09/26/2023

Last Time: Product + Quotient Rule

Tangent lines thru points NOT on the graph.

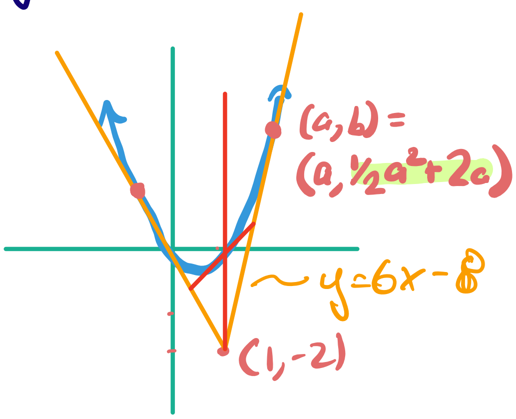
Today: Trig Derivatives

Chain Rule

Tangent lines thru points NOT on the graph

Future: HWs

Find the equation of the lines tangent to $y = \frac{1}{2}x^2 + 2x$ and pass thru $(1,-2)$



Slope between 2 points to equal the derivative at $x=a$.

- $\dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{[\frac{1}{2}a^2 + 2a] - [-2]}{a-1} = \dfrac{\frac{1}{2}a^2 + 2a + 2}{a-1}$

- $y' = x+2 \Rightarrow f' = a+2$

$\Rightarrow \dfrac{\frac{1}{2}a^2 + 2a + 2}{a-1} = a+2 \Rightarrow$

$\frac{1}{2}a^2 + 2a + 2 = a^2 + a - 2$

$0 = \frac{1}{2}a^2 - a - 4$

$= a^2 - 2a - 8 = (a-4)(a+2)$

$\therefore a = 4, -2$

$a=4,\ b=16,\ m=6$

$y+2 = 6(x-1) \rightarrow y = 6x-8$

$y-16 = 6(x-4)$

y-int $\Rightarrow x=0 \Rightarrow y=-8$

x-int $\Rightarrow y=0 \Rightarrow x = 4/3$

$a=-2,\ b=-2,\ m=0$

$y+2 = 0(x+2) \Rightarrow y=-2$

$[\sin(x)]' = \cos(x)$

$[\cos(x)]' = -\sin(x)$

$[\tan(x)]' = \left[\frac{\sin(x)}{\cos(x)}\right]' = $ Q. Rule, Trig $= \sec^2(x)$

$[\sec(x)]' = \left[\frac{1}{\cos(x)}\right]' = $ Q. Rule, Trig $= \sec(x)\tan(x)$

$[\cot(x)]' = \left[\frac{\cos(x)}{\sin(x)}\right]'$ OR $\left[\frac{1}{\tan(x)}\right]' = ???$

$f(x) = x\cos(x) - \sin(x)$, find $f'(0)$ + $f'(\pi/4)$

$= [x]'\cos(x) + x[\cos(x)]' - [\sin(x)]'$

$= \cancel{\cos(x)} - x\sin(x) - \cancel{\cos(x)} = -x\sin(x)$, $f'(\pi/4) = -\frac{\pi}{4}\sin(\pi/4)$

$= -\frac{\pi}{4}\left(\frac{\sqrt{2}}{2}\right) = \boxed{\frac{-\pi\sqrt{2}}{8}}$

$f(x) = \frac{x^2}{\sin(x)+1}$, find $f'(x)$ at $\pi/3$.

$= \frac{[x^2]'(\sin(x)+1) - x^2[\sin(x)+1]'}{(\sin(x)+1)^2} = \frac{2x(\sin(x)+1) - x^2(\cos(x))}{(\sin(x)+1)^2}$

$\sin(\pi/3) = \frac{\sqrt{3}}{2}$

$\cos(\pi/3) = 1/2$

$= \frac{2(\pi/3)\left(\frac{\sqrt{3}}{2}+1\right) - \left(\frac{\pi}{3}\right)^2 \cdot \frac{1}{2}}{\left(\frac{\sqrt{3}}{2}+1\right)^2}$

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2 Functions: $+, -, \times, \div, \circ$

$(f \circ g)(x) = f(g(x))$

Ex: $f(x) = \sqrt{x}$, $g(x) = x^2+1$

Find $f(g(x)) = f(x^2+1) = \sqrt{x^2+1}$

$g(f(x)) = g(\sqrt{x}) = (\sqrt{x})^2 + 1 = x+1$

Chain Rule: $[f(g(x))]' = f'(g(x)) \cdot g'(x)$

$$\left[\sqrt{x^2+1}\right]' = \left[(x^2+1)^{1/2}\right]' = \tfrac{1}{2}(x^2+1)^{-1/2} \cdot [x^2+1]'$$

$$= \cancel{\tfrac{1}{2}}(x^2+1)^{-1/2} \cdot \cancel{2}x$$

$$= \frac{x}{(x^2+1)^{1/2}} = \frac{x}{\sqrt{x^2+1}}$$

$$[\sin(4x^3)]' = \cos(4x^3) \cdot [4x^3]' = \cos(4x^3) \cdot 12x^2$$
$$= 12x^2 \cdot \cos(4x^3)$$

$$[(\cos(x))^2]' = 2 \cdot \cos(x) \cdot [\cos(x)]' = -2\cos(x)\sin(x)$$

$$[x^2 \cdot e^{2x}] = [x^2]'e^{2x} + x^2[e^{2x}]' = 2xe^{2x} + x^2 \cdot e^{2x}[2x]'$$
$$= 2xe^{2x} + 2x^2e^{2x}$$
$$= 2xe^{2x}(x+1)$$

$$[\tan(\tfrac{x}{2})]' = \sec^2(\tfrac{x}{2}) \cdot [\tfrac{x}{2}]'$$

$$= \tfrac{1}{2}\sec^2(\tfrac{x}{2})$$

Question: $[|x|]'$

what is $|x|$?

$$|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

$$|x| = \sqrt{x^2}$$

$$[|x|]' = [\sqrt{x^2}]' = [(x^2)^{1/2}]' = \frac{1}{2}(x^2)^{-1/2} \cdot [x^2]'$$

$$= \cancel{\tfrac{1}{2}}(x^2)^{-1/2} \cdot \cancel{2}x = \frac{x}{\sqrt{x^2}} = \boxed{\frac{x}{|x|}}$$

$$[\sqrt{(2x+1)^2}]'$$

Find $[|2x+1|]' = \frac{2x+1}{|2x+1|}[2x+1]' = 2\frac{2x+1}{|2x+1|}$

Find $[|\sin(x)|]' = \frac{\sin(x)}{|\sin(x)|} \cdot [\sin(x)]' = \frac{\sin(x) \cdot \cos(x)}{|\sin(x)|}$