

§ 2.5 - Continuous Functions, part 2.

● In this video, we will:

- Examine the continuity of $f(g(x))$
- State and use the Intermediate Value Theorem

Q: When is $f(g(x)) = (f \circ g)(x)$ continuous at $x=a$?

If $g(x)$ is continuous at $x=a$ AND
if $f(x)$ is continuous at $x=g(a)$, then

● $f(g(x))$ is continuous at $x=a$.

Ex: Which of the following is/are continuous at $x=1$?

① $\sqrt{2x-3}$ $g(x) = 2x-3$ $f(x) = \sqrt{x}$
 ~~$g(1) = 2-3 = -1$~~ $g(1) = 2-3 = -1$ $f(-1) = \sqrt{-1}$, DNE
NOT continuous at $x=1$

② $\sin\left(\frac{x}{x-1}\right)$ $g(x) = \frac{x}{x-1}$, $g(1) \rightarrow \text{DNE}$
 $\Rightarrow \sin\left(\frac{x}{x-1}\right)$ is not cts at $x=1$

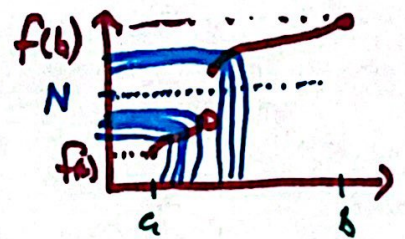
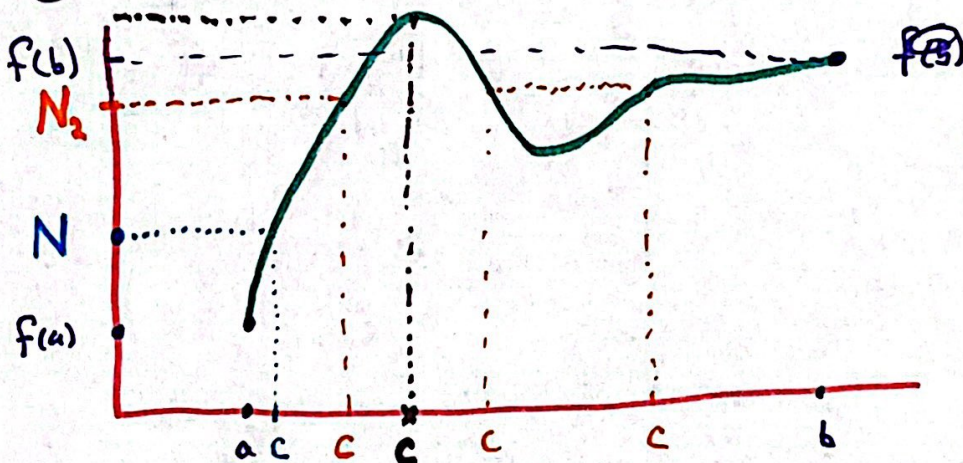
● ③ $e^{\sqrt{x}}$ $g(x) = \sqrt{x}$, $g(1) = \sqrt{1} = 1$
 $f(x) = e^x$, $f(g(1)) = f(1) = e^1 = e$
 $\therefore e^{\sqrt{x}}$ is cts at $x=1$

Intermediate Value Theorem, IVT

Suppose $f(x)$ is a continuous function on the interval $[a, b]$, and N any number between $f(a)$ and $f(b)$, where $f(a) \neq f(b)$. Then there is an x -value c such that:

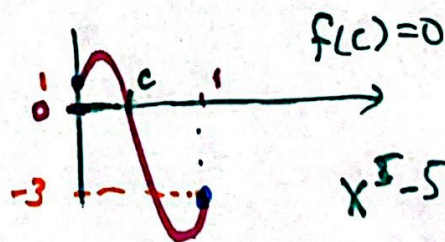
① $a < c < b$

② $f(c) = N$.



Example: Show there is a solution to the equation

$x^5 - 5x + 1 = 0$ between $x=0$ and $x=1$.
polynomial, continuous



$x^5 - 5x + 1 = 0$ has
a soln b/w
 $x=0$ + $x=1$.

What Else: Applications to I.V.T.?