This print-out should have 39 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

## 001 10.0 points

Determine

$$\lim_{x \to 0} \left( \frac{2}{x^2 + 2x} - \frac{1}{x} \right) .$$

- 1. limit =  $-\frac{1}{3}$
- **2.**  $\lim_{x \to 0} 1 = -2$
- **3.** limit =  $\frac{1}{3}$
- **4.** limit =  $\frac{1}{2}$
- 5. limit =  $-\frac{1}{2}$  correct
- 6.  $\lim_{x \to 0} 1 = 2$

#### **Explanation:**

After simplification we see that

$$\frac{2}{x^2 + 2x} - \frac{1}{x} = \frac{2 - (x+2)}{x(x+2)} = -\frac{1}{x+2},$$

for all  $x \neq 0$ . Thus

$$\lim_{x \to 0} -\frac{1}{x+2} = -\frac{1}{2}$$

#### 002 10.0 points

Determine if the limit

$$\lim_{x \to 0} \frac{\frac{6}{x+1} - 6}{x}$$

exists, and if it does, find its value.

- 1. limit = -6 correct
- 2. limit does not exist

- 3.  $\lim_{\to} 1 = 6$
- **4.**  $\lim_{t \to 0} t = 7$
- **5.**  $\lim_{\to} -7$

#### Explanation:

After bringing the numerator to a common denominator and rearranging, we obtain,

$$\frac{\frac{6}{x+1} - 6}{x} = \frac{6 - 6(x+1)}{x(x+1)}$$
$$= -\frac{6x}{x(x+1)} = -\frac{6}{x+1}.$$

Now

$$\lim_{x \to 0} \left( \frac{6}{x+1} \right)$$

exists, so the limit

$$\lim_{x \to 0} \frac{\frac{6}{x+1} - 6}{x}$$

exists and

$$\lim_{x \to 0} \left( \frac{6}{x+1} \right) = -6$$

#### 003 10.0 points

Determine which, if any, of

$$f(x) = 6^{-x} + 3,$$
  

$$g(x) = 6^{3-x},$$
  

$$h(x) = -6^{x-3}.$$

define the same function.

- 1. only f, h
- **2.** only q, h
- **3.** f, g, and h

**4.** only g, f

**5.** no two of f, g, or h correct

## **Explanation:**

By the Laws of Exponents,

$$f(x) = 6^{-x} + 3$$

while

$$g(x) = 6^{3-x} = 6^3 \cdot 6^{-x},$$

and

$$h(x) = -6^{x-3} = -(6^x \cdot 6^{-3}).$$

Consequently,

no two of 
$$f$$
,  $g$ , or  $h$ 

define the same function.

## 004 10.0 points

Determine which, if any, of the following

$$f(x) = 9^x + 9,$$

$$q(x) = 3^{2x+3}$$
,

$$h(x) = 27(9^x) ,$$

define the same function.

- 1. f, g, and h
- **2.** only f, h
- 3. only g, h correct
- **4.** only g, f
- **5.** none of f, g, or h

#### **Explanation:**

By the Laws of Exponents,

$$f(x) = 9^{x} + 9 = (3^{2})^{x} + 9$$
$$= 3^{2x} + 9.$$

while 
$$g(x) = 3^{2x+3}$$
 and

$$h(x) = 27 (9^x) = (3^3)(3^2)^x$$
$$= (3^3)(3^{2x}) = 3^{2x+3}.$$

Thus g and h define the same function. On the other hand,

$$f(0) = 10, \qquad g(0) = 27 = h(0),$$

so neither g nor h can define the same function as f. Consequently

only 
$$g, h$$

define the same function.

## 005 (part 1 of 2) 10.0 points

Write the polynomial

$$6 - 5x + 5x^4 - 7x^9$$

in standard form.

a) What is its degree?

Correct answer: 9.

## **Explanation:**

Standard form is

$$-7x^9 + 5x^4 - 5x + 6$$

The highest power of x is 9.

## 006 (part 2 of 2) 10.0 points

b) What is the leading coefficient?

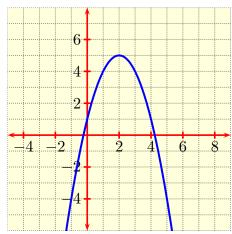
Correct answer: -7.

