10/03/2023

Last Time: Chain Rule
Implicit Differentiation

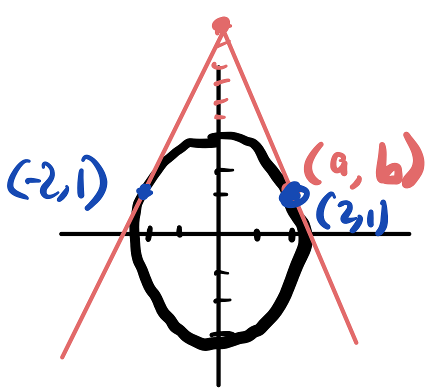
Today: Implicit Differentiation
Derivatives of Logarithms
" of Inverse Trig
" " Inverse Functions

Exponential Growth & Decay

Future: HW Due W

Exam II on Tuesday, Oct 17th

Find the equations of the lines tangent to $2x^2+y^2=9$ that passes thru $(0,9)$.



we want (a,b) s.t. y' at (a,b) equals the slope from (a,b) to $(0,9)$

Implicit Diff: $[2x^2+y^2]'=[9]'$

$$4x+2yy'=0$$

$$y'=\frac{-2x}{y} \Rightarrow y'=\frac{-2a}{b}$$

x-int $\Rightarrow y=0 \Rightarrow x^2=9/2,\ x=\pm\sqrt{4.5}$

y-int $\Rightarrow x=0 \Rightarrow y^2=9,\ y=\pm 3$

Slope btwn pts: $\frac{y_2-y_1}{x_2-x_1}=\frac{9-b}{0-a}=\frac{9-b}{-a}$

$$\Rightarrow \frac{-2a}{b}=\frac{9-b}{-a} \Rightarrow 2a^2=9b-b^2$$

$a=2,\ b=1,\ m=\frac{-2a}{b}=-4$

$y-1=-4(x-2) \Rightarrow \boxed{y=-4x+9}$

$2a^2+b^2=9b \Rightarrow 2a^2=8$

$9=9b$

$b=1$

$a=\pm 2$

$a=-2,\ b=1,\ m=\frac{-2a}{b}=4$

$y-1=4(x+2) \Rightarrow \boxed{y=4x+9}$

$y=\ln(x)$, what is y'?

$\Rightarrow e^y=e^{\ln(x)}=x \Rightarrow x=e^y \xRightarrow{\text{Implicit Diff}} [x]'=[e^y]'$

$\Rightarrow 1=e^y\cdot y'$

$\Rightarrow y'=\frac{1}{e^y}=\frac{1}{x}$

$y=\ln(x),\ y'=\frac{1}{x}$

Ex: Find $f'(x)$:

$f(x)=\ln(x^2+3x) \Rightarrow f'(x)=\frac{1}{x^2+3x}\cdot[x^2+3x]'=\boxed{\frac{2x+3}{x^2+3x}}$

$f(x)=x^2\cdot\ln(3x+1) \Rightarrow 2x\cdot\ln(3x+1)+\frac{3x^2}{3x+1}$

$f(x)=\frac{(\ln(x))^2}{x} \Rightarrow \frac{[2\ln(x)\cdot\frac{1}{x}]x-(\ln(x))^2}{x^2}=\frac{2\ln(x)-(\ln(x))^2}{x^2}$

$f(x)=\ln(10x)=\frac{1}{10x}[10x]'=\frac{1}{10x}\cdot 10=\frac{1}{x}$

$[\ln(c\cdot x)]'=\frac{1}{cx}\cdot c=\frac{1}{x}$

$$f(x) = \ln(|3x|) \Rightarrow \frac{1}{|3x|} \cdot [|3x|]' = \frac{1}{|3x|} \cdot \frac{3x}{|3x|} \cdot 3 = \frac{9x}{(|3x|)^2}$$

$$[|x|]' = \frac{x}{|x|}$$

$y = \sin^{-1}(x)$, find y'

$$\Rightarrow x = \sin(y) \Rightarrow 1 = \cos(y) \cdot y' = y' = \frac{1}{\cos(y)}$$

$$\sin^2(y) + \cos^2(y) = 1$$

$$\cos^2(y) = 1 - \sin^2(y) = 1 - x^2$$

$$\cos(y) = \sqrt{1-x^2}$$

$$y = \sin^{-1}(x)$$

$$y' = \frac{1}{\sqrt{1-x^2}}$$

$y = \tan^{-1}(x)$, find y'

$$x = \tan(y) \Rightarrow 1 = \sec^2(y) \cdot y' \Rightarrow y' = \frac{1}{\sec^2(y)}$$

$$\tan^2(x) + 1 = \sec^2(x)$$

$$\tan^2(y) + 1 = \sec^2(y)$$

$$\Rightarrow \frac{1}{\tan^2(y)+1} = \frac{1}{x^2+1}$$

$$[\sin^{-1}(x/3)]' \Rightarrow \frac{1}{\sqrt{1-(x/3)^2}} \cdot \frac{1}{3} = \frac{1}{3} \cdot \frac{1}{\sqrt{1-\frac{x^2}{9}}}$$

$$= \frac{1}{3} \cdot \frac{1}{\sqrt{\frac{9-x^2}{9}}} = \frac{1}{3} \cdot \frac{1}{\frac{\sqrt{9-x^2}}{\sqrt{9}}}$$

$$[\tan^{-1}(x^2)]'$$

$$= \frac{1}{1+(x^2)^2}[x^2]' \qquad = \frac{1}{\cancel{3}} \cdot \frac{\cancel{3}}{\sqrt{1-x^2}} = \frac{1}{\sqrt{9-x^2}}$$

$$= \frac{2x}{1+x^4}$$

$f(x) = x + \cos(x)$, find $(f^{-1})'(1)$

$x = y + \cos(y)$, we cannot solve for y, but thats ok.

Given $f(x) = y$, find $(f^{-1})'$

$$\Rightarrow x = f^{-1}(y) \Rightarrow 1 = [f^{-1}(y)]' \cdot y'$$

$$\therefore [f^{-1}(y)]' = \frac{1}{y'} = \frac{1}{(f(x))'}$$

$$[f^{-1}(y)]' = \frac{1}{f'(f^{-1}(y))}$$

$$\Rightarrow \boxed{(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}}$$

$f(x) = x + \cos(x)$, find $(f^{-1})'(1)$

$$\Rightarrow \frac{1}{f'(f^{-1}(1))}, \quad f(x) = x + \cos(x)$$

$$1 = x + \cos(x)$$

$$x = 0 \Rightarrow f^{-1}(1) = 0$$

$$\Rightarrow \frac{1}{f'(0)} \rightarrow f(x) = x + \cos(x)$$

$$f'(x) = 1 + \sin(x)$$

$$= \boxed{1} \quad f'(0) = 1 + \sin(0) = 1$$

The Law of Natural Growth.

$P(t)$ is the population of a colony. then

$$P'(t) = a \cdot P(t)$$

$$\Rightarrow P(t) = c \cdot e^{at}$$

Check: $[P(t)]' = [ce^{at}]' = C \cdot e^{at} \cdot [at]'$

$$= C \cdot e^{at} \cdot a$$

$$= a \cdot \underline{Ce^{at}} = a \cdot P(t)$$