§2.8 - The Derivative as a Function - Part 1

In this video, we will:

- Define the derivative of a function, $f'(x)$
- Compare graphs of $f(x)$ and $f'(x)$ $\quad f'(a)$
- Find $f'(x)$

Given a function f, we define a new function called the derivative of $f(x)$, $f'(x)$:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Ex: $f(x) = 2x^2 - x$, find $f'(x)$.

$$f'(x) = \lim_{h \to 0} \frac{[2(x+h)^2 - (x+h)] - [2x^2 - x]}{h} =$$

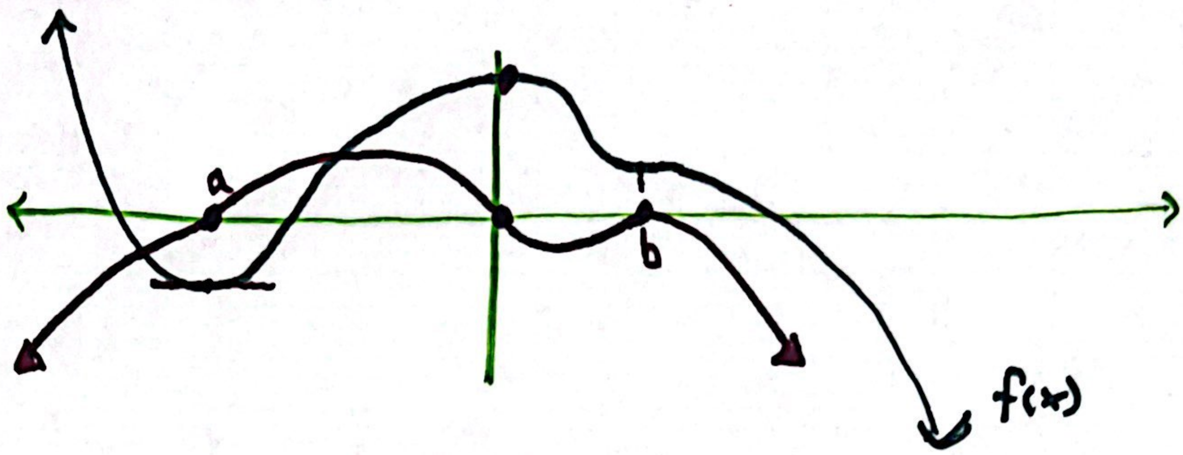
$$\lim_{h \to 0} \frac{2(x^2 + 2xh + h^2) \cancel{-x} - h - 2x^2 \cancel{+x}}{h} = \lim_{h \to 0} \frac{\cancel{2x^2} + 4xh + 2h^2 - h \cancel{-2x^2}}{h}$$

$$= \lim_{h \to 0} \frac{2h^2 - h + 4xh}{h} = \lim_{h \to 0} 2h - 1 + 4x = \boxed{4x - 1}$$

$f(x) = 2x^2 - x$, $\quad f'(1) = 4(1) - 1 = 3$

$\quad f'(10) = 4(10) - 1 = 39$

Graph $f'(x)$ on the same as $f(x)$



$f'(x)$ is the slope of $f(x)$ at x.

$f'(a) = 0$

$f'(0) = 0$

$f'(b) = 0$

from $-\infty < x < a$, $f(x)$ is decreasing

$\Leftrightarrow f'(x)$ is negative

Find $f'(x)$ when $f(x)=\sqrt{x}$

$$f'(x)=\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}\cdot\left(\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}\right)$$

$$=\lim_{h\to 0}\frac{\cancel{(x+h)-x}}{(\sqrt{x+h}+\sqrt{x})\cancel{h}}=\lim_{h\to 0}\frac{1}{\sqrt{x+h}+\sqrt{x}}=\frac{1}{2\sqrt{x}}$$

$$=\frac{1}{2}x^{-1/2}$$