This print-out should have 30 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

001 10.0 points

Find the x- and y-intercepts of the tangent line to the graph of

$$y = (3x + 13)^{1/4}$$

at the point (1, 2).

1. $x\text{-intercept} = -\frac{58}{3}$, $y\text{-intercept} = \frac{29}{16}$

2. $x\text{-intercept} = -20$, $y\text{-intercept} = \frac{31}{16}$

3. $x\text{-intercept} = -\frac{61}{4}$, $y\text{-intercept} = \frac{61}{32}$

4. $x\text{-intercept} = -\frac{13}{3}$, $y\text{-intercept} = \frac{13}{32}$

5. $x\text{-intercept} = -\frac{61}{3}$, $y\text{-intercept} = \frac{61}{32}$ **correct**

Explanation:
By the power rule,

$$\frac{dy}{dx} = \frac{3}{4}(3x + 13)^{1/4-1}.$$

Consequently, the slope of the tangent line at (1, 2) is given by

$$\left.\frac{dy}{dx}\right|_{x=1} = \frac{3}{8(4)},$$

so by the point slope formula, the equation of the tangent line at (1, 2) is

$$y - 2 = \frac{3}{32}(x - 1).$$

After simplification this becomes

$$32y = 3x + 61.$$

Thus

$$\boxed{x\text{-intercept} = -\frac{61}{3}, \quad y\text{-intercept} = \frac{61}{32}}.$$

002 10.0 points

Find the derivative of f when

$$f(x) = 4(\sin^{-1} x)^2.$$

1. $f'(x) = \frac{4\sin^{-1} x}{\sqrt{1-x^2}}$

2. $f'(x) = \frac{8\sin^{-1} x}{1+x^2}$

3. $f'(x) = \frac{4\cos^{-1} x}{1+x^2}$

4. $f'(x) = \frac{8\cos^{-1} x}{\sqrt{1-x^2}}$

5. $f'(x) = \frac{4\cos^{-1} x}{\sqrt{1-x^2}}$

6. $f'(x) = \frac{8\sin^{-1} x}{\sqrt{1-x^2}}$ **correct**

Explanation:
Since

$$\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}},$$

the Chain Rule ensures that

$$\boxed{f'(x) = \frac{8\sin^{-1} x}{\sqrt{1-x^2}}}.$$

003 10.0 points

Find the slope of the line tangent to the graph of

$$\ln(xy) - 2x = 0$$

at the point where $x = -1$.

1. $\text{slope} = \frac{3}{2}e^{-2}$

2. $\text{slope} = -\frac{3}{2}e^{2}$

3. $\text{slope} = \frac{3}{2}e^{2}$

4. slope $= -3e^{-2}$ **correct**

5. slope $= 3e^{-2}$

6. slope $= -3e^{2}$

Explanation:
Differentiating implicitly with respect to x we see that

$$\frac{1}{xy}\Big(y + x\frac{dy}{dx}\Big) - 2 \;=\; 0\,,$$

in which case

$$\frac{dy}{dx} \;=\; -\frac{y(1-2x)}{x} \;=\; -\frac{e^{2x}(1-2x)}{x^2}$$

because, by exponentiation,

$$y \;=\; \frac{e^{2x}}{x}\,.$$

Consequently, at $x = -1$,

$$\boxed{\text{slope} \;=\; \frac{dy}{dx}\Big|_{x=-1} \;=\; -3e^{-2}}\,.$$

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004 10.0 points

Determine $g'(x)$ when

$$g(x) \;=\; \frac{4 + xf(x)}{\sqrt{x}}\,,$$

and f is a differentiable function.

