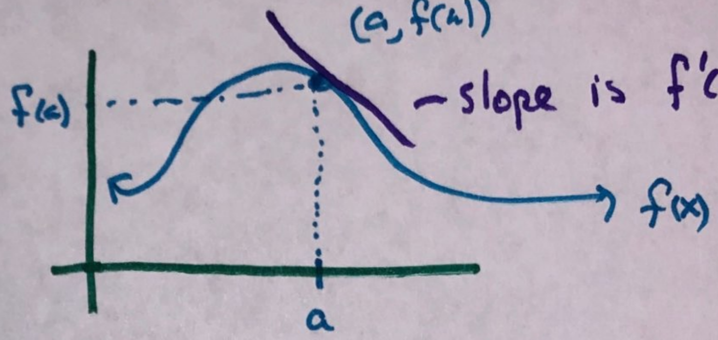
§ 3.1 - Derivatives of Polynomial Functions and e^x

- Limits $\Rightarrow f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$f'(x)$ is the derivative of $f(x)$, and the slope of the line tangent to $f(x)$ at $(a, f(a))$ is $f'(a)$



- $f'(x)$ is also the instantaneous rate of change so

$s'(t) = v(t)$ and $v'(t) = a(t)$

Q.: Any time we need $f'(x)$, do we need to compute the limit above?

Easy Functions:

① If $f(x) = c \Leftrightarrow f'(x) = 0$, $[c]' = 0$

② If $f(x) = mx + c \Leftrightarrow f'(x) = m$, $[mx + b]' = m$

↳ $f(x) = 16x - 4$, $f'(x) = 16$

We will not prove this, but it is true that

If $n \neq 0$, and $f(x) = x^n$, then $f'(x) = n \cdot x^{n-1}$

$$[x^n] = nx^{n-1}, \text{ when } n \neq 0$$

$$[x^4]' = 4x^3, \quad [x^{10}]' = 10x^9, \quad [x^{3/2}]' = \frac{3}{2}x^{1/2}$$

$$[\sqrt{x}]' = [x^{1/2}]' = \frac{1}{2}x^{-1/2} = \frac{1}{2} \cdot \frac{1}{x^{1/2}} = \frac{1}{2\sqrt{x}}$$

$$\cancel{\left[\frac{1}{x^2}\right]' = \frac{1}{2x}}$$ WRONG

$$\left[\frac{1}{x^2}\right]' = [x^{-2}]' = -2 \cdot x^{-3} = \frac{-2}{x^3}$$

$$\left[\frac{1}{\sqrt[3]{x^2}}\right]' = \left[\frac{1}{x^{2/3}}\right]' = \left[x^{-2/3}\right]' = -\frac{2}{3} \cdot x^{-5/3} = \frac{-2}{3x^{5/3}}$$

$$[10^6]' = 0$$

$$[x^\pi]' = \pi \cdot x^{\pi - 1}$$

① $\left[c \cdot f(x)\right]' = c \cdot \left[f(x)\right]' = c \cdot f'(x)$

② + ③ $\left[f(x) \pm g(x)\right]' = f'(x) \pm g'(x)$

$$\left[\frac{3x^4 - 2x}{x^2}\right]' = \left[\frac{3x^4}{x^2} - \frac{2x}{x^2}\right]' =$$

$$= \left[\frac{3x^4}{x^2}\right]' - \left[\frac{2x}{x^2}\right]' = \left[3x^2\right]' - \left[\frac{2}{x}\right]' =$$

$$\left[3x^2\right]' - \left[2x^{-1}\right]' = \cancel{6x}$$

$$= 3\left[x^2\right]' - 2\left[x^{-1}\right]' = 3(2x) - 2(-x^{-2}) = 6x + 2x^{-2}$$

$$= 6x + \frac{2}{x^2} = \frac{6x^3}{x^2} + \frac{2}{x^2} = \frac{6x^3 + 2}{x^2}$$

Exponential Functions

$$\left[a^x\right]' = a^x \cdot \ln(a)$$

$$\hookrightarrow \left[e^x\right]' = e^x \cdot \ln(e) = e^x$$

$$\left[10x^2 - 4\sqrt{x} + e^x\right]' = \left[10x^2\right]' - \left[4x^{1/2}\right]' + \left[e^x\right]' = \boxed{20x - 2x^{-1/2} + e^x}$$

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