

§3.8 - Exponential Growth + Decay

Let $P(t)$ represent the population of a rabbit colony. How fast is the rabbit population increasing?

$$\frac{dP}{dt} = k \cdot P(t), \text{ where } k \text{ is a } \overset{\text{positive}}{\text{constant}}.$$

$$\frac{dy}{dt} = y' = k \cdot y, \text{ where } k \text{ is } \begin{matrix} \text{positive } \# \leftarrow \text{natural growth} \\ \text{negative } \# \leftarrow \text{natural decay} \end{matrix}$$

Qn: If something behaves according to law of natural growth or decay, what is the function itself?

$$\text{If } \underline{\frac{dy}{dt} = k \cdot y} \Rightarrow \underline{y = y(0) \cdot e^{kt}}$$

Suppose a rabbit colony grows according to the law of natural growth, $\frac{dP}{dt} = .15 \cdot P$.

If $P(0) = 100$, find $P(10)$.

$$P(t) = P(0) \cdot e^{kt} = 100 \cdot e^{.15t}, \therefore P(10) = 100 \cdot e^{.15(10)} = 100 \cdot e^{1.5} \approx 448$$