

### § 3.6, Derivatives of Logarithmic Functions

$f(x) = \ln(x)$ , what is  $f'(x)$ .

$$y = \ln(x) \iff x = e^y \iff [x]' = [e^y]'$$

$$\iff 1 = e^y \cdot y'$$

$$y' = \frac{1}{e^y} = \frac{1}{x}$$

$$\boxed{[\ln(x)]' = \frac{1}{x}}, \quad \text{also } [\log_a(x)]' = \frac{1}{x \cdot \ln(a)}$$

$$[\ln(x^2+x)]' = \frac{1}{x^2+x} [x^2+x]' = \frac{2x+1}{x^2+x}$$

$$\begin{aligned} [x^2 \cdot \ln(x)]' &= [x^2]' \ln(x) + x^2 [\ln(x)]' \\ &= 2x \ln(x) + x^2 \cdot \frac{1}{x} \\ &= 2x \ln(x) + x = x(2 \ln(x) + 1) \end{aligned}$$

$$[\ln(10x)]' = \frac{1}{10x} [10x]' = \frac{10}{10x} = \frac{1}{x}$$

$$[\ln(2x)]' = \frac{1}{2x} [2x]' = \frac{2}{2x} = \frac{1}{x}$$

$$[\ln(c \cdot x)]' = [\ln(c) + \ln(x)]' = 0 + \frac{1}{x} = \boxed{\frac{1}{x}}$$