10/10/23

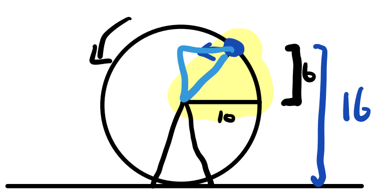
Last Time: Related Rates

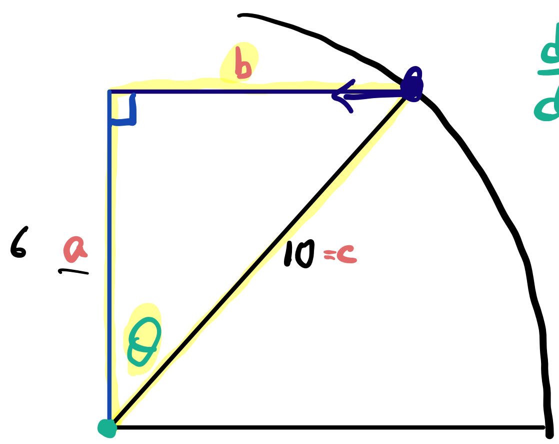
Today: Related Rates
Linearization
Differentials } Ch 3

Min + Max Values
Critical #s. } Ch 4

Future: HW Due M
Exam II on T

A Ferris Wheel with radius 10m is tangent to the ground and makes one rotation every 5 minutes. How fast is a person moving horizontally when they are 16m above the ground?





$$\frac{d\theta}{dt} = \frac{1\text{ rev}}{5\text{ min}} = \frac{2\pi\text{ rad}}{5\text{ min}} = \frac{2\pi}{5}$$

Final Answer: $\frac{db}{dt}$

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{b}{10} \Rightarrow \sin\theta = \frac{b}{10} \Rightarrow \cos(\theta)\cdot\theta' = \frac{1}{10}b'$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{b}{a}$$

$$b' = 10\cdot\cos\theta\cdot\frac{2\pi}{5} = 2\cdot 2\pi\cdot\frac{\text{adj}}{\text{hyp}}$$

$$= 4\pi\cdot\frac{6}{10} = \frac{12\pi}{5}\text{ m/min}$$

The Linearization of $f(x)$ at $x=a$ is:

$$L(x) = f(a) + f'(a)(x-a).$$

Ex: Find the linearization of $f(x) = \sqrt[3]{x}$ at $x=8$.

$$f(8) = \sqrt[3]{8} = 2$$

$$f'(x) = [x^{1/3}]' = \frac{1}{3}x^{-2/3} = \frac{1}{3x^{2/3}}, \quad f'(8) = \frac{1}{3\cdot 8^{2/3}} = \frac{1}{3\cdot 2^2} = \frac{1}{12}$$

$$L(x) = 2 + \frac{1}{12}(x-8)$$

(*) If $x_0 \approx a$, then $f(x_0) \approx L(x_0)$

Use linearization to approx $\sqrt[3]{8.1}$

$$8.1 \approx 8 \Rightarrow \overbrace{f(8.1)}^{\sqrt[3]{8.1}} \approx L(8.1)$$

$$= 2 + \frac{1}{12}(8.1-8)$$

$$= 2 + \frac{1}{12}\left(\frac{1}{10}\right) = 2 + \frac{1}{120} = \frac{241}{120}$$

$$= 2.008333$$

$$\sqrt[3]{8.1} \approx 2.0082198\ldots$$

Differentials: $y = f(x)$,

$$dy = f'(x)\,dx$$

Ex: Find the differential of the Volume function $V(r) = \frac{4}{3}\pi r^3$

$$dV = 4\pi r^2 \cdot dr \Leftrightarrow \frac{dV}{dr} = 4\pi r^2$$

A sphere with radius 3m is painted so that the radius increases 1 mm. Use differentials to approx the amount of paint needed.

