$\int x^k dx = \frac{1}{1+k} x^{k+1}$

Question #1: Evaluate the Integral

$I = \int_0^{\pi/2} 3\sin^3(x)\cos^2(x)\,dx$

$3\int_0^{\pi/2} (1-\cos^2 x)\cos^2 x \sin x\,dx$

Let $u = \cos x$
$du = -\sin x\,dx$
$-du = \sin x\,dx$

$-3\int_0^1 (1-u^2)u^2\,du$

$u^2 - u^4$

$-3\int_1^0 u^2 - u^4\,du$

$\frac13 u^3 - \frac15 u^5 \Big]_1^0$

$-u^3 + \frac35 u^5$

$\left(\cancel{-(0)^3 + \frac35(0)^5}\right) - \left(-(1)^3 + \frac35(1)^5\right) \to 0$

$-\left(-\frac55 + \frac35\right)$

$-\left(-\frac25\right) = \frac25 = I$

Question #2

Determine the indef. integral

$I = \int \sin^2 \cos^3 x\,dx$

$\int (1-\sin^2(x))\sin^2(x)\cos x\,dx$

Let $u = \sin(x)$
$du = \cos x\,dx$

$\int (1-u^2)u^2\,du$

$\int u^2 - u^4\,du$

$\frac13 u^3 - \frac15 u^5 + C$

Question #3

Evaluate the indefinite integral

$I = \int 8\cos^4(2t)\,dt$

$8\int \cos^4(2t)\,dt$

$8\int \frac14(1+\cos 4t)^2\,dt$

$\frac14\left(1 + 2\cos(4t) + \cos^2(4t)\right)dt$

$\frac12(1+\cos(8t))$

$\frac14\left(1 + 2\cos(4t) + \frac12 + \frac12\cos(8t)\right)dt$

$\frac14\left(\frac22 + 2\cos(4t) + \frac12 + \frac12\cos(8t)\right)dt$

$\frac14\left(\frac32 + 2\cos(4t) + \frac12\cos(8t)\right)dt$

$8\left(\frac38 + \frac12\cos(4t) + \frac18\cos(8t)\right)dt$

$\int 3 + 4\cos(4t) + \cos(8t)\,dt$

$\int 3\,dt + \int 4\cos(4t)\,dt + \int \cos(8t)\,dt$

$\boxed{I = 3t + \sin(4t) + \frac18\sin(8t)}$

Question #4

Determine the integral

$I = \int (3\sin(\theta) - 2\sin^3(\theta))\,d\theta$

$\int 3\sin(\theta)\,d\theta - \int 2\sin^3\theta\,d\theta$

$-3\cos(\theta) - 2\int \sin^3\theta\,d\theta$

$-2\int \left((1-\cos^2(\theta))\sin\theta\right)d\theta$

Let $u = \cos(\theta)$
$\frac{du}{-1} = \frac{-\sin\theta}{-1}$
$-du = \sin\theta$

$2\int (1-u^2)\,du$

$2\left(u - \frac13 u^3\right)$

$-3\cos\theta + 2u - \frac23 u^3$

$\cancel{-3\cos\theta + 2\cos\theta} - \frac23\cos^3\theta + C \to -\cos\theta$

$I = -\cos\theta - \frac23\cos^3\theta + C$

Question #5

Evaluate the integral

$I = \int_0^{\pi/2} 4\cos^2(x) + \sin^2(x)\,dx$

$4\int_0^{\pi/2} \cos^2(x) + \int_0^{\pi/2} \sin^2(x)\,dx$

$4\int_0^{\pi/2} \frac12(1+\cos 2x)\,dx + \int_0^{\pi/2} \frac12(1-\cos 2x)\,dx$

$2\int_0^{\pi/2} 1 + \cos 2x\,dx + \frac12\int_0^{\pi/2} 1 - \cos 2x\,dx$

$x + \frac12\cos(2x)\Big]_0^{\pi/2}$ $\quad x - \frac12\sin(2x)\Big]_0^{\pi/2}$

$2\left(\frac{\pi}{2}\right)$ $\quad \left(\frac{\pi}{2} - \cancel{\frac12(\sin(\pi))}\right) - \left[\cancel{0 - \frac12\sin(0)}\right] \to 0$

$\pi \quad + \frac{\pi}{4} \quad \frac12\left(\frac{\pi}{2}\right) = \frac{\pi}{4}$

