# § 4.9 - Antiderivatives, continued.

- $f(x)$ - function , $F(x)$ - Antiderivative of $f(x)$

$f(x) = c \cdot x^n$ , $F(x) = \frac{c}{n+1} x^{n+1} + C$ , $n \neq -1$

$f(x) = \frac{c}{x}$ , $F(x) = c \cdot \ln|x| + C$

$f(x) = \sin(x)$, $F(x) = -\cos(x) + C$

$f(x) = \cos(x)$, $F(x) = \sin(x) + C$

- $f(x) = e^x$ , $F(x) = e^x + C$

Ex: $f(x) = \frac{x^3 - 2x^2 + x - 1}{2x} \Rightarrow F(x) = \frac{\frac{1}{4}x^4 - \frac{2}{3}x^3 + \frac{1}{2}x^2 - x}{x^2} + C$

$= \frac{x^2}{2} - x + \frac{1}{2} - \frac{1}{2x}$ ($-\frac{1}{2} \cdot \frac{1}{x}$) $\Rightarrow F(x) = \frac{1}{2} \cdot \frac{1}{3}x^3 - \frac{1}{2}x^2 + \frac{1}{2}x - \frac{1}{2}\ln|x| + C$

$F(x) = \frac{1}{6}x^3 - \frac{1}{2}x^2 + \frac{1}{2}x - \frac{1}{2}\ln|x| + C$

$= \frac{1}{2}\left(\frac{x^3}{3} - x^2 + x - \ln|x| + C\right)$

$$f(x) = 3x\left(\sqrt{x} + \frac{2}{\sqrt{x}} + \frac{6}{x^{3/2}}\right)$$

$$= 3x^{3/2} + 6x^{1/2} + 18x^{-1/2}$$

$$F(x) = \frac{3x^{5/2}}{5/2} + \frac{6x^{3/2}}{3/2} + \frac{18x^{1/2}}{1/2} + C$$

$$= \frac{6}{5}x^{5/2} + 4x^{3/2} + 36x^{1/2} + C$$

$$= \sqrt{x}\left(\frac{6}{5}x^2 + 4x + 36\right) + C$$

$f(x) = \sin(2x) + \cos(4x)$

$F(x) = -\cos(2x) + \sin(4x) + C$ , Am I right?

Check: $F'(x) = [-\cos(2x) + \sin(4x) + C]'$

$= +\sin(2x)\cdot 2 + \cos(4x)\cdot 4$

$= 2\sin(2x) + 4\cos(4x)$, ¨

$$F(x) = \frac{-\cos(2x)}{2} + \frac{\sin(4x)}{4} + C$$

Check: $F'(x) = \cancel{+\cos x} + \frac{\sin(2x)\cdot 2}{2} + \frac{\cos(4x)\cdot 4}{4} = f(x)$