This print-out should have 46 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

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001 10.0 points

Determine

$$\lim_{x \to 0} \left(\frac{1}{x^2 + x} - \frac{1}{x} \right) .$$

1. limit $= -\dfrac{1}{2}$

2. limit $= 1$

3. limit $= \dfrac{1}{3}$

4. limit $= \dfrac{1}{2}$

5. limit $= -\dfrac{1}{3}$

6. limit $= -1$ **correct**

Explanation:

After simplification we see that

$$\frac{1}{x^2 + x} - \frac{1}{x} = \frac{1 - (x+1)}{x(x+1)} = -\frac{1}{x+1} ,$$

for all $x \neq 0$. Thus

$$\boxed{\text{limit} = \lim_{x \to 0} -\frac{1}{x+1} = -1} .$$

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002 10.0 points

When f is the function defined by

$$f(x) = \begin{cases} 3x - 4, & x \leq 4 , \\ 2x - 1, & x > 4 , \end{cases}$$

determine if

$$\lim_{x \to 4^+} f(x)$$

exists, and if it does, find its value.

1. limit $= 9$

2. limit $= 5$

3. limit does not exist

4. limit $= 6$

5. limit $= 8$

6. limit $= 7$ **correct**

Explanation:

The right hand limit

$$\lim_{x \to 4^+} f(x)$$

depends only on the values of f for $x > 4$. Thus

$$\lim_{x \to 4^+} f(x) = \lim_{x \to 4^+} 2x - 1 .$$

Consequently,

$$\boxed{\text{limit} = 2 \times 4 - 1 = 7} .$$

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003 10.0 points

Consider the function

$$f(x) = \begin{cases} 3 - x, & x < -1 \\ x, & -1 \leq x < 3 \\ (x-1)^2, & x \geq 3 . \end{cases}$$

Find all the values of a for which the limit

$$\lim_{x \to a} f(x)$$

exists, expressing your answer in interval notation.

1. $(-\infty, -1) \cup (-1, 3) \cup (3, \infty)$ **correct**

2. $(-\infty, -1) \cup (-1, \infty)$

3. $(-\infty, 3) \cup (3, \infty)$

4. $(-\infty, -1] \cup [3, \infty)$

5. $(-\infty, \infty)$

Explanation:

The graph of f is a straight line on $(-\infty, -1)$, so

$$\lim_{x \to a} f(x)$$

exists (and $= f(a)$) for all a in $(-\infty, -1)$. Similarly, the graph of f on $(-1, 3)$ is a straight line, so

$$\lim_{x \to a} f(x)$$

exists (and $= f(a)$) for all a in $(-1, 3)$. On $(3, \infty)$, however, the graph of f is a parabola, so

$$\lim_{x \to a} f(x)$$

still exists (and $= f(a)$) for all a in $(3, \infty)$.

On the other hand,

$$\lim_{x \to -1-} f(x) = 4, \quad \lim_{x \to -1+} f(x) = -1,$$

while

$$\lim_{x \to 3-} f(x) = 3, \quad \lim_{x \to 3+} f(x) = 4.$$

Thus neither of the limits

$$\lim_{x \to -1} f(x), \quad \lim_{x \to 3} f(x)$$

exists. Consequently, the limit exists only for values of a in

$$\boxed{(-\infty, -1) \cup (-1, 3) \cup (3, \infty)}.$$

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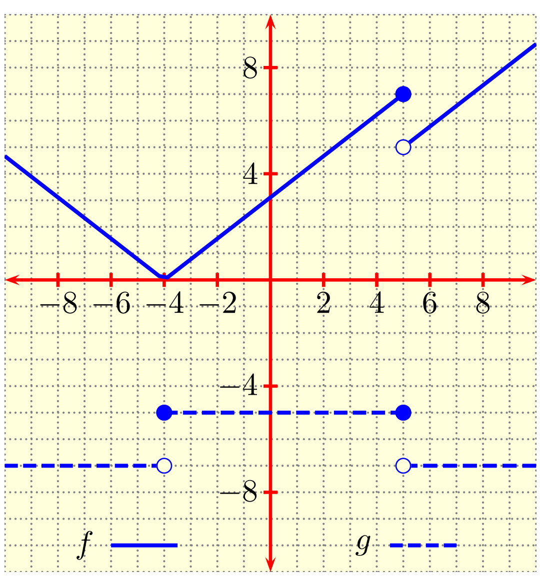
004 10.0 points

Functions f and g are defined on $(-10, 10)$ by their respective graphs in



Find all values of x where the product, fg, of f and g is continuous, expressing your answer in interval notation.

1. $(-10, 5) \bigcup (5, 10)$

2. $(-10, -4] \bigcup [5, 10)$

3. $(-10, -4) \bigcup (-4, 5) \bigcup (5, 10)$

4. $(-10, 10)$ **correct**

5. $(-10, -4) \bigcup (-4, 10)$

Explanation:

