This print-out should have 23 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

---

001 10.0 points

Which one of the points

$$P(-3,\,-4,\,-4),\;\; Q(-6,\,8,\,9),\;\; R(-5,\,3,\,-3)$$

in 3-space is closest to the yz-plane?

1. $P(-3,\,-4,\,-4)$ **correct**

2. $R(-5,\,3,\,-3)$

3. $Q(-6,\,8,\,9)$

Explanation:

The distance of a point $(a,\,b,\,c)$ in 3-space from the yz-plane is given by $|a|$. Consequently, of the three points

$$P(-3,\,-4,\,-4),\;\; Q(-6,\,8,\,9),\;\; R(-5,\,3,\,-3)$$

the one closest to the yz-plane is

$$\boxed{P(-3,\,-4,\,-4)}\,.$$

---

keywords: plane, distance in 3-space,

---

002 10.0 points

A rectangular box is constructed in 3-space with one corner at the origin and other vertices at

$$(6, 0, 0), \quad (0, 3, 0), \quad (0, 0, 2)\,.$$

Find the length of the diagonal of the box.

1. length $= 49$

2. length $= \sqrt{22}$

3. length $= 54$

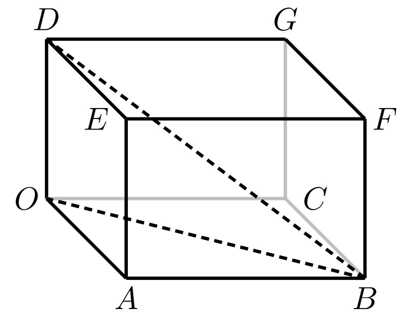
4. length $= 7$ **correct**

5. length $= 3\sqrt{6}$

6. length $= 22$

Explanation:

We have to find the length of BD in the figure



given that

$$OA = 6\,, \quad OC = 3\,, \quad OD = 2\,.$$

Now by Pythagoras' theorem,

$$\text{length } OB = \text{length } AC = 3\sqrt{5}\,.$$

But then, again by Pythagoras,

$$\text{length } BD = 7\,.$$

Consequently,

$$\boxed{\text{length} = 7}\,.$$

---

keywords: length diagonal, rectangular solid, Pythagoras' theorem, ThreeDimSys,

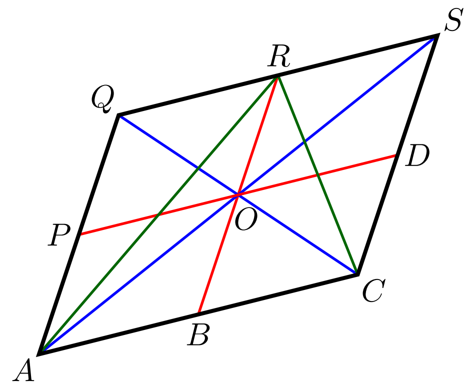
---

003 10.0 points

When $\mathbf{u}$, $\mathbf{v}$ are the displacement vectors

$$\mathbf{u} = \overrightarrow{AB}\,, \qquad \mathbf{v} = \overrightarrow{AP}\,,$$

determined by the parallelogram



express $\overrightarrow{QC}$ in terms of **u** and **v**, where P, B, D and R are the midpoints of $\overline{AQ}$, $\overline{AC}$, $\overline{CS}$ and $\overline{SQ}$, respectively.

1. $\overrightarrow{QC} = 2(\mathbf{u}+\mathbf{v})$
2. $\overrightarrow{QC} = 2(\mathbf{u}-\mathbf{v})$ **correct**
3. $\overrightarrow{QC} = \mathbf{u}+2\mathbf{v}$
4. $\overrightarrow{QC} = 2\mathbf{v}-\mathbf{u}$
5. $\overrightarrow{QC} = 2\mathbf{v}$
6. $\overrightarrow{QC} = 2\mathbf{u}$

Explanation:

By the parallelogram law for the addition of vectors we see that

$$\boxed{\overrightarrow{QC} = 2(\mathbf{u}-\mathbf{v})}.$$

---

keywords: vectors, linear combination, vector sum displacement vector, parallelogram

