§ 3.5, Implicit Differentiation

In some sense, you have been lied to!

$$y = \text{Bunch of } x\text{'s}$$

Ex: $x^3 + 2xy + y^2 = 4$, Find $\frac{dy}{dx}$.

$[x]' = 1$ $\quad$ $[y]' = y'$ $\quad$ $[x \cdot y]' = [x]'y + x[y]' = y + xy'$ $\quad$ $[y^2]' = 2y[y]' = 2yy'$

$$[x^3 + 2xy + y^2]' = [4]'$$

$$= 3x^2 + [2x]'y + 2x[y]' + 2y[y]' = 0$$

$$3x^2 + 2y + 2xy' + 2yy' = 0$$

$$3x^2 + 2y = -2xy' - 2yy'$$

$$3x^2 + 2y = y'(-2x - 2y)$$

$$\therefore \boxed{y' = \frac{dy}{dx} = \frac{3x^2 + 2y}{-2x - 2y}}$$

$\sin(x+y) + y = 1$, find $\frac{dy}{dx}$.

$$[\sin(x+y) + y]' = [1]'$$

$$\cos(x+y)[x+y]' + y' = 0$$

$$\cos(x+y)(1 + y') + y' = 0$$

$$\cos(x+y) + y' \cdot \cos(x+y) + y' = 0$$

$$\cos(x+y) = -y'\cos(x+y) - y'$$

$$\cos(x+y) = y'(-\cos(x+y) - 1)$$

$$\therefore\ y' = \frac{\cos(x+y)}{-\cos(x+y)-1} = \frac{-\cos(x+y)}{\cos(x+y)+1}$$

Find the equation of the line tangent to $x^2+3xy-y^3=9$ at $(2,1)$

$y-y_0=m(x-x_0)$

$y-1=m(x-2)$, $m=\left.\frac{dy}{dx}\right|_{(2,1)}$

$[x^2+3xy-y^3]'=[9]'$

$=2x+3([x]'y+x[y]')-3y^2[y]'=0$

$2x+3y+3xy'-3y^2y'=0$

$\therefore\ 2x+3y=3y^2y'-3xy'$

$\therefore\ \frac{2x+3y}{3y^2-3x}=y'$

Plug in $(2,1) \Rightarrow \frac{2(2)+3(1)}{3\cdot(1)^2-3(2)}=\frac{7}{3-6}=-\frac{7}{3}$

$$\boxed{y-1=-\frac{7}{3}(x-2)}$$

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