11/07/2023

Last Time: Distance + Area

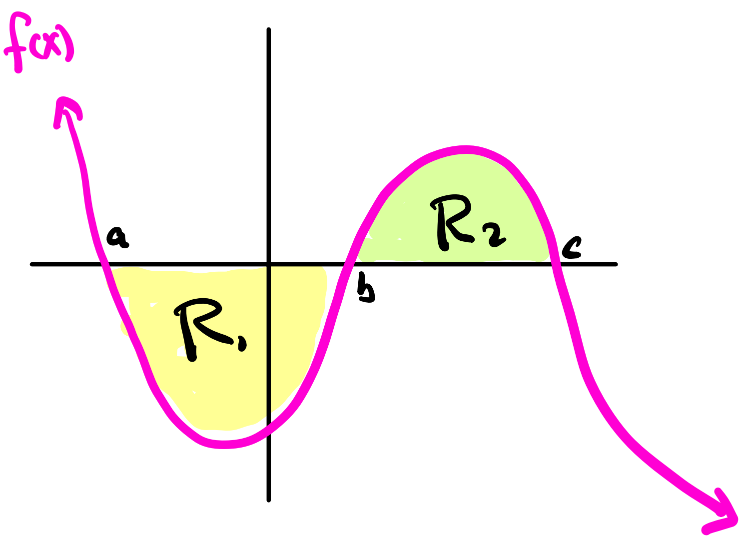
The Definite Integral

Today: The Fundamental Theorem of Calculus

↳ FTC pt 1

↳ FTC pt 2

Future: HWs



$$\text{Area}(R_1) = 6 \iff \int_a^b f(x)\,dx = -6$$

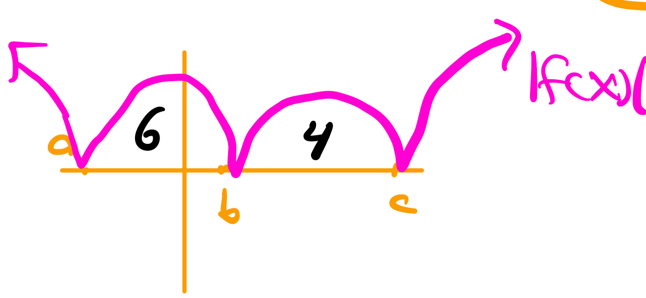
$$\text{Area}(R_2) = 4 \iff \int_b^c f(x)\,dx = 4$$

$$\int_a^c 3\cdot f(x)\,dx + \int_c^b 2f(x)\,dx = 3\int_a^c f(x)\,dx - 2\int_b^c f(x)\,dx$$

$$= 3(-6+4) - 2(4) = -6-8$$

$$-14$$

$$\int_a^c 2|f(x)| + \left|\int_a^c 5f(x)\,dx\right|$$



$$2(6+4) + 5\cdot\left|(-6+4)\right|$$

$$2(10) + 5\cdot|-2| =$$

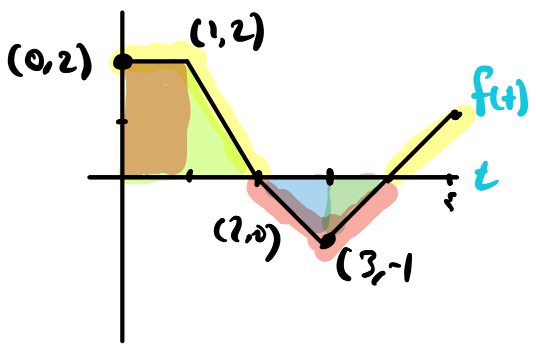
$$20 + 10 = 30$$

FTC I - Theoretical, not very practical

FTC II - Not Theoretical, very practical.

Area Function (not a great name)

$$F(x) = \int_a^x f(t)\,dt$$



$$F(x) = \int_0^x f(t)\,dt$$

$$F(0) = \int_0^0 f(t)\,dt = 0$$

$$F(1) = \int_0^1 f(t)\,dt = 2\cdot 1 = 2$$

$$F(2) = \int_0^2 f(t)\,dt = 2 + \tfrac{1}{2}(1)(2) = 3$$

$$F(3) = \int_0^3 f(t)\,dt = 3 - \tfrac{1}{2}(1)(1) = 5/2$$

$$F(4) = \int_0^4 f(t)\,dt = 5/2 - \tfrac{1}{2}(1)(1) = 2$$

$$F(5) = \int_0^5 f(t)\,dt = 2 + \tfrac{1}{2}(1)(1) = 5/2$$

FTC I: If f is continuous from $[a,b]$ then $g(x) = F(x) = \int_a^x f(t)\,dt$ is

(a) continuous on $[a,b]$

(b) differentiable on (a,b)

