M408D – Final Exam Review Problems

1. Evaluate $\int t \sin 2t \, dt$

2. Evaluate $\int \dfrac{e^{\sin x}}{\sec x} \, dx$

3. Evaluate $\int_0^{\pi/2} \sin^3 \theta \cos^2 \theta \, d\theta$

4. Evaluate $\int_1^2 \dfrac{(x+1)^2}{x} \, dx$

5. Evaluate $\int \dfrac{1}{x\sqrt{x^2+1}} \, dx$

6. Evaluate $\int \dfrac{x-1}{x^2+2x} \, dx$

7. Evaluate $\int_1^\infty \dfrac{1}{(2x+1)^3} \, dx$

8. Solve the initial value problem: $\dfrac{dr}{dt} + 2tr = r$, $r(0) = 5$.

9. Find the Cartesian equation for $x = t^2 + 4t$, $y = 2 - t$.

10. Find a polar equation for $x + y = 2$.

11. Determine if the series converges or diverges $\displaystyle\sum_{n=1}^\infty \dfrac{n}{n^3+1}$

12. Determine if the series converges or diverges $\displaystyle\sum_{n=1}^\infty \dfrac{n^3}{5^n}$

13. Determine if the series converges or diverges $\displaystyle\sum_{n=1}^\infty \dfrac{\sqrt{n}}{n+1}$

14. Determine if the series converges absolutely, converges conditionally or diverges $\displaystyle\sum_{n=1}^\infty \dfrac{(-3)^{n-1}}{2^n}$

15. Determine if the series converges absolutely, converges conditionally or diverges $\displaystyle\sum_{n=1}^\infty \dfrac{(-1)^n(n+1)3^n}{2^{2n+1}}$

16. Determine if the series converges absolutely, converges conditionally or diverges $\displaystyle\sum_{n=1}^\infty \dfrac{(-1)^{n-1}}{\sqrt[3]{n}}$

17. Find the interval of convergence for $\displaystyle\sum_{n=1}^\infty \dfrac{(x+2)^n}{n4^n}$

18. Find the interval of convergence for $\displaystyle\sum_{n=1}^\infty \dfrac{2^n(x-3)^n}{\sqrt{n+3}}$

19. Find the Taylor's series expansion of $f(x) = e^{2x}$ for $a = 3$.

20. Find the MacLauren series expansion of $f(x) = x \cos x$.

21. Find $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$ for $f(x, y) = (5y^3 + 2x^2y)^8$

22. Find z_{xy} for $z = xe^{-2y}$.

23. Use Chain Rule to find $\dfrac{du}{dp}$ for $u = x^2y^3 + z^4$ where $x = p + 3p^2$, $y = pe^p$, $z = p \sin p$.

24. Suppose $z = f(x, y)$ with $x = g(s, t)$, $y = h(s, t)$, find $\dfrac{\partial z}{\partial s}$ and $\dfrac{\partial z}{\partial t}$ at the point $(s, t) = (1, 2)$ given you know the following information:

$$g(1, 2) = 3, \quad g_s(1, 2) = -1, \quad g_t(1, 2) = 4$$
$$h(1, 2) = 6, \quad h_s(1, 2) = -5, \quad h_t(1, 2) = 10$$
$$f_x(3, 6) = 7 \quad f_y(3, 6) = 8$$

25. Evaluate $\displaystyle\iint_R (6x^2y - 2x)\, dA$ for $R : 1 \le x \le 4,\ 0 \le y \le 2$

26. Evaluate $\displaystyle\iint_R x \cos y\, dA$ for R bounded by $y = 0, y = x^2, x = 1$.

27. Evaluate $\displaystyle\int_0^2 \int_0^{\sqrt{4-x^2}} e^{-x^2-y^2}\, dy dx$ using polar coordinates.

Answers

1. $-\frac{t}{2} \cos 2t + \frac{1}{4} \sin 2t$

2. $e^{\sin x} + C$

3. $\frac{2}{15}$

4. $\frac{7}{2} + \ln 2$

5. $\ln \left| \dfrac{\sqrt{x^2 + 1}}{x} - \dfrac{1}{x} \right| + C$

6. $-\frac{1}{2} \ln |x| + \frac{3}{2} \ln |x + 2| + C$

7. $\frac{1}{36}$

8. $r(t) = 5e^{t-t^2}$

9. $x = y^2 - 8y + 12$

10. $r = \dfrac{2}{\cos \theta + \sin \theta}$

11. converges

12. converges

13. diverges

14. diverges

15. converges absolutely

16. converges conditionally

17. $[-6, 2)$

18. $[2.5, 3.5)$

19. $\displaystyle\sum_{n=0}^{\infty} \frac{2^n e^6 (x-3)^n}{n!}$

20. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n)!}$

21. $f_x = 32xy(5y^3 + 2x^2y)^7, \quad f_y = (16x2 + 120y^2)(5y^3 + 2x^2y)^7$

22. $z_{xy} = -2e^{-2y}$

23. $2(p + 3p^2)(pe^p)^3(1 + 6p) + 3(p + 3p^2)^2(pe^p)^2(pe^p + e^p) + 4(p\sin p)^3(p\cos p + \sin p)$

24. $-47, 108$

25. 222

26. $\frac{1}{2}(1 - \cos 1)$

27. $\frac{\pi}{4}(1 - e^{-4})$