1. $g'(x) \;=\; \dfrac{2xf(x) + x^2f'(x) - 4}{x\sqrt{x}}$

2. $g'(x) \;=\; \dfrac{xf(x) + 2x^2f'(x) + 4}{\sqrt{x}}$

3. $g'(x) \;=\; \dfrac{xf(x) - x^2f'(x) + 4}{x\sqrt{x}}$

4. $g'(x) \;=\; \dfrac{xf(x) + 2x^2f'(x) - 4}{2x\sqrt{x}}$ **correct**

5. $g'(x) \;=\; \dfrac{xf(x) - 2x^2f'(x) + 4}{2x\sqrt{x}}$

6. $g'(x) \;=\; \dfrac{2xf(x) + x^2f'(x) - 4}{\sqrt{x}}$

Explanation:
By the Quotient and Power Rules

$$g'(x) \;=\; \frac{\sqrt{x}(f(x) + xf'(x)) - \dfrac{4 + xf(x)}{2\sqrt{x}}}{(\sqrt{x})^2}\,.$$

But after bringing the numerator to a common denominator and simplifying, the right hand side becomes

$$\frac{2x(f(x) + xf'(x)) - (4 + xf(x))}{2x\sqrt{x}}\,.$$

Consequently,

$$\boxed{g'(x) \;=\; \frac{xf(x) + 2x^2f'(x) - 4}{2x\sqrt{x}}}\,.$$

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005 10.0 points

Find the derivative of g when

$$g(x) \;=\; x^4\cos(x)\,.$$

1. $g'(x) \;=\; x^3\,(4\cos(x) + x\sin(x))$

2. $g'(x) \;=\; x^3\,(4\sin(x) - x\cos(x))$

3. $g'(x) \;=\; x^3\,(4\cos(x) - x\sin(x))$ **correct**

4. $g'(x) \;=\; x^3\,(4\sin(x) + x\cos(x))$

5. $g'(x) \;=\; x^4\,(3\sin(x) - \cos(x))$

6. $g'(x) \;=\; x^4\,(3\cos(x) - \sin(x))$

Explanation:
By the Product rule,

$$g'(x) \;=\; x^4\,(-\sin(x)) + (\cos(x))\cdot 4x^3\,.$$

Consequently,

$$\boxed{g'(x) \;=\; x^3\,(4\cos(x) - x\sin(x))}\,.$$

006 10.0 points

Values of m and b can be chosen so that the function f defined by

$$f(x) = \begin{cases} 3x^2+8, & x \le 2\,, \\ mx+b, & x > 2\,, \end{cases}$$

is differentiable for all values of x.

What is the value of b?

1. $b = -6$

2. $b = -2$

3. $b = -7$

4. $b = -4$ **correct**

5. $b = -8$

Explanation:

Since a polynomial is differentiable everywhere, the function f will be differentiable for all $x \neq 2$ without any restrictions on m or b. So we have to concentrate on $x = 2$.

Now if f is differentiable at $x = 2$, it must be continuous at $x = 2$, so

$$\lim_{x \to 2+} f(x) = \lim_{x \to 2-} f(x) = f(2)$$

i.e., $2m + b = 20$.

But if f is differentiable at $x = 2$, then the left and right hand derivatives of f must be equal at $x = 2$; so

$$\lim_{h \to 0-} \frac{f(2+h)-f(2)}{h} = 6\,x\Big|_{x=2}$$

$$= \lim_{h \to 0+} \frac{f(2+h)-f(2)}{h} = m\,.$$

Thus $m = 12$. Consequently,

$$\boxed{b = 20 - 24 = -4}.$$

007 10.0 points

Determine the value of the third derivative of f at $x = 1$ when

$$f(x) = 3\ln(3x+2),$$

1. $f'''(x) = \dfrac{162}{125}$ **correct**

2. $f'''(x) = \dfrac{486}{125}$

3. $f'''(x) = -\dfrac{162}{125}$

4. $f'''(x) = -\dfrac{81}{125}$

5. $f'''(x) = \dfrac{81}{125}$

Explanation:

