§1.5. Inverse Functions + Logarithms, part II

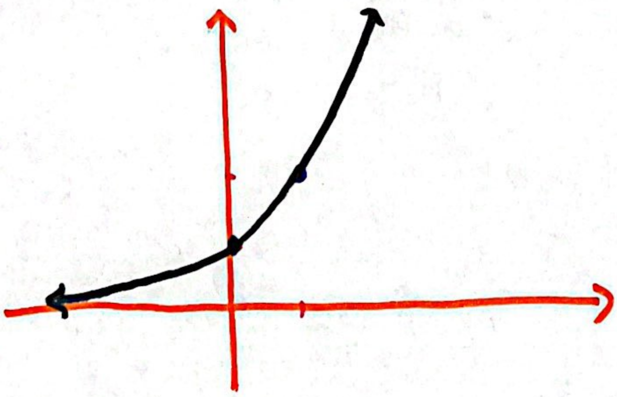
In this video, we will:

① Define the logarithm function
  ↳ the natural log function

② List the Rules of Logarithms

③ Graph the natural logarithm function.

Consider the graph of $f(x) = 2^x$



$f(0) = 2^0 = 1$

$f(1) = 2$

This graph passes the H.L.T., $\therefore$ it is 1-1,

$\therefore$ $f^{-1}(x)$ exists... but what is it?

$2^x = 3$, $x = ?$

$f^{-1}(3) = ?$

This is where logarithms come from.

These mean the same thing:

$2^x = 3 \iff \boxed{\log_2(3)} = x$

$$a^x = y \iff \log_a(y) = x$$

Ex: $2^4 = 16 \iff \log_2(16) = 4$

$5^x = 25 \iff \log_5(25) = x = 2 \qquad \log_5(25) = 2$

$\log_4(64) = x \iff 4^x = 64 = 3 \qquad \log_4(64) = 3$

* $y = 2^x$, to find the inverse we switch x + y, solve for y:

$$x = 2^y \Rightarrow \log_2(x) = y$$

$$f(x) = 2^x, \quad f^{-1}(x) = \log_2(x)$$

$(x+5) - 5 = x$

$(x \cdot 5)/5 = x$

$$\log_2(2^x) = x \text{ AND } 2^{\log_2(x)} = x$$

$$f(x)=a^x, \quad f^{-1}(x)=\log_a(x)$$

$f(x)=e^x,$

$f^{-1}(x)=\log_e(x)=\ln(x)$

Rules of Logarithms

- $\log_a(xy)=\log_a(x)+\log_a(y)$
- $\log_a(x/y)=\log_a(x)-\log_a(y)$
- $\log_a(x^y)=y\cdot\log_a(x)$ $\qquad \log_3(100)=\log_3(10^2)=2\cdot\log_3(10)$
- $\log_a(a)=1$ $\qquad \ln(e)=1$
- $\log_a(1)=0$ $\qquad \ln(1)=0$
- $\log_a(0)$ undefined (V. asy)
- $\log_a(\text{negative number})$ undefined

Ex: Find $\log_5(5ab^2)$ if $\log_5(a)=8$, $\log_5(b)=10$.

$$\underbrace{\log_5(5)}_{1}+\underbrace{\log_5(a)}_{8}+\log_5(b^2)$$

$$1+8+2\cdot\log_5(b)=1+8+2(10)=\boxed{29}$$

Ex: Find x such that $e^{x+1} = 9$

$\log_e(9) = x+1$

$\ln(9) = x+1$

$x = \underline{\ln(9)+1}$

or

$\ln(e^{x+1}) = \ln(9)$

$x+1 = \ln(9)$

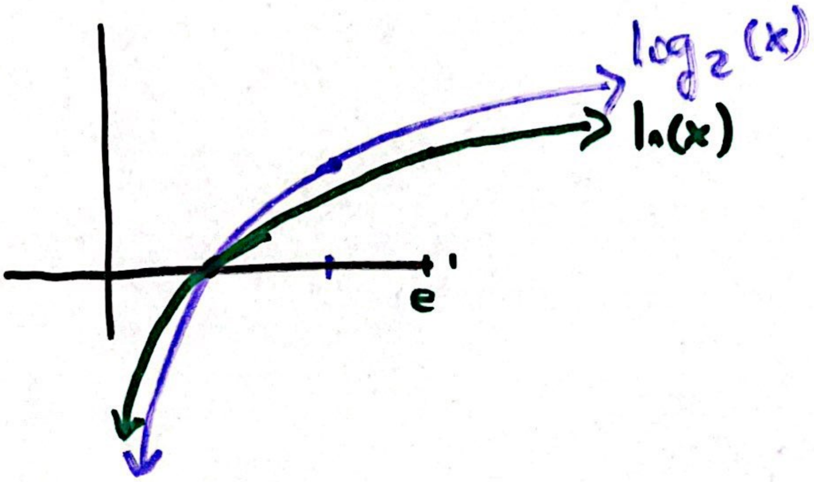
$x = \underline{\ln(9) - 1}$

Graph $y = \log_2 x$

$\log_2(1) = 0$
$\log_2(2) = 1$

$y = \ln(x)$

$\ln(1) = 0$
$\ln(e) = 1$



What Else:

- Solving more exponential equations
- Real-World Applications of logarithms
- Different way to rewrite $\log_a(x)$