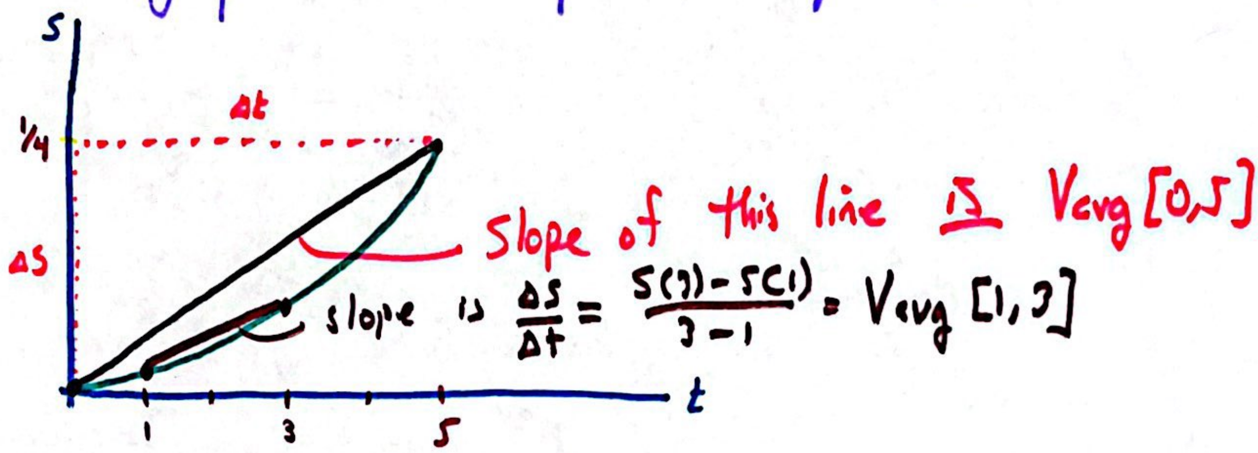
§ 2.1. The Tangent + Velocity Problems

- In this video, we will:
  - Compute average velocities of position function $s(t)$
  - Try to find the instantaneous velocity
  - Relate this to tangent lines

Consider a Drag Race with racer Bob the Driver. A graph of his position might be:



The average velocity from start to finish

" " " " $t=1$ to $t=3$

- $\cancel{V_{avg}}$ $V_{avg}[t_1, t_2] = \dfrac{S(t_2) - S(t_1)}{t_2 - t_1} = \dfrac{\Delta S}{\Delta t} \approx \dfrac{\Delta y}{\Delta x} = \text{slope}$

$V_{avg}[0,5] = \dfrac{1/4 - 0}{5 - 0} = \dfrac{1/4}{5} = \frac{1}{4} \cdot \frac{1}{5} = 1/20$ miles/s.

The average velocity between two times **is** the slope of the line connecting the two points.

Ex: If $s(t) = 3t^2$, find the average velocity from $t=1$ to $t=3$.

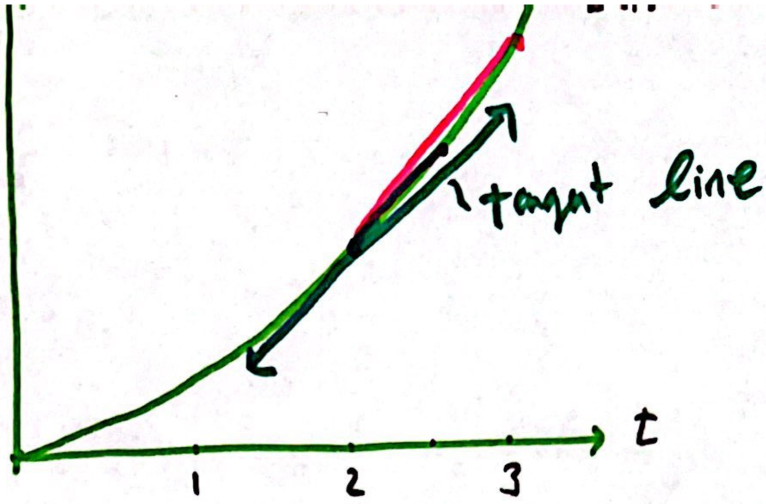
$$V_{avg}[1,3] = \frac{s(3)-s(1)}{3-1} = \frac{3(3)^2-3(1)^2}{3-1} = \frac{27-3}{2} = \frac{24}{2} = \boxed{12}$$

This is neat, but what is the velocity at exactly $t=2$?

Guess: Find $V_{avg}[2,2] = \frac{s(2)-s(2)}{2-2} = \boxed{\frac{0}{0}}$ Nonsense.

The velocity at $t=2$ is Not $V_{avg}[2,2]$

V_{avg} [2,2] no good



f_{avg} from $t=2$ to $t=3$, $\dfrac{S(3)-S(2)}{3-2} = 15$

f_{avg} from $t=2$ to $t=2.5$, $\dfrac{S(2.5)-S(2)}{2.5-2} = 13.5$

f_{avg} from $t=2$ to $t=2.1$, $\dfrac{S(2.1)-S(2)}{2.1-2} = 12.3$

f_{avg} from $t=2$ to $t=2.001$, $\dfrac{S(2.001)-S(2)}{.001} = 12.003$

is $V(2) = 12$?

The instantaneous velocity is the slope of the Tangent Line... but is there a better way to find it than what we did above.