10/24/2023

Last Time: f', f'', the shape of f.

Today: L'Hospital's Rule

Curve sketching

Future: HWs

For what values of a, b does $f(x) = x^3 - ax^2 + bx$ have a local extrema at $(-1, -4)$? Is it a Local min or max?

$f(-1) = -4 \Rightarrow (-1)^3 - a(-1)^2 + b(-1) = -1 - a - b = -4 \Rightarrow -a - b = -3$

$3 = a + b$

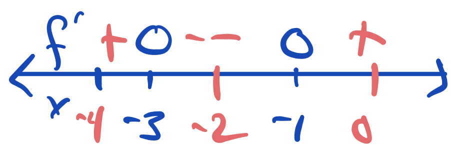
$f'(-1) = 0 \Rightarrow 3(-1)^2 - 2a(-1) + b = 3 + 2a + b = 0 \Rightarrow 3 = -2a - b$

$$\begin{array}{rl} & 3 = a + b \\ + & 3 = -2a - b \\ \hline & 6 = -a + 0 \Rightarrow a = -6 \Rightarrow b = 9 \end{array}$$

① $f'(x) = 3x^2 + 12x + 9 = 3(x^2 + 4x + 3)$

$3(x+1)(x+3)$

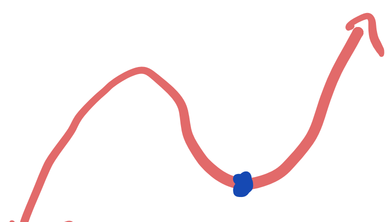
$x = -1, -3$



$f'(-4) = (-)(-) = +$

$f'(-2) = (-)(+) = -$

$f'(0) = (+)(+) = +$



L. Min

② $f''(x) = 6x + 12 = 0,$

$x = -2$

f'' - - - - - 0 + + + + + +

x ⟵——|——|——|——⟶
$-3 \quad -2 \quad -1$

$$\lim_{x \to \pi} \frac{\sin(2x)}{x - \pi} \to \frac{\sin(2\pi)}{\pi - \pi} \to \frac{0}{0} \quad \cancel{\quad}$$

L'Hospital's Rule: $\lim_{x \to a} \frac{f(x)}{g(x)} \to \frac{0}{0}$ or $\frac{\infty}{\infty}$, then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

$$\lim_{x \to \pi} \frac{\sin(2x)}{x - \pi} \overset{L.R.}{=} \lim_{x \to \pi} \frac{2\cos(2x)}{1} = \frac{2\cos(2\pi)}{1} = \boxed{2}$$

$$\lim_{x \to 2} \frac{x^2 + 2x - 8}{x - 2} = \lim_{x \to 2} \frac{\cancel{(x-2)}(x+4)}{\cancel{x-2}} = \boxed{6}$$

L.R. $\lim_{x \to 2} \frac{2x + 2}{1} = \boxed{6}$

why does this work.

$$f(a) = g(a) = 0, \quad \lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f(x) - f(a)}{g(x) - g(a)} = \lim_{x \to a} \frac{\frac{f(x) - f(a)}{x - a}}{\frac{g(x) - g(a)}{x - a}}$$

$$= \frac{f'(a)}{g'(a)}$$

$$\lim_{x\to e} \frac{\ln(x)-1}{x-e} \to \frac{\ln(e)-1}{e-e} \to \frac{0}{0}, \therefore L.R.: \lim_{x\to e} \frac{\frac{1}{x}-0}{1-0} = \frac{1}{e}$$

$$\lim_{x\to 1} \frac{x^7-1}{x^4-1} \to \frac{0}{0}, \therefore L.R.: \lim_{x\to 1} \frac{7x^6}{4x^3} = 7/4$$

$$\lim_{x\to\infty} \frac{\sqrt{x}}{1+e^x} \to \frac{\infty}{\infty} \therefore L.R.: \lim_{x\to\infty} \frac{\frac{1}{2}x^{-1/2}}{e^x} = \lim_{x\to\infty} \frac{1}{2\sqrt{x}e^x} = \boxed{0}$$

$$\lim_{x\to 0} \frac{e^{2x}-1-2x}{8x^2} \xRightarrow{L.R.} \lim_{x\to 0} \frac{2e^{2x}-2}{16x} \xRightarrow{LR} \lim_{x\to 0} \frac{4e^{2x}}{16} = \boxed{1/4}$$

