10/19/2023

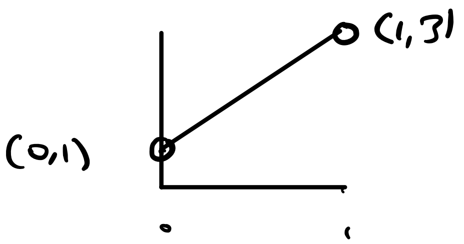
Last Time: Exam II

Today: f', f'', + the shape of $f(x)$, 4.3

Future: HWs

Exam III on T, Nov 28th, after Thanksgiving.

Ex: Find the absolute min, max of $f(x) = 2x+1$ on $(0,1)$.



$2.\overline{99999} = y \Rightarrow 10y = 29.\overline{99999}$

$y = 2.\overline{999}$

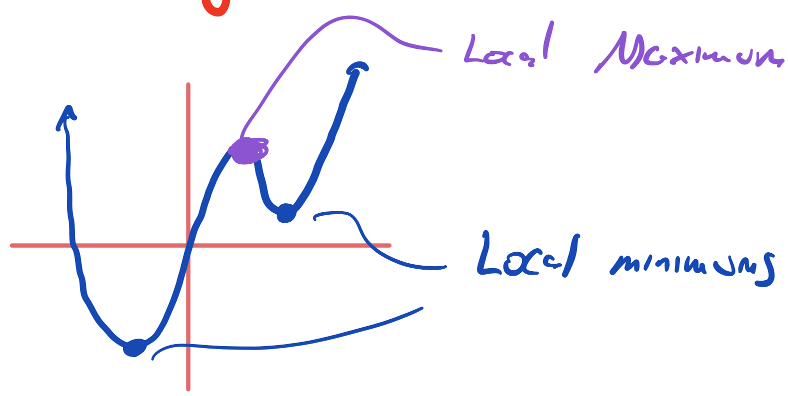
$9y = 27$

$y = 3$

Is no abs min or max.

$\Rightarrow$ open intervals are tricky.

OR $(-\infty, \infty)$



Our goal is to find local mins + max's

⊛ $f'(x) > 0$, f is increasing

$f'(x) < 0$, f is decreasing.

Find the interval where f, g are inc. and dec.

$f(x) = 3x^4 - 16x^3 + 24x^2$

$g(x) = 5x^6 + 6x^5 - 45x^4$

$f'(-1) = 12(-1)(-3)^2 = NEG$

$f'(1) = 12 \Rightarrow +$

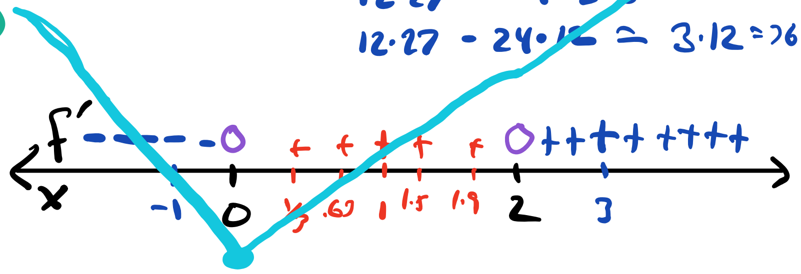
$f'(3) = 12\cdot 27 - 48\cdot 9 + 48\cdot 3$

$12\cdot 27 - 48\cdot 6$

$12\cdot 27 - 24\cdot 2\cdot 6$

$12\cdot 27 - 24\cdot 12 = 3\cdot 12 = 36 > 0$

$$f'(x) = 12x^3 - 48x^2 + 48x = 0$$
$$12x(x^2 - 4x + 4) = 0$$
$$12x(x-2)(x-2) = 0$$
$$12x(x-2)^2 = 0$$
$$x = 0, 2$$

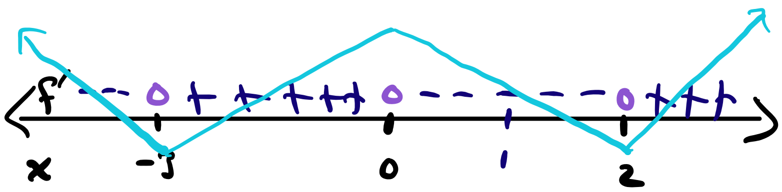


once we find $f'(x_0)$, the sign of $f(x)$ will not change between the critical numbers

Inc: $(0,2) \cup (2,\infty)$; Dec: $(-\infty, 0)$

$0 < x < 2,\ 2 < x$, Dec: $x < 0$

$$g'(x) = 30x^5 + 30x^4 - 180x^3$$
$$= 30x^3(x^2 + x - 6)$$
$$= 30x^3(x+3)(x-2)$$
$$x = -3, 0, 2$$



$f'(x) = 30x^3(x+3)(x-2)$

$f'(1) \Rightarrow (+)(+)(-) = -$

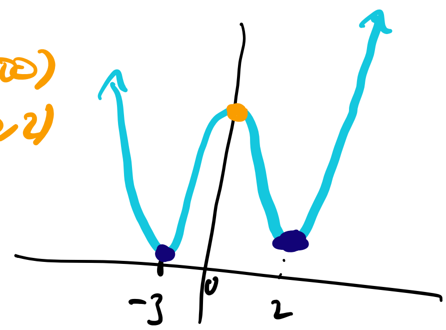
$f(3) \Rightarrow (+)(+)(+) = +$

$f'(-1) \Rightarrow (-1)(+1)(-1) = +$

$f'(-4) = (-)(-)(-) = -$

Inc: $(-3, 0) \cup (2, \infty)$

Dec: $(-\infty, -3) \cup (0, 2)$



First Derivative Test: If $f(x)$ is continuous, then

① f' changes from + to −, we have a local max

② f " " − to +, " " " local min

Use the 1st Derivative Test to find the x,y values of all local min, maxs.

