11. $g(x) = 4x^{-2/3} - 2x^{5/3}$

12. $h(z) = 3z^{0.8} + z^{-2.5}$

13. $f(x) = 3\sqrt{x} - 2\sqrt[3]{x}$

14. $g(x) = \sqrt{x}\,(2 - x + 6x^2)$

15. $f(t) = \dfrac{2t - 4 + 3\sqrt{t}}{\sqrt{t}}$

16. $f(x) = \sqrt[4]{5} + \sqrt[4]{x}$

17. $f(x) = \dfrac{2}{5x} - \dfrac{3}{x^2}$

18. $f(x) = \dfrac{5x^2 - 6x + 4}{x^2}, \quad x > 0$

19. $g(t) = 7e^t - e^3$

20. $f(x) = \dfrac{10}{x^6} - 2e^x + 3$

21. $f(\theta) = 2\sin\theta - 3\sec\theta\tan\theta$

22. $h(x) = \sec^2 x + \pi\cos x$

23. $f(r) = \dfrac{4}{1 + r^2} - \sqrt[5]{r^4}$

24. $g(v) = 2\cos v - \dfrac{3}{\sqrt{1 - v^2}}$

25. $f(x) = 2^x + 4\sinh x$

26. $f(x) = \dfrac{2x^2 + 5}{x^2 + 1}$

27–28 Find the antiderivative F of f that satisfies the given condition. Check your answer by comparing the graphs of f and F.

27. $f(x) = 2e^x - 6x, \quad F(0) = 1$

28. $f(x) = 4 - 3(1 + x^2)^{-1}, \quad F(1) = 0$

29–54 Find f.

29. $f''(x) = 24x$

30. $f''(t) = t^2 - 4$

31. $f''(x) = 4x^3 + 24x - 1$

32. $f''(x) = 6x - x^4 + 3x^5$

33. $f''(x) = 2x + 3e^x$

34. $f''(x) = 1/x^2, \quad x > 0$

35. $f'''(t) = 12 + \sin t$

36. $f'''(t) = \sqrt{t} - 2\cos t$

37. $f'(x) = 8x^3 + \dfrac{1}{x}, \quad x > 0, \quad f(1) = -3$

38. $f'(x) = \sqrt{x} - 2, \quad f(9) = 4$

39. $f'(t) = 4/(1 + t^2), \quad f(1) = 0$

40. $f'(t) = t + 1/t^3, \quad t > 0, \quad f(1) = 6$

41. $f'(x) = 5x^{2/3}, \quad f(8) = 21$

42. $f'(x) = (x + 1)/\sqrt{x}, \quad f(1) = 5$

43. $f'(t) = \sec t\,(\sec t + \tan t), \quad -\pi/2 < t < \pi/2,$ $f(\pi/4) = -1$

44. $f'(t) = 3^t - 3/t, \quad f(1) = 2, \quad f(-1) = 1$

45. $f''(x) = -2 + 12x - 12x^2, \quad f(0) = 4, \quad f'(0) = 12$

46. $f''(x) = 8x^3 + 5, \quad f(1) = 0, \quad f'(1) = 8$

47. $f''(\theta) = \sin\theta + \cos\theta, \quad f(0) = 3, \quad f'(0) = 4$

48. $f''(t) = t^2 + 1/t^2, \quad t > 0, \quad f(2) = 3, \quad f'(1) = 2$

49. $f''(x) = 4 + 6x + 24x^2, \quad f(0) = 3, \quad f(1) = 10$

50. $f''(x) = x^3 + \sinh x, \quad f(0) = 1, \quad f(2) = 2.6$

51. $f''(x) = e^x - 2\sin x, \quad f(0) = 3, \quad f(\pi/2) = 0$

52. $f''(t) = \sqrt[3]{t} - \cos t, \quad f(0) = 2, \quad f(1) = 2$

53. $f''(x) = x^{-2}, \quad x > 0, \quad f(1) = 0, \quad f(2) = 0$

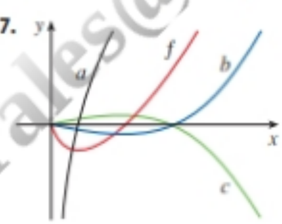
54. $f'''(x) = \cos x, \quad f(0) = 1, \quad f'(0) = 2, \quad f''(0) = 3$

55. Given that the graph of f passes through the point (2, 5) and that the slope of its tangent line at $(x, f(x))$ is $3 - 4x$, find $f(1)$.