---

004 10.0 points

Determine the vector $\mathbf{c} = 2\mathbf{a} + \mathbf{b}$ when

$$\mathbf{a} = \langle 1, 3, 2 \rangle, \quad \mathbf{b} = \langle 2, 1, -1 \rangle.$$

1. $\mathbf{c} = \langle 3, 7, 4 \rangle$
2. $\mathbf{c} = \langle 4, 7, 4 \rangle$
3. $\mathbf{c} = \langle 3, 8, 4 \rangle$
4. $\mathbf{c} = \langle 4, 8, 3 \rangle$
5. $\mathbf{c} = \langle 4, 7, 3 \rangle$ **correct**
6. $\mathbf{c} = \langle 3, 8, 3 \rangle$

Explanation:

The sum of vectors

$$\mathbf{a} = \langle a_1, a_2, a_3 \rangle, \quad \mathbf{b} = \langle b_1, b_2, b_3 \rangle$$

is defined componentwise:

$$\mathbf{a}+\mathbf{b} = \langle a_1+b_1,\ a_2+b_2, a_3+b_3 \rangle;$$

similarly, multiplication by a scalar λ also is defined componentwise:

$$\lambda\mathbf{a} = \langle \lambda a_1,\ \lambda a_2, \lambda a_3 \rangle.$$

Consequently, when

$$\mathbf{a} = \langle 1, 3, 2 \rangle, \quad \mathbf{b} = \langle 2, 1, -1 \rangle,$$

we see that

$$\boxed{\mathbf{c} = \langle 4, 7, 3 \rangle}.$$

---

005 10.0 points

Determine the vector $\mathbf{c} = \mathbf{a} + 2\mathbf{b}$ when

$$\mathbf{a} = 3\mathbf{i} + 2\mathbf{j} + \mathbf{k}, \quad \mathbf{b} = 2\mathbf{i} + \mathbf{j} + 2\mathbf{k}.$$

1. $\mathbf{c} = 8\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$
2. $\mathbf{c} = 7\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}$ **correct**
3. $\mathbf{c} = 8\mathbf{i} - 3\mathbf{j} - 4\mathbf{k}$
4. $\mathbf{c} = 8\mathbf{i} + 4\mathbf{j} - 4\mathbf{k}$
5. $\mathbf{c} = 7\mathbf{i} + 4\mathbf{j} - 4\mathbf{k}$
6. $\mathbf{c} = 7\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$

Explanation:

The sum of vectors

$$\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}, \quad \mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$$

is defined componentwise:

$$\mathbf{a}+\mathbf{b} = (a_1+b_1)\mathbf{i} + (a_2+b_2)\mathbf{j} + (a_3+b_3)\mathbf{k};$$

similarly, multiplication by a scalar λ also is defined componentwise:

$$\lambda\mathbf{a} = (\lambda a_1)\mathbf{i} + (\lambda a_2)\mathbf{j} + (\lambda a_3)\mathbf{k}.$$

When

$$\mathbf{a} = 3\mathbf{i} + 2\mathbf{j} - \mathbf{k}, \quad \mathbf{b} = 2\mathbf{i} - \mathbf{j} + 2\mathbf{k},$$

therefore, we see that

$$\mathbf{c} = \Big((1)(3) + (2)(2)\Big)\mathbf{i} + \Big((1)(2) + (2)(1)\Big)\mathbf{j} + \Big((1)(1) + (2)(2)\Big)\mathbf{k}.$$

Consequently,

$$\boxed{\mathbf{c} = 7\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}}.$$

---

006 10.0 points

Determine the length of the vector $-2\mathbf{a}+\mathbf{b}$ when

$$\mathbf{a} = \langle 1, 2, -1 \rangle, \quad \mathbf{b} = \langle -3, -1, -2 \rangle.$$

1. length $= 2\sqrt{11}$

2. length $= \sqrt{46}$

3. length $= 4\sqrt{3}$

4. length $= 5\sqrt{2}$ **correct**

5. length $= 2\sqrt{13}$

Explanation:

