HW 05 Solutions

3.2 #46) a) $h(x) = 3f(x) + 8g(x)$

$h'(x) = 3f'(x) + 8g'(x)$

$h'(4) = 3(6) + 8(-3) = 18 - 24 = -6$

b) $h(x) = f(x) \cdot g(x)$

$h'(x) = f'(x)g(x) + g'(x)f(x)$

$h'(4) = 6 \cdot 5 + -3 \cdot 2 = 30 - 6 = 24$

c) $h(x) = \frac{f(x)}{g(x)}$

$h'(x) = \frac{f'(x)g(x) - g'(x)f(x)}{[g(x)]^2}$

$h'(4) = \frac{6(5) - (-3)(2)}{(5)^2} = \frac{24}{25}$

d) $h(x) = \frac{g(x)}{f(x)+g(x)}$

$h'(x) = \frac{g'(x)[f(x)+g(x)] - g(x)(f'(x)+g'(x))}{(f(x)+g(x))^2}$

$h'(4) = \frac{(-3)(7) - 5(3)}{(7)^2} = \frac{-21-15}{49} = \frac{-36}{49}$

3.2, #51) $u(x) = f(x) \cdot g(x)$

$u'(x) = f'g + g'f$

$u'(1) = f'(1)g(1) + g'(1)f(1)$

$= (\frac{1}{3})(3) + (1)(2) = 1 + 2 = 3$

$v(x) = \frac{f'g - g'f}{g^2}$, $v'(4) = \frac{(\frac{2}{3})(2) - (1)(3)}{4} = \frac{\frac{2}{3} - 3}{4} = -\frac{7}{12}$

3.2 #52) $P'(x) = F'(x)G(x) + G'(x)F(x)$

$P'(2) = F'(2)G(2) + G'(2)F(2)$

$= 0(2) + (\frac{1}{2})(3) = 3/2$

$Q'(7) = \frac{F'(7)G(7) - G'(7)F(7)}{(G(7))^2} = \frac{\frac{1}{4}\cdot 1 - (-\frac{2}{3})(5)}{1}$

$= \frac{1}{4} + \frac{10}{3} = \frac{3}{12} + \frac{40}{12} = \boxed{\frac{43}{12}}$

3.2, #64) $F = f \cdot g$

$F' = f'g + g'f$

$F'' = f''g + f'g' + g'f' + g''f$

$= f''g + 2f'g' + g''f$

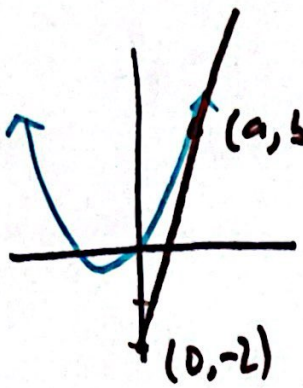
$F'''(x) = f'''g + f''g' + 2f''g' + 2f'g'' + g'''f + g''f'$

$= f'''g + 3f''g + 3f'g'' + f'''g$

$F''''(x) = f''''g + 4f'''g' + 6f''g'' + 4f'g''' + fg''''$

Additional Problem #1)

$y = \frac{1}{2}x^2 + 2x, \quad (0,-2)$



We want $f'(a)$ to equal the slope between (a,b) and $(0,-2)$.

$$f'(a) = a + 2$$

$$\frac{f(y_2) - f(y_1)}{x_2 - x_1} = \frac{\frac{1}{2}a^2 + 2a - (-2)}{a - 0}$$

$$\Rightarrow a + 2 = \frac{\frac{1}{2}a^2 + 2a + 2}{a}$$

$$\Rightarrow a^2 + 2a = \frac{1}{2}a^2 + 2a + 2$$

$$\frac{1}{2}a^2 = 2$$

$$a^2 = 4$$

$$a = \pm 2$$

$y + 2 = 4(x - 0)$

$y - 6 = 4(x - 2)$

$a = 2 \Rightarrow b = 6 \Rightarrow m = 4 \Rightarrow y - 6 = 4(x-2) \Rightarrow \boxed{y = 4x - 2}$

$a = -2 \Rightarrow b = -2 \Rightarrow m = 0 \Rightarrow y - (-2) = 0(x+2) \Rightarrow \boxed{y = 2}$