§4.1 - Minimum + Maximum Values

Def 2: A __critical number__ (or critical point) of a function $f(x)$ is an x-value, c, such that:

$$f'(c) = 0 \quad \text{or} \quad f'(c) \text{ DNE}$$

Ex: Find the critical numbers of

ⓐ $f(x) = 2x^3 - 3x^2 - 36x$

$$f'(x) = 6x^2 - 6x - 36$$
$$= 6(x^2 - x - 6)$$
$$= 6(x-3)(x+2)$$

$\Rightarrow \boxed{x = 3, -2}$

ⓑ $g(x) = x^{3/5}(4-x)$

$$g'(x) = \frac{3}{5}x^{-2/5}(4-x) + x^{3/5}(-1)$$

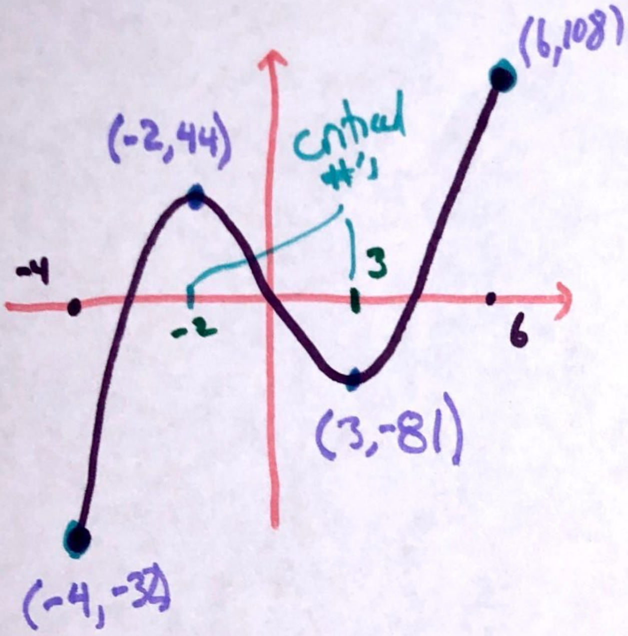
$$= \frac{3(4-x)}{5x^{2/5}} - x^{3/5}$$

$$= \frac{12 - 3x - 5x}{5x^{2/5}} = \frac{12-8x}{5x^{2/5}} \Rightarrow \boxed{x = \frac{3}{2}, 0}$$

So, why do we care about critical numbers?
(critical points?)

One Answer: It helps us find the minimum + maximum values of a function

Ex: Find the minimum + maximum values of $f(x) = 2x^3 - 3x^2 - 36x$ on the interval $[-4, 6]$



$f'(x) = 6(x-3)(x+2)$

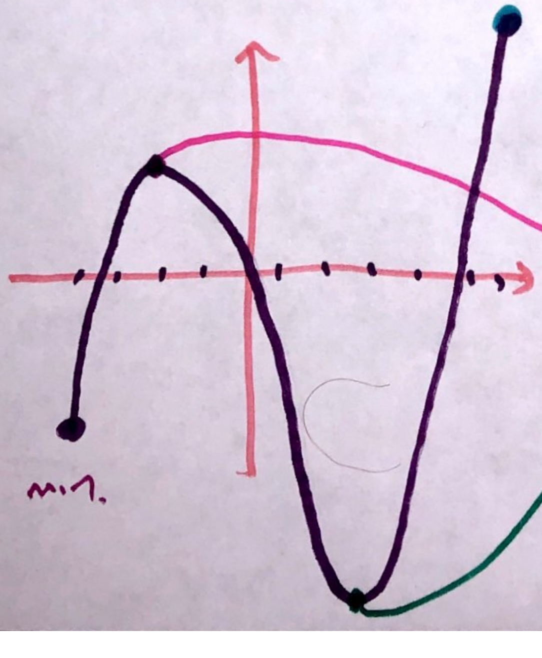
$\therefore$ cr. #'s $x = 3, -2$

$f(-2) = 2(-2)^3 - 3(-2)^2 - 36(-2) = \boxed{44}$

$f(3) = 2(3)^3 - 3(3)^2 - 36(3) = \boxed{-81}$

$f(-4) = \boxed{-32}$

$f(6) = \boxed{108}$



Absolute Maximum of 108 at $x=6$

Local Maximum of 44 at $x=-2$

Absolute + Local Min of -81 at $x=3$

Local min.