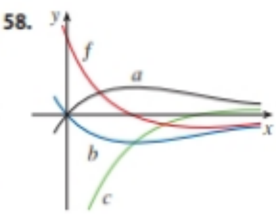
56. Find a function f such that $f'(x) = x^3$ and the line $x + y = 0$ is tangent to the graph of f.

57–58 The graph of a function f is shown. Which graph is an antiderivative of f and why?

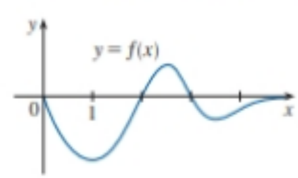
57.



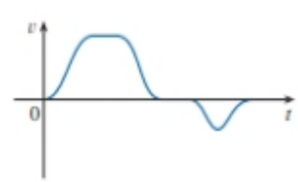
58.



59. The graph of a function is shown in the figure. Make a rough sketch of an antiderivative F, given that $F(0) = 1$.



60. The graph of the velocity function of a particle is shown in the figure. Sketch the graph of a position function.



8. Evaluate the upper and lower sums for

$$f(x) = 1 + \cos(x/2) \qquad -\pi \leqslant x \leqslant \pi$$

with $n = 3$, 4, and 6. Illustrate with diagrams like Figure 14.

9. The speed of a runner increased steadily during the first three seconds of a race. Her speed at half-second intervals is given in the table. Find lower and upper estimates for the distance that she traveled during these three seconds.

t (s)	0	0.5	1.0	1.5	2.0	2.5	3.0
v (ft/s)	0	6.2	10.8	14.9	18.1	19.4	20.2

10. The table shows speedometer readings at 10-second intervals during a 1-minute period for a car racing at the Daytona International Speedway in Florida.

(a) Estimate the distance the race car traveled during this time period using the velocities at the beginning of the time intervals.

(b) Give another estimate using the velocities at the end of the time periods.

(c) Are your estimates in parts (a) and (b) upper and lower estimates? Explain.

Time(s)	Velocity (mi/h)
0	182.9
10	168.0
20	106.6
30	99.8
40	124.5
50	176.1
60	175.6

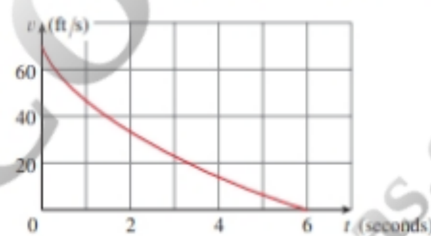
11. Oil leaked from a tank at a rate of $r(t)$ liters per hour. The rate decreased as time passed and values of the rate at two-hour time intervals are shown in the table. Find lower and upper estimates for the total amount of oil that leaked out.

t (h)	0	2	4	6	8	10
$r(t)$ (L/h)	8.7	7.6	6.8	6.2	5.7	5.3

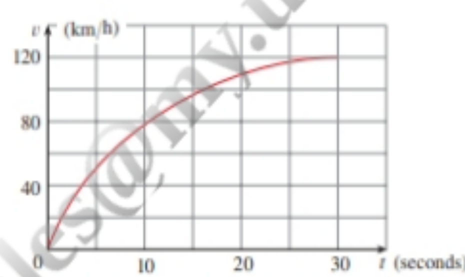
12. When we estimate distances from velocity data, it is sometimes necessary to use times $t_0, t_1, t_2, t_3, \ldots$ that are not equally spaced. We can still estimate distances using the time periods $\Delta t_i = t_i - t_{i-1}$. For example, in 1992 the space shuttle *Endeavour* was launched on mission STS-49 in order to install a new perigee kick motor in an Intelsat communications satellite. The table, provided by NASA, gives the velocity data for the shuttle between liftoff and the jettisoning of the solid rocket boosters. Use these data to estimate the height above the earth's surface of the *Endeavour*, 62 seconds after liftoff.

Event	Time (s)	Velocity (ft/s)
Launch	0	0
Begin roll maneuver	10	185
End roll maneuver	15	319
Throttle to 89%	20	447
Throttle to 67%	32	742
Throttle to 104%	59	1325
Maximum dynamic pressure	62	1445
Solid rocket booster separation	125	4151

13. The velocity graph of a braking car is shown. Use it to estimate the distance traveled by the car while the brakes are applied.



14. The velocity graph of a car accelerating from rest to a speed of 120 km/h over a period of 30 seconds is shown. Estimate the distance traveled during this period.



