§ 2.5 - Continuous Functions, part 1.

In this video, we will:

- Define a continuous function
- List Properties of continuous functions
- Describe types of continuous functions.

Qn: What does it mean for a function to be continuous?

Answer: It must be continuous at everypoint.

Qn: Ok, fine, but what does it mean to be continuous at a point?

Answer: You don't have to pick up your pencil.

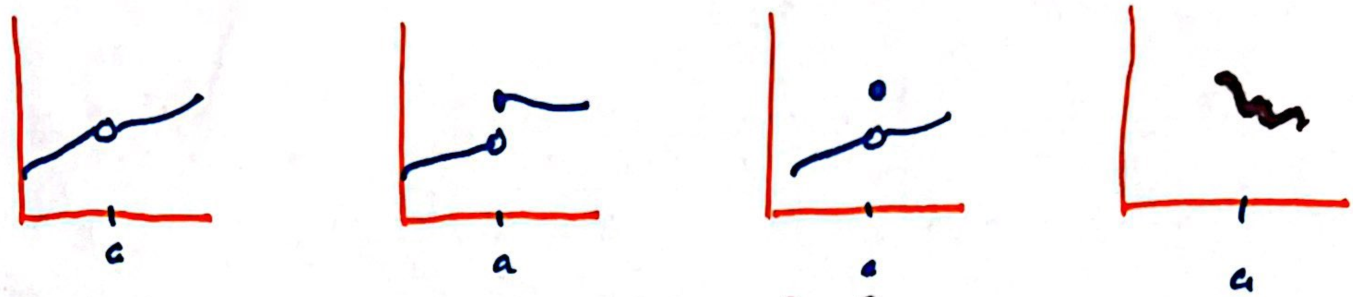
Better Answer: f is continuous at $x=a$ if

$$\lim_{x \to a} f(x) = f(a)$$

$\Rightarrow$ ① $f(a)$ exists and is finite

② $\lim_{x \to a} f(x)$ exists $\Leftrightarrow$ Left + Right hand limits exist and equal each other

③ $\lim_{x \to a} f(x) = f(a)$



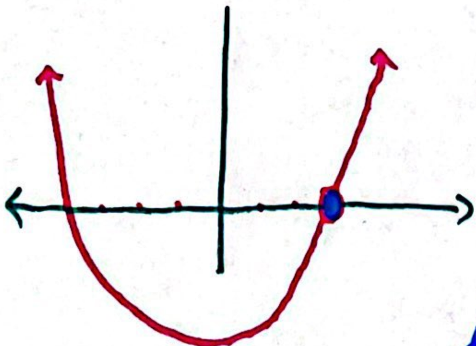
f(a) DNE

$\lim_{x\to a} f(x)$ DNE

$\lim_{x\to a} f(x) \neq f(a)$

Limit exists

Example: $f(x) = \begin{cases} x^2+x-12 & x\neq 3 \\ B & x=3 \end{cases}$. Find B such that f(x) is continuous at x=3.



$x^2+x-12=(x-3)(x+4)=0$

$x=3,4$

B=3, that makes

$\lim_{x\to 3} f(x) = \lim_{x\to 3} x^2+x-12 = B = 0$
$= f(3)$

Additional properties of continuous functions.

- If $f(x)$ + $g(x)$ are continuous at $x=a$, so are:

- $f(x)+g(x)$
- $f(x)-g(x)$
- $f(x)\cdot g(x)$
- $c\cdot f(x)$
- $\frac{f(x)}{g(x)}$ when $g(a)\neq 0$

- These properties, plus a few more, mean the following function are continuous everywhere the function is defined

- polynomials
- rational functions - $\frac{f(x)}{g(x)}$, $g(x)\neq 0$
- root functions - $\sqrt[n]{x}$
- Trig functions
- exponential functions
- Inverse Trig Functions
- logarithmic Functions