After successive applications of the Chain Rule

$$f'(x) = \frac{9}{3x+2}, \quad f''(x) = -\frac{27}{(3x+2)^2},$$

and

$$f'''(x) = \frac{162}{(3x+2)^3}.$$

The value of f''' at $x = 1$ is thus given by

$$\boxed{f'''(1) = \frac{162}{125}.}$$

008 10.0 points

Find the derivative of

$$f(t) = \frac{3\ln t}{2+\ln t}\,.$$

1. $f'(t) = \dfrac{6}{t(2+\ln t)^2}$ **correct**

2. $f'(t) = \dfrac{6\ln t}{(2+\ln t)^2}$

3. $f'(t) = \dfrac{3}{t(2+\ln t)^2}$

4. $f'(t) = \dfrac{3\ln t}{2+\ln t}$

5. $f'(t) = \dfrac{6\ln t}{2+\ln t}$

6. $f'(t) = \dfrac{3}{t(2+\ln t)}$

Explanation:
By the Quotient Rule,

$$f'(t) = \frac{3(2+\ln t)(1/t) - (3\ln t)(1/t)}{(2+\ln t)^2}.$$

Consequently,

$$\boxed{f'(t) = \frac{6}{t(2+\ln t)^2}}.$$

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009 10.0 points

Find the value of $F'(7)$ when

$$F(x) = \frac{f(x)}{f(x)-g(x)}$$

and

$$f(7) = 3, \qquad f'(7) = 4,$$
$$g(7) = 2, \qquad g'(7) = 5.$$

1. $F'(7) = -7$

2. $F'(7) = 7$ **correct**

3. $F'(7) = -23$

4. $F'(7) = 23$

5. $F'(7) = 22$

Explanation:
By the Quotient Rule,

$$F'(x)$$
$$= \frac{f'(x)\,(f(x)-g(x)) - f(x)\,(f'(x)-g'(x))}{(f(x)-g(x))^2}$$
$$= \frac{f(x)\,g'(x) - f'(x)\,g(x)}{(f(x)-g(x))^2},$$

Consequently,

$$\boxed{F'(7) = 7}.$$

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010 10.0 points

Determine the derivative of

$$f(x) = \frac{x+3}{\sqrt{x-2}}.$$

1. $f'(x) = \dfrac{x-1}{2(x-2)^{1/2}}$

2. $f'(x) = \dfrac{x-7}{2(x-2)^{3/2}}$ **correct**

3. $f'(x) = \dfrac{x-1}{(x-2)^{1/2}}$

4. $f'(x) = \dfrac{x-7}{2(x-2)^{1/2}}$

5. $f'(x) = \dfrac{x-7}{(x-2)^{3/2}}$

6. $f'(x) = \dfrac{x-1}{(x-2)^{3/2}}$

Explanation:
By the Quotient and Chain Rules,

$$f'(x) = \frac{\sqrt{x-2} - \dfrac{x+3}{2\sqrt{x-2}}}{x-2}$$
$$= \frac{2(x-2)-(x+3)}{2(x-2)^{3/2}}.$$

Consequently,

$$\boxed{f'(x) = \frac{x-7}{2(x-2)^{3/2}}}.$$

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keywords: Quotient Rule, Chain Rule, Power Rule, square root function,

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011 10.0 points

Find the derivative of

$$f(x) = 4x^{\frac{1}{4}} - 3x^{-\frac{1}{4}} + 3.$$

1. $f'(x) = \dfrac{4x^{\frac{1}{2}} + 3}{3x^{\frac{5}{4}}}$

2. $f'(x) = \dfrac{4x^{\frac{1}{4}} + 3}{4x^{\frac{3}{4}}}$

3. $f'(x) = \dfrac{4x^{\frac{1}{2}} + 3}{4x^{\frac{5}{4}}}$ **correct**

4. $f'(x) = \dfrac{4x^{\frac{1}{2}} - 3}{4x^{\frac{3}{4}}}$

5. $f'(x) = \dfrac{4x^{\frac{1}{2}} - 3}{4x^{\frac{5}{4}}}$

Explanation:
Since

$$\frac{d}{dx}(x^r) = rx^{r-1},$$

we see that

$$f'(x) = \frac{1}{4}\left(\frac{4}{x^{\frac{3}{4}}} + \frac{3}{x^{\frac{5}{4}}}\right).$$

Consequently,

$$\boxed{f'(x) = \frac{4x^{\frac{1}{2}} + 3}{4x^{\frac{5}{4}}}}.$$

012 10.0 points

Find the derivative of f when

$$f(x) = x^{\frac{3}{2}} + 2x^{-\frac{5}{2}} - \frac{1}{x}.$$

1. $f'(x) = \dfrac{3x^{\frac{5}{2}} - 10x^{-\frac{3}{2}} + 2}{2\,x^2}$ **correct**