Since f and g are piecewise linear, they are continuous *individually* on $(-10, 10)$ except at their 'jumps'; *i.e.*, at $x = 5$ in the case of f and $x = 5, -4$ in the case of g. But the product of continuous functions is again continuous, so fg is certainly continuous on

$$(-10, -4) \bigcup (-4, 5) \bigcup (5, 10).$$

The only question is what happens at $x_0 = 5, -4$. To do that we have to check if

$$\lim_{x \to x_0-} \{f(x)g(x)\}$$

$$= f(x_0)g(x_0)$$

$$= \lim_{x \to x_0+} \{f(x)g(x)\}.$$

Now at $x_0 = 5$,

$$\lim_{x \to 5-} \{f(x)g(x)\} = -35 = f(5)g(5)$$

$$= \lim_{x \to 5+} \{f(x)g(x)\},$$

while at $x_0 = -4$,

$$\lim_{x \to -4-} \{f(x)g(x)\} = 0 = f(-4)g(-4)$$

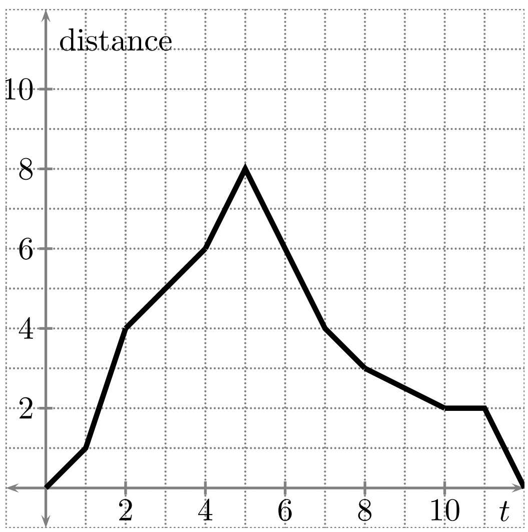
$$= \lim_{x \to -4+} \{f(x)g(x)\}.$$

Thus, fg is continuous at $x = 5$ and at $x = -4$. Consequently, the product fg is continuous at all x in

$$\boxed{(-10,\ 10)}\,.$$

005 (part 1 of 3) 10.0 points

A Calculus student leaves the RLM building and walks in a straight line to the PCL Library. His distance (in multiples of 40 yards) from RLM after t minutes is given by the graph



i) What is his speed after 3 minutes, and in what direction is he heading at that time?

1. away from RLM at 30 yds/min
2. away from RLM at 40 yds/min **correct**
3. away from RLM at 20 yds/min
4. towards RLM at 20 yds/min
5. towards RLM at 40 yds/min

Explanation:

The graph is linear and has positive slope on [2, 4], so the speed of the student at time $t = 3$ coincides with the slope of the line on [2, 4]. Hence

$$\text{speed} = 40 \cdot \frac{6-4}{4-2} = \boxed{40 \text{ yds/min}\,.}$$

As the distance from RLM is increasing on [2, 4] the student is thus moving away from the RLM.

006 (part 2 of 3) 10.0 points

ii) What is his speed after 9 minutes, and in what direction is he heading at that time?

1. towards RLM at 40 yds/min
2. away from RLM at 5 yds/min.
3. away from RLM at 10 yds/min.
4. away from RLM at 20 yds/min.
5. towards RLM at 20 yds/min. **correct**

Explanation:

The graph is linear on [8, 10], so the student's speed at time $t = 9$ is the (absolute value of the) slope of this line. Hence

$$\text{slope} = 40 \cdot \frac{2-3}{10-8} = \boxed{-20 \text{ yards/min}\,.}$$

The fact that the distance is decreasing at $t = 9$ indicates that the student is walking towards RLM at that time.

007 (part 3 of 3) 10.0 points

iii) How far is he from RLM when he turns back?

1. distance = 320 yards **correct**
2. distance = 160 yards
3. distance = 200 yards
4. distance = 240 yards
5. distance = 280 yards

Explanation:

The graph achieves its maximum 8 at $t = 5$, at which point his distance from RLM begins decreasing. Thus at $t = 5$ the student has started walking back to RLM. Hence the student is $\boxed{\text{320 yards}}$ away from the RLM when he turns back.

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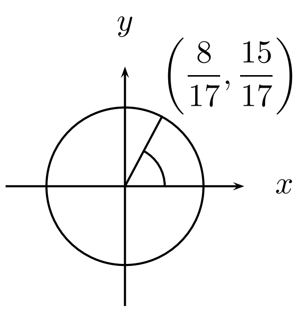
008 (part 1 of 6) 10.0 points

Consider the angle t defined by the point $\left(\frac{8}{17}, \frac{15}{17}\right)$



on the unit circle.

Find $\sin(t)$.

1. $\frac{8}{15}$
2. None of these
3. $\frac{17}{15}$
4. $\frac{17}{8}$
5. $\frac{8}{17}$
6. $\frac{15}{8}$
7. $\frac{15}{17}$ **correct**

Explanation:

$$(x, y) = \left(\frac{8}{17}, \frac{15}{17}\right)$$

$$\sin(t) = y = \frac{15}{17}$$

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009 (part 2 of 6) 10.0 points

Find $\cos(t)$.

1. $\frac{17}{8}$
2. $\frac{15}{17}$
3. $\frac{8}{15}$
4. None of these
5. $\frac{8}{17}$ **correct**
6. $\frac{17}{15}$
7. $\frac{15}{8}$

Explanation:

$$\cos(t) = x = \frac{8}{17}$$

---

010 (part 3 of 6) 10.0 points

Find $\tan(t)$.

1. $\frac{8}{17}$
2. None of these
3. $\frac{8}{15}$
4. $\frac{17}{15}$
5. $\frac{15}{17}$
6. $\frac{17}{8}$
7. $\frac{15}{8}$ **correct**

Explanation:

$$\tan(t) = \frac{y}{x} = \frac{\frac{15}{17}}{\frac{8}{17}} = \frac{15}{8}$$

---

011 (part 4 of 6) 10.0 points

Find $\csc(t)$.

