Summary of Tests and Equations for Series and Sequences

1. Basic Definitions

- **Sequence Convergence**: A sequence $\{a_n\}$ converges to L if $\lim_{n\to\infty} a_n = L$.
- **Series Convergence**: The series $\sum a_n$ converges if the partial sums $S_n = \sum_{k=1}^{n} a_k$ approach a finite limit as $n \to \infty$.

2. Tests for Series Convergence and Divergence

Divergence Test (Test for Divergence)

If $\lim_{n\to\infty} a_n \neq 0$, then the series $\sum a_n$ **diverges**. If $\lim_{n\to\infty} a_n = 0$, this test is inconclusive.

Geometric Series Test

For a series $\sum ar^n$, it converges if $|r| < 1$ and diverges if $|r| \geq 1$.

- **Sum for a convergent geometric series**: $S = \frac{a}{1-r}$.

p-Series Test

A p-series $\sum \frac{1}{n^p}$ converges if $p > 1$ and diverges if $p \leq 1$.

Comparison Test

For $\sum a_n$ and a positive comparison series $\sum b_n$:

- If $0 \leq a_n \leq b_n$ and $\sum b_n$ converges, then $\sum a_n$ converges.
- If $0 \leq b_n \leq a_n$ and $\sum b_n$ diverges, then $\sum a_n$ diverges.

Limit Comparison Test

If $\lim_{n\to\infty} \frac{a_n}{b_n} = L$, where $0 < L < \infty$, then $\sum a_n$ and $\sum b_n$ both converge or diverge.

Ratio Test

For a series $\sum a_n$, calculate $L = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$:

- If $L < 1$, the series converges absolutely.
- If $L > 1$, the series diverges.
- If $L = 1$, the test is inconclusive.

Root Test

For a series $\sum a_n$, calculate $L = \lim_{n\to\infty} \sqrt[n]{|a_n|}$:

- If $L < 1$, the series converges absolutely.
- If $L > 1$, the series diverges.
- If $L = 1$, the test is inconclusive.

Alternating Series Test (Leibniz Test)

For an alternating series $\sum(-1)^{n-1}b_n$:

- The series converges if b_n is decreasing and $\lim_{n\to\infty} b_n = 0$.

3. Absolute and Conditional Convergence

- **Absolute Convergence**: If $\sum |a_n|$ converges, then $\sum a_n$ converges absolutely.
- **Conditional Convergence**: If $\sum a_n$ converges but $\sum |a_n|$ diverges, then $\sum a_n$ converges conditionally.

4. Power Series

A power series $\sum a_n x^n$ converges within its **radius of convergence** R.

- **Interval of Convergence**: $(-R, R)$
- **Ratio Test for Radius**: $R = \lim_{n\to\infty} \left|\frac{a_n}{a_{n+1}}\right|$ if this limit exists.