The length, $|\mathbf{c}|$, of the vector

$$\mathbf{c} = \langle c_1, c_2, c_3 \rangle$$

is defined by

$$|\mathbf{c}| = \sqrt{c_1^2 + c_2^2 + c_3^2}.$$

Consequently, when

$$\mathbf{a} = \langle 1, 2, -1 \rangle, \quad \mathbf{b} = \langle -3, -1, -2 \rangle,$$

and

$$\mathbf{c} = -2\mathbf{a} + \mathbf{b} = \langle -5, -5, 0 \rangle,$$

we see that

$$\boxed{|-2\mathbf{a} + \mathbf{b}| = 5\sqrt{2}}.$$

---

007 10.0 points

Find all scalars λ so that $\lambda(\mathbf{a}+2\mathbf{b})$ is a unit vector when

$$\mathbf{a} = \langle 1, 2 \rangle, \quad \mathbf{b} = \langle 1, -2 \rangle.$$

1. $\lambda = -\frac{1}{13}$

2. $\lambda = \frac{1}{13}$

3. $\lambda = -\frac{1}{\sqrt{13}}$

4. $\lambda = \frac{1}{\sqrt{13}}$

5. $\lambda = \pm\frac{1}{13}$

6. $\lambda = \pm\frac{1}{\sqrt{13}}$ **correct**

Explanation:

A vector

$$\mathbf{c} = \langle c_1, c_2 \rangle$$

is said to be a unit vector when

$$|\mathbf{c}| = \sqrt{c_1^2 + c_2^2} = 1.$$

But for the given vectors $\mathbf{a}$ and $\mathbf{b}$,

$$\lambda(\mathbf{a} + 2\mathbf{b}) = \lambda\langle 3, -2 \rangle = \langle 3\lambda, -2\lambda \rangle.$$

Thus

$$|\lambda(\mathbf{a} + 2\mathbf{b})| = \sqrt{\lambda^2((3)^2 + (-2)^2)} = |\lambda|\sqrt{(3)^2 + (-2)^2} = |\lambda|\sqrt{13}.$$

Consequently, $\lambda(\mathbf{a} + 2\mathbf{b})$ will be a unit vector if and only if

$$\boxed{\lambda = \pm\frac{1}{\sqrt{13}}}.$$

---

keywords: vector sum, length, linear combination, unit vector,

008 10.0 points

Find a unit vector $\mathbf{n}$ with the same direction as the vector

$$\mathbf{v} = 3\,\mathbf{i} + 6\,\mathbf{j} - 2\,\mathbf{k}\,.$$

1. $\mathbf{n} = \frac{3}{7}\,\mathbf{i} + \frac{6}{7}\,\mathbf{j} - \frac{2}{7}\,\mathbf{k}$ **correct**

2. $\mathbf{n} = \frac{1}{3}\,\mathbf{i} - \frac{2}{3}\,\mathbf{j} + \frac{2}{9}\,\mathbf{k}$

3. $\mathbf{n} = \frac{1}{3}\,\mathbf{i} + \frac{2}{3}\,\mathbf{j} - \frac{2}{9}\,\mathbf{k}$

4. $\mathbf{n} = \frac{3}{10}\,\mathbf{i} - \frac{3}{5}\,\mathbf{j} + \frac{1}{5}\,\mathbf{k}$

5. $\mathbf{n} = \frac{3}{7}\,\mathbf{i} - \frac{6}{7}\,\mathbf{j} + \frac{2}{7}\,\mathbf{k}$

6. $\mathbf{n} = \frac{3}{10}\,\mathbf{i} + \frac{3}{5}\,\mathbf{j} - \frac{1}{5}\,\mathbf{k}$

Explanation:

The vector

$$\mathbf{v} = 3\,\mathbf{i} + 6\,\mathbf{j} - 2\,\mathbf{k}\,.$$

has length

$$\|\mathbf{v}\| = \sqrt{3^2 + 6^2 + 2^2} = \sqrt{49} = 7\,.$$

Consequently,

$$\boxed{\mathbf{n} = \frac{\mathbf{v}}{\|\mathbf{v}\|} = \frac{3}{7}\,\mathbf{i} + \frac{6}{7}\,\mathbf{j} - \frac{2}{7}\,\mathbf{k}}$$

is a unit vector with the same direction as $\mathbf{v}$.

