

This print-out should have 5 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

001 10.0 points

Evaluate the expression

$$E = \begin{vmatrix} 1 & -3 \\ 2 & 2 \end{vmatrix} + 2 \begin{vmatrix} 1 & 2 \\ -3 & 2 \end{vmatrix}.$$

1. $E = 24$ **correct**

2. $E = 25$

3. $E = 22$

4. $E = 23$

5. $E = 26$

Explanation:

For a 2×2 determinant,

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc.$$

Thus

$$\begin{aligned} E &= \begin{vmatrix} 1 & -3 \\ 2 & 2 \end{vmatrix} + 2 \begin{vmatrix} 1 & 2 \\ -3 & 2 \end{vmatrix} \\ &= (2 + 6) + 2(2 + 6). \end{aligned}$$

Consequently,

$E = 24$

keywords: matrix, determinant

002 10.0 points

By evaluating the determinant, express

$$f(x) = \begin{vmatrix} 1 & x & x^2 \\ 0 & -4 & 2 \\ 4 & 1 & 0 \end{vmatrix}$$

as a quadratic function in x .

1. $f(x) = -2 + 16x + 8x^2$

2. $f(x) = -2 + 8x + 16x^2$ **correct**

3. $f(x) = 2 - 8x - 16x^2$

4. $f(x) = 2 + 8x - 16x^2$

5. $f(x) = -2 - 8x + 16x^2$

6. $f(x) = 2 - 16x - 8x^2$

Explanation:

For this 3×3 determinant, use expansion by minors along the top row:

$$f(x) = \begin{vmatrix} -4 & 2 \\ 1 & 0 \end{vmatrix} - \begin{vmatrix} 0 & 2 \\ 4 & 0 \end{vmatrix} x + \begin{vmatrix} 0 & -4 \\ 4 & 1 \end{vmatrix} x^2.$$

Evaluating the 2×2 determinants, we thus see that

$f(x) = -2 + 8x + 16x^2$

keywords: matrix, determinant, quadratic function, expansion by minors

003 10.0 points

Find the value of the determinant

$$D = \begin{vmatrix} 3 & x & -2 \\ 1 & y & -3 \\ -2 & z & 1 \end{vmatrix}.$$

1. $D = -5x + y - 7z$

2. $D = 5x + y - 7z$

3. $D = 5x + y + 7z$

4. $D = -5x - y + 7z$

5. $D = 5x - y + 7z$ **correct**

6. $D = -5x - y - 7z$

Explanation:

For any 3×3 determinant

$$\begin{vmatrix} A & B & C \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = A \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix} - B \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix} + C \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}.$$

Thus

$$\begin{aligned} D &= \begin{vmatrix} 3 & x & -2 \\ 1 & y & -3 \\ -2 & z & 1 \end{vmatrix} \\ &= 3 \begin{vmatrix} y & -3 \\ z & 1 \end{vmatrix} - x \begin{vmatrix} 1 & -3 \\ -2 & 1 \end{vmatrix} - 2 \begin{vmatrix} 1 & y \\ -2 & z \end{vmatrix} \\ &= 3(y3z) - x(-5) - 2(+z + 2y). \end{aligned}$$

Consequently,

$$\boxed{D = 5x - y + 7z}.$$

004 10.0 points

Find the value of the determinant

$$D = \begin{vmatrix} 3 & 2 & 1 \\ x & y & z \\ 1 & 2 & -3 \end{vmatrix}.$$

1. $D = 8x + 10y + 4z$
2. $D = -8x - 10y + 4z$
3. $D = 8x + 10y - 4z$
4. $D = 8x - 10y - 4z$ **correct**
5. $D = -8x - 10y - 4z$
6. $D = -8x + 10y + 4z$

Explanation:

For any 3×3 determinant

$$\begin{vmatrix} A & B & C \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = A \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix} - B \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix} + C \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}.$$

Thus

$$\begin{aligned} D &= \begin{vmatrix} 3 & 2 & 1 \\ x & y & z \\ 1 & 2 & -3 \end{vmatrix} \\ &= 3 \begin{vmatrix} y & z \\ 2 & -3 \end{vmatrix} - 2 \begin{vmatrix} x & z \\ 1 & -3 \end{vmatrix} + \begin{vmatrix} x & y \\ 1 & 2 \end{vmatrix} \\ &= 3(-3y - 2z) - 2(-3x - z) + (2x - y). \end{aligned}$$

Consequently,

$$\boxed{D = 8x - 10y - 4z}.$$

keywords: determinant

005 10.0 points

Find the value of the determinant

$$D = \begin{vmatrix} 1 & -1 & x \\ -2 & 3 & y \\ -3 & -2 & z \end{vmatrix}.$$

1. $D = 13x - 5y + z$
2. $D = 13x + 5y + z$ **correct**
3. $D = -13x - 5y - z$
4. $D = -13x + 5y - z$
5. $D = 13x - 5y - z$
6. $D = -13x + 5y + z$

Explanation:

For any 3×3 determinant

$$\begin{vmatrix} A & B & C \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = A \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix} \\ - B \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix} + C \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}.$$

Thus

$$D = \begin{vmatrix} 1 & -1 & x \\ -2 & 3 & y \\ -3 & -2 & z \end{vmatrix} \\ = \begin{vmatrix} 3 & y \\ -2 & z \end{vmatrix} + \begin{vmatrix} -2 & y \\ -3 & z \end{vmatrix} + x \begin{vmatrix} -2 & 3 \\ -3 & -2 \end{vmatrix} \\ = (3z + 2y) + (-2z + 3y) + x(13).$$

Consequently,

$$\boxed{D = 13x + 5y + z}.$$

keywords: determinant