14.2 Exercises

1. Suppose that $\lim_{(x, y)\to(3, 1)} f(x, y) = 6$. What can you say about the value of $f(3, 1)$? What if f is continuous?

2. Explain why each function is continuous or discontinuous.
(a) The outdoor temperature as a function of longitude, latitude, and time
(b) Elevation (height above sea level) as a function of longitude, latitude, and time
(c) The cost of a taxi ride as a function of distance traveled and time

3–4 Use a table of numerical values of $f(x, y)$ for (x, y) near the origin to make a conjecture about the value of the limit of $f(x, y)$ as $(x, y) \to (0, 0)$. Then explain why your guess is correct.

3. $f(x, y) = \dfrac{x^2y^3 + x^3y^2 - 5}{2 - xy}$ **4.** $f(x, y) = \dfrac{2xy}{x^2 + 2y^2}$

5–12 Find the limit.

5. $\lim_{(x, y)\to(3, 2)} (x^2y^3 - 4y^2)$

6. $\lim_{(x, y)\to(5, -2)} (x^2y + 3xy^2 + 4)$

7. $\lim_{(x, y)\to(-3, 1)} \dfrac{x^2y - xy^3}{x - y + 2}$ **8.** $\lim_{(x, y)\to(2, -1)} \dfrac{x^2y + xy^2}{x^2 - y^2}$

9. $\lim_{(x, y)\to(\pi, \pi/2)} y\sin(x - y)$ **10.** $\lim_{(x, y)\to(3, 2)} e^{\sqrt{2x-y}}$

11. $\lim_{(x, y)\to(1, 1)} \left(\dfrac{x^2y^3 - x^3y^2}{x^2 - y^2}\right)$ **12.** $\lim_{(x, y)\to(\pi, \pi/2)} \dfrac{\cos y - \sin 2y}{\cos x \cos y}$

13–18 Show that the limit does not exist.

13. $\lim_{(x, y)\to(0, 0)} \dfrac{y^2}{x^2 + y^2}$ **14.** $\lim_{(x, y)\to(0, 0)} \dfrac{2xy}{x^2 + 3y^2}$

15. $\lim_{(x, y)\to(0, 0)} \dfrac{(x + y)^2}{x^2 + y^2}$ **16.** $\lim_{(x, y)\to(0, 0)} \dfrac{x^2 + xy^2}{x^4 + y^2}$

17. $\lim_{(x, y)\to(0, 0)} \dfrac{y^2\sin^2 x}{x^4 + y^4}$ **18.** $\lim_{(x, y)\to(1, 1)} \dfrac{y - x}{1 - y + \ln x}$

19–30 Find the limit, if it exists, or show that the limit does not exist.

19. $\lim_{(x, y)\to(-1, -2)} (x^2y - xy^2 + 3)^3$

20. $\lim_{(x, y)\to(\pi, 1/2)} e^{xy}\sin xy$

21. $\lim_{(x, y)\to(2, 3)} \dfrac{3x - 2y}{4x^2 - y^2}$ **22.** $\lim_{(x, y)\to(1, 2)} \dfrac{2x - y}{4x^2 - y^2}$

23. $\lim_{(x, y)\to(0, 0)} \dfrac{xy^2\cos y}{x^2 + y^4}$ **24.** $\lim_{(x, y)\to(0, 0)} \dfrac{x^3 - y^3}{x^2 + xy + y^2}$

25. $\lim_{(x, y)\to(0, 0)} \dfrac{x^2 + y^2}{\sqrt{x^2 + y^2 + 1} - 1}$

26. $\lim_{(x, y)\to(0, 0)} \dfrac{xy^4}{x^2 + y^8}$

27. $\lim_{(x, y, z)\to(6, 1, -2)} \sqrt{x + z}\cos(\pi y)$

28. $\lim_{(x, y, z)\to(0, 0, 0)} \dfrac{xy + yz}{x^2 + y^2 + z^2}$

29. $\lim_{(x, y, z)\to(0, 0, 0)} \dfrac{xy + yz^2 + xz^2}{x^2 + y^2 + z^4}$

30. $\lim_{(x, y, z)\to(0, 0, 0)} \dfrac{x^4 + y^2 + z^3}{x^4 + 2y^2 + z}$

31–34 Use the Squeeze Theorem to find the limit.

31. $\lim_{(x, y)\to(0, 0)} xy\sin\dfrac{1}{x^2 + y^2}$ **32.** $\lim_{(x, y)\to(0, 0)} \dfrac{xy}{\sqrt{x^2 + y^2}}$

33. $\lim_{(x, y)\to(0, 0)} \dfrac{xy^4}{x^4 + y^4}$

34. $\lim_{(x, y, z)\to(0, 0, 0)} \dfrac{x^2y^2z^2}{x^2 + y^2 + z^2}$

35–36 Use a graph of the function to explain why the limit does not exist.

