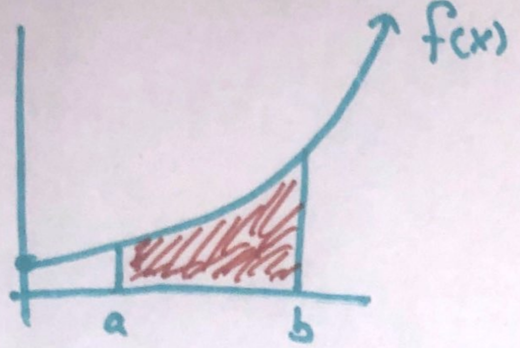
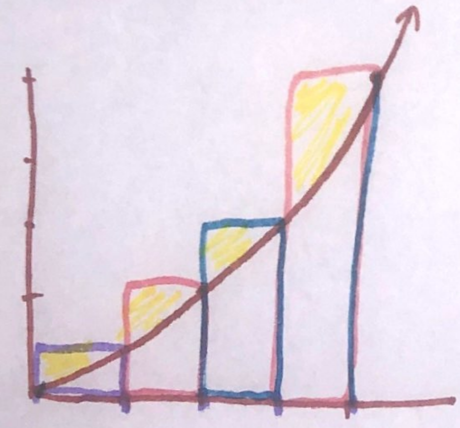
§ 5.1 - Areas + Distance

- Big Unanswered question for us as math students: How do we compute the area under a graph?



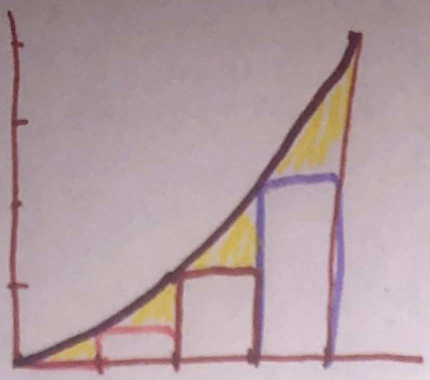
First, we will try to make an accurate guess.

- Ex: Approximate the area under $y = x^2$ between $x = 0$ and $x = 2$



I will approx this area with 4 rectangles + right endpoints.

$$\text{Area} = \text{Sum of 4 rectangles}$$
$$= b_1h_1 + b_2h_2 + b_3h_3 + b_4h_4$$
$$= \tfrac{1}{2}\left(\left(\tfrac{1}{2}\right)^2 + (1)^2 + \left(\tfrac{3}{2}\right)^2 + 2^2\right)$$
$$= \tfrac{1}{2}\left(\tfrac{1}{4} + \tfrac{4}{4} + \tfrac{9}{4} + \tfrac{16}{4}\right) = \tfrac{1}{2}\left(\tfrac{30}{4}\right) =$$
$$= \tfrac{15}{4} = \boxed{3.75}$$



Same thing, but left-endpoints

Area = Sum of rectangles

$= \frac{1}{2}(0^2 + 1/4 + 1 + 9/4)$

$= \frac{1}{2}\left(\frac{14}{4}\right) = 7/4 = 1.75$

~~Actual~~

1.75 < Actual Area < 3.75

Our ultimate goal is to make this guess exactly correct.

Area is approx the sum of rectangles

$$= \sum_{i=1}^{n} b_i \cdot h_i \quad - \quad b_i = \Delta x = \frac{b-a}{n}$$

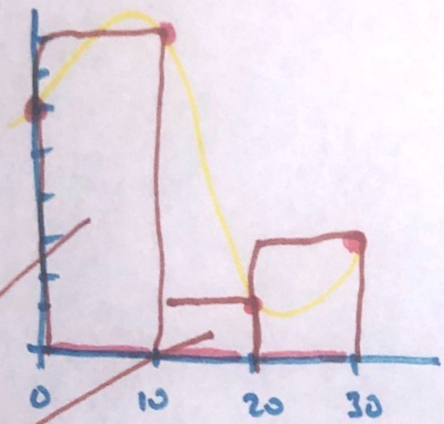
$$= \sum_{i=1}^{n} f(x_i^*)\Delta x \text{ , } x_i^* \text{ is any point in the } i^{th} \text{ subinterval.}$$

The <u>Riemann Sum</u> is $\underline{\sum_{i=1}^{n} f(x_i^*)\Delta x}$, $\Delta x = \frac{b-a}{n}$, x_i^* any x-value in i^{th} interval, and it approximates the area under f(x) between x=a + x=b.

Ex: Suppose the velocity of a car at certain times is given in the chart:

t	0 s	10 s	20 s	30 s
$v(t)$	6 m/s	8 m/s	1 m/s	3 m/s

Use 3 equal subintervals and <u>right</u> <u>endpoints</u> to approximate the area under the velocity graph



$$\sum_{i=1}^{3} f(x_i^*)\cdot\frac{30-0}{3} = \sum_{i=1}^{3} f(x_i^*)\cdot 10$$

$$= \big(v(10)+v(20)+v(30)\big)\,10$$

$$= (8+1+3)\cdot 10$$

$$= 12\cdot 10$$

$$= \underline{120}\text{ m}$$

$b\cdot h = (10\,s)\cdot(8\,m/s) = 80\,m$

$b\cdot h = (10\,s)(1\,m/s) = 10\,m$

$= 30\,m$

$120\,m$

If $v(t)$ is the velocity function, and $v(t)$ is positive, then the area under $v(t)$ is distance traveled.

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