2. $f'(x) = \dfrac{3x^{\frac{5}{2}} - 10x^{-\frac{3}{2}} - 1}{x^2}$

3. $f'(x) = \dfrac{3x^{\frac{3}{2}} - 6x^{-\frac{3}{2}} - 2}{2\,x^2}$

4. $f'(x) = \dfrac{x^{\frac{5}{2}} + 10x^{-\frac{5}{2}} + 2}{2\,x^2}$

5. $f'(x) = \dfrac{x^{\frac{3}{2}} - 6x^{-\frac{5}{2}} + 1}{2\,x^2}$

Explanation:
Since

$$\frac{d}{dx}x^r = rx^{r-1}$$

holds for all real numbers r, we see that

$$f'(x) = \frac{3}{2}x^{\frac{1}{2}} - 5x^{-\frac{7}{2}} + \frac{1}{x^2}.$$

To simplify this expression we bring the right hand side to a common denominator so that

$$\boxed{f'(x) = \frac{3x^{\frac{5}{2}} - 10x^{-\frac{3}{2}} + 2}{2\,x^2}}.$$

013 10.0 points

Use linear approximation with $a = 4$ to estimate the number $\sqrt{4.5}$ as a fraction.

1. $\sqrt{4.5} \approx 2\dfrac{1}{10}$

2. $\sqrt{4.5} \approx 2\dfrac{1}{40}$

3. $\sqrt{4.5} \approx 2\dfrac{1}{8}$ **correct**

4. $\sqrt{4.5} \approx 2\dfrac{1}{20}$

5. $\sqrt{4.5} \approx 2\dfrac{3}{40}$

Explanation:
For a general function f, its linear approximation at $x = a$ is defined by

$$L(x) = f(a) + f'(a)(x - a)$$

and for values of x near a

$$f(x) \approx L(x) = f(a) + f'(a)(x - a)$$

provides a reasonable approximation for $f(x)$.

Now set

$$f(x) = \sqrt{x}, \qquad f'(x) = \frac{1}{2\sqrt{x}}.$$

Then, if we can calculate $\sqrt{a}$ easily, the linear approximation

$$\sqrt{a+h} \approx \sqrt{a} + \frac{h}{2\sqrt{a}}$$

provides a very simple method via calculus for computing a good estimate of the value of $\sqrt{a+h}$ for small values of h.

In the given example we can thus set

$$a = 4, \qquad h = \frac{5}{10}.$$

For then

$$\boxed{\sqrt{4.5} \approx 2\frac{1}{8}}.$$

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014 10.0 points

Find $f'(x)$ when

$$f(x) = \sqrt{x^2+6x}.$$

1. $f'(x) = \dfrac{2(x+3)}{\sqrt{x^2+6x}}$
2. $f'(x) = \dfrac{x+3}{2\sqrt{x^2+6x}}$
3. $f'(x) = (x+3)\sqrt{x^2+6x}$
4. $f'(x) = \dfrac{1}{2}(x+3)\sqrt{x^2+6x}$
5. $f'(x) = 2(x+3)\sqrt{x^2+6x}$
6. $f'(x) = \dfrac{x+3}{\sqrt{x^2+6x}}$ **correct**

Explanation:
By the Chain Rule,

$$f'(x) = \frac{1}{2\sqrt{x^2+6x}}(2x+6).$$

Consequently,

$$\boxed{f'(x) = \frac{x+3}{\sqrt{x^2+6x}}}.$$

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015 10.0 points

Find the derivative of f when

$$f(x) = 5\tan^{-1}\left(e^{-x}\right) + 6e^{x}.$$

1. $f'(x) = \dfrac{e^{-x}+6e^{x}}{\sqrt{1-e^{-2x}}}$
2. $f'(x) = \dfrac{e^{x}-6e^{-x}}{\sqrt{1-e^{-2x}}}$
3. $f'(x) = \dfrac{6e^{-x}+e^{x}}{1+e^{-2x}}$
4. $f'(x) = \dfrac{6e^{x}+e^{-x}}{1+e^{-2x}}$ **correct**
5. $f'(x) = \dfrac{6e^{x}-5e^{-x}}{1+e^{2x}}$
6. $f'(x) = \dfrac{e^{-x}+6e^{x}}{\sqrt{1-e^{2x}}}$

