§ 3.10 - Differentials

- Sometimes we need $\Delta y = f(x+\Delta x) - f(x)$

Ex: How much will $f(x) = x^4 + 3x^2 - 4x - 1$

Change if x changes from $x = 2$ to $x = 2.01$?

$x = 2$, $\Delta x = 0.01$, $f(2.01) - f(2) = [(2.01)^4 + 3(2.01)^2 - 4(2.01) - 1] -$

$[2^4 + 3(2)^2 - 4(2) - 1$

$= 0.40270801$

That was a bunch of work just of find Δy.

- Insted, we can find the diffrential dy:

① $dy = f'(x)dx$

② $\boxed{dy \approx \Delta y}$, $dx \approx \Delta X$

OR $dx = \Delta X$

So $\Delta y \approx dy = (4x^3 + 6x - 4) \cdot (0.01)$

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$f(x) = x^4 + 3x^2 - 4x - 1$

$dy = (4x^3 + 6x - 4) \cdot dx$

↳ approximate Δy when $x = 2$, $\Delta x = dx = 0.01$

$\Rightarrow (4 \cdot 8 + 6 \cdot 2 - 4) \cdot (0.01)$

$= (32 + 12 - 4)\left(\frac{1}{100}\right)$

$= \frac{40}{100} = \frac{4}{10} = \frac{2}{5} = 0.4$

The radius of a sphere is 21cm, with a possible error of 0.05 cm. Use differentials to approximate the max possible error of the volume. $\leftrightarrow$ Error = |(Actual volume) − (Measured Volume)|

Exact Answer : $\left(\frac{4}{3}\cdot\pi(\underline{21.05})^3\right) - \left(\frac{4}{3}\cdot\pi(\underline{21})^3\right) = A$

$$\left(\frac{4}{3}\cdot\pi(\underline{21})^3 - \frac{4}{3}\cdot\pi(\underline{20.95})\right)^3 = B$$

max error is max of $\{A, B\}$

FYI: 277.749 cm^3

Instead we will use differentials:

$$\Delta V \approx dV = \left[\tfrac{4}{3}\pi r^3\right]' \cdot dr$$

$$= 4\pi r^2 \cdot dr \quad \leftrightarrow \quad r = 21$$

$$dr = \Delta r = 0.05$$

$$= 4\cdot\pi\cdot(21)^2\cdot(0.05)$$

$$= 277.088\ cm^3$$