M408D Final Exam Review – Day 3

1) Christo, the installation artist, is putting up his new piece called "Very Long Fence." The first fence post he puts up is 3m tall and each subsequent fence post is 60% of the height of the previous one.

a) Assuming he can go on forever (and that we don't have to worry about atomic physics), what is the limit of the heights of these fence posts going to?
b) What is the total length of fencing material he used assuming the base of each post starts at the ground?

Determine whether the following series converge or diverge, justify your answer.

If applicable, say if it converges conditionally. (Unless otherwise stated, assume $\sum = \sum_{n=1}^{\infty}$)

2) $\sum \frac{\cos n\pi}{n}$

3) $\sum \frac{n!}{(n+2)!}$

4) $\sum \frac{\ln n}{n}$

5) $\sum \frac{1}{\sqrt{2n^2+n}}$

6) $\sum \frac{(n^3-1)^{1/2}}{n^3}$

7) $\sum \frac{(-1)^n n}{(n+1)^3}$

8) $\sum \frac{3^n n^n}{n!}$

9) Find the sum of the series: $\sum_{n=0}^{\infty} \frac{2^n+3^n}{5^n}$

10) True or False.

A) The basic comparison test can be used to show that the series $\sum_{n=1}^{\infty} \frac{1}{2\sqrt{n}+6}$ diverges by comparing it to the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ with $p = \frac{1}{2}$

B) The limit comparison test can be used to show that the series $\sum_{n=1}^{\infty} \frac{5n}{2n+n^3}$ converges by comparing it to the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ with $p = 2$

C) The integral test can be used to show that the series $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$ converges to the value $\frac{1}{\ln 2}$ by comparing it to the improper integral $\int_2^{\infty} \frac{1}{x(\ln x)^2} dx$

D) The Harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$ can be shown to diverge using the p-series test, the integral test, and the geometric series test

E) If an alternating series $\sum_{n=1}^{\infty} (-1)^n a_n$ fails to satisfy either of the two criteria of the alternating series test, 1) $\lim_{n\to\infty} |a_n| = 0$ or 2) $|a_{n+1}| < |a_n|$ then it is known to diverge.

Compute the following Taylor Polynomials, centered at zero:

11) $T_2(x)$ for $f(x) = \dfrac{x}{x^2+1}$

12) $T_3(x)$ for $f(x) = e^{2x}$

Find the interval of convergence for the following:

13) $\sum \dfrac{2^k}{(2k)!}(x+1)^k$

14) $\sum \dfrac{k+1}{k^2 3^k}(x-3)^k$

15) $\sum \dfrac{(-1)^k}{5^{k+1}}(3x)^k$

Determine the power series representations for the following functions.

16) $f(y) = \dfrac{5}{4+y^2}$

17) $f(x) = \ln\left(1-x^2\right)$

18) $f(t) = \left(\dfrac{2t}{4-t}\right)^2$