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Exam 2 Question 2 427J

2. Use the correct set notation to express the set $\ker(A)$ when

$$A = \begin{bmatrix} 5 & -20 & 2 & -3 & 23 \\ 2 & -8 & 1 & -2 & 10 \\ 5 & -20 & 3 & -7 & 27 \end{bmatrix}$$

$\text{rref}(A)$: $\begin{bmatrix} 5 & -20 & 2 & -3 & 23 \\ 2 & -8 & 1 & -2 & 10 \\ 5 & -20 & 3 & -7 & 27 \end{bmatrix} \xrightarrow[R_3-R_1]{R_1-2R_2} \begin{bmatrix} 1 & -4 & 0 & 1 & 3 \\ 2 & -8 & 1 & -2 & 10 \\ 0 & 0 & 1 & -4 & 4 \end{bmatrix} \xrightarrow{R_2-2R_1} \begin{bmatrix} 1 & -4 & 0 & 1 & 3 \\ 0 & 0 & 1 & -4 & 4 \\ 0 & 0 & 1 & -4 & 4 \end{bmatrix} \xrightarrow[R_3-R_2]{} \begin{bmatrix} 1 & -4 & 0 & 1 & 3 \\ 0 & 0 & 1 & -4 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

$\text{rref}(A) = \begin{bmatrix} 1 & -4 & 0 & 1 & 3 \\ 0 & 0 & 1 & -4 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

Based on the rref(A), we can declare x_2, x_4, and x_5 as free variables.

To continue:

$\text{rref}(A) \longrightarrow \begin{cases} x_1 - 4x_2 + x_4 + 3x_5 = 0 \\ x_2 = x_2 \\ x_3 - 4x_4 + 4x_5 = 0 \\ x_4 = x_4 \\ x_5 = x_5 \end{cases} \longmapsto \begin{cases} x_1 = 4x_2 - x_4 - 3x_5 \\ x_2 = x_2 \\ x_3 = 4x_4 - 4x_5 \\ x_4 = x_4 \\ x_5 = x_5 \end{cases}$

$$\text{Null}(A) = \ker(A) = \left\{ x_2 \begin{bmatrix} 4 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -1 \\ 0 \\ 4 \\ 1 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} -3 \\ 0 \\ -4 \\ 0 \\ 1 \end{bmatrix} \middle| \; x_2, x_4, x_5 \in \mathbb{R} \right\}$$