§ 5.4 - Net-Change Theorem

FTC II: $\int_a^b f(x)\,dx = F(b) - F(a)$, where $F'(x) = f(x)$

Net-Change Theorem: $\int_a^b f'(x)\,dx = f(b) - f(a)$

Suppose an oil rig is pumping oil out at a rate of $R(t) = \dfrac{10}{(1+t)^2}$ gal/hr

If there are 30 gal of ~~oil~~ oil at time = 0, how many will there be at $t = 2$?

Let $f(t)$ be the # of gal of oil at time t.

$f(0) = 0$. We want $f(2)$.

$f'(t) = R(t)$ $\therefore$ $\int_0^2 f'(t)\,dt = f(2) - f(0)$

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$$\boxed{\int_0^2 10(1+t)^{-2}\,dt} + 30 = f(2)$$

$$\frac{10(1+t)^{-1}}{-1}\Bigg|_0^2 = \frac{-10}{1+t}\Bigg|_0^2 = \frac{-10}{3} - \frac{-10}{1} = 10 - \frac{10}{3} = \boxed{\frac{20}{3}}$$

$$\therefore\ f(2) = 30 + \frac{20}{3} = \frac{60}{3} + \frac{20}{3} = \frac{80}{3} \text{ gal.}$$

Let $s(t)$ be the position of a particle

$$s'(t) = v(t)$$

$$\int_a^b v(t)\,dt = \int_a^b s'(t) = \underline{s(b) - s(a)} \quad \text{Displacement}$$

$$\int_a^b |v(t)|\,dt = \text{Distance}$$

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