



This exam consists of 4 questions located in 2 pages. Attempt all the questions and assume any missing data or logical assumptions.

Question (1): (25 marks)

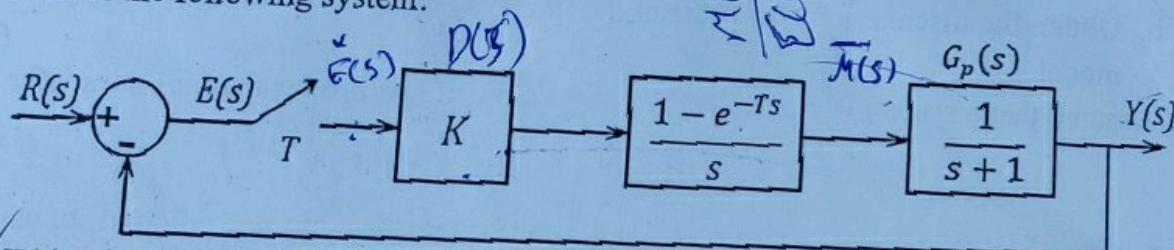
1. Draw the block diagram of a generic digital control system. Include all blocks, interfaces, and all relevant continuous and discrete signals. (5 marks)
2. What is the challenge that arises in real-time implementation of digital control systems related to varying computation time? Discuss a simple approach to solve it. (5 marks)
3. Assume that a controller is required to be implemented on a microprocessor and the controller continuous transfer function is defined as:

$$C(s) = \frac{2s+1}{s+\alpha}, \quad \text{whereas } \alpha \text{ is a tuning parameter.}$$

- a. Obtain the discrete transfer function of the controller $C(z)$ using backward Euler emulation technique for a generic sampling time T . (8 marks)
- b. What is the range of the tuning parameter α to produce a stable discrete transfer function for the controller using backward Euler emulation $C(z)$? (4 marks)
- c. What is the condition on α that makes both continuous transfer function $C(s)$ and discrete transfer function using backward Euler $C(z)$ for the controller stable? (3 marks)

Question (2): (25 marks)

1. Consider the following system:



- a. Write the discrete closed-loop system characteristic equation as a function of the sampling time T . (5 marks)
 - b. Determine ranges of $K > 0$ for stability for sampling times $T = 1 \text{ sec}$, $T = 0.1 \text{ sec}$, $T = 0.01 \text{ sec}$. (5 marks)
 - c. How does reducing sampling time affect system stability? (5 marks)
- Plot the root locus for $K > 0$ for a system with an open-loop transfer function as: (10 marks)

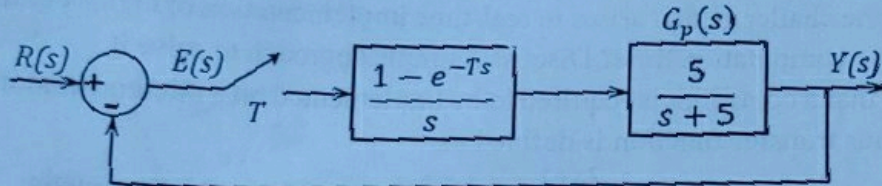
$$G(z)H(z) = \frac{K(z + 0.995)}{(z - 1)(z - 0.905)}$$

Question (3): (25 marks)

1. Consider a unity feedback discrete-time system with a sampling time $T = 2 \text{ sec}$ and an open-loop transfer function:

$$G(z) = \frac{K(z + 0.8)}{(z - 1)(z - 0.6)}$$

- Determine the range of K for stability using the Routh-Hurwitz criterion. (10 marks)
 - Verify the obtained range of K for stability using the Jury test. (5 marks)
2. Consider the following closed-loop digital control system. Draw the Bode diagram assume that $T = 0.1 \text{ sec}$. For what range of radial frequencies does this plot hold? (10 marks)



Question (4): (25 marks)

1. Consider the following continuous-time state variable model for a second-order system:

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

- Compute a discrete-time state variable model for a sampling time $T = 1$. (5 marks)
- Obtain the discrete transfer function for the system from the discrete-time state variable model. (5 marks)

Assume that a system is controlled by continuous PID controller that has a transfer function:

$$PID(s) = 1.5 + \frac{0.16}{s} + 0.45s$$

- Calculate a difference equation for a digital PID implementation of continuous PID using sampling time $T = 1$. (5 marks)
- Draw a block diagram of the obtained digital PID controller. (5 marks)
- Write a C code that computes the control action of this digital PID. (5 marks)

With my best wishes
Dr. Ahmed Mahmoud