§ 3.2 - The Product Rule

$[x \cdot e^x]' \stackrel{?}{=} [x]' \cdot [e^x]'$ WRONG

$[x^4]' = 4x^3$ ← Yikes

↳ $[x^2 \cdot x^2]' = [x^2]' \cdot [x^2]' = (2x)(2x) = 4x^2$

Product Rule

$$[f(x) \cdot g(x)]' = f'(x)g(x) + g'(x)f(x)$$

$$f'(x)g(x) + f(x)g'(x)$$

$$g'(x)f(x) + f'(x)g(x)$$

$[x^4]' = 4x^3$

↳ $[x^2 \cdot x^2]' = [x^2]'x^2 + x^2[x^2]' = (2x)(x^2) + x^2(2x)$

$= 2x^3 + 2x^3$

$= 4x^3$

↳ $[x \cdot x^3]' = [x]'x^3 + x[x^3]'$

$= 1 \cdot x^3 + x \cdot 3x^2$

$= x^3 + 3x^3$

$= 4x^3$ ✓

$[x \cdot e^x]' = [x]' \cdot e^x + x \cdot [e^x]' = e^x + xe^x = e^x(x+1)$

Find the derivative of

$$f(x) = (x^3 + 3x^2 - 5x - 1)(x^4 - 3x^2 - x + 2)$$

at $x = 1$.

$$f'(x) = [x^3 + 3x^2 - 5x - 1]'(x^4 - 3x^2 - x + 2) + (x^3 + 3x^2 - 5x - 1)[x^4 - 3x^2 - x + 2]'$$

$$= (3x^2 + 6x - 5)(x^4 - 3x^2 - x + 2) + (x^3 + 3x^2 - 5x - 1)(4x^3 - 6x - 1)$$

$$\xrightarrow{x=1} (3 + 6 - 5)(1 - 3 - 1 + 2) + (1 + 3 - 5 - 1)(4 - 6 - 1)$$

$$(4)(-1) + (-2)(-3) = -4 + 6 = \boxed{2}$$