

Discrete Dynamical System

$$\vec{x}_{k+1} = A\vec{x}_k, \vec{x}_0 \text{ is known}$$

Differential System

$$\vec{x}'(t) = A\vec{x}(t), \vec{x}(0) = \vec{x}_0$$

$\lambda_1, \vec{v}_1; \lambda_2, \vec{v}_2$ (Eigenvalues & Eigenvectors)

$$\vec{x}_0 = c_1 \vec{v}_1 + c_2 \vec{v}_2$$

Solution Form:

$$\vec{x}_k = c_1 \lambda_1^k \vec{v}_1 + c_2 \lambda_2^k \vec{v}_2$$

$\{|\lambda_1|, |\lambda_2|\} < 1$ (an attractor)

$\{|\lambda_1|, |\lambda_2|\} > 1$ (a repeller)

$|\lambda_1| < 1, |\lambda_2| > 1$ (A saddle point)

$$\vec{x}(t) = e^{c_1 \lambda_1 t} \vec{v}_1 + c_2 e^{\lambda_2 t} \vec{v}_2$$

$\{\operatorname{Re} \lambda_1, \operatorname{Re} \lambda_2\} < 0$ (an attractor)

$\{\operatorname{Re} \lambda_1, \operatorname{Re} \lambda_2\} > 0$ (a repeller)

$\operatorname{Re} \lambda_1 < 0, \operatorname{Re} \lambda_2 > 0$ (a saddle point)