§ 5.5 - u-sub + Definite Integrals

$$\int_0^2 \frac{10x}{(1+x^2)^3}\,dx = \int_{0=x}^{2=x} \frac{10\cancel{x}}{u^3}\cdot\frac{du}{2\cancel{x}} = 5\cdot\int_{x=0}^{x=2} u^{-3}\,du$$

$u = 1+x^2$

$du = 2x\,dx$

$dx = \frac{du}{2x}$

$$= -\frac{5}{2}u^{-2}\Big|_{x=0}^{x=2}$$

$$\cancel{\frac{-5}{2(2)^2} - \frac{-5}{2(0)}}$$

$$= \frac{-5}{2u^2} = \frac{-5}{2(1+x^2)^2}\Big|_0^2$$

$$= \left(\frac{-5}{2(5)^2} - \frac{-5}{2(1)^2}\right) = \frac{5}{2} - \frac{1}{10} = \frac{25}{10} - \frac{1}{10} = \frac{24}{10} = \boxed{\frac{12}{5}}$$

$$\int_0^2 \frac{10x}{(1+x^2)^3}\,dx = \int_1^5 \frac{10x}{u^3}\,\frac{du}{2x} = 5\int_1^5 u^{-3}\,du$$

$u = 1+x^2$

$du = 2x\,dx$

$dx = \frac{du}{2x}$

$x=0 \Rightarrow u=1$

$x=2 \Rightarrow u=5$

$$= \frac{5}{-2}u^{-2}\Big|_1^5 =$$

$$= \frac{-5}{2u^2}\Big|_1^5 = \left(\frac{-5}{2\cdot 5^2} - \frac{-5}{2\cdot 1^2}\right)$$

$$= \boxed{\frac{12}{5}}$$

$$\int_0^{\pi/6} \frac{\sin(t)}{\cos^2(t)}\,dt = \int_1^{\sqrt{3}/2} \frac{\sin(t)}{u^2}\,\frac{du}{-\sin(t)} = \int_1^{\sqrt{3}/2} -u^{-2}\,du$$

$u = \cos(t)$

$du = -\sin(t)\,dt$

$dt = \dfrac{du}{-\sin(t)}$

$t = 0 \Rightarrow u = \cos(0) = 1$

$t = \pi/6 \Rightarrow u = \cos(\pi/6) = \dfrac{\sqrt{3}}{2}$

$$= \frac{1}{u}\Big|_1^{\sqrt{3}/2}$$

$$= \frac{1}{\sqrt{3}/2} - \frac{1}{1}$$

$$= \frac{2}{\sqrt{3}} - 1$$

$$= \frac{2-\sqrt{3}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{2\sqrt{3}-3}{3}$$

$$= \frac{2\sqrt{3}}{3} - 1$$