Hwk#19 Sec 3.11 (#3,7,8,9,25,27ab,35,37,39,41,45)

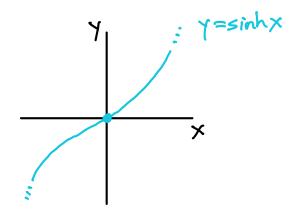
Hyperbolic trig functions

Defn The hyperbolic trig funcs are

$$sinh x = \frac{e^{x} - e^{-x}}{2}$$
, $cosh x = \frac{e^{x} + e^{-x}}{2}$, $tanh x = \frac{sinh x}{cosh x}$

$$csch x = \frac{1}{sinhx}$$
, $sech x = \frac{1}{ceshx}$, $coth x = \frac{1}{tenhx}$

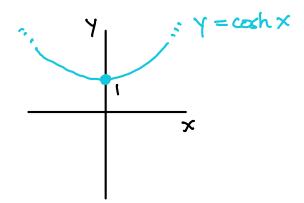
Pictures



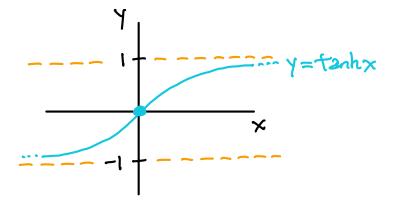
$$D_{om} = (-\alpha_{1} \omega)$$

$$R_{ng} = (-\omega_{1} \omega)$$

$$Sinh_{x} \rightarrow \pm \infty \text{ as } x \rightarrow \pm \infty$$



$$Dom = (-\infty, \infty)$$
 $Rng = [1, \infty)$
 $coshx \rightarrow \infty$ as $x \rightarrow foo_{le}$



$$Dom = (-09,00)$$

$$Rng = (-1,1)$$

$$tanh x \rightarrow 11 \ 2s \ x \rightarrow 100/l_{6}$$

Examples (1) Verify sinhx > foo as x > 100.

$$\lim_{x\to\infty} \sinh x = \lim_{x\to\infty} \frac{e^x - \overline{e}^x}{2} = \frac{\infty - 0}{2} = \infty$$

Recall:
$$e^{\infty} = 0$$

$$\lim_{x \to -\infty} \sinh x = \lim_{x \to -\infty} \frac{e^x - e^x}{2} = \frac{0 - \infty}{2} = -\infty / \ell_4$$

②
$$\cosh (3) = ?$$
, $\cosh (\ln 3) = ?$

$$\cosh (3) = \frac{e^3 + \overline{e}^3}{2} = \frac{e^3 + \overline{e}^3}{2} = 10.07$$

$$\cosh(\ln 3) = \frac{e^{\ln 3} + e^{\ln 3}}{2} = \frac{e^{\ln 3} + e^{\ln 3}}{2} = \frac{3 + \frac{1}{3}}{2} = \frac{5}{3} \frac{1}{4}$$

Hyperbolic trig identities

$$sinh(-x) = -sinh(x)$$
 $sinh(-x) = cosh(x)$

$$\cosh^2(x) - \sinh^2(x) = 1$$
, $1 - \tanh^2(x) = \operatorname{sech}^2(x)$

sinh (A+B) = sinh A cosh B + sinh B cosh A

coch (A+B) = cosh A cosh B + sinh A sinh B

Examples () Verify sinh(-x) = - sinh(x).

$$sinh(-x) = \frac{e^{(-x)} - \bar{e}^{(-x)}}{2} = \frac{\bar{e}^{-x} - e^{x}}{2} = -\left(\frac{e^{x} - \bar{e}^{x}}{2}\right)$$

$$= - sinh(x) / u$$

② Verify
$$\cosh^2(x) - \sinh^2(x) = 1$$
.

$$\cosh^{2}(x) - \sinh^{2}(x) = \left(\frac{e^{x} + \overline{e}^{x}}{2}\right)^{2} - \left(\frac{e^{x} - \overline{e}^{x}}{2}\right)^{2}$$

$$= \frac{e^{2x} + 2 + \overline{e}^{2x}}{4} - \frac{e^{x} - 2 + \overline{e}^{2x}}{4}$$

$$= \frac{2 + 2}{4} = \frac{1}{4}$$

3) If
$$sinhx = 2$$
, then $coshx = ?$, $tonhx = ?$

$$\cosh^{2}x - \sinh^{2}x = 1$$

$$\cosh^{2}x = 5$$

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$$\cosh x = 175$$

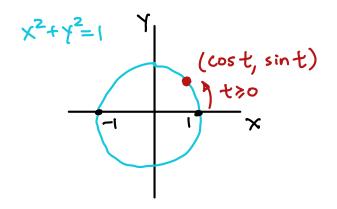
$$\cosh x = 175$$

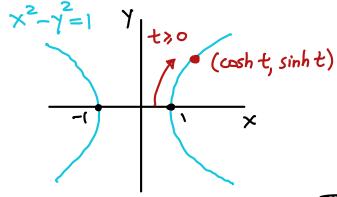
$$\cosh x = +75$$

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$$(4c \cosh x > 1)$$

Relation to hyperbola Just as (cost, sint) is a point on the unit circle, (cosh t, sinht) is a point on the unit hyperbola.







Hyperbolic trig derivatives

,
$$[\operatorname{csch} x] = -\operatorname{csch} x$$
 coth x

,
$$[coth \times]' = -csch^2 \times$$

Examples () Verify [sinhx] = coshx.

$$sinhx = \frac{1}{2} \left(e^{x} - \overline{e}^{x} \right)$$

$$\left[\sinh x\right]' = \frac{1}{2} \left[e^{x} - \overline{e}^{x}\right]'$$

$$= \frac{1}{2} \left[e^{x} - \overline{e}^{x}\right]'$$

$$= \frac{1}{2} \left(e^{\times} + \overline{e}^{\times} \right)$$

②
$$f(x) = x + anh(2x+1)$$
. $f(-\frac{1}{2}) = ?$

$$f'(x) = 1 \cdot fanh(2xt1) + x \cdot sech^2(2x+1) \cdot 2$$

$$f(-\frac{1}{2}) = \frac{1^2}{(-\frac{1}{2})^2} = \frac{1^2}{(-\frac{1}{2})^2}$$

sinh (0) =0