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The absolute max of $f(x)$ on Domain D

is $f(c)$ s.t. $f(c) \geq f(x)$ for all x in D

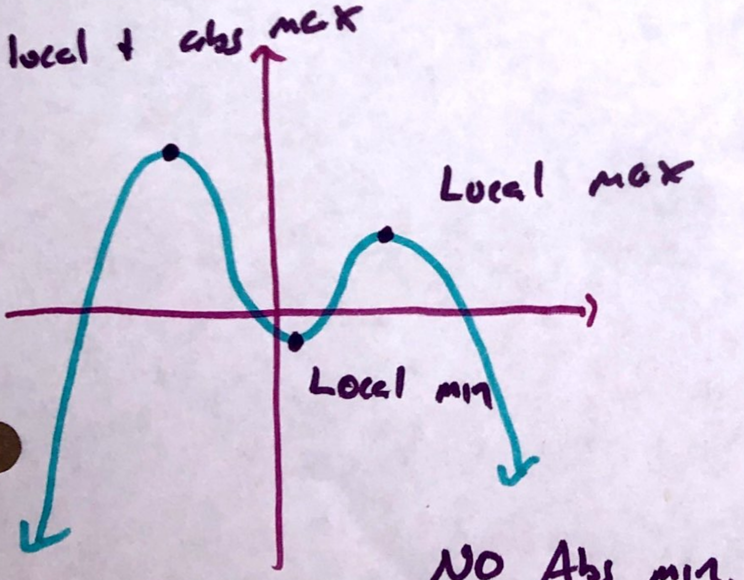
The absolute min of $f(x)$ on Domain D

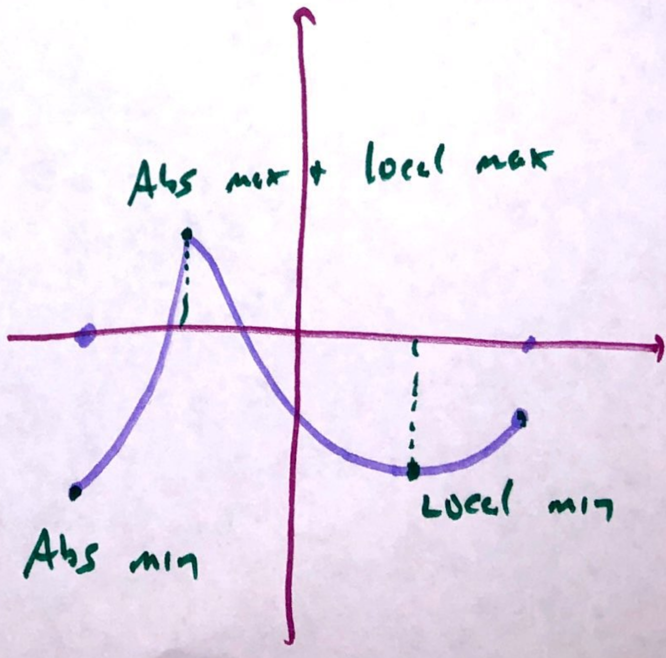
is $f(c)$ s.t. $f(c) \leq f(x)$ for all x in D

The number $f(c)$ is a:

- local max value of f if $f(c) \geq f(x)$ for x near c

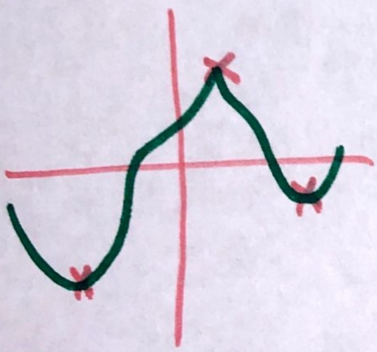
- local min value of f if $f(c) \leq f(x)$ for x near c.

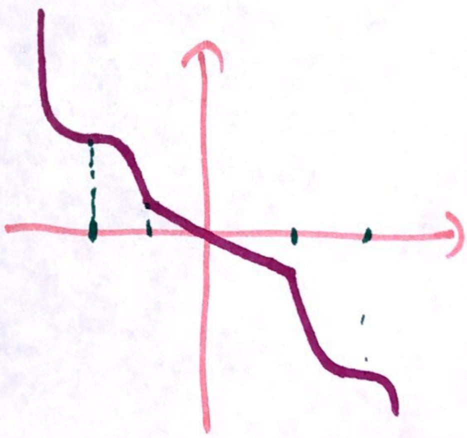




Extreme Value Theorem: If f is continuous on a closed interval $[a, b]$, then f attains an absolute min value and absolute max value for some numbers in $[a, b]$.

Fermat's Theorem: If f has a local min or max, then it occurs at at critical number





Find the absolute min + max of

$f(x) = x + \frac{1}{x}$ on the interval $[1/2, 4]$

Abs min and max occur at $x = 1/2$ or

$x = 4$ or

cr #'s

$$f'(x) = 1 + [x^{-1}]'$$

$$= 1 - x^{-2}$$

$$= 1 - \frac{1}{x^2}$$

$$= \frac{x^2 - 1}{x^2}$$

$x^2 - 1 = 0 \Rightarrow x = \boxed{1}, -1$

$x^2 = 0 \Rightarrow x = 0$

$f(1/2) = \frac{1}{2} + \frac{1}{1/2} = 5/2$

$f(1) = 1 + \frac{1}{1} = 2$ Abs min of 2 at $x = 1$

$f(4) = 4 + \frac{1}{4} = 17/4$ Abs max of $\frac{17}{4}$ at $x = 4$