1. $\frac{17}{15}$ **correct**

2. $\dfrac{8}{15}$

3. $\dfrac{8}{17}$

4. None of these

5. $\dfrac{17}{8}$

6. $\dfrac{15}{8}$

7. $\dfrac{15}{17}$

Explanation:

$$\csc(t) = \frac{1}{y} = \frac{1}{\frac{15}{17}} = \frac{17}{15}$$

---

012 (part 5 of 6) 10.0 points

Find $\sec(t)$.

1. $\dfrac{15}{8}$

2. None of these

3. $\dfrac{15}{17}$

4. $\dfrac{8}{15}$

5. $\dfrac{17}{15}$

6. $\dfrac{17}{8}$ **correct**

7. $\dfrac{8}{17}$

Explanation:

$$\sec(t) = \frac{1}{x} = \frac{1}{\frac{8}{17}} = \frac{17}{8}$$

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013 (part 6 of 6) 10.0 points

Find $\cot(t)$.

1. $\dfrac{17}{15}$

2. $\dfrac{8}{15}$ **correct**

3. $\dfrac{8}{17}$

4. $\dfrac{15}{17}$

5. None of these

6. $\dfrac{15}{8}$

7. $\dfrac{17}{8}$

Explanation:

$$\cot(t) = \frac{x}{y} = \frac{\frac{8}{17}}{\frac{15}{17}} = \frac{8}{15}$$

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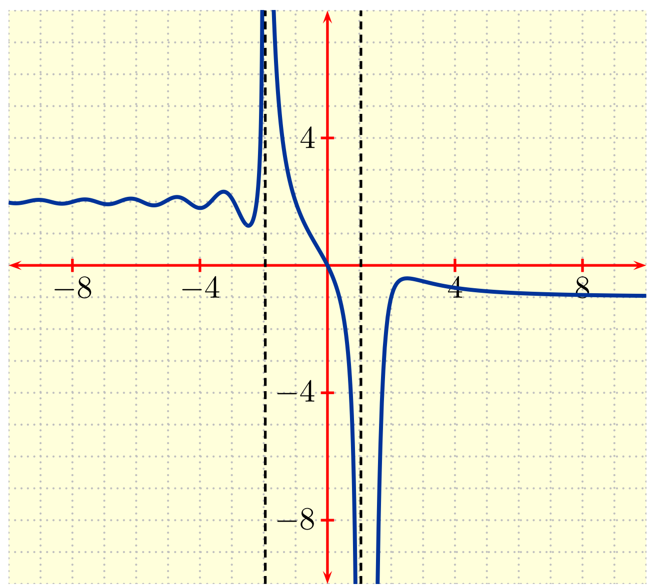
014 (part 1 of 3) 10.0 points

A certain function f is given by the graph



(i) What is the value of

$$\lim_{x \to -\infty} f(x)$$

1. limit $= -1$

2. limit $= -2$

3. limit does not exist

4. limit $= 1$

5. limit $=$ 2 **correct**

Explanation:
To the left of $x = -2$ the graph of f oscillates about the line $y = 2$ and as x approaches $-\infty$ the oscillations become smaller and smaller. Thus

$$\boxed{\text{limit} = 2}.$$

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015 (part 2 of 3) 10.0 points

(ii) What is the value of

$$\lim_{x \to \infty} f(x)?$$

1. limit does not exist

2. limit $=$ 1

3. limit $= -1$ **correct**

4. limit $= -2$

5. limit $=$ 2

Explanation:
To the right of $x = 1$ the graph of f is asymptotic to the line $y = -1$. Thus

$$\boxed{\text{limit} = -1}.$$

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016 (part 3 of 3) 10.0 points

(iii) What is the value of

$$\lim_{x \to -2} f(x)?$$

1. limit $= -2$

2. limit $= -1$

3. limit $=$ 2

4. limit $= \infty$ **correct**

5. limit $=$ 1

Explanation:
From the graph of f the left hand limit

$$\lim_{x \to -2-} f(x) = \infty,$$

while the right hand limit

$$\lim_{x \to -2+} f(x) = \infty.$$

Thus the two-sided limit

$$\boxed{\lim_{x \to -2} f(x) = \infty}.$$

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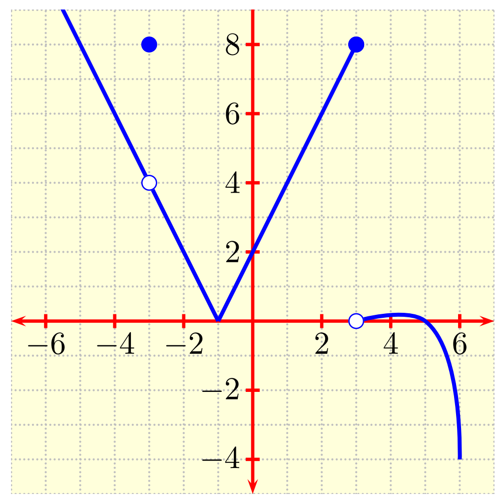
017 10.0 points

Below is the graph of a function f.



Use the graph to determine all the values of x on $(-6, 6)$ at which f fails to be continuous.

