09/14/2023

Last Time: $f(x)$ as a Function

↳ Definition

↳ graphs

Today: Derivatives Rules

- Power Functions
- Polynomials
- e^x

Future: Quest Due M at 11:30

Gradescope Due M at 11:30

Exam I on Tuesday [Wed TA goes over the exam.]

- In-Class
- 75 min
- Bring something to write with. I'll bring everything else.
- Sit as far back as you can, no empty seats!
- 1.4, 1.5, 2.1 – 2.8 (not 2.4), 3.1
- Class Notes, Quest, Gradescope.

[≈ 12 Multiple Choice Questions

[≈ 4 True / False

[≈ 2 Graphing

105 Possible, graded out of 100.

Last Class:

$[c]' = 0$

$[mx]' = m$

$[f(x) \pm g(x)]' = f'(x) \pm g'(x)$

$[10x - 25 + \pi]' =$

$[10x]' - [25]' + [\pi]'$

$= 10 - 0 + 0$

$= 10$

$[x^{10}]'$ at $x = a$

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a} = \lim_{x \to a} \frac{x^{10} - a^{10}}{x - a} =$$

$$x^{10} - a^{10} = (x - a)(x^9 + x^8 a + x^7 a^2 + x^6 a^3 + \cdots + x a^8 + a^9)$$

$$= \lim_{x \to a} \frac{\cancel{(x-a)}(x^9 + x^8 a + \cdots + x a^8 + a^9)}{\cancel{x - a}} = x^9 + x^9 + x^9 + \cdots + x^9$$

$10x^9 = [x^{10}]'$

Power Rule: $[x^n]' = n x^{n-1}$

Ex: $[x^2 - x^7]' = 2x - 7x^6$

Ex: $[\sqrt{x}]' = [x^{1/2}]' = \frac{1}{2}x^{-1/2} = \frac{1}{2x^{1/2}} = \frac{1}{2\sqrt{x}}$

Ex: $\left[\frac{1}{x^4}\right]' =$ Not $\frac{1}{4x^3}$

$\hookrightarrow [x^{-4}]' = -4x^{-5} = -4/x^5$

Ex: $\left[x^3 + 1/x^2\right]' = 3x^2 + [x^{-2}]' = 3x^2 - 2x^{-3} = 3x^2 - \frac{2}{x^3} = \frac{3x^5 - 2}{x^3}$

Ex: $\left[\sqrt{x} + \frac{1}{\sqrt{x}}\right]' = [x^{1/2}]' + [x^{-1/2}]' = \frac{1}{2}x^{-1/2} - \frac{1}{2}x^{-3/2} = \frac{1}{2x^{1/2}} - \frac{1}{2x^{3/2}}$

$= \boxed{\frac{x-1}{2x^{3/2}}} = \frac{x-1}{2\cdot\sqrt{x^3}}$

$= \frac{x-1}{2x\sqrt{x}}$

Ex: $f(x) = x^3 + x^n$, $f'(1) = 12$, find n.

$f'(x) = 3x^2 + nx^{n-1}$

$f'(1) = 3\cdot(1)^2 + n(1)^{n-1} = 3 + n = 12 \Rightarrow n = 9$

Ex: $[10x^3 + 4x^2 - 8x + 1]' =$ $\quad [c\cdot f(x)]' = c\cdot f'(x)$

$10\cdot[x^3]' + 4[x^2]' - 8[x] + [1]'$

$= 30x^2 + 8x - 8$

Find the derivative of $f(x) = e^x$

Def^n of $f'(0)$ when $f(x) = e^x$

$\lim \frac{e^h - e^0}{h - 0} = 1$

$[e^x]' = \lim_{h\to 0} \frac{e^{x+h} - e^x}{h} = \lim_{h\to 0} \frac{e^x\cdot e^h - e^x}{h} = \lim_{h\to 0} e^x \cdot \left[\frac{e^h - 1}{h}\right] =$

[$f(x)=e^x$ is the function such that the slope at 0 is 1]

