Class 06

Last Time: Continuity, limits at $\pm\infty$

Today: limits at $\pm\infty$

Derivatives at a point, $f'(a)$

Future: Quest HW Due M

Gradescope HW Due W

Find $\displaystyle\lim_{x\to\infty} \frac{\sqrt{3x^6-4x^2}}{2x^3}$;

$$= \lim_{x\to\infty} \frac{\sqrt{x^6(3-4/x^4)}}{2x^3} = \lim_{x\to\infty} \frac{\sqrt{x^6}\sqrt{3-4/x^4}}{2x^3} =$$

$$\lim_{x\to\infty} \frac{\cancel{x^3}\sqrt{3-4/x^4}}{2\cancel{x^3}} \to \frac{\sqrt{3-0}}{2} = \frac{\sqrt{3}}{2}$$

$$\lim_{x\to-\infty} \frac{\sqrt{3x^6-4x^2}}{2x^3} \Rightarrow \lim_{x\to-\infty} \frac{\sqrt{x^6}\sqrt{3-4/x^4}}{2x^3} =$$

$$\lim_{x\to\infty} \frac{-\cancel{x^3}\sqrt{3-4/x^4}}{2\cancel{x^3}} \to \frac{-\sqrt{3-0}}{2} = \frac{-\sqrt{3}}{2}$$

$(a-b)(a+b) = a^2-b^2$

$$\lim_{x\to\infty} \sqrt{x^2+2x+2} - \sqrt{x^2-3x+3}\left(\frac{\sqrt{x^2+2x+2}+\sqrt{x^2-3x+3}}{\sqrt{x^2+2x+2}+\sqrt{x^2-3x+3}}\right) =$$

$$\lim_{x\to\infty}\frac{(x^2+2x+2)-(x^2-3x+3)}{\sqrt{x^2+2x+2}+\sqrt{x^2-3x+3}} = \lim_{x\to\infty}\frac{5x-1}{\sqrt{x^2+2x+2}+\sqrt{x^2-3x+3}}$$

$$=\lim_{x\to\infty}\frac{x(5-1/x)}{\sqrt{x^2(1+2/x+2/x^2)}+\sqrt{x^2(1-3/x+3/x^2)}}$$

$$=\lim_{x\to\infty}\frac{x(5-1/x)}{x\cdot\sqrt{1+2/x+2/x^2}+x\sqrt{1-3/x+3/x^2}}$$

$$=\lim_{x\to\infty}\frac{\cancel{x}(5-\cancel{1/x})}{\cancel{x}\left(\sqrt{1+\cancel{2/x}+\cancel{2/x^2}}+\sqrt{1-\cancel{3/x}+\cancel{3/x^2}}\right)} = \frac{5}{\sqrt{1}+\sqrt{1}} = \boxed{\frac{5}{2}}$$

The Derivative of $f(x)$ at $x=a$ is:

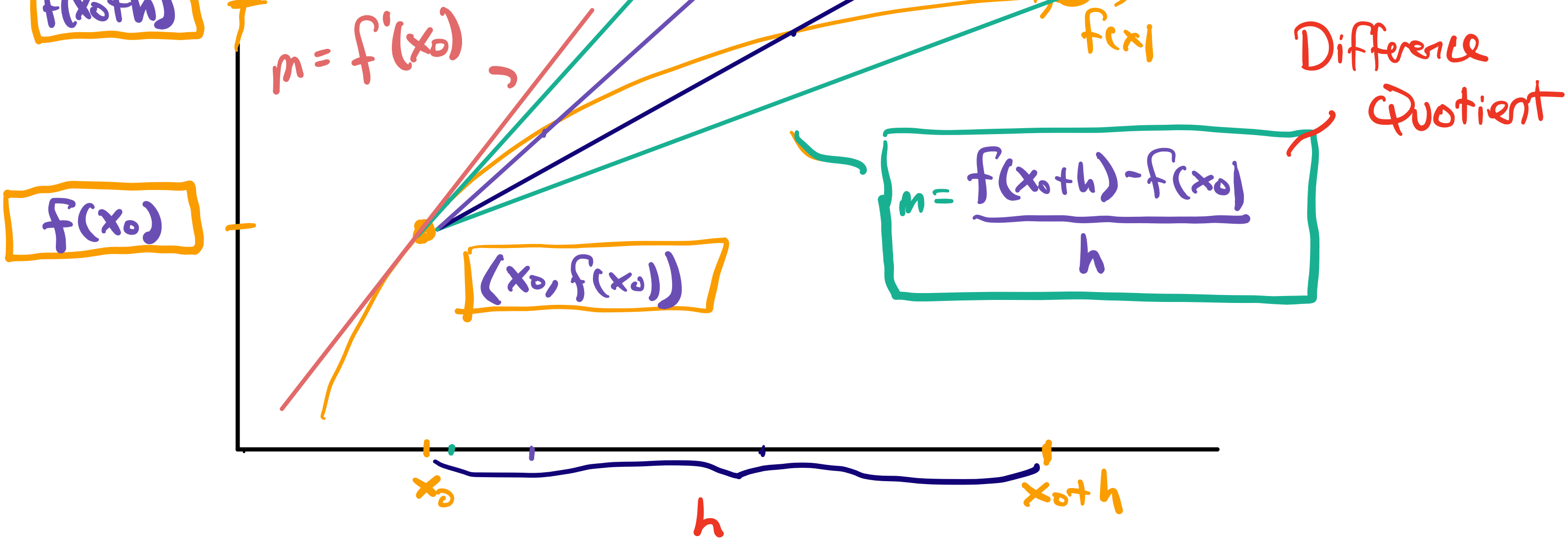
① $$\lim_{x\to a}\frac{f(x)-f(a)}{x-a} = f'(a) \qquad (x\to a)\text{-def}^n$$

