Hwk#19 Sec 3.11 (#3,7,8,9,25,27ab,35,37,39,41,45)

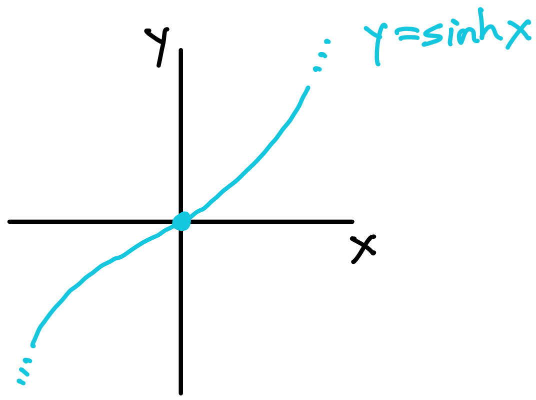
Hyperbolic trig functions

Defn The hyperbolic trig funcs are

$$\sinh x = \frac{e^x - e^{-x}}{2}, \quad \cosh x = \frac{e^x + e^{-x}}{2}, \quad \tanh x = \frac{\sinh x}{\cosh x}$$

$$\operatorname{csch} x = \frac{1}{\sinh x}, \quad \operatorname{sech} x = \frac{1}{\cosh x}, \quad \coth x = \frac{1}{\tanh x}$$

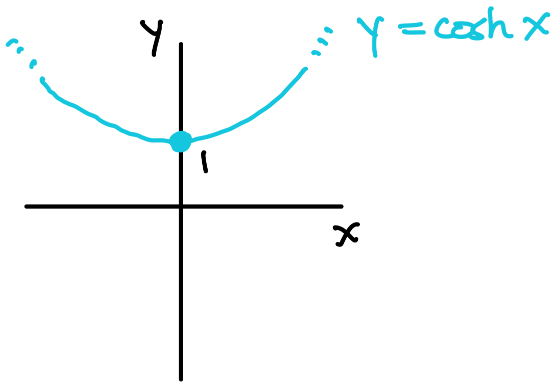
Pictures



Dom $= (-\infty, \infty)$

Rng $= (-\infty, \infty)$

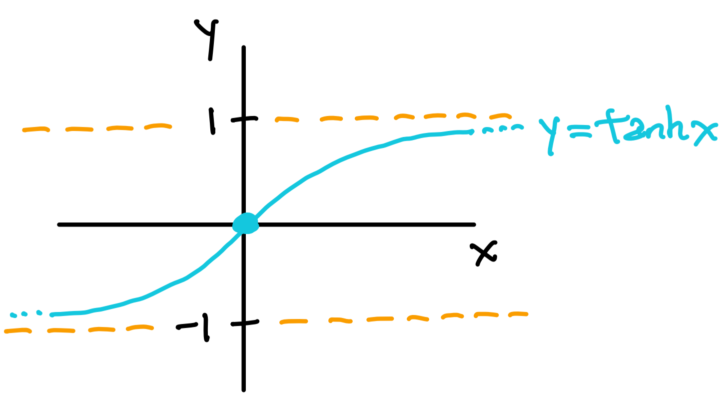
$\sinh x \to \pm\infty$ as $x \to \pm\infty$



Dom $= (-\infty, \infty)$

Rng $= [1, \infty)$

$\cosh x \to \infty$ as $x \to \pm\infty$



Dom $= (-\infty, \infty)$

Rng $= (-1, 1)$

$\tanh x \to \pm 1$ as $x \to \pm\infty$

Examples ① Verify $\sinh x \to \pm\infty$ as $x \to \pm\infty$.

$$\lim_{x\to\infty} \sinh x = \lim_{x\to\infty} \frac{e^x - e^{-x}}{2} = \frac{\infty - 0}{2} = \infty$$

Recall: $e^{\infty} = \infty$, $e^{-\infty} = 0$

$$\lim_{x\to-\infty} \sinh x = \lim_{x\to-\infty} \frac{e^x - e^{-x}}{2} = \frac{0-\infty}{2} = -\infty \;//$$

② $\cosh(3) = ?$, $\cosh(\ln 3) = ?$

$$\cosh(3) = \frac{e^3 + e^{-3}}{2} = \frac{e^3 + \frac{1}{e^3}}{2} = 10.07$$

$$\cosh(\ln 3) = \frac{e^{\ln 3} + e^{-\ln 3}}{2} = \frac{e^{\ln 3} + \frac{1}{e^{\ln 3}}}{2} = \frac{3 + \frac{1}{3}}{2} = \frac{5}{3} \;//$$