$4\pi + \frac{\pi}{4}$

$\boxed{I = \frac54\pi} = \frac{5\pi}{4}$

Question #6

$I = \int_0^{\frac{\pi}{4}} \sec^2 x(3 - 2\tan x)\,dx$

$\int_0^{\frac{\pi}{4}} 3\sec^2 x - 2\tan x \sec^2 x\,dx$

$\int_0^{\pi/4} 3\sec^2 x - \int_0^{\pi/4} 2\tan x \sec^2 x$

$3\int_0^{\pi/4} \sec^2 x - 2\int_0^{\frac{\pi}{4}} \tan x \sec^2 x\,dx$

$\tan x\Big]_0^{\frac{\pi}{4}}$

$\left(\tan\left(\frac{\pi}{4}\right) - \tan(0)\right)$

$(1-0) = (1)3$

$3 - 1 = 2$

Let $u = \tan x$
$du = \sec^2 x\,dx$

$2\int_0^1 u\,du$

$2\left(\frac12 u^2\Big]_0^1\right)$

$2\left(\frac12 - 0\right)$

$2\left(\frac12\right)$

Question #7

Find the value of the definite integral

$I = \int_0^{\pi/4} (8\sec^4(x) - 5\sec^2(x))\tan(x)\,dx$.

$\int_0^{\pi/4} 8\sec^4(x)\tan(x) - \int_0^{\pi/4} 5\sec^2(x)\tan(x)\,dx$

$8\int_0^{\pi/4} \sec^4(x)\tan(x)\,dx - 5\int_0^{\pi/4} \sec^2(x)\tan(x)\,dx$

Let $u = \sec(x)$
$du = \sec(x)\tan(x)$

$8\int_1^{\sqrt2} u^3\,du$

$\frac14 u^4\Big]_1^{\sqrt2}$

$(\sqrt2)^4$

$\cancel{\frac44}\ 1 - \frac14$

$8\left(\frac34\right)$

$6 - \frac52$

$\frac{12}{2} - \frac52 = \frac72 = I$

Let $u = \tan(x)$
$du = \sec^2(x)\,dx$

$\int_0^1 u\,du$

$\frac12 u^2\Big]$

$5\left(\frac12\right)$

$\frac52$

Question #8

Evaluate the integral

$I = \int_0^{\pi/3} \frac{\sec(x)\tan(x)}{5 + 2\sec(x)}\,dx$.

Let $u = 5 + 2\sec(x)$
$\frac{du}{2} = \frac{\cancel{2}\sec(x)\tan(x)\,dx}{\cancel{2}}$

$\frac12\int_0^{\pi/3} \frac1u\,du$

$\frac12\left(\ln|5 + 2\sec(x)|\right)\Big]_0^{\pi/3}$

$\left(\ln\left|5 + 2\sec\left(\frac{\pi}{3}\right)\right| - \ln|5 + 2\sec(0)|\right)$

$(\ln|9| - \ln|7|)$

$\frac12\ln\left(\frac97\right) = I$

Question #9

Find the value of

$\int_0^{\pi/4} 4\tan^4 x\,dx$.

$4\int_0^{\frac{\pi}{4}} \tan^4 x\,dx$

$4\int_0^{\pi/4} \tan^2(x)\tan^2(x)\,dx$

$4\int_0^{\pi/4} (\sec^2 x - 1)\tan^2(x)\,dx$

$4\int_0^{\pi/4} \tan^2 x \sec^2 x - \tan^2 x\,dx$

$\int_0^{\pi/4} \tan^2 x \sec^2 x\,dx - \int_0^{\pi/4} \tan^2 x\,dx$

Let $u = \tan x$
$du = \sec^2 x\,dx$

$\int u^2\,du$

$\frac13 u^3\Big]_0^1$

$\int_0^{\pi/4} \sec^2 x - 1\,dx$

$\tan x - x\Big]_0^{\pi/4}$

$1 - \frac{\pi}{4}$

$\frac44 - \frac{\pi}{4}$

$\frac{4-\pi}{4}$

$\frac13 - \left(1 - \frac{\pi}{4}\right)$

$4\left(\frac13 - 1 + \frac{\pi}{4}\right)$

$\frac43 - 4 + \frac{4\pi}{4}$

$\frac43 - 4 + \pi$

$\frac43 - \frac{12}{3} + \frac{3\pi}{3}$

$\left(-\frac83 + \frac{3\pi}{3}\right)\frac13$

$\frac13(3\pi - 8) = I$