§4.4, I.F. + L.R, part 3

I.F.: $0^0, \infty^0, 1^\infty$

Ex: $\lim\limits_{x\to\infty} \left(1+\frac{3}{x}\right)^x \to 1^\infty$, I.F.

$$L = \lim_{x\to\infty} \left(1+\frac{3}{x}\right)^x$$

$$\boxed{\ln(L)} = \lim_{x\to\infty} \overset{\infty\cdot 0}{x\cdot\ln\left(1+\tfrac{3}{x}\right)} = \lim_{x\to\infty} \frac{\ln(1+3/x)}{1/x} \to \frac{0}{0}$$

$$\overset{L.R.}{=} \lim_{x\to\infty} \frac{\frac{1}{1+3/x}\cdot \cancel{-}3\cancel{x^{-2}}}{\cancel{-x^{-2}}} = \lim_{x\to\infty} \frac{3}{1+3/x} = \boxed{3}$$

$$\ln(L) = 3 \Rightarrow \boxed{L = e^3}$$

$$\lim_{x\to\infty}\left(1+\frac{a}{x}\right)^{bx} = \cdots = e^{a\cdot b}$$

$$\lim_{x\to\infty}\left(1-\frac{2}{x}\right)^{4x} = e^{(-2)(4)} = e^{-8}$$

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x} = e^{1} = \boxed{e}$$

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