This print-out should have 15 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

001 10.0 points

For which value(s) of λ will the vectors

$$\mathbf{v_1} = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}, \quad \mathbf{v_2} = \begin{bmatrix} -2 \\ 3 \\ 2 \end{bmatrix}, \quad \mathbf{v_3} = \begin{bmatrix} 6 \\ 6 \\ \lambda + 2 \end{bmatrix},$$

be linearly dependent?

1. $\lambda \neq -2$

2. $\lambda = -3$ **correct**

3. $\lambda \neq -3$

4. $\lambda = -2$

5. $\lambda \neq 1$

6. $\lambda = 1$

Explanation:

The vectors $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$ will be linearly dependent if the vector equation

$$x_1\mathbf{v}_1 + x_2\mathbf{v}_2 + x_3\mathbf{v}_3 = \mathbf{0}$$

has a non-trivial solution, *i.e.*, when the homogeneous system

$$\begin{aligned} 4x_1 - 2x_2 + 6x_3 &= 0, \\ 9x_1 + 3x_2 + 6x_3 &= 0, \\ x_1 + 2x_2 + (\lambda + 2)x_3 &= 0, \end{aligned}$$

has a non-trivial solution. This occurs when the augmented matrix

$$A = \begin{bmatrix} 4 & -2 & 6 & 0 \\ 9 & 3 & 6 & 0 \\ 1 & 2 & \lambda + 2 & 0 \end{bmatrix}$$

associated with this system has a free variable. Now

$$\text{rref}(A) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{3}{5}(\lambda + 3) & 0 \end{bmatrix},$$

so A will have a free variable only when $\lambda + 3 = 0$.

Consequently, $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$ will be linearly dependent only when

$$\boxed{\lambda = -3}.$$

002 5.0 points

If $\mathbf{u}_1$, $\mathbf{u}_2$, $\mathbf{u}_3$ and $\mathbf{u}_4$ are vectors in $\mathbb{R}^7$ and $\mathbf{u}_2 = 0$, then the set $S = \{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ is linearly dependent.

True or False?

1. TRUE **correct**

2. FALSE

Explanation:

Any set containing the zero vector is linearly dependent.

Consequently, the statement is

$$\boxed{\text{TRUE}}.$$

003 5.0 points

The columns of a 5x6 matrix A are linearly dependent.

True or False?

1. FALSE

2. TRUE **correct**

Explanation:

The columns of a 5x6 matrix A are linearly dependent because the set whose vectors are the columns of A, contains more vectors than there are entries in each vector.

Consequently, the statement is

$$\boxed{\text{TRUE}}.$$

004 5.0 points

If a set of vectors contains fewer vectors than there are entries in the vectors, then the set must be linearly independent.

True or False?

1. FALSE **correct**

2. TRUE

Explanation:
This is not true because, for instance, the set of two parallel vectors in $\mathbb{R}^5$ is linearly dependent in spite there are only two vectors with 5 entries.

Consequently, the statement is

$$\boxed{\text{FALSE}}.$$

005 10.0 points

If $T : \mathbb{R}^2 \to \mathbb{R}^2$ is the linear transformation such that

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = x_1 \begin{bmatrix} -1 \\ -2 \end{bmatrix} + x_2 \begin{bmatrix} 2 \\ 1 \end{bmatrix},$$

determine $T(\mathbf{x})$ when $\mathbf{x} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$.

1. $T(\mathbf{x}) = \begin{bmatrix} 6 \\ 1 \end{bmatrix}$

2. $T(\mathbf{x}) = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

3. $T(\mathbf{x}) = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$ **correct**

4. $T(\mathbf{x}) = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$

5. $T(\mathbf{x}) = \begin{bmatrix} 6 \\ 0 \end{bmatrix}$

Explanation:
When

$$\mathbf{x} = \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 3 \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

then

$$\begin{aligned} T(\mathbf{x}) &= T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) + 3\,T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) \\ &= \begin{bmatrix} -1 \\ -2 \end{bmatrix} + 3 \begin{bmatrix} 2 \\ 1 \end{bmatrix}. \end{aligned}$$

Thus

$$\boxed{T(\mathbf{x}) = \begin{bmatrix} -1 \\ -2 \end{bmatrix} + \begin{bmatrix} 6 \\ 3 \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \end{bmatrix}}.$$

006 10.0 points

If $T : \mathbb{R}^2 \to \mathbb{R}^2$ is the linear transformation such that

$$T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \quad T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -2 \\ 2 \end{bmatrix},$$

determine $T(\mathbf{x})$ when $\mathbf{x} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$.