1. $x = -3, 3$ **correct**

2. none of the other answers

3. no values of x

4. $x = 3$

5. $x = -3$

Explanation:
Since $f(x)$ is defined for all values of x on $(-6, 6)$, the only values of x in $(-6, 6)$ at which the function f is discontinuous are those for which

$$\lim_{x \to x_0} f(x) \neq f(x_0)$$

or

$$\lim_{x \to x_0-} f(x) \neq \lim_{x \to x_0+} f(x).$$

The only possible candidates here are $x_0 = -3$ and $x_0 = 3$. But at $x_0 = -3$

$$f(-3) = 8 \neq \lim_{x \to -3} f(x) = 4,$$

while at $x_0 = 3$

$$\lim_{x \to 3-} f(x) = 8 \neq \lim_{x \to 3+} f(x) = 0.$$

Consequently, on $(-6, 6)$ the function f fails to be continuous only at

$$\boxed{\text{at } x = -3,\ 3}.$$

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018 (part 1 of 3) 10.0 points

Determine the value of

$$\lim_{x \to 5+} \frac{x-6}{x-5}.$$

1. limit $= \infty$

2. limit $= -\infty$ **correct**

3. none of the other answers

4. limit $= -\dfrac{6}{5}$

5. limit $= \dfrac{6}{5}$

Explanation:

For $5 < x < 6$ we see that

$$\frac{x-6}{x-5} < 0.$$

On the other hand,

$$\lim_{x \to 5+} x - 5 = 0.$$

Thus, by properties of limits,

$$\boxed{\lim_{x \to 5+} \frac{x-6}{x-5} = -\infty}.$$

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019 (part 2 of 3) 10.0 points

Determine the value of

$$\lim_{x \to 5-} \frac{x-6}{x-5}.$$

1. limit $= -\dfrac{6}{5}$

2. none of the other answers

3. limit $= \dfrac{6}{5}$

4. limit $= -\infty$

5. limit $= \infty$ **correct**

Explanation:

For $x < 5 < 6$ we see that

$$\frac{x-6}{x-5} > 0.$$

On the other hand,

$$\lim_{x \to 5-} x - 5 = 0.$$

Thus, by properties of limits,

$$\boxed{\lim_{x \to 5-} \frac{x-6}{x-5} = \infty}.$$

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020 (part 3 of 3) 10.0 points

Determine the value of

$$\lim_{x \to 5} \frac{x-6}{x-5}.$$

1. limit $= -\infty$

2. limit $= \dfrac{6}{5}$

3. limit $= \infty$

4. limit $= -\dfrac{6}{5}$

5. none of the other answers **correct**

Explanation:

If

$$\lim_{x \to 5} \frac{x-6}{x-5}$$

exists, then

$$\lim_{x \to 5+} \frac{x-6}{x-5} = \lim_{x \to 5-} \frac{x-6}{x-5}.$$

But as parts (i) and (ii) show,

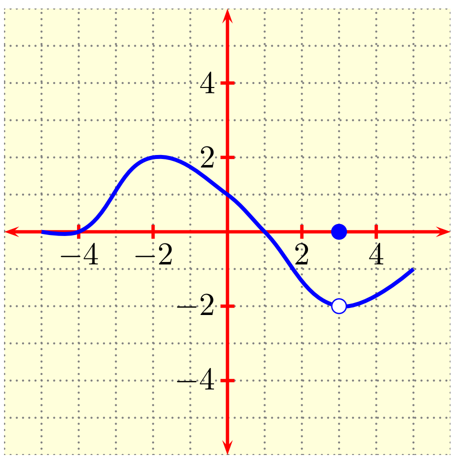
$$\lim_{x \to 5+} \frac{x-6}{x-5} \neq \lim_{x \to 5-} \frac{x-6}{x-5}.$$

Consequently,

$$\boxed{\lim_{x \to 5} \frac{x-6}{x-5} \text{ does not exist}}.$$

021 10.0 points

Below is the graph of a function f.



Use the graph to determine $\lim_{x \to 3} f(x)$.

1. limit $= 0$

2. does not exist

3. limit $= 1$

4. limit $= -2$ **correct**

5. limit $= -1$

Explanation:

From the graph it is clear that the limit

$$\lim_{x \to 3-} f(x) = -2,$$

from the left and the limit

$$\lim_{x \to 3+} f(x) = -2,$$

from the right exist and coincide in value. Thus the two-sided limit exists and

$$\boxed{\lim_{x \to 3} f(x) = -2}.$$

022 10.0 points

Find the value of

$$\lim_{x \to \infty} \frac{2+3x+2x^4}{3-5x^3}.$$

1. none of the other answers

2. limit $= -\infty$ **correct**

3. limit $= \infty$

4. limit $= \frac{2}{3}$

5. limit $= -\frac{2}{5}$

6. limit $= 0$

Explanation:

Dividing the numerator and denominator by x^4 we see that

$$\frac{2+3x+2x^4}{3-5x^3} = \frac{\frac{2}{x^4} + \frac{3}{x^3} + 2}{\frac{1}{x}\left(\frac{3}{x^3} - 5\right)}.$$

With $s = 1/x$, therefore,

$$\frac{2+3x+2x^4}{3-5x^3} = \frac{1}{s}\left(\frac{2s^4+3s^3+2}{3s^3-5}\right).$$

Thus

$$\lim_{x \to \infty} \frac{2+3x+2x^4}{3-5x^3} = \lim_{s \to 0+} \frac{1}{s}\left(\frac{2s^4+3s^3+2}{3s^3-5}\right).$$

