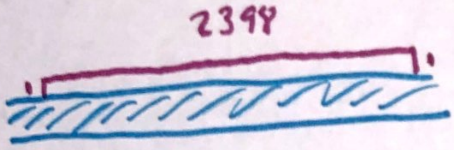
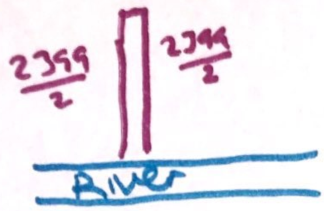
§4.7 - Optimization Problems

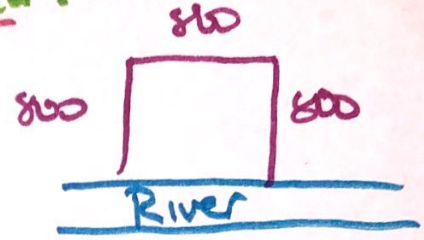
A farmer wants to build a rectangular fence that boarders a straight river (no fence needed here). She has 2400 ft of fence. What are the dimensions of the ~~largest fence~~ fence which enclose the largest Area?



$$\text{Area} = (2398)(1) = 2398\text{ ft}^2$$



Area = $\frac{2399}{2} = 1199.5$

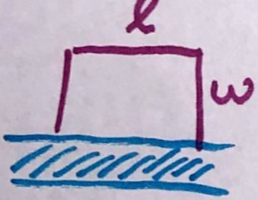


$$\text{Area} = (800)(800) = 640{,}000\text{ ft}^2$$

① Slowly check every set of dimensions (∞ many to check)

② Calculus

Largest Area $\Rightarrow$ Maximize Area $\Rightarrow$ Find formula for Area, take derivative, set equal to 0.



$$\Rightarrow A = l \cdot w$$
$$2400 = l + 2w$$
$$2400 - 2w = l$$

$$\Rightarrow A = (2400 - 2w)w = 2400w - 2w^2$$

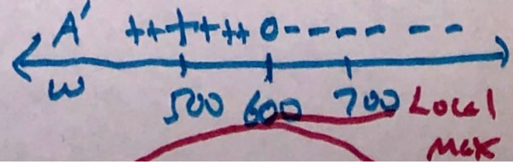
$$\therefore A' = 2400 - 4w = 0$$
$$2400 = 4w$$
$$\therefore w = 600$$

Dim that maximize area are:

$w: 600$

$l = 1200$

$\Rightarrow 720{,}000\text{ ft}^2$



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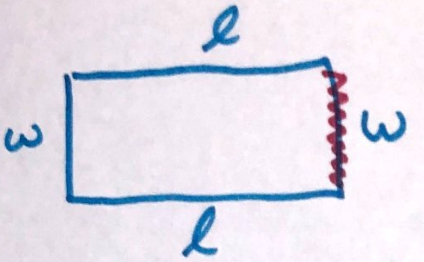
Ex: A rancher is building a 12,000 ft^2 rectangular fence. 3 sides are built with cheap fencing material, \$2/ft, but the 4^{th} side is built with a expensive material that costs \$6/ft. What is the cheapest cost for this fence?



Minimize cost,

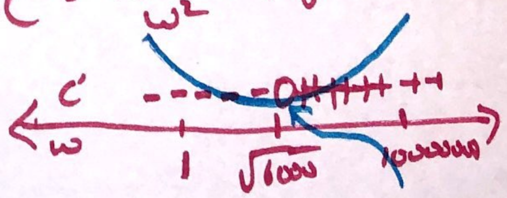
$$C = 2l + 2w + 2l + 6w$$

$$= 4l + 8w$$

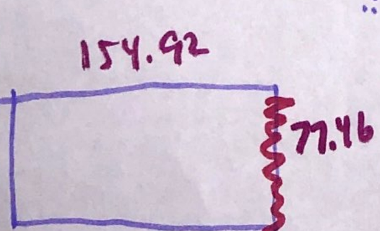
$$12{,}000 = l \cdot w \Rightarrow \boxed{l = \frac{12{,}000}{w}}$$

$$\therefore C = 4\left(\frac{12{,}000}{w}\right) + 8w = 48{,}000 \cdot w^{-1} + 8w \longrightarrow C' = \frac{-48000}{w^2} + 8$$

$$\therefore C' = -48{,}000w^{-2} + 8 = 0$$



$$\therefore 8 = \frac{48000}{w^2}$$



$$w^2 = \frac{48000}{8} = \frac{24{,}000}{4} = \frac{12{,}000}{2} = 6000$$

$$\therefore w = \sqrt{6000}$$

But $l = \frac{12{,}000}{\sqrt{6000}} \Rightarrow \boxed{4\left(\frac{12000}{\sqrt{6000}}\right) + 8\sqrt{6000}}$