F23 - M408C
Exam 1 - Name. EID:

Instructions:

- Write your name and EID on **every page**.
- Put your answers on the last sheet of paper.
- This exam is **only** for students in the 12:30 class.
- No other outside resources, such as books, notes, the internet, or other people, are allowed.
- The max possible score is 105. It will be graded out of 100.

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1. (6 points) Which of the following is equal to $\dfrac{4x^{-2}}{x+x^{-2}}$.

   (A) $\dfrac{4}{x^3}+4$ (B) $\dfrac{4}{x^2-1}$ (C) $\dfrac{4}{x^3+x}$ (D) $\dfrac{4}{x^2+x}$ (E) $\dfrac{4}{x^3+1}$ (F) $\dfrac{4}{x^3+x^2}$

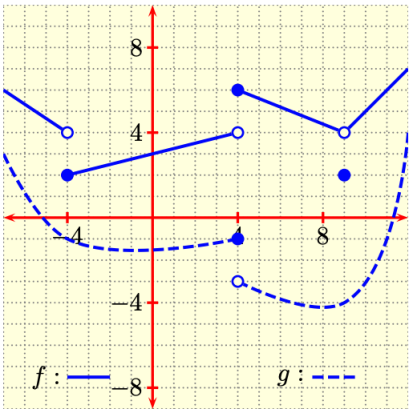
2. (6 points) The expression $2\left(\dfrac{1}{4}\right)^{2x-1}$ can be written as $a(2^{bx})$. Which of the following equals $2a+2b$?

   (A) 0 (B) 4 (C) 8 (D) 12 (E) 16 (F) 20

3. (6 points) The position function of an object moving into Houston is given by $s(t)=t^2-4t$. Find the average velocity $v_{[3,3+h]}$.

   (A) 4 (B) $4+h$ (C) 2 (D) $2+h$ (E) 3 (F) $3+h$

4. (6 points) The graphs of f and g are shown below:



Find $\lim_{x\to 4}[f(x)+g(x)]$

(A) 3 (B) 0 (C) 1 (D) 2 (E) Does not Exist

5. (6 points). Find $\lim_{x\to 3}\dfrac{x^2-x-3}{x^2-3x+2}$.

(A) $\dfrac{5}{3}$ (B) $\dfrac{7}{4}$ (C) 1 (D) $\dfrac{3}{2}$ (E) 2 (F) 3

6. (6 points) Find $\lim_{x\to 0}\left(\dfrac{1}{x}-\dfrac{1}{x-3x^2}\right)$

(A) 0 (B) -1 (C) -2 (D) -3 (E) -4

7. (6 points) The function $f(x)$ is continuous:

$$f(x)\begin{cases} x+a & x<2 \\ B & x=2 \\ x^2-a & x>2 \end{cases}$$

Find B.

(A) -4 (B) -1 (C) 0 (D) 6 (E) 2 (F) 3

8. (6 points) Find $\lim_{x\to\infty} \cos\left(\frac{1}{x^2}\right)$

(A) Does not Exist (B) $\frac{1}{2}$ (C) $\frac{\sqrt{3}}{2}$ (D) 0 (E) 1

9. (6 points) Find $\lim_{x\to\infty} \left(\sqrt{4x^2+16x}-2x\right)$.

(A) 16 (B) 4 (C) 1 (D) 0 (E) 8 (F) 2

10. (6 points) Find $\lim_{x\to-\infty} \frac{\sqrt{4x^2+x+3}}{3-x}$

(A) 2 (B) 1 (C) 0 (D) -1 (E) -2 (F) 4

11. (6 points) Given the function $f(x) = \dfrac{2}{x} - \dfrac{3}{x^2}$, find $f'(x)$

(A) $\dfrac{6-2x}{x^3}$ (B) $\dfrac{7-2x}{x^3}$ (C) $\dfrac{8-2x}{x^3}$ (D) $\dfrac{9-2x}{x^3}$ (E) $\dfrac{10-2x}{x^3}$

12. (6 points) The function $f(x) = x^5 + 2x$ is a 1-1 function. Find $f^{-1}(-3)$.

(A)3 (B) 1 (C) 0 (D) -1 (E) -2

13. (3 points): True or False: If $\lim\limits_{x\to 2} \dfrac{f(x)-13}{x-2} = 10$, then $\lim\limits_{x\to 2} f(x) = 13$.

14. (3 points): True or False: If $f(x) = 3x^3 - 5x + e^x$, then $f'(0) = -5$.

