§ 3.5, Implicit Differentiation, Inverse Trig Application.

If $y = \sin^{-1}(x)$, what is $y' = \frac{dy}{dx}$?

$\sin^{-1}(x) \neq \frac{1}{\sin(x)}$

$y = \sin^{-1}(x) \iff \sin(y) = x$

$[\sin(y)]' = [x]'$

$\cos(y) \cdot [y]' = 1$

$\cos(y) \cdot y' = 1$

$y' = \frac{1}{\cos(y)}$

$\sin^2(y) + \cos^2(y) = 1$

$\cos^2(y) = 1 - \sin^2(y)$

$\cos(y) = \sqrt{1 - \sin^2(y)}$

$\cos(y) = \sqrt{1 - x^2}$

$\boxed{\text{If } y = \sin^{-1}(x),\ y' = \frac{1}{\sqrt{1-x^2}}}$

$\boxed{\text{If } y = \tan^{-1}(x),\ y' = \frac{1}{1+x^2}}$

$$[x \cdot \sin^{-1}(x)]' = [x]' \sin^{-1}(x) + x\,[\sin^{-1}(x)]'$$

$$= \sin^{-1}(x) + x \cdot \frac{1}{\sqrt{1-x^2}} = \sin^{-1}(x) + \frac{x}{\sqrt{1-x^2}}$$