§2.3. Calculating Limits Using Limit Laws, part 2

- In this video, we will:
  - Compute limits of the form $0/0$.
  - Relate this to graphing

Find $\lim_{x\to 1} \frac{x^2-1}{x-1} \to \frac{1^2-1}{1-1} = \frac{0}{0}$, Ind. Form

- A limit approaching $\frac{0}{0}$ or $\frac{\infty}{\infty}$ is in indeterminate form, meaning we cannot determine the limit as is.

To get it out of I.F., we will use algebra:
  - Factoring
  - Adding or Subtracting Fractions
  - Using the conjugate

$$\lim_{x\to 1} \underbrace{\frac{x^2-1}{x-1}} = \lim_{x\to 1} \frac{(x+1)\cancel{(x-1)}}{\cancel{(x-1)}} = \lim_{x\to 1} \underbrace{x+1} = 1+1 = \boxed{2}$$

Question: are $f(x) = \frac{x^2-1}{x-1}$ and $g(x) = x+1$ the same function? No

$$\lim_{x\to 2} \frac{\frac{1}{2} - \frac{1}{x}}{x-2} \to \frac{\frac{1}{2} - \frac{1}{2}}{2-2} = \frac{0}{0}, \text{ I.F.}$$

$$\lim_{x\to 2} \frac{\frac{x}{2x} - \frac{2}{2x}}{x-2} = \lim_{x\to 2} \frac{\frac{x-2}{2x}}{x-2} =$$

$$= \lim_{x\to 2} \frac{\cancel{x-2}}{2x} \cdot \frac{1}{\cancel{x-2}} = \lim_{x\to 2} \frac{1}{2x} = \frac{1}{2\cdot 2} = \boxed{\frac{1}{4}}$$

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Find $\lim_{t \to 0} \frac{\sqrt{1+2t} - \sqrt{1-2t}}{t} \to \frac{\sqrt{1}-\sqrt{1}}{0} = \frac{0}{0}$

$$\lim_{t \to 0} \frac{\sqrt{1+2t} - \sqrt{1-2t}}{t} \left(\frac{\sqrt{1+2t} + \sqrt{1-2t}}{\sqrt{1+2t} + \sqrt{1-2t}} \right)$$

$(a-b)(a+b) =$
$a^2 \cancel{+ab} \cancel{-ab} - b^2$
$a^2 - b^2$

$$= \lim_{t \to 0} \frac{(1+2t) - (1-2t)}{t(\sqrt{1+2t} + \sqrt{1-2t})} = \lim \frac{4\cancel{t}}{\cancel{t}(\sqrt{1+2t} + \sqrt{1-2t})}$$

$$\lim_{t \to 0} \frac{4}{\sqrt{1+2t} + \sqrt{1-2t}} = \frac{4}{\sqrt{1}+\sqrt{1}} = \frac{4}{1+1} = \frac{4}{2} = \boxed{2}$$

What Else: Are there more Indet Forms. $\frac{\infty}{\infty}$, ...
What about absolute values?

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