$\blacksquare$

Hyperbolic trig identities

$\sinh(-x) = -\sinh(x)$, $\cosh(-x) = \cosh(x)$

$\cosh^2(x) - \sinh^2(x) = 1$, $1 - \tanh^2(x) = \operatorname{sech}^2(x)$

$$\sinh(A+B) = \sinh A \cosh B + \sinh B \cosh A$$
$$\cosh(A+B) = \cosh A \cosh B + \sinh A \sinh B$$

$\blacksquare$

Examples ① Verify $\sinh(-x) = -\sinh(x)$.

$$\sinh(-x) = \frac{e^{(-x)} - e^{-(-x)}}{2} = \frac{e^{-x} - e^{x}}{2} = -\left(\frac{e^x - e^{-x}}{2}\right)$$
$$= -\sinh(x) \;//$$

② Verify $\cosh^2(x) - \sinh^2(x) = 1$.

$$\cosh^2(x) - \sinh^2(x) = \left(\frac{e^x + e^{-x}}{2}\right)^2 - \left(\frac{e^x - e^{-x}}{2}\right)^2$$

$$= \frac{e^{2x} + 2 + e^{-2x}}{4} - \frac{e^{2x} - 2 + e^{-2x}}{4}$$

$$= \frac{2+2}{4} = 1$$

③ If $\sinh x = 2$, then $\cosh x = ?$, $\tanh x = ?$

$$\cosh^2 x - \underbrace{\sinh^2 x}_{4} = 1$$

$$\cosh^2 x = 5$$

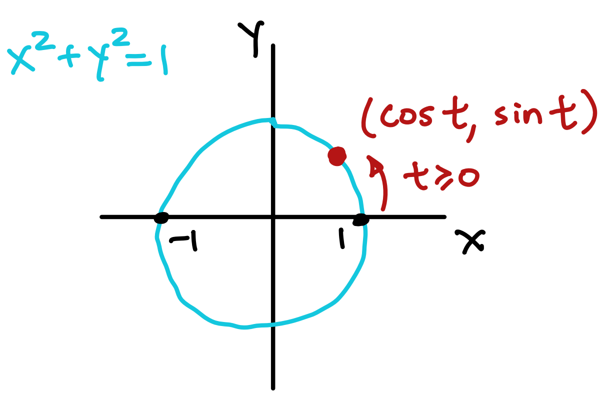
$$\cosh x = \pm\sqrt{5}$$

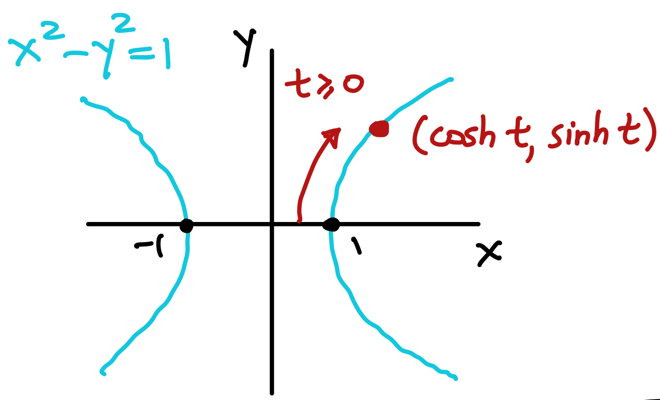
$$\cosh x = +\sqrt{5}$$

(b/c $\cosh x \geq 1$)

$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{2}{\sqrt{5}}$$

<u>Relation to hyperbola</u> Just as $(\cos t, \sin t)$ is a point on the unit circle, $(\cosh t, \sinh t)$ is a point on the unit hyperbola.





Hyperbolic trig derivatives

$[\sinh x]' = \cosh x$, $[\operatorname{csch} x]' = -\operatorname{csch} x \coth x$

$[\cosh x]' = \sinh x$, $[\operatorname{sech} x]' = -\operatorname{sech} x \tanh x$

$[\tanh x]' = \operatorname{sech}^2 x$, $[\coth x]' = -\operatorname{csch}^2 x$

Examples ① Verify $[\sinh x]' = \cosh x$.

$$\sinh x = \frac{1}{2}(e^x - e^{-x})$$

$$[\sinh x]' = \frac{1}{2}[\underbrace{e^x}_{e^x} - \underbrace{e^{-x}}_{e^{-x}(-1)}]'$$

$$= \frac{1}{2}(e^x + e^{-x})$$

$$= \cosh x$$

② $f(x) = x \tanh(2x+1)$. $f'(-\frac{1}{2}) = ?$

$$f'(x) = 1 \cdot \tanh(2x+1) + x \cdot \operatorname{sech}^2(2x+1) \cdot 2$$

$$f'(-\tfrac{1}{2}) = \underbrace{\tanh(0)}_{0} - \underbrace{\operatorname{sech}^2(0)}_{1^2}$$

Note:
$\sinh(0) = 0$
$\cosh(0) = 1$

$$= -1$$