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Exam 1 Question 2 427J

2. Consider the following linear system:

$$\begin{aligned} -2x_1 - x_2 + 10x_3 &= 1, \\ 3x_1 + x_2 - 13x_3 &= 0, \\ -2x_1 + 6x_3 &= -2, \end{aligned} \rightarrow \left[\begin{array}{ccc|c} -2 & -1 & 10 & 1 \\ 3 & 1 & -13 & 0 \\ -2 & 0 & 6 & -2 \end{array}\right] = A$$

in variables x_1, x_2, and x_3. Write the general solution in parametric form.

(a) What is the RREF of the augmented matrix?

$\text{rref}(A) \rightarrow R_1 \leftrightarrow R_2 \left[\begin{array}{ccc|c} 3 & 1 & -13 & 0 \\ -2 & -1 & 10 & 1 \\ -2 & 0 & 6 & -2 \end{array}\right] R_1 + R_2 \left[\begin{array}{ccc|c} 1 & 0 & -3 & 1 \\ -2 & -1 & 10 & 1 \\ -2 & 0 & 6 & -2 \end{array}\right] R_2 + R_3 \left[\begin{array}{ccc|c} 1 & 0 & -3 & 1 \\ 0 & -1 & 4 & 3 \\ -2 & 0 & 6 & -2 \end{array}\right] -R_2 \; R_3 + 2R_1$

(b) What is the rank of the augmented matrix?

rank(A) = 2 due to the number of pivots in rref(A).

(c) Give the solution to the system in parametric form.

$x_1 - 3x_3 = 1 \rightarrow x_1 = 1 + 3x_3$

$-x_2 - 4x_3 = -3 \rightarrow x_2 = 3 + 4x_3$

$x_3 = x_3 \rightarrow x_3 = x_3$

$$\therefore \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 + 3x_3 \\ 3 + 4x_3 \\ x_3 \end{bmatrix} \xrightarrow{\text{Param.}} \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}$$

a) continued

$$\left[\begin{array}{ccc|c} 1 & 0 & -3 & 1 \\ 0 & 1 & -4 & -3 \\ 0 & 0 & 0 & 0 \end{array}\right] = \text{rref}(A)$$