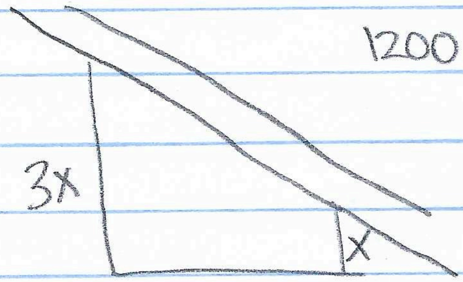
Abdan Morales

Nov 1st
1979

Question 1A



1200 ft = Fencing enclosed = P

$A_T = \frac{1}{2}h(b_1 + b_2)$

$A_T = \frac{1}{2}h(x+3x)$

$A_T = \frac{1}{2}h\ (4x)$

$A_T = 2hx$

$P = 3x + h + x$

$P = 4x + h$

$1200 = 4x + h$

$-4x$

$1200 - 4x = h$

$A_T = 2(1200-4x)\,x$

$(2400-8x)\,x$

$A_T = [2400 - 8x^2]$

$\frac{dA}{dx} = 2400 - 16x$

$0 = 2400 - 16x$

$+16x$

$\frac{\cancel{16}x}{\cancel{16}} = \frac{2400}{16} = 150$

$A_T = 2(150)(1200 - 4(150))$

$300(1200-600)$

$300(600)$

$\boxed{180{,}000\ ft^2}$

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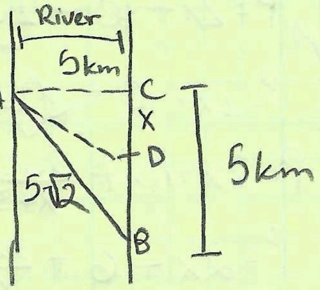
Question 55



$DB = 5 - X$

$AD = \sqrt{25 + x^2}$

$CB = 5$

$AC = 5$ $\quad r_r = 8$

$AB = 5\sqrt{2}$ $\quad r_{wr} = 6$

$CD = X$

$5^2 + x^2 = c^2$

$\sqrt{25 + x^2} = \sqrt{c^2}$

$\sqrt{25 + x^2} = AD$

$5^2 + 5^2 = c^2$

$25 + 25 = c^2$

$\sqrt{50} = \sqrt{c^2}$

$\sqrt{50} = c$

$5\sqrt{2} = c$

$$\left[\frac{\sqrt{25 + x^2}}{6}\right]' + \left[\frac{5 - x}{8}\right]'$$

$$\downarrow$$

$$\frac{x}{6\sqrt{25 + x^2}} - \frac{1}{8} = 0$$

$$+\frac{1}{8} \qquad +\frac{1}{8}$$

$$\frac{x}{6\sqrt{25 + x^2}} = \frac{1}{8}$$

$$\frac{8x}{2} = \frac{6\sqrt{25 + x^2}}{2}$$

$$(4x)^2 = \left(3\sqrt{25 + x^2}\right)^2$$

$$16x^2 = 9(25 + x^2)$$

$$16x^2 = 9x^2 + 225$$

$$-9x^2 \qquad -9x^2$$

$$\frac{\cancel{7}x^2}{\cancel{7}} = \frac{225}{7}$$

$$\sqrt{x^2} = \sqrt{\frac{225}{7}}$$

$$\boxed{x = \frac{15\sqrt{7}}{7}}$$

$X = 0, 1.458$

$X = 5, 1.179$

$x = \frac{15\sqrt{7}}{7}, 1.176$] Lowest value

Since the smallest of these values of T occurs when $x = \frac{15\sqrt{7}}{7}$, the absolute minimum value of T must occur there. Thus, the women should land the boat at point $\frac{15\sqrt{7}}{7}$ km downstream from her starting point

Question 27

$$\lim_{x\to 0}\left[\frac{e^x+e^{-x}-2}{e^x-x-1}\right]' \overset{\text{I.F}}{\underline{\underline{x=0}}} \frac{0}{0}$$

$$\xrightarrow{LH} \left[\frac{e^x-e^{-x}}{e^x-1}\right]' \underset{\text{I.F}}{\underline{\underline{x=0}}} \frac{0}{0}$$

$$\rightarrow \frac{e^x+e^{-x}}{e^x} \underline{\underline{x=0}} \frac{2}{1} = \boxed{2}$$

Question 85

$$y = \lim_{x\to a}\left[\frac{\sqrt{2a^3x-x^4}-a\sqrt[3]{aax}}{a-\sqrt[4]{ax^3}}\right]' \underline{\underline{x\to a}} \frac{0}{0}$$

$$\xrightarrow{LH} \frac{\frac{1}{2}(2a^3-x^4)^{-1/2}(2a^3-4x^3)-a}{-\frac{1}{4}(ax^3)^{-3/4}\cdot 3ax^2}$$

$$\frac{-(aa^3)^{-3/4}\cdot 3aa^2}{4} = \frac{-(a^4)^{-3/4}\cdot 3a^3}{4} = \frac{-a^{-3}\cdot 3a^3}{4} = \frac{-3}{4}$$

