This print-out should have 5 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

001 10.0 points

For the matrix

$$A = \begin{bmatrix} 2 & 4 \\ -7 & -8 \end{bmatrix}$$

find a C matrix in the decomposition where $A = PCP^{-1}$.

1.

$$C = \begin{bmatrix} 3 & -\sqrt{3} \\ \sqrt{3} & 3 \end{bmatrix}$$

2.

$$C = \begin{bmatrix} 4 & 0 \\ -5 & \sqrt{3} \end{bmatrix}$$

3.

$$C = \begin{bmatrix} 3 & -3 \\ 3 & 3 \end{bmatrix}$$

4.

$$C = \begin{bmatrix} -3 & -\sqrt{3} \\ \sqrt{3} & -3 \end{bmatrix}$$

002 10.0 points

For the matrix

$$A = \begin{bmatrix} 5 & -3 \\ 7 & -1 \end{bmatrix}$$

Find one possible P matrix in the decomposition $A = PCP^{-1}$.

1.

$$P = \begin{bmatrix} 0 & 3 \\ 2\sqrt{3} & 2 \end{bmatrix}$$

2.

$$P = \begin{bmatrix} 3 & 3 \\ 3 - 2\sqrt{3}i & 3 + 2\sqrt{3}i \end{bmatrix}$$

3.

$$P = \begin{bmatrix} 3 & 0 \\ 3 & 2\sqrt{3} \end{bmatrix}$$

4.

$$P = \begin{bmatrix} 2 & -2\sqrt{3} \\ 2\sqrt{3} & 2 \end{bmatrix}$$

003 10.0 points

Solve $\mathrm{x}_{k+1} = A\mathrm{x}_k$ for $k = 0, 1, 2, \ldots$, where $\mathrm{x}_0 = [1, 1]^T$ and

$$A = \begin{bmatrix} 4.4 & 4 \\ -5 & -4.6 \end{bmatrix}$$

1.

$$\mathrm{x}_k = 9(0.4)^k \begin{bmatrix} 1 \\ -1 \end{bmatrix} + 2(-0.6)^k \begin{bmatrix} -4 \\ 5 \end{bmatrix}$$

~~**2.**~~

$$\mathrm{x}_k = (3.6)^k \begin{bmatrix} 1 \\ -1 \end{bmatrix} + (-1.2)^k \begin{bmatrix} -4 \\ 5 \end{bmatrix}$$

~~**3.**~~

$$\mathrm{x}_k = (0.4)^k \begin{bmatrix} 1 \\ 1 \end{bmatrix} + 0(-0.6)^k \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

~~**4.**~~

$$\mathrm{x}_k = (0.4)^k \begin{bmatrix} 1 \\ -1 \end{bmatrix} + (-0.6)^k \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

004 10.0 points

Solve the following system of ordinary differential equations. Classify the origin as an attractor, repeller, or saddle point.

$$\mathrm{x}'(t) = \begin{bmatrix} 5.8 & 4.4 \\ -5.5 & -4.1 \end{bmatrix} \mathrm{x}(t), \; \mathrm{x}(0) = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

1.

$\mathrm{x}(t) = 22e^{1.4t}\begin{bmatrix}1\\-1\end{bmatrix} + 5e^{0.3t}\begin{bmatrix}-4\\5\end{bmatrix}$. It is a repeller.

2.

~~$\mathrm{x}(t) = 22e^{1.4t}\begin{bmatrix}1\\-1\end{bmatrix} + 5e^{0.3t}\begin{bmatrix}-4\\5\end{bmatrix}$. It is a saddle point.~~

3.

~~$\mathrm{x}(t) = 2e^{1.4t}\begin{bmatrix}1\\1\end{bmatrix} + e^{0.3t}\begin{bmatrix}4\\5\end{bmatrix}$. It is a saddle point.~~

4.

~~$\mathrm{x}(t) = 2e^{1.4t}\begin{bmatrix}1\\1\end{bmatrix} + e^{0.3t}\begin{bmatrix}4\\5\end{bmatrix}$. It is a repeller.~~

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005 10.0 points

Solve the following system of ordinary differential equations. Classify the origin as an attractor, repeller, or saddle point.

$$\mathrm{x}'(t) = \begin{bmatrix}1 & -1.4\\0.7 & -1.1\end{bmatrix}\mathrm{x}(t),\ \mathrm{x}(0) = \begin{bmatrix}3\\4\end{bmatrix}$$

1.

~~$\mathrm{x}(t) = 5e^{0.4t}\begin{bmatrix}1\\1\end{bmatrix} - e^{0.3t}\begin{bmatrix}2\\1\end{bmatrix}$. It is an attractor.~~

2.

$\mathrm{x}(t) = 5e^{-0.4t}\begin{bmatrix}1\\1\end{bmatrix} - e^{0.3t}\begin{bmatrix}2\\1\end{bmatrix}$. It is a saddle point.

3.

~~$\mathrm{x}(t) = 11e^{0.4t}\begin{bmatrix}-1\\1\end{bmatrix} - 7e^{0.3t}\begin{bmatrix}-2\\1\end{bmatrix}$. It is an attractor.~~

4.

~~$\mathrm{x}(t) = 11e^{-0.4t}\begin{bmatrix}-1\\1\end{bmatrix} - 7e^{0.3t}\begin{bmatrix}-2\\1\end{bmatrix}$. It is a saddle point.~~

All the work is done in freeform.