## Explanation:

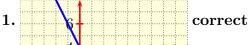
The coefficient of the highest power of x is -7.

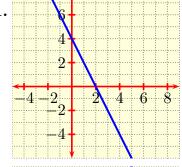
## 007 10.0 points

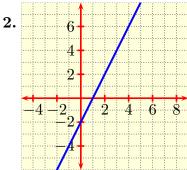
If f is a function having

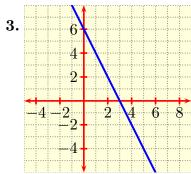


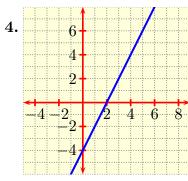
as its graph, which of the following could be the graph of f'?

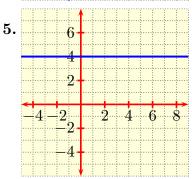






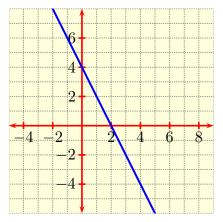






## **Explanation:**

The slope of the tangent to the graph is zero at x=2, is positive to the left of x=2 and is negative to the right of x = 2. In addition, the slope of the tangent is decreasing as xincreases, so the graph of f' is



00810.0 points

Determine

$$\lim_{x \to 0} \frac{x-1}{x^2(x+8)}.$$

**1.**  $\lim_{t \to 0} 1$ 

2. limit = 
$$-\frac{1}{8}$$

3.  $\lim_{n \to \infty} -\infty$  correct

4. 
$$\lim_{n \to \infty} 1$$

5. 
$$\lim_{\to} 1 = 0$$

**6.** none of the other answers

#### **Explanation:**

Now

$$\lim_{x \to 0} x - 1 = -1.$$

On the other hand,  $x^2(x+8) > 0$  for all small x, both positive and negative, while

$$\lim_{x \to 0} x^2(x+8) = 0.$$

Consequently,

$$\lim_{n \to \infty} 1$$

keywords: evaluate limit, rational function

## 009 10.0 points

If a, b are the solutions of the exponential equation

$$3^{x^2} = 9^{-\frac{3}{2}x+9}$$

calculate the value of |a+b|.

1. 
$$|a+b| = -3$$

**2.** 
$$|a+b| = 3$$
 **correct**

3. 
$$|a+b| = 5$$

**4.** 
$$|a+b| = 11$$

**5.** 
$$|a+b|=4$$

## **Explanation:**

By properties of exponents,

$$9^{-\frac{3}{2}x+9} = 3^{-3x+18}.$$

Thus the equation can be rewritten as

$$3^{x^2} = 3^{-3x+18}.$$

which after taking logs to the base 3 becomes

$$x^2 = -3x + 18.$$

This equation factors as

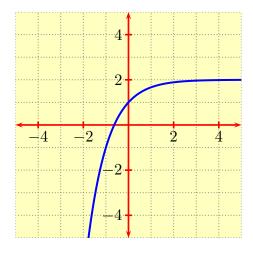
$$(x-3)(x+6) = 0,$$

and so its solutions are 3, -6. Hence

$$|a+b|=3.$$

## 010 10.0 points

Which function has



as its graph?

1. 
$$f(x) = 3^x - 3$$

**2.** 
$$f(x) = 3^{-x} - 2$$

3. 
$$f(x) = 2 - 3^{-x}$$
 correct

**4.** 
$$f(x) = 2^{-x-1} - 2$$

5. 
$$f(x) = 2^{x-1} - 3$$

**6.** 
$$f(x) = 2 - 2^{-x-1}$$

#### **Explanation:**

The given graph has the property that

$$\lim_{x \to \infty} f(x) = 2.$$

But

$$\lim_{x \to \infty} 2^{-x} = 0 = \lim_{x \to \infty} 3^{-x},$$

while

$$\lim_{x \to -\infty} 2^x = 0 = \lim_{x \to -\infty} 3^x,$$

so f(x) must be one of

$$2 - 3^{-x}$$
,  $2 - 2^{-x-1}$ .

On the other hand, the y-intercept of the given graph is at y = 1.

