This print-out should have 9 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

001 10.0 points

Determine a so that the vector

$$\mathbf{u} = \begin{bmatrix} 1 \\ 7 \end{bmatrix}$$

is a linear combination $\mathbf{u} = a\mathbf{v} + b\mathbf{w}$ of vectors

$$\mathbf{v} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}, \qquad \mathbf{w} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}.$$

1. $a = -1$

2. $a = 1$

3. $a = -3$ **correct**

4. $a = -2$

5. $a = 3$

Explanation:

Since addition and scalar multiplication of vectors is carried out componentwise, we see that

$$\mathbf{u} = \begin{bmatrix} 1 \\ 7 \end{bmatrix} = a \begin{bmatrix} 1 \\ -3 \end{bmatrix} + b \begin{bmatrix} -2 \\ 1 \end{bmatrix} = \begin{bmatrix} a - 2b \\ -3a + b \end{bmatrix}.$$

Thus

$$a - 2b = 1, \qquad -3a + b = 7.$$

Consequently, solving for a gives

$$\boxed{a = -3}.$$

002 10.0 points

Given

$$\mathbf{v_1} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \quad \mathbf{v_2} = \begin{bmatrix} 2 \\ 4 \\ 2 \end{bmatrix}, \quad \mathbf{v_3} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix},$$

determine all values of λ for which

$$\mathbf{w} = \begin{bmatrix} -3 \\ 1 \\ \lambda \end{bmatrix}$$

is a vector in Span$\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$?

1. $\lambda = -4$

2. $\lambda = -2$

3. $\lambda = 4, -2$

4. $\lambda = 4, -4$

5. $\lambda = 4$ **correct**

6. $\lambda = -4, -2$

Explanation:

The vector $\mathbf{w}$ is in Span$\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$ if there exist weights x_1, x_2, x_3 such that

$$x_1\mathbf{v}_1 + x_2\mathbf{v}_2 + x_3\mathbf{v}_3 = \mathbf{w}.$$

Such weights exist when the rightmost column in the augmented matrix

$$[\mathbf{v}_1 \ \mathbf{v}_2 \ \mathbf{v}_3 \ \mathbf{w}] = \begin{bmatrix} 1 & 2 & 1 & -3 \\ 1 & 4 & 0 & 1 \\ 0 & 2 & -1 & \lambda \end{bmatrix}$$

is not a pivot column. But

$$\begin{bmatrix} 1 & 2 & 1 & -3 \\ 1 & 4 & 0 & 1 \\ 0 & 2 & -1 & \lambda \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 1 & -3 \\ 0 & 2 & -1 & 4 \\ 0 & 2 & -1 & \lambda \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & -3 \\ 0 & 2 & -1 & 4 \\ 0 & 0 & 0 & \lambda - 4 \end{bmatrix}$$

Thus the rightmost column is not a pivot column when $\lambda - 4 = 0$. Consequently, $\mathbf{w}$ lies in Span$\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$ when

$$\boxed{\lambda = 4}.$$

003 10.0 points

Under what conditions on b_1, b_2 does the equation

$$\begin{bmatrix} -3 & -2 \\ -12 & -8 \end{bmatrix} \mathbf{x} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

not have a solution in $\mathbb{R}^2$?

1. $-3b_2 + 2b_1 = 0$

2. $-3b_2 - 2b_1 = 0$

3. $b_2 - 4b_1 = 0$

4. $-3b_2 - 2b_1 \neq 0$

5. $b_2 - 4b_1 \neq 0$ **correct**

6. $b_2 + 4b_1 \neq 0$

Explanation:

A matrix equation $A\mathbf{x} = \mathbf{b}$ fails to have a solution, *i.e.*, is inconsistent, when the last column of the associated augmented matrix $[A\ \ \mathbf{b}]$ is a pivot column.

