§3.9 - Related Rates

Ex: You drop a rock into a giant pool of water. When you do this, a perfectly circular wave forms. The wave moves out from the center of the circle at 2 ft/s.

Qn: What is happening to the Area of the circle?

t	$r(t)$	$A(t) = \pi r^2$	$\frac{\Delta A}{\Delta t} \rightsquigarrow \frac{dA}{dt}$
0	0 $\Delta r = 2$	0 4π	—
1	2 $\Delta r = 2$	4π 12π	4π
2	4 $\Delta r = 2$	16π 20π	8π
3	6 $\Delta r = 2$	36π 28π	12π
4	8	64π	16π

Just because one variable changes at a constant rate, like $r(t)$, does not mean everything changes at a constant rate, like $A(t)$.

But, I bet they are related

Key is to remember our old constants are really functions, often functions of time.

$$Ex \Rightarrow A = \pi r^2 \Rightarrow A(t) = \pi \cdot (r(t))^2 \qquad \boxed{\frac{dA}{dt} = 4 \cdot \pi \cdot r(t)}$$

~~Real~~ Recall implicit differentiation

$$[A]' = [\pi r^2]' \Rightarrow \boxed{\frac{dA}{dt}} = \pi \cdot [r(t)^2]' = \pi \cdot 2 \cdot r(t) \cdot [r(t)]'$$

$$\frac{dA}{dt} = 2 \cdot \pi \cdot r(t) \cdot \frac{dr}{dt} = 2 \cdot \pi \cdot r(t) \cdot 2$$