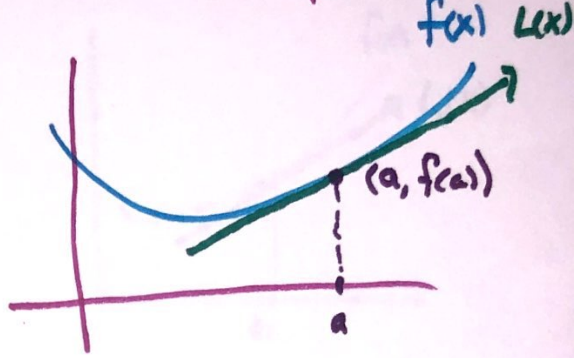
— §3.10 – Linearization + Linear Approximation

- Let $f(x)$ be a function, $(a, f(a))$ is a point on $f(x)$. The line tangent to $f(x)$ at $x=a$ is:



$$y - y_0 = m(x - x_0)$$
$$y - f(a) = f'(a)(x-a)$$
$$y = f(a) + f'(a)(x-a)$$

The linearization of $f(x)$ at $x=a$ is:

$$L(x) = f(a) + f'(a)(x-a)$$

Ex: Find the linearization of $f(x) = x^{1/3} = \sqrt[3]{x}$ at $x=8$

$$L(x) = f(8) + f'(8)(x-8)$$
$$= 2 + \tfrac{1}{12}(x-8)$$
$$= 2 + \tfrac{1}{12}x - \tfrac{8}{12}$$
$$= 2 + \tfrac{1}{12}x - \tfrac{2}{3}$$
$$= \tfrac{1}{12}x + \tfrac{4}{3}$$

$f(x) = x^{1/3} \Rightarrow f(8) = 2$

$f'(x) = \frac{1}{3}x^{-2/3} = \frac{1}{3 \cdot x^{2/3}}$

$f'(8) = \frac{1}{3 \cdot 8^{2/3}}$

$= \frac{1}{3 \cdot 2^2}$

$= \frac{1}{12}$

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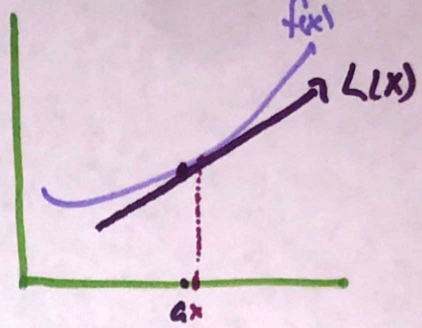
What do we use linearization for ?

a is going to be a "nice" number.

when $x \approx a \Rightarrow f(x) \approx L(x)$



Use the linearization of $f(x) = x^{1/3}$

at $x = 8$ to approximate $f(8.5) = (8.5)^{1/3}$

$$a = 8, \quad 8.5 \approx 8 \Rightarrow (8.5)^{1/3} \approx L(8.5)$$

$$(8.5)^{1/3} \approx L(8.5) = 2 + \tfrac{1}{12}(8.5 - 8)$$

$$= 2 + \tfrac{1}{12}\left(\tfrac{1}{2}\right)$$

$$= 2 + \tfrac{1}{24}$$

$$= \frac{49}{24} \approx 2.041\overline{6}$$

$$(8.5)^{1/3} \approx 2.040828$$