This print-out should have 11 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

001 10.0 points

Find a basis for the eigenspace of the matrix

$$A = \begin{bmatrix} 1 & 3 & 3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{bmatrix},$$

corresponding to the eigenvalue $\lambda = 1$.

1. $\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

2. $\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$

3. $\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

4. $\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

5. $\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

002 10.0 points

If the matrix

$$A = \begin{bmatrix} 1 & 4 \\ 1 & -2 \end{bmatrix}$$

is diagonalizable, *i.e.*, $A = PDP^{-1}$ with P invertible and D diagonal, which of the following is a choice for P?

1. $P = \begin{bmatrix} -1 & 1 \\ 1 & 4 \end{bmatrix}$

2. $P = \begin{bmatrix} -1 & 4 \\ 2 & 1 \end{bmatrix}$

3. $P = \begin{bmatrix} -1 & 4 \\ 1 & 1 \end{bmatrix}$

4. $P = \begin{bmatrix} 2 & 4 \\ -1 & 1 \end{bmatrix}$

5. A is not diagonalizable

003 10.0 points

If the matrix

$$A = \begin{bmatrix} 4 & 9 \\ -1 & -2 \end{bmatrix}$$

is diagonalizable, *i.e.*, $A = PDP^{-1}$ with P invertible and

$$D = \begin{bmatrix} d_0 & 0 \\ 0 & d_1 \end{bmatrix}, \quad d_0 \geq d_1,$$

diagonal, which of the following is a choice for P?

1. $P = \begin{bmatrix} -3 & 2 \\ -1 & 1 \end{bmatrix}$

2. $P = \begin{bmatrix} -3 & 1 \\ -1 & 2 \end{bmatrix}$

3. A is not diagonalizable

4. $P = \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$

5. $P = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$

004 10.0 points

When

$$A = \begin{bmatrix} 2 & 0 \\ 3 & -1 \end{bmatrix}$$

find matrices D and P in a diagonalization of A given that $\lambda_1 > \lambda_2$.

1. $D = \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}, \quad P = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}$

2. $D = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}, \quad P = \begin{bmatrix} -1 & -1 \\ -1 & 0 \end{bmatrix}$

3. $D = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}, \quad P = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}$

4. $D = \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}$, $P = \begin{bmatrix} -1 & -1 \\ -1 & 0 \end{bmatrix}$

5. $D = \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}$, $P = \begin{bmatrix} 1 & 3 \\ -1 & -1 \end{bmatrix}$

6. $D = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}$, $P = \begin{bmatrix} 1 & 3 \\ -1 & -1 \end{bmatrix}$

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005 10.0 points

Find a matrix P so that

$$P \begin{bmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{bmatrix} P^{-1}, \quad d_1 \geq d_2 \geq d_3 ,$$

is a diagonalization of the matrix

$$A = \begin{bmatrix} -11 & -15 & 0 \\ 10 & 14 & 0 \\ -6 & -6 & -3 \end{bmatrix} .$$

1. $P = \begin{bmatrix} 1 & 0 & 3 \\ -1 & 0 & -2 \\ 0 & 1 & -3 \end{bmatrix}$

2. $P = \begin{bmatrix} 0 & -2 & -1 \\ 0 & 3 & 1 \\ 1 & -3 & 0 \end{bmatrix}$

3. $P = \begin{bmatrix} 1 & 3 & 0 \\ 0 & -3 & 1 \\ -1 & -2 & 0 \end{bmatrix}$

4. $P = \begin{bmatrix} -1 & -2 & 0 \\ 0 & -3 & 1 \\ 1 & 3 & 0 \end{bmatrix}$

5. $P = \begin{bmatrix} 3 & 1 & 0 \\ -2 & -1 & 0 \\ -3 & 0 & 1 \end{bmatrix}$

6. $P = \begin{bmatrix} 1 & 3 & 0 \\ -1 & -2 & 0 \\ 0 & -3 & 1 \end{bmatrix}$

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006 10.0 points

For the following matrix, is it invertible? Is it diagonalizable?

$$\begin{bmatrix} 0 & -1 & -1 \\ -2 & 1 & 2 \\ 2 & -2 & -3 \end{bmatrix}$$

1. Both invertible and diagonalizable.

2. Invertible, but not diagonalizable.

3. Neither invertible nor diagonalizable.

4. Diagonalizable, but not invertible.

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007 10.0 points

For the following matrix, is it invertible? Is it diagonalizable?

$$\begin{bmatrix} -4 & 3 & 2 \\ -3 & 2 & 2 \\ -4 & 4 & 1 \end{bmatrix}$$

1. Neither invertible nor diagonalizable.

2. Both invertible and diagonalizable.

3. Diagonalizable, but not invertible.

4. Invertible, but not diagonalizable.

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008 10.0 points

Find a basis for R^2 under which the transformation

$$T(\mathbf{x}) = \begin{bmatrix} 4 & 3 \\ 2 & -1 \end{bmatrix} \mathbf{x}$$

is multiplication by a diagonal matrix.

