

This print-out should have 27 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

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**001 10.0 points**

Find the derivative of  $f$  when

$$f(\theta) = \ln(\sin 6\theta) .$$

1.  $f'(\theta) = 6 \tan 6\theta$
2.  $f'(\theta) = \frac{6}{\sin 6\theta}$
3.  $f'(\theta) = \frac{1}{\cos 6\theta}$
4.  $f'(\theta) = 6 \cot 6\theta$
5.  $f'(\theta) = \cot 6\theta$
6.  $f'(\theta) = -\tan 6\theta$

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**002 10.0 points**

Differentiate the function

$$f(x) = \cos(\ln 4x) .$$

1.  $f'(x) = -\sin(\ln 4x)$
2.  $f'(x) = \frac{1}{\cos(\ln 4x)}$
3.  $f'(x) = -\frac{\sin(\ln 4x)}{x}$
4.  $f'(x) = \frac{4 \sin(\ln 4x)}{x}$
5.  $f'(x) = -\frac{4 \sin(\ln 4x)}{x}$
6.  $f'(x) = \frac{\sin(\ln 4x)}{x}$

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**003 10.0 points**

Find the slope of the line tangent to the graph of

$$\ln(xy) + 2x = 0$$

at the point where  $x = 1$ .

1. slope =  $3e^{-2}$
2. slope =  $\frac{3}{2}e^{-2}$
3. slope =  $-3e^{-2}$
4. slope =  $\frac{3}{2}e^2$
5. slope =  $-3e^2$
6. slope =  $-\frac{3}{2}e^2$

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**004 10.0 points**

Find the derivative of

$$f(t) = \frac{3 \ln t}{4 - \ln t} .$$

1.  $f'(t) = \frac{3}{t(4 - \ln t)}$
2.  $f'(t) = \frac{12 \ln t}{(4 - \ln t)^2}$
3.  $f'(t) = \frac{3}{t(4 - \ln t)^2}$
4.  $f'(t) = \frac{12}{t(4 - \ln t)^2}$
5.  $f'(t) = \frac{12 \ln t}{4 - \ln t}$
6.  $f'(t) = \frac{3 \ln t}{4 - \ln t}$

