§ 3.9, Related Rates, Examples

A perfectly spherical balloon is being filled with water at a rate of $2\ ft^3/min$. How fast is the radius of the balloon increasing when the radius is $\frac{1}{2}$ ft?

We need an equation that relates Volume of sphere w/ its radius.

$$V = \frac{4}{3}\pi r^3 \Rightarrow V(t) = \frac{4}{3}\cdot\pi\cdot(r(t))^3 \quad , \quad \frac{dV}{dt} = 2$$

$$\frac{dr}{dt}$$

$$\frac{dV}{dt} = \frac{4}{\cancel{3}}\cdot\pi\cdot\cancel{3}\cdot(r(t))^2\cdot\frac{dr}{dt}$$

$$\frac{dV}{dt} = 4\cdot\pi\,(r(t))^2\cdot\frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{dV/dt}{4\cdot\pi\cdot(r(t))^2} = \frac{2}{4\cdot\pi\cdot\left(\frac{1}{2}\right)^2} = \boxed{\frac{2}{\pi}\ ft/s}$$

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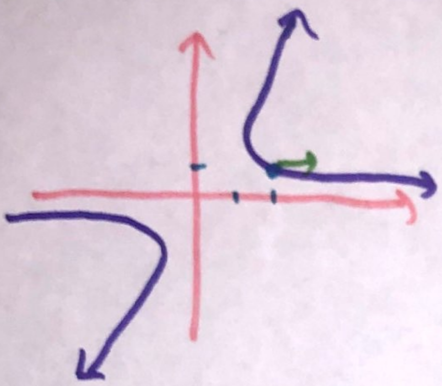
A partical is moving along the graph $3xy - y^2 = 5$. At the point $(2,1)$, the x-coordinate is increasing at 3 units per second. How fest is the y-coordinate moving?



$$3xy - y^2 = 5$$

$$3 \cdot x(t) \cdot y(t) - (y(t))^2 = 5$$

$x=2$, $y=1$

$\frac{dx}{dt} = 3$, $\boxed{\frac{dy}{dt}}$

$$3\left(\frac{dx}{dt} \cdot y(t) + x(t) \cdot \frac{dy}{dt}\right) - 2 \cdot y(t) \cdot \frac{dy}{dt} = 0$$

$$3\left(3 \cdot 1 + 2\frac{dy}{dt}\right) - 2 \cdot 1 \frac{dy}{dt} = 0$$

$$9 + 6\frac{dy}{dt} - 2\frac{dy}{dt} = 0$$

$$4\frac{dy}{dt} = -9$$

$\therefore \frac{dy}{dt} = -9/4$ units/sec

or $9/4$ units/sec, decreasing