Formula for integration by parts

$\int u\, dv = uv - \int v\, du$

Question #1

Evaluate the integral

$I = \int_0^2 te^{-t}dt$

Let $u = t$ $\quad dv = e^{-t}dt$

$du = dt$ $\quad v = -e^{-t}$

$\int_0^2 te^{-t}dt = t \cdot -e^{-t}\Big]_0^2 - \int_0^2 -e^{-t}dt$

$\left((2 \cdot -e^{-2}) - \cancel{(0 \cdot -e^{0})}\right) - 1\int_0^2 e^{-t}$

$e^{-t}\Big]_0^2$

$e^{-2} - e^{-0}$

$(2 \cdot -e^{-2}) - (e^{-2} - 1)$

$(2 \cdot -e^{-2}) - e^{-2} + 1$

$2\left(\frac{-1}{e^2}\right) -$

$\frac{-2}{e^2} - \frac{1}{e^2} + 1$

$\frac{-3}{e^2} + 1$

Question #8

Evaluate the integral

$I = \int_0^{\pi} 2x \cos x \, dx.$

Let

$u = 2x$ $\quad dv = \cos x\, dx$

$du = 2\, dx$ $\quad v = \sin x$

$2x \sin x\Big]_0^{\pi} - \int_0^{\pi} \sin x \; 2\, dx$

$\left((2\pi \cdot \sin(\pi)) - \cancel{(2(0) \cdot \sin(0))}\right)$

$2\int_0^{\pi} \sin x\, dx$

$2\left(-\cos x \;\Big]_0^{\pi}\right)$

$(-\cos(\pi)) - (-\cos(0))$

$1 - (-1)$

$2(2)$

$-(4)$

$0 - 4 = 4 = I$

Question #14

Evaluate the integral

$I = \int_0^{\pi/4} x \sec^2 x\, dx$

Let $u = x$

$du = dx$

$dv = \sec^2 x\, dx$

$v = \tan x$

$x \tan x\Big]_0^{\frac{\pi}{4}} - \int_0^{\pi/4} \tan x\, dx$

$\frac{\pi}{4}\tan\left(\frac{\pi}{4}\right)$

$\frac{\pi}{4}$

$\int_0^{\pi/4} \frac{\sin(x)}{\cos(x)}dx$

Let $u = \cos(x)$

$-1\, du = -\sin(x)\, dx$ $\quad -1$

$-\int_0^{\pi/4} \frac{1}{u}\, du$

$-1\int_1^{\sqrt{2}/2} \frac{1}{u}\, du$

$-\ln(u)\, du\Big]_1^{\sqrt{2}/2}$

$\left(-\ln\left(\frac{\sqrt{2}}{2}\right)\right) - \cancel{(-\ln(1))}^{0}$

$\frac{\pi}{4} - \ln\left(\frac{\sqrt{2}}{2}\right)$

$-\ln\left(\frac{(2)^{1/2}}{2}\right)$

$\ln(2^{1/2}) - \ln(2)$

$\frac{1}{2}\ln(2) - \frac{2}{2}\ln(2)$

$-1\left(-1\left(-\frac{1}{2}\ln(2)\right)\right)$

$\frac{\pi}{4} - \frac{1}{2}\ln(2)$

Question #7

Determine the indefinite integral

$I = \int e^{-4x}\cos x\, dx$

Let $u = e^{-4x}$

$du = -4e^{-4x}dx$

$dv = \cos x\, dx$

$v = \sin(x)$

$e^{-4x}\sin x - \int -4\sin(x)e^{-4x}dx$

$e^{-4x}\sin(x) + 4\int \sin(x)e^{-4x}dx$

$e^{-4x}\sin(x) + 4\int e^{-4x}\sin(x)\, dx$

Let $u = e^{-4x}$

$du = -4e^{-4x}dx$

$dv = \sin(x)\, dx$

$v = -\cos(x)$

$\left(-e^{-4x}\cos(x) - \int 4\cos(x)e^{-4x}dx\right)$

$4\left(-e^{-4x}\cos(x) - 4\int e^{-4x}\cos(x)\, dx\right)$

$1\int e^{-4x}\cos x\, dx = e^{-4x}\sin(x) - 4e^{-4x}\cos(x) - 16\int e^{-4x}\cos(x)\, dx$

