§1.5 - Inverse Functions + Logarithms, pt. I

- In this video, we will:

  ① Define + describe one-to-one functions

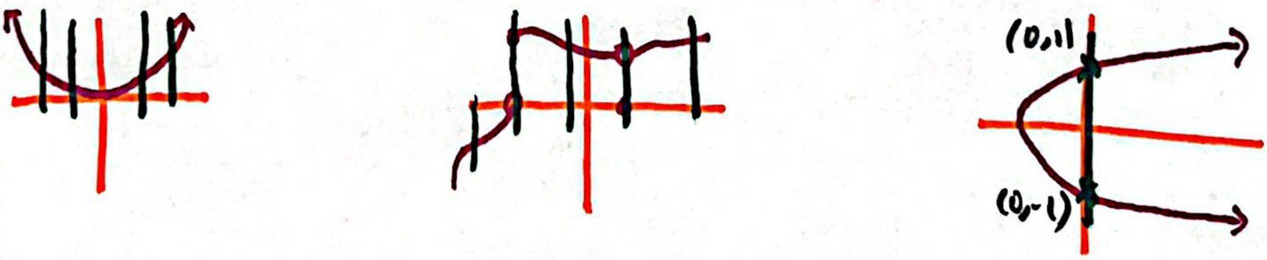
  ② Define + describe inverse functions

  ③ Find inverse functions

One reason functions are great is because for each input you get exactly one output.

Ex: $f(x) = x^2$, $f(2) = 4$, only one number

- As a graph, a function passes the Vertical Line Test:



$f(0) = 1$ or -1

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Next Question: If $f(x)=x^2$ and $f(x)=4$, what is x?

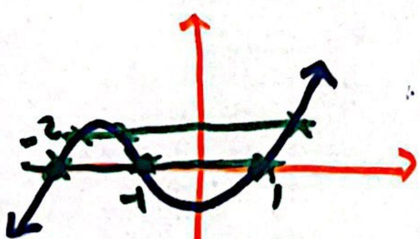
Potential Headache Looming... there might be more than one x-value

$f(x)=x^2$, $f(x)=4 \Rightarrow$ $f(2)=2^2=4$
$f(-2)=(-2)^2=4$

Sometimes we want to know each output of a function has only one input $f(x)=x^3$, $f(x)=8$, $x=2$

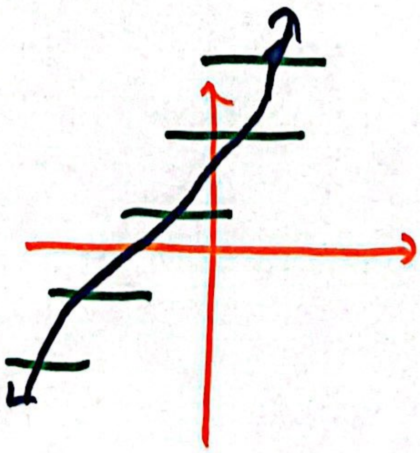
Def^n: A function $f(x)$ is called <u>one-to-one</u>, or <u>1-1</u>, when each output has exactly one input $\Longleftrightarrow$ if $f(x_1)\neq f(x_2)$ then $x_1 \neq x_2$

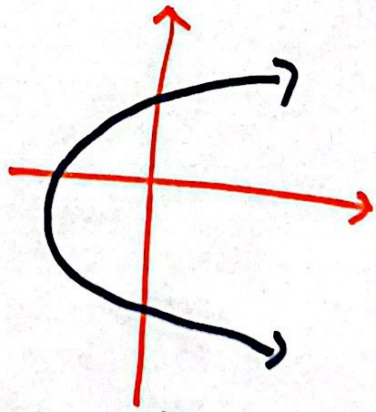
Important: One-to-one functions pass the Horizontal Line Test



NOT 1-1

~~$f(x)=0$, $x=-2,-1,1$~~





Not a Fn, $\therefore$ Not 1-1 Fn.

If $f(x)$ is a 1-1 function, then there is a function called the inverse function, $f^{-1}(x)$, such that $f(a)=b \iff f^{-1}(b)=a$.

Important Notation: $f^{-1}(x) \neq \frac{1}{f(x)}$

$$\frac{1}{f(x)} = (f(x))^{-1}$$

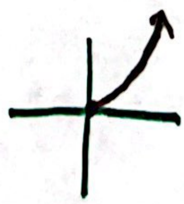
Ex: If $f(x)$ is 1-1 function and $f(7)=21$,

then $f^{-1}(21)=7$

If $f(x)$ is a 1-1 fn with Domain D and Range R,

then f^{-1} is a 1-1 fn with Domain R + Range D.

This can get tricky: $f(x)=x^2$ is not 1-1



But $f(x)=x^2$, $x \geq 0$,

it is 1-1.

Important : $f(a)=b$, $f^{-1}(b)=a$

Find $f(f^{-1}(b)) = f(a) = b$

$f^{-1}(f(a)) = f^{-1}(b) = a$

Given a 1-1 function $f(x)$, how do we find $f^{-1}(x)$?

Ex: $f(x) = x^3+1$, it is 1-1, find $f^{-1}(x)$

$y = x^3+1 \Leftrightarrow x = y^3+1$

$x-1 = y^3$

$y = \boxed{\sqrt[3]{x-1} = f^{-1}(x)}$

$f(3) = 3^3+1 = 27+1 = 28$

$f^{-1}(28) = \sqrt[3]{28-1} = \sqrt[3]{27} = 3$, ◡̈

Example: $f(x) = \frac{2x+1}{x-3}$, it is 1-1, find $f^{-1}(x)$.

$$y = \frac{2x+1}{x-3} \iff x = \frac{2y+1}{y-3}$$

$$x(y-3) = 2y+1$$

$$xy - 3x = 2y + 1$$

$$xy - 2y = 3x + 1$$

$$y(x-2) = 3x+1$$

$$y = \boxed{\frac{3x+1}{x-2} = f^{-1}(x)}$$

$$f(\underline{4}) = \frac{2(4)+1}{4-3} = \underline{9}$$

$$f^{-1}(\underline{9}) = \frac{3(9)+1}{9-2} = \frac{28}{7} = \underline{4}$$