§4.4 - I.F. + L.R., part 2

- I.F.: $0/0$, ∞/∞, L.R.

I.F.: $\infty - \infty$, $0 \cdot \infty$, Not initially L.R.

Ex: $\lim\limits_{x \to 1} \frac{x}{x-1} - \frac{1}{\ln(x)} \to \frac{1}{0} - \frac{1}{0} = \infty - \infty$, I.F.

$$\lim_{x \to 1} \frac{x\ln(x) - (x-1)}{(x-1)\ln(x)} \to \frac{1 \cdot 0 - 0}{0 \cdot 0} = \frac{0}{0}, \text{ I.F.}$$

$$\overset{\text{L.R.}}{=} \lim_{x \to 1} \frac{\ln(x) + x\left(\frac{1}{x}\right) - 1}{\ln(x) + (x-1)\frac{1}{x}} = \lim_{x \to 1} \frac{\ln(x)}{\ln(x) + 1 - \frac{1}{x}} \to \frac{0}{0}$$

$$= \lim_{x \to 1} \frac{1/x}{\frac{1}{x} - \left(\frac{-1}{x^2}\right)} = \frac{1/x}{\frac{1}{x} + \frac{1}{x^2}} = \frac{1}{1+1} = \boxed{\frac{1}{2}}$$

Ex: $\lim\limits_{x \to \infty} x \cdot \ln\left(1 - 4/x\right) \to \infty \cdot \ln\left(1 - \frac{4}{\infty}\right) = \infty \cdot \ln(1) = \infty \cdot 0$

$$= \lim_{x \to \infty} \frac{\ln(1 - 4/x)}{1/x} \to \frac{\ln(1)}{1/\infty} = \frac{0}{0},$$

$$\overset{\text{L.R.}}{=} \lim_{x \to \infty} \frac{\frac{1}{1 - 4/x} \cdot \left[1 - 4x^{-1}\right]'}{-\frac{1}{x^2}} = \lim_{x \to \infty} \frac{\frac{1}{1-4/x} \cdot 4\cancel{x^{-2}}}{-\cancel{x^{-2}}}$$

$$= \lim_{x \to \infty} -\frac{4}{1 - 4/x} = -\frac{4}{1-0} = \boxed{-4}$$