Consequently, the graph is that of

$$f(x) = 2 - 3^{-x} \ .$$

The straight line  $\ell$  is parallel to y+4x=5 and passes through the point P(4,3). Find its y-intercept.

- 1. y-intercept = 20
- **2.** y-intercept = 21
- 3. y-intercept = 19 correct
- 4. y-intercept = -13
- 5. y-intercept = -12

## **Explanation:**

Since  $\ell$  is parallel to the line y + 4x = 5, these lines have the same slope -4, Thus by the point-slope formula the equation of  $\ell$  is given by

$$y-3 = -4(x-4)$$
.

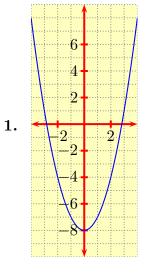
Now the y-intercept of  $\ell$  occurs at x = 0. Consequently,

$$y$$
-intercept = 19.

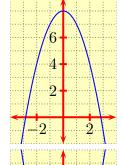
## 012 10.0 points

Sketch the graph of the function

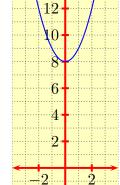
$$f(x) = (x+8)^2.$$



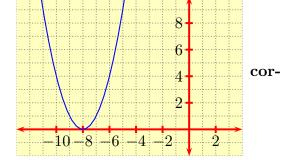




# 3.

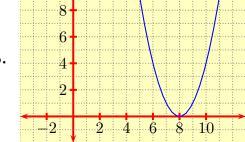


4.



rect





**6.** None of these

## **Explanation:**

The graph of the function

$$f(x) = (x+8)^2 = [x-(-8)]^2$$

can be obtained from the graph of the function

 $g(x) = x^2$  by shifting it 8 units to the left.

## 013 10.0 points

Find all values of x at which the function f defined by

$$f(x) = \frac{x-8}{x^2+9}$$

is not continuous?

- 1. x = 3
- **2.** x = -3, 8
- 3. x = -3, 3
- **4.** x = 8
- 5. x = -3
- **6.** no values of x correct

## **Explanation:**

Because f is a rational function it will fail to be continuous only at zeros of the denominator. Since there are no real solutions to

$$x^2 = -9$$

the function is continuous everywhere; put another way, f fails to be continuous at

no values of x

#### 014 10.0 points

Determine which of the following functions (if any) are the same.

$$f(x) = 9^{-x} + 7$$
$$g(x) = 9^{7-x}$$
$$h(x) = -9^{x-7}$$

- **1.** f(x) = g(x) = h(x)
- 2. None of these **correct**
- **3.** q(x) = f(x) only

**4.** 
$$g(x) = h(x)$$
 only

**5.** 
$$f(x) = h(x)$$
 only

## **Explanation:**

$$f(x) = 9^{-x} + 7$$

$$g(x) = 9^{7-x} = 9^7 \cdot 9^{-x}$$

$$h(x) = -9^{x-7} = -(9^x \cdot 9^{-7})$$

Thus f(x), g(x) and h(x) are all distinct.

## 015 10.0 points

Find all values of x at which the function f defined by

$$f(x) = \frac{x - 7}{x^2 - x - 42}$$

is continuous, expressing your answer in interval notation.

1. 
$$(-\infty, -6) \cup (-6, \infty)$$

**2.** 
$$(-\infty, -7) \cup (-7, 6) \cup (6, \infty)$$

**3.** 
$$(-\infty, 7) \cup (7, \infty)$$

**4.** 
$$(-\infty, -6) \cup (-6, 7) \cup (7, \infty)$$
 **correct**

5. 
$$(-\infty, -6) \cup (-6, -7) \cup (-7, \infty)$$

## Explanation:

After factorization the denominator becomes

$$x^2 - x - 42 = (x - 7)(x + 6),$$

so f can be rewritten as

$$f(x) = \frac{x-7}{(x-7)(x+6)} = \frac{1}{(x+6)}$$

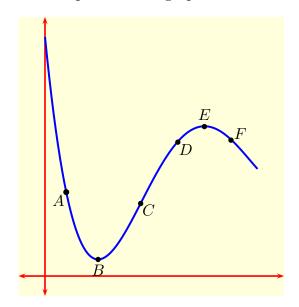
whenever  $x \neq 7$ . At x = 7 both the numerator and denominator will be zero; thus f will not be defined, hence not continuous, at x = 7. Elsewhere f is a ratio of polynomial functions and so will be continuous except at

zeros of its denominator. Thus f will continuous except at x=7,-6. Consequently, in interval notation f will be continuous on

$$(-\infty, -6) \cup (-6, 7) \cup (7, \infty)$$

## 016 (part 1 of 5) 10.0 points

At which point on the graph



is the slope greatest (i.e., most positive)?