Now

$$\lim_{s \to 0+} \left(\frac{2s^4+3s^3+2}{3s^3-5}\right) = -\frac{2}{5}.$$

On the other hand,

$$\lim_{s \to 0+} \frac{1}{s} = \infty .$$

Consequently, by properties of limits,

$$\boxed{\text{limit} = -\infty} .$$

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023 (part 1 of 3) 10.0 points

If $t = \dfrac{\pi}{4}$, evaluate (if possible)
a) $\sin t$

1. $\frac{1}{2}$

2. $-\frac{\sqrt{3}}{2}$

3. 1

4. None of these

5. $\frac{\sqrt{3}}{2}$

6. $\frac{1}{\sqrt{2}}$ **correct**

7. 0

Explanation:
$t = \dfrac{\pi}{4}$ corresponds to the point:

$$(x, y) = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$$

$$\sin\left(\frac{\pi}{4}\right) = y = \frac{1}{\sqrt{2}}$$

---

024 (part 2 of 3) 10.0 points

b) $\cos t$

1. 0

2. -1

3. $\frac{1}{2}$

4. None of these

5. $\frac{\sqrt{3}}{2}$

6. $-\frac{\sqrt{3}}{2}$

7. $\frac{1}{\sqrt{2}}$ **correct**

Explanation:

$$\cos\left(\frac{\pi}{4}\right) = x = \frac{1}{\sqrt{2}}$$

---

025 (part 3 of 3) 10.0 points

c) $\tan t$

1. -1

2. $-\frac{\sqrt{3}}{2}$

3. None of these

4. $\frac{1}{2}$

5. 0

6. 1 **correct**

7. $\frac{\sqrt{3}}{2}$

Explanation:

$$\tan\left(\frac{\pi}{4}\right) = \frac{y}{x} = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = 1$$

---

026 10.0 points

Determine where

$$f(x) = \begin{cases} 20 - x, & x \le -5, \\ x^2, & -5 < x < 2, \\ 2 + x, & x \ge 2. \end{cases}$$

is continuous, expressing your answer in interval notation.

1. $(-\infty, -5) \cup (2, \infty)$

2. $(-\infty, \infty)$ **correct**

3. $(-\infty, -5) \cup (-5, 2) \cup (2, \infty)$

4. $(-\infty, 2) \cup (2, \infty)$

5. $(-\infty, -5) \cup (-5, \infty)$

Explanation:

The function is piecewise continuous, so we have to check the left and right hand limits at the points where the definition of f changes, *i.e.*, at $x = -5$ and $x = 2$. Now at $x = -5$

$$\lim_{x \to -5-} f(x) = \lim_{x \to -5-} 20 - x = 25,$$

$$\lim_{x \to -5+} f(x) = \lim_{x \to -5+} x^2 = 25,$$

hence, the function is continuous at the point $x = -5$. On the other hand, at $x = 2$

$$\lim_{x \to 2-} f(x) = \lim_{x \to 2-} x^2 = (2)^2 = 4,$$

$$\lim_{x \to 2+} f(x) = \lim_{x \to 2+} 2 + x = 4.$$

Thus, the function is also continuous at the point $x = 2$, and so interval notation, f is continuous for x in

$$\boxed{(-\infty, \infty)}.$$

keywords: continuous, limit, piecewise defined function,

027 10.0 points

Find the largest value of c so that the function g defined by

$$g(x) = \begin{cases} x^2 + x - c^2, & x > -1, \\ cx - 12, & x \le -1, \end{cases}$$

is continuous for all x.

1. $c = 7$

2. $c = -3$

3. $c = 3$

4. none of these **correct**

5. $c = -7$

Explanation:

Since g is linear for $x \le -1$ and quadratic for $x > -1$, g is continuous for all $x \ne -1$. On the other hand,

$$\lim_{x \to -1+} g(x) = -c^2$$

while

$$\lim_{x \to -1-} g(x) = -c - 12 = g(-1).$$

Thus g is continuous at $x = -1$ when

$$-c^2 = -c - 12, \quad i.e., \quad c^2 - c - 12 = 0.$$

But

$$c^2 - c - 12 = 0 = (c+3)(c-4),$$

so g is continuous for all x when

$$\boxed{c = -3, 4}.$$

028 10.0 points

Find the solution of the exponential equation

$$3^{2x} = 9^{\frac{5}{2}x - 3}.$$

1. none of these

2. $x = 3$

3. $x = -2$

4. $x = 2$ **correct**

5. $x = -3$

Explanation:

By properties of exponents,

$$9^{\frac{5}{2}x-3} = 3^{5x-6}.$$

Thus the equation can be rewritten as

$$3^{2x} = 3^{5x-6},$$

which after taking logs to the base 3 of both sides becomes

$$2x = 5x-6.$$

Rearranging and solving we thus find that

$$\boxed{x = 2}.$$

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029 10.0 points

Let F be the function defined by

$$F(x) = \frac{x^2-4}{|x-2|}.$$

Determine if

$$\lim_{x \to 2^-} F(x)$$

exists, and if it does, find its value.