$+16\int e^{-4x}\cos x\, dx$ $\quad +16\int e^{-4x}\cos(x)\, dx$

$17\int e^{-4x}\cos x\, dx = e^{-4x}\sin(x) - 4e^{-4x}\cos(x)$

$17\int e^{-4x}\cos x\, dx = e^{-4x}(\sin(x) - 4\cos(x)) \cdot \frac{1}{17}$

$\frac{1}{17} \cdot$

$I = \frac{1}{17}e^{-4x}(\sin(x) - 4\cos(x)) + C$

Question #2

Evaluate the integral

$I = \int_0^1 6x\, e^{2x}dx$

$u = 2x$ $\quad dv = e^{2x}dx$

$\frac{1}{2}du = dx$ $\quad v = \frac{1}{2}e^{2x}$

$du = 2\, dx$

3 $\quad 2x\, \frac{1}{2}e^{2x}\Big]_0^1 - \int_0^1 \frac{1}{2}e^{2x}\, dx \cdot 2$

$2(1)\frac{1}{2}e^{2(1)}$

e^2

$\int_0^1 e^{2x}$

$\frac{1}{2}e^{2x}$

$e^2 - \frac{1}{2}e^2 + \frac{1}{2}$

$3\left(\frac{1}{2}e^2 + \frac{1}{2}\right)$

$\frac{3}{2}(e^2 + 1)$

Question # 5

Evaluate the integral $I = \int_0^1 (7x^2 - 5)e^x\, dx$

Let

$u = 7x^2 - 5$

$du = 14x\, dx$

$dv = e^x\, dx$

$v = e^x$

$e^x(7x^2 - 5)\Big]_0^1 - \int_0^1 e^x\, 14x\, dx$

$e^1(7(1)^2 - 5)$ $\quad e^0(\cancel{7(0)^2} - 5) = -5$

$e^1(2)$

$2e^1 - (-5)$

$\int_0^1 e^x\, 14x\, dx$

$2e + 5 - 14$

$\underline{I = 2e - 9}$

Let $u_2 = 14x$

$du_2 = 14\, dx$

$dv_2 = e^x\, dx$

$v_2 = e^x$

$14xe^x\Big]_0^1 - \int_0^1 e^x\, 14\, dx$

$14e^x\Big]_0^1 - (14e^1 - 14)$

$14xe^x\Big]_0^1 - (14e^x)\Big]_0^1$

$\cancel{14e^1} - \cancel{14e^1} + 14$

$-(14)$

Question #3

Evaluate the integral

$I = \int_0^{\ln(3)} 5(3 - xe^x)\, dx$

$5\int_0^{\ln(3)} 3 - xe^x\, dx$

$5\left(\int_0^{\ln(3)} 3\, dx - \int_0^{\ln(3)} xe^x\, dx\right)$

$3(\ln(3) - 0)$

$3\ln(3)$

Let $u = x$ $\quad dv = e^x\, dx$

$du = dx$ $\quad v = e^x$

$xe^x\Big]_0^{\ln(3)} - \int_0^{\ln(3)} e^x\, dx\Big]$ $\quad e^{\ln(3)} - 1$

$\ln(3)e^{\ln(3)} - e^{\ln(3)} + 1$

$-3 + 1 = -2$

$-(3\ln(3) - 2)$

$\cancel{3\ln(3)} - \cancel{3\ln(3)} + 2$

$I = 10 = 5(2)$

Question #6

Evaluate the definite integral

$I = \int_1^9 e^{\sqrt{t}}\, dt$

Let $u = (t)^{1/2} \equiv \sqrt{t}$

$u^2 = t$

$2u\, du = dt$

$\int_1^9 e^u\; 2u\, du$

$2\int_1^3 e^u\, u\, du$

$v = u$ $\quad dw = e^u\, du$

$dv = du$ $\quad w = e^u$

$2\left([ue^u] - \int e^u\, du\right)$

$(ue^u)\Big]_1^3 - \left[e^u\right]_1^3$

$3e^3 - e - (e^3 - e)$

$3e^3 - \cancel{e^0} - e^3 + \cancel{e^0}$

$2(2e^3)$

$4e^3$

Question #15

Evaluate the integral

$I = \int_0^1 x\, f(x)\, dx$

Let $u = f(x)$

$du = f'(x)\, dx$

$dv = x\, dx$

$v = \frac{1}{2}x^2$

$\frac{1}{2}x^2(f(x))\Big]_0^1 - \int_0^1 \frac{1}{2}x^2 f'(x)$

$\left(\frac{1}{2}(1)(f(1))\right) - \cancel{\left(\frac{1}{2}(0)^2 f(0)\right)}^{0}$ $\quad \frac{1}{2}\int_0^1 x^2 f'(x)\, dx$

