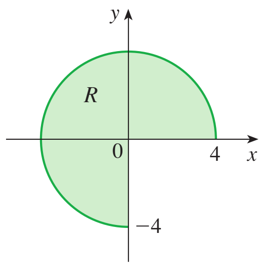
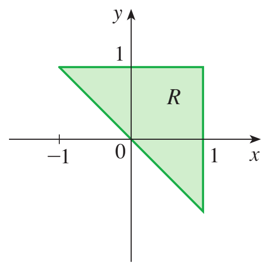
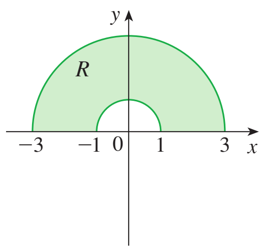
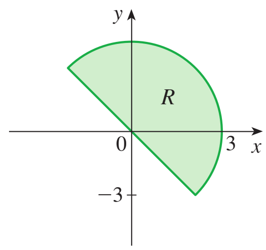
15.3 Exercises

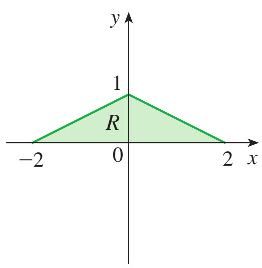
1–6 A region R is shown. Decide whether to use polar coordinates or rectangular coordinates and write $\iint_R f(x, y)\, dA$ as an iterated integral, where f is an arbitrary continuous function on R.

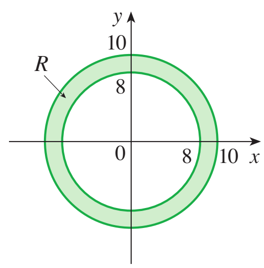
1.


2.


3.


4.


5.


6.


7–8 Sketch the region whose area is given by the integral and evaluate the integral.

7. $\displaystyle\int_{\pi/4}^{3\pi/4} \int_1^2 r\, dr\, d\theta$

8. $\displaystyle\int_{\pi/2}^{\pi} \int_0^{2\sin\theta} r\, dr\, d\theta$

9–16 Evaluate the given integral by changing to polar coordinates.

9. $\iint_D x^2 y\, dA$, where D is the top half of the disk with center the origin and radius 5

10. $\iint_R (2x - y)\, dA$, where R is the region in the first quadrant enclosed by the circle $x^2 + y^2 = 4$ and the lines $x = 0$ and $y = x$

11. $\iint_R \sin(x^2 + y^2)\, dA$, where R is the region in the first quadrant between the circles with center the origin and radii 1 and 3

12. $\displaystyle\iint_R \frac{y^2}{x^2 + y^2}\, dA$, where R is the region that lies between the circles $x^2 + y^2 = a^2$ and $x^2 + y^2 = b^2$ with $0 < a < b$

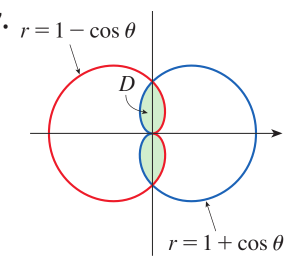
13. $\iint_D e^{-x^2-y^2}\, dA$, where D is the region bounded by the semicircle $x = \sqrt{4 - y^2}$ and the y-axis

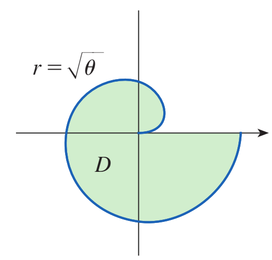
14. $\iint_D \cos\sqrt{x^2 + y^2}\, dA$, where D is the disk with center the origin and radius 2

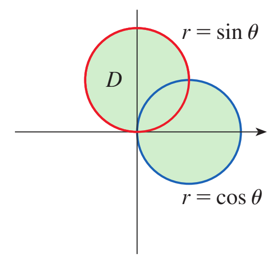
15. $\iint_R \arctan(y/x)\, dA$,
where $R = \{(x, y) \mid 1 \leqslant x^2 + y^2 \leqslant 4,\ 0 \leqslant y \leqslant x\}$

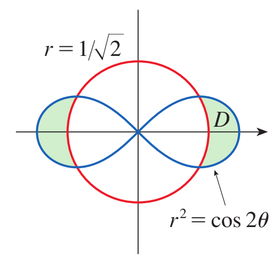
16. $\iint_D x\, dA$, where D is the region in the first quadrant that lies between the circles $x^2 + y^2 = 4$ and $x^2 + y^2 = 2x$

17–22 Use a double integral to find the area of the region D.

17.


18.


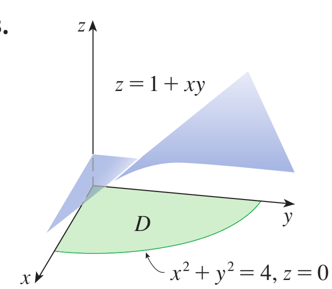
19.


20.


