4.2 - Mean Value Theorem, MVT

Rolle's Theorem: If $f(x)$ is a fn s.t.

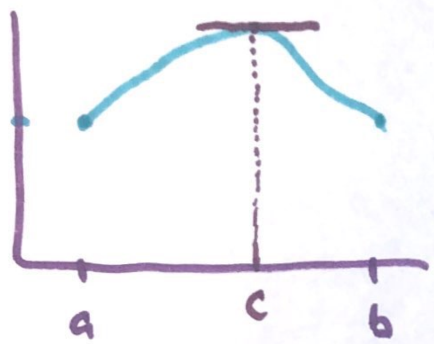
① $f(x)$ is continuous on $[a,b]$

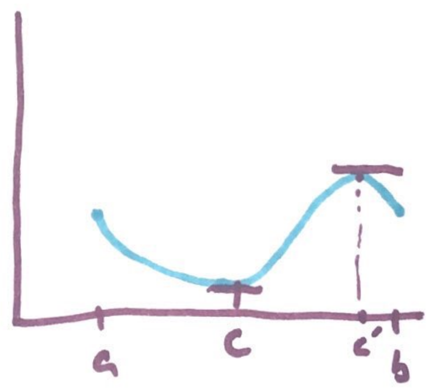
② $f(x)$ is differentiable on (a,b)

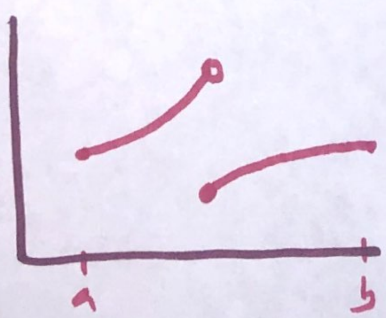
③ $f(a) = f(b)$

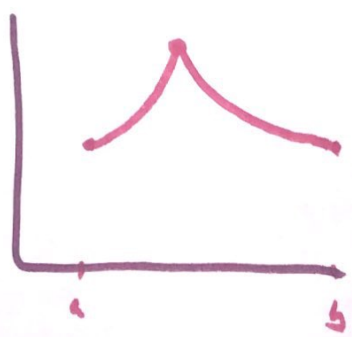
Then there is an x-value c such that

$$a < c < b \text{ AND } f'(c) = 0$$









Mean Value Theorem (MVT): If $f(x)$ is a function such that:

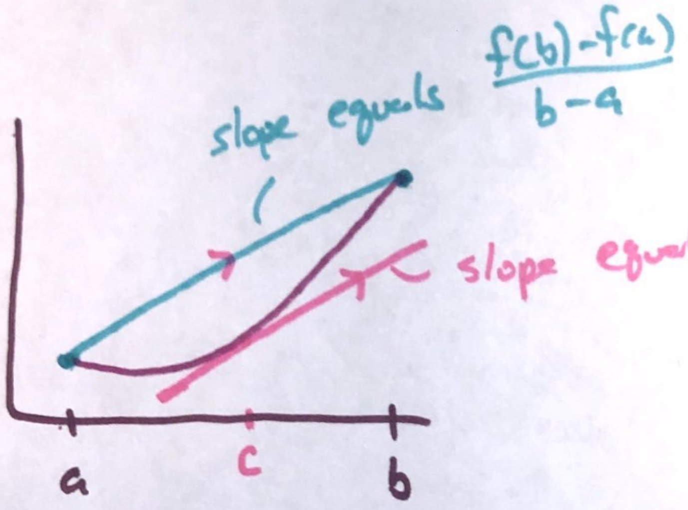
① $f(x)$ is continuous on $[a, b]$

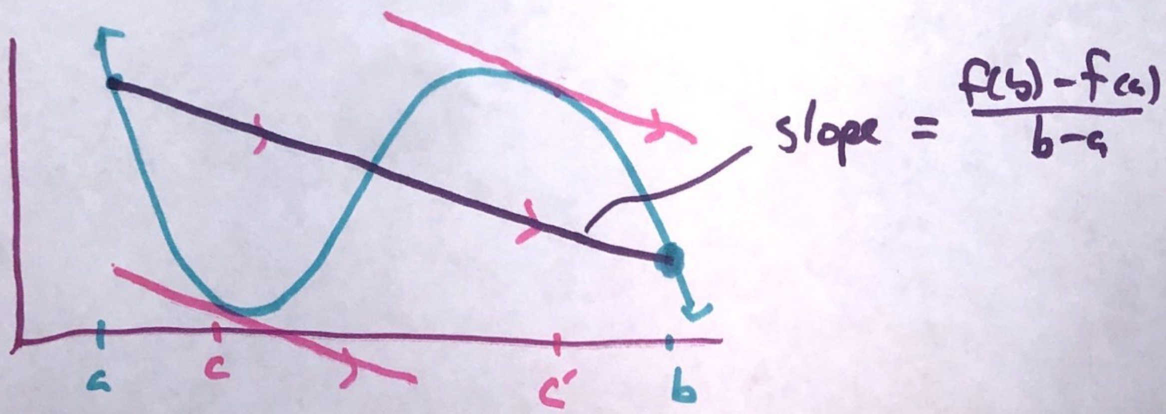
② $f(x)$ is differentiable on (a, b)

Then there is an x-value c such that

$a < c < b$ AND $f'(c) = \dfrac{f(b) - f(a)}{b - a}$ $\Leftrightarrow$

$f(b) - f(a) = f'(c)(b-a)$



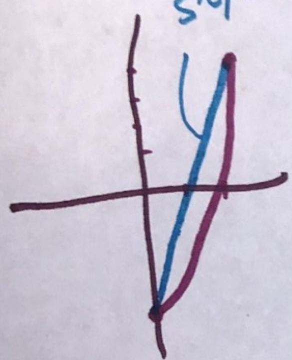


Determine if $f(x) = x^2+2x-4$ satisfies the MVT on the interval $[0,2]$ and if it does find all values of c which satisfy the MVT

on $[0,2]$, $f(x)$ is cts + differentiable $\therefore$ satisfies MVT

find x-value c s.t. $f'(c) = \frac{f(2)-f(0)}{2-0}$

$$\frac{f(2)-f(0)}{2-0} = \frac{[4+4-4]-[0+0-4]}{2} = \frac{4-(-4)}{2} = \frac{8}{2} = 4$$



where is c s.t. $f'(c) = 4$

$f'(c) = 2c+2 = 4$

$\therefore 2c = 2$

$\therefore c = 1$

Check: $f'(1) = 2(1)+2 = \underline{4}$