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**005 10.0 points**

Determine the value of the third derivative of  $f$  at  $x = 1$  when

$$f(x) = 4 \ln(2x + 3),$$

$$1. f'''(x) = \frac{64}{125}$$

$$2. f'''(x) = \frac{128}{125}$$

$$3. f'''(x) = -\frac{32}{125}$$

$$4. f'''(x) = -\frac{64}{125}$$

$$5. f'''(x) = \frac{32}{125}$$

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**006 10.0 points**

Find the value of  $f'(e^5)$  when

$$f(x) = \frac{x}{\ln x}.$$

$$1. f'(e^5) = \frac{4}{25}$$

$$2. f'(e^5) = -\frac{4}{25}$$

$$3. f'(e^5) = \frac{9}{25}$$

$$4. f'(e^5) = -\frac{9}{25}$$

$$5. f'(e^5) = -\frac{8}{25}$$

$$6. f'(e^5) = \frac{8}{25}$$

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**007 10.0 points**

Determine  $y'$  when

$$y = \ln(3x^2 + 2y^2).$$

$$1. y' = \frac{6y}{3x^2 + 2y^2 - 2y}$$

$$2. y' = \frac{4x}{3x^2 + 2y^2 - 4y}$$

$$3. y' = \frac{4y}{3x^2 - 2y^2 + 2y}$$

$$4. y' = \frac{4x}{3x^2 - 2y^2 + 4y}$$

$$5. y' = \frac{6x}{3x^2 + 2y^2 - 4y}$$

$$6. y' = \frac{6x}{3x^2 - 2y^2 + 2y}$$

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**008 10.0 points**

Find  $\frac{dy}{dx}$  when

$$\ln(xy) + x = 4.$$

$$1. \frac{dy}{dx} = \frac{y(x+1)}{x}$$

$$2. \frac{dy}{dx} = -\frac{x+1}{xy}$$

$$3. \frac{dy}{dx} = -\frac{y(x-1)}{x}$$

$$4. \frac{dy}{dx} = 2$$

$$5. \frac{dy}{dx} = -\frac{y(x+1)}{x}$$

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**009 10.0 points**

Determine the derivative of

$$f(x) = 5 \arcsin\left(\frac{x}{3}\right).$$

$$1. f'(x) = \frac{3}{\sqrt{9-x^2}}$$

$$2. f'(x) = \frac{3}{\sqrt{1-x^2}}$$

$$3. f'(x) = \frac{15}{\sqrt{9-x^2}}$$

$$4. f'(x) = \frac{5}{\sqrt{9-x^2}}$$

$$5. f'(x) = \frac{5}{\sqrt{1-x^2}}$$

6.  $f'(x) = \frac{15}{\sqrt{1-x^2}}$

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**010 10.0 points**

Find the derivative of

$$f(x) = \frac{1}{3} \left( \arctan(3x) \right)^2.$$

1.  $f'(x) = \frac{2}{1+9x^2} \arctan(3x)$

2.  $f'(x) = \frac{2}{9+x^2} \arctan(3x)$

3.  $f'(x) = \frac{1}{3} \sec^2(3x) \tan(3x)$

4.  $f'(x) = 2 \sec^2(3x) \tan(3x)$

5.  $f'(x) = \frac{1}{1+9x^2} \arctan(3x)$

6.  $f'(x) = \frac{1}{9+x^2} \arctan(3x)$

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**011 10.0 points**

Find the derivative of  $f$  when

$$f(x) = 3(\sin^{-1} x)^2.$$

1.  $f'(x) = \frac{6 \sin^{-1} x}{\sqrt{1-x^2}}$

2.  $f'(x) = \frac{6 \sin^{-1} x}{1+x^2}$

3.  $f'(x) = \frac{3 \cos^{-1} x}{\sqrt{1-x^2}}$

4.  $f'(x) = \frac{6 \cos^{-1} x}{\sqrt{1-x^2}}$

5.  $f'(x) = \frac{3 \sin^{-1} x}{\sqrt{1-x^2}}$

6.  $f'(x) = \frac{3 \cos^{-1} x}{1+x^2}$

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**012 10.0 points**

Find the derivative of

$$f(x) = \sin^{-1}(e^{3x}).$$

1.  $f'(x) = \frac{e^{3x}}{\sqrt{1-e^{6x}}}$

2.  $f'(x) = \frac{1}{\sqrt{1-e^{6x}}}$

3.  $f'(x) = \frac{1}{1+e^{6x}}$

4.  $f'(x) = \frac{3}{\sqrt{1-e^{6x}}}$

5.  $f'(x) = \frac{3e^{3x}}{1+e^{6x}}$

6.  $f'(x) = \frac{e^{3x}}{1+e^{6x}}$

7.  $f'(x) = \frac{3}{1+e^{6x}}$

8.  $f'(x) = \frac{3e^{3x}}{\sqrt{1-e^{6x}}}$

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**013 10.0 points**

Find the derivative of

$$f(x) = 3 \tan^{-1}(e^x).$$

1.  $f'(x) = \frac{3}{\sqrt{1-e^{2x}}}$

2.  $f'(x) = \frac{1}{\sqrt{1-e^{2x}}}$

3.  $f'(x) = \frac{3e^x}{1+e^{2x}}$

4.  $f'(x) = \frac{e^x}{\sqrt{1-e^{2x}}}$

5.  $f'(x) = \frac{1}{1+e^{2x}}$

6.  $f'(x) = \frac{3}{1+e^{2x}}$

7.  $f'(x) = \frac{e^x}{1 + e^{2x}}$

8.  $f'(x) = \frac{3e^x}{\sqrt{1 - e^{2x}}}$

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**014 10.0 points**

Determine  $f'(x)$  when

$$f(x) = \tan^{-1}\left(\frac{x}{\sqrt{6 - x^2}}\right).$$

(Hint : first simplify  $f$ .)

