

Name: Please print your name clearly in all capital letters in the boxes below.

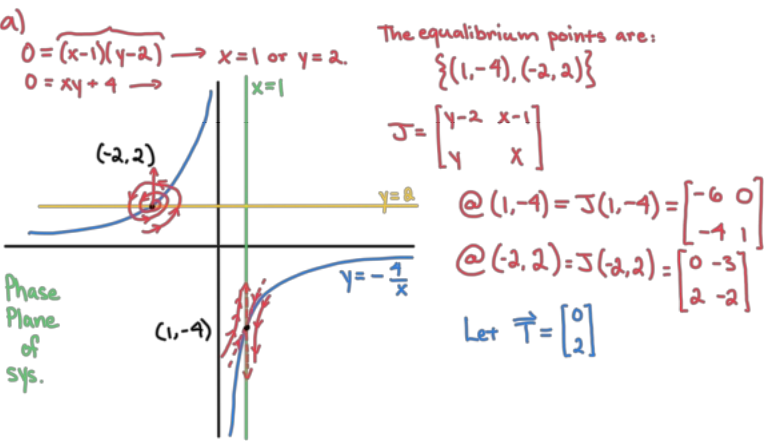
FIRST NAME	Abdon
LAST NAME	Morales

Quiz 427J

Consider the nonlinear system

$$\begin{aligned}x' &= (x - 1)(y - 2) \\ y' &= xy + 4\end{aligned}$$

- (a) For each equilibrium point, give (i) the equilibrium point, (ii) the linear system at that point, (iii) its eigenvalues, (iv) the real eigenvectors, and (v) the type of phase portrait.
- (b) Graph the phase plane of the nonlinear system.



@ $(1, -4), \frac{d}{dt} \vec{x} = \begin{bmatrix} -6 & 0 \\ -4 & 1 \end{bmatrix} \vec{x}$

$p(\lambda) = \det \begin{bmatrix} -6-\lambda & 0 \\ -4 & 1-\lambda \end{bmatrix} = (-6-\lambda)(1-\lambda)$
 $= (\lambda-1)(\lambda+6)$
 $\lambda = \{1, -6\}$ Saddle

$\lambda = 1: A - I = \begin{bmatrix} -7 & 0 \\ -4 & 0 \end{bmatrix} \xrightarrow{\text{RREF}} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \begin{cases} x=0 \\ y=y \end{cases}$
 $\therefore \vec{v}_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$\lambda = -6: A + 6I = \begin{bmatrix} 0 & 0 \\ -4 & 7 \end{bmatrix} \xrightarrow{\text{RREF}} \begin{bmatrix} 1 & -7/4 \\ 0 & 0 \end{bmatrix} = \begin{cases} x - 7/4 y = 0 \\ y=y \end{cases} \rightarrow x = 7/4 y$
 $\therefore \vec{v}_2 = \begin{bmatrix} 7/4 \\ 1 \end{bmatrix}$

@ $(-2, 2), \frac{d}{dt} \vec{x} = \begin{bmatrix} 0 & -3 \\ 2 & -2 \end{bmatrix} \vec{x}$

$p(\lambda) = \det \begin{bmatrix} -\lambda & -3 \\ 2 & -2-\lambda \end{bmatrix} = (-\lambda)(-2-\lambda) - (-3)(2) = \lambda^2 + 2\lambda + 6$

$\lambda = \frac{-2 \pm \sqrt{2^2 - 4(1)(6)}}{2} = \frac{-2 \pm \sqrt{4 - 24}}{2} = \frac{-2 \pm i\sqrt{20}}{2} = -1 \pm i\sqrt{5}$

$\lambda = -1 \pm i\sqrt{5}$ Spiral sink