10.3 Exercises

1–2 Plot the point whose polar coordinates are given. Then find two other pairs of polar coordinates of this point, one with $r > 0$ and one with $r < 0$.

1. (a) $(1, \pi/4)$ (b) $(-2, 3\pi/2)$ (c) $(3, -\pi/3)$

2. (a) $(2, 5\pi/6)$ (b) $(1, -2\pi/3)$ (c) $(-1, 5\pi/4)$

3–4 Plot the point whose polar coordinates are given. Then find the Cartesian coordinates of the point.

3. (a) $(2, 3\pi/2)$ (b) $(\sqrt{2}, \pi/4)$ (c) $(-1, -\pi/6)$

4. (a) $(4, 4\pi/3)$ (b) $(-2, 3\pi/4)$ (c) $(-3, -\pi/3)$

5–6 The Cartesian coordinates of a point are given.
(i) Find polar coordinates (r, θ) of the point, where $r > 0$ and $0 \leqslant \theta < 2\pi$.
(ii) Find polar coordinates (r, θ) of the point, where $r < 0$ and $0 \leqslant \theta < 2\pi$.

5. (a) $(-4, 4)$ (b) $(3, 3\sqrt{3})$

6. (a) $(\sqrt{3}, -1)$ (b) $(-6, 0)$

7–12 Sketch the region in the plane consisting of points whose polar coordinates satisfy the given conditions.

7. $1 < r \leqslant 3$

8. $r \geqslant 2, \quad 0 \leqslant \theta \leqslant \pi$

9. $0 \leqslant r \leqslant 1, \quad -\pi/2 \leqslant \theta \leqslant \pi/2$

10. $3 < r < 5, \quad 2\pi/3 \leqslant \theta \leqslant 4\pi/3$

11. $2 \leqslant r < 4, \quad 3\pi/4 \leqslant \theta \leqslant 7\pi/4$

12. $r \geqslant 0, \quad \pi \leqslant \theta \leqslant 5\pi/2$

13. Find the distance between the points with polar coordinates $(4, 4\pi/3)$ and $(6, 5\pi/3)$.

14. Find a formula for the distance between the points with polar coordinates (r_1, θ_1) and (r_2, θ_2).

15–20 Identify the curve by finding a Cartesian equation for the curve.

15. $r^2 = 5$

16. $r = 4 \sec\theta$

17. $r = 5 \cos\theta$

18. $\theta = \pi/3$

19. $r^2 \cos 2\theta = 1$

20. $r^2 \sin 2\theta = 1$

21–26 Find a polar equation for the curve represented by the given Cartesian equation.

21. $x^2 + y^2 = 7$

22. $x = -1$

23. $y = \sqrt{3}x$

24. $y = -2x^2$

25. $x^2 + y^2 = 4y$

26. $x^2 - y^2 = 4$

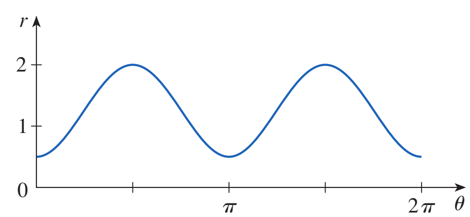
27–28 For each of the described curves, decide if the curve would be more easily given by a polar equation or a Cartesian equation. Then write an equation for the curve.

27. (a) A line through the origin that makes an angle of $\pi/6$ with the positive x-axis
(b) A vertical line through the point $(3, 3)$

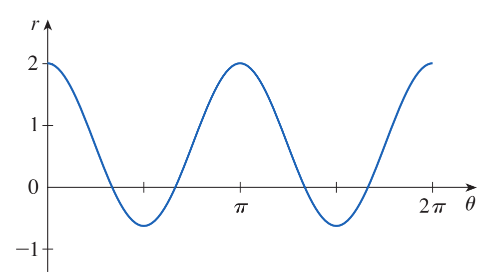
28. (a) A circle with radius 5 and center $(2, 3)$
(b) A circle centered at the origin with radius 4

29–32 The figure shows a graph of r as a function of θ in Cartesian coordinates. Use it to sketch the corresponding polar curve.

29.



30.



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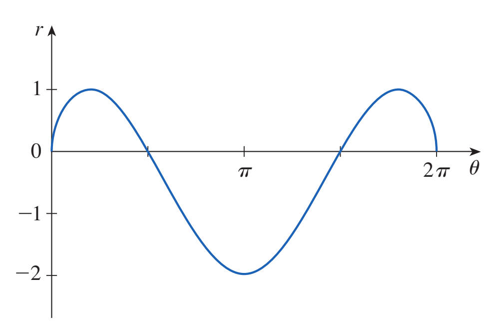

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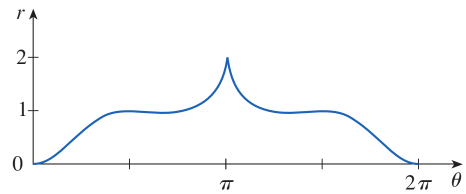


33–50 Sketch the curve with the given polar equation by first sketching the graph of r as a function of θ in Cartesian coordinates.

