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Quiz 427J

1. Let

$$A = \begin{bmatrix} 1 & -4 \\ 1 & -3 \end{bmatrix}$$

(a) What is the general solution to the system

$$\frac{d}{dt}\vec{x} = A\,\vec{x}\ ?$$

(b) What is e^{At}?

a) $\frac{d}{dt}\vec{x} = A\vec{x}$

$$\begin{array}{ccc} & 1 & -\lambda \\ -3 & -3 & 3\lambda \\ -\lambda & -\lambda & \lambda^2 \end{array}$$

$\frac{d}{dt}\vec{x} = \begin{bmatrix} 1 & -4 \\ 1 & -3 \end{bmatrix}\vec{x}$

$p(\lambda) = \det\begin{bmatrix} 1-\lambda & -4 \\ 1 & -3-\lambda \end{bmatrix} = (1-\lambda)(-3-\lambda)+4$

$= \lambda^2 + 2\lambda - 3 + 4 = \lambda^2 + 2\lambda + 1$

$= (\lambda+1)(\lambda+1)$

$\therefore \lambda = -1$, double

$\lambda = -1$: $A + I$

$= \begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix} \xrightarrow{RREF} \begin{bmatrix} 1 & -2 \\ 0 & 0 \end{bmatrix} \mapsto \begin{cases} x - 2y = 0 \\ y = y \end{cases} \longmapsto \begin{cases} x = 2y \\ y = y \end{cases}$

$\equiv y\begin{bmatrix} 2 \\ 1 \end{bmatrix} \therefore \vec{v}_{-1} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \vec{x}^1 = e^{-t}\begin{bmatrix} 2 \\ 1 \end{bmatrix}$

Jordan Cycle:

Solve $(A - \lambda I)\vec{J} = \vec{v}_\lambda$

$\begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix}\vec{J} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$\Longrightarrow \left[\begin{array}{cc|c} 2 & -4 & 2 \\ 1 & -2 & 1 \end{array}\right] \xrightarrow{RREF} \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 0 & 0 \end{array}\right] \mapsto \begin{cases} x - 2y = 1 \\ y = y \end{cases} \rightarrow \begin{cases} x = 2y + 1 \\ y = y \end{cases}$

$\Rightarrow \vec{J} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad \equiv y\begin{bmatrix} 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$\Longrightarrow \vec{x}^2 = e^{\lambda t}(\vec{J} + t\vec{v}_\lambda)$

$= e^{-t}\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} 2 \\ 1 \end{bmatrix}\right)$

$\vec{x}^2 = e^{-t}\begin{bmatrix} 1+2t \\ t \end{bmatrix}$

$\boxed{\vec{X} = c_1 e^{-t}\begin{bmatrix} 2 \\ 1 \end{bmatrix} + c_2 e^{-t}\begin{bmatrix} 1+2t \\ t \end{bmatrix}}$ Gen. sol for $\frac{d}{dt}\vec{x} = A\vec{x}$

b) What is e^{At}? $\rightarrow \frac{1}{\det A}\cdot\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$e^{At} = X(t)\cdot(X(0))^{-1}$

$X(0) = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}$, $\det(X(0)) = -1$

$(X(0))^{-1} = (-1)\cdot\begin{bmatrix} 0 & -1 \\ -1 & 2 \end{bmatrix}$ $\quad X(t) = e^{-t}\begin{bmatrix} 2 & 1+2t \\ 1 & t \end{bmatrix}$

$= \begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix}$

$e^{At} = e^{-t}\begin{bmatrix} 2 & 1+2t \\ 1 & t \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix}$

$\boxed{= e^{-t}\begin{bmatrix} 2t+1 & -4t \\ t & 1-2t \end{bmatrix}}$ Sol. of e^{At}