HW 03 Solutions

§2.5 #48)

$$f(x) = \begin{cases} \frac{x^2-4}{x-2} & x < 2 \\ ax^2 - bx + 3 & 2 \le x < 3 \\ 2x - a + b & x \ge 3 \end{cases}$$

To be continuous at $x=2$, $\lim_{x \to 2^-} = f(2)$

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2} \frac{(x+2)(x-2)}{x-2} = 4 \; ; \; f(2) = 4a - 2b + 3$$

$$\therefore \; 4 = 4a - 2b + 3 \Rightarrow 1 = 4a - 2b$$

To be continuous at $x=3$, $\lim_{x\to 3^-} f(x) = f(3)$

$$\lim_{x\to 3^-} f(x) = 9a - 3b + 3, \quad f(3) = 6 - a + b$$

$$\therefore \; 9a - 3b + 3 = 6 - a + b$$

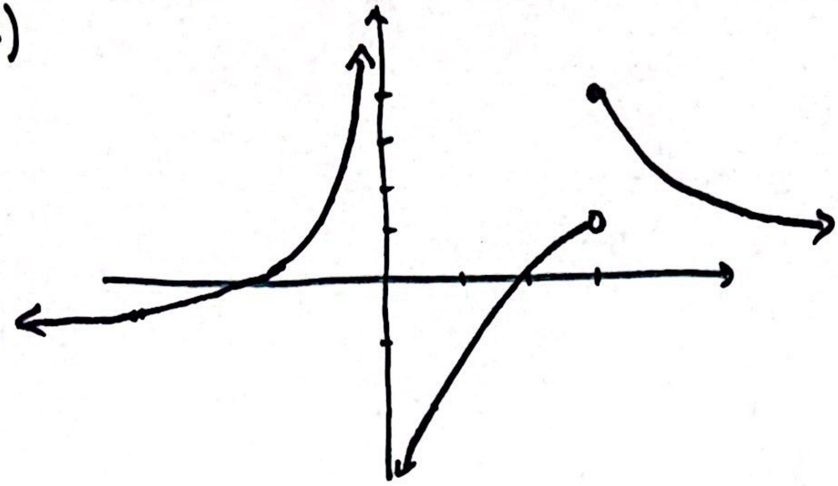
$$10a - 4b = 3$$

Solve: $\begin{matrix} 4a - 2b = 1 \\ 10a - 4b = 3 \end{matrix} \Rightarrow 2a = 1 \quad \therefore a = \frac{1}{2}$

$$\therefore \; 2 - 2b = 1 \quad \therefore \; b = \frac{1}{2}$$

$$\boxed{a = b = \frac{1}{2}}$$

§ 2.6, #10)



§ 2.6, #23) $\lim\limits_{x\to\infty} \dfrac{\sqrt{x+3x^2}}{4x-1} = \lim\limits_{x\to\infty} \dfrac{\sqrt{x^2}\sqrt{1/x+3}}{x(4-1/x)} = \dfrac{\sqrt{3}}{4}$

2.6, #25) $\lim\limits_{x\to\infty} \dfrac{\sqrt{1+4x^6}}{2-x^3} = \lim\limits_{x\to\infty} \dfrac{\sqrt{x^6}\sqrt{1/x^6+4}}{x^3(2/x^3-1)} = -2$

2.6, #26) $\lim\limits_{x\to-\infty} \dfrac{\sqrt{1+4x^6}}{2-x^3} = \lim\limits_{x\to-\infty} \dfrac{\sqrt{x^6}\sqrt{1/x^6+4}}{x^3(2/x^3-1)}$

$= \lim\limits_{x\to-\infty} \dfrac{-\cancel{x^3}}{\cancel{x^3}} \dfrac{\sqrt{1/x^6+4}}{(2/x^3-1)} = \boxed{2}$

§ 2.7, # 43) $\lim\limits_{h\to 0} \dfrac{\sqrt{9+h}-3}{h}$, $f(x)=\sqrt{x}$, $a=9$

(h-def^n)

44) $\lim\limits_{h\to 0} \dfrac{e^{-2+h}-e^{-2}}{h}$, $f(x)=e^x$, $a=-2$

(h-def^n)

45) $\lim\limits_{h\to 2} \dfrac{x^6-64}{x-2}$, $f(x)=x^6$, $a=2$

(x-a def^n)

Additional # 1) $f'(2)=\lim\limits_{x\to 2} \dfrac{\sqrt{x}-\sqrt{2}}{x-2} \cdot \dfrac{(\sqrt{x}+\sqrt{2})}{(\sqrt{x}+\sqrt{2})} =$

$$=\lim_{x\to 2} \frac{x-2}{(x-2)(\sqrt{x}+\sqrt{2})} = \frac{1}{\sqrt{2}+\sqrt{2}} = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}$$

Additional # 2) $f'(2)-\lim\limits_{h\to 0} \dfrac{\sqrt{2+h}-\sqrt{2}}{h} \cdot \left(\dfrac{\sqrt{2+h}+\sqrt{2}}{\sqrt{2+h}+\sqrt{2}}\right) =$

$$=\lim_{h\to 0} \frac{(2+h)-(2)}{h(\sqrt{2+h}+\sqrt{2}} = \frac{1}{\sqrt{2}+\sqrt{2}} = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}$$

Additional # 3) $\lim\limits_{x\to 2} \dfrac{f(x)-f(2)}{x-2}$ is finite, so

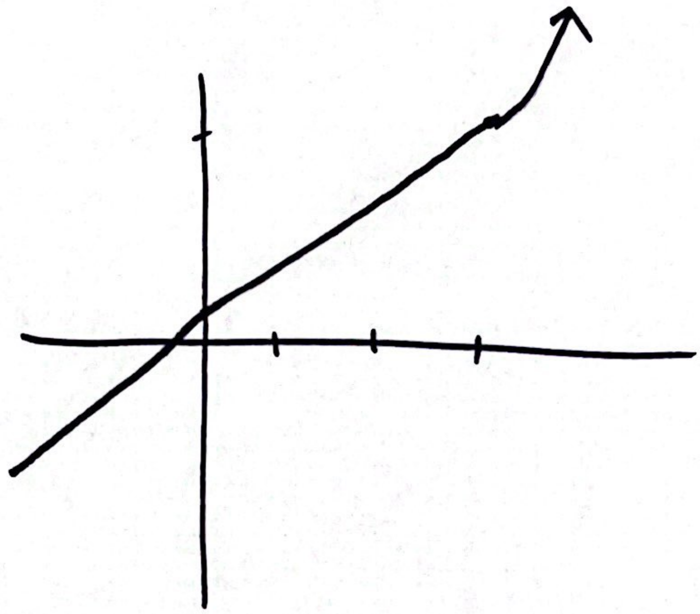
$$\lim_{x\to 2} f(x)-f(2)=0 \Rightarrow \lim_{x\to 2} f(x) = f(2)$$

Additional #c1) $f(x) = \begin{cases} 2x+3 & x \le a \\ x^2 & x > a \end{cases}$

The graph is continuous when $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$

$\Rightarrow 2a + 3 = a^2 \Rightarrow a^2 - 2a - 3 = 0 \Rightarrow (a-3)(a+1) = 0$

$a = 3$



$a = -1$