33. $r = -2\sin\theta$

34. $r = 1 - \cos\theta$

35. $r = 2(1 + \cos\theta)$

36. $r = 1 + 2\cos\theta$

37. $r = \theta, \ \theta \geqslant 0$

38. $r = \theta^2, \ -2\pi \leqslant \theta \leqslant 2\pi$

39. $r = 3\cos 3\theta$

40. $r = -\sin 5\theta$

41. $r = 2\cos 4\theta$

42. $r = 2\sin 6\theta$

43. $r = 1 + 3\cos\theta$

44. $r = 1 + 5\sin\theta$

45. $r^2 = 9\sin 2\theta$

46. $r^2 = \cos 4\theta$

47. $r = 2 + \sin 3\theta$

48. $r^2\theta = 1$

49. $r = \sin(\theta/2)$

50. $r = \cos(\theta/3)$

51. Show that the polar curve $r = 4 + 2\sec\theta$ (called a **conchoid**) has the line $x = 2$ as a vertical asymptote by showing that $\lim_{r\to\pm\infty} x = 2$. Use this fact to help sketch the conchoid.

52. Show that the curve $r = 2 - \csc\theta$ (a conchoid) has the line $y = -1$ as a horizontal asymptote by showing that $\lim_{r\to\pm\infty} y = -1$. Use this fact to help sketch the conchoid.

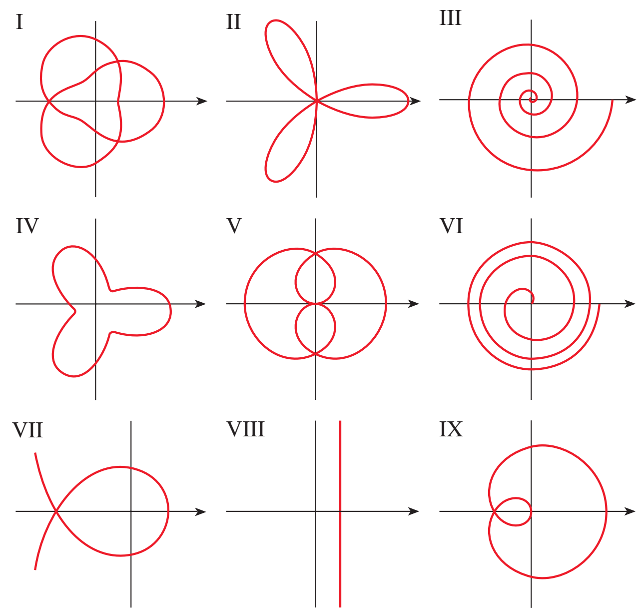
53. Show that the curve $r = \sin\theta\tan\theta$ (called a **cissoid of Diocles**) has the line $x = 1$ as a vertical asymptote. Show also that the curve lies entirely within the vertical strip $0 \leqslant x < 1$. Use these facts to help sketch the cissoid.

54. Sketch the curve $(x^2 + y^2)^3 = 4x^2y^2$.

55. (a) In Example 10 the graphs suggest that the limaçon $r = 1 + c\sin\theta$ has an inner loop when $|c| > 1$. Prove that this is true, and find the values of θ that correspond to the inner loop.
(b) From Figure 18 it appears that the limaçon loses its dimple when $c = \frac{1}{2}$. Prove this.

56. Match the polar equations with the graphs labeled I–IX. Give reasons for your choices.

(a) $r = \cos 3\theta$ (b) $r = \ln\theta, \ 1 \leqslant \theta \leqslant 6\pi$
(c) $r = \cos(\theta/2)$ (d) $r = \cos(\theta/3)$
(e) $r = \sec(\theta/3)$ (f) $r = \sec\theta$
(g) $r = \theta^2, \ 0 \leqslant \theta \leqslant 8\pi$ (h) $r = 2 + \cos 3\theta$
(i) $r = 2 + \cos(3\theta/2)$



57. Show that the polar equation $r = a\sin\theta + b\cos\theta$, where $ab \neq 0$, represents a circle. Find its center and radius.

58. Show that the curves $r = a\sin\theta$ and $r = a\cos\theta$ intersect at right angles.

59–64 Graph the polar curve. Choose a parameter interval that produces the entire curve.

59. $r = 1 + 2\sin(\theta/2)$ (nephroid of Freeth)

60. $r = \sqrt{1 - 0.8\sin^2\theta}$ (hippopede)

61. $r = e^{\sin\theta} - 2\cos(4\theta)$ (butterfly curve)

62. $r = |\tan\theta|^{|\cot\theta|}$ (valentine curve)

63. $r = 1 + \cos^{999}\theta$ (Pac-Man curve)

64. $r = 2 + \cos(9\theta/4)$

65. How are the graphs of $r = 1 + \sin(\theta - \pi/6)$ and $r = 1 + \sin(\theta - \pi/3)$ related to the graph of $r = 1 + \sin\theta$? In general, how is the graph of $r = f(\theta - \alpha)$ related to the graph of $r = f(\theta)$?

66. Use a graph to estimate the y-coordinate of the highest points on the curve $r = \sin 2\theta$. Then use calculus to find the exact value.