1.

$$\begin{bmatrix} -3 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

2.

$$\begin{bmatrix} 5 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -2 \end{bmatrix}$$

3.

$$\begin{bmatrix} 3 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

4.

$$\begin{bmatrix} 4 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

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009 10.0 points

Find the vector $\mathbf{x}$ in $\mathbb{R}^2$ having coordinate vector

$$[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} 8 \\ -5 \end{bmatrix}$$

with respect to the basis

$$\mathcal{B} = \left\{ \begin{bmatrix} 4 \\ 5 \end{bmatrix}, \begin{bmatrix} 6 \\ 7 \end{bmatrix} \right\}$$

for $\mathbb{R}^2$.

1. no such $\mathbf{x}$ exists

2. $\mathbf{x} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$

3. $\mathbf{x} = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$

4. $\mathbf{x} = \begin{bmatrix} -5 \\ -2 \end{bmatrix}$

5. $\mathbf{x} = \begin{bmatrix} -2 \\ -5 \end{bmatrix}$

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010 (part 1 of 2) 10.0 points

Determine the change of coordinates matrix $P_{\mathcal{C}\leftarrow\mathcal{B}}$ from basis $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$ to $\mathcal{C} = \{\mathbf{c}_1, \mathbf{c}_2\}$ in $\mathbb{R}^2$ when

$$\mathbf{b}_1 = \begin{bmatrix} -6 \\ -1 \end{bmatrix}, \qquad \mathbf{b}_2 = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

and

$$\mathbf{c}_1 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}, \qquad \mathbf{c}_2 = \begin{bmatrix} 6 \\ -2 \end{bmatrix}.$$

1. $P_{\mathcal{C}\leftarrow\mathcal{B}} = \begin{bmatrix} 9 & -2 \\ -4 & 1 \end{bmatrix}$

2. $P_{\mathcal{C}\leftarrow\mathcal{B}} = \begin{bmatrix} 9 & -2 \\ 4 & -1 \end{bmatrix}$

3. $P_{\mathcal{C}\leftarrow\mathcal{B}} = \begin{bmatrix} -1 & 2 \\ 4 & -9 \end{bmatrix}$

4. $P_{\mathcal{C}\leftarrow\mathcal{B}} = \begin{bmatrix} 9 & 2 \\ -4 & -1 \end{bmatrix}$

5. $P_{\mathcal{C}\leftarrow\mathcal{B}} = \begin{bmatrix} 1 & 2 \\ 4 & 9 \end{bmatrix}$

6. $P_{\mathcal{C}\leftarrow\mathcal{B}} = \begin{bmatrix} 1 & -2 \\ -4 & 9 \end{bmatrix}$

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011 (part 2 of 2) 10.0 points

Determine the change of coordinates matrix $P_{\mathcal{B}\leftarrow\mathcal{C}}$ from basis $\mathcal{C} = \{\mathbf{c}_1, \mathbf{c}_2\}$ to $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$.

1. $P_{\mathcal{B}\leftarrow\mathcal{C}} = \begin{bmatrix} 1 & 2 \\ 4 & 9 \end{bmatrix}$

2. $P_{\mathcal{B}\leftarrow\mathcal{C}} = \begin{bmatrix} 9 & -2 \\ -4 & 1 \end{bmatrix}$

3. $P_{\mathcal{B}\leftarrow\mathcal{C}} = \begin{bmatrix} 9 & 2 \\ -4 & -1 \end{bmatrix}$

4. $P_{\mathcal{B}\leftarrow\mathcal{C}} = \begin{bmatrix} 1 & -2 \\ -4 & 9 \end{bmatrix}$

5. $P_{\mathcal{B}\leftarrow\mathcal{C}} = \begin{bmatrix} -1 & 2 \\ 4 & -9 \end{bmatrix}$

6. $P_{\mathcal{B}\leftarrow\mathcal{C}} = \begin{bmatrix} 9 & -2 \\ 4 & -1 \end{bmatrix}$