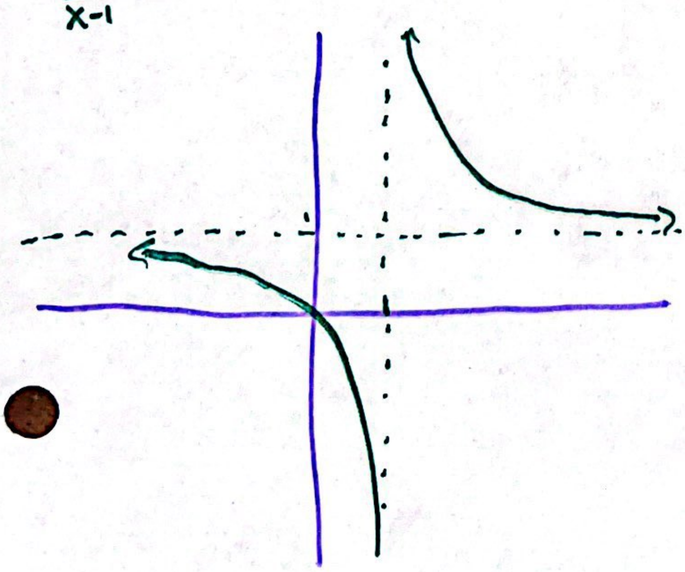
§2.6 - Limits at Infinity; Horizontal Asymptotes

- In this video, we will:
    - Describe what $\lim_{x \to \infty} f(x)$ means
    - Compute infinite limits not in I.F.
    - Compute infinite limits in I.F.

What does $\lim_{x \to \infty} f(x) = L$ or $\lim_{x \to -\infty} f(x) = L$ mean?

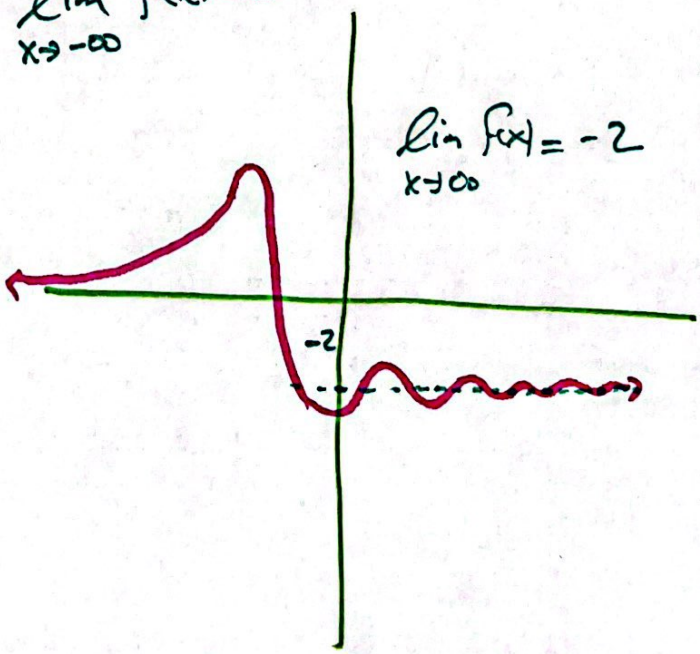
- We say $\lim_{x \to \infty} f(x) = L$ whenever values of $f(x)$ get arbitrarily close to L by requiring x to be arbitrarily large.

$\frac{x}{x-1}$



$\lim_{x \to -\infty} f(x) = 0$

$\lim_{x \to \infty} f(x) = -2$



If $\lim_{x\to\infty} f(x) = L$ or $\lim_{x\to -\infty} f(x) = L$, then the graph $y=L$ is called a horizontal asymptote of L.

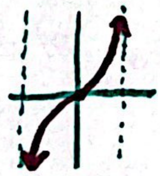
Computing infinite limits, case 1:

① $\lim_{x\to\infty} \sqrt{8x^2+x} \to \sqrt{8\cdot\infty^2+\infty} = \sqrt{8\infty+\infty} = \sqrt{\infty+\infty} = \sqrt{\infty}$

$= \boxed{\infty}$

② $\lim_{x\to\infty} \frac{5}{x^3} \to \frac{5}{\infty} = 0$

tan(x)



$\tan(x) = \infty$

$\tan(x) = -\infty$

$-\frac{\pi}{2}$ $\frac{\pi}{2}$

③ $\lim_{x\to\infty} \tan^{-1}(x) = \frac{\pi}{2}$

=

④ $\lim_{x\to -\infty} \tan^{-1}(x) = -\frac{\pi}{2}$

$$\lim_{x\to\infty} \frac{3x^2-x+3}{5x^2+10x+100} \to \frac{\infty}{\infty}, \text{ I.F.}$$

$$\lim_{x\to\infty} \frac{\cancel{x^2}\left(3-\frac{1}{x}+\frac{3}{x^2}\right)}{\cancel{x^2}\left(5+\frac{10}{x}+\frac{100}{x^2}\right)} = \lim_{x\to\infty} \frac{3-\cancelto{0}{\frac{1}{x}}+\cancelto{0}{\frac{3}{x^2}}}{5+\cancelto{0}{\frac{10}{x}}+\cancelto{0}{\frac{100}{x^2}}} = \frac{3}{5}$$

$$\lim_{x\to\infty} \frac{4x-1}{3x^2-4x} = \lim_{x\to\infty} \frac{x\left(4-\frac{1}{x}\right)}{x^2\left(3-\frac{4}{x}\right)} = \lim_{x\to\infty} \frac{1\left(4-\frac{1}{x}\right)}{x\left(3-\frac{4}{x}\right)} \to \frac{4}{3\cdot\infty}$$

$$= \boxed{0}$$

$$\lim_{x\to\infty} \frac{\sqrt{8x^2+x+1}}{2x+9} \to \lim_{x\to\infty} \frac{\sqrt{x^2\left(8+\frac{1}{x}+\frac{1}{x^2}\right)}}{x\left(2+\frac{9}{x}\right)}$$

$\oplus$

$$\lim_{x\to\infty} \frac{\sqrt{x^2}\sqrt{8+\frac{1}{x}+\frac{1}{x^2}}}{x\left(2+\frac{9}{x}\right)} = \lim_{x\to\infty} \frac{\cancel{x}\sqrt{8+\cancel{\frac{1}{x}}+\cancel{\frac{1}{x^2}}}}{\cancel{x}\left(2+\cancel{\frac{9}{x}}\right)} = \frac{\sqrt{8+0+0}}{2}$$

$$= \frac{\sqrt{8}}{2} = \frac{\sqrt{4\cdot 2}}{2} = \frac{\cancel{\sqrt{4}}\cdot\sqrt{2}}{\cancel{2}} = \boxed{\sqrt{2}}$$

What else: $\lim_{x\to\underline{-\infty}} f(x)$