Abdan Morales | 2.7 + 2.6 #43, 44, 45 | 9/11 2023

2.7

#43

$\boxed{\begin{matrix}\sqrt{x} = f(x) \\ 9 = a\end{matrix}}$ $\lim_{x \to 0} \frac{\sqrt{9+h} - 3}{}$

#44

$\boxed{\begin{matrix}e^x = f(x) \\ -1 = a\end{matrix}}$ $\frac{e^{-1+h} - e^{-1}}{h}$

#45

$\boxed{\begin{matrix}x^6 = f \\ a = 2\end{matrix}}$ $\lim_{x \to 2} \frac{x^6 - 64}{x - 2}$

2.6 Questions

#23

$\lim_{x \to \infty} \frac{\sqrt{x + 3x^2}}{4x - 1} = \frac{\sqrt{x^2(3 + \frac{1}{x})}}{4x - 1} = \frac{\sqrt{x^2}\sqrt{3 + 1/x}}{x(4 - 1/x)} = \frac{\cancel{x}\sqrt{3 + 1/x}}{\cancel{x}(4 - 1/x)}$

$\frac{\sqrt{3+0}}{4-0} = \boxed{\frac{\sqrt{3}}{4}}$ $\frac{1}{\infty} = 0$

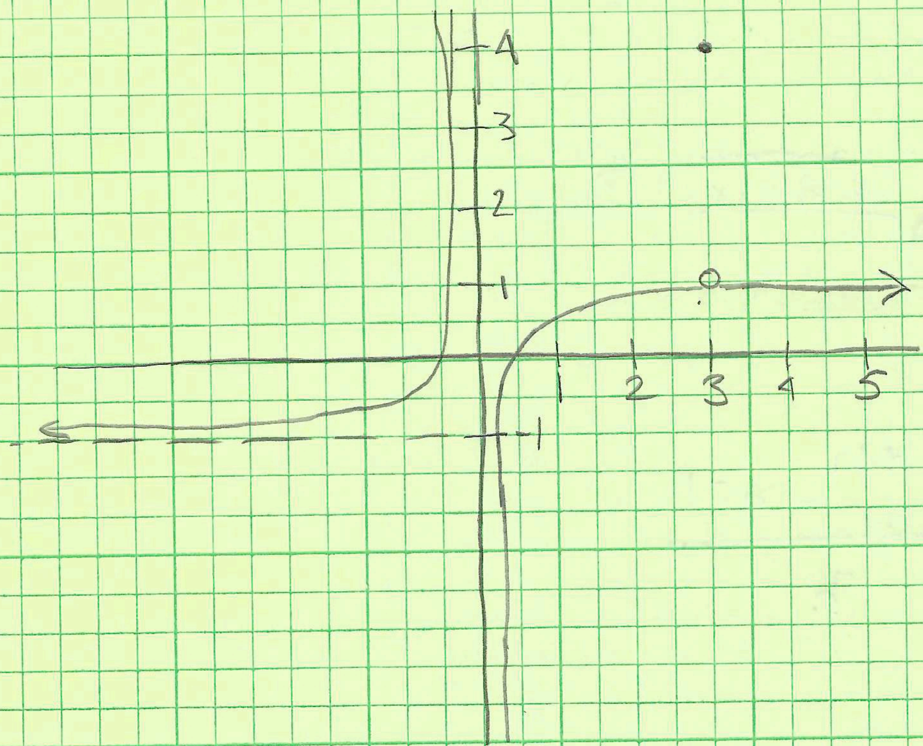
#25

$\lim_{x \to \infty} \frac{\sqrt{1 + 4x^6}}{2 - x^3} \to \frac{\sqrt{x^6}\sqrt{(4 + 1/x^6)}}{x^3(-1 + 2/x^3)} = \frac{\cancel{x^3}\sqrt{4 + 1/x^6}}{\cancel{x^3}(-1 + 2/x^3)} = \frac{\sqrt{4 + 1/x^6}}{(-1 + 2/x^3)}$

$\frac{\sqrt{4+0}}{-1+0} = \frac{\sqrt{4}}{-1} = \frac{2}{-1} = \boxed{-2}$

#26 $\frac{-(\cancel{x^3})\sqrt{4 + 1/x^6}}{\cancel{x^3}(-1 + 2/x^3)} = \frac{-\sqrt{4 + 1/x^6}}{(-1 + 2/x^3)} = \frac{-\sqrt{4+0}}{-1+0} = \frac{-2}{-1} = \boxed{2}$

Question 10d



Abdon Morales | Question 48 2.5 | 9/11 2023

48. $f(x) = \begin{cases} \frac{x^2-4}{x-2} & \text{if } x<2 \\ ax^2+bx+3 & \text{if } 2 \le x < 3 \\ 2x-a+b & \text{if } x \ge 3 \end{cases}$

$\lim_{x \to 2^-} \frac{x^2-4}{x-2} \rightarrow \frac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}} = x+2$

$2+2=4$

$\lim_{x\to 2} f(2) = a(2^2) - b(2) + 3$ Left-side

$4 = 4a - 2b + 3$
$-3 \qquad\quad -3$

$1 = 4a - 2b$ (Eq 1)