1.  $f'(x) = \frac{x}{x^2 + 6}$

2.  $f'(x) = \frac{1}{\sqrt{6 - x^2}}$

3.  $f'(x) = \frac{\sqrt{6}}{\sqrt{6 + x^2}}$

4.  $f'(x) = \frac{x}{\sqrt{x^2 - 6}}$

5.  $f'(x) = \frac{\sqrt{6}}{\sqrt{6 - x^2}}$

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**015 10.0 points**

Find the derivative of  $f$  when

$$f(x) = 3 \tan^{-1}(e^{-x}) + 6e^x.$$

1.  $f'(x) = \frac{3e^{-x} + 6e^x}{\sqrt{1 - e^{2x}}}$

2.  $f'(x) = \frac{6e^x + 3e^{-x}}{1 + e^{-2x}}$

3.  $f'(x) = \frac{3e^x - 6e^{-x}}{\sqrt{1 - e^{-2x}}}$

4.  $f'(x) = \frac{3e^{-x} + 6e^x}{\sqrt{1 - e^{-2x}}}$

5.  $f'(x) = \frac{6e^x - 3e^{-x}}{1 + e^{2x}}$

6.  $f'(x) = \frac{6e^{-x} + 3e^x}{1 + e^{-2x}}$

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**016 10.0 points**

A 10 foot ladder is leaning against a wall. If the foot of the ladder is sliding away from the wall at a rate of 9 ft/sec, at what speed is the top of the ladder falling when the foot of the ladder is 8 feet away from the base of the wall?

1. speed =  $\frac{35}{3}$  ft/sec

2. speed = 13 ft/sec

3. speed =  $\frac{37}{3}$  ft/sec

4. speed =  $\frac{38}{3}$  ft/sec

5. speed = 12 ft/sec

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**017 10.0 points**

The radius of a circle is increasing at a constant rate of 2 ft/sec.

Express the rate at which the area of the circle is changing in terms of the circumference,  $C$  of the circle.

1. rate =  $4C$  sq. ft./sec

2. rate =  $2\pi C$  sq. ft./sec

3. rate =  $\pi C$  sq. ft./sec

4. rate =  $4\pi C$  sq. ft./sec

5. rate =  $C$  sq. ft./sec

6. rate =  $2C$  sq. ft./sec

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**018 10.0 points**

Determine the value of  $dy/dt$  at  $x = 2$  when

$$y = x^2 - 3x$$

and  $dx/dt = 3$ .

1.  $\left. \frac{dy}{dt} \right|_{x=2} = 3$

2.  $\left. \frac{dy}{dt} \right|_{x=2} = 7$

3.  $\left. \frac{dy}{dt} \right|_{x=2} = 9$

4.  $\left. \frac{dy}{dt} \right|_{x=2} = 5$

5.  $\left. \frac{dy}{dt} \right|_{x=2} = 11$

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**019 10.0 points**

A point is moving on the graph of  $xy = 5$ . When the point is at  $\left(3, \frac{5}{3}\right)$ , its  $x$ -coordinate is increasing at a rate of 4 units per second.

What is the speed of the  $y$ -coordinate at that moment and in which direction is it moving?

1. speed =  $\frac{29}{9}$  units/sec, increasing  $y$

2. speed =  $\frac{38}{9}$  units/sec, decreasing  $y$

3. speed =  $\frac{20}{9}$  units/sec, decreasing  $y$

4. speed =  $\frac{29}{9}$  units/sec, decreasing  $y$

5. speed =  $\frac{38}{9}$  units/sec, increasing  $y$

6. speed =  $\frac{20}{9}$  units/sec, increasing  $y$

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**020 10.0 points**

If the radius of a melting snowball decreases at a rate of 5 ins/min, find the rate at which the volume is decreasing when the snowball has diameter 3 inches.

1. rate =  $42\pi$  cu.ins/min

2. rate =  $41\pi$  cu.ins/min

3. rate =  $43\pi$  cu.ins/min

4. rate =  $45\pi$  cu.ins/min

5. rate =  $44\pi$  cu.ins/min

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**021 10.0 points**

Gravel is being dumped from a conveyor belt at a rate of  $20 \text{ ft}^3/\text{min}$  and its coarseness is such that it forms a pile in the shape of a cone whose base diameter and height are always equal.