$f(x)=2^x$

$f(x)=3^x$

$$[e^x]' = e^x$$

$f(x) = 16\sqrt{x} + \frac{8}{x} - e^x$, find $f'(4)$

$= 16x^{1/2} + 8x^{-1} - e^x$, $f'(x) = 8x^{-1/2} - 8x^{-2} - e^x = \frac{8}{\sqrt{x}} - \frac{8}{x^2} - e^x$

$\Rightarrow f'(4) = \frac{8}{2} - \frac{8}{16} - e^4 = 4 - \frac{1}{2} - e^4 = \frac{7}{2} - e^4 = \frac{7-2e^4}{2}$

Find the y-intercept of the line tangent to

$f(x) = \frac{1}{2}x^2 - 3x + e^x$ at $x=0$.

$y - y_0 = m(x - x_0)$ $\quad x_0 = 0 \Rightarrow y_0 = f(0) = 0 - 0 + e^0 = 1$

$y - 1 = -2(x-0)$ $\quad m = x - 3 + e^x \Rightarrow 0 - 3 + e^0 = -2$

$y = -2x + 1$, 1

Does the line tangent to $y=x^2$ at $x=1$ pass thru $(3,10)$?

$y - y_0 = m(x - x_0)$ $\quad x_0 = 1$, $y_0 = 1^2 = 1$, $f'(x) = 2x$, $f'(1) = 2$

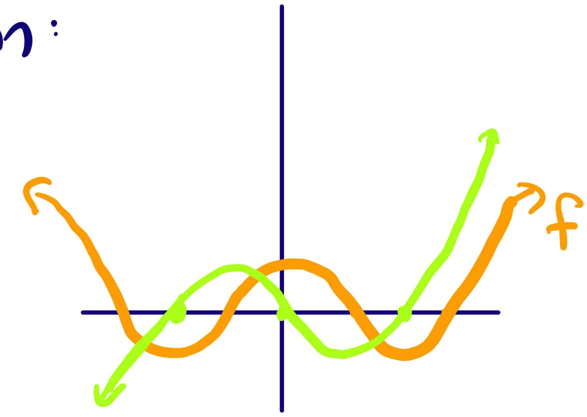
$y - 1 = 2(x-1) \rightarrow y - 1 = 2x - 2 \rightarrow y = 2x - 1$

Check $(3,10) \Rightarrow 10 = 2(3) - 1$ False

Not on the line.

$(5,9)$, $(3,5)$, $(\frac{11}{2}, 10)$

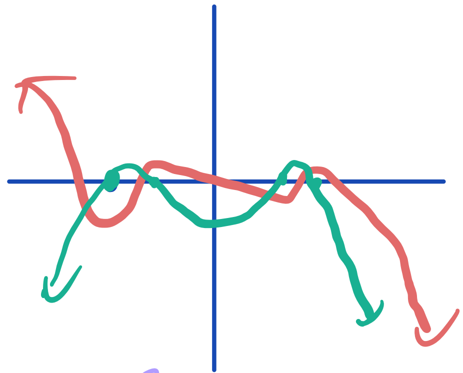
Even Function:



$$f(x) = f(-x)$$

$$x^2 = (-x)^2$$

Odd Function:



$$f(-x) = -f(x)$$

$$(-x)^3 = -x^3$$

$$[x^3]' = 3x^2$$

[even Function]' $\stackrel{?}{=}$ odd Function

[odd Function]' $\stackrel{?}{=}$ even Function

Let $f(x)$ be an even Function: $f(-x) = f(x)$

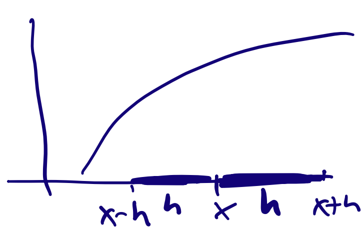
what do I want: $f'(x)$ is odd

$$f'(-x) = -f'(x)$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(-x) = \lim_{h \to 0} \frac{f(-x+h) - f(-x)}{h} = \lim_{h \to 0} \frac{f(x-h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{-[f(x) - f(x-h)]}{h} = -f'(x)$$



$$\frac{f(x) - f(x-h)}{x-(x-h)} = \frac{f(x) - f(x-h)}{h}$$