$f(\boxed{3}) = a(3^2) - b(3) + 3$ Middle

$9a - 3b + 3$

$f(\boxed{3}) = 2(3) - a + b$

$6 - a + b$

Cont @ 3 $\rightarrow 9a - 3b + 3 = 6 - a + b$
$+a - b - 3 \quad -3 + a - b$

(Eq 2) $10a - 4b = 3$

$1 = 4a - 2b$
$+2b$

$\frac{2b+1}{4} = \frac{\cancel{4}a}{\cancel{4}}$

$\frac{2b+1}{4} = a$

$10\left(\frac{2b+1}{4}\right) - 4b = 3$

$\frac{20b+10}{4} - 4b = 3$
$+4b \qquad +4b$

$\cancel{(4)}\frac{20b+10}{\cancel{4}} = 3 + 4b(4)$

$20b + \cancel{10} = 12 + \cancel{16b}$
$-16b - \cancel{10} \quad -10 - \cancel{16b}$

$\frac{4b}{4} = \frac{2}{4} = \boxed{\frac{1}{2} = b}$

$1 = 4a - \left(\frac{1}{2}\right)2$

$1 = 4a - \cancel{1}$
$+1 \qquad \cancel{+1}$

$\frac{2}{4} = \frac{\cancel{4}a}{\cancel{4}} = \boxed{a = \frac{1}{2}}$

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Question #1

$$\lim_{x\to a}\frac{f(x)-f(a)}{x-a}=\frac{\sqrt{x}-\sqrt{2}}{x-2}\cdot\frac{\sqrt{x}+\sqrt{2}}{\sqrt{x}+\sqrt{2}}=\frac{x-2}{x\sqrt{x}+x\sqrt{2}-2\sqrt{x}-2\sqrt{2}}$$

$$(\sqrt{x}+\sqrt{2})(x-2)$$

$$\frac{\cancel{x-2}}{(\sqrt{x}+\sqrt{2})\cancel{(x-2)}}=\frac{1}{\sqrt{x}+\sqrt{2}}\rightarrow\lim_{x\to 2}\frac{1}{\sqrt{2}+\sqrt{2}}=\boxed{\frac{1}{2\sqrt{2}}}$$

Question #2

$$\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}=\frac{\sqrt{2+h}-\sqrt{2}}{h}\cdot\frac{\sqrt{2+h}+\sqrt{2}}{\sqrt{2+h}+\sqrt{2}}=\frac{\cancel{2}+\cancel{h}-\cancel{2}}{\cancel{h}(\sqrt{2+h}+\sqrt{2})}$$

$$\frac{h\,1}{(\sqrt{2+h}+\sqrt{2})}=\boxed{\frac{1}{\sqrt{2+h}+\sqrt{2}}}$$

$$\lim_{h\to 0}\frac{1}{\sqrt{2+0}+\sqrt{2}}=\frac{1}{\sqrt{2}+\sqrt{2}}=\boxed{\frac{1}{2\sqrt{2}}}$$

Question #3

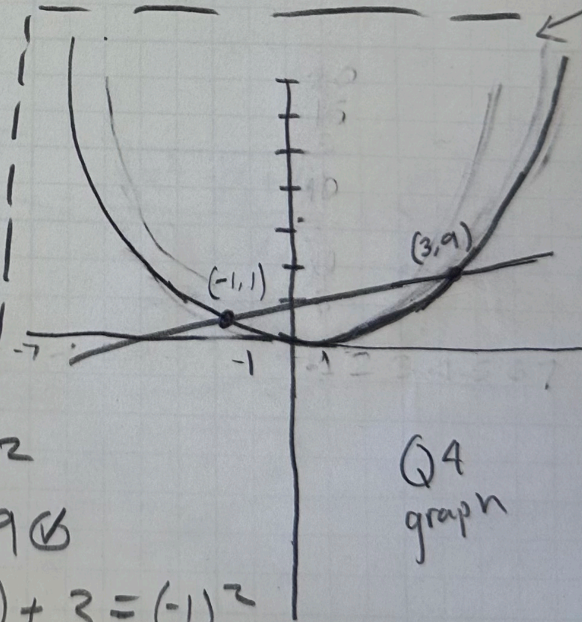
$$\lim_{x\to 2}\frac{f(x)-f(2)}{x-2}=21\longrightarrow\lim_{x\to 2}f(x)\rightarrow f(x)-f(2)=21(x-2)$$

$$f(x)=21x-42+f(2)$$

$$\lim_{x\to 2}f(x)=21\times 2-42+f(2)$$

$$42-42+f(2)$$

$$\boxed{\lim_{x\to 2}f(x)=f(2)}$$



Q4 graph

Question 4

$$2a+3=a^2 \qquad f(3)=2(3)+3=3^2$$

$$9=9\checkmark$$

$$a^2-2a-3=0$$

$$(a-3)(a+1)=0 \qquad f(-1)=2(-1)+3=(-1)^2$$

$$\boxed{a=3\quad a=-1} \qquad 1=1\checkmark$$