Now for the given equation,

$$[A\ \ \mathbf{b}] = \begin{bmatrix} -3 & -2 & b_1 \\ -12 & -8 & b_2 \end{bmatrix}$$

$R_2' \to R_2 - 4R_1$

$$\sim \begin{bmatrix} -3 & -2 & b_1 \\ 0 & 0 & b_2 - 4b_1 \end{bmatrix}.$$

Thus the equation fails to have a solution when

$$\boxed{b_2 - 4b_1 \neq 0}.$$

004 5.0 points

Any five vectors in $\mathbb{R}^6$ span $\mathbb{R}^6$.

True or False?

1. FALSE **correct**

2. TRUE

Explanation:

A set of five vectors in $\mathbb{R}^6$ cannot span $\mathbb{R}^6$ because the matrix A whose columns are these five vectors has six rows. To span $\mathbb{R}^6$ the matrix A has to have six pivot rows. However, A has only five columns, so A cannot have a pivot in every row.

Consequently, the statement is

$$\boxed{\text{FALSE}}.$$

005 5.0 points

If the equation $A\,\mathbf{x} = \mathbf{b}$ is consistent, then $\mathbf{b}$ is in the set spanned by the columns of A.

True or False?

1. TRUE **correct**

2. FALSE

Explanation:

If

$$A = [\,\mathbf{a}_1\ \mathbf{a}_2\ \ldots\ \mathbf{a}_n\,]$$

is an $m \times n$ matrix and $\mathbf{b}$ is in $\mathbb{R}^m$, then the matrix equation $A\,\mathbf{x} = \mathbf{b}$ has the same solution set as the vector equation

$$x_1\mathbf{a}_1 + x_2\mathbf{a}_2 + \ldots + x_n\mathbf{a}_n = \mathbf{b}\,.$$

If $A\,\mathbf{x} = \mathbf{b}$ is consistent, then there exist solutions $x_1, x_2, \ldots, x_n$ for this vector equation. But $\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_n$ are the columns of A.

Thus if $A\,\mathbf{x} = \mathbf{b}$ is consistent, then $\mathbf{b}$ belongs to the set spanned by the **columns** of A. Consequently, the statement is

$$\boxed{\text{TRUE}}.$$

006 5.0 points

For an $m \times n$ matrix A the equation $A\mathbf{x} = \mathbf{b}$ is inconsistent for some $\mathbf{b}$ in R^m if A has a pivot position in every row.

True or False?

1. TRUE

2. FALSE **correct**

Explanation:

If U is an echelon form of A and $\mathbf{b}$ is a vector in R^m, then the augmented matrix $[\,A\,|\,\mathbf{b}\,]$ can be reduced to $[\,U\,|\,\mathbf{d}\,]$ for some $\mathbf{d}$ in R^m.

$$A = [\,\mathbf{a}_1\ \mathbf{a}_2\ \ldots\ \mathbf{a}_n\,]$$

is an $m \times n$ matrix, and $\mathbf{b}$ is in R^m, then the matrix equation $A\mathbf{x} = \mathbf{b}$ has the same solution set as the vector equation

$$x_1\mathbf{a}_1 + x_2\mathbf{a}_2 + \ldots + x_n\mathbf{a}_n = \mathbf{b}\,.$$

a lime has colorBut if $A\mathbf{x} = \mathbf{b}$ is consistent, then there exist solutions $x_1, x_2, \ldots, x_n$ for this vector equation. Since $\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_n$ are the columns of A, $\mathbf{b}$ thus belongs to the set spanned by the inconsistent for some $\mathbf{b}$ in R^m if A has a pivot position in every rowof A when the system is consistent.

Consequently, the statement is

$$\boxed{\text{FALSE}}\,.$$

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007 10.0 points

Find the solution set of the following homogeneous system in parametric vector form.

$$\begin{aligned} 4x_1 + 3x_2 - 2x_3 &= 0 \\ -x_1 + x_2 - 3x_3 &= 0 \\ -2x_1 - 2x_3 &= 0 \end{aligned}$$

1.

$$x = \begin{bmatrix} 2 \\ -6 \\ 7 \end{bmatrix}$$

2.

$$x = s\begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$$ correct

3.

$$x = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

4.

$$x = s\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

Explanation:

