

§ 5.3, The Fundamental Theorem of Calculus, part 2.

FTC I: If $g(x) = \int_a^x f(t) dt$, then $g'(x) = f(x)$

FTC II: $\int_a^b f(x) dx = F(b) - F(a)$, where $F'(x) = f(x)$

$$\begin{aligned} \text{Ex: } \int_1^4 2x + 6x^2 dx &= F(4) - F(1) = (16 + 2 \cdot 64 + C) - (1 + 2 + C) \\ &= 16 + 128 + \cancel{C} - 1 - 2 - \cancel{C} \\ &= 144 - 3 = \boxed{141} \\ F(x) &= x^2 + 2x^3 + C \end{aligned}$$

$$F(b) - F(a) = F(x) \Big|_a^b$$

$$\begin{aligned} \text{Ex: } \int_0^2 e^x - x dx &= \left[e^x - \frac{1}{2}x^2 \right]_0^2 = \left(e^2 - \frac{1}{2}(2)^2 \right) - \left(e^0 - \frac{1}{2}0^2 \right) \\ &= (e^2 - 2) - (1 - 0) \\ &= \boxed{e^2 - 3} \end{aligned}$$

$$\int_a^b f'(x) dx = f(b) - f(a)$$