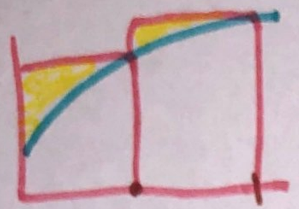
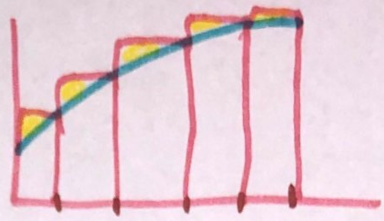
§ 5.2 - The Definite Integral

- Approximate the area under the curve from $x=0$ to $x=5$





Observation: Bigger $n \Rightarrow$ Better Approximation

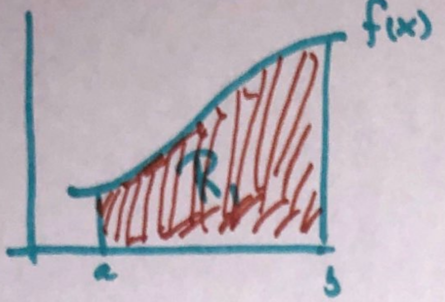
$\Rightarrow$ $n \to \infty \Rightarrow$ Exact Area

- Let $f(x)$ be a function on $[a,b]$, and $\Delta x = \frac{b-a}{n}$, and $[a,b]$ divided into n subintervals. Let x_i^* be an x-value in the i^{th} sub interval. Then the <u>definite integral</u> of $f(x)$ from a to b is:

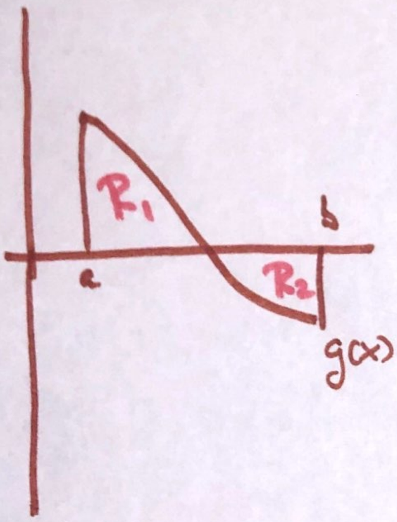
$$\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*) \overset{\frac{b-a}{n}}{\Delta x} = \int_a^b f(x)\,dx$$

- $$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = f'(x) = \frac{df}{dx}$$

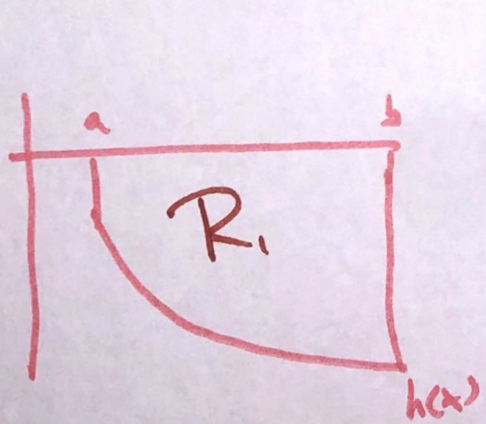
The definite integral is the area under a graph f(x), sometimes.

① 

$$\int_a^b f(x)\,dx = \text{Area}(R_1)$$

② 

$$\int_a^b g(x)\,dx = \text{Area}(R_1) - \text{Area}(R_2)$$

③ 

$$\int_a^b h(x)\,dx = -\text{Area}(R_1)$$

$$\int_0^{2\pi} \sin(x)\,dx = \text{Area}(R_1) - \text{Area}(R_2)$$
$$= 0$$