$$\begin{array}{c|cc} & -1 & -\lambda \\ \hline 4 & -4 & -4\lambda \\ -\lambda & \lambda & \lambda^2 \end{array}$$

Find a basis for $\mathbb{R}^2$ for

$T(\vec{x}) = \begin{bmatrix} 4 & 3 \\ 2 & -1 \end{bmatrix} \vec{x}$

$\begin{bmatrix} 4 & 3 \\ 2 & -1 \end{bmatrix} \longrightarrow A - \lambda I = \begin{bmatrix} 4-\lambda & 3 \\ 2 & -1-\lambda \end{bmatrix} \xrightarrow{\det(A-\lambda I)} (4-\lambda)(-1-\lambda) - 6 = 0$

$\lambda^2 - 3\lambda - 4 - 6 = 0$

$\lambda^2 - 3\lambda - 10 = 0$

$(\lambda - 5)(\lambda + 2) = 0$

$\lambda_s \{5, -2\}$

Eigenvector for $\lambda_1 = 5$; $A - 5I =$

$\begin{bmatrix} 4 & 3 \\ 2 & -1 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} -1 & 3 \\ 2 & -6 \end{bmatrix} \xrightarrow{RREF} \begin{bmatrix} -1 & 3 \\ 2 & -6 \end{bmatrix} \sim R_1 \begin{bmatrix} 1 & -3 \\ 2 & -6 \end{bmatrix} R_2 - 2R_1 \underbrace{\begin{bmatrix} 1 & -3 \\ 0 & 0 \end{bmatrix}}_{\vec{v}_1 - 3\vec{v}_2 = 0} \vec{v} = \vec{0}$

$+3v_2$

$\vec{v}_1 = 3v_2$

$\vec{v}_2 = 1$

$\frac{2}{2} - \frac{3}{2} = -\frac{1}{2}$

The eigenvector when $\lambda_1 = 5$ is

$\begin{bmatrix} 3 \\ 1 \end{bmatrix}$ ✓

Eigenvector for $\lambda_2 = -2$; $A + 2I$

$6\vec{v}_1 + 3\vec{v}_2 = -3v^2$

$\frac{6\vec{v}_1}{-3} = \frac{-3\vec{v}_2}{-3}$

$-2v_1 = \vec{v}_2$ ✓

$v_1 = 1$

$\begin{bmatrix} 4 & 3 \\ 2 & -1 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \underbrace{\begin{bmatrix} 6 & 3 \\ 2 & 1 \end{bmatrix} \vec{v} = \vec{0}}_{6\vec{v}_1 + 3v_2 = 0}$

The eigenvector when $\lambda = -2$ is:

$\begin{bmatrix} -1 \\ 2 \end{bmatrix}$

$6\vec{v}_1 + 3v_2 = 0 \longrightarrow$

$2\vec{v}_1 + \vec{v}_2 = 0 \longrightarrow \vec{v}_2 = -2\vec{v}_1$