1. limit $= 2$
2. limit $= -2$
3. limit $= 4$
4. limit does not exist
5. limit $= -4$ **correct**

Explanation:

After factorization,

$$\frac{x^2-4}{|x-2|} = \frac{(x+2)(x-2)}{|x-2|}.$$

But, for $x < 2$,

$$|x-2| = -(x-2).$$

Thus

$$F(x) = -(x+2), \quad x < 2,$$

By properties of limits, therefore, the limit exists and

$$\boxed{\lim_{x \to 2^-} F(x) = -4}.$$

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030 10.0 points

Find all values of x at which the function f defined by

$$f(x) = \frac{x-6}{x^2-4x-12}$$

is continuous, expressing your answer in interval notation.

1. $(-\infty, -2) \cup (-2, \infty)$
2. $(-\infty, -2) \cup (-2, 6) \cup (6, \infty)$ **correct**
3. $(-\infty, 6) \cup (6, \infty)$
4. $(-\infty, -2) \cup (-2, -6) \cup (-6, \infty)$
5. $(-\infty, -6) \cup (-6, 2) \cup (2, \infty)$

Explanation:

After factorization the denominator becomes

$$x^2 - 4x - 12 = (x-6)(x+2),$$

so f can be rewritten as

$$f(x) = \frac{x-6}{(x-6)(x+2)} = \frac{1}{(x+2)}$$

whenever $x \neq 6$. At $x = 6$ both the numerator and denominator will be zero; thus f will not be defined, hence not continuous, at $x = 6$. Elsewhere f is a ratio of polynomial functions and so will be continuous except at zeros of its denominator. Thus f will continuous except at $x = 6, -2$. Consequently, in interval notation f will be continuous on

$$\boxed{(-\infty, -2) \cup (-2, 6) \cup (6, \infty)}.$$

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031 10.0 points

Find the value of

$$\lim_{x \to 3} \frac{2x - 6}{\sqrt{x} - \sqrt{3}}$$

if the limit exists.

1. limit $= 3\sqrt{3}$

2. limit $= 2\sqrt{3}$

3. limit $= 12$

4. limit $= 6\sqrt{3}$

5. limit $= 4\sqrt{3}$ **correct**

6. limit does not exist

Explanation:

Since

$$x - 3 = (\sqrt{x} + \sqrt{3})(\sqrt{x} - \sqrt{3}),$$

we can rewrite the given expression as

$$\frac{2(\sqrt{x} + \sqrt{3})(\sqrt{x} - \sqrt{3})}{\sqrt{x} - \sqrt{3}} = 2(\sqrt{x} + \sqrt{3})$$

for $x \neq 3$. Thus

$$\boxed{\lim_{x \to 3} \frac{2x - 6}{\sqrt{x} - \sqrt{3}} = 4\sqrt{3}}.$$

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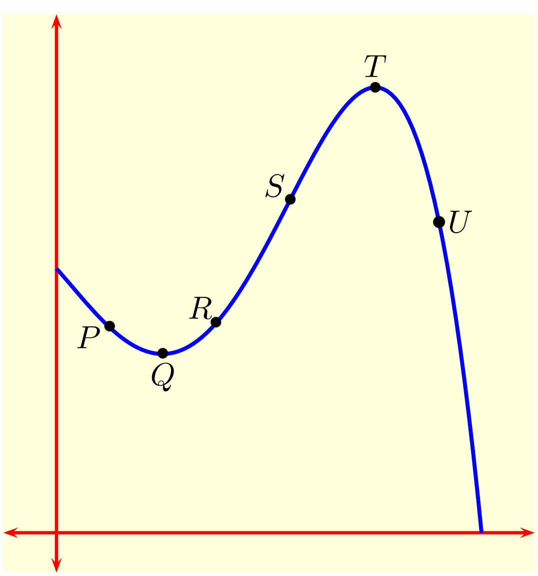
032 (part 1 of 5) 10.0 points

At which point on the graph



is the slope greatest (*i.e.*, most positive)?

1. S **correct**

2. P

3. R

4. U

5. T

6. Q

Explanation:

By inspection the point is S.

---

033 (part 2 of 5) 10.0 points

At which point is the slope smallest (*i.e.*, most negative)?

1. U **correct**

2. S

3. P

4. R

5. T

6. Q

Explanation:

By inspection the point is U.

---

034 (part 3 of 5) 10.0 points

At which point does the slope change from positive to negative?

1. P

2. T **correct**

3. U

4. Q

5. R

6. S

Explanation:
By inspection the point is T.

---

035 (part 4 of 5) 10.0 points

At which point does the slope change from negative to positive?

1. P

2. R

3. U

4. Q **correct**

5. T

6. S

Explanation:
By inspection the point is Q.

---

036 (part 5 of 5) 10.0 points

At which point is the tangent line parallel to the secant line $\overline{PT}$?

1. S

2. P

3. R **correct**

4. U

5. Q

6. T

Explanation:
By inspection the point is R.

---

keywords: slope, graph, change of slope

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037 10.0 points

Determine

$$\lim_{x \to 3} \left\{ \frac{1}{x-3} - \frac{3}{x^2 - 3x} \right\}.$$

1. limit does not exist

2. limit $= -3$

3. limit $= \dfrac{1}{2}$

4. limit $= -\dfrac{1}{2}$

5. limit $= 3$

6. limit $= \dfrac{1}{3}$ **correct**

7. limit $= -\dfrac{1}{3}$

Explanation:
After simplification we see that

$$\frac{1}{x-3} - \frac{3}{x^2-3x} = \frac{x-3}{x(x-3)} = \frac{1}{x},$$

for all $x \neq 3$. Thus

$$\boxed{\text{limit} = \lim_{x \to 3} \frac{1}{x} = \frac{1}{3}}.$$

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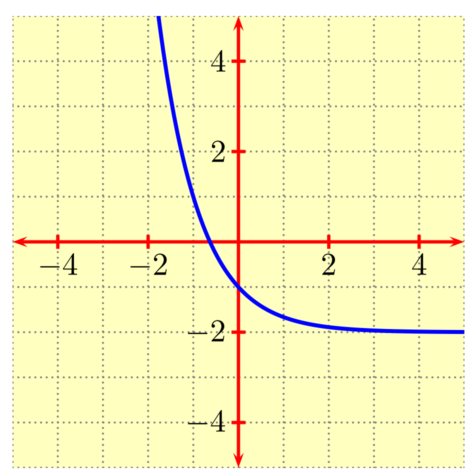
038 10.0 points

Which function has



as its graph?