If you set up the augmented system and reduce, you get the matrix

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

x_3 is the free variable here. Let $x_3 = s$, then that gives that $x_1 = -s$ and $x_2 = 2s$, so the parametric vector representation of the solution is

$$\mathbf{x} = s\begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}.$$

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008 10.0 points

Describe the solution set to the system $Ax = 0$ in parametric vector form, given that A is row equivalent to the matrix

$$\begin{bmatrix} 2 & -2 & 0 & 4 \\ 3 & -3 & -1 & 9 \\ -2 & 2 & -1 & -1 \end{bmatrix}$$

1.

$$x = s\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + t\begin{bmatrix} 2 \\ -3 \\ 0 \end{bmatrix}$$

2.

$$\mathbf{x} = s\begin{bmatrix}1\\-1\\0\\2\end{bmatrix} + t\begin{bmatrix}0\\0\\1\\-3\end{bmatrix}$$

3.

$$\mathbf{x} = s\begin{bmatrix}2\\3\\-2\end{bmatrix} + t\begin{bmatrix}0\\-1\\-1\end{bmatrix}$$

4.

$$\mathbf{x} = s\begin{bmatrix}1\\1\\0\\0\end{bmatrix} + t\begin{bmatrix}-2\\0\\3\\1\end{bmatrix}$$ correct

Explanation:

When fully reduced, this matrix becomes

$$\begin{bmatrix}1 & -1 & 0 & 2\\0 & 0 & 1 & -3\\0 & 0 & 0 & 0\end{bmatrix}$$

This means that x_2 and x_4 are free variables. So call $x_2 = s$ and $x_4 = t$, and the first two rows of the matrix gives the equations $x_1 = s - 2t$ and $x_3 = 3t$, or in vector form,

$$\mathbf{x} = s\begin{bmatrix}1\\1\\0\\0\end{bmatrix} + t\begin{bmatrix}-2\\0\\3\\1\end{bmatrix}$$

009 10.0 points

Describe the solution set to the system $A\mathbf{x} = 0$ in parametric vector form, given that A is row equivalent to the matrix

$$\begin{bmatrix}1 & 2 & 3 & 0 & 3 & 4\\0 & 0 & 1 & 0 & -2 & -5\\0 & 0 & 0 & 0 & 1 & 3\\0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}$$

1.

$$\mathbf{x} = r\begin{bmatrix}1\\0\\0\\0\end{bmatrix} + s\begin{bmatrix}3\\1\\0\\0\end{bmatrix} + t\begin{bmatrix}3\\-2\\1\\0\end{bmatrix}$$

2.

$$\mathbf{x} = r\begin{bmatrix}-2\\1\\0\\0\\0\\0\end{bmatrix} + s\begin{bmatrix}0\\0\\0\\1\\0\\0\end{bmatrix} + t\begin{bmatrix}8\\0\\-1\\0\\-3\\1\end{bmatrix}$$ correct

3.

$$\mathbf{x} = s\begin{bmatrix}-2\\1\\0\\0\\0\\0\end{bmatrix} + t\begin{bmatrix}8\\0\\-1\\0\\-3\\1\end{bmatrix}$$

4.

$$\mathbf{x} = r\begin{bmatrix}2\\0\\0\\0\end{bmatrix} + s\begin{bmatrix}0\\0\\0\\0\end{bmatrix} + t\begin{bmatrix}4\\-5\\3\\0\end{bmatrix}$$

Explanation:

When fully reduced, this matrix becomes

$$\begin{bmatrix} 1 & 2 & 0 & 0 & 0 & -8 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

This means that x_2, x_4, and x_6 are free variables. Don't let that column of zeros throw you off, it still counts as a free variable since there is no pivot in that column. So call $x_2 = r$, $x_4 = s$, $x_6 = t$. The nonzero rows of the matrix gives the equations $x_1 = -2r + 8t$, $x_3 = t$, and $x_5 = -3t$, or in vector form,

$$\mathbf{x} = r \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} 8 \\ 0 \\ -1 \\ 0 \\ -3 \\ 1 \end{bmatrix}$$