

Question 69

Implicit Differentiation

(12, 3)

 $x_1, y_1$ 

$$\boxed{x^2 + 4y^2 = 36} \rightarrow 2x + 8y \cdot y' = 0$$

- 2x

- 2x

$$\cancel{8y \cdot y'} = -\frac{2x}{2}$$

$$\frac{8y}{8y} = 2$$

$$y' = \frac{-x}{4y}$$

$$\frac{-12}{4(3)} = -1$$

$$\cancel{\frac{y-3}{x-12} = \frac{4(3)-x}{4y}} \quad (\text{Ans})$$

$$\frac{y-3}{x-12} = \frac{-x}{4y}$$

$$-\frac{x}{x-12}$$

$$-x - x^2 + 12x$$

$$x^2 + 4y^2 = 36$$

$$-4y^2 \quad x^2 = 36 - 4y^2$$

$$-x^2 + 12x = 4y^2 - 12$$

$$\frac{3-3}{0-12} = \frac{0}{-12} = \frac{0}{a}$$

$$\frac{b-3}{a-12}$$

$$-(36 - 4y^2) \quad y = x^2 + 12x$$

$$4b^2 - 12b = -a^2 + 12a$$

$$-36 + 4y^2 + 12x = 4y^2 - 12$$

+ 12

$$x=0$$

$$y=3$$

$$y-3 = \frac{0-0}{x-0} (x-0)$$

$$\frac{y-3}{x-0} = 0$$

$$\frac{+3+3}{y-3}$$

$$\boxed{(0, 3)}$$

$$\left(\frac{24}{5}, -\frac{9}{5}\right)$$

$$-24 + 4y^2 + 12x = 4y^2$$

$$-4y^2$$

$$-4y^2$$

$$-24 + 12x = 0$$

# Textbook HW07 - 3.6

Question 85

$$[f(x)] = [x + e^x], \text{ find } f^{-1}'(1)$$

$$f(x) = 1 \quad f^{-1}(y) = \frac{1}{f'(f^{-1}(y))}$$

$$1 = x + e^x, \quad x = 0$$

$$f'(x) = 1 + e^x$$

$$f'(0) = 1 + e^0 = 2$$

$$f^{-1}'(1) = \frac{1}{f'(f^{-1}(1))} = \frac{1}{f'(0)} = \boxed{\frac{1}{2}}$$

$$f'(0) = 3(0)^2 + 3\cos(0) + 2\sin(0)$$

$$0 + 3(1) = 0$$

$$0 + 3$$

$$3 - 0$$

$$f'(0) = 3$$

$$[f(x)] = [x^3 + 3\sin(x) + 2\cos(x)] \text{ find } f''(2)$$

$$2 = x^3 + 3\sin(x) + 2\cos(x)$$

$$x = 0$$

$$0^3 + 3\sin(0) + 2\cos(0)$$

$$0 + 0 + 2(1) = 2 = f(0)$$

$$f'(x) = 3x^2 + 3\cos(x) - 2\sin(x)$$

$$\frac{1}{f'(0)} = \boxed{\frac{1}{3}}$$

## Chapter 3.9

Question 29

$$\frac{dV}{dt} = 30 \text{ ft}^3/\text{min}$$

$$h = 10 \text{ ft}, \quad d = h$$

$$\frac{dV}{dt} = \frac{1}{4}\pi h^2 \cdot \frac{dh}{dt}$$

$$30 = \frac{1}{4}\pi (10)^2 \cdot \frac{dh}{dt} \quad (1)$$

$$\frac{12\phi}{100\pi} = \frac{100\pi \cdot \frac{dh}{dt}}{100\pi}$$

$$\frac{12}{10\pi} = \frac{dh}{dt} \rightarrow \boxed{\frac{dh}{dt} = \frac{6}{5\pi} = 0.38197}$$

$$\theta = 1 \cdot \left(\frac{1}{\frac{3}{110}}\right)^2$$

$$\frac{80}{3\sqrt{240}}$$

Question 48

$$[\sin \theta]' = [\frac{y}{10}]' \quad \frac{d\theta}{dt} = \pi/\text{min} = \theta'$$