Explanation:
By the Chain Rule,

$$f'(x) = \frac{-5e^{-x}}{1+e^{-2x}} + 6e^{x}$$

since

$$\frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2}, \qquad \left(e^{-x}\right)^2 = e^{-2x}.$$

The expression for f' can now be simplified by bringing the right hand side to a common denominator. For then

$$f'(x) = \frac{-5e^{-x}+6e^{x}(1+e^{-2x})}{1+e^{-2x}} = \frac{6e^{x}+6e^{-x}-5e^{-x}}{1+e^{-2x}}.$$

Consequently,

$$\boxed{f'(x) = \frac{6e^{x}+e^{-x}}{1+e^{-2x}}}.$$

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016 10.0 points

There is one point in the first quadrant at which the tangent line to the graph of

$$y = 5 + 2x + \frac{3}{2}x^2 - \frac{2}{3}x^3$$

is horizontal. Find the y-coordinate of this point.

1. $y = \dfrac{38}{3}$

2. $y = \dfrac{29}{3}$ **correct**

3. $y = \dfrac{35}{3}$

4. $y = \dfrac{26}{3}$

5. $y = \dfrac{32}{3}$

Explanation:

The tangent line to the graph will be horizontal when

$$\frac{dy}{dx} = 2 + 3x - 2x^2 = (2x+1)(2-x) = 0.$$

The only solution of this for which $x > 0$ occurs at $x = 2$. But at $x = 2$ the corresponding value of y is $y = \dfrac{29}{3}$. Since this value of y is positive, the only point in the first quadrant at which the tangent line is horizontal is the point

$$\boxed{P = \left(2, \frac{29}{3}\right)}.$$

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017 10.0 points

Find $\dfrac{dy}{dx}$ when

$$\ln(xy) + x = 4.$$

1. $\dfrac{dy}{dx} = \dfrac{y(x+1)}{x}$

2. $\dfrac{dy}{dx} = 2$

3. $\dfrac{dy}{dx} = -\dfrac{y(x-1)}{x}$

4. $\dfrac{dy}{dx} = -\dfrac{y(x+1)}{x}$ **correct**

5. $\dfrac{dy}{dx} = -\dfrac{x+1}{xy}$

Explanation:

By properties of logs the equation

$$\ln(xy) + x = 4$$

can be written as

$$\ln x + \ln y + x = 4.$$

Differentiating implicitly with respect to x we now get

$$\frac{1}{x} + \frac{1}{y}\frac{dy}{dx} + 1 = 0.$$

Consequently,

$$\boxed{\frac{dy}{dx} = -\frac{y(x+1)}{x}}.$$

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018 10.0 points

Find $f'(x)$ when

$$f(x) = \frac{5x-1}{6x-1}.$$

1. $f'(x) = \dfrac{30x-5}{(6x-1)^2}$

2. $f'(x) = \dfrac{6-5x}{(6x-1)^2}$

3. $f'(x) = -\dfrac{1}{(6x-1)^2}$

4. $f'(x) = \dfrac{1}{(6x-1)^2}$ **correct**

5. $f'(x) = \dfrac{1}{6x-1}$

Explanation:

Using the Quotient Rule for differentiation we see that

$$f'(x) = \frac{5(6x-1) - 6(5x-1)}{(6x-1)^2}.$$

Consequently,

$$\boxed{f'(x) = \frac{1}{(6x-1)^2}}.$$

019 10.0 points

Find y' when

$$xy + 5x + 4x^2 = 5.$$

1. $y' = -\dfrac{y+5+8x}{x}$ **correct**

2. $y' = -(y+5+8x)$

3. $y' = -\dfrac{y+5+4x}{x}$

4. $y' = \dfrac{5+4x-y}{x}$

5. $y' = \dfrac{y+5+8x}{x}$

6. $y' = \dfrac{y+5+4x}{x}$

Explanation:

