§5.2 - Properties of Integrals

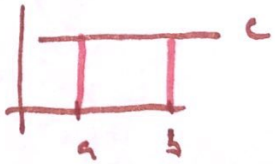
① $\int_a^a f(x)\,dx = 0$

② $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

③ $\int_a^b c\cdot f(x)\,dx = c\cdot\int_a^b f(x)\,dx$

④ $\int_a^b f(x) \pm g(x)\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$

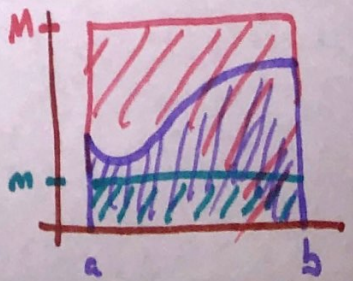
⑤ $\int_a^b c\,dx = (b-a)c$



⑥ $\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$

⑦ If $m \le f(x) \le M$ for $a \le x \le b$, then:

$$(b-a)m \le \int_a^b f(x)\,dx \le (b-a)M$$



Find $\int_0^2 x^2\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\,\Delta x$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{2i}{n}\right)^2 \cdot \left(\frac{2-0}{n}\right)$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{4i^2}{n^2} \cdot \frac{2}{n}$$

$$= \lim_{n\to\infty} \frac{8}{n^3} \sum_{i=1}^{n} i^2$$

$$= \lim_{n\to\infty} \frac{8}{n^3} \left(\frac{n(n+1)(2n+1)}{6}\right)$$

$$= \lim_{n\to\infty} \frac{8}{6n^3} \left(2n^3 + \cdots\right)$$

$$= \lim_{n\to\infty} \frac{4}{3} \left(\frac{2n^3 + \cdots}{n^3}\right)$$

$$= \frac{4}{3}(2) = \frac{8}{3} = 2.\overline{66}$$