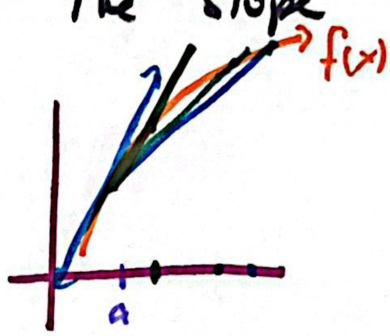
§27- Derivatives and Rates of Change, part 1

In this video, we will:

- State 2 definitions of a derivative
- Relate position, velocity, and derivatives

In the previous video we found the slope of the tangent line using:

$$\lim_{x \to a} \frac{f(x)-f(a)}{x-a}$$



This expression is very, Very, VERY important, so much so that it gets its own name.

The derivative of function f at $x=a$ is

$$f'(a) = \lim_{x \to a} \frac{f(x)-f(a)}{x-a}.$$

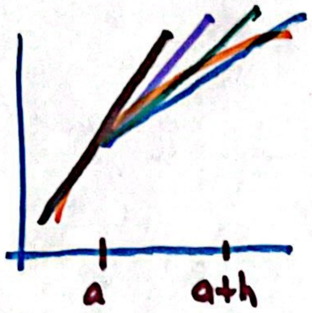
Let $f(x) = 2x^2 - x$, find $f'(3)$

$$f'(3) = \lim_{x \to 3} \frac{f(x)-f(3)}{x-3} = \lim_{x \to 3} \frac{2x^2 - x - [2(3)^2-3]}{x-3} =$$

$$= \lim_{x \to 3} \frac{2x^2-x-15}{x-3} = \lim_{x \to 3} \frac{(2x+5)\cancel{(x-3)}}{\cancel{x-3}} = \lim_{x \to 3} 2x+5 = \boxed{11}$$

There is a second definition for $f'(a)$:

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{(a+h) - a}$$



$$= \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Same Qn as before, but use the new def^n of $f'(a)$.

$f(x) = 2x^2 - x$

$$f'(3) = \lim_{h \to 0} \frac{f(3+h) - f(3)}{(3+h) - 3} =$$

$$= \lim_{h \to 0} \frac{[2(3+h)^2 - (3+h)] - [2(3)^2 - 3]}{h}$$

$$= \lim_{h \to 0} \frac{[2(9 + 6h + h^2) - 3 - h] - [15]}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{18} + 12h + 2h^2 \cancel{- 3} - h \cancel{- 15}}{h} = \lim_{h \to 0} \frac{2h^2 + 11h}{h} =$$

$$= \lim_{h \to 0} \frac{\cancel{h}(2h + 11)}{\cancel{h}} = \lim_{h \to 0} 2h + 11 = 0 + 11 =$$

$$= \boxed{11}$$

So what is $f'(a)$?

- $f'(a)$ is the slope of $f(x)$ at $x=a$.
- $f'(a)$ is the instantaneous rate of change of $f(x)$ at $x=a$.

⇒ If $s(t)$ is a position function, $s'(t)$ is the instantaneous rate of change of $s(t)$, but that is the instantaneous velocity!

$$s'(t) = v(t)$$

Also, $v'(t) = a(t)$

Ex: A car is traveling with position $s(t) = 2t^2 - t$, how fast is it traveling at $t=3$.

$$v(3) = s'(3) = \boxed{11}$$

Problem: Computing $f'(a)$ for 10 different values of a means computing 10 different limits.....