15. In a person infected with measles, the virus level N (measured in number of infected cells per mL of blood plasma) reaches a peak density at about $t = 12$ days (when a rash appears) and then decreases fairly rapidly as a result of immune response. The area under the graph of $N(t)$ from $t = 0$ to $t = 12$ (as shown in the figure) is equal to the total amount of infection needed to develop symptoms (measured in density of infected cells $\times$ time). The function N has been modeled by the function

$$f(t) = -t(t - 21)(t + 1)$$

Use this model with six subintervals and their midpoints to

EXAMPLE 9 Use Property 8 to estimate $\int_0^1 e^{-x^2}\,dx$.

SOLUTION Because $f(x) = e^{-x^2}$ is a decreasing function on $[0, 1]$, its absolute maximum value is $M = f(0) = 1$ and its absolute minimum value is $m = f(1) = e^{-1}$. Thus, by Property 8,

$$e^{-1}(1 - 0) \leqslant \int_0^1 e^{-x^2}\,dx \leqslant 1(1 - 0)$$

or

$$e^{-1} \leqslant \int_0^1 e^{-x^2}\,dx \leqslant 1$$

Since $e^{-1} \approx 0.3679$, we can write

$$0.367 \leqslant \int_0^1 e^{-x^2}\,dx \leqslant 1$$

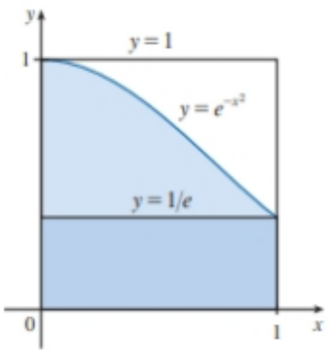


FIGURE 18

The result of Example 9 is illustrated in Figure 18. The integral is greater than the area of the lower rectangle and less than the area of the square.

5.2 Exercises

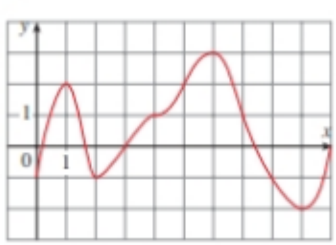
1. Evaluate the Riemann sum for $f(x) = x - 1$, $-6 \leqslant x \leqslant 4$, with five subintervals, taking the sample points to be right endpoints. Explain, with the aid of a diagram, what the Riemann sum represents.

2. If
$$f(x) = \cos x \qquad 0 \leqslant x \leqslant 3\pi/4$$
evaluate the Riemann sum with $n = 6$, taking the sample points to be left endpoints. (Give your answer correct to six decimal places.) What does the Riemann sum represent? Illustrate with a diagram.

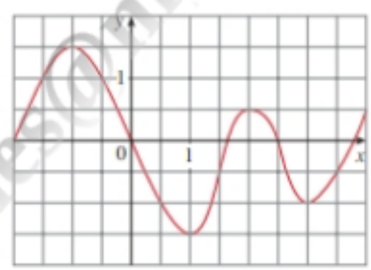
3. If $f(x) = x^2 - 4$, $0 \leqslant x \leqslant 3$, evaluate the Riemann sum with $n = 6$, taking the sample points to be midpoints. What does the Riemann sum represent? Illustrate with a diagram.

4. (a) Evaluate the Riemann sum for $f(x) = 1/x$, $1 \leqslant x \leqslant 2$, with four terms, taking the sample points to be right endpoints. (Give your answer correct to six decimal places.) Explain what the Riemann sum represents with the aid of a sketch.
(b) Repeat part (a) with midpoints as the sample points.

5. The graph of a function f is given. Estimate $\int_0^{10} f(x)\,dx$ using five subintervals with (a) right endpoints, (b) left endpoints, and (c) midpoints.



6. The graph of a function g is shown. Estimate $\int_{-2}^{4} g(x)\,dx$ with six subintervals using (a) right endpoints, (b) left endpoints, and (c) midpoints.



7. A table of values of an increasing function f is shown. Use the table to find lower and upper estimates for $\int_{10}^{30} f(x)\,dx$.

x	10	14	18	22	26	30
$f(x)$	-12	-6	-2	1	3	8

8. The table gives the values of a function obtained from an experiment. Use them to estimate $\int_3^9 f(x)\,dx$ using three equal subintervals with (a) right endpoints, (b) left endpoints, and (c) midpoints. If the function is known to be an increasing function, can you say whether your estimates are less than or greater than the exact value of the integral?

x	3	4	5	6	7	8	9
$f(x)$	-3.4	-2.1	-0.6	0.3	0.9	1.4	1.8

9–10 Use the Midpoint Rule with $n = 4$ to approximate the integral.