009 10.0 points

Determine the dot product of the vectors

$$\mathbf{a} = \langle -1\,, -2\,, 3\,\rangle\,, \quad \mathbf{b} = \langle\, 1\,, -3\,, 1\,\rangle\,.$$

1. $\mathbf{a}\cdot\mathbf{b} = 6$

2. $\mathbf{a}\cdot\mathbf{b} = 2$

3. $\mathbf{a}\cdot\mathbf{b} = 8$ **correct**

4. $\mathbf{a}\cdot\mathbf{b} = 4$

5. $\mathbf{a}\cdot\mathbf{b} = 0$

Explanation:

The dot product, $\mathbf{a}\cdot\mathbf{b}$, of vectors

$$\mathbf{a} = \langle\, a_1\,, a_2\,, a_3\,\rangle\,, \quad \mathbf{b} = \langle\, b_1\,, b_2\,, b_3\,\rangle$$

is defined by

$$\mathbf{a}\cdot\mathbf{b} = a_1\,b_1 + a_2\,b_2 + a_3\,b_3\,.$$

Consequently, when

$$\mathbf{a} = \langle -1\,, -2\,, 3\,\rangle\,, \quad \mathbf{b} = \langle\, 1\,, -3\,, 1\,\rangle\,,$$

we see that

$$\boxed{\mathbf{a}\cdot\mathbf{b} = 8}\,.$$

010 10.0 points

Determine the dot product of the vectors

$$\mathbf{a} = 2\mathbf{i} + \mathbf{j} - 3\mathbf{k}, \quad \mathbf{b} = 3\mathbf{i} + 2\mathbf{j} - \mathbf{k}\,.$$

1. $\mathbf{a}\cdot\mathbf{b} = 9$

2. $\mathbf{a}\cdot\mathbf{b} = 13$

3. $\mathbf{a}\cdot\mathbf{b} = 15$

4. $\mathbf{a}\cdot\mathbf{b} = 17$

5. $\mathbf{a}\cdot\mathbf{b} = 11$ **correct**

Explanation:

The dot product, $\mathbf{a}\cdot\mathbf{b}$, of vectors

$$\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}\,, \quad \mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$$

is defined by

$$\mathbf{a}\cdot\mathbf{b} = a_1\,b_1 + a_2\,b_2 + a_3\,b_3\,.$$

Consequently, when

$$\mathbf{a} = 2\mathbf{i} + \mathbf{j} - 3\mathbf{k}, \quad \mathbf{b} = 3\mathbf{i} + 2\mathbf{j} - \mathbf{k}\,,$$

we see that

$$\boxed{\mathbf{a}\cdot\mathbf{b} \;=\; 11}\,.$$

---

011 10.0 points

Determine the dot product of vectors **a**, **b** when

$$|\mathbf{a}| \;=\; 3\,, \qquad |\mathbf{b}| \;=\; 6$$

and the angle between **a** and **b** is $\pi/3$.

1. $\mathbf{a}\cdot\mathbf{b} \;=\; \dfrac{19}{2}$

2. $\mathbf{a}\cdot\mathbf{b} \;=\; 9$ **correct**

3. $\mathbf{a}\cdot\mathbf{b} \;=\; 10$

4. $\mathbf{a}\cdot\mathbf{b} \;=\; \dfrac{21}{2}$

5. $\mathbf{a}\cdot\mathbf{b} \;=\; \dfrac{17}{2}$

Explanation:

The dot product of vectors **a**, **b** is defined in coordinate-free form by

$$\mathbf{a}\cdot\mathbf{b} \;=\; |a|\,|b|\,\cos\theta$$

where θ is the angle between **a** and **b** . For the given vectors, therefore,

$$\boxed{\mathbf{a}\cdot\mathbf{b} \;=\; 18\,\cos\frac{\pi}{3} \;=\; 9}\,.$$

---

012 10.0 points

Find the angle between the vectors

$$\mathbf{a} \;=\; \langle\, -2\sqrt{3},\, 1\,\rangle\,, \quad \mathbf{b} \;=\; \langle\, -3\sqrt{3},\, -5\,\rangle\,.$$

1. angle $\;=\; \dfrac{\pi}{4}$

2. angle $\;=\; \dfrac{5\pi}{6}$

3. angle $\;=\; \dfrac{3\pi}{4}$

4. angle $\;=\; \dfrac{\pi}{6}$

5. angle $\;=\; \dfrac{2\pi}{3}$

6. angle $\;=\; \dfrac{\pi}{3}$ **correct**

Explanation:

Since the dot product of vectors **a** and **b** can be written as

$$\mathbf{a}.\mathbf{b} \;=\; \|\mathbf{a}\|\,\|\mathbf{b}\|\cos\theta\,, \quad 0 \le \theta \le \pi,$$

where θ is the angle between the vectors, we see that

$$\cos\theta \;=\; \frac{\mathbf{a}.\mathbf{b}}{\|\mathbf{a}\|\,\|\mathbf{b}\|}\,, \quad 0 \le \theta \le \pi\,.$$

But for the given vectors,

$$\mathbf{a}\cdot\mathbf{b} \;=\; (-2\sqrt{3})(-3\sqrt{3}) + (1)(-5) \;=\; 13\,,$$

while

$$\|\mathbf{a}\| \;=\; \sqrt{13}\,, \qquad \|\mathbf{b}\| \;=\; \sqrt{52}\,.$$

Consequently,

$$\cos\theta \;=\; \frac{13}{\sqrt{13}\cdot 2\sqrt{13}} \;=\; \frac{1}{2}$$

where $0 \le \theta \le \pi$. Thus

$$\boxed{\text{angle} \;=\; \frac{\pi}{3}}\,.$$

---

013 10.0 points

Which, if any, of the following pairs of vectors are perpendicular?

I. $\langle\, 3,\, 2\,\rangle\,, \quad \langle\, 4,\, -6\,\rangle\,,$

II. $\mathbf{i} + 5\mathbf{j} - 2\mathbf{k}\,, \quad 3\mathbf{i} - 2\mathbf{j} - 4\mathbf{k}\,.$

1. both of them

2. II only

3. I only **correct**

4. neither of them

Explanation:

I Since the dot product

$$\langle 3, 2\rangle\cdot\langle 4, -6\rangle = (3)(4) + (2)(-6) = 0,$$

the vectors are perpendicular.

II Since the dot product

$$(\mathbf{i} + 5\mathbf{j} - 2\mathbf{k}) \cdot (3\mathbf{i} - 2\mathbf{j} - 4\mathbf{k}) = 1,$$

the vectors are not perpendicular.

---

014 10.0 points

Find the scalar projection of **b** onto **a** when

$$\mathbf{b} = \langle -5, 4\rangle, \qquad \mathbf{a} = \langle 4, -3\rangle.$$

1. scalar projection $= -\dfrac{31}{5}$

2. scalar projection $= -\dfrac{32}{5}$ **correct**

3. scalar projection $= -\dfrac{33}{5}$

4. scalar projection $= -\dfrac{29}{5}$

5. scalar projection $= -6$

Explanation:

The scalar projection of **b** onto **a** is given in terms of the dot product by

$$\text{comp}_{\mathbf{a}}\mathbf{b} = \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|}.$$

Now when

$$\mathbf{b} = \langle -5, 4\rangle, \qquad \mathbf{a} = \langle 4, -3\rangle,$$

we see that

$$\mathbf{a}\cdot\mathbf{b} = -32, \qquad |\mathbf{a}| = \sqrt{(4)^2 + (-3)^2}.$$

Consequently,

$$\boxed{\text{comp}_{\mathbf{a}}\mathbf{b} = -\frac{32}{5}}.$$

---

keywords:

