Quiz 07/29

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Consider the following

$O: \{(x,y,z): x - y^2 = 1,\ z = y^2\}$

From point $(1,0,0)$ to $P(x_0, y_0, z_0)$. Let C denote this oriented path.

a) Find a parametrization for C, clearly indicating its domain.

Let $x = 1 + y^2$, $z = y^2$

$\boxed{C: C(y) = (1+y^2, y, y^2), \text{ from } y = 0 \text{ to } y = y_0.}$

$c'(y) = (2y, 1, 2y)$

b) Now, consider the force field given by

$$F(x,y,z) = (y, x-1, z - y^2)$$

Using local coordinates, find an expression for the work done by F to move a particle along the curve C going from $(1,0,0)$ to $P(x_0, y_0, z_0)$.

If $C(y) = (1+y^2, y, y^2)$ and $C'(y) = (2y, 1, 2y)$

$$\Longrightarrow \int_C y\,dx + x-1\,dy + z-y^2\,dz = \int_{y=0}^{y=y_0} y(2y) + y^2 + 0\cdot(2y)$$

$$= \int_{y=0}^{y_0} 3y^2\,dy \longrightarrow \left[y^3\right]_0^{y_0} = (y_0^3) - (0^3)$$

$$= \boxed{y_0^3}$$

c) Suppose our path is electrically charged with charge density given by:

$$\mu(x,y,z) = \sqrt{x + 3y^2 + 4z}.$$

Let's find the total charge between the points $(1,0,0)$ and $(2,-1,1)$. In other words, compute:

$$\int_C \mu\,ds,$$

where $ds = \sqrt{dx^2 + dy^2 + dz^2}$

$c(y) = (1+y^2, y, y^2),\ y \in [\overset{f}{0}, \overset{+}{-1}]$

$\mu(x,y,z) = \sqrt{x + 3y^2 + 4z} \longrightarrow \mu(c(y)) = \sqrt{(1+y^2) + 3y^2 + 4(y^2)}$

$= \sqrt{1 + 8y^2}$

$c'(y) = (2y, 1, 2y),\ \|c'(y)\| = \sqrt{(2y)^2 + 1^2 + (2y)^2}$

$= \sqrt{8y^2 + 1}$

$\Longrightarrow ds = \|c'(y)\|\,|dy|$

$$\therefore \int_C \mu\,ds = \int_0^{-1} \mu(c(y))\cdot\|c'(y)\|\,|dy| = \int_0^{-1} \sqrt{1+8y^2}\cdot\sqrt{8y^2+1}\,|dy|$$

$$= \int_0^{-1} 1 + 8y^2\,(-dy) \Longrightarrow \int_{-1}^{0} 1 + 8y^2\,dy = \left[y + \frac{8}{3}y^3\right]_{-1}^{0}$$

$= \boxed{\frac{11}{3}}$ Total charge