$$\cos \theta \cdot \theta' = \frac{1}{10} y' (40)$$

$$y = 8 \quad c = 10 \quad 10 \cos \theta \cdot \theta' = y' \quad 10 \left(\frac{8}{10}\right) \cdot \pi = y' = \boxed{8\pi}$$

$$\begin{aligned} & \text{Diagram: A right triangle OPL with vertex P at the bottom left. The vertical leg OP is labeled 1 km, the horizontal leg OL is labeled 3 km, and the hypotenuse PL is labeled L.} \\ & [\tan \theta]' = \frac{1}{3} \quad \frac{8\pi}{\frac{d\theta}{dt}} = \frac{d\theta}{dt} \\ & \sec^2 \theta \cdot \theta' = \frac{1}{3} (3) \end{aligned}$$

$$3 \sec^2 \theta \cdot \theta' =$$

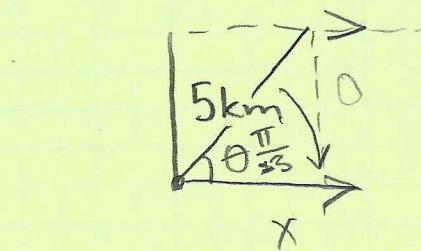
$$\left( \frac{1}{\cos \theta} \right)^2 \cdot (\pi) \cdot 3 = x' \quad \left( \frac{1}{\cos \theta} \right) \cdot 24\pi$$

$$\frac{240\pi}{9} = \boxed{\frac{80\pi}{3} = x'}$$

Andrea Morales

Oct 11

Question 47



$$\pi/6 \text{ / min} = \frac{d\theta}{dt}$$

$$[\tan(\theta)] = \left[ \frac{n}{x} \right]$$

$$5 = n$$

$$\sec^2 \theta \cdot \theta' = \frac{-h}{x^2} \cdot x'$$

$$\tan \theta = \frac{h}{x} (x)$$

$$\frac{x + \tan \theta}{\tan \theta} = \frac{h}{\tan \theta} = x = \frac{5}{\tan(\frac{\pi}{3})} = \frac{5}{\sqrt{3}}$$

$$\left( \frac{1}{\left( \frac{1}{2} \right)} \right)^2 = \left( \frac{1}{\cos(\frac{\pi}{3})} \right)^2$$

$$(2)^2 = 4$$

$$\frac{x^2 \sec^2 \theta \cdot \theta'}{-h} = \frac{-h \cdot x'}{-h}$$

$$\frac{x^2 \sec^2 \theta \cdot \theta'}{-h} = x'$$

$$\frac{\frac{25}{3}(1) \cdot \frac{\pi}{6}}{-5} = \frac{\frac{100\pi}{18}}{-5} = \frac{\frac{100\pi}{18} \cdot \frac{1}{-5}}{-5} = \frac{100\pi}{-90}$$

$$\boxed{\frac{10\pi}{9} = x' = -3.49 \text{ km/min}}$$

# Additional Questions

## HW 07

**Question #3**

$$\frac{dP}{dt} = k \cdot P(t) \rightarrow \frac{dP}{dt} = k \cdot y \rightarrow y = y(0) \cdot e^{kt}$$

$$k = 0.7 \\ P(0) \text{ or } P_I = 1000 \rightarrow P(t) = 1000 \cdot e^{0.7t}$$

$$[f(x)]' = [|\ln(0.5x + 0.5e^{-1})|]' \rightarrow [-(\ln(0.5x + 0.5e^{-1}))] =$$

$$\frac{-1}{0.5x + 0.5e^{-1}} \cdot [0.5x + 0.5e^{-1}] = \frac{-0.5}{0.5x + 0.5e^{-1}} \rightarrow \frac{-1}{x + e^{-1}} = \frac{dy}{dx}$$

$$x = e^{-1} \text{ value}$$

$$y = 1 = |\ln(0.5(e^{-1}) + 0.5e^{-1})| = |\ln(e^{-1})| = |-1| = 1$$

$$y - 1 = \frac{-1}{2e^{-1}}(x - e^{-1}) + 1 \\ y = \frac{-1}{2e^{-1}}(x - e^{-1}) + 1$$

$$0 = \frac{-1}{2e^{-1}}(x - e^{-1}) + 1 \\ -1 = \frac{-1}{2e^{-1}}(x - e^{-1})$$