1. $T(\mathbf{x}) = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

2. $T(\mathbf{x}) = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

3. $T(\mathbf{x}) = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

4. $T(\mathbf{x}) = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$

5. $T(\mathbf{x}) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ **correct**

Explanation:
Since

$$\mathbf{x} = \begin{bmatrix} 3 \\ 1 \end{bmatrix} = 3 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

and T is a linear transformation,

$$T(\mathbf{x}) = T\left(3\begin{bmatrix}1\\0\end{bmatrix} + \begin{bmatrix}0\\1\end{bmatrix}\right)$$
$$= 3\,T\left(\begin{bmatrix}1\\0\end{bmatrix}\right) + T\left(\begin{bmatrix}0\\1\end{bmatrix}\right)$$
$$= 3\begin{bmatrix}1\\-1\end{bmatrix} + \begin{bmatrix}-2\\2\end{bmatrix}.$$

Consequently

$$\boxed{T(\mathbf{x}) = \begin{bmatrix}3\\-3\end{bmatrix} + \begin{bmatrix}-2\\2\end{bmatrix} = \begin{bmatrix}1\\-1\end{bmatrix}}.$$

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007 10.0 points

Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation such that

$$T(x_1,\, x_2) \;=\; (4x_1 + x_2,\, -2x_1 - 2x_2)\,.$$

Determine A so that T can be written as the matrix transformation $T_A : \mathbb{R}^2 \to \mathbb{R}^2$.

1. $A \;=\; \begin{bmatrix}4 & 0 & 1\\0 & 1 & -2\end{bmatrix}$

2. $A \;=\; \begin{bmatrix}4 & 0 & -2\\0 & 1 & 1\end{bmatrix}$

3. $A \;=\; \begin{bmatrix}4 & 1\\-2 & -2\end{bmatrix}$ **correct**

4. $A \;=\; \begin{bmatrix}4 & -2\\1 & -2\end{bmatrix}$

Explanation:

We can write $\mathbb{R}^2$ both as rows and column vectors

$$\text{(i)}\quad (x_1,\, x_2)\,, \qquad \text{(ii)}\quad \begin{bmatrix}x_1\\x_2\end{bmatrix}.$$

In row form

$$T(x_1,\, x_2) \;=\; (4x_1 + x_2,\, -2x_1 - 2x_2)\,,$$

while in column vector form

$$T\left(\begin{bmatrix}x_1\\x_2\end{bmatrix}\right) \;=\; A\begin{bmatrix}x_1\\x_2\end{bmatrix}$$

where A is the 2×2 matrix standard matrix

$$[T(\mathbf{e}_1)\;\; T(\mathbf{e}_2)]$$

of T. Now

$$\mathbf{e}_1 \;=\; \begin{bmatrix}1\\0\end{bmatrix} \;=\; (1,\, 0)\,, \quad \mathbf{e}_2 \;=\; \begin{bmatrix}0\\1\end{bmatrix} \;=\; (0,\, 1)\,,$$

so that

$$T(1,\, 0) \;=\; (4,\, -2) \;=\; \begin{bmatrix}4\\-2\end{bmatrix} \;=\; T(\mathbf{e}_1)\,,$$

and

$$T(0,\, 1) \;=\; (1,\, -2) \;=\; \begin{bmatrix}1\\-2\end{bmatrix} \;=\; T(\mathbf{e}_2)\,.$$

Consequently,

$$\boxed{A \;=\; \begin{bmatrix}4 & 1\\-2 & -2\end{bmatrix}}.$$

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008 10.0 points

Consider the matrix

$$A = \begin{bmatrix}2 & 1 & 3\\1 & -1 & 1\end{bmatrix}$$

If a transformation $T\colon \mathrm{R}^3 \to \mathrm{R}^2$ is defined as $T(\mathbf{x}) = A\mathbf{x}$, is this transformation one-to-one or onto?

