§2.3 - Calculating Limits using Limit Laws, part 1

In this video, we will:

- List the limit laws
- Compute limits except one case, $\frac{0}{0}$

Suppose $\lim_{x\to a} f(x) = L$, $\lim_{x\to a} g(x) = M$, where L, M are finite numbers. Also, c is a constant:

1. $\lim_{x\to a} f(x) \pm g(x) = L \pm M$
2. $\lim_{x\to a} c \cdot f(x) = c \cdot L$
3. $\lim_{x\to a} f(x) \cdot g(x) = L \cdot M$
4. $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{L}{M}$ unless $M = 0$

Note: We will consider $a = \infty$, $a = -\infty$ later

Ex: $\lim_{x \to 4} 3x^2 - 2x + 3\sqrt{x}$

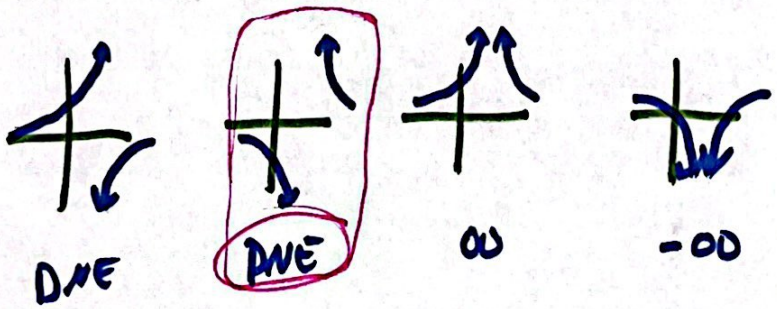
$3(4)^2 - 2(4) + 3\cdot\sqrt{4}$

$3(16) - 8 + 6 = 48 - 8 + 6 = \underline{46}$

Ex: $\lim_{x \to \pi} x^2 \cdot \cos(x) \Rightarrow \pi^2 \cdot \cos(\pi)$

$= \pi^2(-1) = \underline{-\pi^2}$

Ex: $\lim_{x \to 2} \frac{x}{x-2} \rightarrow \frac{2}{0}$, stop $\frac{\text{non-zero}}{\text{zero}} \Rightarrow$ V. Asy.



$\lim_{x \to 2^+} \frac{x}{x-2} \Rightarrow$ Try $2.1 \Rightarrow \frac{2.1}{2.1-2} = \frac{2.1}{0.1} \Rightarrow \frac{+}{+} \Rightarrow +\infty$.

$\lim_{x \to 2^-} \frac{x}{x-2} \Rightarrow$ Try $1.9 \Rightarrow \frac{1.9}{1.9-2} \Rightarrow \frac{1.9}{-0.1} \Rightarrow \frac{+}{-} \Rightarrow -\infty$ ..

Find $\lim_{x\to 0} f(x)$ and $\lim_{x\to 4} f(x)$, where

$\lim_{x\to 0} f(x) = 1$ $\quad$ $\lim_{x\to 4} f(x)$ DNE

$$f(x) = \begin{cases} x+1 & 0 \le x \\ x^2+1 & 0 < x < 4 \\ \sqrt{x} & x \ge 4 \end{cases}$$

$$\lim_{x\to 0} f(x) \Rightarrow \lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} x^2+1 = 0^2+1 = 1$$

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} x+1 = 0+1 = 1$$

$$\lim_{x\to 4} f(x) \Rightarrow \lim_{x\to 4^+} f(x) = \lim_{x\to 4^+} \sqrt{x} = \sqrt{4} = \underline{2}$$

$$\lim_{x\to 4^-} f(x) \Rightarrow \lim_{x\to 4^-} x^2+1 = 4^2+1 = \underline{17}$$