① $f(x) = x^3 - 3x^2 - 9x$

② $g(x) = x + \sqrt{2}\cos(x)$ $(0, \pi)$

① $f'(x) = 3x^2 - 6x - 9 = 0$

$3(x^2 - 2x - 3) = 0$

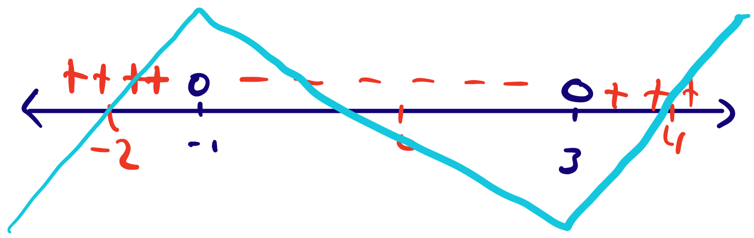
$3(x-3)(x+1) = 0$

$x = -1, 3$

$f(0) = \text{NEG}$

$f(4) = \text{POS}$

$f(-2) = (-)(-) = \text{POS}$



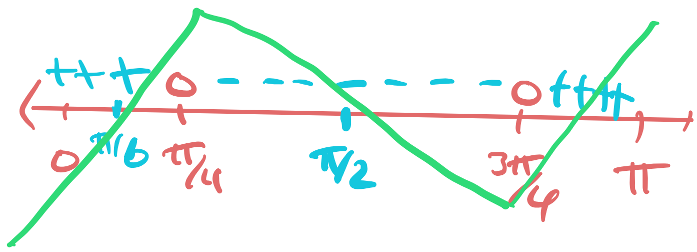
local max at $x = -1$, $y = 5$, $(-1, 5)$

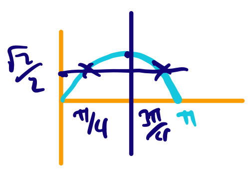
local min at $x = 3$ $y = -27$ $(3, -27)$

② $f'(x) = 1 - \sqrt{2}\sin(x) = 0$

$\frac{\sqrt{2}}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} = \sin(x)$

$\sin(x) = \frac{\sqrt{2}}{2} \Rightarrow \pi/4, \frac{3\pi}{4}$





$f'(\pi/2) = 1 - \sqrt{2}\sin(\pi/2) = 1 - \sqrt{2}$, NEG

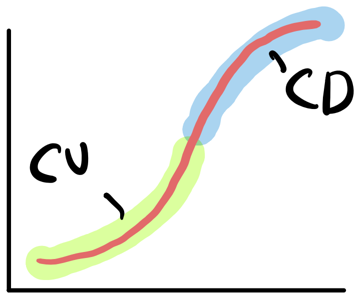
$f'(\pi/6) = 1 - \sqrt{2}\sin(\pi/6) = 1 - \frac{\sqrt{2}}{2}$, Pos

$f'(5\pi/6)$ ...... POS

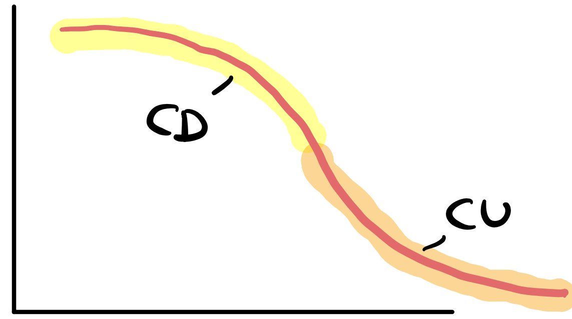
Local max $x = \pi/4$, $f(\pi/4) = \pi/4 + \sqrt{2}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4} + 1$

Local min $x = 3\pi/4$, $f\left(\frac{3\pi}{4}\right) = \frac{3\pi}{4} + \sqrt{2}\left(\frac{-\sqrt{2}}{2}\right) = \frac{3\pi}{4} - 1$

Increase:



Decrease



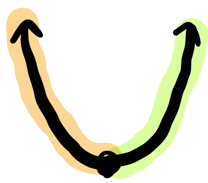
If $f''(x) > 0$, Concave up

If $f''(x) < 0$, Concave down

A graph changes concavity at an inflection point

(often at $f''(x) = 0$)

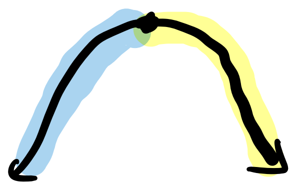
CU:



Local min

CD:

Local max



2nd Derivative Test:

$f'(c) = 0$, $f''(c) < 0$, Local max

$f'(c) = 0$, $f''(c) > 0$, local min.

Q1: For what values of a, b does

$f(x) = ax \cdot e^{bx^2}$ have a local max $f(2) = 1$?

$f(2) = 2a \cdot e^{4b} = 1$ $\qquad$ $f''(2) = NEG$

$f'(2) = 0$

Q4:

x	f(x)	f'(x)	f''(x)
x<0		+	−
x=0	0	0	−
0<x<1		−	−
x=1	-1	DNE	DNE
x>1		+	+