$$\lim_{x\to\pi/2} \frac{1-\sin(x)}{\cos(2x)+x} \xRightarrow{L.R.} \lim_{x\to\pi/2} \frac{-\cos(x)}{-2\sin(2x)+1} = \frac{-1}{-2(0)+1} = \boxed{-1}$$

$$\hookrightarrow \frac{1-\sin(\pi/2)}{\cos(\pi)+\pi/2} = \frac{0}{-1+\pi/2} = 0$$

$$\lim_{x\to\infty} \frac{x}{6}\cdot\sin\left(\frac{2}{x}\right) = (\infty)\cdot(0) \leftarrow \text{I.F.} \left[L.R: \frac{0}{0}, \frac{\infty}{\infty}\right]$$

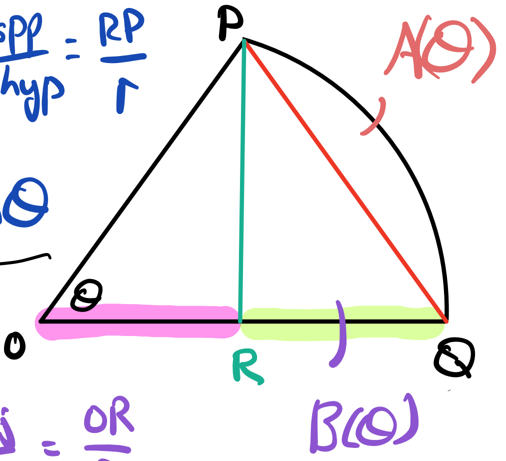
$$= \lim_{x\to\infty} \frac{\sin\left(\frac{2}{x}\right)}{\frac{6}{x}} \xRightarrow{L.R.} \lim_{x\to\infty} \frac{\cos\left(\frac{2}{x}\right)\cdot\cancel{-}2\cancel{x^{-2}}}{\cancel{-}6\cancel{x^{-2}}} =$$

$$= \lim_{x\to\infty} \frac{1}{3}\cdot\cos\left(\frac{2}{x}\right) = \boxed{1/3}$$

- $\infty - \infty$
- 1^∞
- 0^0
- ∞^0

$\sin\theta = \frac{opp}{hyp} = \frac{RP}{r}$

$RP = r\cdot\sin\theta$



$\cos\theta = \frac{adj}{hyp} = \frac{OR}{r}$

$OR = r\cos\theta$

$$\lim_{\theta\to 0} \frac{A(\theta)}{B(\theta)} \to \frac{0}{0}$$

Sector of a circle given θ is a "pie slice" with area

$$A = \tfrac{1}{2}r^2\theta$$

$\boxed{\text{if } \theta = 2\pi \Rightarrow A = \tfrac{1}{2}r^2(2\pi) = \pi r^2}$

$$\begin{aligned} A(\theta) &= \text{Sector} - \Delta\, OPQ \\ &= \tfrac{1}{2}r^2\theta - \tfrac{1}{2}(OQ)(RP) \\ &= \tfrac{1}{2}r^2\theta - \tfrac{1}{2}r\cdot r\sin\theta \\ &= \tfrac{1}{2}r^2\theta - \tfrac{1}{2}r^2\sin\theta \end{aligned}$$

$$B(\theta) = \tfrac{1}{2}(RP)(RQ)$$

$$= \tfrac{1}{2}(r\cdot\sin\theta)(r - OR) = \tfrac{1}{2}(r\sin\theta)(r - r\cos\theta)$$

$$\Rightarrow \lim_{\theta\to 0} \frac{\tfrac{1}{2}r^2\theta - \tfrac{1}{2}r^2\sin\theta}{\tfrac{1}{2}r(r\sin\theta)(r - r\cos\theta)} = \lim_{\theta\to 0} \frac{\tfrac{1}{2}r^2[\theta - \sin\theta]}{\tfrac{1}{2}r^2[(\sin\theta)(1-\cos\theta)]}$$

$$= \lim_{\theta\to 0} \frac{\theta - \sin\theta}{\sin\theta - \sin\theta\cos\theta} = \lim_{\theta\to 0} \frac{1-\cos\theta}{\cos\theta - [\cos^2\theta - \sin^2\theta]}$$

$$= \lim_{\theta\to 0} \frac{1-\cos\theta}{\cos\theta - \cos^2\theta + \sin^2\theta} \xrightarrow{L.R.} \lim_{\theta\to 0}$$