- **1.** *B*
- **2.** A
- 3. C correct
- **4.** E
- **5.** *F*
- **6.** *D*

#### **Explanation:**

By inspection the point is C.

## 017 (part 2 of 5) 10.0 points

At which point is the slope smallest (*i.e.*, most negative)?

**1.** *D* 

- **2.** E
- **3.** C
- **4.** *F*
- **5.** B
- 6. A correct

## **Explanation:**

By inspection the point is A.

## 018 (part 3 of 5) 10.0 points

At which point does the slope change from positive to negative?

- 1. E correct
- **2.** C
- **3.** *F*
- **4.** A
- **5.** D
- **6.** *B*

#### **Explanation:**

By inspection the point is E.

#### 019 (part 4 of 5) 10.0 points

At which point does the slope change from negative to positive?

- **1.** *E*
- 2. B correct
- **3.** *D*
- **4.** *F*
- **5.** A
- **6.** C

## **Explanation:**

By inspection the point is B.

## 020 (part 5 of 5) 10.0 points

At which point is the tangent line parallel to the secant line  $\overline{BF}$ ?

- **1.** *E*
- **2.** C
- **3.** *B*
- **4.** *F*
- **5.** *A*
- 6. D correct

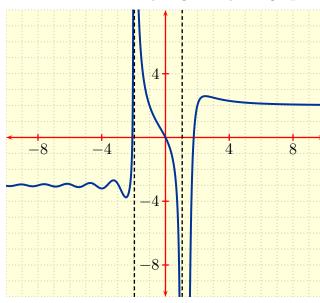
## **Explanation:**

By inspection the point is D.

keywords: slope, graph, change of slope

## 021 (part 1 of 3) 10.0 points

A certain function f is given by the graph



(i) What is the value of

$$\lim_{x \to -\infty} f(x)$$

1. limit does not exist

- **2.**  $\lim_{x \to 0} 1 = 2$
- 3.  $\lim_{\to} 1 = -2$
- **4.**  $\lim_{x \to 0} 1 = 3$
- 5.  $\lim_{t \to 0} t = -3$  correct

## **Explanation:**

To the left of x = -2 the graph of f oscillates about the line y = -3 and as x approaches  $-\infty$  the oscillations become smaller and smaller. Thus

$$limit = -3$$

## 022 (part 2 of 3) 10.0 points

(ii) What is the value of

$$\lim_{x \to \infty} f(x)$$
?

- 1. limit = 2 correct
- 2. limit does not exist
- 3.  $\lim_{\to} 1 = -3$
- 4.  $\lim_{x \to 0} 1 = 3$
- **5.**  $\lim_{x \to 0} 1 = -2$

#### Explanation:

To the right of x = 1 the graph of f is asymptotic to the line y = 2. Thus

## 023 (part 3 of 3) 10.0 points

(iii) What is the value of

$$\lim_{x \to -2} f(x)?$$

- 1.  $\lim_{\to} 1 = -2$
- 2.  $\lim_{n \to \infty} \mathbf{correct}$

3.  $\lim_{x \to 0} 1 = 2$ 

4.  $\lim_{x \to a} 1 = 3$ 

**5.**  $\lim_{x \to 0} 1 = -3$ 

## **Explanation:**

From the graph of f the left hand limit

$$\lim_{x \to -2-} f(x) = \infty,$$

while the right hand limit

$$\lim_{x \to -2+} f(x) = \infty.$$

Thus the two-sided limit

$$\lim_{x \to -2} f(x) = \infty .$$

#### 024 10.0 points

Find the largest value of c so that the function g defined by

$$g(x) = \begin{cases} x^2 - 3x - c^2, & x > 3, \\ cx - 4, & x \le 3, \end{cases}$$

is continuous for all x.

