

Invertibility

$$A = \begin{bmatrix} 0 & -1 & -1 \\ -2 & 1 & 2 \\ 2 & -2 & -3 \end{bmatrix}$$

$$\rightarrow \det A = 0 \cdot \det \begin{bmatrix} 1 & 2 \\ -2 & -3 \end{bmatrix} - (-1) \det \begin{bmatrix} -2 & 2 \\ 2 & -3 \end{bmatrix} + (-1) \det \begin{bmatrix} -2 & 1 \\ 2 & -2 \end{bmatrix}$$

$$0 - (-2) + (-2)$$

$$2 + (-2) = 0 = \det(A) \therefore \text{not invertible}$$

Diagonalizability

$$A - \lambda I = \begin{bmatrix} -\lambda & -1 & -1 \\ -2 & 1-\lambda & 2 \\ 2 & -2 & -3-\lambda \end{bmatrix}$$

$$\det(A - \lambda I) = -\lambda \det \begin{bmatrix} 1-\lambda & 2 \\ -2 & -3-\lambda \end{bmatrix} - (-1) \det \begin{bmatrix} -2 & 2 \\ 2 & -3-\lambda \end{bmatrix} + (-1) \det \begin{bmatrix} -2 & 1-\lambda \\ 2 & -2 \end{bmatrix}$$

$$\begin{array}{r|rr} & 1-\lambda & \\ -3 & -3 & 3\lambda \\ -\lambda & -\lambda & \lambda^2 \end{array}$$

$$-\lambda (\lambda^2 + 2\lambda - 3) - (-1)(2\lambda + 2) + (-1)(2 - 2\lambda)$$

$$\cancel{6} + 2\lambda - \cancel{4}$$

$$2\lambda + 2$$