Information Technology

Spring Semester

Mobile and Sensor Networks

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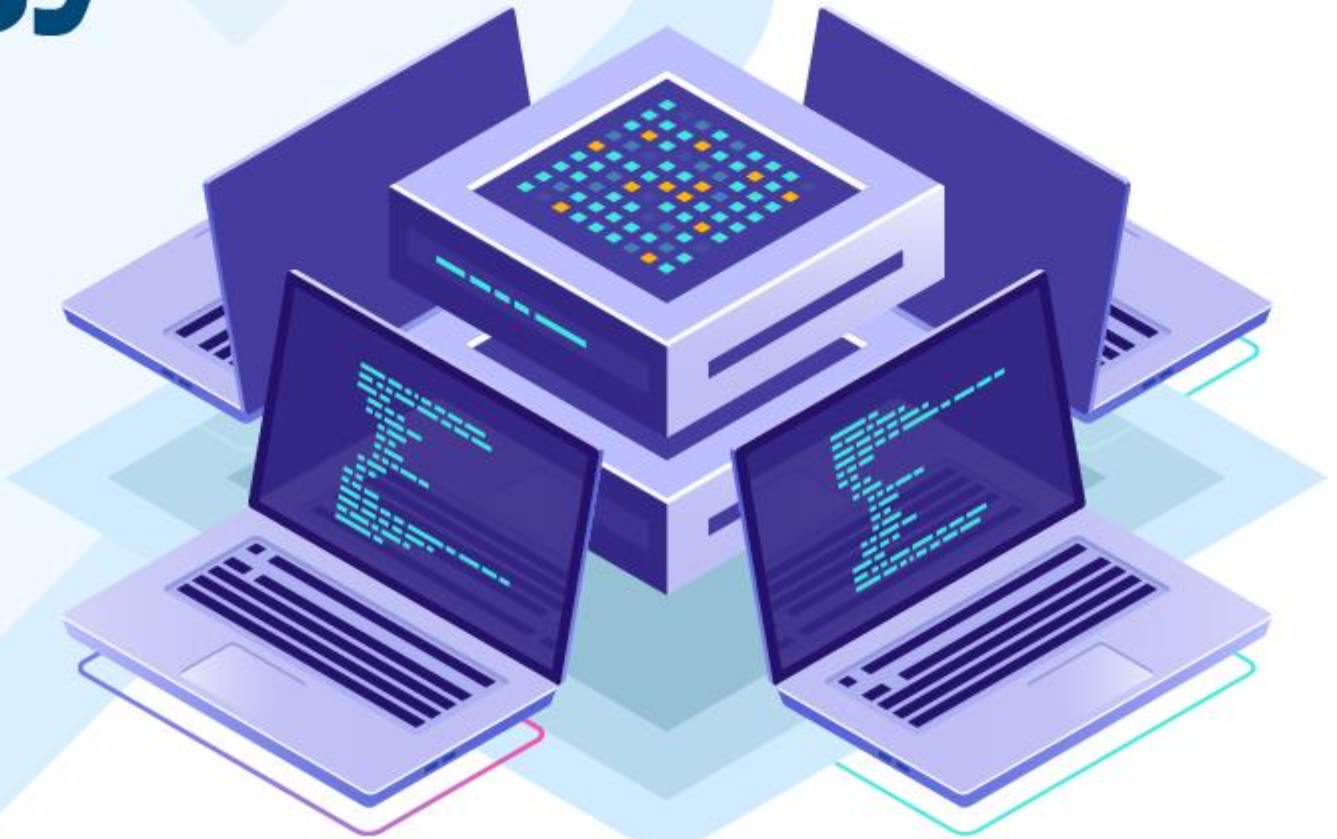
Lec_9