1. c = 4

**2.** c = 5

3. c = -5

4. none of these correct

5. c = -4

#### **Explanation:**

Since g is linear for  $x \leq 3$  and quadratic for x > 3, g is continuous for all  $x \neq 3$ . On the other hand,

$$\lim_{x \to 3+} g(x) = -c^2$$

while

$$\lim_{x \to 3^{-}} g(x) = 3c - 4 = g(3).$$

Thus g is continuous at x = 3 when

$$-c^2 = 3c - 4$$
, i.e.,  $c^2 + 3c - 4 = 0$ .

But

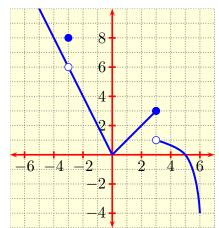
$$c^2 + 3c - 4 = 0 = (c+4)(c-1),$$

so g is continuous for all x when

$$\boxed{c = -4, 1}.$$

#### 025 10.0 points

Below is the graph of a function f.



Use the graph to determine

$$\lim_{x \to 3} f(x).$$

1.  $\lim_{n \to \infty} 1 = 8$ 

**2.**  $\lim_{x \to 0} 12$ 

**3.**  $\lim_{x \to 0} 1 = 6$ 

4. limit does not exist correct

5.  $\lim_{x \to 0} 1 = 3$ 

#### Explanation:

From the graph it is clear the f has a left hand limit at x = 3 which is equal to 3; and

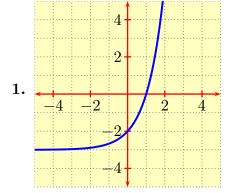
a right hand limit which is equal to 1. Since the two numbers do not coincide, the

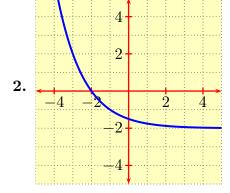
limit does not exist

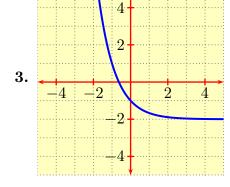
## 026 10.0 points

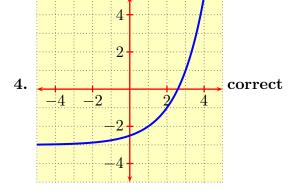
Which of the following is the graph of

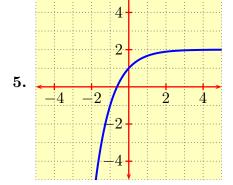
$$f(x) = 2^{x-1} - 3?$$

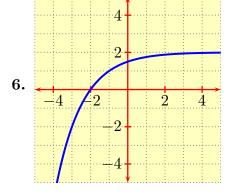












## **Explanation:**

Since

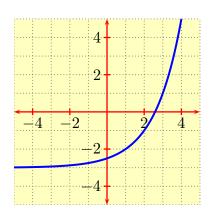
$$\lim_{x \to -\infty} 2^x = 0,$$

we see that

$$\lim_{x \to -\infty} f(x) = -3,$$

in particular, f has a horizontal asymptote y=-3. This eliminates all but two of the graphs. On the other hand,  $f(0)=-\frac{5}{2}$ , so the y-intercept of the given graph must occur at  $y=-\frac{5}{2}$ .

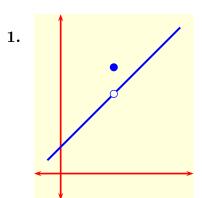
Consequently, the graph is of f is

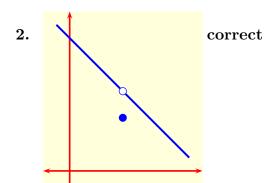


027 10.0 points

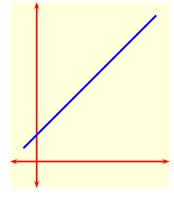
Determine which of the following could be the graph of f near the origin when

$$f(x) = \begin{cases} \frac{x^2 - 7x + 10}{2 - x}, & x \neq 2, \\ 2, & x = 2. \end{cases}$$

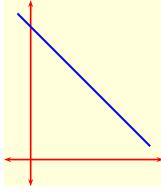




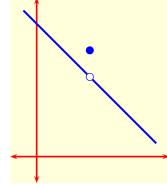
3.