$$\frac{-2e^{-1}}{-1} = \frac{-(x - e^{-1})}{-1}$$

$$2e^{-1} = x - e^{-1}$$

$$+e^{-1} +e^{-1}$$

$$3e^{-1} = x$$

$$x\text{-intercept} = (3e^{-1}, 0) \\ y\text{-intercept} = (0, 1.5)$$

$$y = \frac{-1}{2e^{-1}}(-e^{-1}) + 1$$

$$y = \frac{1}{2} + 1$$

$$y = 1.5$$

$$\text{When } x=0; y=1.5$$

**Question #2**

$$y^3 - 4xy = 8$$

$$3y^2 \cdot y' - 4y - 4xy' = 0 \\ +4y \\ +4y$$

$$3y^2 y' - 4xy' = 4y$$

$$\frac{y'(3y^2 - 4x)}{(3y^2 - 4x)} = \frac{4y}{(3y^2 - 4x)}$$

$$y' = \frac{4y}{3y^2 - 4x}$$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{b - 0}{a + 3} \rightarrow \frac{b}{a + 3} = \frac{4b}{3b^2 - 4a} \rightarrow \frac{4ab + 12b}{-4ab - 12b} = \frac{3b^3 - 4ab}{-4ab}$$

$$0 = 3b^3 - 8ab - 12b$$

$$b^3 - 4ab = 8$$

$$-b^3 + 4ab = -b^3$$

$$\frac{b^3 - 4ab}{-1 - 1} = \frac{8 - b^3}{A}$$

$$\frac{4ab}{A} = \frac{-8 - b^3}{A}$$

Next  
page

$$3(4ab + 8) - 8ab - 12b$$

$$0 = 12ab + 24 - 8ab - 12b$$

$$ab = \frac{-8 - b^3}{-4} \quad 12 \left( \frac{-8 - b^3}{A} \right) + 24 - 8 \left( \frac{-8 - b^3}{A} \right)$$

↓ Next page

$$b^3 - 4ab = 8 - 1, \quad 0 = 3b^3 - 8ab - 12b$$

$\frac{12}{5}$	$\frac{72}{30} \div 2$	$\frac{36}{15} \div 3$
----------------	------------------------	------------------------

$$\frac{-4ab}{-1} = \frac{8-b^3}{-1}$$

$$(2)4ab = -8 + b^3(2)$$

$$8ab = -16 + 2b^3$$

$$3b^3 + 16 - 2b^3 - 12b$$

$$0 = b^3 - 12b + 16$$

$$-16 = b(b^2 - 12)$$

$$\boxed{b = -2, a = 0}$$

$$\boxed{b = -4, a = \frac{9}{2}}$$

$$x = 0, y = 2$$

$$x = \frac{9}{2}, y = -4$$

$$y - 2 = \frac{4(2)}{3(-2)^2 - 4(0)} (x - 0)$$

$$\frac{8}{-12} \quad \frac{4}{6} \quad \frac{2}{3}(x - 0)$$

$$\boxed{y = \frac{2}{3}x + 2}$$

$$(2)^3 - 4a(2) = 8$$

$$8 - 4a(2)$$

$$8 - 8a = 8$$

$$-8 \quad -8$$

$$\frac{-8a}{-8} = 0$$

$$\boxed{(a=0)}$$

$$(-4)^3$$

$$-64 + 16a = 8$$

$$+64 \quad +64$$

$$16a = 72 \div 2$$

$$\frac{16a}{16} = \frac{72}{16} \div 2$$

$$\frac{16a}{16} = \frac{36}{8} \div 4$$

$$\frac{16a}{16} = \frac{9}{2}$$

$$a = \frac{9}{2}$$

$$y + 4 = \frac{4(-4)}{3(-4)^2 - 1(\frac{9}{2})} (x - \frac{9}{2})$$

$$\frac{-16}{48 - 18} = \frac{-16}{30} = \frac{-8}{15}$$

$$y + 4 = \frac{-8}{15} (x - \frac{9}{2})$$

$$y + 4 = -\frac{8}{15}x + \frac{12}{5} - \frac{40}{5}$$

$$\boxed{y = -\frac{8}{15}x - \frac{8}{5}}$$



# 2011 INTRAMURAL