الجامعة المصرية للتعلم الإلكتروني الأهلية

THE EGYPTIAN E-LEARNING UNIVERSITY

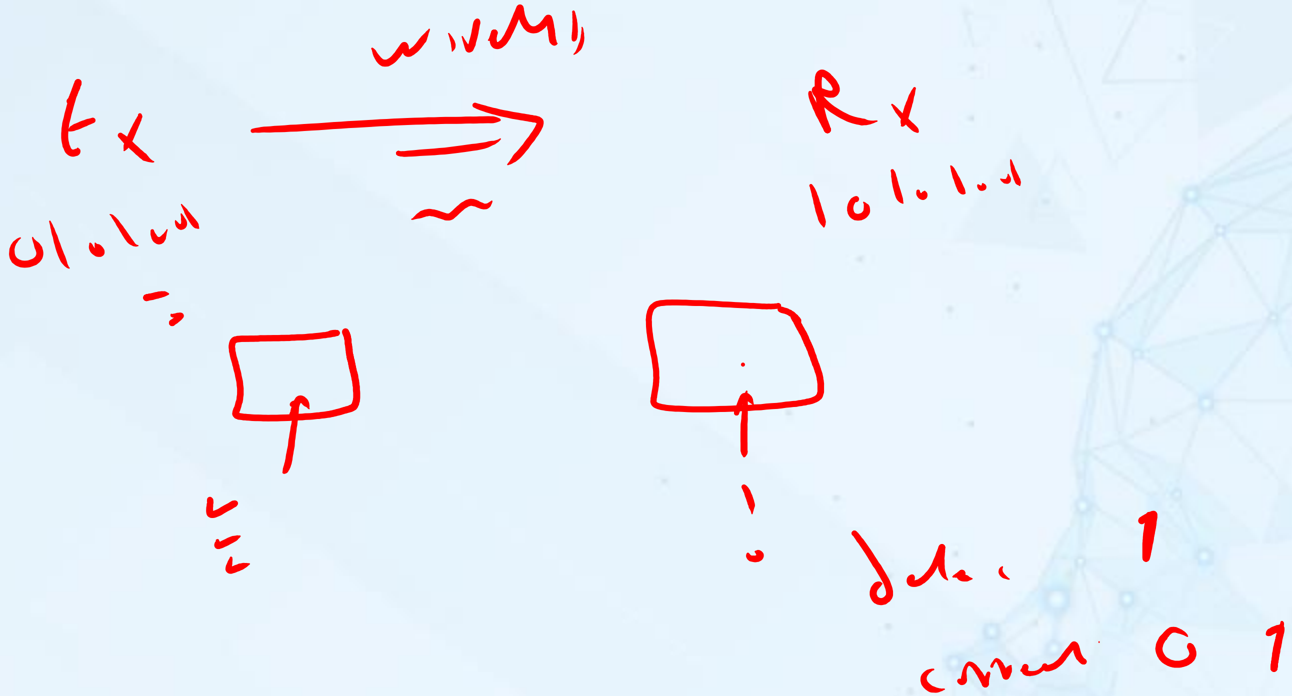
$$\boxed{C = mG}$$

$$G = [P \vdots I_k] = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1,n-k} & 1 & 0 & \cdots & 0 \\ p_{21} & p_{22} & \cdots & p_{2,n-k} & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ p_{k1} & p_{k2} & \cdots & p_{k,n-k} & 0 & 0 & \cdots & 1 \end{bmatrix}$$

$\mathbf{G} = [\mathbf{P} \quad \mathbf{I}_k]_{k \times n}$ Generator matrix

$\mathbf{I}_k = k \times k$ identity matrix

$\mathbf{P} = k \times (n-k)$ Parity matrix


tx
Rx

Ch

$t_x \xrightarrow{n} R_x$

0101001101001

3 bits

$k = 3$ bits

$m = n - k$

$m = n - k = 3$

$n = 6$ bits

(1) $k = 2^m - m - 1 = 2^3 - 3 - 1 = 4$

(2) $n = 2^m - 1 = 2^3 - 1 = 7$

$n = 6$

$k = 3$

$$G = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right]$$

I P

$G = [\,I \;\; P\,]$

3×3

$C = [\,I \;\; P\,]$

k n

C H

tx → rx

$c = mG$

$C = mG$

$d_{min} = 3$

m	c
000	000 000
001	001 101
100	

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

$H =$

$$H^T = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$cH^T = 0$

$G = [I \quad P]$

$H = [P^T \quad I]$

$$cH^T = 0$$

Ex $c \rightarrow r$

$$r = c + e$$

$$S = eH^T$$

$$011 \times G \quad \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right]$$

