Class 05

Last Time: Find $\lim_{x\to a} f(x)$ given
- ↪ graph
- ↳ expression

a is a <u>number</u>

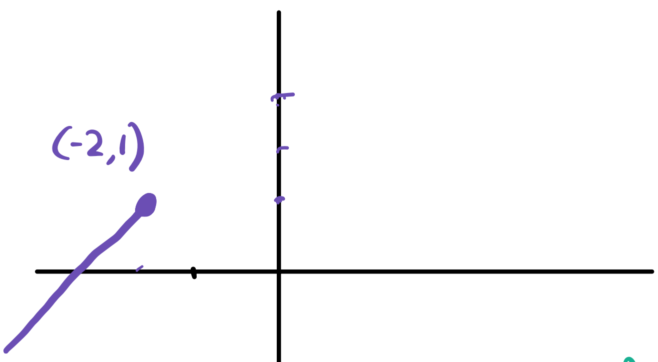
Today: Continuous Functions
- IVT
- Limits at $\pm\infty$, $\lim_{x\to\infty} f(x)$, $\lim_{x\to-\infty} f(x)$

Future: HW02 Due W

Find values of a, b such that $f(x)$ is continuous:

$$f(x) = \begin{cases} x+3 & x \le -2 \\ x^2+4x+5 & -2 < x \le b \\ -2x+32 & x > b \end{cases}$$

$f(-2) = 1$



Continuous: $\lim_{x\to a^-} = \lim_{x\to a^+} = f(a)$

$\Rightarrow \lim_{x\to -2^+} f(x) = 1$

$\lim_{x\to 2^+} x^2+4x+a = 1$

$(-2)^2 + 4(-2) + a = 1$

$4 - 8 + a = 1$

$a = 5$

$x^2+4x+5 = -2x+32$

left limit

$b^2+4b+5 = -2b+32$

$b^2+6b-27 = 0$

$(b-3)(b+9) = 0$

$b = 3, \cancel{-9}$

$b = 3$

$$\lim_{x\to 2} \frac{4}{1+\sin\left(\frac{\pi(x^2-3x+2)}{4x(x-2)}\right)} \; ; \; \lim_{x\to 1} \sqrt{\cos\left(\frac{\pi x-\pi}{x-1}\right)}$$

$$\lim_{x\to 2} \frac{\pi(x^2-3x+2)}{2x(x-2)} = \lim_{x\to 2} \frac{\pi\cancel{(x-2)}(x-1)}{2x\cancel{(x-2)}} = \frac{\pi(1)}{2\cdot 2} = \pi/4$$

$$\frac{4}{1+\sin(\pi/4)} = \frac{4}{1+\frac{\sqrt{2}}{2}} = \frac{4}{\frac{2}{2}+\frac{\sqrt{2}}{2}} = \frac{4}{\frac{2+\sqrt{2}}{2}} = \frac{8}{2+\sqrt{2}}$$

$$\underbrace{\lim_{x\to 1} \sqrt{\cos\left(\frac{\pi x-\pi}{x-1}\right)}}_{= \text{DNE}} \Rightarrow \lim_{x\to 1} \frac{\pi x-\pi}{x-1} = \lim_{x\to 1} \frac{\pi(x-1)}{x-1} = \pi$$

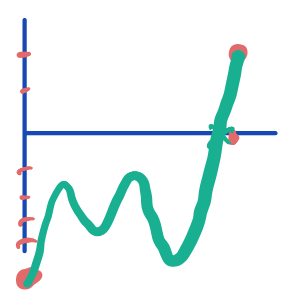
$$\Rightarrow \sqrt{\cos(\pi)} = \sqrt{-1} \leftarrow \text{DNE}$$

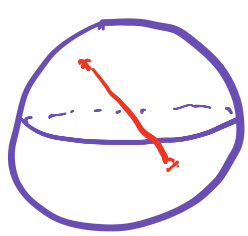
IVT: Watch this: why does $f(x) = 4x^3+3x^2-5$ equal 0 between $x=0$ and $x=1$?