Differentiating implicitly with respect to x we see that

$$\frac{d}{dx}\left(xy + 5x + 4x^2\right) = \frac{d}{dx}(5).$$

Thus

$$(xy' + y) + 5 + 8x = 0,$$

and so

$$xy' = -y - 5 - 8x.$$

Consequently,

$$\boxed{y' = -\frac{y+5+8x}{x}}.$$

020 10.0 points

Find the derivative of f when

$$f(x) = \frac{(5+x^2)^{1/2}}{x+3}.$$

1. $f'(x) = \dfrac{(3x-5)(5+x^2)^{1/2}}{(x+3)^2}$

2. $f'(x) = \dfrac{3x-5}{(x+3)(5+x^2)^{1/2}}$

3. $f'(x) = \dfrac{x-15}{(x+3)^2(5+x^2)^{1/2}}$

4. $f'(x) = \dfrac{3x-5}{(x+3)^2(5+x^2)^{1/2}}$ **correct**

5. $f'(x) = \dfrac{1-15x}{(x+3)^2(5+x^2)^{1/2}}$

Explanation:

By the Chain and Product Rules,

$$\begin{aligned} f'(x) &= \frac{x}{(x+3)\,(5+x^2)^{1/2}} \\ &\quad - \frac{(5+x^2)^{1/2}}{(x+3)^2} \\ &= \frac{x(x+3)-(5+x^2)}{(x+3)^2(5+x^2)^{1/2}}. \end{aligned}$$

Consequently,

$$\boxed{f'(x) = \frac{3x-5}{(x+3)^2(5+x^2)^{1/2}}}.$$

021 10.0 points

Find the value of $f'(0)$ when

$$f(x) = \frac{1}{4}e^{4x} + \frac{1}{4}e^{-x}.$$

1. $f'(0) = \dfrac{3}{4}$ **correct**

2. $f'(0) = \dfrac{15}{16}$

3. $f'(0) = \dfrac{9}{16}$

4. $f'(0) = \dfrac{13}{16}$

5. $f'(0) = \dfrac{7}{8}$

Explanation:

By the Chain rule,

$$f'(x) = e^{4x} - \frac{1}{4}e^{-x}.$$

Consequently,

$$\boxed{f'(0) = \frac{3}{4}}.$$

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022 10.0 points

Find the x-intercept of the tangent line to the graph of

$$f(x) = 3\sin(x) + \cos(x)$$

at the point $(0, f(0))$.

1. $x\text{-intercept} = \frac{1}{4}$

2. $x\text{-intercept} = 3$

3. $x\text{-intercept} = \frac{1}{3}$

4. $x\text{-intercept} = -\frac{3}{4}$

5. $x\text{-intercept} = -3$

6. $x\text{-intercept} = -\frac{1}{3}$ **correct**

Explanation:

When

$$f(x) = 3\sin(x) + \cos(x),$$

then

$$f'(x) = 3\cos(x) - \sin(x).$$

Thus at $x = 0$,

$$f(0) = 1, \qquad f'(0) = 3.$$

So by the Point-Slope formula, an equation for the tangent line at $(0, f(0))$ is

$$y - 1 = 3(x - 0),$$

which after rearranging become

$$y = 3x + 1.$$

Consequently, the tangent line to the graph of f at $(0, f(0))$ has

$$\boxed{x\text{-intercept} = -\frac{1}{3}}.$$

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keywords: tangent, trig function, sin, cos, trig derivative, intercept, point-slope formula,

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023 10.0 points

Determine $f'(x)$ when

$$f(x) = \frac{\sin(x) - 4}{\sin(x) + 2}.$$

1. $f'(x) = \frac{6\sin(x)\cos(x)}{\sin(x) + 2}$

2. $f'(x) = \frac{2\cos(x)}{\sin(x) + 2}$

3. $f'(x) = \frac{6\cos(x)}{(\sin(x) + 2)^2}$ **correct**

4. $f'(x) = -\frac{2\sin(x)\cos(x)}{\sin(x) + 2}$

5. $f'(x) = -\frac{2\cos(x)}{(\sin(x) + 2)^2}$

6. $f'(x) = -\frac{6\cos(x)}{(\sin(x) + 2)^2}$

Explanation:

