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Quiz 7 427J

1. Solve the system

$$\frac{d}{dt}\vec{x} = A\,\vec{x} \text{ where } A = \begin{bmatrix} 11 & -12 \\ 6 & -7 \end{bmatrix}$$

$p(\lambda) = \det(A-\lambda I)$, where

$$\begin{array}{cc} 11 & -\lambda \\ -7\;-77 & 7\lambda \\ -\lambda\;-11\lambda & \lambda^2 \end{array}$$

$$A-\lambda I = \begin{bmatrix} 11 & -12 \\ 6 & -7 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = \begin{bmatrix} 11-\lambda & -12 \\ 6 & -7-\lambda \end{bmatrix}$$

$$\begin{aligned} \det(A-\lambda I) &= (11-\lambda)(-7-\lambda) - (-12)(6) \\ &= \lambda^2 - 4\lambda - 77 + 72 \\ &= \lambda^2 - 4\lambda - 5 \\ &= (\lambda-5)(\lambda+1) \\ &= \lambda = \left\{\begin{matrix} 5 \\ -1 \end{matrix}\right\} \text{ (eigenvals.)} \end{aligned}$$

Eigenvectors

$\lambda = 5$

$$A-\lambda I = \begin{bmatrix} 11 & -12 \\ 6 & -7 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 6 & -12 \\ 6 & -12 \end{bmatrix}$$

$$\operatorname{rref}\left(\begin{bmatrix} 6 & -12 \\ 6 & -12 \end{bmatrix}\right) = \begin{bmatrix} 1 & -2 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{cases} x - 2y = 0 \\ y = y \end{cases} \rightarrow \left.\begin{matrix} x = 2y \\ y = y \end{matrix}\right\} \Longrightarrow y\begin{bmatrix} 2 \\ 1 \end{bmatrix} \Longrightarrow \vec{v}_5 = \begin{bmatrix} 2 \\ 1 \end{bmatrix},\ \vec{x}^1 = e^{5t}\begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\boxed{\therefore\ \vec{x} = c_1 e^{5t}\begin{bmatrix} 2 \\ 1 \end{bmatrix} + c_2 e^{-t}\begin{bmatrix} 1 \\ 1 \end{bmatrix}}$$

$\lambda = -1$

$$A-\lambda I = \begin{bmatrix} 11 & -12 \\ 6 & -7 \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 12 & -12 \\ 6 & -6 \end{bmatrix}$$

$$\operatorname{rref}\left(\begin{bmatrix} 12 & -12 \\ 6 & -6 \end{bmatrix}\right) = \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix} \longrightarrow \begin{cases} x - y = 0 \\ y = y \end{cases} \longmapsto \left.\begin{matrix} x = y \\ y = y \end{matrix}\right\} \Rightarrow y\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\Longrightarrow \vec{v}_{-1} = \begin{bmatrix} 1 \\ 1 \end{bmatrix},\ \vec{x}^2 = e^{-t}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$