1. one-to-one

2. neither one-to-one nor onto

3. onto **correct**

4. both one-to-one and onto

Explanation:

This transformation is onto but not one-to-one since it has a pivot in every row but does not have a pivot in every column.

009 10.0 points

Consider the matrix

$$A = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

If a transformation $T\colon \mathrm{R}^3 \to \mathrm{R}^4$ is defined as $T(\mathbf{x}) = A\mathbf{x}$, is this transformation one-to-one or onto?

1. onto

2. one-to-one **correct**

3. both one-to-one and onto

4. neither one-to-one nor onto

Explanation:

This transformation is one-to-one but not onto since it has a pivot in every column but does not have a pivot in every row.

010 10.0 points

Consider the matrix

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 2 & -1 & -3 \\ -3 & 3 & 3 \end{bmatrix}$$

If a transformation $T\colon \mathrm{R}^3 \to \mathrm{R}^3$ is defined as $T(\mathbf{x}) = A\mathbf{x}$, is this transformation one-to-one or onto?

1. onto

2. neither one-to-one nor onto

3. both one-to-one and onto **correct**

4. one-to-one

Explanation:

This transformation is both one-to-one and onto since it has three pivots.

011 10.0 points

Consider the matrix

$$A = \begin{bmatrix} 1 & -2 & 3 \\ 3 & -3 & -3 \\ -1 & 1 & 1 \end{bmatrix}$$

If a transformation $T\colon \mathrm{R}^3 \to \mathrm{R}^3$ is defined as $T(\mathbf{x}) = A\mathbf{x}$, is this transformation one-to-one or onto?

1. both one-to-one and onto

2. neither one-to-one nor onto **correct**

3. onto

4. one-to-one

Explanation:

This transformation is neither one-to-one nor onto since the matrix has only two pivots.

012 10.0 points

Compute the expression $AB - BA$ when

$$A = \begin{bmatrix} 1 & 3 \\ 5 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} -2 & 2 \\ 2 & -4 \end{bmatrix}.$$

1. $AB - BA = \begin{bmatrix} -16 & 4 \\ 16 & 4 \end{bmatrix}$

2. $AB - BA = \begin{bmatrix} -4 & 0 \\ 16 & 16 \end{bmatrix}$

3. $AB - BA = \begin{bmatrix} -4 & -12 \\ 0 & 4 \end{bmatrix}$

4. $AB - BA = \begin{bmatrix} -4 & -12 \\ 16 & 4 \end{bmatrix}$ **correct**

5. $AB - BA = \begin{bmatrix} 16 & 0 \\ 16 & 4 \end{bmatrix}$

Explanation:
Each of the products AB, BA is defined because A, B are 2×2-matrices, so each of AB, BA is a 2×2 matrix and the difference $AB - BA$ also is defined.

Now by the row-column rule for the product of matrices, the $(i,j)^{\text{th}}$ entry in AB is given by

$$(AB)_{ij} \; = \; a_{i1}b_{1j} + a_{2i}b_{2j} \, .$$

Thus

$$AB = \begin{bmatrix} 1 & 3 \\ 5 & 4 \end{bmatrix} \begin{bmatrix} -2 & 2 \\ 2 & -4 \end{bmatrix} = \begin{bmatrix} 4 & -10 \\ -2 & -6 \end{bmatrix} .$$

On the other hand, the $(i,j)^{\text{th}}$ entry in BA is given by

$$(BA)_{ij} \; = \; b_{i1}a_{1j} + b_{2i}a_{2j} \, .$$

Thus

$$BA = \begin{bmatrix} -2 & 2 \\ 2 & -4 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 5 & 4 \end{bmatrix} = \begin{bmatrix} 8 & 2 \\ -18 & -10 \end{bmatrix} .$$

Consequently,

$$\boxed{AB - BA \; = \; \begin{bmatrix} -4 & -12 \\ 16 & 4 \end{bmatrix}} .$$

Note that $AB \neq BA$ for these choices of matrices A, B. So the multiplication of matrices does not have all of the properties of multiplication of numbers.

