This print-out should have 5 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

001 10.0 points

Find a matrix A so that $\mathrm{Nul}(A)$ is the set of all vectors

$$H = \left\{ \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} : \quad \begin{array}{c} a + 2b \ = \ 4c\,, \\ a \ = \ c - 3d\,, \end{array} \right\}$$

in $\mathbb{R}^4$.

1. $A = \begin{bmatrix} 1 & 2 & -4 & 0 \\ 1 & 0 & -1 & 3 \end{bmatrix}$ **correct**

2. $A = \begin{bmatrix} 1 & -2 & 4 & 0 \\ 1 & 0 & 1 & -3 \end{bmatrix}$

3. $A = \begin{bmatrix} 1 & -2 & -4 & 0 \\ 1 & 0 & 1 & -3 \end{bmatrix}$

4. $A = \begin{bmatrix} 1 & 2 & 4 & 0 \\ 1 & 0 & 1 & -3 \end{bmatrix}$

5. $A = \begin{bmatrix} 1 & 2 & -4 & 0 \\ 1 & 0 & -1 & -3 \end{bmatrix}$

6. $A = \begin{bmatrix} 1 & -2 & 4 & 0 \\ 1 & 0 & -1 & 3 \end{bmatrix}$

Explanation:

Rewrite the conditions

$$a + 2b = 4c, \quad a = c - 3d$$

as

$$a + 2b - 4c = 0,$$
$$a - c + 3d = 0\,,$$

and set

$$A = \begin{bmatrix} 1 & 2 & -4 & 0 \\ 1 & 0 & -1 & 3 \end{bmatrix}.$$

Then

$$A \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 1 & 2 & -4 & 0 \\ 1 & 0 & -1 & 3 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

$$= \begin{bmatrix} a + 2b - 4c \\ a - c - 3d \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

if and only if

$$a + 2b - 4c = 0,$$
$$a - c + 3d = 0\,.$$

Consequently,

$$\boxed{\mathrm{Nul}(A) = H}\,.$$

002 10.0 points

When A is a 5×7 matrix, then Row$A$ is a subspace of $\mathbb{R}^p$ and Col$A$ is a subspace of $\mathbb{R}^q$ for which values of p and q.

1. $p = 5, \quad q = 7$

2. $p = 5, \quad q = 5$

3. $p = 7, \quad q = 7$

4. $p = 7, \quad q = 5$ **correct**

Explanation:

Because A is a 5×7 matrix, it has 5 rows and 7 columns. Now each row has 7 entries, so as vectors the rows belong to $\mathbb{R}^7$. On the other hand, each column has 5 entries, so as vectors the columns belong to $\mathbb{R}^5$.

Thus Row$A$ is a subspace of $\mathbb{R}^7$ and Col$A$ is a subspace of $\mathbb{R}^5$. Consequently,

$$\boxed{p = 7, \quad q = 5}\,.$$

003 10.0 points

Let H be the set of all vectors

$$\begin{bmatrix} a - 2b \\ ab + 3a \\ b \end{bmatrix}$$

where a and b are real. Determine if H is a subspace of $\mathbb{R}^3$, and then check the correct answer below.

1. *H is not a subspace of $\mathbb{R}^3$ because it is not closed under vector addition.* **correct**

2. *H is not a subspace of $\mathbb{R}^3$ because it does not contain $\mathbf{0}$.*

3. *H is a subspace of $\mathbb{R}^3$ because it can be written as Span$\{\mathbf{v}_1, \mathbf{v}_2\}$ with $\mathbf{v}_1, \mathbf{v}_2$ in $\mathbb{R}^3$.*

4. *H is a subspace of $\mathbb{R}^3$ because it can be written as Nul(A) for some matrix A.*

Explanation:

To check if the set H of all vectors

$$\begin{bmatrix} a-2b \\ ab+3a \\ b \end{bmatrix}$$

is a subspace of $\mathbb{R}^3$ we check the properties defining a subspace:

1. *the zero vector $\mathbf{0}$ is in H*: set $a = b = 0$. Then

$$\begin{bmatrix} 0-0 \\ 0+0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},$$

so H contains $\mathbf{0}$.

2. *for each $\mathbf{u}$, $\mathbf{v}$ in H the sum $\mathbf{u}+\mathbf{v}$ is in H*: set

$$\mathbf{v}_1 = \begin{bmatrix} a_1-2b_1 \\ a_1b_1+3a_1 \\ b_1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} a_2-2b_2 \\ a_2b_2+3a_2 \\ b_2 \end{bmatrix},$$

in H. Then

$$\mathbf{v}_1+\mathbf{v}_2 = \begin{bmatrix} a_1-2b_1 \\ a_1b_1+3a_1 \\ b_1 \end{bmatrix} + \begin{bmatrix} a_2-2b_2 \\ a_2b_2+3a_2 \\ b_2 \end{bmatrix}$$

$$= \begin{bmatrix} (a_1+a_2)-2(b_1+b_2) \\ a_1b_1+a_2b_2+3(a_1+a_2) \\ (b_1+b_2) \end{bmatrix}.$$

But in general,

$$a_1b_1 + a_2b_2 \neq (a_1+a_2)(b_1+b_2),$$

in which case $\mathbf{u}+\mathbf{v}$ is not in H.

Consequently, H is not a subspace of $\mathbb{R}^3$ because it is

$$\boxed{\textit{not closed under vector addition}}.$$

004 10.0 points

Determine the rank of the matrix

$$A = \begin{bmatrix} 3 & 6 & -3 \\ -3 & -3 & -6 \\ 2 & 1 & 10 \end{bmatrix}.$$

1. rank(A) = 4

2. rank(A) = 5

3. rank(A) = 1

4. rank(A) = 2

5. rank(A) = 3 **correct**

Explanation:

Since

$$\text{rref}(A) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

all three rows of rref(A) contain leading 1's, so

$$\boxed{\text{Rank}(A) = 3}.$$

005 10.0 points

Determine the rank of the matrix

$$A = \begin{bmatrix} 3 & -3 & 3 & -3 \\ -3 & 5 & -9 & 1 \\ -1 & -2 & 8 & 4 \end{bmatrix}.$$

1. rank(A) = 1

2. $\text{rank}(A) = 5$

3. $\text{rank}(A) = 4$

4. $\text{rank}(A) = 3$

5. $\text{rank}(A) = 2$ **correct**

Explanation:
Since

$$\text{rref}(A) = \begin{bmatrix} 1 & 0 & -2 & -2 \\ 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

only the first two rows of rref(A) contain leading 1's, so

$$\boxed{\text{Rank}(A) = 2}.$$