§4.4 - Indeterminate Form + L'Hospital's Rule

- Recall: $\lim_{x \to 4} \frac{x^2 - x - 2}{x^2 - 5x + 6} = \frac{16-4-2}{16-20+6} = \frac{10}{2} = 5$

$$\lim_{x \to 2} \frac{x^2 - x - 2}{x^2 - 5x + 6} = \frac{4-2-2}{4-10+6} = \frac{0}{0}, \text{ I.F}$$

$$\searrow \lim_{x \to 2} \frac{\cancel{(x-2)}(x+1)}{\cancel{(x-2)}(x-3)} = \lim_{x \to 2} \frac{x+1}{x-3} = \frac{2+1}{2-3} = \boxed{-3}$$

What about $\lim_{x \to 2} \frac{\ln(x-1)}{x-2} = \frac{\ln(1)}{0} = \frac{0}{0}, \text{ I.F.}$

L'Hospital's Rule: If $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{0}{0}$ or $\frac{\infty}{\infty}$,

Then $\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$

$$\lim_{x \to 2} \frac{\ln(x-1)}{x-2} \overset{L.R.}{=} \lim_{x \to 2} \frac{1/(x-1)}{1} = \lim_{x \to 2} \frac{1}{x-1} = \frac{1}{2-1} = \boxed{1}$$

$$\lim_{x \to 2} \frac{x^2 - x - 2}{x^2 - 5x + 6} \overset{L.R.}{=} \lim_{x \to 2} \frac{2x-1}{2x-5} = \frac{4-1}{4-5} = \frac{3}{-1} = \boxed{-3}$$

$$\lim_{x\to 0} \frac{\sin(4x)}{4x} \rightarrow \frac{\sin(0)}{4\cdot 0} = \frac{0}{0}, \text{ I.F.}$$

$$\overset{L.R.}{=} \lim_{x\to 0} \frac{\cos(4x)\cdot 4}{4} = \frac{\cos(0)\cdot 4}{4} = \boxed{1}$$

$$\lim_{x\to\infty} \frac{\ln(x)}{\sqrt{x}} \rightarrow \frac{\ln(\infty)}{\sqrt{\infty}} \rightarrow \frac{\infty}{\infty}, \text{ I.F.}$$

$$\overset{L.R.}{=} \lim_{x\to\infty} \frac{1/x}{\frac{1}{2}x^{-1/2}} = \lim_{x\to\infty} \frac{\frac{1}{x}}{\frac{1}{2\sqrt{x}}} = \lim_{x\to\infty} \frac{2\sqrt{x}}{x} =$$

$$\lim_{x\to\infty} \frac{2}{\sqrt{x}} = \frac{2}{\infty} = \boxed{0}$$

$$\lim_{x\to 0} \frac{\sin(x) - x}{x^3} = \frac{\sin(0) - 0}{0^3} = \frac{0}{0}, \text{ I.F.}$$

$$\overset{L.R.}{=} \lim_{x\to 0} \frac{\cos(x) - 1}{3x^2} \rightarrow \frac{\cos(0) - 1}{0} = \frac{0}{0}, \text{ I.F}$$

$$\overset{L.R.}{=} \lim_{x\to 0} \frac{-\sin(x)}{6x} \rightarrow \frac{0}{0}, \text{ I.F.}$$

$$\overset{L.R.}{=} \lim_{x\to 0} \frac{-\cos(x)}{6} \rightarrow \frac{-\cos(0)}{6} = \boxed{-1/6}$$

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