① $4x^3+3x^2-5$ is Continuous on $[0,1]$

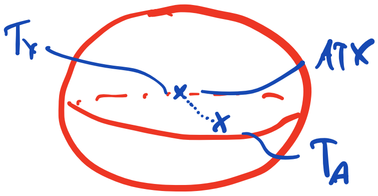
② $f(0) = -5$

③ $f(1) = 2$

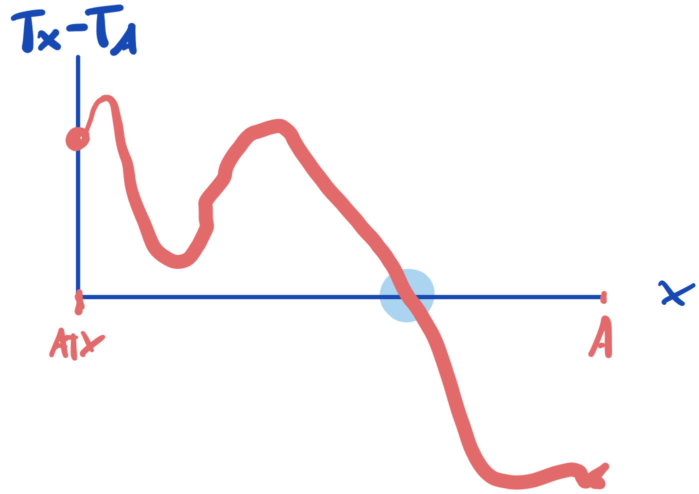




Claim: Somewhere on earth the antipodal points have the same temp.



T_x



$\frac{0}{0}, \infty - \infty, \frac{\infty}{\infty}$

$$\lim_{x\to\infty} \frac{3x^2+x-1}{2x^2-1} \to \left.\frac{\infty}{\infty}\right\} \text{I.F.}$$

$$\hookrightarrow \lim_{x\to\infty} = \frac{\cancel{x^2}\left(3+\frac{1}{x}-\frac{1}{x^2}\right)}{\cancel{x^2}\left(2-\frac{1}{x^2}\right)} = \lim_{x\to\infty} \frac{3+\cancel{\frac{1}{x}}-\cancel{\frac{1}{x^2}}}{2-\cancel{\frac{1}{x^2}}} = \frac{3}{2}$$

$$\lim_{x\to\infty} \frac{\sqrt{6x^2+x-10}}{x+1} = \lim_{x\to\infty} \frac{\sqrt{x^2\left(6+\frac{1}{x}-\frac{10}{x^2}\right)}}{x\left(1+\frac{1}{x}\right)} = \lim_{x\to\infty} \frac{\cancel{\sqrt{x^2}}\cdot\sqrt{6+\cancel{\frac{1}{x}}-\cancel{\frac{10}{x^2}}}}{\cancel{x}\left(1+\cancel{\frac{1}{x}}\right)}$$

$$= \sqrt{6}$$

$$\lim_{x\to-\infty} \frac{\sqrt{6x^2+x-10}}{x+1} = \lim_{x\to-\infty} \frac{\sqrt{x^2}\sqrt{6+\frac{1}{x}-\frac{10}{x^2}}}{x\left(1+\frac{1}{x}\right)} = \lim_{x\to\infty} \frac{-\cancel{x}\sqrt{6+\cancel{\frac{1}{x}}-\cancel{\frac{10}{x^2}}}}{\cancel{x}\left(1+\cancel{\frac{1}{x}}\right)}$$

$$= -\sqrt{6}$$

when $x<0$, $\sqrt{x^2} = -x$

$\sqrt{(-4)^2} = -(-4)$

$x \geq 0$, $\sqrt{x^2} = x$

$$\lim_{x\to-\infty} \frac{\sqrt{4x^2-5}}{3-4x} = \lim_{x\to\infty} \frac{\sqrt{x^2\left(4-\frac{5}{x^2}\right)}}{x\left(\frac{3}{x}-4\right)}$$

$$= \lim_{x \to -\infty} \frac{\sqrt{x^2} \cdot \sqrt{4 - 5/x^2}}{x\left(3/x - 4\right)} = \lim_{x \to -\infty} \frac{-\cancel{x}\sqrt{4 - \cancel{5/x^2}}}{\cancel{x}\left(\cancel{3/x} - 4\right)}$$

$$= \frac{-\sqrt{4-0}}{(0-4)} = \frac{-2}{-4} = 1/2$$