Linear Independence and Dependence

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Lecture 5

If $s = \{\vec{v})_1, \vec{v}_2, ..., \vec{v}_k\}$ is a set of vectors in R^n, then s is linearly independent if

$$c_1\vec{v}_1 + c_2\vec{v}_2 + ... + c_k\vec{v}_k = \vec{0} \tag{1}$$

can only be solved when $c_1 = c_2 = ... = c_k = 0$

The set s is linearly dependent if the equation $c_1\vec{v}_1 + c_2\vec{v}_2 + ... + c_k\vec{v}_k = \vec{0}$ can be solved where at least one of the c's is not zero.

Example #1

Let's pretend $\{\vec{v}_1, ..., \vec{v}_k\}$ is linearly dependent and $c_k \neq 0$

$\{\ _1, \vec{v}_2, ..., \vec{v}_k\} \rightarrow$ linearly independent $\vee$ linearly dependent
$\mathrm{c}_1\vec{v}_1 + c_2\vec{v}_2 + ... + c_k\vec{v}_k = \vec{0}$

$$\begin{bmatrix} \vec{v}_1 & \vec{v}_2 & ... & \vec{v}_k \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ ... \\ c_3 \end{bmatrix} = \ \rightarrow A\vec{x} = \vec{0}$$

$\{\vec{v}_1, \vec{v}_2, ..., \vec{v}_k\}$ is a linear independence set of vectors $\iff A\vec{x} = \vec{0}$ where $A = \begin{bmatrix} \vec{v}_1 & \vec{v}_2 & ... & \vec{v}_k \end{bmatrix}$ only has the trivial solution $\iff A$ has a pivot in every column

$\{\vec{v}_1, \vec{v}_2, ..., \vec{v}_k\}$ is a linear dependent set of vectors $\longleftrightarrow A\vec{x} = \vec{0}$ has a non-trivial solutions $\longleftrightarrow$ there is at least one free variable in $A\vec{x} = \vec{0} \longleftrightarrow A$ does not have pivot in every column.

$$\{\begin{bmatrix}1\\-3\\2\end{bmatrix},\begin{bmatrix}5\\1\\7\end{bmatrix},\begin{bmatrix}0\\0\\0\end{bmatrix}\}\ c_1\vec{v}_1+c_2\vec{v}_2+c_3\vec{v}_3=\vec{0}$$

If 0 is in your set $\rightarrow$ is linearly dependent

$$\{\vec{v}_1,\vec{v}_2\}$$
$$c_1\vec{v}_1+c_2\vec{2}=\vec{0}\text{if}c_2\neq 0,\vec{v}_2=\frac{-c_1}{c_2}\vec{v}_1$$

two vectors are linearly dependent $\longleftrightarrow$ they are parallel i.e one is a scalar multiple of the other.

$$\{\begin{bmatrix}2\\7\\-5\end{bmatrix},\begin{bmatrix}6\\21\\-15\end{bmatrix}\}\rightarrow\text{linear dependence}\tag{2}$$

$$\{\begin{bmatrix}2\\7\\-5\end{bmatrix},\begin{bmatrix}6\\21\\4\end{bmatrix}\}\rightarrow\text{linear independence}\tag{3}$$