09/12/2023

Last Time: The derivative at $x=a$, $f'(a)$

Today: The derivative as a function, $f'(x)$

↳ Defn

↳ Graphs

might start 3.1

Future: HW03 Due Wednesday

Quest 04 Due Monday

HW04 Due Monday

Exam I on Tuesday the 19th

↳ I'll answer Questions on Thursday.

Use the $x-a$ def'n of derivative to find $f'(0)$ where $f(x) = \begin{cases} x^2 \cdot \sin\left(\frac{1}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x-a}$$

$$f'(0) = \lim_{x \to 0} \frac{f(x) - f(0)}{x-0} = \lim_{x \to 0} \frac{x^2 \cdot \sin\left(\frac{1}{x}\right) - 0}{x}$$

$$= \lim_{x \to 0} \frac{x^2 \cdot \sin\left(\frac{1}{x}\right)}{x} = \lim_{x \to 0} x \cdot \sin\left(\frac{1}{x}\right) = 0 \cdot L, \; -1 \leq L \leq 1$$

$$\therefore f'(0) = 0 \cdot L = \boxed{0}$$

$$g(x) = \begin{cases} x \cdot \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0 & x = 0, \end{cases}$$

find $g'(0)$ using x-a defn.

$$g'(0) = \lim_{x \to 0} \frac{g(x) - g(0)}{x - 0} = \lim_{x \to 0} \frac{x \sin\left(\frac{1}{x}\right) - 0}{x - 0}$$

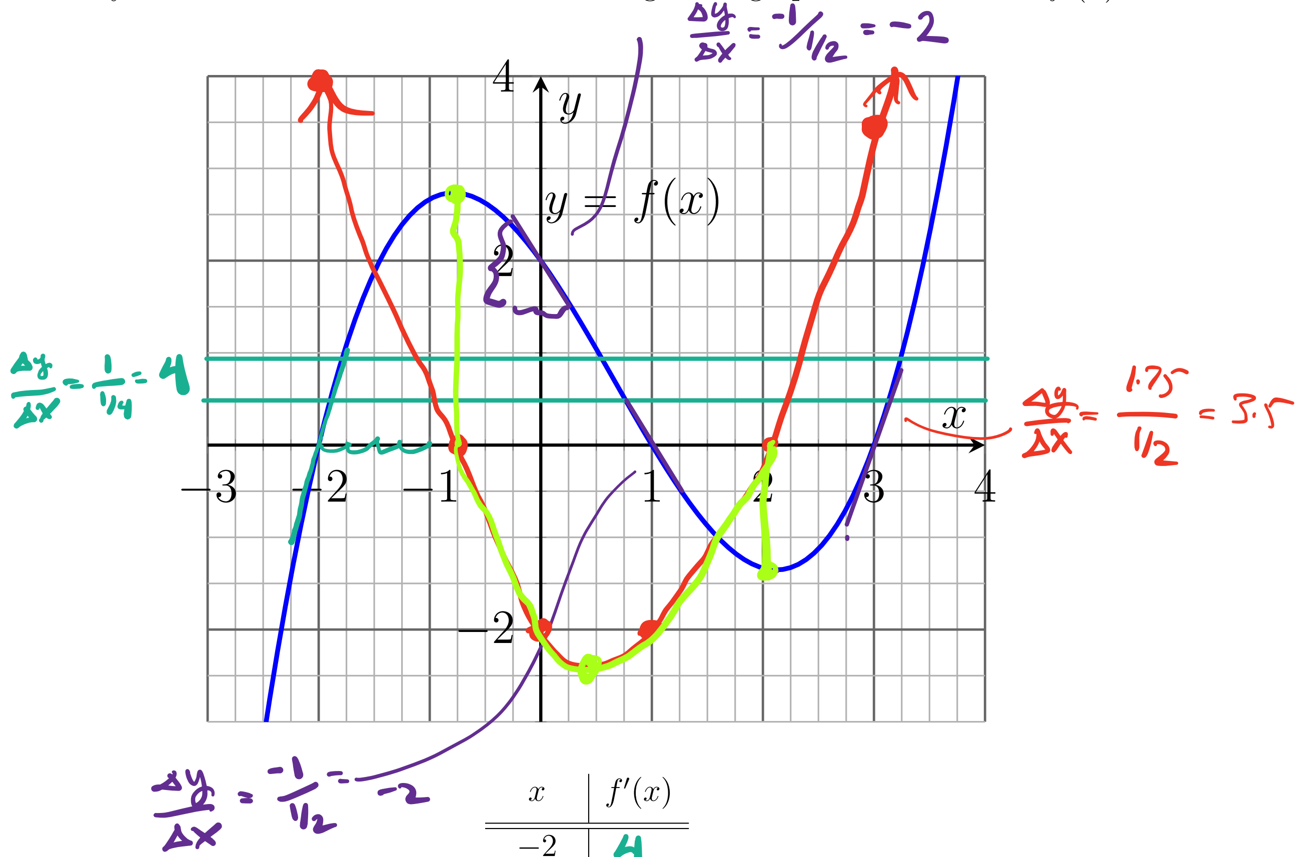
$$= \lim_{x \to 0} \frac{x \sin\left(\frac{1}{x}\right)}{x} = \lim_{x \to 0} \sin\left(\frac{1}{x}\right) = L, \; -1 \leq L \leq 1,$$

oscillates.

$\therefore$ limit D.N.E.

