Course: Digital Control | **Mid-Term Exam (1) Model Answer**
Date: 12/ 12 / 2020 | **Time: 1.00 H**

Question (1): **(10 marks):**

1. The Euler backward emulation technique is applied by using the substitution:

$$s = \frac{z-1}{Tz}$$

Inserting this equation into the controller $C(s)$ yields:

$$C(z) = \frac{2\frac{z-1}{Tz}+1}{\frac{z-1}{Tz}+\alpha} = \frac{z(2+T)-2}{z(1+\alpha T)-1}$$

2.

The controller $C(z)$ is stable if its pole z_p fulfills the condition $|z_p| < 1$.

The pole z_p is obtained from the equation $z(1 + \alpha T) - 1 = 0$.

$$z_p = \frac{1}{1+\alpha T}$$

From the condition above, α must satisfy:

$$\left|\frac{1}{1+\alpha T}\right| < 1$$

$$|1+\alpha T| > 1$$

$$1+\alpha T > 1 \qquad \text{or} \qquad 1+\alpha T < -1$$

These inequalities constraints lead to the solutions $\alpha > 0$ or $\alpha < -\frac{2}{T}$.

3.
The digital controller is: $C(z) = \frac{z(2+T)-2}{z(1+\alpha T)-1}$

We simplify the controller as:

$$C(z) = \left(\frac{2+T}{1+\alpha T}\right)\frac{z-\left(\frac{2}{2+T}\right)}{z-\left(\frac{1}{1+\alpha T}\right)} = K\frac{z-a}{z-b}$$

Where $K = \frac{2+T}{1+\alpha T}, \quad a = \frac{2}{2+T}, \quad b = \frac{1}{1+\alpha T}$

Then,

$$\frac{U(z)}{E(z)} = K\frac{z-a}{z-b}$$

$$z.U(z) - b.U(z) = z.K.E(z) - K.a.E(z)$$
$$u(k+1) - b.u(k) = K.e(k+1) - K.a.e(k)$$
$$u(k) = b.u(k-1) + K.e(k) - K.a.e(k-1)$$

Code:

Parameters b, a, K

Variable e_prev=0, e, u_prev=0, u.

Loop:

Read e.

Compute u = b*u_prev + K*e + K*a*e_prev.

Output u.

e_prev = e.

u_prev =u.

End loop.

Question (2):

$$G(s) = \frac{V(s)}{U(s)} = \frac{1}{Ms+b} = \frac{K/\tau}{s+1/\tau}$$

where $K = 1/b$ and $\tau = M/b$

For a zero-order hold:

$$G(z) = (1-z^{-1})Z\left[\frac{G(s)}{s}\right]$$

$$\frac{G(s)}{s} = \frac{K/\tau}{s(s+1/\tau)} = (K/\tau)\left[\frac{\tau}{s} - \frac{\tau}{s+1/\tau}\right]$$

Thereby

$$G(z) = (1-z^{-1})Z\left[\left(\frac{K}{\tau}\right)\left[\frac{\tau}{s} - \frac{\tau}{s+1/\tau}\right]\right]$$

$$= (1-z^{-1})Z\left[K\left[\frac{1}{s} - \frac{1}{s+1/\tau}\right]\right]$$

$$\Rightarrow G(z) = \frac{z-1}{z}K\left[\frac{z}{z-1} - \frac{z}{z-e^{-T/\tau}}\right] = K\left[1 - \frac{z-1}{z-e^{-T/\tau}}\right]$$

or

$$G(z) = K\left[\frac{z-e^{-T/\tau}-z+1}{z-e^{-T/\tau}}\right] = K\left[\frac{1-e^{-T/\tau}}{z-e^{-T/\tau}}\right]$$

Question (3): **(10 marks).**

The maximum frequency can be determined from Nyquist frequency:

$$f < \frac{f_s}{2}$$

Therefore, $\omega < \frac{2\pi}{2T} = \frac{\pi}{0.1} = 31.4159\, rad/sec$

$$G(z) = Z\left\{\frac{1-e^{-sT}}{s}\frac{5}{s+5}\right\} = \frac{1-e^{-0.5}}{z-e^{-0.5}},$$

$$G(z) = \frac{0.393}{z-0.606}.$$

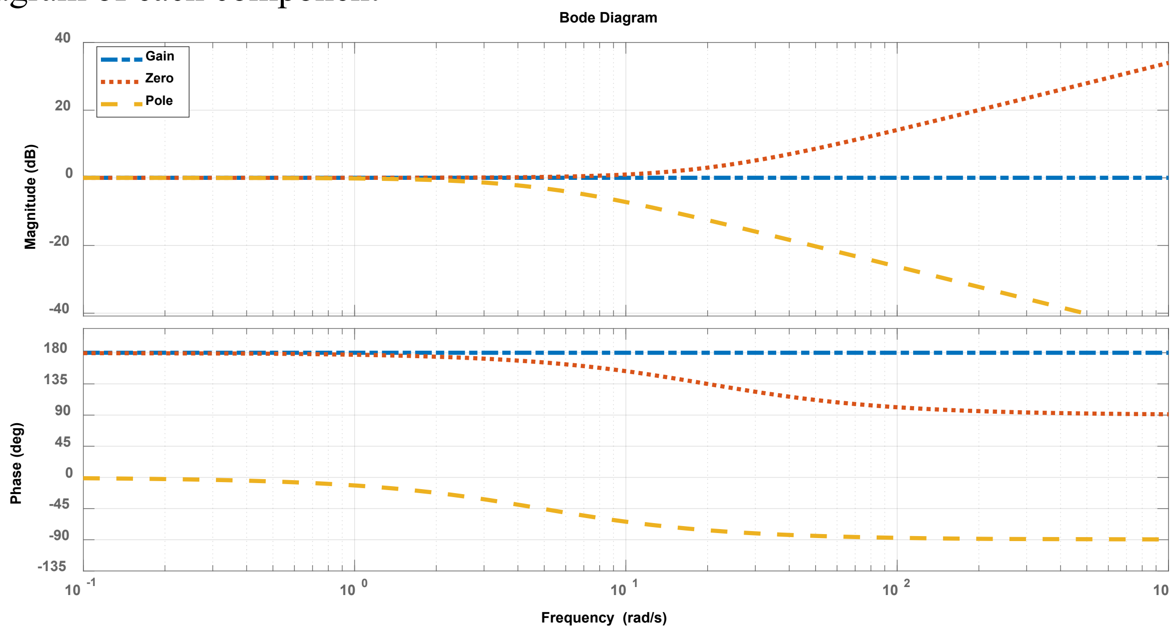
Applying bilinear transform:

$$z = \frac{1 + \left(\frac{T}{2}\right) w}{1 - \left(\frac{T}{2}\right) w} = \frac{1 + 0.05w}{1 - 0.05w}$$

$$G(w) = \frac{0.3935}{\frac{1 + 0.05w}{1 - 0.05w} - 0.6065} = \frac{-0.01968w + 0.3935}{0.08033w + 0.3935}$$

$$G(w) = \frac{-0.24494(w - 20)}{(w + 4.899)} = \frac{-1.0020\left(\frac{j\omega}{20} - 1\right)}{\left(\frac{j\omega}{4.899} + 1\right)}$$

Bode diagram of each component



Complete bode diagram: