NAME: Abdon Morales EID: am226923

Quiz 1 427J

1. Consider the O.D.E.

$$\frac{dy}{dt} + y = te^t$$

(a) What is the integrating factor, $\mu(t)$?

$\mu(t) = e^{\int a(t)\,dt} \rightarrow \boxed{\mu(t) = e^{\int dt} = e^t}$

(b) Use $\mu(t)$ to solve the O.D.E.

$\mu(t)\left(\frac{dy}{dt} + y\right) = (te^t)\mu(t) \rightarrow \mu(t)\frac{dy}{dt} + \mu(t)y = te^t \cdot \mu(t)$

$\rightarrow e^t\frac{dy}{dt} + e^t y = te^{2t} \rightarrow \frac{d}{dt}(e^t y) = \int te^{2t}\,dt + C$ (I.B.P)

I.B.P

Let $u = t$ $dv = te^{2t}dt$

$du = dt$ $v = \frac{1}{2}e^{2t}$

$\int te^{2t}dt = \frac{t}{2}e^{2t} - \frac{1}{2}\int e^{2t}dt$

$= \frac{t}{2}e^{2t} - \frac{1}{4}e^{2t}$

(c) Use your solution in part b to solve the I.V.P.

$$\frac{dy}{dt} + y = te^t, \quad y(0) = -1/4$$

$\frac{e^t y}{e^t} = \frac{\frac{t}{2}e^{2t} - \frac{1}{4}e^{2t} + C}{e^t}$

$\boxed{y(t) = \frac{te^t}{2} - \frac{e^t}{4} + \frac{C}{e^t}}$

Using my solution in part B:

$y(0) = -1/4 = \frac{0 \cdot e^0}{2} - \frac{e^0}{4} + \frac{C}{e^0}$

$-\frac{1}{4} = -\frac{1}{4} + C$

$+1/4 \quad +1/4$

$0 = C$

$\therefore \boxed{y(t) = \frac{te^t}{2} - \frac{e^t}{4}}$