15. (3 points): True or False: $\lim\limits_{x\to 2^+} \dfrac{x^2-5}{x-2} = -\infty$

16. (3 points): True or False: If $f(x) = x^{10}$, then $f'(1) = \lim\limits_{x\to 1} \dfrac{x^{10}+1}{x-1}$.

17. (11 points) Draw a graph of a function on the interval $[-3, 3]$ that has each of the following properties:

1) $f(-3) = 1$

2) $\lim_{x \to -1} f(x) = -2$

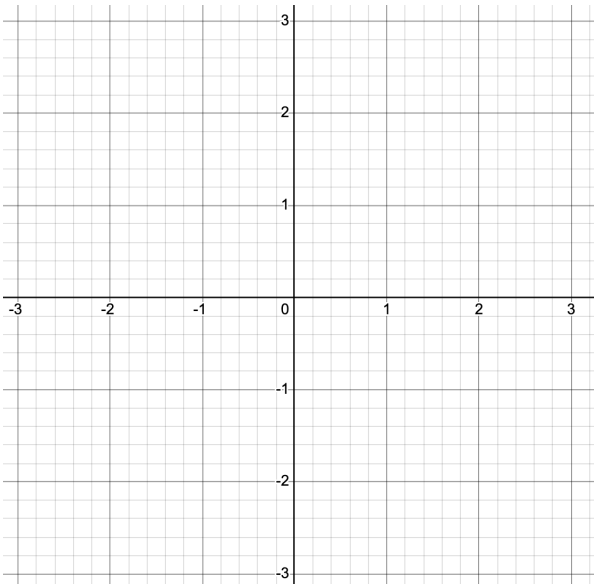
3) $f(-1) = -3$

4) $\lim_{x \to 2^-} f(x) = -1$

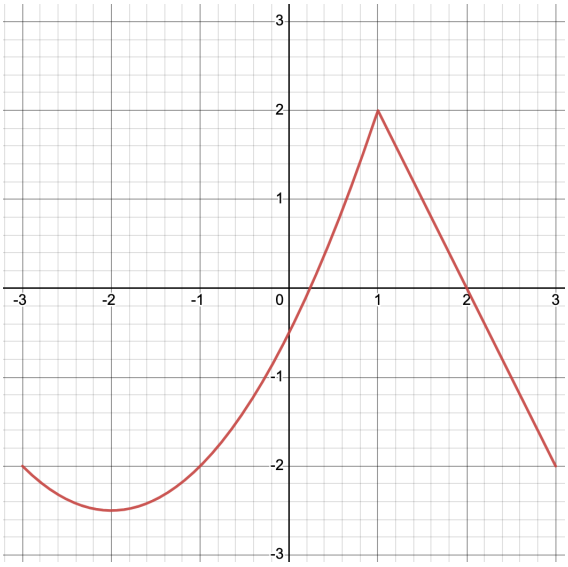
5) $\lim_{x \to 2^+} f(x) = f(2) = 3$

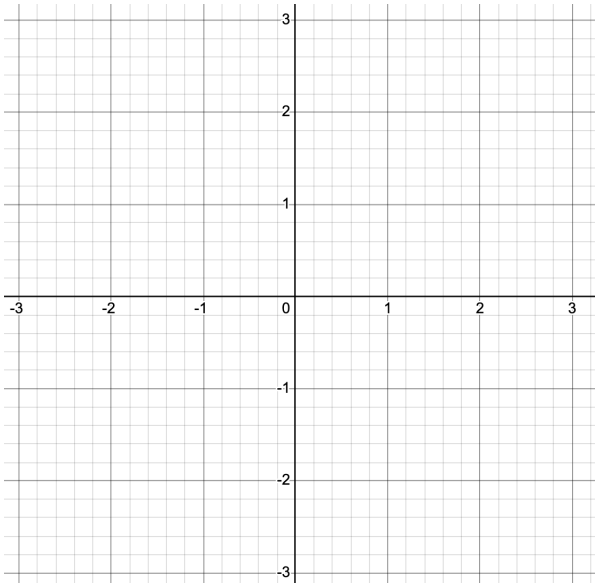
6) $f(3) = 2$

7) $f(x)$ is continuous everywhere except $x = -1, 2$



18. (10 points) The graph of $f(x)$ is on the left. Sketch a graph of $f'(x)$ on the xy-axis on the right.





Multiple Choice (6 points each):

1) **7)**

2) **8)**

3) **9)**

4) **10)**

5) **11)**

6) **12)**

True or False (3 points each):

13) **15)**

14) **16)**