---

015 10.0 points

Find the scalar projection of **b** onto **a** when

$$\mathbf{b} = 2\,\mathbf{i} + \mathbf{j} + 3\,\mathbf{k}, \quad \mathbf{a} = 2\,\mathbf{i} - 2\,\mathbf{j} - \mathbf{k}.$$

1. scalar projection $= \dfrac{2}{3}$

2. scalar projection $= -\dfrac{1}{3}$ **correct**

3. scalar projection $= 0$

4. scalar projection $= 1$

5. scalar projection $= \dfrac{1}{3}$

Explanation:

The scalar projection of **b** onto **a** is given in terms of the dot product by

$$\text{comp}_{\mathbf{a}}\mathbf{b} = \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|}.$$

Now when

$$\mathbf{b} = 2\,\mathbf{i} + \mathbf{j} + 3\,\mathbf{k}, \quad \mathbf{a} = 2\,\mathbf{i} - 2\,\mathbf{j} - \mathbf{k},$$

we see that

$$\mathbf{a}\cdot\mathbf{b} = -1, \qquad |\mathbf{a}| = \sqrt{(2)^2 + (-2)^2 + (-1)^2}.$$

Consequently,

$$\boxed{\text{comp}_{\mathbf{a}}\mathbf{b} = -\frac{1}{3}}.$$

---

keywords:

---

016 10.0 points

Find the vector projection of **b** onto **a** when

$$\mathbf{b} = \langle -2, -1\rangle, \qquad \mathbf{a} = \langle -1, -3\rangle.$$

1. vector proj. $= \dfrac{7}{10}\langle -1, -3\rangle$

2. vector proj. $= \dfrac{1}{2}\langle -2, -1 \rangle$

3. vector proj. $= \dfrac{7}{\sqrt{10}}\langle -1, -3 \rangle$

4. vector proj. $= \dfrac{1}{2}\langle -1, -3 \rangle$ **correct**

5. vector proj. $= \dfrac{7}{\sqrt{10}}\langle -2, -1 \rangle$

6. vector proj. $= \dfrac{5}{\sqrt{10}}\langle -2, -1 \rangle$

Explanation:

The vector projection of **b** onto **a** is given in terms of the dot product by

$$\text{proj}_{\mathbf{a}}\mathbf{b} = \left(\frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|^2}\right)\mathbf{a}.$$

But when

$$\mathbf{b} = \langle -2, -1 \rangle, \qquad \mathbf{a} = \langle -1, -3 \rangle,$$

we see that

$$\mathbf{a}\cdot\mathbf{b} = (-1)(-2) + (-3)(-1) = 5,$$

while

$$|\mathbf{a}|^2 = (-1)^2 + (-3)^2 = 10.$$

Consequently,

$$\boxed{\text{proj}_{\mathbf{a}}\mathbf{b} = \frac{1}{2}\langle -1, -3 \rangle}.$$

---

keywords:

---

017 10.0 points

Find the vector projection of **b** onto **a** when

$$\mathbf{b} = 2\,\mathbf{i} + 2\,\mathbf{j} + \mathbf{k}, \quad \mathbf{a} = 3\,\mathbf{i} - \mathbf{j} + 2\,\mathbf{k}.$$

1. vector proj. $= \dfrac{14}{9}(2\,\mathbf{i} + 2\,\mathbf{j} + \mathbf{k})$

2. vector proj. $= \dfrac{2}{3}(2\,\mathbf{i} + 2\,\mathbf{j} + \mathbf{k})$

3. vector proj. $= \dfrac{2}{3}(3\,\mathbf{i} - \mathbf{j} + 2\,\mathbf{k})$

4. vector proj. $= \dfrac{14}{9}(3\,\mathbf{i} - \mathbf{j} + 2\,\mathbf{k})$

5. vector proj. $= \dfrac{3}{7}(3\,\mathbf{i} - \mathbf{j} + 2\,\mathbf{k})$ **correct**