1. $f(x) = 2 - 2^{-x-1}$

2. $f(x) = 2^{x-1} - 3$

3. $f(x) = 2 - 3^{-x}$

4. $f(x) = 2^{-x-1} - 2$

5. $f(x) = 3^{-x} - 2$ **correct**

6. $f(x) = 3^{x} - 3$

Explanation:

The given graph has the property that

$$\lim_{x \to \infty} f(x) = -2 .$$

But

$$\lim_{x \to \infty} 2^{-x} = 0 = \lim_{x \to \infty} 3^{-x} ,$$

while

$$\lim_{x \to -\infty} 2^{x} = 0 = \lim_{x \to -\infty} 3^{x} ,$$

so $f(x)$ must be one of

$$3^{-x} - 2, \qquad 2^{-x-1} - 2 .$$

On the other hand, the y-intercept of the given graph is at $y = -1$.

Consequently, the graph is that of

$$\boxed{f(x) = 3^{-x} - 2} .$$

---

039 10.0 points

If the function f is continuous everywhere and

$$f(x) = \frac{x^2 - 16}{x + 4}$$

when $x \neq -4$, find the value of $f(-4)$.

1. $f(-4) = -4$

2. $f(-4) = 8$

3. $f(-4) = 16$

4. $f(-4) = -16$

5. $f(-4) = -8$ **correct**

6. $f(-4) = 4$

Explanation:

Since f is continuous at $x = -4$,

$$f(-4) = \lim_{x \to -4} f(x) .$$

But, after factorization,

$$\frac{x^2 - 16}{x + 4} = \frac{(x-4)(x+4)}{x+4} = x - 4 ,$$

whenever $x \neq -4$. Thus

$$f(x) = x - 4$$

for all $x \neq -4$. Consequently,

$$\boxed{f(-4) = \lim_{x \to -4} (x - 4) = -8} .$$

---

040 (part 1 of 2) 10.0 points

Write the polynomial

$$1 - 2x + 8x^2 - 4x^3$$

in standard form.

a) What is its degree?

Correct answer: 3.

Explanation:

Standard form is

$$-4x^3 + 8x^2 - 2x + 1$$

The highest power of x is 3.

---

041 (part 2 of 2) 10.0 points

b) What is the leading coefficient?

Correct answer: -4.

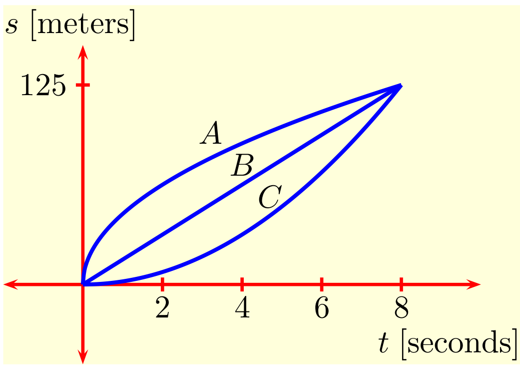
Explanation:

The coefficient of the highest power of x is -4.

---

042 10.0 points

Shown are the graphs of distance versus time for three runners A, B, and C who run a 125 -m race and finish in tie. Which of the following statements about the runners is **false**?



1. Runner C gradually speeds up throughout the race.

2. At $t = 7$, runner B has a lower velocity than runner A. **correct**

3. At $t = 1$, runner A has a higher velocity than B.

4. Runner B runs as a constant speed throughout the race.

5. Runner A gradually slows down throughout the race.

Explanation:

Statement (A) is false. At $t = 7$, the graph associated to runner B is steeper than the graph associated to runner A. Thus runner B has a higher velocity at $t = 7$ than does runner A.

---

keywords: velocity, position function

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043 10.0 points

Find the value of

$$\lim_{x \to 2} \frac{2}{x-2}\left(1 + \frac{6}{x-8}\right)$$

if the limit exists.

