This print-out should have 6 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

001 10.0 points

Given that $\{\mathbf{x}_1, \mathbf{x}_2\}$ is a basis for W, find an orthogonal basis $\{\mathbf{v}_1, \mathbf{v}_2\}$ for W when

$$\mathbf{x}_1 = \begin{bmatrix} -2 \\ 2 \\ 2 \end{bmatrix}, \quad \mathbf{x}_2 = \begin{bmatrix} 5 \\ -3 \\ -4 \end{bmatrix}.$$

1. $\mathbf{v}_1 = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 9 \\ -7 \\ -8 \end{bmatrix}$

2. $\mathbf{v}_1 = \begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

3. $\mathbf{v}_1 = \begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$

4. $\mathbf{v}_1 = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

5. $\mathbf{v}_1 = \begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}.$

002 10.0 points

Find a set of orthogonal vectors which spans the same space as the following vectors:

$$\begin{bmatrix} 1 \\ 2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 8 \\ 1 \\ 5 \end{bmatrix}, \begin{bmatrix} -6 \\ -3 \\ -7 \\ 5 \end{bmatrix}$$

1. $\begin{bmatrix} 1 \\ 2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -4 \\ 2 \\ -2 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ -2 \\ 4 \end{bmatrix}$

2. $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$

3. $\begin{bmatrix} 1 \\ 2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -4 \\ 2 \\ -2 \\ 2 \end{bmatrix}, \begin{bmatrix} -4 \\ 1 \\ -5 \\ 7 \end{bmatrix}$

4. $\begin{bmatrix} 1 \\ 2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} -3 \\ -1 \\ 0 \\ 5 \end{bmatrix}$

003 10.0 points

Find a set of orthogonal vectors which spans the same space as the following vectors:

$$\begin{bmatrix} 1 \\ 1 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ -1 \\ 9 \\ 5 \end{bmatrix}, \begin{bmatrix} 3 \\ -6 \\ -8 \\ 3 \end{bmatrix}$$

1. $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$

2.

3.
$$\begin{bmatrix}1\\1\\3\\1\end{bmatrix}, \begin{bmatrix}2\\-4\\0\\2\end{bmatrix}, \begin{bmatrix}2\\2\\-2\\2\end{bmatrix}$$

4.
$$\begin{bmatrix}1\\1\\3\\1\end{bmatrix}, \begin{bmatrix}2\\-4\\0\\2\end{bmatrix}, \begin{bmatrix}70\\-47\\5\\70\end{bmatrix}$$

4.
$$\begin{bmatrix}1\\1\\3\\1\end{bmatrix}, \begin{bmatrix}-1\\1\\1\\-3\end{bmatrix}, \begin{bmatrix}-1\\-2\\1\\0\end{bmatrix}$$

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004 10.0 points

Construct the normal equations for the least-squares solution of $A\mathbf{x} = \mathbf{b}$ when

$$A = \begin{bmatrix}-1 & -2\\-1 & -1\\1 & 1\end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix}7\\5\\6\end{bmatrix}.$$

1.
$$\begin{bmatrix}3 & 4\\4 & 6\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}-6\\-13\end{bmatrix}$$

2.
$$\begin{bmatrix}6 & -4\\-4 & 3\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}8\\-\frac{15}{2}\end{bmatrix}$$

3.
$$\begin{bmatrix}-1 & -2\\-1 & -1\\1 & 1\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}8\\-\frac{15}{2}\\6\end{bmatrix}$$

4.
$$\begin{bmatrix}-1 & -2\\-1 & -1\\1 & 1\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}7\\5\\6\end{bmatrix}$$

5.
$$\begin{bmatrix}3 & 4\\4 & 6\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}8\\-\frac{15}{2}\end{bmatrix}$$

6.
$$\begin{bmatrix}6 & -4\\-4 & 3\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}-6\\-13\end{bmatrix}$$

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005 10.0 points

Find the least-squares solution of $A\mathbf{x} = \mathbf{b}$ when

$$A = \begin{bmatrix}2 & -2\\-1 & 1\\-1 & 2\end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix}1\\4\\4\end{bmatrix}.$$

1. $\frac{1}{5}\begin{bmatrix}16\\18\end{bmatrix}$

2. $\frac{1}{5}\begin{bmatrix}19\\20\end{bmatrix}$

3. $\frac{1}{5}\begin{bmatrix}1\\-3\end{bmatrix}$

4. $\begin{bmatrix}1\\4\end{bmatrix}$

5. $\begin{bmatrix}-6\\10\end{bmatrix}$

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006 10.0 points

Find the least-squares solution of $A\mathbf{x} = \mathbf{b}$ when

$$A = \begin{bmatrix} -1 & -2 \\ 0 & 0 \\ 2 & 3 \\ 1 & 1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 1 \\ -7 \\ 4 \\ 3 \end{bmatrix}.$$

1. $\frac{1}{3}\begin{bmatrix} 18 \\ 16 \end{bmatrix}$

2. $\frac{1}{3}\begin{bmatrix} 23 \\ -12 \end{bmatrix}$

3. $\begin{bmatrix} 1 \\ -7 \end{bmatrix}$

4. $\begin{bmatrix} 10 \\ 13 \end{bmatrix}$

5. $\frac{1}{3}\begin{bmatrix} 19 \\ 22 \end{bmatrix}$