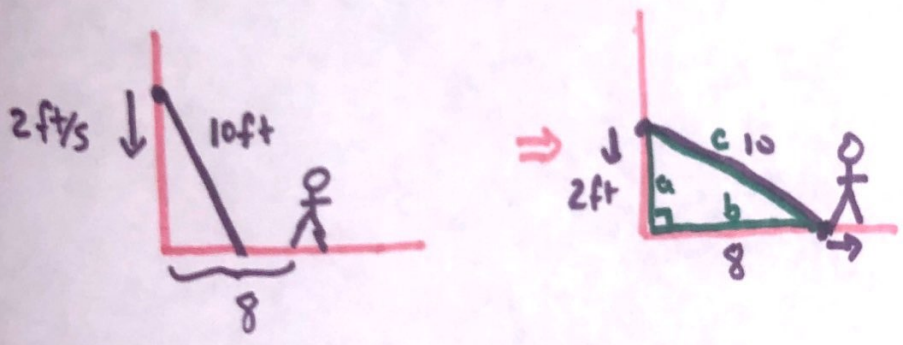
§3.9 - Related Rates, More Examples

A 10 foot ladder is sliding down a wall at 2 ft/sec. You are standing 8 feet away from the wall when the ladder hits your foot. How fast is the base of the ladder moving when it hits your foot?



$a^2+b^2=c^2 \Rightarrow (a(t))^2+(b(t))^2=10$

$a^2+8^2=10^2$

$a^2=100-64=36$

$a=6$

$2\cdot a(t)\cdot\frac{da}{dt}+2b(t)\cdot\frac{db}{dt}=0$

$6(-2)+8\cdot\frac{db}{dt}=0$

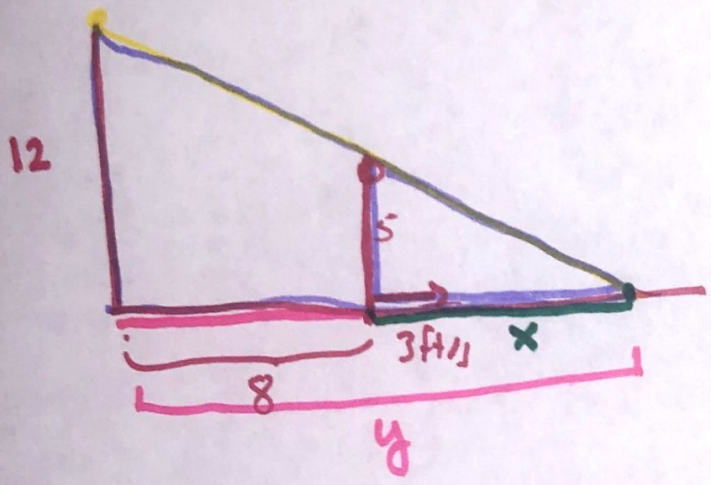
$8\frac{db}{dt}=12$

$\therefore \frac{db}{dt}=\frac{12}{8}=3/2 \text{ ft/s}$

A streetlight is on top of a 12ft pole. You, a 5 ft tall person, walk away from the streetlight at 3ft/s. When you are 8 ft away from the pole

(a) How fast is your shadow growing → $\frac{15}{7}$ ft/s

(b) " " " the tip of your shadow moving away from the pole. → $\frac{26}{7}$ ft/s

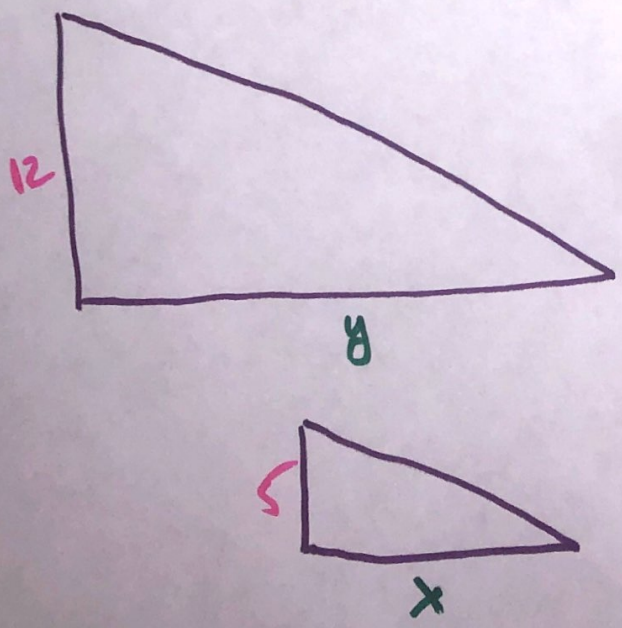


These two right triangles are similar, aka they have equal ∠'s, aka their sides are proportional.

$$\frac{12}{y} = \frac{5}{x} \Rightarrow 12x = 5y$$

$$\Rightarrow 12 \cdot x(t) = 5 \cdot y(t)$$

$$\therefore 12 \cdot \frac{dx}{dt} = 5 \cdot \frac{dy}{dt}$$



$$y - x = 8$$

$$\frac{dy}{dt} - \frac{dx}{dt} = 3$$

$$\frac{dy}{dt} = 3 + \frac{dx}{dt}$$

$$12\frac{dx}{dt} = 5\left(3 + \frac{dx}{dt}\right)$$

$$12\frac{dx}{dt} = 15 + 5\frac{dx}{dt}$$

$$7\frac{dx}{dt} = 15$$

$$\therefore \boxed{\frac{dx}{dt} = \frac{15}{7}} \quad \frac{dy}{dt} = 3 + \frac{15}{7} = \frac{36}{7}$$