M408D Final Exam Review – Day 2

1) If y_0 is the particular solution of the differential equation $\frac{dy}{dx} - \frac{\ln x}{xy} = 0$ satisfying the initial condition $y(1) = 2$, find the value of $y_0(3)$.

2) A tank contains a mixture of 500 L of water and 40 L of alcohol. A solution of 70% water and 30% alcohol enters the tank at a rate of 5 L/min. The solution is continuously mixed with the fluid in the tank, which is drained from the tank at a rate of 5 L/min. What volume of alcohol remains in the tank after 10 minutes?

3) Find the general solution for the differential equation $(y+1)\frac{dy}{dx} = xy\sin x$

4) A function $y(t)$ satisfies the differential equation $\frac{dy}{dt} = e^y(y^2 - 3y - 10)$. Find the intervals on which $y(t)$ is increasing, and those on which it is decreasing.

5) A function $y(t)$ satisfies the differential equation $\frac{dy}{dt} = y^3 - y$. Find $\lim_{t\to\infty} y(t)$.

6) For what values of *k* does the function $y(t) = \cos kt + \sin kt$ satisfy the differential equation $5y'' + 16y = 0$?

7) A freshly-poured cup of coffee has a temperature of 90° C in a room with temperature 20° C. After 5 minutes, the temperature of the coffee was 85° C. Newton's law of cooling states that the rate of cooling of an object is proportional to the difference in temperature between the object and its surroundings. Find the function $y(t)$ representing the temperature of the coffee at time *t*, where *t* = 0 is the time at which the coffee is poured.

8) Find the particular solution, $y_0(t)$ of the differential equation $(1+\cos x)y' = (1+e^{-y})\sin x$ satisfying the initial condition $y(0) = 0$.

9) Use Euler's method with step size 0.2 to estimate $y(1)$, where $y(x)$ is the solution to the initial-value problem $y' = y + xy,\ \ y(0) = 1$.

10) Find the solution to the linear initial-value problem $x^2 y' + 2xy = \ln x,\ y(1) = 2$.

11) Sketch the slope field corresponding to the differential equation $\frac{dy}{dx} = 2 - y$, and draw the solution curve passing through the point $(0,1)$.

12) Eliminate the parameter to find a Cartesian equation of the curve given by the parametric equations $x(t)=3\sin 2t,\ y(t)=5\cos 2t,\ 0\le t\le \pi$. Then graph the curve, indicating orientation.

13) Eliminate the parameter to find a Cartesian equation of the curve given by the parametric equations $x(t)=\tan^2 t,\ y(t)=\sec t,\ -\frac{\pi}{2}<t<\frac{\pi}{2}$. Then graph the curve, indicating orientation.

14) Write parametric equations of the line segment starting at the point $P(-1,3)$, and ending at the point $Q(2,-6)$, with parameter t in $[0,1]$.

15) Write parametric equations corresponding to the path of the particle traversing the circle of radius 2, centered at $(0,1)$, counterclockwise, completing the circle once in the interval $0\le t<4\pi$.

16) Find the length of the curve $x(t)=e^t+e^{-t},\ y(t)=1-2t,\ 1\le t\le 3$.

17) Graph the curve and find its length $x(t)=e^t\sin t, y(t)=e^t\cos t,\ 0\le t\le 2$.

18) Find the points on the curve $x(t)=t^3-4t,\ y(t)=t^2-t$ where the tangent line is vertical.

19) Find the points on the curve $x(t)=e^{-t}+e^t,\ y(t)=\sin t+\cos t,\ 0\le t\le 2\pi$, corresponding to a horizontal tangent line.

20) Find the equation of the tangent line to the graph of $x(t)=2t^2-1,\ y(t)=3t-2$ at the point $(7,-8)$.

21) Find $\frac{d^2y}{dx^2}$ for the parametric curve given by $x(t)=t^2+5,\ y(t)=\sin 2t$.

22) Find the area under the curve enclosed by the *x*-axis and the curve $x(t)=e^t-1,\ y(t)=t^2-4$.

23) The equation in polar coordinates $r=\cos^2\theta$ defines a curve in the xy plane. Find the equation of the line tangent to the curve at the point corresponding to $\theta=\pi/4$.

Sketch the bounded region and then calculate the area.

24) $r=\tan\theta,\ \alpha=0$ and $\beta=\pi/4$

25) One loop of $r=\sin 3\theta$

26) Inside $r=\sin\theta$ and outside $r=\cos\theta$

27) One loop of $r=2\cos(3\theta)$