Question #1: Find all nonzero values of k for which the function $y = A \sin kt \; (+) \; B \cos kt$ satisfies the differential equation

$$y'' + 9y = 0$$

for all values of A and B

$y = A \sin kt \;(+)\; B \cos kt$

$y' = Ak \cos kt - Bk \sin kt$

$y'' = -k^2 A \sin kt - k^2 B \cos kt$

$\qquad -k^2 (A \sin kt + B \cos kt)$

$y'' = -k^2 y$

$-k^2 y + 9y = 0$

$\qquad -9y \quad -9y$

$$\frac{-k^2 y}{y} = \frac{-9y}{y}$$

$$\frac{-k^2}{-1} = \frac{-9}{-1}$$

$\sqrt{k^2} = \sqrt{9}$

$k = \sqrt{9}$ or ± 3

Question #2: The family of solutions to the differential equation $y' = -10xy$ is $y = Ce^{-5x^2}$. Find the solution that satisfies $y(0) = 1$.

$y = 1 \cdot e^{-5\cos z^2}$ ^1

$y(0) = 1 \qquad\qquad y = 1 \cdot 1$

$1 = Ce^{-5(0)^2}$^1 $\qquad \boxed{y = 1 \checkmark}$

$\boxed{1 = C}$

$\boxed{\therefore y = e^{-9x^2}}$

Question #3: Which of the following answers lists all constant solutions to the equation:

$\frac{dy}{dt} = y^4 - 5y^3 + 6y^2 \qquad \boxed{y = \{0, 3, 2\}}$

$0 = y^4 - 5y^3 + 6y^2$

$\quad y^2(y^2 - 5y + 6)$

$\quad y^2 = 0 \rightarrow y = \sqrt{0} = 0$

$\quad y - 3 = 0 \rightarrow y = 3$

$\quad y - 2 = 0 \rightarrow y = 2$

Question #4: Find all values of r for which the function $y = e^{rt}$ satisfies the differential equation:

$$y'' - 4y' - 12y = 0$$

$e^{rt} = y$

$r e^{rt} = y'$

$r^2 e^{rt} = y''$

$r^2 y - 4ry - 12y = 0$

$\qquad\qquad\quad + 12y \quad +12y$

$$\frac{y(r^2 \cancel{A} - 4r \cancel{y})}{y} = \frac{12\cancel{y}}{\cancel{A}}$$

$r^2 - 4r = 12$

$\qquad\qquad -12$

$r^2 - 4r - 12 = 0 \rightarrow \begin{array}{l} r - 6 = 0 \rightarrow \boxed{r = 6} \\ r + 2 = 0 \quad\;\; \boxed{r = -2} \end{array}$

$\boxed{r = \{-2, 6\}}$

Question #5: Find all values of k that don't result in a zero function for which the function $y = \sin kt$ satisfies the differential equation:

$$y'' + 36y = 0 \rightarrow -k^2 y + 36y = 0$$

$\qquad\qquad\qquad\qquad\qquad\quad -36y \quad -36y$

$y = \sin kt \qquad\qquad \dfrac{-k^2 y}{y} = \dfrac{-36y}{y}$

$y' = k \cos kt \qquad\qquad \dfrac{-k^2}{-1} = \dfrac{-36}{-1}$

$y'' = -k^2 \sin kt \qquad \sqrt{k^2} = \sqrt{36}$

$\qquad\qquad\qquad \boxed{k = \sqrt{36} \text{ or } \pm 6}$

$\qquad\qquad\qquad \boxed{k = \{6, -6\}}$