$\frac{1}{2}(7)$

$\frac{7}{2} -$

Let $u = f'(x)$

$du = f''(x)$

$dv = x^2\, dx$

$v = \frac{1}{3}x^3$

$\frac{1}{3}x^3(f'(x))\Big]_0^1 - \int_0^1 \frac{1}{3}x^3 f''(x)\, dx$

$\frac{1}{3}(6)$

$\frac{1}{2}\left(2 - \frac{1}{3}\int_0^1 x^3 f''(x)\, dx\right)$

$-\left(1 - \frac{1}{6}\int_0^1 x^3 f''(x)\, dx\right)$

$\frac{7}{2} - \frac{2}{2}$

$\left(\frac{7}{2} - 1 + \frac{1}{6}\int_0^1 x^3 f''(x)\, dx\right)$

$I = \frac{5}{2} + \frac{1}{6}\int_0^1 x^3 f''(x)\, dx$

Question #4

Determine the integral

$I = \int (6x + 7)e^{2x}\, dx.$

Let

$u = 6x + 7$ $\quad dv = e^{2x}\, dx$

$du = 6\, dx$ $\quad v = \frac{1}{2}e^{2x}$

$-6e^{2x}$

$\frac{1}{2}e^{2x}(6x + 7) - \int \frac{1}{2}e^{2x}\, 6\, dx$

$\frac{1}{2}(6x + 7)e^{2x} - \frac{1}{2} \cdot 6\int e^{2x}$

$3\left(\frac{1}{2}e^{2x}\right)$

$\frac{1}{2}(6x + 7)e^{2x} - \frac{3}{2}e^{2x}$

$\frac{1}{2}\left((6x + 7)e^{2x} - 3e^{2x}\right)$

$6xe^{2x} + 7e^{2x} - 3e^{2x}$

$6xe^{2x} + 4e^{2x}$

$\frac{1}{2} \cdot 2e^{2x}(3x + 2)$

$e^{2x}(3x + 2) + C = I$

Question #9

Evaluate the integral

$I = \int_0^{\pi/2} (x^2 + 4)\sin x\, dx$

Let

$u = x^2 + 4$

$du = 2x\, dx$

$dv = \sin x\, dx$

$v = -\cos x$

$(x^2 + 4)(-\cos x)\Big]_0^{\pi/2} - \int_0^{\pi/2} -\cos x\; 2x\, dx$

$\left(\left(\frac{\pi}{2}\right)^2 + 4\right)\left(-\cos\left(\frac{\pi}{2}\right)\right) \cdots$ IBP

$-\left((0^2 + 4)(-\cos(0))\right)$ $\quad -1\int_0^{\pi/2} \cos x\; 2x\, dx$

4 $\quad -(1)$

$4(-1)$

$0 - (-4)$

$0 + 4$

Let $u = 2x$

$du = 2\, dx$

$dv = \cos x\, dx$

$v = \sin x$

$-1\left(2x \sin x\Big]_0^{\pi/2} - \int_0^{\pi/2} \sin x\; 2\, dx\right)$

$\left(\frac{\pi}{2}\sin\left(\frac{\pi}{2}\right)\right) - 0$ $\quad 2\int \sin x\, dx$

