{Y}_{zi}(s) = \frac{2}{s+2} \leftrightarrow 2e^{-2t}u(t) = y_{zi}(t);$$

1. c. zero-state response:

$$\hat{Y}_{zs}(s) = \frac{2}{5(s+10)s} = \frac{1}{5s} - \frac{1}{5(s+2)} \leftrightarrow [\frac{1}{5} - \frac{1}{5}e^{-2t}]u(t) = y_{zs}(t)$$

1. d. To determine steady state and transient responses:
The total response is:

$$y(t) = y_{zi}(t) + y_{zs}(t) = \left(2e^{-2t} - \frac{1}{5}e^{-2t} + \frac{1}{5}\right)u(t) = \left(\frac{9}{5}e^{-2t} + \frac{1}{5}\right)u(t)$$

Then

$$y_{ss} = \frac{1}{5}u(t)$$
$$y_{tr} = \frac{9}{5}e^{-2t}u(t)$$

2.

. Since

$$\frac{(z+2)^2}{(z+1)(z+3)} = 1 + \frac{1/2}{z+1} + \frac{-1/2}{z+3}$$

has a pole at $z = -1$ and $z = -3$, there are three possible Roc.

Case 1. $RoC(x) = \{z \mid |z| < 1\}$. x is anti-causal,

$$\forall n, \quad x(n) = \delta(n) + \frac{1}{2}(-1)^n u(-n) + \frac{1}{2}(-3)^{n-1}u(-n).$$

Case 2. $Roc(x) = \{z \mid 1 < |z| < 3\}$. x is the two-signal

$$\forall n, \quad x(n) = \delta(n) + \frac{1}{2}(-1)^{n-1}u(n-1) + \frac{1}{2}(-3)^n u(-n).$$

Case 3. $RoC(x) = \{z \mid |z| > 3\}$. x is the causal signal

$$\forall n, \quad x(n) = \delta(n) + \frac{1}{2}(-1)^{n-1}u(n-1) - \frac{1}{2}(-3)^{n-1}u(n-1).$$