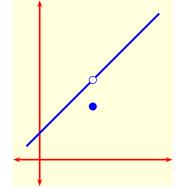
4.



**5.** 



6.



## **Explanation:**

Since

$$\frac{x^2 - 7x + 10}{2 - x} = \frac{(x - 2)(x + 5)}{2 - x} = 5 - x,$$

for  $x \neq 2$ , we see that f is linear on

$$(-\infty, 2) \bigcup (2, \infty),$$

while

$$\lim_{x \to 2} f(x) = 3 \neq f(2).$$

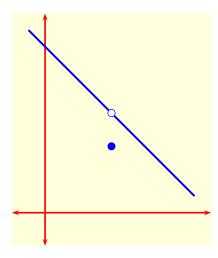
Thus the graph of f will be a straight line of slope -1, having a hole at x=2. This eliminates four of the possible graphs. But the two remaining graphs are the same except that in one

$$f(2) > \lim_{x \to 2} f(x),$$

while in the other

$$f(2) < \lim_{x \to 2} f(x).$$

Consequently,



must be the graph of f near the origin.

## 028 10.0 points

Evaluate

$$\lim_{x \to -2} \frac{x+2}{x^2 - 4x - 12}.$$

- 1.  $\lim_{\to} \frac{1}{4}$
- 2. limit =  $-\frac{1}{8}$  correct
- **3.** limit does not exist

**4.** limit = 
$$-\frac{1}{4}$$

5. limit = 
$$\frac{1}{8}$$

**6.** 
$$\lim_{x \to 0} 1 = -4$$

## **Explanation:**

Since

$$x^2 - 4x - 12 = (x - 6)(x + 2),$$

the expression above can be rewritten as

$$\frac{x+2}{x^2-4x-12} = \frac{1}{x-6}$$

for  $x \neq -2$ . Thus

$$\lim_{x \to -2} \frac{x+2}{x^2 - 4x - 12} = -\frac{1}{8}$$

#### 029 10.0 points

Find the value of

$$\lim_{x \to 0} \frac{(3x-2)^2 - 4}{5x}$$

if the limit exists.

- 1. limit =  $-\frac{12}{5}$  correct
- **2.** limit =  $\frac{12}{5}$
- 3. limit =  $\frac{6}{5}$
- 4. limit does not exist
- 5. limit =  $-\frac{6}{5}$

#### **Explanation:**

After expansion,

$$(3x-2)^2-4 = (9x^2-12x+4)-4$$
.

Thus

$$\frac{(3x-2)^2-4}{5x} = \frac{x(9x-12)}{5x}$$
$$= \frac{9}{5}x - \frac{12}{5},$$

when  $x \neq 0$ . Consequently,

$$\lim_{x \to 0} \frac{(3x-2)^2 - 4}{5x} = -\frac{12}{5}$$

#### 030 10.0 points

Let F be the function defined by

$$F(x) = \frac{x^2 - 9}{|x - 3|}.$$

Determine if

$$\lim_{x \to 3^{-}} F(x)$$

exists, and if it does, find its value.

- 1. limit does not exist
- **2.**  $\lim_{x \to 0} 1 = -3$
- 3.  $\lim_{x \to a} 1 = 3$
- 4.  $\lim_{t \to 0} t = -6$  correct
- 5.  $\lim_{\to} = 6$

#### **Explanation:**

After factorization,

$$\frac{x^2 - 9}{|x - 3|} = \frac{(x + 3)(x - 3)}{|x - 3|}.$$

But, for x < 3,

$$|x-3| = -(x-3)$$
.

Thus

$$F(x) = -(x+3), \quad x < 3,$$

By properties of limits, therefore, the limit exists and

$$\lim_{x \to 3^-} F(x) = -6$$

## 031 10.0 points

If the function f defined by

$$f(x) = \begin{cases} cx + 4, & x < 2, \\ 4x^2 - 4, & x \ge 2, \end{cases}$$

is continuous everywhere on  $(-\infty, \infty)$ , what is the value of f(1)?