Exact: $\Delta V = V(3.001) - V(3) = \frac{4}{3}\pi(3.001)^3 - \frac{4}{3}\pi \cdot 3^3 = \ ?\ 0.113135$

$\hookrightarrow \Delta r = .001 = 1\text{mm}$

Approx using differentials: $dV = 4\cdot\pi r^2 \cdot dr = 4\cdot\pi\cdot 3^2\cdot(.001)$

$dx = .001$

$= 36\cdot .001\cdot\pi$

$= .036\pi \approx .113097\ m^3$

hyperbolic sine, $\sinh(x)$

hyperbolic cosine, $\cosh(x)$

$\sinh(0) = 0 \qquad [\sinh(x)]' = \cosh(x)$

$\cosh(0) = 1 \qquad [\cosh(x)]' = \sinh(x)$

$[\sinh(x)\cdot\cosh(3x)]' =$

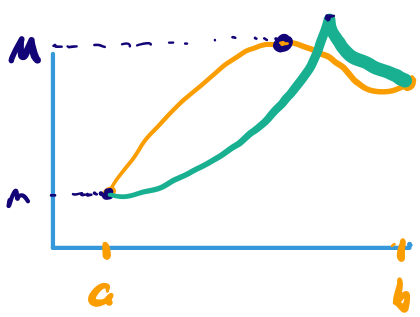
$= [\sinh(x)]'\cosh(3x) + \sinh(x)[\cosh(3x)]'$

$= \boxed{\cosh(x)\cosh(3x) + \sinh(x)\cdot\sinh(3x)\cdot 3}$

Ch 3

Ch 4 - Applications of Derivatives

Let $f(x)$ be a continuous function on the interval $[a,b]$



Where does $f(x)$ have the max value and min value.

A min or max occurs

- Endpoints
- $f'(x) = 0$
- $f'(x)$ DNE

$\}$ x-values are called critical numbers

Find the critical numbers of:

$f(x) = 2x^3 - 3x^2 - 12x + 3 \rightarrow f'(x) = 6x^2 - 6x - 12 = 6(x^2 - x - 2)$

$6(x-2)(x+1)$

$g(x) = x^{3/5}(4-x) \Rightarrow \frac{3}{5}x^{-2/5}(4-x) + x^{3/5}(-1) = \frac{3(4-x)}{5x^{2/5}} - \frac{x^{3/5}}{1} = \frac{12-3x-5x}{5x^{2/5}}$ $x = -1, 2$

$= \frac{12-8x}{5x^{2/5}}$, $x = 3/2, 0$

$h(x) = \sin^2(x) - \sqrt{3}\sin(x)$, $0 \le x \le \pi/2$

$h'(x) = 2\sin(x)\cdot\cos(x) - \sqrt{3}\cos(x) = \underbrace{\cos(x)}_{\pi/2}\underbrace{[2\sin(x) - \sqrt{3}]}_{\cancel{2}\sin(x) = \frac{\sqrt{3}}{2},\ x = \frac{\pi}{3}} = 0$

$\boxed{x = \pi/3, \pi/2}$

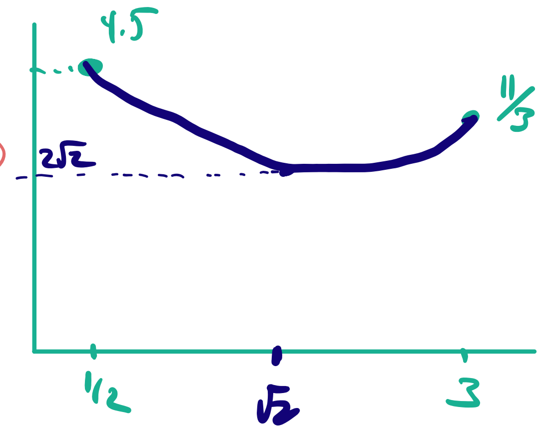
Find the absolute min and max: $1\ 2\sqrt{2}$ $1\ 9/2$

$f(x) = x + \frac{2}{x}$, $[1/2, 3]$

$f(\frac{1}{2}) = \frac{1}{2} + \frac{2}{\frac{1}{2}} = \frac{1}{2} + 4 = 4.5$

$f(3) = 3 + \frac{2}{3} = 11/3$

$f'(x) = 1 - \frac{2}{x^2} = 0 \Rightarrow \frac{x^2-2}{x^2}$ $x = \sqrt{2}$ $\cancel{x = -\sqrt{2}}$ $\cancel{x = 0}$



$f(\sqrt{2}) = \sqrt{2} + \frac{2}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}$

$= \sqrt{2} + \frac{2\sqrt{2}}{2} = \sqrt{2} + \sqrt{2} = 2\sqrt{2}$