

NAME: \_\_\_\_\_ EID: \_\_\_\_\_

**Exam 4 427J**

1. (10 points) Consider the system

$$\frac{d}{dt}\vec{x} = A\vec{x} \quad \text{where} \quad A = \begin{bmatrix} 4 & 13 \\ -2 & -6 \end{bmatrix}$$

- (a) Compute the general solution to the system.  
(b) Compute the matrix exponential  $e^{At}$ .

2. (15 points) Consider the Heat Equation

$$u_t = \alpha^2 u_{xx}$$

$$u(x, 0) = \begin{cases} -1, & 0 < x \leq \pi \\ 1, & \pi < x < 2\pi \end{cases}$$

$$u(0, t) = u(2\pi, t) = 0, \quad t \geq 0$$

- (a) Give the **Sine Series** for  $u(x, 0)$ . Include the first four nonzero terms.  
**Each coefficient should be a single reduced fraction with no trig functions and no decimals.**
- (b) Give the solution to the Heat Equation,  $u(x, t)$ . Include the first four nonzero terms.  
**Each coefficient should be a single reduced fraction with no trig functions and no decimals.**