Course: System Analysis **Mid-Term Exam (2) Solution Manual**
Date: 22 / 12 / 2020 **Time: 30 min.**

1. a.

The output is $\forall t, y(t) = H(\omega)e^{i\omega t}$

Substituting into DE: $i\omega H(\omega)e^{i\omega t} + 0.5H(\omega)e^{i\omega t} = e^{i\omega t}$

$$H(\omega) = \frac{1}{0.5 + i\omega}$$

$$y(t) = \frac{1}{0.5 + i\omega} e^{i\omega t}$$

1. b.

$$H(\omega) = \frac{1}{0.5 + i\omega}$$

1. c. $|H(0.5)| = \left|\frac{1}{0.5+i0.5}\right| = \sqrt{2}$, $\angle H(0.5) = -\frac{\pi}{4}$

2.

Period of x is the smallest p such that $2\pi p$ and $3\pi p$ are integer multiples of 2π

Then, $p = 2$.

Angular frequency is $\omega = \frac{2\pi}{p} = \pi$

To save time, we apply Euler's identities:

$$\cos(2\pi t) = \frac{e^{i2\pi t} + e^{-i2\pi t}}{2}, \qquad \sin(3\pi t) = \frac{e^{i3\pi t} + e^{-i3\pi t}}{2i}$$

Then, $x(t) = \frac{i}{2}e^{-i3\pi t} + \frac{1}{2}e^{-i2\pi t} + \frac{1}{2}e^{i2\pi t} - \frac{i}{2}e^{i3\pi t}$

Let $3\pi t = 3\omega_0 t$, $2\pi t = 2\omega_0 t$:

$$x_m = \begin{cases} 1/2 & \text{when } m = \pm 2 \\ i/2 & \text{when } m = -3 \\ -i/2 & \text{when } m = 3 \\ 0 & \text{otherwise} \end{cases}$$