I

$$c = 011\,\boxed{}$$

$$011 \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$rH^T \neq 0$$

$$S = rH^T$$

$$[c+e]H^T$$

$$S = cH^T + eH^T$$

EELU

- The generator matrix is a compact description of how codewords are generated from information bits in a linear block code.
- The design goal in linear block codes is to find generator matrices such that
  - the corresponding codes are easy to encode and decode
  - yet have powerful error correction/detection capabilities.
- Tradeoffs between
  - Efficiency
  - Reliability
  - Encoding/Decoding complexity

- Parity Check Matrix: Associated to each (n,k) linear block code there is a parity check matrix H at the decoder.

$$H = \left[I_{n-k} \vdots P^T \right]_{n-k,n}$$

- Parity check matrix H has its rows orthogonal to rows of G such that

$$GH^T = 0$$

- The parity check matrix H is used to detect errors in the received code
- C is the codeword in (n,k) block code generated by G iff

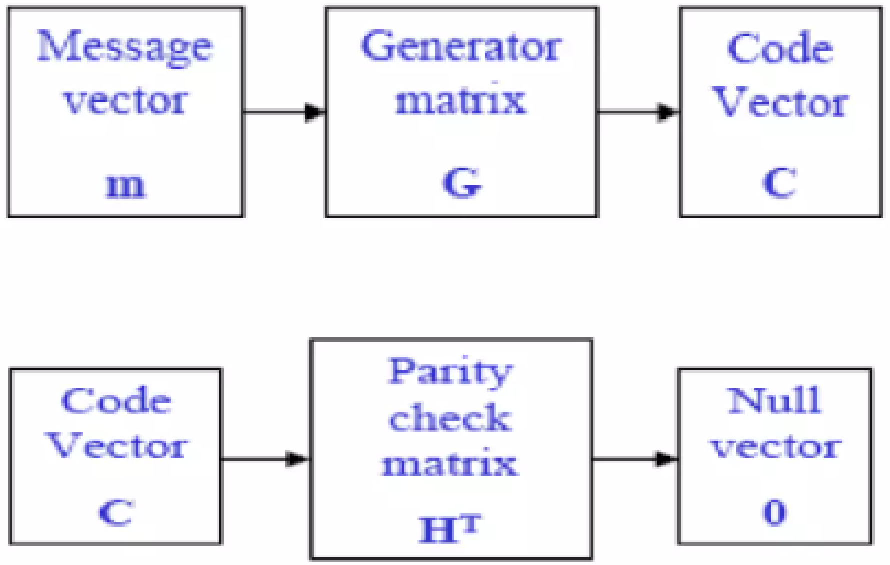
$$CH^T = 0$$

EELU

- Let R = C + E be the received message where C is the correct code and E is the error
- Decoding operation is done by determining syndrome vector S as

$$
\begin{aligned}
S &= RH^T \\
&= CH^T + EH^T \\
&= EH^T
\end{aligned}
$$

- If R is a valid vector then syndrome of received vector S is 0
- Nonzero S indicates that there are errors in transmission
- Thus S is used to detect the errors. It is also used to correct the errors.
- The errors can be corrected at the receiver by comparing S with the rows of H^T and correcting the i^{th} received bit if S matches with the i^{th} row of H^T



Operations of the generator matrix and the parity check matrix

- Example1: For a (6,3)LBC with generator matrix G, find codeword for all the distinct messages.

$$\mathbf{G} = \begin{bmatrix} 1\,1\,0\,1\,0\,0 \\ 0\,1\,1\,0\,1\,0 \\ 1\,0\,1\,0\,0\,1 \end{bmatrix}$$

- Example: Block code (6,3)

$$\mathbf{G} = \begin{bmatrix} 1\,1\,0\,1\,0\,0 \\ 0\,1\,1\,0\,1\,0 \\ 1\,0\,1\,0\,0\,1 \end{bmatrix}$$

Message vector	Codeword
000	000000
100	110100
010	011010
110	101110
001	101001
101	011101
011	110011
111	000111

(n, k) linear block code
n = number of bits in code word
k = number of message bits
$(n-k)$ = number of parity bits or parity check bits

N.B. parity bits are computed from message bits according to encoding rule. Encoding rule decides mathematical structure of the code.

