10/05/2023

Last Time: Implicit Differentiation
Derivative of $\ln(x)$
inverse Trig
$f^{-1}(x)$

Today: Related Rates
Linearization

Future: HW's

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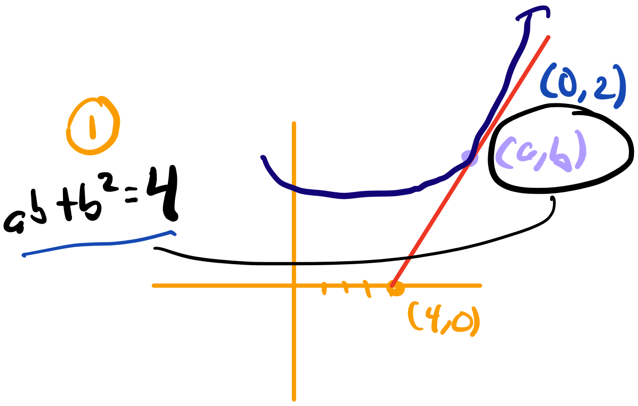
① There is one line tangent to $xy+y^2=4$ that passes thru the point $(4,0)$. Find the point of intersection and the slope.

② Let $f(x)=x^5+2x^3+3x$, find $(f^{-1})'(6)$



y' at (a,b) to equal slope btwn (a,b) + $(4,0)$.

$$y+xy'+2yy'=0 \Rightarrow y'=\frac{-y}{x+2y} \Rightarrow \frac{-b}{a+2b}$$

$$\frac{y_2-y_1}{x_2-x_1}=\frac{b-0}{a-4}=\frac{b}{a-4}$$

$$\Rightarrow \frac{-b}{a+2b}=\frac{b}{a-4} \Rightarrow -ab+4b=ab+2b^2 \Rightarrow 4b=2ab+2b^2$$

$$4b=2(ab+b^2)$$

$$4b=2\cdot 4=8$$

$\therefore$ $b=2$

$(a,b)=(0,2)$

$\therefore$ $a(2)+2^2=4$

$2a=0$

$a=0$

$$y'=\frac{-2}{0+2(2)}=-\tfrac{1}{2}$$

$$y-2=-\tfrac{1}{2}(x-0)$$

$y = -\frac{1}{2}x + 2$

Let $f(x) = x^5 + 2x^3 + 3x$, find $(f^{-1})'(6)$

① $(f^{-1})'(6) = \dfrac{1}{f'(f^{-1}(6))}$ $\qquad f^{-1}(6) = 1 \Leftrightarrow f(1) = 6$

$= \dfrac{1}{f'(1)} = \dfrac{1}{5(1)^4 + 6(1)^2 + 3} = \dfrac{1}{14}$

$y = x^5 + 2x^3 + 3x \Leftrightarrow \boxed{x = y^5 + 2y^3 + 3y}$, $y'(6)$ x

$\Leftrightarrow 1 = 5y^4 \cdot y' + 6y^2 y' + 3y'$

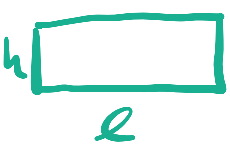
$x = 6,\ 6 = y^5 + 2y^3 + 3y$ $\qquad = y'(5y^4 + 6y^2 + 3)$

$y = 1$ $\qquad \therefore\ y' = \dfrac{1}{5y^4 + 6y^2 + 3} \overset{y=1}{\Rightarrow} \dfrac{1}{14}$

The length of a rectangle increases 2 ft/min

The height " " " decreases 3 ft/min.

At what rate is the area changing when $h = 30$ and $\ell = 10$?

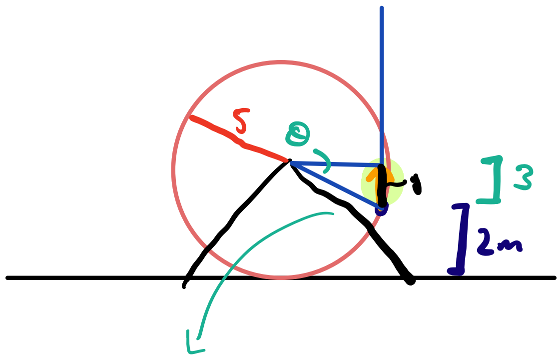


, $A = \ell \cdot h = \ell(t) \cdot h(t)$

$A' = \ell'(t) \cdot h(t) + \ell(t) \cdot h'(t) \Rightarrow A' = (2)(30) + (10)(-3)$

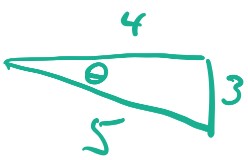
$= 60 - 30 = 30 \, ft^2/min$

A Ferris Wheel w/ radius 5m makes one revolution every 2 minutes. How fast is a person moving vertically when they are 2m off the ground?