9. $\int_0^8 x^2\,dx$

10. $\int_0^2 (8x + 3)\,dx$

11–14 Use the Midpoint Rule with the given value of n to approximate the integral. Round the answer to four decimal places.

11. $\int_0^3 e^{\sqrt{x}}\,dx, \quad n = 6$

12. $\int_0^1 \sqrt{x^3 + 1}\,dx, \quad n = 5$

13. $\int_1^3 \dfrac{x}{x^2 + 8}\,dx, \quad n = 5$

14. $\int_0^\pi x \sin^2 x\,dx, \quad n = 4$

T **15.** Use a computer algebra system that evaluates midpoint approximations and graphs the corresponding rectangles (use `RiemannSum` or `middlesum` and `middlebox` commands in Maple) to check the answer to Exercise 13 and illustrate with a graph. Then repeat with $n = 10$ and $n = 20$.

T **16.** Use a computer algebra system to compute the left and right Riemann sums for the function $f(x) = x/(x + 1)$ on the interval $[0, 2]$ with $n = 100$. Explain why these estimates show that

$$0.8946 < \int_0^2 \frac{x}{x + 1}\,dx < 0.9081$$

T **17.** Use a calculator or computer to make a table of values of right Riemann sums R_n for the integral $\int_0^\pi \sin x\,dx$ with $n = 5, 10, 50,$ and 100. What value do these numbers appear to be approaching?

T **18.** Use a calculator or computer to make a table of values of left and right Riemann sums L_n and R_n for the integral $\int_0^2 e^{-x^2}\,dx$ with $n = 5, 10, 50,$ and 100. Between what two numbers must the value of the integral lie? Can you make a similar statement for the integral $\int_{-1}^2 e^{-x^2}\,dx$? Explain.

19–22 Express the limit as a definite integral on the given interval.

19. $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} \frac{e^{x_i}}{1 + x_i}\,\Delta x, \quad [0, 1]$

20. $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} x_i\sqrt{1 + x_i^3}\,\Delta x, \quad [2, 5]$

21. $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} [5(x_i^*)^3 - 4x_i^*]\,\Delta x, \quad [2, 7]$

22. $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} \frac{x_i^*}{(x_i^*)^2 + 4}\,\Delta x, \quad [1, 3]$

23–24 Show that the definite integral is equal to $\lim_{n\to\infty} R_n$ and then evaluate the limit.

23. $\displaystyle\int_0^4 (x - x^2)\,dx, \quad R_n = \frac{4}{n}\sum_{i=1}^{n}\left[\frac{4i}{n} - \frac{16i^2}{n^2}\right]$

24. $\displaystyle\int_1^3 (x^3 + 5x^2)\,dx, \quad R_n = \frac{2}{n}\sum_{i=1}^{n}\left[6 + \frac{26i}{n} + \frac{32i^2}{n^2} + \frac{8i^3}{n^3}\right]$

25–26 Express the integral as a limit of Riemann sums using right endpoints. Do not evaluate the limit.

25. $\int_1^3 \sqrt{4 + x^2}\,dx$

26. $\displaystyle\int_2^5 \left(x^2 + \frac{1}{x}\right)dx$

27–34 Use the form of the definition of the integral given in Theorem 4 to evaluate the integral.

27. $\int_0^2 3x\,dx$

28. $\int_0^3 x^2\,dx$

29. $\int_0^3 (5x + 2)\,dx$

30. $\int_0^4 (6 - x^2)\,dx$

31. $\int_1^5 (3x^2 + 7x)\,dx$

32. $\int_{-1}^2 (4x^2 + x + 2)\,dx$

33. $\int_0^1 (x^3 - 3x^2)\,dx$

34. $\int_0^2 (2x - x^3)\,dx$

35. The graph of f is shown. Evaluate each integral by interpreting it in terms of areas.

(a) $\int_0^2 f(x)\,dx$

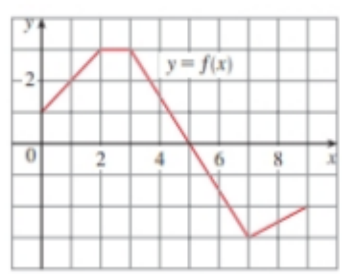
(b) $\int_0^5 f(x)\,dx$

(c) $\int_5^7 f(x)\,dx$

(d) $\int_3^7 f(x)\,dx$

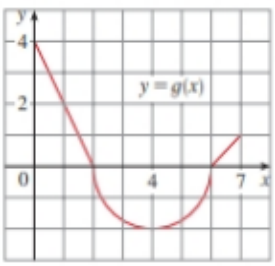
(e) $\int_3^7 |f(x)|\,dx$

(f) $\int_2^0 f(x)\,dx$



36. The graph of g consists of two straight lines and a semicircle. Evaluate each integral by interpreting it in terms of areas.