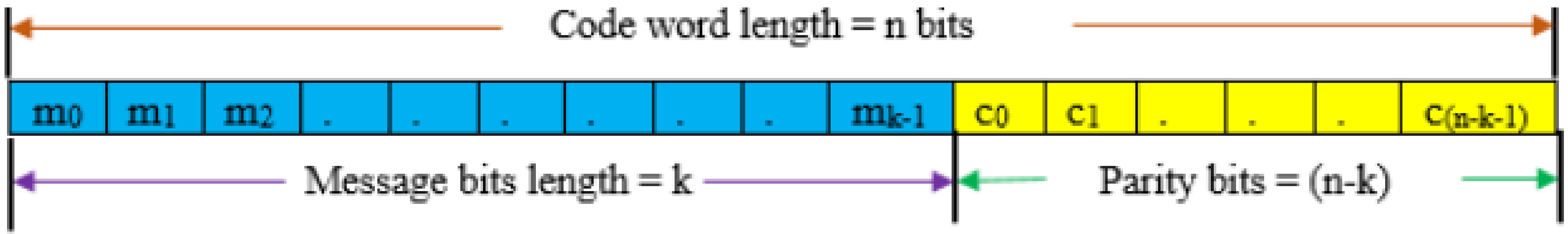


Fig: Structure of a codeword

$$[X]_{1xn} = [M]_{1xk}\,[G]_{kxn}$$

The generator matrix is dependent on the type of linear block code used:

$$[G] = [I_k \mid P]$$

Where I_k = kxk identity matrix
P = kx(n-k) coefficient matrix

For example, (5,3) code:

n =5, k = 3
(n-k) = (5-3) =2

$$I_k = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P = \begin{bmatrix} p_{00} & p_{01} \\ p_{10} & p_{11} \\ p_{20} & p_{21} \end{bmatrix}$$

Now parity vector C can be computed as C = MP

$$C = \begin{bmatrix} m_0 & m_1 & m_2 \end{bmatrix} \begin{bmatrix} p_{00} & p_{01} \\ p_{10} & p_{11} \\ p_{20} & p_{21} \end{bmatrix}$$

Now solve for c_0, c_1, $c_2 \ldots c_{n-k}$

$$c_0 = m_0\, p_{00} \oplus m_1\, p_{10} \oplus m_2\, p_{20}$$

$$c_1 = m_0\, p_{01} \oplus m_1\, p_{11} \oplus m_2\, p_{21}$$

Q. The generator matrix for a (6,3) block code is given below. Find all the code vectors of this code.

$$G = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{array}\right]$$

$C = mG$

$G = [I \quad P]$

$G \ (6,3)$

$k = 3$

$n = 6$

$k = 3$

$r = \frac{k}{n} = \frac{1}{2}$

$r = \frac{3}{6} = \frac{1}{2}$

EELU

Q. The generator matrix for a (6,3) block code is given below. Find all the code vectors of this code.

$$G = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{array}\right]$$

Sol. (n, k) = (6,3)

n = 6

k = 3

n – k = 6 - 3 =3 number of parity bits.

Step1:

Separate the identity matrix and coefficient matrix

Generator matrix is given by:

$$[G] = [I_k \mid P]$$

$$\therefore I_k = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3x3} \qquad P = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}_{3x3}$$

(000)
(001)
(010)
(011)
(100)
(101)
(110)
(111)

m = 8 = 2^k = 8

As the size of message block k =3, there are 8 possible message sequences

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Q. The generator matrix for a (6,3) block code is given below. Find all the code vectors of this code.

$$G = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{array}\right]$$

Step 2: Obtain parity vector C = MP

$$\therefore\ [C] = [c_0 \quad c_1 \quad c_2] = [m_0 \quad m_1 \quad m_2] \begin{array}{c} P \\ \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \end{array}$$

$$c_0 = (m_0 \,.\, 0) \oplus (m_1 \,.\, 1) \oplus (m_2 . 1)$$

$$= 0 \oplus m_1 \oplus m_2$$

$$= m_1 \oplus m_2$$

Similarly,

$$c_1 = m_0 \oplus m_2$$

$$c_2 = m_0 \oplus m_1$$

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Q. The generator matrix for a (6,3) block code is given below. Find all the code vectors of this code.