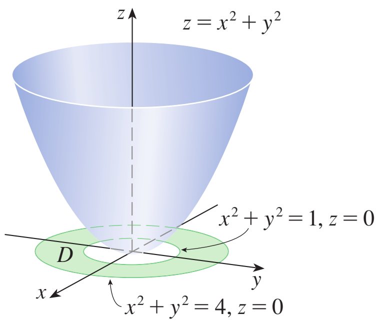
21. D is the loop of the rose $r = \sin 3\theta$ in the first quadrant.

22. D is the region inside the circle $(x - 1)^2 + y^2 = 1$ and outside the circle $x^2 + y^2 = 1$.

23–24
(a) Set up an iterated integral in polar coordinates for the volume of the solid under the surface and above the region D.
(b) Evaluate the iterated integral to find the volume of the solid.

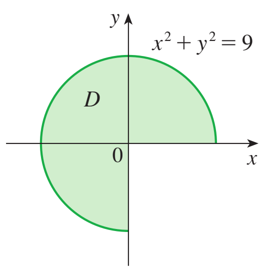
23.


24.

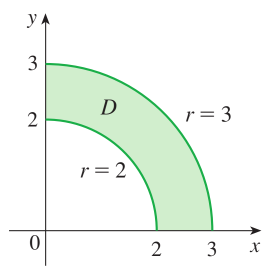


25–28
(a) Set up an iterated integral in polar coordinates for the volume of the solid under the graph of the given function and above the region D.
(b) Evaluate the iterated integral to find the volume of the solid.

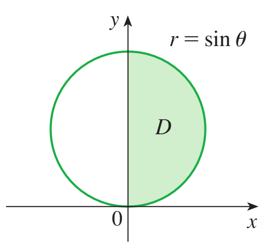
25. $f(x, y) = y$



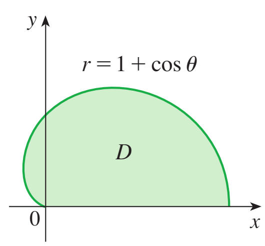
26. $f(x, y) = xy^2$



27. $f(x, y) = x$



28. $f(x, y) = 1$



29–37 Use polar coordinates to find the volume of the given solid.

29. Under the paraboloid $z = x^2 + y^2$ and above the disk $x^2 + y^2 \leqslant 25$

30. Below the cone $z = \sqrt{x^2 + y^2}$ and above the ring $1 \leqslant x^2 + y^2 \leqslant 4$

31. Below the plane $2x + y + z = 4$ and above the disk $x^2 + y^2 \leqslant 1$

32. Inside the sphere $x^2 + y^2 + z^2 = 16$ and outside the cylinder $x^2 + y^2 = 4$

33. A sphere of radius a

34. Bounded by the paraboloid $z = 1 + 2x^2 + 2y^2$ and the plane $z = 7$ in the first octant

35. Above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = 1$

36. Bounded by the paraboloids $z = 6 - x^2 - y^2$ and $z = 2x^2 + 2y^2$

37. Inside both the cylinder $x^2 + y^2 = 4$ and the ellipsoid $4x^2 + 4y^2 + z^2 = 64$

38. (a) A cylindrical drill with radius r_1 is used to bore a hole through the center of a sphere of radius r_2. Find the volume of the ring-shaped solid that remains.
(b) Express the volume in part (a) in terms of the height h of the ring. Notice that the volume depends only on h, not on r_1 or r_2.

39–42 Evaluate the iterated integral by converting to polar coordinates.

39. $\displaystyle\int_0^2 \int_0^{\sqrt{4-x^2}} e^{-x^2-y^2}\, dy\, dx$

40. $\displaystyle\int_0^a \int_{-\sqrt{a^2-y^2}}^{\sqrt{a^2-y^2}} (2x + y)\, dx\, dy$

41. $\displaystyle\int_0^{1/2} \int_{\sqrt{3}\,y}^{\sqrt{1-y^2}} xy^2\, dx\, dy$

42. $\displaystyle\int_0^2 \int_0^{\sqrt{2x-x^2}} \sqrt{x^2 + y^2}\, dy\, dx$

T **43–44** Express the double integral in terms of a single integral with respect to r. Then use a calculator (or computer) to evaluate the integral correct to four decimal places.

43. $\iint_D e^{(x^2+y^2)^2}\, dA$, where D is the disk with center the origin and radius 1

44. $\iint_D xy\sqrt{1 + x^2 + y^2}\, dA$, where D is the portion of the disk $x^2 + y^2 \leqslant 1$ that lies in the first quadrant

45. A swimming pool is circular with a 40-ft diameter. The depth is constant along east-west lines and increases linearly from 2 ft at the south end to 7 ft at the north end. Find the volume of water in the pool.

46. An agricultural sprinkler distributes water in a circular pattern of radius 100 ft. It supplies water to a depth of e^{-r} feet per hour at a distance of r feet from the sprinkler.
(a) If $0 < R \leqslant 100$, what is the total amount of water supplied per hour to the region inside the circle of radius R centered at the sprinkler?
(b) Determine an expression for the average amount of water per hour per square foot supplied to the region inside the circle of radius R.

47. Find the average value of the function $f(x, y) = 1/\sqrt{x^2 + y^2}$ on the annular region $a^2 \leqslant x^2 + y^2 \leqslant b^2$, where $0 < a < b$.