(a) $\int_0^2 g(x)\,dx$ (b) $\int_2^6 g(x)\,dx$ (c) $\int_0^7 g(x)\,dx$



37–38

(a) Use the form of the definition of the integral given in Theorem 4 to evaluate the given integral.

(b) Confirm your answer to part (a) graphically by interpreting the integral in terms of areas.

37. $\int_0^3 4x\,dx$ **38.** $\int_{-1}^4 (2 - \frac{1}{2}x)\,dx$

39–40

(a) Find an approximation to the integral using a Riemann sum with right endpoints and $n = 8$.

(b) Draw a diagram like Figure 3 to illustrate the approximation in part (a).

(c) Use Theorem 4 to evaluate the integral.

(d) Interpret the integral in part (c) as a difference of areas and illustrate with a diagram like Figure 4.

39. $\int_0^8 (3 - 2x)\,dx$ **40.** $\int_0^4 (x^2 - 3x)\,dx$

41–46 Evaluate the integral by interpreting it in terms of areas.

41. $\int_{-2}^5 (10 - 5x)\,dx$ **42.** $\int_{-1}^3 (2x - 1)\,dx$

43. $\int_{-4}^3 \left|\frac{1}{2}x\right|\,dx$ **44.** $\int_0^1 |2x - 1|\,dx$

45. $\int_{-3}^0 \left(1 + \sqrt{9 - x^2}\right)\,dx$ **46.** $\int_{-4}^4 \left(2x - \sqrt{16 - x^2}\right)\,dx$

47. Prove that $\displaystyle\int_a^b x\,dx = \frac{b^2 - a^2}{2}$.

48. Prove that $\displaystyle\int_a^b x^2\,dx = \frac{b^3 - a^3}{3}$.

T **49–50** Express the integral as a limit of sums. Then evaluate, using a computer algebra system to find both the sum and the limit.

49. $\int_0^\pi \sin 5x\,dx$ **50.** $\int_2^{10} x^6\,dx$

51. Evaluate $\int_1^1 \sqrt{1 + x^4}\,dx$.

52. Given that $\int_0^\pi \sin^4 x\,dx = \frac{3}{8}\pi$, what is $\int_\pi^0 \sin^4 \theta\,d\theta$?

53. In Example 5.1.2 we showed that $\int_0^1 x^2\,dx = \frac{1}{3}$. Use this fact and the properties of integrals to evaluate $\int_0^1 (5 - 6x^2)\,dx$.

54. Use the properties of integrals and the result of Example 4 to evaluate $\int_1^3 (2e^x - 1)\,dx$.

55. Use the result of Example 4 to evaluate $\int_1^3 e^{x+2}\,dx$.

56. Use the result of Exercise 47 and the fact that $\int_0^{\pi/2} \cos x\,dx = 1$ (from Exercise 5.1.33), together with the properties of integrals, to evaluate $\int_0^{\pi/2} (2\cos x - 5x)\,dx$.

57. Write as a single integral in the form $\int_a^b f(x)\,dx$:

$$\int_{-2}^2 f(x)\,dx + \int_2^5 f(x)\,dx - \int_{-2}^{-1} f(x)\,dx$$

58. If $\int_2^8 f(x)\,dx = 7.3$ and $\int_2^4 f(x)\,dx = 5.9$, find $\int_4^8 f(x)\,dx$.

59. If $\int_0^9 f(x)\,dx = 37$ and $\int_0^9 g(x)\,dx = 16$, find

$$\int_0^9 [2f(x) + 3g(x)]\,dx$$

60. Find $\int_0^5 f(x)\,dx$ if

$$f(x) = \begin{cases} 3 & \text{for } x < 3 \\ x & \text{for } x \geq 3 \end{cases}$$

61. For the function f whose graph is shown, list the following quantities in increasing order, from smallest to largest, and explain your reasoning.

(A) $\int_0^8 f(x)\,dx$ (B) $\int_0^3 f(x)\,dx$ (C) $\int_3^8 f(x)\,dx$

(D) $\int_4^8 f(x)\,dx$ (E) $f'(1)$