1. 
$$f(1) = 9$$

**2.** 
$$f(1) = 12$$

3. 
$$f(1) = 10$$

**4.** 
$$f(1) = 8$$
 **correct**

5. 
$$f(1) = 11$$

## **Explanation:**

Since f is linear to the left of x=2 and quadratic to the right of x=2, it is certainly continuous on  $(-\infty, 2) \cup (2, \infty)$ . But at x=2,

$$\lim_{x \to 2^{-}} f(x) = 2c + 4,$$

while

$$\lim_{x \to 2+} f(x) = 16 - 4 = 12,$$

Thus the continuity at x = 2 ensures that

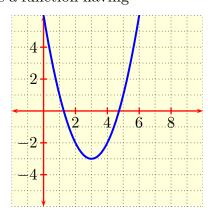
$$2c + 4 = 12$$
, i.e.,  $c = 4$ .

Consequently,

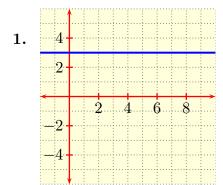
$$f(1) = c + 4 = 8$$

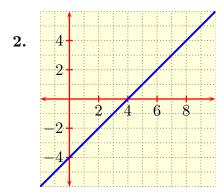
#### 032 10.0 points

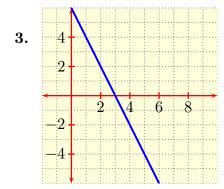
If f is a function having

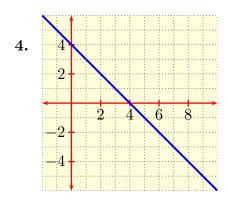


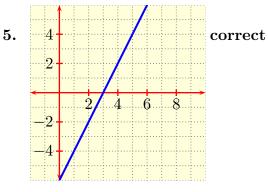
as its graph, which of the following is the graph of the derivative f' of f?





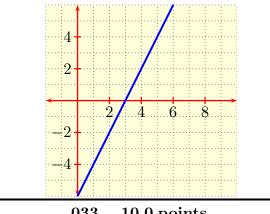






## **Explanation:**

The slope of the tangent to the graph is zero at x = 3, is negative to the left of x = 3 and is positive to the right of x = 3. But it is increasing for all x, so the graph of f' is



033 10.0 points

If the function f is continuous everywhere and

$$f(x) = \frac{x^2 - 9}{x - 3}$$

when  $x \neq 3$ , find the value of f(3).

1. 
$$f(3) = 3$$

**2.** 
$$f(3) = 6$$
 **correct**

3. 
$$f(3) = -3$$

**4.** 
$$f(3) = -9$$

**5.** 
$$f(3) = 9$$

**6.** 
$$f(3) = -6$$

## **Explanation:**

Since f is continuous at x = 3,

$$f(3) = \lim_{x \to 3} f(x).$$

But, after factorization,

$$\frac{x^2 - 9}{x - 3} = \frac{(x - 3)(x + 3)}{x - 3} = x + 3,$$

whenever  $x \neq 3$ . Thus

$$f(x) = x + 3$$

for all  $x \neq 3$ . Consequently,

$$f(3) = \lim_{x \to 3} (x+3) = 6$$

## 034 (part 1 of 3) 10.0 points

Determine the value of

$$\lim_{x \to 2+} \frac{x-8}{x-2}.$$

- 1. none of the other answers
- **2.**  $\lim_{x \to a} 1 = 4$
- 3.  $\lim_{n \to \infty} 1$
- **4.**  $\lim_{x \to 0} 1 = -4$
- 5.  $\lim_{n \to \infty} -\infty$  correct

#### **Explanation:**

For 2 < x < 8 we see that

$$\frac{x-8}{x-2} < 0.$$

On the other hand,

$$\lim_{x \to 2+} x - 2 = 0.$$

Thus, by properties of limits,

$$\lim_{x \to 2+} \frac{x-8}{x-2} = -\infty$$

## 035 (part 2 of 3) 10.0 points

Determine the value of

$$\lim_{x \to 2-} \frac{x-8}{x-2}.$$

1.  $\lim_{n \to \infty} 1 = -\infty$ 

- **2.**  $\lim_{x \to a} 1 = 4$
- 3.  $\lim_{n \to \infty} \mathbf{correct}$
- 4. none of the other answers
- **5.**  $\lim_{x \to 0} 1 = -4$

