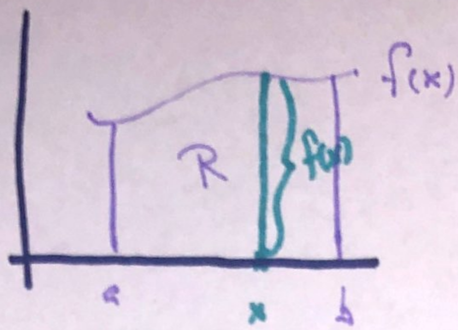
§ 6.2 - Volumes

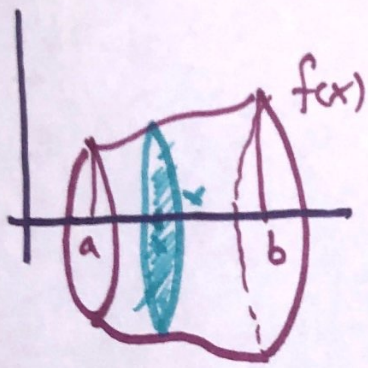
Before:



$$\text{Area}(R) = \int_a^b f(x)\,dx$$

$$= \int_a^b \underbrace{\text{lengths}}_{\text{1-dim}}\,dx = \text{Area}$$

Now



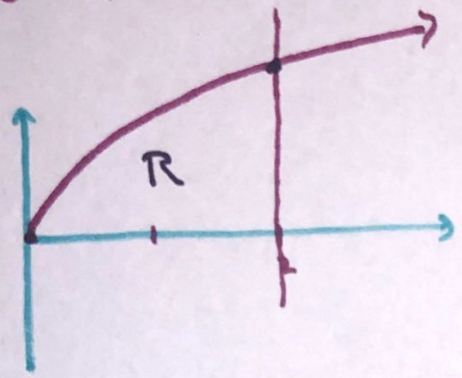
$$\int_a^b \text{Areas}\,dx = \text{Volume}$$

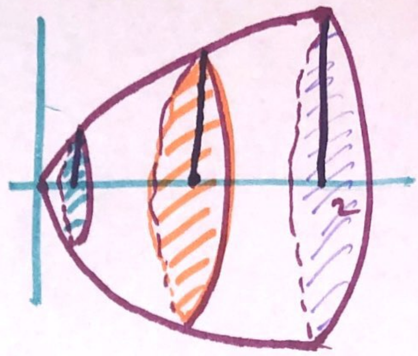
Let S be a solid that lies between $x=a$ & $x=b$. If the cross-sectional area of S is $\perp$ to the x-axis, and has area $A(x)$, then

$$\text{Volume}(S) = \int_a^b A(x)\,dx$$

Scanned with CamScanner

Ex: Let R be the region bounded by $y = 2\sqrt{x}$, x-axis, and $y = 2$. Find the volume of the solid obtained by rotating R about the x-axis.





$$V = \int_0^2 A(x)\,dx \quad , \quad A(x) = \text{Area of cross-sectional shapes}$$

$$= \text{Circle} = \pi r^2$$

$$= \pi y^2$$

$$= \pi (2\sqrt{x})^2$$

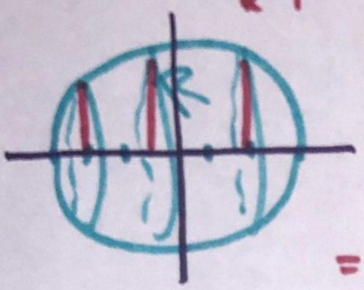
$$= 4\pi x$$

$$= \int_0^2 4\pi x \, dx$$

$$= 2\pi x^2 \Big|_0^2 = 2\pi \cdot 2^2 - 2\pi \cdot 0^2$$

$$= 8\pi$$

Ex: Let R be the region bounded by $y=\sqrt{9-x^2}$ and the x-axis. Find the volume of the solid obtained by rotating R about the x-axis.



← sphere w/ radius 3.

$$\frac{4}{3}\pi r^3$$
$$=\frac{4}{3}\pi \cdot 3^3$$
$$=\frac{4}{3}\pi 27$$
$$=\boxed{36\pi}$$

$$V=\int_{-3}^{3} A(x)\,dx=$$

$A(x)$ = Area of circle
$$=\pi r^2$$
$$=\pi y^2$$
$$=\pi\left(\sqrt{9-x^2}\right)^2$$
$$=\pi(9-x^2)$$

$$=\pi\int_{-3}^{3} 9-x^2\,dx=\pi\left[9x-\tfrac{1}{3}x^3\right]_{-3}^{3}=$$

$$=\pi\left[\left(27-\tfrac{1}{3}(27)\right)-\left(-27-\tfrac{1}{3}(-27)\right)\right]$$

$$=\pi\left[18-(-18)\right]=\pi\cdot 36=36\pi$$

$V=\cancel{\int_{-R}^{R}\sqrt{\;}}$ $\quad V=\int_{-R}^{R} A(x)\,dx,$ ~~A(x)=~~ $y=\sqrt{R^2-x^2}$

$=\boxed{4/3\,\pi R^3}$