6. vector proj. $= \dfrac{3}{7}(2\,\mathbf{i} + 2\,\mathbf{j} + \mathbf{k})$

Explanation:

The vector projection of **b** onto **a** is given in terms of the dot product by

$$\text{proj}_{\mathbf{a}}\mathbf{b} = \left(\frac{\mathbf{a}\cdot\mathbf{b}}{\|\mathbf{a}\|^2}\right)\mathbf{a}.$$

Now when

$$\mathbf{b} = 2\,\mathbf{i} + 2\,\mathbf{j} + \mathbf{k}, \quad \mathbf{a} = 3\,\mathbf{i} - \mathbf{j} + 2\,\mathbf{k}.$$

we see that

$$\mathbf{a}\cdot\mathbf{b} = (3)(2) + (-1)(2) + (2)(1) = 6,$$

while

$$\|\mathbf{a}\|^2 = (3)^2 + (-1)^2 + (2)^2 = 14.$$

Consequently,

$$\boxed{\text{proj}_{\mathbf{a}}\mathbf{b} = \frac{3}{7}(3\,\mathbf{i} - \mathbf{j} + 2\,\mathbf{k})}.$$

---

keywords: vector projection, vectors in space,

---

018 10.0 points

Find the value of the determinant

$$D = \begin{vmatrix} 1 & 2 & -1 \\ -3 & -2 & -2 \\ -1 & 1 & -3 \end{vmatrix}.$$

1. $D = -3$

2. $D = -1$ **correct**

3. $D = 5$

4. $D = 3$

5. $D = 1$

Explanation:

For any 3×3 determinant

$$\begin{vmatrix} A & B & C \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = A \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix} - B \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix} + C \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}.$$

Thus

$$D = \begin{vmatrix} 1 & 2 & -1 \\ -3 & -2 & -2 \\ -1 & 1 & -3 \end{vmatrix}$$

$$= \begin{vmatrix} -2 & -2 \\ 1 & -3 \end{vmatrix} - 2 \begin{vmatrix} -3 & -2 \\ -1 & -3 \end{vmatrix} - \begin{vmatrix} -3 & -2 \\ -1 & 1 \end{vmatrix}$$

$$= (-2)(-3) - (1)(-2) - 2\left((-3)(-3) - (-1)(-2)\right) - \left((-3)(1) - (-1)(-2)\right).$$

Consequently,

$$\boxed{D = -1}.$$

keywords: determinant

019 10.0 points

Find the cross product of the vectors

$$\mathbf{a} = 2\mathbf{i} - 3\mathbf{j} - 2\mathbf{k}, \quad \mathbf{b} = \mathbf{i} + \mathbf{j} + 2\mathbf{k}.$$

1. $\mathbf{a} \times \mathbf{b} = -4\mathbf{i} - 5\mathbf{j} + 5\mathbf{k}$

2. $\mathbf{a} \times \mathbf{b} = -3\mathbf{i} - 6\mathbf{j} + 4\mathbf{k}$

3. $\mathbf{a} \times \mathbf{b} = -3\mathbf{i} - 5\mathbf{j} + 4\mathbf{k}$

4. $\mathbf{a} \times \mathbf{b} = -4\mathbf{i} - 6\mathbf{j} + 4\mathbf{k}$

5. $\mathbf{a} \times \mathbf{b} = -3\mathbf{i} - 6\mathbf{j} + 5\mathbf{k}$

6. $\mathbf{a} \times \mathbf{b} = -4\mathbf{i} - 6\mathbf{j} + 5\mathbf{k}$ **correct**

Explanation:

One way of computing the cross product

$$(2\mathbf{i} - 3\mathbf{j} - 2\mathbf{k}) \times (\mathbf{i} + \mathbf{j} + 2\mathbf{k})$$

is to use the fact that

$$\mathbf{i} \times \mathbf{j} = \mathbf{k}, \quad \mathbf{j} \times \mathbf{k} = \mathbf{i}, \quad \mathbf{k} \times \mathbf{i} = \mathbf{j},$$

while

$$\mathbf{i} \times \mathbf{i} = 0, \quad \mathbf{j} \times \mathbf{j} = 0, \quad \mathbf{k} \times \mathbf{k} = 0.$$