35. $\lim_{(x, y)\to(0, 0)} \dfrac{2x^2 + 3xy + 4y^2}{3x^2 + 5y^2}$ **36.** $\lim_{(x, y)\to(0, 0)} \dfrac{xy^3}{x^2 + y^6}$

37–38 Find $h(x, y) = g(f(x, y))$ and the set of points at which h is continuous.

37. $g(t) = t^2 + \sqrt{t}$, $f(x, y) = 2x + 3y - 6$

38. $g(t) = t + \ln t$, $f(x, y) = \dfrac{1 - xy}{1 + x^2y^2}$

39–40 Graph the function and observe where it is discontinuous. Then use the formula to explain what you have observed.

39. $f(x, y) = e^{1/(x-y)}$ **40.** $f(x, y) = \dfrac{1}{1 - x^2 - y^2}$

41–50 Determine the set of points at which the function is continuous.

41. $F(x, y) = \dfrac{xy}{1 + e^{x-y}}$ **42.** $F(x, y) = \cos\sqrt{1 + x - y}$

43. $F(x, y) = \dfrac{1 + x^2 + y^2}{1 - x^2 - y^2}$ **44.** $H(x, y) = \dfrac{e^x + e^y}{e^{xy} - 1}$

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45. $G(x, y) = \sqrt{x} + \sqrt{1 - x^2 - y^2}$

46. $G(x, y) = \ln(1 + x - y)$

47. $f(x, y, z) = \arcsin(x^2 + y^2 + z^2)$

48. $f(x, y, z) = \sqrt{y - x^2} \ln z$

49. $f(x, y) = \begin{cases} \dfrac{x^2 y^3}{2x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\ 1 & \text{if } (x, y) = (0, 0) \end{cases}$

50. $f(x, y) = \begin{cases} \dfrac{xy}{x^2 + xy + y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$

51–53 Use polar coordinates to find the limit. [If (r, θ) are polar coordinates of the point (x, y) with $r \geqslant 0$, note that $r \to 0^+$ as $(x, y) \to (0, 0)$.]

51. $\displaystyle\lim_{(x, y) \to (0, 0)} \frac{x^3 + y^3}{x^2 + y^2}$

52. $\displaystyle\lim_{(x, y) \to (0, 0)} (x^2 + y^2) \ln(x^2 + y^2)$

53. $\displaystyle\lim_{(x, y) \to (0, 0)} \frac{e^{-x^2 - y^2} - 1}{x^2 + y^2}$

54. Prove the three special limits in (2).

55. At the beginning of this section we considered the function

$$f(x, y) = \frac{\sin(x^2 + y^2)}{x^2 + y^2}$$

and guessed on the basis of numerical evidence that $f(x, y) \to 1$ as $(x, y) \to (0, 0)$. Use polar coordinates to confirm the value of the limit. Then graph the function.

56. Graph and discuss the continuity of the function

$$f(x, y) = \begin{cases} \dfrac{\sin xy}{xy} & \text{if } xy \neq 0 \\ 1 & \text{if } xy = 0 \end{cases}$$

57. Let

$$f(x, y) = \begin{cases} 0 & \text{if } y \leqslant 0 \quad \text{or} \quad y \geqslant x^4 \\ 1 & \text{if } 0 < y < x^4 \end{cases}$$

(a) Show that $f(x, y) \to 0$ as $(x, y) \to (0, 0)$ along any path through $(0, 0)$ of the form $y = mx^a$ with $0 < a < 4$.
(b) Despite part (a), show that f is discontinuous at $(0, 0)$.
(c) Show that f is discontinuous on two entire curves.

58. Show that the function f given by $f(\mathbf{x}) = |\mathbf{x}|$ is continuous on $\mathbb{R}^n$. [*Hint:* Consider $|\mathbf{x} - \mathbf{a}|^2 = (\mathbf{x} - \mathbf{a}) \cdot (\mathbf{x} - \mathbf{a})$.]

59. If $\mathbf{c} \in V_n$, show that the function f given by $f(\mathbf{x}) = \mathbf{c} \cdot \mathbf{x}$ is continuous on $\mathbb{R}^n$.

14.3 Partial Derivatives

Partial Derivatives of Functions of Two Variables

On a hot day, extreme humidity makes us think the temperature is higher than it really is, whereas in very dry air we perceive the temperature to be lower than the thermometer indicates. The National Weather Service has devised the *heat index* (also called the temperature-humidity index, or humidex, in some countries) to describe the combined effects of temperature and humidity. The heat index I is the perceived air temperature when the actual temperature is T and the relative humidity is H. So I is a function of T and H and we can write $I = f(T, H)$. The following table of values of I is an excerpt from a table compiled by the National Weather Service.

Table 1 Heat index I as a function of temperature and humidity

Relative humidity (%)

Actual temperature (°F)

T \ H	50	55	60	65	70	75	80	85	90
90	96	98	100	103	106	109	112	115	119
92	100	103	105	108	112	115	119	123	128
94	104	107	111	114	118	122	127	132	137
96	109	113	116	121	125	130	135	141	146
98	114	118	123	127	133	138	144	150	157
100	119	124	129	135	141	147	154	161	168

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