(c) $g'(x) = F'(x) = \left[\int_a^x f(t)\,dt\right]' = f(x)$

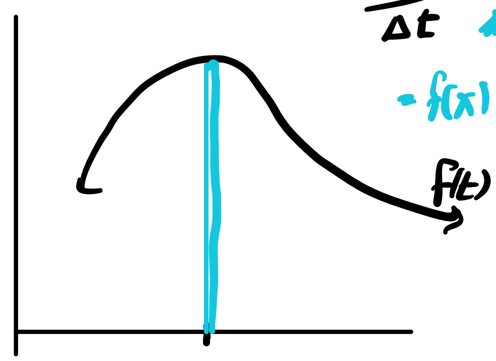
$$g(x) = \int_2^x t\cdot\sin(t^2)\,dt, \quad g'(x) = x\sin(x^2)$$

$$F(x) = \int_x^{-1} e^{4-t^3}\,dt, \quad F'(x) = -e^{4-x^3}$$

$$= -\int_{-1}^{x} e^{4-t^3}\,dt$$

$F'(x)$ measures $\frac{\Delta A}{\Delta t} \approx$

$= f(x)$



$$F(x) = \int_1^{x^2} t\cdot\ln\sqrt{t}\,dt, \quad F'(x) = \left[x^2\cdot\ln\sqrt{x^2}\right]\cdot 2x = 2x^3\cdot\ln x$$

↳ $[g(x^2)]' = g'(x^2)\cdot 2x$

$$g(x) = \int_0^{\sqrt{x}} 3t\cos(\pi t^2)\,dt, \text{ find } g'(2)$$

$$f(x) = \int_0^{g(x)} \frac{1}{\sqrt{1+t^3}}\,dt, \quad g(x) = \int_0^{\cos(x)} [1+\sin(t^2)]\,dt, \text{ find } f'(\pi/2)$$

$$g'(x) = \left[3\cdot\sqrt{x}\cdot\cos(\pi\cdot x)\right]\cdot\left[x^{1/2}\right]' = 3\cancel{\sqrt{x}}\cdot\cos(\pi x)\cdot\frac{1}{2\cancel{\sqrt{x}}}$$

$$= \frac{3}{2}\cos(\pi x), \quad g'(2) = \frac{3}{2}\cos(2\pi) = 3/2$$

$$\rightarrow f'(x) = \frac{1}{\sqrt{1+[g(x)]^3}}\, g'(x), \quad g'(x) = \left[1+\sin((\cos^2(x))\right]\cdot\left[\cos(x)\right]'$$

$$= -\sin(x)\left[1+\sin(\cos^2(x))\right]$$

$$f'(\pi/2) =$$

FTC II - If f is continuous on $[a, b]$ then

$$\int_a^b f(x)\,dx = F(b) - F(a), \text{ where } F'(x) = f(x)$$

$$= F(x)\Big|_a^b$$

$$\int_0^2 2e^x + 3x^2\,dx = \left[2e^x + x^3 + C\right]_0^2 = \left[2e^2 + 8 + \cancel{C}\right] - \left[2e^0 + 0 + \cancel{C}\right]$$

$$= \left[2e^2 + 8\right] - \left[2\right]$$

$$= 2e^2 + 6$$

$$\int_1^4 6\sqrt{x} - \frac{4}{\sqrt{x}}\,dx = \int_1^4 6x^{1/2} - 4x^{-1/2}\,dx = \frac{6}{3/2}x^{3/2} - \frac{4}{1/2}x^{1/2} -$$

$$= 4x^{3/2} - 8x^{1/2}\Big|_1^4 = \left[4(4)^{3/2} - 8\sqrt{4}\right] - \left[4(1)^{3/2} - 8(1)^{1/2}\right]$$

$$= \left[4(8) - 8(2)\right] - \left[4 - 8\right] = \boxed{20}$$

$$\int_{-3}^{-1} e^x - \frac{1}{x}\,dx$$

$$\hookrightarrow \left[e^x - \ln|x|\right]\Big|_{-3}^{-1} = \left[e^{-1} - \ln|-1|\right] - \left[e^{-3} - \ln|-3|\right]$$

$$= \left[\frac{1}{e} - 0\right] - \left[\frac{1}{e^3} - \ln(3)\right]$$

$$= \frac{1}{e} - \frac{1}{e^3} + \ln(3)$$

$$= \frac{e^2 - 1}{e^3} + \ln(3)$$

$$= \frac{e^2 - 1 + e^3\ln(3)}{e^3}$$

$$\int_0^3 f(x)\,dx \qquad f(x) = \begin{cases} 2x & 0 \le x \le 1 \\ 3 - x^2 & 1 < x \le 3 \end{cases}$$

$$\int_0^4 x^2 - |x^2 - 4|\,dx$$