② $$\lim_{h\to 0}\frac{f(a+h)-f(a)}{h} = f'(a) \qquad h\text{-def}^n$$

$f'(a)$ is the slope of the line tangent to f at $x=a$.

$f'(a)$ is also the instantaneous rate of change of f at $x=a$.

↳ $S'(t_0)=V(t_0)$ AND $V'(t_0)=a(t_0)$



Find $f'(2)$ when $f(x) = \frac{1}{x}$

Use $x-a$ - formula: $f'(2) = \lim_{x\to 2} \frac{f(x)-f(2)}{x-2}$

$$= \lim_{x\to 2} \frac{\frac{1}{x} - \frac{1}{2}}{x-2} = \lim_{x\to 2} \frac{\frac{2}{2x} - \frac{x}{2x}}{x-2} = \lim_{x\to 2} \frac{\frac{2-x}{2x}}{x-2} =$$

$$= \lim_{x\to 2} \frac{2-x}{2x} \div x-2 = \lim_{x\to 2} \frac{2-x}{2x} \cdot \frac{1}{x-2} = \lim_{x\to 2} \frac{-\cancel{(x-2)}}{2x\cancel{(x-2)}}$$

$$= \lim_{x\to 2} \frac{-1}{2x} = \boxed{-\frac{1}{4}}$$

Same Question, h-defn: $f'(2) = \lim_{h\to 0} \frac{f(2+h)-f(2)}{h}$

$$\lim_{h\to 0} \frac{\frac{1}{2+h} - \frac{1}{2}}{h} = \lim_{h\to 0} \frac{\frac{2}{2(2+h)} - \frac{2+h}{2(2+h)}}{h} = \lim_{h\to 0} \frac{\frac{2-(2+h)}{2(2+h)}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{-h}{2(2+h)}}{h} = \lim_{h\to 0} \frac{-\cancel{h}}{2(2+h)} \cdot \frac{1}{\cancel{h}} = \lim_{h\to 0} \frac{-1}{2(2+h)} = -\frac{1}{4}$$

Find the eqn of the line tangent to $f(x) = \frac{1}{x}$ at $x = 2$.

$y_0 = f(x_0)$

point-slope $\Rightarrow$ $y - y_0 = m(x - x_0)$

$m = f'(x_0)$
$= f'(2)$

$y - \frac{1}{2} = -\frac{1}{4}(x - 2)$

$y - \frac{1}{2} = -\frac{1}{4}x + \frac{1}{2}$

$y = -\frac{1}{4}x + 1$

$\lim_{x \to 2} \frac{x^5 - 32}{x - 2} = f'(a)$, where $f(x) = x^5$
$a = 2$

$\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

$32 = 2^5 = 32$ ✓

$\lim_{h \to 0} \frac{\cos(\pi + h) + 1}{h}$ $\quad - (-1)$

$f(x) = \cos(x)$
$a = \pi$

$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

$f(a) = \cos(\pi) = -1$

Find the derivative of $x \cdot f(x)$ at $x = 5$ using h-defn.

$$\lim_{h \to 0} \frac{\left[(5+h)\, f(5+h)\right] - \left[5 f(5)\right]}{h} =$$