# §3.4, The Chain Rule

We know how to take the derivative of:

$\{x^n, e^x, \sin(x), \cos(x), \tan(x), \sec(x)$

$\hookrightarrow f(x) \pm g(x),\ f(x)\cdot g(x),\ \dfrac{f(x)}{g(x)},\ c\cdot f(x)$

There is one more way we can combine functions [one more function operation]:

$$f(g(x)) = (f\circ g)(x)$$

Ex: $(x^2+10x)^{10}$ $\hookrightarrow u = x^2+10x$, $u^{10}$

$f(x) = x^{10}$
$g(x) = x^2+10x$,

$f(g(x)) = f(x^2+10x) = (x^2+10x)^{10}$

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$[(3x)^2]' = [9x^2]' = 18x$

? $\hookrightarrow 2(3x) = 6x$, wrong

? $\hookrightarrow (3)^2 = 9$, wrong

$\hookrightarrow 2(3x)\cdot[3x]' = 2(3x)\cdot 3 = 18x$ ✓

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$$\left[(x^2+10x)^{10}\right]' = 10(x^2+10x)^9 \cdot \left[x^2+10x\right]'$$
$$= 10(x^2+10x)^9(2x+10)$$

$$\left[\sqrt{x^3-2x}\right]' = \left[(x^3-2x)^{1/2}\right]' = \tfrac{1}{2}(x^3-2x)^{-1/2}\left[x^3-2x\right]' =$$
$$\frac{1}{2\sqrt{x^3-2x}} \cdot 3x^2-2$$
$$= \frac{3x^2-2}{2\sqrt{x^3-2x}}$$

$$\left[e^{10x}\right]' = e^{10x}\left[10x\right]' = 10e^{10x}$$

$$\left[\cos(3x)\right]' = -\sin(3x)\cdot\left[3x\right]' = -3\sin(3x)$$

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$$\left[\sin(x^2)\right]' = \cos(x^2)\cdot\left[x^2\right]'$$
$$= 2x\cos(x^2)$$

$$\left[\sin^2(x)\right]' = \left[(\sin(x))^2\right]'$$
$$= 2\sin(x)\left[\sin(x)\right]'$$
$$= 2\sin(x)\cos(x)$$

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Find $f'(0)$ when $f(x) = e^{2x}\cdot\sin(x^2)$

$$f'(x) = \left[e^{2x}\right]'\sin(x^2) + e^{2x}\left[\sin(x^2)\right]' = e^{2x}\left[2x\right]'\sin(x^2) + e^{2x}\cos(x^2)\cdot\left[x^2\right]'$$
$$= 2e^{2x}\sin(x^2) + e^{2x}\cdot\cos(x^2)\cdot 2x = 2e^{2x}\left(\sin(x^2) + x\cos(x^2)\right)$$

$$\therefore f'(0) = 2e^0\left(\sin(0) + 0\cdot\cos(0)\right)$$
$$= 2(1)(0+0\cdot 1) = \boxed{0}$$

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