

Question #1

$$I = \int_{\pi/6}^{\pi/3} (6 \sin 2x + 2 \cos 2x) dx$$

Let  $u = 2x$

$$2\left(\frac{\pi}{3}\right) = \frac{2\pi}{3}$$

$$\frac{du}{2} = \frac{dx}{1}$$

$$2\left(\frac{\pi}{6}\right) = \frac{2\pi}{6} = \frac{\pi}{3}$$

$$\int_{\pi/3}^{2\pi/3} -3 \cos(u) + \int_{\pi/3}^{2\pi/3} 2 \sin(u)$$

$$-3 \left( \cos\left(\frac{2\pi}{3}\right) - \cos\left(\frac{\pi}{3}\right) \right) + 2 \left( \sin\left(\frac{2\pi}{3}\right) - \sin\left(\frac{\pi}{3}\right) \right)$$

$$0 = 2\left(0\right) = \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}$$

$$-\frac{1}{2} - \frac{1}{2} = -1$$

$$-3(-1) = 3 \quad 3 + 0 = 3$$

Question #2

$$I = \int_0^1 5x(1-x^2)^4 dx$$

Let  $u = 1-x^2$   $1-(0)^2$   $1-(1)^2$

$$du = -2x dx \quad 1 = u_1 \quad 0 = u_2$$

$$-\frac{1}{2} du = x dx$$

$$5 \int_1^0 (u)^4 \cdot -\frac{1}{2} du \leftarrow 5 \int_1^0 (u)^4 x dx$$

$$-\frac{5}{2} \int_1^0 (u)^4 du \xrightarrow{\text{change}} \frac{5}{2} \int_0^1 \frac{u^5}{5} du$$

$$I = 1/2$$

$$\frac{5}{2} \left( \frac{1^5}{5} - \frac{0^5}{5} \right)$$

$$\frac{5}{2} \left( \frac{1}{5} \right) = \frac{5}{10} = 1/2$$

Question #3

Determine integral

$$I = \int 4x(3+2x^2)^4 dx$$

Let  $u = 3+2x^2$

$$\frac{du}{4} = \frac{4x dx}{4}$$

$$\frac{1}{4} du = x dx$$

$$4 \int u^4 x dx \rightarrow 4 \int u^4 \cdot \frac{1}{4} du$$

$$I = \int u^4 = \frac{u^5}{5} = \frac{1}{5} (3+2x^2)^5 + C$$

Question #4

The graph of  $f$  has the slope:

$$\frac{df}{dx} = x\sqrt{2x^2+1}$$

and passes through the point  $(2, 2)$ . Find the y-intercept of this graph.

Let's say,

$$I = \int x\sqrt{2x^2+1} \rightarrow \frac{1}{4} \int u^{1/2} \cdot \frac{1}{1+1/2} \cdot \frac{2}{3} u^{3/2}$$

Set  $u = 2x^2+1$  where

$$du = 4x dx$$

$$\frac{1}{4} du = x dx$$

$$f(0) = \frac{1}{6} - \frac{5}{2}$$

$$-\frac{14}{6} = \left( \frac{7}{3} \right) = f(0)$$

Question #5

Evaluate the integral

$$I = \int x^2 \sqrt{x^2+7} dx$$

Let  $u = x^2+7$

$$\frac{1}{3} du = \frac{2x dx}{3}$$

$$\frac{1}{3} du = x^2 dx$$

$$\frac{1}{3} \int \sqrt{u} du = I = \frac{1}{3} \int u^{1/2}$$

$$= \frac{1}{3} \int \frac{2}{3} u^{3/2}$$

$$I = \frac{2}{9} (x^2+7)^{3/2} + C = \frac{2}{9} u^{3/2} + C$$

Question #6

Determine the integral

$$I = \int \frac{2}{(1+4x)^3} dx$$

Let  $u = 1+4x$

$$\frac{1}{4} du = dx$$

$$I = \frac{1}{4} \int \frac{2}{u^3} du = \frac{1}{4} \cdot \frac{2}{-1} \int \frac{1}{u^3}$$

$$I = \frac{1}{-4(1+4x)^2} + C = -\frac{1}{4} \int u^{-2}$$

$$= \frac{1}{2} \cdot \frac{1}{-2} \int u^{-2}$$

$$= \frac{1}{-4} u^{-2} \rightarrow \frac{1}{-4(1+4x)^2} + C$$

Question #7

Evaluate the definite integral

$$I = \int_1^5 \frac{2x-7}{\sqrt{7x-x^2}} dx \quad \frac{1}{2} \cdot \frac{2}{-1/2+1/2} u^{-1/2+1/2}$$

Let  $u = 7x-x^2$   $7(1)-1$

$$du = 7-2x dx$$

$$(7-2x) du = dx$$

$$7-1=6$$

$$7(5)-(5)^2$$

$$35-25=10$$

Question #8

Determine the integral

$$I = \int t^2 \cos(3-t^3) dt$$

Let  $u = 3-t^3$

$$\frac{du}{-3} = \frac{-3t^2 dt}{-3}$$

$$-\frac{1}{3} du = t^2 dt$$

$$I = -\frac{1}{3} \int \cos(u) du$$

$$I = -\frac{1}{3} \sin(3-t^3) + C = I$$

Question #9

Determine the integral

$$I = \int \cos^5 x \sin x dx$$

Let  $u = \cos x$

$$\frac{du}{-1} = \frac{-\sin x dx}{-1}$$

$$-du = \sin x dx$$

$$I = -\int u^4 du$$

$$= -\frac{1}{5} \cos^5(x) + C$$

Question #10

Determine the integral

$$I = \int \frac{x-4}{(x^2-8x-6)^4} dx$$

Let  $u = x^2-8x-6$

$$du = 2x-8 dx$$

$$\frac{du}{2} = x-4 dx$$

$$I = \int \frac{1}{2u^4} du$$

$$\rightarrow \frac{1}{2} \int u^{-4} du$$

$$\rightarrow \frac{1}{2} \int -\frac{1}{3} u^{-3} du$$

$$I = -\frac{1}{6} \left( \frac{1}{u^3} \right)$$

$$I = -\frac{1}{6} \left( \frac{1}{x^2-8x-6} \right) + C$$

Review

Question #11

$$I = \int \frac{1}{\theta^2} \left( 5 \cos\left(\frac{1}{\theta}\right) - \frac{2}{\theta} \right) d\theta$$

$$= \int_0^{\pi/4} \frac{5 \cos\left(\frac{1}{\theta}\right)}{\theta^2} - \int \frac{2}{\theta^3} d\theta$$

Let  $u = \frac{1}{\theta}$   $\theta^{-1}$

$$=$$

$$-5(-\sin(u))$$

$$- \frac{1}{\theta^2} + 5 \sin\left(\frac{1}{\theta}\right) + C$$

$$5 \int \cos(u)$$

$$5 \sin\left(\frac{1}{\theta}\right) = I_1$$

$$5 \sin(u) + \frac{1}{\theta^2}$$

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