$$\frac{d\theta}{dt} = \frac{1\,rev}{2\,min} = \frac{2\pi\,rad}{2\,min} = \pi\,\frac{rad}{min}$$

$$\sin(\theta) = \frac{y}{5} \qquad \tan(\theta) = \frac{y}{x}$$



$$\Rightarrow \cos(\theta)\,\theta' = \frac{1}{5}y'$$

$$\Rightarrow \cos(\theta)\cdot\pi\cdot 5 = y'$$

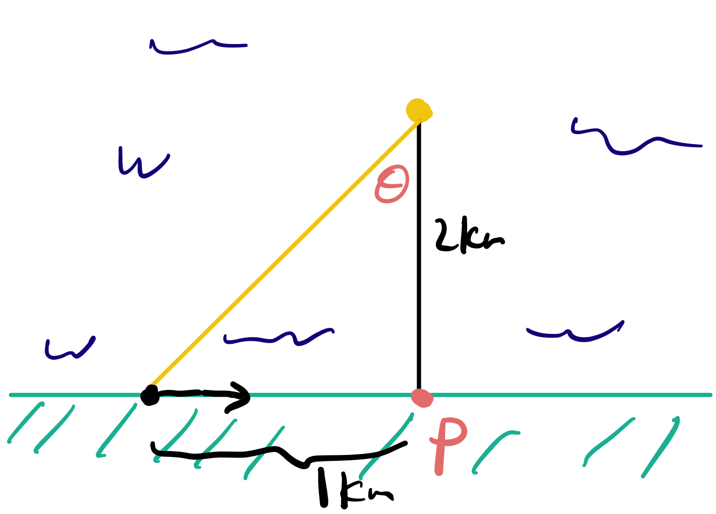
$\cos(\theta) = \frac{adj}{hyp}$

$= 4/5$

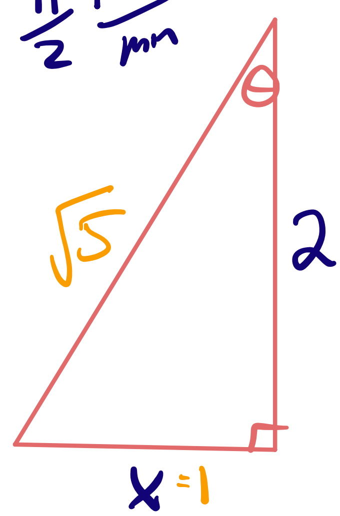
$$\Rightarrow \frac{4}{5}\cdot 5\cdot\pi = 4\pi\,m/min$$

A lighthouse is 2km off the coast of a perfectly straight shoreline. The lighthouse casts a beam of light onto the coast. The beam rotates once every 4 minutes. How fast is the beam moving along the coast when it is 1km from P

Aerial view:



$$\frac{d\theta}{dt} = \frac{2\pi}{4} = \frac{\pi}{2}\ \frac{\text{rad}}{\text{min}}$$



$$\boxed{\frac{dx}{dt}}$$

$$\tan(\theta) = \frac{\text{opp}}{\text{adj}} = \frac{x}{2}$$

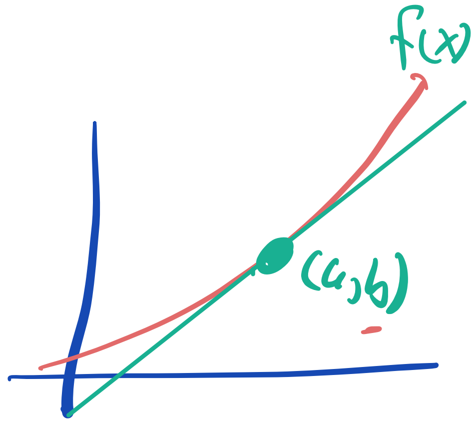
$$\cos(\theta) = \frac{2}{\sqrt{5}}$$

$$\left(\frac{1}{\cos(\theta)}\right)^2 = \frac{5}{4}$$

$$\sec^2(\theta) \cdot \theta' = \frac{x'}{2}$$

$$\left(\frac{1}{\cos\theta}\right)^2 \cdot \frac{\pi}{2} \cdot 2 = x' \Rightarrow x' = \frac{5\pi}{4}$$

The line tangent to $f(x)$ at $x = a$:



$$y - b = f'(a)(x-a)$$

$$y = b + f'(a)(x-a)$$

$$y = f(a) + f'(a)(x-a)$$

$$\boxed{L(x) = f(a) + f'(a)(x-a)},$$ The linearization of $f(x)$ at $x = a$.