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Exam 3 Question 1 427J

1. Use parametric form to produce a linear dependence relation for the set

$$S = \left\{ \begin{bmatrix} -1 \\ -2 \\ 3 \\ 5 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -2 \\ -1 \end{bmatrix}, \begin{bmatrix} -4 \\ -5 \\ 9 \\ 8 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 0 \\ -7 \end{bmatrix} \right\}$$

Let $A = \begin{bmatrix} -1 & 1 & -4 & -1 \\ -2 & 1 & -5 & 1 \\ 3 & -2 & 9 & 0 \\ 5 & -1 & 8 & -7 \end{bmatrix}$

$\text{rref}(A) = \begin{bmatrix} 1 & 0 & 1 & -2 \\ 0 & 1 & -3 & -3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ (x_3, x_4) $\longrightarrow$

$x_1 + x_3 - 2x_4 = 0 \quad x_1 = -x_3 + 2x_4$

$x_2 - 3x_3 - 3x_4 = 0 \longrightarrow x_2 = 3x_3 + 3x_4$

$x_3 = x_3 \quad x_3 = x_3$

$x_4 = x_4 \quad x_4 = x_4$

$$\text{Ker}(A) = \left\{ x_3 \begin{bmatrix} -1 \\ 3 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 2 \\ 3 \\ 0 \\ 1 \end{bmatrix} \;\middle|\; x_3, x_4 \in \mathbb{R} \right\}$$

Sols. to linear dependence relation for S is: $\left\{ \begin{bmatrix} -1 \\ 3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 0 \\ 1 \end{bmatrix} \right\}$

Verification:

$$\underbrace{(-1)\begin{bmatrix} -1 \\ -2 \\ 3 \\ 5 \end{bmatrix}}_{\begin{bmatrix} 1 \\ 2 \\ -3 \\ -5 \end{bmatrix}} + \underbrace{(3)\begin{bmatrix} 1 \\ 1 \\ -2 \\ -1 \end{bmatrix}}_{\begin{bmatrix} 3 \\ 3 \\ -6 \\ -3 \end{bmatrix}} + (1)\begin{bmatrix} -4 \\ -5 \\ 9 \\ 8 \end{bmatrix} + \cancel{(0)\begin{bmatrix} -1 \\ 1 \\ 0 \\ -7 \end{bmatrix}} = \vec{\emptyset}$$

$$\underbrace{\begin{bmatrix} 4 \\ 5 \\ -9 \\ -8 \end{bmatrix}} + \begin{bmatrix} -4 \\ -5 \\ 9 \\ 8 \end{bmatrix} = \vec{0}$$

$\vec{0} \overset{\checkmark}{=} \vec{0}$

$$\underbrace{(2)\begin{bmatrix} -1 \\ -2 \\ 3 \\ 5 \end{bmatrix}}_{\begin{bmatrix} -2 \\ -4 \\ 6 \\ 10 \end{bmatrix}} + \underbrace{(3)\begin{bmatrix} 1 \\ 1 \\ -2 \\ -1 \end{bmatrix}}_{\begin{bmatrix} 3 \\ 3 \\ -6 \\ -3 \end{bmatrix}} + \cancel{(0)\begin{bmatrix} -4 \\ -5 \\ 9 \\ 8 \end{bmatrix}} + (1)\begin{bmatrix} -1 \\ 1 \\ 0 \\ -7 \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} 1 \\ -1 \\ 0 \\ -7 \end{bmatrix}} + \begin{bmatrix} -1 \\ 1 \\ 0 \\ -7 \end{bmatrix} = \vec{0} \longrightarrow \vec{0} \overset{\checkmark}{=} \vec{0}$$