

§ 3.2 - Quotient Rule

$$\left[\frac{x^4 - x}{x^2} \right]' = [x^2 - x^{-1}]' = 2x - (-x^{-2}) = 2x + \frac{1}{x^2} = \dots$$

$$\left[\frac{x^2}{x^4 - x} \right]' \Rightarrow \text{Quotient Rule: } \left[\frac{f(x)}{g(x)} \right]' = \frac{f'(x)g(x) - g'(x)f(x)}{[g(x)]^2}$$

$$\left[\frac{x^4 - x}{x^2} \right]' = \frac{[x^4 - x]'x^2 - [x^2]'(x^4 - x)}{(x^2)^2} = \frac{(4x^3 - 1)x^2 - (2x)(x^4 - x)}{x^4}$$

$$= \frac{4x^5 - x^2 - 2x^5 + 2x^2}{x^4} = \frac{2x^5 + x^2}{x^4} = \boxed{2x + \frac{1}{x^2}}$$

$$\left[\frac{x^2}{x^4 - x} \right]' = \frac{[x^2]'(x^4 - x) - [x^4 - x]'x^2}{(x^4 - x)^2} =$$

$$= \frac{2x(x^4 - x) - (4x^3 - 1)x^2}{(x^4 - x)^2} = \frac{2x^5 - 2x^2 - [4x^5 - x^2]}{(x^4 - x)^2}$$

$$= \frac{2x^5 - 2x^2 - 4x^5 + x^2}{(x^4 - x)^2} = \frac{-2x^5 - x^2}{(x^4 - x)^2}$$

$$\left[\frac{x^2 e^x}{2x+1} \right]' = \frac{[x^2 e^x]'(2x+1) - [2x+1]'(x^2 e^x)}{(2x+1)^2} =$$

$$= \frac{([x^2]'e^x + x^2[e^x]')(2x+1) - 2x^2 e^x}{(2x+1)^2}$$

$$= \frac{(2x e^x + x^2 e^x)(2x+1) - 2x^2 e^x}{(2x+1)^2}$$

$$= \frac{4x^2 e^x + 2x e^x + 2x^3 e^x + x^2 e^x - 2x^2 e^x}{(2x+1)^2}$$

$$\frac{3x^2 e^x + 2x e^x + 2x^3 e^x}{(2x+1)^2}$$

$$= \frac{x e^x (2x^2 + 3x + 2)}{(2x+1)^2}$$