1. limit $= -\dfrac{1}{3}$ **correct**

2. limit $= \dfrac{1}{2}$

3. limit $= -\dfrac{1}{2}$

4. limit does not exist

5. limit $= \dfrac{1}{3}$

Explanation:

After the second term in the product is brought to a common denominator it becomes

$$\frac{x+6-8}{x-8} = \frac{x-2}{x-8}.$$

Thus the given expression can be written as

$$\frac{2(x-2)}{(x-2)(x-8)} = \frac{2}{x-8}$$

so long as $x \neq 2$. Consequently,

$$\lim_{x \to 2} \frac{2}{x-2}\left(1 + \frac{6}{x-8}\right) = \lim_{x \to 2} \frac{2}{x-8}.$$

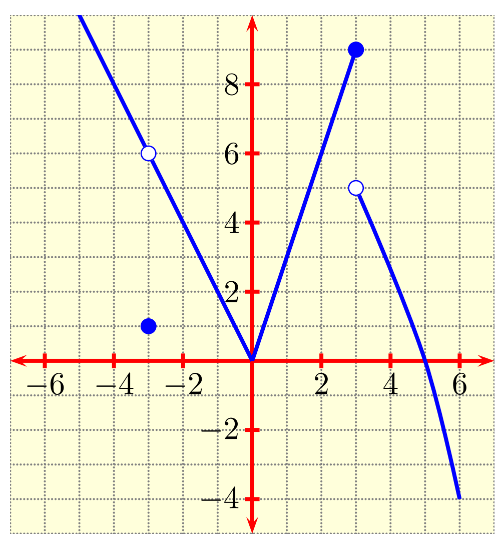
By properties of limits, therefore,

$$\boxed{\text{limit} = -\frac{1}{3}}.$$

---

044 10.0 points

Below is the graph of a function f.



Use the graph to determine $\lim_{x \to -3} f(x)$.

1. $\lim_{x \to -3} f(x) = 1$

2. $\lim_{x \to -3} f(x) = 9$

3. $\lim_{x \to -3} f(x) = 12$

4. $\lim_{x \to -3} f(x)$ does not exist

5. $\lim_{x \to -3} f(x) = 6$ **correct**

Explanation:

From the graph it is clear the f has both a left hand limit and a right hand limit at $x = -3$; in addition, these limits coincide. Thus

$$\boxed{\lim_{x \to -3} f(x) = 6}.$$

---

045 10.0 points

Find the value of b, $b \geq 0$, for which

$$\lim_{x \to 0} \left\{ \frac{\sqrt{6x+b} - 1}{x} \right\}$$

exists.

1. $b = 3$

2. $b = 4$

3. $b = 2$

4. $b = 1$ **correct**

5. $b = 0$

Explanation:

We are told that

$$\lim_{x \to 0} \left\{ \frac{\sqrt{6x+b} - 1}{x} \right\} = A$$

for some value A, but we aren't told what the particular value of A is. The question requires us to see exactly what value b must take for A to exist.

To begin, note that

$$\sqrt{x+y} - \sqrt{z} = \frac{x+y-z}{\sqrt{x+y} + \sqrt{z}}.$$

Thus

$$\frac{\sqrt{6x+b} - 1}{x} = \frac{6x+b-1}{x(\sqrt{6x+b}+1)}$$

$$= \frac{6x}{x(\sqrt{6x+b}+1)} + \frac{b-1}{x(\sqrt{6x+b}+1)}.$$

Now

$$\lim_{x \to 0} \sqrt{6x+b} + 1 = \sqrt{b} + 1,$$

so by properties of limits,

$$\lim_{x \to 0} \frac{6x}{x(\sqrt{6x+b}+1)}$$

$$= \lim_{x \to 0} \frac{6}{\sqrt{6x+b}+1} = \frac{6}{\sqrt{b}+1}.$$

Consequently, once again by the properties of limits,

$$\lim_{x \to 0} \left\{ \frac{b-1}{x\,(\sqrt{6x+b}+1)} \right\}$$

$$= \lim_{x \to 0} \left(\frac{\sqrt{6x+b} - 1}{x} - \frac{6x}{x(\sqrt{6x+b}+1)} \right)$$

$$= A - \frac{6}{\sqrt{b}+1}.$$

But

$$\lim_{x \to 0} \left\{ \frac{b-1}{x\,(\sqrt{6x+b}+1)} \right\}$$

$$= (b-1) \lim_{x \to 0} \left\{ \frac{1}{x\,(\sqrt{6x+b}+1)} \right\}.$$

Since

$$\lim_{x \to 0} \frac{1}{x\,(\sqrt{6x+b}+1)}$$

doesn't exist, however, the only way

$$\lim_{x \to 0} \left\{ \frac{b-1}{x\,(\sqrt{6x+b}+1)} \right\} = A - \frac{6}{\sqrt{b}+1}$$

can hold is if

$$\boxed{b-1 = 0}.$$

Notice that this means that

$$A - \frac{6}{\sqrt{b}+1} = 0,$$

so the previously unknown value of A is thus found to be $A = 3$.

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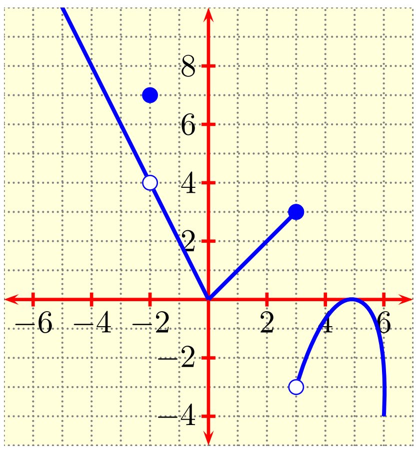
046 10.0 points

Below is the graph of a function f.



Use the graph to determine

$$\lim_{x \to 3} f(x).$$

1. limit does not exist **correct**

2. limit $= 7$

3. limit $= 4$

4. limit $= 3$

5. limit $= 12$

Explanation:

From the graph it is clear the f has a left hand limit at $x = 3$ which is equal to 3; and a right hand limit which is equal to -3. Since the two numbers do not coincide, the

$$\boxed{\text{limit does not exist}}.$$