$\therefore$ $g'(0)$ DNE

1. Carefully fill out the table below and use it to great a graph of the function $f'(x)$.

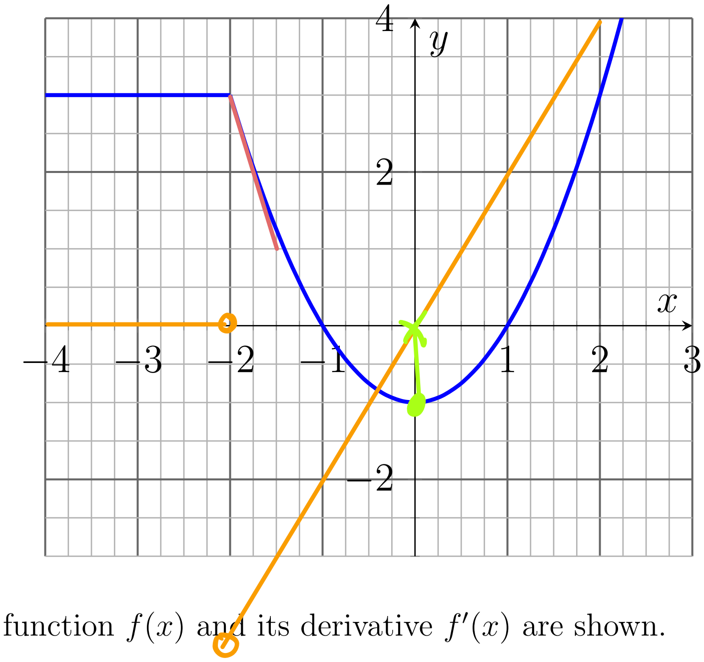


x	$f'(x)$
-2	4
$-3/4$	0
0	-2
1	-2
2.1	0
3	3.5

2. Neatly sketch the derivative of the following function.

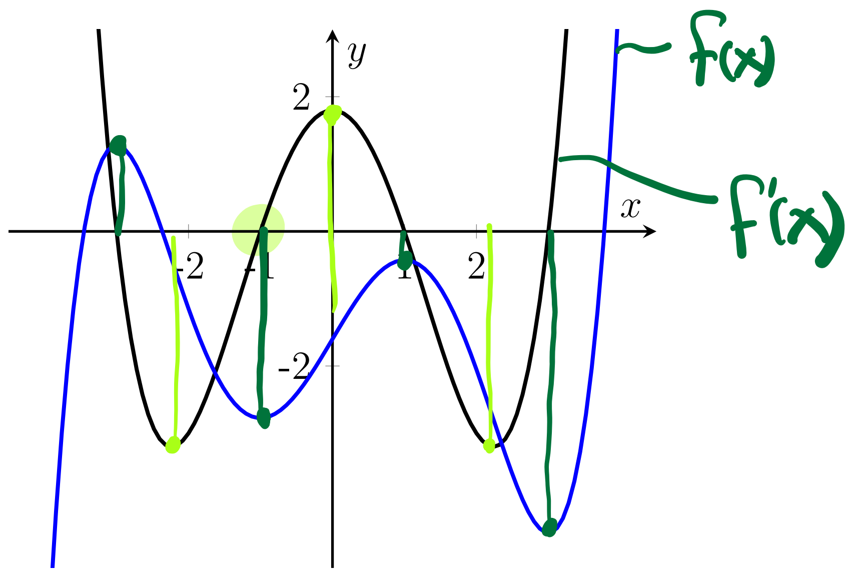
$f'(a)$ exists if left hand derivative equals right hand derivative

x	$f'(x)$
-4	0
-3	0
-2.1	0
-2	DNE
-1.99	-4
-1	-2
0	0
1	2
2	4



3. The graphs of a function $f(x)$ and its derivative $f'(x)$ are shown.

zero slope of f $\Leftrightarrow$ x-int of f'



(a) Clearly label which graph is $f(x)$ and which graph is $f'(x)$.

(b) Which is bigger, $\underbrace{f'(-1)}_{=0}$ or $f''(1)$? $[f'(x)]'$ - Neg. $f'(-1) > f''(1)$

4. Find the derivative of $f(x) = \dfrac{1}{x}$ by using the definition of the derivative.

Chapter 2 $\Rightarrow$ Limits $\to$ .... $\to$ Derivative $f'(x)$

Chapter 3 $\Rightarrow$ How do we find $f'(x)$ more quickly.

Suppose $f(x) = c$, the constant function. Find $f'(x)$

Def^n $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{c-c}{h} = \lim_{h\to 0} \frac{0}{h} = 0$

$$[c]' = 0$$

Suppose $f(x) = mx$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{m(x+h)-mx}{h} =$$

$$\lim_{h\to 0} \frac{\cancel{mx}+mh-\cancel{mx}}{h} = \lim_{h\to 0} \frac{m\cancel{h}}{\cancel{h}} = m.$$

$h(x) = f(x) + g(x)$. Find $h'(x)$.

$$h'(x) = \lim_{h\to 0} \frac{h(x+h)-h(x)}{h} = \lim_{h\to 0} \frac{[f(x+h)+g(x+h)]-[f(x)+g(x)]}{h}$$

$$= \lim_{h\to 0} \frac{f(x+h)+g(x+h)-f(x)-g(x)}{h} =$$

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} + \frac{g(x+h)-g(x)}{h} = f'(x)+g'(x)$$