By the Quotient Rule,

$$f'(x) = \frac{(\sin(x) + 2)\cos(x) - (\sin(x) - 4)\cos(x)}{(\sin(x) + 2)^2}.$$

But

$$(\sin(x)+2)\cos(x) - (\sin(x)-4)\cos(x) = 6\cos(x).$$

Thus

$$\boxed{f'(x) = \frac{6\cos(x)}{(\sin(x) + 2)^2}}.$$

keywords: derivative of trig functions, derivative, quotient rule

024 10.0 points

Find the derivative of

$$g(x) = \left(\frac{x+2}{x+3}\right)(2x-7).$$

1. $g'(x) = \dfrac{2x^2-12x-5}{x+3}$

2. $g'(x) = \dfrac{2x^2+12x+5}{x+3}$

3. $g'(x) = \dfrac{x^2+12x-5}{(x+3)^2}$

4. $g'(x) = \dfrac{2x^2-12x-5}{(x+3)^2}$

5. $g'(x) = \dfrac{x^2-12x+5}{x+3}$

6. $g'(x) = \dfrac{2x^2+12x+5}{(x+3)^2}$ **correct**

Explanation:

By the Quotient and Product Rules we see that

$$\begin{aligned} g'(x) &= 2\left(\frac{x+2}{x+3}\right) \\ &\qquad + (2x-7)\left(\frac{(x+3)-(x+2)}{(x+3)^2}\right) \\ &= 2\left(\frac{x+2}{x+3}\right) + \left(\frac{2x-7}{(x+3)^2}\right) \\ &= \frac{2(x+2)(x+3)+(2x-7)}{(x+3)^2}. \end{aligned}$$

But

$$\begin{aligned} &2(x+2)(x+3)+(2x-7) \\ &\qquad = 2x^2+12x+5. \end{aligned}$$

Consequently

$$\boxed{g'(x) = \frac{2x^2+12x+5}{(x+3)^2}}.$$

025 10.0 points

Find $f'(x)$ when

$$f(x) = \frac{1-x}{2(1+x)}.$$

1. $f'(x) = -\dfrac{1}{(1+x)^2}$ **correct**

2. $f'(x) = \dfrac{2}{(1+x)^2}$

3. $f'(x) = -\dfrac{2}{(1+x)^2}$

4. $f'(x) = \dfrac{3}{(1+x)^2}$

5. $f'(x) = \dfrac{1}{(1+x)^2}$

6. $f'(x) = -\dfrac{3}{(1+x)^2}$

Explanation:

By the Quotient Rule

$$f'(x) = \frac{-2(1+x)-2(1-x)}{4(1+x)^2}.$$

Consequently,

$$\boxed{f'(x) = -\frac{1}{(1+x)^2}}.$$

026 10.0 points

If $y = y(x)$ is defined implicitly by

$$3y^2 + xy + 2 = 0,$$

find the value of dy/dx at the point $(5, -1)$.