## **Explanation:**

For x < 2 < 8 we see that

$$\frac{x-8}{x-2} > 0.$$

On the other hand,

$$\lim_{x \to 2-} x - 2 = 0.$$

Thus, by properties of limits,

$$\lim_{x \to 2-} \frac{x-8}{x-2} = \infty$$

## 036 (part 3 of 3) 10.0 points

Determine the value of

$$\lim_{x \to 2} \frac{x-8}{x-2}.$$

- 1.  $\lim_{n \to \infty} 1$
- **2.**  $\lim_{x \to a} 1 = 4$
- 3.  $\lim_{n \to \infty} 1$
- **4.**  $\lim_{x \to 0} 1 = -4$
- **5.** none of the other answers **correct**

#### **Explanation:**

If

$$\lim_{x \to 2} \frac{x-8}{x-2}$$

exists, then

$$\lim_{x \to 2+} \frac{x-8}{x-2} = \lim_{x \to 2-} \frac{x-8}{x-2}.$$

But as parts (i) and (ii) show,

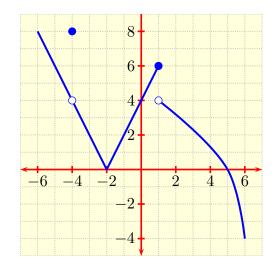
$$\lim_{x \to 2+} \frac{x-8}{x-2} \neq \lim_{x \to 2-} \frac{x-8}{x-2}.$$

Consequently,

$$\lim_{x \to 2} \frac{x - 8}{x - 2} \text{ does not exist} \quad .$$

## 037 10.0 points

Below is the graph of a function f.



Use the graph to determine all the values of x on (-6, 6) at which f fails to be continuous.

- 1. x = -4, 1 correct
- **2.** none of the other answers
- **3.** x = 1
- **4.** x = -4
- **5.** no values of x

#### **Explanation:**

Since f(x) is defined for all values of x on (-6, 6), the only values of x in (-6, 6) at which the function f is discontinuous are those for which

$$\lim_{x \to x_0} f(x) \neq f(x_0)$$

or

$$\lim_{x \to x_0-} f(x) \neq \lim_{x \to x_0+} f(x).$$

The only possible candidates here are  $x_0 = -4$  and  $x_0 = 1$ . But at  $x_0 = -4$ 

$$f(-4) = 8 \neq \lim_{x \to -4} f(x) = 4,$$

while at  $x_0 = 1$ 

$$\lim_{x \to 1^{-}} f(x) = 6 \neq \lim_{x \to 1^{+}} f(x) = 4.$$

Consequently, on (-6, 6) the function f fails to be continuous only at

at 
$$x = -4, 1$$

## 038 10.0 points

Find the solution of the exponential equation

$$4^{15x} = 16^{\frac{9}{2}x - 4}.$$

Correct answer: -8/6.

## **Explanation:**

By properties of exponents,

$$16^{\frac{9}{2}x-4} = 4^{9x-8}.$$

Thus the equation can be rewritten as

$$4^{15x} = 4^{9x-8}$$

which, after taking logs to the base 4, becomes

$$15x = 9x - 8$$
.

Rearranging and solving we thus find that

$$x = -8/6.$$

#### 039 10.0 points

A tank holds 1000 gallons of water, which drains from the bottom of the tank in half an hour. The values in the table

t  (min)	5	10	15	20	25	30
V (gal)	644	466	212	116	19	0

show the volume, V(t), of water remaining in the tank (in gallons) after t minutes.

If P is the point (15, V(15)) on the graph of V as a function of time t, find the slope of the secant line PQ when Q = (25, V(25)).

- 1. slope = -38.6
- 2. slope = -19.3 correct
- 3. slope = -43.2
- **4.** slope = -9.6
- 5. slope = -25.4

## **Explanation:**

When

$$P = (15, V(15)), Q = (25, V(25))$$

the slope of the secant line PQ is given by

$$\frac{\text{rise}}{\text{run}} = \frac{V(25) - V(15)}{25 - 15}.$$

From the table of values, therefore, we see that

slope = 
$$\frac{19 - 212}{25 - 15} = -19.3$$
.