Line 1 $\frac{(2a^3-x^4)^{-1/2}(2a^3-4x^3)}{2} - a = \frac{(2a^4-a^4)^{-1/2}\cdot 2a^3-4a^3}{2} - a$

Line 2

Line 3 $\frac{(a^4)^{-1/2}-2a^3}{2} - a = \frac{a^{-2}(-2a^3)}{2} = \frac{\cancel{2}a}{\cancel{2}} - a \left(\frac{-3}{4}\right)$

4

5 $\frac{-\frac{4}{3}a}{\frac{-3}{4}} = \left(\frac{-4a}{3}\cdot\frac{4}{-3}\right) = \boxed{\frac{16a}{9}}$

6

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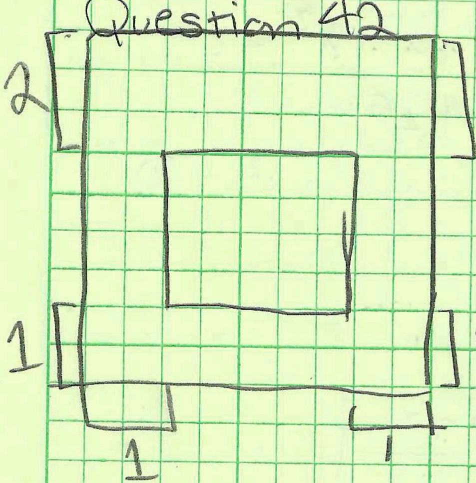
Question 42



$$\frac{180}{x+3} = \frac{(x+3)(2+y)}{(x+3)}$$

$$\frac{180}{x+3} = 2+y \qquad -2$$

$$\frac{180}{x+3} - 2 = y$$

$$P = 2(2+y) + 2(3+x)$$

$$P = 2x + 10 + 2y$$

$$P = 2x + 10 + 2\left(\frac{180}{x+3} - 2\right)$$

$$P = 2x + 10 + \frac{360}{x+3} - 4$$

$$P = 2x + 6 + \frac{360}{x+3}$$

$$\frac{dP}{dx} = 2 + \frac{-360}{(x+3)^2}$$

$$0 = 2 + \frac{-360}{(x+3)^2}$$

$$-2 = \frac{-360}{\cancel{(x+3)^2}} \cancel{(x+3)^2}$$

$$\frac{-2(x+3)^2}{-2} = \frac{-360}{-2}$$

$$\sqrt{(x+3)^2} = \sqrt{180}$$

$$x+3 = 6\sqrt{5} \qquad -3$$

$$x = -3 \pm 6\sqrt{5}$$

$$\boxed{x = -3 + 6\sqrt{5}}$$

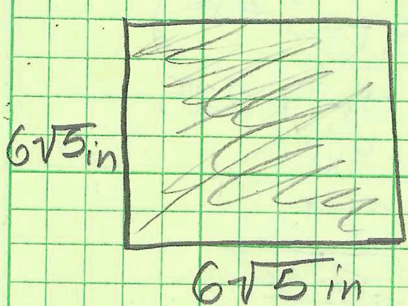
$$y = \frac{180}{\cancel{-3} + 6\sqrt{5} \cancel{+3}} - 2$$

$$y = \frac{180}{6\sqrt{5}} - 2$$

$$\frac{30}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$$

$$y = \frac{30\sqrt{5}}{5} - 2$$

$$\boxed{y = 6\sqrt{5} - 2}$$



The dimensions on (x, y) to ~~maximize~~ get the largest printed area is +6√5 on both the x and y axis.

Question 53

$$\lim_{x \to 0^+} \left(\frac{1}{x} - \frac{1}{e^x - 1}\right) \longrightarrow \left[\frac{e^x - 1 - x}{(e^x - 1)x}\right]' \overset{I.F}{=} \frac{0}{0} \longrightarrow \left[\frac{e^x - 1}{e^x + xe^x - 1}\right]' \overset{I.F}{=} \frac{0}{0}$$

Next line

$$\frac{e^x}{e^x + e^x + xe^x} = \frac{e^x}{2e^x + xe^x} \overset{x=0^+}{=} \boxed{\frac{1}{2}}$$

