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Exam 2 Question 1 427J

1. Let

$$\vec{v} = \begin{bmatrix} -17 \\ -7 \\ -10 \\ 15 \end{bmatrix} \text{ and } S = \left\{ \begin{bmatrix} 5 \\ 2 \\ 3 \\ -4 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 1 \\ -3 \end{bmatrix}, \begin{bmatrix} -8 \\ -3 \\ -5 \\ 5 \end{bmatrix} \right\}.$$

Use parametric form to show $\vec{v} \in \text{Span}(S)$.

Setup matrix to solve:

Let $A = \left[\begin{array}{ccc|c} 5 & 2 & -8 & -17 \\ 2 & 1 & -3 & -7 \\ 3 & 1 & -5 & -10 \\ -4 & -3 & 5 & 15 \end{array}\right]$

rref(A): $\left[\begin{array}{ccc|c} 5 & 2 & -8 & -17 \\ 2 & 1 & -3 & -7 \\ 3 & 1 & -5 & -10 \\ -4 & -3 & 5 & 15 \end{array}\right] \xrightarrow[R_3 - R_2,\ R_4 + 2R_2]{R_1 + R_4} \left[\begin{array}{ccc|c} 1 & -1 & -3 & -2 \\ 2 & 1 & -3 & -7 \\ 1 & 0 & -2 & -3 \\ 0 & -1 & -1 & 1 \end{array}\right] \xrightarrow[R_3 - R_1]{R_2 - 2R_1} \left[\begin{array}{ccc|c} 1 & -1 & -3 & -2 \\ 0 & 3 & 3 & -3 \\ 0 & 1 & 1 & -1 \\ 0 & -1 & -1 & 1 \end{array}\right] \xrightarrow[R_3 + R_4]{\frac{1}{3} R_2} \left[\begin{array}{ccc|c} 1 & -1 & -3 & -2 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & -1 & 1 \end{array}\right] \begin{array}{l} R_1 + R_2 \\ \longrightarrow \text{newline} \\ \\ R_4 + R_2 \end{array}$

$\left[\begin{array}{ccc|c} 1 & 0 & -2 & -3 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] = \text{rref}(A)$; x_3 is free

$\left.\begin{array}{l} x_1 - 2x_3 = -3 \\ x_2 + x_3 = -1 \\ x_3 = x_3 \end{array}\right\} \begin{array}{l} x_1 = -3 + 2x_3 \\ x_2 = -1 - x_3 \\ x_3 = x_3 \end{array} \xrightarrow{\text{Param.}} \begin{bmatrix} -3 \\ -1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} = \text{solution}$

To verify this is a valid solution:

$-3 \cdot \begin{bmatrix} 5 \\ 2 \\ 3 \\ -4 \end{bmatrix} + (-1) \begin{bmatrix} 2 \\ 1 \\ 1 \\ -3 \end{bmatrix} + (0) \begin{bmatrix} -8 \\ -3 \\ -5 \\ 5 \end{bmatrix} \Rightarrow \vec{0} \Rightarrow \begin{bmatrix} -17 \\ -7 \\ -10 \\ 15 \end{bmatrix} = \vec{v}$

$= \begin{bmatrix} -15 \\ -6 \\ -9 \\ 12 \end{bmatrix} + \begin{bmatrix} -2 \\ -1 \\ -1 \\ 3 \end{bmatrix} = \begin{bmatrix} -17 \\ -7 \\ -10 \\ 15 \end{bmatrix} = \vec{v}$ ✓