$\pi - 0 = \pi$

$-\left(-2\cos x\Big]_0^{\pi/2}\right)$

$\left(2\cos\left(\frac{\pi}{2}\right)\right) - (2\cos(0))$

$4 + (-1(-\pi + 2))$

$4 + \pi - 2 = \boxed{\pi + 2}$

$-1(\pi - 2)$ $\quad 0 - (-2)$

$0 + 2 = 2$

Question #12

Evaluate the definite integral

$I = \int_0^2 \sin^{-1}\left(\frac{x}{2}\right)dx$

$w = \frac{x}{2}$

$2w = x$

$2\, dw = dx$

$2\int_0^1 \sin^{-1}(w)\, dw$

Let $u = \sin^{-1}(w)$

$du = \frac{1}{\sqrt{1 - w^2}}\, dw$

$v = w$ $\quad dv = dw$

$2\left([w \sin^{-1}(w)]_0^1 - \int_0^1 \frac{w}{\sqrt{1 - w^2}}\, dw\right)$

Let $y = 1 - w^2$

$dy = -2w\, dw$

$\sin^{-1}(1) + 1$

$-\int_1^0 -\frac{1}{2}\frac{1}{\sqrt{y}}\, dy$

$+\frac{1}{2}\left[2y^{1/2}\right]$

$2\left(\frac{\pi}{2} + 1\right)$

$\pi + 2$

Question #10

Determine the indefinite integral

$I = \int e^{-x}\sin 3x\, dx$

Let $u = e^{-x}$

$du = -e^{-x}dx$

$dv = \sin 3x\, dx$

$v = -\frac{1}{3}\cos(3x)$

$-\frac{1}{3}\cos(3x)(-e^{-x}) - \int -\frac{1}{3}\cos(3x) - e^{-x}\, dx$

$\frac{1}{3}\int e^{-x}\cos(3x)\, dx$

Let $u = e^{-x}$

$du = -e^{-x}\, dx$

$dv = \cos(3x)\, dx$

$v = \frac{1}{3}\sin(3x)$

$\frac{1}{3}\sin(3x)e^{-x} - \int -\frac{1}{3}\sin(3x)e^{-x}\, dx$

$-\frac{1}{3}\left(\frac{1}{3}\sin(3x)e^{-x} + \frac{1}{3}\int \sin(3x)e^{-x}\, dx\right)$

$-9\int e^{-x}\sin(3x)\, dx = \left(\frac{1}{3}\cos(3x)e^{-x} - \frac{1}{9}\sin(3x)e^{-x} - \frac{1}{9}\int \sin(3x)e^{-x}\, dx\right)(-9)$

$-9\int e^{-x}\sin(3x)\, dx = -3\cos(3x)e^{-x} + \sin(3x)e^{-x} + \cancel{\int e^{-x}\sin(3x)\, dx}^{0}$

$-\int e^{-x}\sin(3x)\, dx$ $\quad -\cancel{\int e^{-x}\sin(3x)\, dx}$

$-10\int e^{-x}\sin(3x)\, dx = -3\cos(3x)e^{-x} + \sin(3x)e^{-x}$

(shorthand) $-10\, I = e^{-x}(\sin(3x) - 3\cos(3x))\;\left(-\frac{1}{10}\right)$

$-\frac{1}{10} \cdot \boxed{I = -\frac{1}{10}e^{-x}(\sin(3x) - 3\cos(3x)) + C}$

Question #13

Evaluate the integral

$I = \int_1^e 2x \ln(x)\, dx$

Let $u = \ln(x)$

$du = \frac{1}{x}\, dx$

$dv = 2x\, dx$

$v = x^2$

$x^2\ln(x)\Big]_1^e - \int_1^e x^2\frac{1}{x}\, dx$

$e^2\ln(e)$

$\int_1^e x^1\, dx$

$\frac{1}{1+1}x^{1+1}$

$\frac{1}{2}x^2\Big]_1^e$

$-1\left(\frac{1}{2}e^2 - \frac{1}{2}\right)$

$e^2 - \frac{1}{2}e^2 + \frac{1}{2}$

$\frac{1}{2}e^2 + \frac{1}{2}$

$I = \frac{1}{2}(e^2 + 1)$

Question #11

Evaluate the definite integral

$I = \int_1^e 4x^3\ln(x)\, dx$

Let $u = \ln(x)$

$du = \frac{1}{x}\, dx$

$dv = 4x^3\, dx$

$v = x^4$

$x^4\ln(x)\Big]_1^e - \int_1^e x^4\frac{1}{x}\, dx$

$(e^4\ln(e)) - (\ln(1))$

$e^4 - 0$

$\int_1^e x^3\, dx$

$\frac{1}{4}x^4\Big]_1^e \rightarrow \left(\frac{1}{4}e^4 - \frac{1}{4}\right) - 1$

$-\frac{1}{4}e^4 + \frac{1}{4}$

$e^4 - \frac{1}{4}e^4 + \frac{1}{4}$

$\frac{3}{4}e^4 + \frac{1}{4}$

$\frac{1}{4}(3e^4 + 1) = I$