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013 10.0 points

Compute the product of the matrices

$$A = \begin{bmatrix} 3 & -4 & 2 \\ 3 & 4 & -3 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & -2 \\ -5 & 3 \\ 3 & -4 \end{bmatrix} .$$

1. $\begin{bmatrix} -32 & -26 \\ -23 & -18 \end{bmatrix}$

2. $\begin{bmatrix} 32 & -26 \end{bmatrix}$

3. $\begin{bmatrix} 32 & 26 \\ 23 & 18 \end{bmatrix}$

4. $\begin{bmatrix} 32 & -26 \\ -23 & 18 \end{bmatrix}$ **correct**

5. $\begin{bmatrix} 32 & -26 & -23 \\ -23 & 18 & 2 \end{bmatrix}$

Explanation:
Since A is 2×3 and B is 3×2, the product AB is defined and is a 2×2-matrix. But by the row-column rule for computing products of matrices, the $(i,j)^{\text{th}}$-entry in the product of A and B is

$$(AB)_{ij} \; = \; a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} \, .$$

Thus

$$AB \; = \; \begin{bmatrix} 3 & -4 & 2 \\ 3 & 4 & -3 \end{bmatrix} \begin{bmatrix} 2 & -2 \\ -5 & 3 \\ 3 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 32 & -26 \\ -23 & 18 \end{bmatrix} .$$

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014 10.0 points

Determine the matrix product

$$\begin{bmatrix} 3 & 2 & -1 \\ 4 & 10 & 0 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

1. $\begin{bmatrix} 6 & 1 & 3 & -1 \\ 8 & 10 & 4 & 0 \\ 0 & 3 & 0 & 2 \end{bmatrix}$ **correct**

2. It is undetermined.

3. $\begin{bmatrix} 1 & 3 & -1 \\ 8 & 10 & 4 \\ 3 & 0 & 2 \end{bmatrix}$

4. $\begin{bmatrix} 6 & 8 & 0 \\ 1 & 10 & 3 \\ 3 & 4 & 0 \\ -1 & 0 & 2 \end{bmatrix}$

Explanation:

By the row-column definition of the product of matrices,

$$\begin{bmatrix} 3 & 2 & -1 \\ 4 & 10 & 0 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 6+0+0 & 0+2-1 & 3+0+0 & 0+0-1 \\ 8+0+0 & 0+10+0 & 4+0+0 & 0+0+0 \\ 0+0+0 & 0+1+2 & 0+0+0 & 0+0+2 \end{bmatrix}$$
$$= \begin{bmatrix} 6 & 1 & 3 & -1 \\ 8 & 10 & 4 & 0 \\ 0 & 3 & 0 & 2 \end{bmatrix}$$

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015 10.0 points

Compute the product AB of the matrices

$$A = \begin{bmatrix} -2 & 1 & 1 \\ 1 & 4 & 1 \\ 3 & -2 & -1 \end{bmatrix}, \; B = \begin{bmatrix} 2 & 3 & 2 \\ 5 & 2 & -2 \\ 3 & -4 & 2 \end{bmatrix}.$$

1. $AB = \begin{bmatrix} 3 & 25 & -7 \\ -8 & 5 & 9 \\ -4 & -4 & 5 \end{bmatrix}$

2. $AB = \begin{bmatrix} 4 & 25 & -7 \\ -8 & 7 & 9 \\ -4 & -4 & 8 \end{bmatrix}$

3. $AB = \begin{bmatrix} 4 & -8 & -4 \\ 25 & 7 & -4 \\ -7 & 9 & 8 \end{bmatrix}$ **correct**

4. $AB = \begin{bmatrix} 3 & -8 & -4 \\ 25 & 5 & -4 \\ -7 & 9 & 5 \end{bmatrix}$

Explanation:

By definition of the product of matrices,

$$AB = \begin{bmatrix} -2 & 1 & 1 \\ 1 & 4 & 1 \\ 3 & -2 & -1 \end{bmatrix} \begin{bmatrix} 2 & 3 & 2 \\ 5 & 2 & -2 \\ 3 & -4 & 2 \end{bmatrix}$$
$$= \begin{bmatrix} 4 & -8 & -4 \\ 25 & 7 & -4 \\ -7 & 9 & 8 \end{bmatrix}.$$