§5.5 - The Substitution Rule (u-sub)

$$\int 6x\sqrt{x^2+1}\,dx = \int \cancel{6x}\sqrt{u}\cdot\frac{du}{\cancel{2x}}$$

$u = x^2+1$

$du = 2x\,dx$

$dx = \frac{du}{2x}$

$$= \int 3\sqrt{u}\,du = 3\int u^{1/2}\,du$$

$$= \frac{3\cdot u^{3/2}}{3/2} + C = 2u^{3/2} + C$$

$$\boxed{2(x^2+1)^{3/2} + C}$$

$$\left[2(x^2+1)^{3/2}+C\right]' = \cancel{2}\cdot\frac{3}{\cancel{2}}(x^2+1)^{1/2}\cdot(2x) = \boxed{6x\sqrt{x^2+1}}$$

u-sub undoes the chain rule.

Q1: If we want to use u-sub, how do we find u?

① derivative of u should cancel with other stuff.

② Function inside another function.

$$\int t^2 \sin(2t^3)\,dt = \int \cancel{t^2} \cdot \sin(u) \frac{du}{6\cancel{t^2}}$$

$u = 2t^3$

$du = 6t^2 dt$

$dt = \frac{du}{6t^2}$

$$= \frac{1}{6}\int \sin(u)\,du$$

$$= -\frac{1}{6}\cos(u) + C$$

$$= -\frac{1}{6}\cos(2t^3) + C$$

$u = t^3$

$du = 3t^2 dt$

$dt = \frac{du}{3t^2} \Rightarrow \int \cancel{t^2} \sin(2u) \frac{du}{3\cancel{t^2}} = \frac{1}{3}\int \sin(2u)\,du$ pause

$$= -\frac{1}{6}\cos(2u) + C$$

$$= -\frac{1}{6}\cos(2t^3) + C$$

$$\int e^{5x}\,dx = \int e^u \frac{du}{5} = \frac{1}{5}\int e^u\,du = \frac{1}{5}e^u + C$$

$u = 5x$

$du = 5\,dx$

$dx = \frac{du}{5}$

$$= \boxed{\frac{1}{5}e^{5x} + C}$$