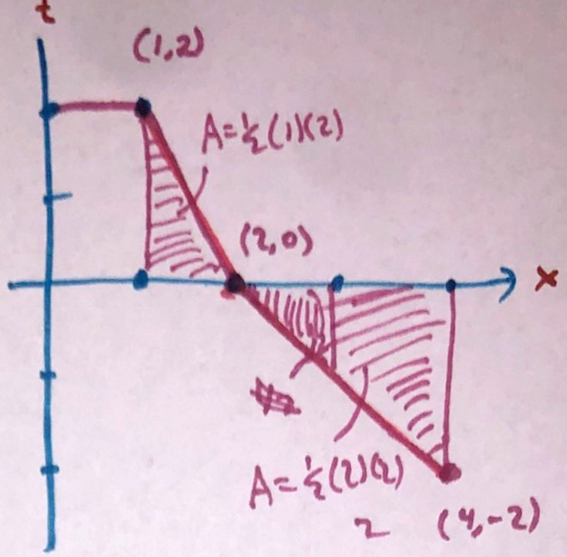
§5.3 - The Fundamental Theorem of Calculus.

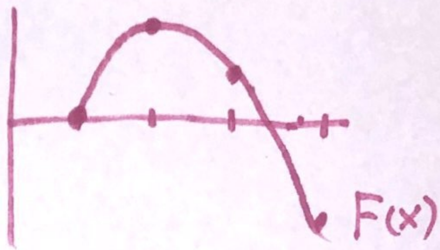
Start with a function $f(t)$:



$$F(x) = \int_1^x f(t)\,dt$$

$$F(1) = \int_1^1 f(t)\,dt = 0$$

$$F(2) = \int_1^2 f(t)\,dt = 1$$



$$F(3) = \int_1^3 f(t)\,dt = 1 - \tfrac{1}{2} = \tfrac{1}{2}$$

$$F(4) = \int_1^4 f(t)\,dt = 1 - 2 = -1$$

Q1: What is $F'(x)$?

The fundamental Theorem of Calculus, part I.

If $f(t)$ is continuous on $[a,b]$, and $F(x) = \int_a^x f(t)\,dt$,

then $\boxed{F'(x) = f(x)}$

$$F(x) = \int_0^x \sin(t)\,dt, \quad F'(x) = \sin(x)$$

$$F(x) = \int_{-2}^{x} 3t^2 - t + 4\,dt, \quad F'(x) = 3x^2 - x + 4$$

$$F(x) = \int_x^7 e^{2t} - 4t^2\,dt = -\int_7^x e^{2t} - 4t^2\,dt \quad \therefore F'(t) = -(e^{2x} - 4x^2)$$

$$= 4x^2 - e^{2x}$$

$$F(x) = \int_0^{x^2} \cos(t)\,dt, \quad F'(x) = \cos(x^2) \cdot 2x$$

$$= 2x\cos(x^2)$$

$$F(x) = \int_1^{x^4} \sec(t^2)\,dt, \quad F'(x) = \sec((x^4)^2) \cdot 4x^3$$

$$= 4x^3 \cdot \sec(x^8)$$

$+, -$, $f(x)$, $f^{-1}(x)$

$\times, \div$, $\int$, $\frac{df}{dx}$

e^x, $\ln(x)$,