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**Quiz 6 427J**

Use parametric form to give a linear dependence relation for the set

$$\left\{ \begin{bmatrix} 3 \\ 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -14 \\ -11 \\ -12 \end{bmatrix} \right\}$$

Let $S$ be the set of vectors:

$S = \left\{ \begin{bmatrix} 3 \\ 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -14 \\ -11 \\ -12 \end{bmatrix} \right\}$

and let $A$ be the matrix of column vectors of set $S$.

$$A = \begin{bmatrix} 3 & 1 & -14 \\ 2 & 1 & -11 \\ 4 & 0 & -12 \end{bmatrix}$$

rref $(A)$:

$$\begin{bmatrix} 3 & 1 & -14 \\ 2 & 1 & -11 \\ 4 & 0 & -12 \end{bmatrix} R_1 - R_2 \begin{bmatrix} 1 & 0 & -3 \\ 2 & 1 & -11 \\ 4 & 0 & -12 \end{bmatrix} R_2 - 2R_1 \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & -5 \\ 4 & 0 & -12 \end{bmatrix} R_3 - 4R_1 \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & -5 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\text{rref}(A) = \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & -5 \\ 0 & 0 & 0 \end{bmatrix} \Longrightarrow \begin{cases} x_1 - 3x_3 = 0 \\ x_3 = x_3 \\ x_2 - 5x_3 = 0 \end{cases} \longrightarrow \begin{cases} x_1 = 3x_3 \\ x_2 = 5x_3 \\ x_3 = x_3 \end{cases}$$

$$\ker(A) = x_3 \begin{bmatrix} 3 \\ 5 \\ 1 \end{bmatrix} \longmapsto \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} 3 \\ 5 \\ 1 \end{bmatrix}, \; x_3 \in \mathbb{R} \right\}$$

Our answer

Verify the answer:

$$3\begin{bmatrix} 3 \\ 2 \\ 4 \end{bmatrix} + 5\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + (1)\begin{bmatrix} -14 \\ -11 \\ -12 \end{bmatrix} = \vec{0}$$

$$\begin{bmatrix} 9 \\ 6 \\ 12 \end{bmatrix} + \begin{bmatrix} 5 \\ 5 \\ 0 \end{bmatrix} + \begin{bmatrix} -14 \\ -11 \\ -12 \end{bmatrix} = \vec{0}$$

$$\begin{bmatrix} 14 \\ 11 \\ 12 \end{bmatrix} + \begin{bmatrix} -14 \\ -11 \\ -12 \end{bmatrix} = \vec{0}$$

$$\vec{0} \overset{\checkmark}{=} \vec{0}$$

$\therefore \left\{ x_3 \begin{bmatrix} 3 \\ 5 \\ 1 \end{bmatrix} \right\}$ demonstrates that it is our solution for the relation of linear dependence of the set $S$.