1. $\left.\dfrac{dy}{dx}\right|_{(5,-1)} = -1$ **correct**

2. $\left.\dfrac{dy}{dx}\right|_{(5,-1)} = 2$

3. $\left.\dfrac{dy}{dx}\right|_{(5,\,-1)} = -3$

4. $\left.\dfrac{dy}{dx}\right|_{(5,\,-1)} = 3$

5. $\left.\dfrac{dy}{dx}\right|_{(5,\,-1)} = 1$

6. $\left.\dfrac{dy}{dx}\right|_{(5,\,-1)} = -2$

Explanation:
Differentiating implicitly with respect to x we see that

$$6y\frac{dy}{dx} + y + x\frac{dy}{dx} = 0,$$

so

$$\frac{dy}{dx} = -\frac{y}{6y+x}.$$

At $(5,\,-1)$, therefore,

$$\boxed{\left.\frac{dy}{dx}\right|_{(5,\,-1)} = -1}.$$

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027 10.0 points

Find the derivative of

$$f(x) = 2\sin^{-1}(e^{3x}).$$

1. $f'(x) = \dfrac{2}{1+e^{6x}}$

2. $f'(x) = \dfrac{2e^{3x}}{1+e^{6x}}$

3. $f'(x) = \dfrac{6e^{3x}}{1+e^{6x}}$

4. $f'(x) = \dfrac{6}{1+e^{6x}}$

5. $f'(x) = \dfrac{6e^{3x}}{\sqrt{1-e^{6x}}}$ **correct**

6. $f'(x) = \dfrac{6}{\sqrt{1-e^{6x}}}$

7. $f'(x) = \dfrac{2}{\sqrt{1-e^{6x}}}$

8. $f'(x) = \dfrac{2e^{3x}}{\sqrt{1-e^{6x}}}$

Explanation:
Since

$$\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}}, \quad \frac{d}{dx}e^{ax} = ae^{ax},$$

the Chain Rule ensures that

$$\boxed{f'(x) = \frac{6e^{3x}}{\sqrt{1-e^{6x}}}}.$$

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028 10.0 points

Determine the derivative of

$$f(x) = 5\arcsin\left(\frac{x}{3}\right).$$

1. $f'(x) = \dfrac{5}{\sqrt{9-x^2}}$ **correct**

2. $f'(x) = \dfrac{15}{\sqrt{1-x^2}}$

3. $f'(x) = \dfrac{15}{\sqrt{9-x^2}}$

4. $f'(x) = \dfrac{5}{\sqrt{1-x^2}}$

5. $f'(x) = \dfrac{3}{\sqrt{9-x^2}}$

6. $f'(x) = \dfrac{3}{\sqrt{1-x^2}}$

Explanation:
Use of

$$\frac{d}{dx}\arcsin(x) = \frac{1}{\sqrt{1-x^2}},$$

together with the Chain Rule shows that

$$f'(x) = \frac{5}{\sqrt{1-(x/3)^2}}\left(\frac{1}{3}\right).$$

Consequently,

$$\boxed{f'(x) = \frac{5}{\sqrt{9-x^2}}}.$$

029 10.0 points

Find the x-intercept of the tangent line to the graph of

$$f(x) = x + 3\cos(x)$$

at the point $(0, f(0))$.

1. $x\text{-intercept} = \dfrac{1}{3}$

2. $x\text{-intercept} = -3$ **correct**

3. $x\text{-intercept} = \dfrac{3}{4}$

4. $x\text{-intercept} = -\dfrac{1}{3}$

5. $x\text{-intercept} = 3$

6. $x\text{-intercept} = -\dfrac{1}{4}$

Explanation:
When

$$f(x) = x + 3\cos(x),$$

then

$$f'(x) = 1 - 3\sin(x).$$

Thus at $x = 0$,

$$f(0) = 3, \qquad f'(0) = 1.$$

So by the Point-Slope formula, an equation for the tangent line at $(0, f(0))$ is

$$y - 3 = 1(x - 0),$$

which after rearranging become

$$y = x + 3.$$

Consequently, the tangent line to the graph of f at $(0, f(0))$ has

$$\boxed{x\text{-intercept} = -3}.$$

keywords: tangent, trig function, sin, cos, trig derivative, intercept, point-slope formula,

030 10.0 points

Differentiate the function

$$f(x) = \cos(\ln 5x).$$

1. $f'(x) = \dfrac{1}{\cos(\ln 5x)}$

2. $f'(x) = \dfrac{\sin(\ln 5x)}{x}$

3. $f'(x) = -\sin(\ln 5x)$

4. $f'(x) = -\dfrac{\sin(\ln 5x)}{x}$ **correct**

5. $f'(x) = -\dfrac{5\sin(\ln 5x)}{x}$

6. $f'(x) = \dfrac{5\sin(\ln 5x)}{x}$

Explanation:
By the Chain Rule

$$\boxed{f'(x) = -\frac{\sin(\ln 5x)}{x}}.$$