How fast is the height of the pile increasing when the pile is 7 ft high?

1.  $\frac{dh}{dt} \approx 0.51969 \text{ ft/min}$

2.  $\frac{dh}{dt} \approx 0.56969 \text{ ft/min}$

3.  $\frac{dh}{dt} \approx 0.44969 \text{ ft/min}$

4.  $\frac{dh}{dt} \approx 0.50969 \text{ ft/min}$

5.  $\frac{dh}{dt} \approx 0.55969 \text{ ft/min}$

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**022 10.0 points**

Find the linearization of

$$f(x) = \frac{1}{\sqrt{3+x}}$$

at  $x = 0$ .

1.  $L(x) = \frac{1}{\sqrt{3}} - \frac{1}{3}x$

2.  $L(x) = \frac{1}{\sqrt{3}} \left(1 - \frac{1}{6}x\right)$

3.  $L(x) = \frac{1}{3} \left(1 - \frac{1}{3}x\right)$

4.  $L(x) = \frac{1}{\sqrt{3}} + \frac{1}{3}x$

5.  $L(x) = \frac{1}{\sqrt{3}} \left(1 + \frac{1}{6}x\right)$

6.  $L(x) = \frac{1}{3} \left(1 + \frac{1}{6}x\right)$

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**023 10.0 points**

Use linear approximation with  $a = 9$  to estimate the number  $\sqrt{8.6}$  as a fraction.

1.  $\sqrt{8.6} \approx 2\frac{9}{10}$
2.  $\sqrt{8.6} \approx 2\frac{29}{30}$
3.  $\sqrt{8.6} \approx 2\frac{14}{15}$
4.  $\sqrt{8.6} \approx 2\frac{19}{20}$
5.  $\sqrt{8.6} \approx 2\frac{11}{12}$

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**024 10.0 points**

Find the differential,  $dy$ , of

$$y = f(x) = \tan(3x^2).$$

1.  $dy = 3\sec^2(3x^2)\tan(3x^2)$
2.  $dy = 6x\sec^2(3x) + dx$
3.  $dy = 6x\sec^2(3x^2)$
4.  $dy = 3\sec^2(3x^2)\tan(3x^2) + dx$
5.  $dy = 3\sec^2(3x^2)\tan(3x^2) dx$
6.  $dy = 6x\sec^2(3x^2) dx$

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**025 10.0 points**

Use differentials to estimate the amount of paint needed to apply a  $1/8$  cms thick coat of paint to a sphere having diameter 12 cms.

1. amount  $\approx 6\pi \text{ cm}^3$
2. amount  $\approx 18\pi \text{ cm}^3$
3. amount  $\approx 9\pi \text{ cm}^3$

4. amount  $\approx 18 \text{ cm}^3$

5. amount  $\approx 6 \text{ cm}^3$

6. amount  $\approx 9 \text{ cm}^3$

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**026 10.0 points**

After spending  $\$x$  in advertising per day, a McDonalds restaurant finds that it sells  $N$  hamburgers where

$$N = 1800 + 240x - 5x^2.$$

Estimate using differentials how many more hamburgers the restaurant will sell if it increases its daily spending on advertising from \$10 to \$10.70.

1. 96 more hamburgers
2. 97 more hamburgers
3. 100 more hamburgers
4. 98 more hamburgers
5. 99 more hamburgers

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**027 10.0 points**

At the H-E-B stores throughout Texas the daily demand (in pounds) for candy at  $\$x$  per pound is given by

$$D = 7000 - 50x^2, \quad 1 \leq x \leq 7.$$

If the price of a pound of candy is increased from \$3 to \$3.07, use differentials to estimate the change in demand for candy.

1. 24 pound increase in demand
2. 24 pound decrease in demand
3. 23 pound increase in demand
4. 21 pound increase in demand

5. 21 pound decrease in demand
6. 23 pound decrease in demand