$$G = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{array}\right]$$

Parity bits for message word ($m_0\ m_1\ m_2 = 0\ 1\ 0$)

$$[C] = [c_0 \quad c_1 \quad c_2] = [m_0 \quad m_1 \quad m_2] \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$c_0 = m_1 \oplus m_2$$

$$c_1 = m_0 \oplus m_2$$

$$c_2 = m_0 \oplus m_1$$

$m_0 = 0, \ m_1 = 1, \ m_2 = 0$

$c_0 = m_1 \oplus m_2 = 1 \oplus 0 = 1$

$c_1 = m_0 \oplus m_2 = 0 \oplus 0 = 0$

$c_2 = m_0 \oplus m_1 = 0 \oplus 1 = 1$

Complete codeword for message block (001)

m_0	m_1	m_2	c_0	c_1	c_2
0	1	0	1	0	1

Q. The generator matrix for a (6,3) block code is given below. Find all the code vectors of this code.

$$G = \left[\begin{array}{ccc:ccc} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{array}\right]$$

Parity bits for message word ($m_0\ m_1\ m_2 = 0\ 1\ 1$)

$$[C] = [c_0 \quad c_1 \quad c_2] = [m_0 \quad m_1 \quad m_2] \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$c_0 = m_1 \oplus m_2$$

$$c_1 = m_0 \oplus m_2$$

$$c_2 = m_0 \oplus m_1$$

$m_0 = 0,\ m_1 = 1,\ m_2 = 1$

$c_0 = m_1 \oplus m_2 = 1 \oplus 1 = 0$

$c_1 = m_0 \oplus m_2 = 0 \oplus 1 = 1$

$c_2 = m_0 \oplus m_1 = 0 \oplus 1 = 1$

Complete codeword for message block (001)

m_0	m_1	m_2	c_0	c_1	c_2
0	1	1	0	1	1

Q. The generator matrix for a (6,3) block code is given below. Find all the code vectors of this code.

$$G = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{array}\right]$$

Parity bits for message word ($m_0\ m_1\ m_2 = 1\ 1\ 0$)

$$[C] = [c_0 \quad c_1 \quad c_2] = [m_0 \quad m_1 \quad m_2] \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$c_0 = m_1 \oplus m_2$$

$$c_1 = m_0 \oplus m_2$$

$$c_2 = m_0 \oplus m_1$$

$m_0 = 1,\ m_1 = 1,\ m_2 = 0$

$c_0 = m_1 \oplus m_2 = 1 \oplus 0 = 1$

$c_1 = m_0 \oplus m_2 = 1 \oplus 0 = 1$

$c_2 = m_0 \oplus m_1 = 1 \oplus 1 = 0$

Complete codeword for message block (001)

m_0	m_1	m_2	c_0	c_1	c_2
1	1	0	1	1	0

Q. The generator matrix for a (6,3) block code is given below. Find all the code vectors of this code.

$$G = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{array}\right]$$

Code vectors for (6,3) linear block code

S.No.	Message vectors	Parity bits	Code vectors
1	000	000	000000
2	001	110	001110
3	010	101	010101
4	011	011	011011
5	100	011	100011
6	101	101	101101
7	110	110	110110
8	111	000	111000

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Code vectors for (6,3) linear block code

S.No.	Message vectors	Parity bits	Code vectors
1	000	000	000000
2	001	110	001110
3	010	101	010101
4	011	011	011011
5	100	011	100011
6	101	101	101101
7	110	110	110110
8	111	000	111000

Q. Check linearity of above (6,3) code

Add codewords at S.No. 2 & 3

0	0	1	1	1	0
$\oplus$	Mod-2 addition				
0	1	0	1	0	1
= 0	1	1	0	1	1

This codeword exists at S.No. 4

Add codewords at S.No. 7 & 8

1	1	0	1	1	0
$\oplus$	Mod-2 addition				
1	1	1	0	0	0
= 0	0	1	1	1	0

This codeword exists at S.No. 2

EELU

Q. The parity check matrix of a particular (7,4) linear block code is given by:

$$H = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

i. Find the generator matrix G

ii. List all the codewords C $m\,G$

iii. What is the minimum distance between the code vectors? d_{min}

iv. How many errors can be detected? How many errors can be corrected?

$G = [\,I \quad P\,]$,

$H = [\,P^T \quad I\,]$

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THANK YOU FOR WATCHING

QUESTIONS?