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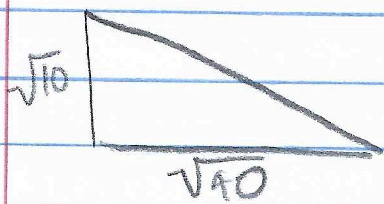
Additional Question #1



$$(\sqrt{10})^2 + (\sqrt{90})^2 = 100$$
$$= 10^2$$

$$V(h) = \pi\left(100 - \frac{h^2}{4}\right)h$$

$$V(h) = \left(100\pi - \frac{\pi h^2}{4}\right)h$$

$$V(h) = \left[100\pi h - \frac{\pi h^3}{4}\right]'$$

$$\frac{dV}{dh} = 100\pi - \frac{3\pi h^2}{4}$$

$$0 = 100\pi - \frac{3\pi h^2}{4}$$
$$+\frac{3\pi h^2}{4}$$

$$\cancel{(4)}\ \frac{3\cancel{\pi} h^2}{\cancel{4}} = 100\cancel{\pi}\ (4)$$

$$\frac{\cancel{3}h^2}{\cancel{3}} = \frac{400}{3} = \sqrt{h^2} = \sqrt{\frac{400}{3}} = h = \frac{20\sqrt{3}}{3}$$

$$100\pi\left(\frac{20\sqrt{3}}{3}\right) - \frac{\pi(20\sqrt{3})^3}{4} = V\left(\frac{20\sqrt{3}}{3}\right) = \boxed{\frac{4000\pi\sqrt{3}}{9}}$$

Q26

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \rightarrow d = \sqrt{(x-3)^2 + (\sqrt{x} - 0)^2}$$

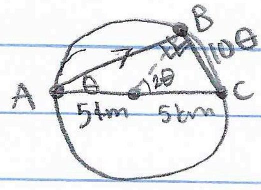
$$D'(x) = \frac{-5+2x}{2\sqrt{9+(-5+x)x}} \qquad \left[D(x)\right]' = \left[\boxed{\sqrt{x^2 - 6x + 9 + x}}\right]'$$

$$-5 + 2x = 0$$
$$-2x \quad -2x$$
$$\frac{-5}{-2} = \frac{-2x}{-2}$$

$$\boxed{X_{CR} = \frac{5}{2}}$$

Point closest to the point $P = (3,0)$ is $\left(\frac{5}{2}, \sqrt{\frac{5}{2}}\right)$

Additional Question #2



Run $= \frac{4km}{hr}$

Swim $= \frac{2km}{hr}$

$r = 5km$

$\overset{\frown}{BC} = \frac{2\theta}{\cancel{2\pi}}(\cancel{2\pi} r) = 2\theta r$

$\overset{\frown}{BC} = 2\theta(5) = 10\theta$

$10\cos\theta = \frac{AB}{\cancel{10}}\cancel{(10)}$

$10\cos\theta = AB$

$T(\theta) = t_{water} + t_{run} = \frac{d_w}{r_w} + \frac{d_r}{d_r}$

$T(\theta) = \frac{10\cos\theta}{2} + \frac{10\theta}{4}$

$T(\theta) = \left[5\cos\theta + \frac{5}{2}\theta\right]'$

$T'(\theta)$ or $\frac{dT}{d\theta} = -5\sin\theta + \frac{5}{2}$

$0 = -\sin\theta + \cancel{\frac{5}{2}}$

$\cancel{(2)} - \frac{5}{2} = -5\sin\theta \, \cancel{(2)} - \cancel{\frac{5}{2}}$

$\frac{-5}{-10} = \frac{-10\sin\theta}{-10}$

$\frac{1}{2} = \sin\theta$

$\theta = \frac{\pi}{6}$

Plug-in to $T(x)$

$T(\pi/6) = 5\cos\left(\frac{\pi}{6}\right) + \frac{5}{2}\left(\frac{\pi}{6}\right)$

$T(\pi/6) = \frac{5\sqrt{3}}{2} + \frac{5\pi}{12}$

$T(\pi/6) \approx 5.64$ hr

Slowest path

Swim only $= \frac{10}{2} = 5$hrs

~~$T(0) =$~~

Run only $= \frac{5\pi}{6} = 2.617$

Fastest path

The slowest path is from A to B (swim) and from B to C (run) would take 5.64 hours compared to the fastest which would be around the lake from A to C (run) which would take 2.617 hours.

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Additional Thing

$$\lim_{x\to 0}\left[\frac{ax^2+\sin bx+\sin cx+\sin dx}{3x^2+5x^4+7x^6}\right]' \underset{x=0}{\overset{\text{I.F}}{=}} \frac{0}{0}$$

$$\lim_{x\to 0}\left[\frac{2ax+b\cos bx+c\cos cx+d\cos dx}{6x+20x^3+42x^5}\right]' = \frac{0}{0}$$

$$\lim_{x\to 0}\left[\frac{2a-b^2\sin(bx)-c^2\sin(cx)-d^2\sin dx}{6+60x^2+210x^4}\right]$$

c, d, and b are ~~begin~~ always going to zero when $x=0$; no matter the value in the variable.

$$\boxed{\sum abcd = \overset{24+0+0+0}{\cancel{0+0+0}} = 24}$$

$$(6)\,\frac{2a}{6} = 8\,(6)$$

$$\frac{2a}{2} = \frac{48}{2}$$

$$\underline{\underline{a = 24}}$$