Quiz 6 – M427L

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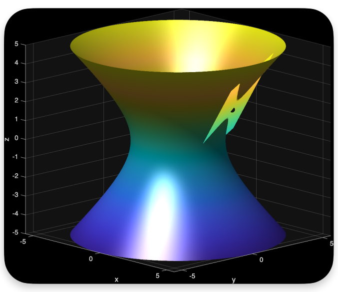
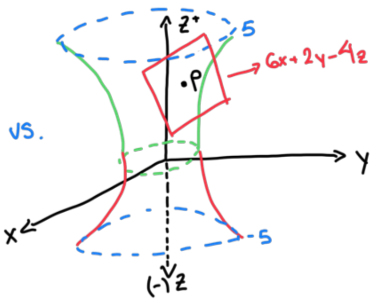
Q1: Consider the hyperboloid given by

$$x^2+y^2-z^2=6$$

Sketch the surface and the tangent plane at $P=(3,1,2)$.



vs.




$3^2+1^2-2^2=6$

$9+1-4$

$10-4=6$

$6=6$ ✓

Tangent Plane @ P.

$\therefore \boxed{6x+2y-4z=12}$

$\nabla f=(2x,2y,-2z)$

$\nabla f(P)=(6,2,-4)$

$0=\nabla f(P)\cdot(\Delta x,\Delta y)$

$6(x-3)+2(y-1)-4(z-2)=0$

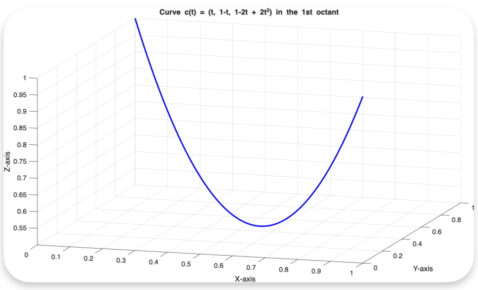
$6x-18+2y-2-4z+8=0$

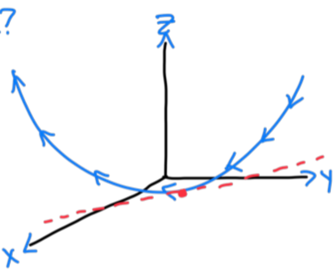
$6x+2y-4z-12=0$

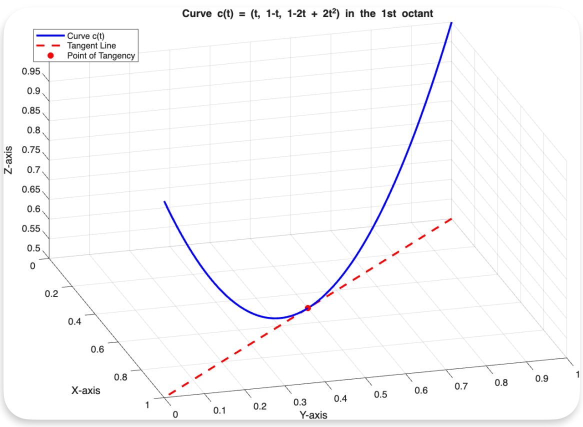
Q2: Consider the curve C parametrized by

$$C(t)=(t,1-t,1-2t+2t^2);\ t\in\mathbb{R}.$$

Sketch the portion of the curve in the 1st Octant, and compute $c'(1/2)$, which is a vector tangent to C at the point $c(1/2)=(1/2,1/2,1/2)$. Use the direction of the tangent vector $c'(1/2)$ to determine the orientation of C. Indicate this orientation by drawing arrows along the trace of the curve. Finally, sketch the tangent line to C at the point $c(1/2)=(1/2,1/2,1/2)$; what is the parametrization $L(t)$ for this tangent line?




