Q1: Use partial fractions to rewrite the expression:

$$\frac{***}{x^2(x+1)(x^2+x+1)^2}$$

$$\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x^3}+\frac{D}{x+1}+\frac{Ex+C}{(x^2+x+1)^2}$$

Q2: Evaluate $\int_{-\infty}^{0} \frac{1}{x+3}$

$$\int_{-\infty}^{-3}\frac{1}{x+3}+\int_{-3}^{0}\frac{1}{x+3}$$

Q3: Find all constant solutions to $y' = y^3 - 4y$.

$y' = y(y^2-4)$

$y=0$
$y=-2$
$y=2$

$y+2$
$y \quad y^2 \quad \cancel{2y} \; 0$
$-2 \quad \cancel{-2y} \quad -4$

Free response

Q4: Evaluate the following integral $\int x^2 \sin x \, dx$

$u = x^2 \quad v = -\cos x$
$du = 2x\,dx \quad dv = \sin x \, dx$

$-x^2\cos x - \int -2x\cos x \, dx$
$-x^2\cos x + \int 2x \cos x \, dx$

Let $u = 2x \quad v = \sin x$
$du = dx \quad dv = \cos x$

$2x\sin x - \int \sin x \, dx$

$$\boxed{-x^2\cos x + 2x\sin x + 2\cos x + C}$$

$\sin^2\theta + \cos^2\theta = 1 - \cos^2\theta$
$- \cos^2\theta$

Q5: Evaluate the following integral: $\int \sin^5 \cos^2 x \, dx$

$\int \underline{\sin^2 x \, \sin^2 x} \cdot \sin x \cdot \cos^2 x$

$\int (1-\cos^2 x)(1-\cos^2 x)\cos^2 x \, \sin x \, dx$

Let $u = \cos x$
$-1\,du = \frac{-\sin x\,dx}{-1}$
$-1\,du = \sin x\,dx$

$-\int (1-u^2)(1-u^2)u^2 \, du$

$1 - u^2$
$1 \quad 1 \quad -u^2$
$-u^2 \; -u^2 \quad u^4$

$(1-2u^2+u^4)u^2$
$\int u^2 - 2u^4 + u^6 \, du$
$\frac{1}{3}u^3 - \frac{2}{5}u^5 + \frac{1}{7}u^7$

$$\boxed{-\frac{1}{3}\cos^3 x + \frac{2}{5}\cos^5 x + \frac{1}{7}\cos^7 x + C}$$

Q6: Evaluate the following integral: $\int \frac{1}{x^2\sqrt{x^2-16}} \, dx$

$\sqrt{x^2-a^2} = a\sec\theta$

Let $x = 4\sec\theta$
$dx = 4\sec\theta\tan\theta \, d\theta$

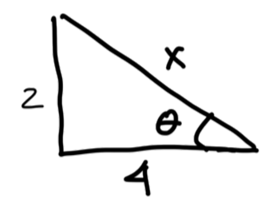
$\cancel{\sec^2}$

$16\sec^2\theta \sqrt{16\sec^2\theta - 16}$
$16\sec^2\theta(4)\sqrt{\sec^2\theta - 1}$
$16\sec^2\theta(4)\tan\theta$

$\frac{\cancel{4\sec\theta\tan\theta}\,d\theta}{16\sec^{\cancel{2}}\theta\,\cancel{4\tan\theta}}$

$\int \frac{1}{16\sec\theta}\,d\theta \rightarrow \int \frac{\cos\theta}{16}\,d\theta$

$\frac{1}{16}\int \cos\theta \, d\theta$



$\frac{1}{16}\sin\theta + C$

$$\boxed{\frac{1}{16}\left(\frac{\sqrt{x^2-16}}{x}\right)+C}$$

$\sec\theta = \frac{x}{4}$

$\frac{1}{\cos\theta} = \frac{1}{\frac{A}{H}} \rightarrow \frac{H}{A}$

SOH
CAH

$2^2 + 4^2 = x^2 - 4^2$
-4^2
$\sqrt{z^2} = \sqrt{x^2-16}$
$z = \sqrt{x^2-16}$

Q7: Solve the following differential equation: $2yy' = xy^2 + x$

$2y\frac{dy}{dx} = xy^2 + x$

$2y\frac{dy}{dx} = \frac{x(y^2+1)}{y^2+1}$

$\int \frac{2y}{y^2+1}\,dy = \int x\,dx$

Let $u = y^2+1$
$du = 2y\,dy$

$\int \frac{1}{u} = \int x\,dx$

$\ln|y^2+1| = \frac{1}{2}x^2 + C$

$e^{\ln|y^2+1|} = e^{\frac{1}{2}x^2 + C}$

$y^2 + 1 = 2e^{\frac{1}{2}x^2} \quad e^{\frac{1}{2}x^2} \quad \cancel{e^c} \, 2$

$y = \sqrt{2e^{\frac{1}{2}x^2} - 1}$

Q8: Solve the following differential equation: $\frac{dy}{dx} + 3y = 2xe^{-3x}(e^{3x})$

(e^{3x})

$e^{\int P(x)}$
$e^{\int 3\,dx}$
e^{3x}
e^{3x}

$e^{3x}y' + 3ye^{3x} = 2x$

$\int \frac{d}{dx}(e^{3x} \cdot y) = \int 2x \, dx$

$\frac{e^{3x}y}{e^{3x}} = \frac{x^2+C}{e^{3x}}$

$$\boxed{y = \frac{x^2+C}{e^{3x}}}$$