1st Octant $\Longrightarrow \{x,y,z\geq 0\}$

$\rightarrow x(t)=t\geq 0$

$y(t)=1-t\geq 0\rightarrow t\leq 1$ $\Big\}\ 0\leq t\leq 1\rightarrow[0,1]$

$z(t)=1-2t+2t^2\equiv 2t^2-2t+1$

$z(t)\geq 0;[0,1]$ iff $T\rightarrow z(t)\geq 1/2>0\ \forall t\in[0,1]$

Vertex theorem: $\frac{-b}{2a}$ when $z(t)=at^2+bt+c$

$t=\frac{-(-2)}{2\cdot 2}=\frac{1}{2}$

$z(1/2)=2(1/2)^2-2(1/2)+1=1/2$

$C'(t)=(x'(t),y'(t))=x'(t)i+y'(t)j$

For $\mathbb{R}^3$, in this case, is $c't=x'(t)i+y'(t)j+z'(t)k$

$\therefore C'(t)=\langle 1,-1,-2+4t\rangle$

$C'(1/2)=\langle 1,-1,-2+4(1/2)\rangle=\langle 1,-1,0\rangle$

When $t=1/2$,

$\vec{v}=\langle 1,-1,0\rangle\rightarrow$ Increasing $x$, decreasing $y$

$l(t)=C(t_0)+(t-t_0)c'(t_0)$

$=(1/2,1/2,1/2)+(t-1/2)(1,-1,0)$

$(1/2,1/2,1/2)+(t-1/2,1/2-t,0)$

$\boxed{l(t)=(t,1-t,1/2)}$

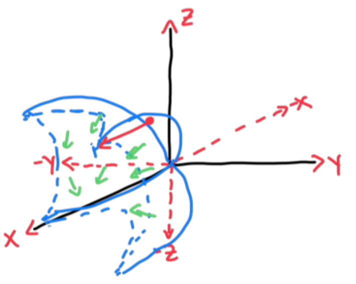
Q3: Consider the surface S given by $e^x=1+y^2+z^2$. Since we can solve for $x$, a parametrization for S is the smooth map $\Phi:\mathbb{R}^2\longrightarrow\mathbb{R}^3$ given by

$$\Phi(y,z)=(\ln(1+y^2+z^2),y,z)$$

Compute the tangent vectors $\Phi_y$ and $\Phi_z$. It should be clear from their expressions that one is not a multiple of the other, which means that $\Phi$ is a regular parametrization. Write down the expression for the nowhere-vanishing normal vector field:

$$N=\Phi_y\times\Phi_z$$

and compute $N(0,1)$ to find normal vector S at the point $P=\Phi(0,1)=\langle\ln(2),0,1\rangle$. Use this normal vector to find the cartesian equation for the tangent plane to S at the point P. Sketch surface S and indicate the orientation associated with our choice of N by drawing arrows on the surface. Would you call this orientation inward or outward?




ad-bc

$\frac{2y}{1+y^2+z^2}$

$-\frac{2z}{1+y^2+z^2}$

$\Phi_y(y,z)=\left(\frac{2y}{1+y^2+z^2},1,0\right)$

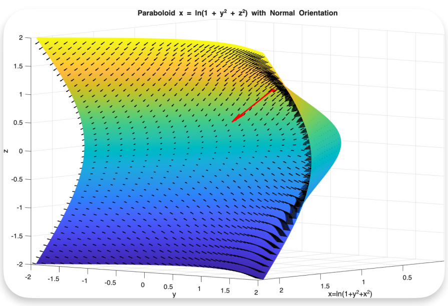
$\Phi_z(y,z)=\left(\frac{2z}{1+y^2+z^2},0,1\right)$

$N=\text{cross}(\Phi_y,\Phi_z)=\begin{vmatrix} i & j & k\\ \frac{2y}{1+y^2+z^2} & 1 & 0\\ \frac{2z}{1+y^2+z^2} & 0 & 1\end{vmatrix}=i\det\begin{pmatrix}1&0\\0&1\end{pmatrix}-j\det\begin{pmatrix}\frac{2y}{1+y^2+z^2}&0\\ \frac{2z}{1+y^2+z^2}&1\end{pmatrix}+k\det\begin{pmatrix}\frac{2y}{1+y^2+z^2}&1\\ \frac{2z}{1+y^2+z^2}&0\end{pmatrix}$

$i-j\frac{2y}{1+y^2+z^2}-k\frac{2z}{1+y^2+z^2}$

$N=\left(1,-\frac{2y}{1+y^2+z^2},-\frac{2z}{1+y^2+z^2}\right)$

$N(0,1)=\left(1,0,-\frac{2(1)}{1+0^2+1}\right)=(1,0,-1);\ P=\Phi(0,1)=(\ln 2,0,1)$




Cartesian $\rightarrow$ $1(x-\ln 2)+0(y-0)+(-1)(z-1)=x-\ln 2-z+1=0$

$\boxed{x-z=\ln 2-1}$ Inward Orientation