For then

$$\boxed{\mathbf{a} \times \mathbf{b} = -4\mathbf{i} - 6\mathbf{j} + 5\mathbf{k}}.$$

Alternatively, we can use the definition

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -3 & -2 \\ 1 & 1 & 2 \end{vmatrix}$$

$$= \begin{vmatrix} -3 & -2 \\ 1 & 2 \end{vmatrix} \mathbf{i} - \begin{vmatrix} 2 & -2 \\ 1 & 2 \end{vmatrix} \mathbf{j} + \begin{vmatrix} 2 & -3 \\ 1 & 1 \end{vmatrix} \mathbf{k}$$

to determine $\mathbf{a} \times \mathbf{b}$.

020 10.0 points

Find the cross product of the vectors

$$\mathbf{a} = \langle -1, 1, 3 \rangle, \quad \mathbf{b} = \langle 1, -3, 1 \rangle.$$

1. $\mathbf{a} \times \mathbf{b} = \langle 11, 4, 1 \rangle$

2. $\mathbf{a} \times \mathbf{b} = \langle 11, -7, 1 \rangle$

3. $\mathbf{a} \times \mathbf{b} = \langle 11, 4, 2 \rangle$

4. $\mathbf{a} \times \mathbf{b} = \langle 10, 4, 2 \rangle$ **correct**

5. $\mathbf{a} \times \mathbf{b} = \langle 10, -7, 1 \rangle$

6. $\mathbf{a} \times \mathbf{b} = \langle 10, -7, 2 \rangle$

Explanation:

By definition

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & 1 & 3 \\ 1 & -3 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 3 \\ -3 & 1 \end{vmatrix} \mathbf{i} - \begin{vmatrix} -1 & 3 \\ 1 & 1 \end{vmatrix} \mathbf{j} + \begin{vmatrix} -1 & 1 \\ 1 & -3 \end{vmatrix} \mathbf{k}.$$

Consequently,

$$\boxed{\mathbf{a} \times \mathbf{b} = \langle 10, 4, 2 \rangle}.$$

keywords: vectors, cross product

021 10.0 points

Determine all unit vectors $\mathbf{v}$ orthogonal to

$$\mathbf{a} = \mathbf{i} + 4\mathbf{j} + 3\mathbf{k}, \quad \mathbf{b} = 2\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}.$$

1. $\mathbf{v} = \pm\left(\frac{3}{7}\mathbf{i} - \frac{6}{7}\mathbf{j} - \frac{2}{7}\mathbf{k}\right)$

2. $\mathbf{v} = -\frac{6}{7}\mathbf{i} + \frac{3}{7}\mathbf{j} - \frac{2}{7}\mathbf{k}$

3. $\mathbf{v} = -3\mathbf{i} + 6\mathbf{j} + 2\mathbf{k}$

4. $\mathbf{v} = -\frac{3}{7}\mathbf{i} + \frac{6}{7}\mathbf{j} - \frac{2}{7}\mathbf{k}$

5. $\mathbf{v} = \pm\left(\frac{6}{7}\mathbf{i} - \frac{3}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}\right)$ **correct**

6. $\mathbf{v} = -6\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$

Explanation:

The non-zero vectors orthogonal to $\mathbf{a}$ and $\mathbf{b}$ are all of the form

$$\mathbf{v} = \lambda(\mathbf{a} \times \mathbf{b}), \quad \lambda \neq 0,$$

with λ a scalar. The only unit vectors orthogonal to $\mathbf{a}$, $\mathbf{b}$ are thus

$$\mathbf{v} = \pm\frac{\mathbf{a} \times \mathbf{b}}{\|\mathbf{a} \times \mathbf{b}\|}.$$

But for the given vectors $\mathbf{a}$ and $\mathbf{b}$,

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 4 & 3 \\ 2 & 6 & 3 \end{vmatrix}$$

$$= \begin{vmatrix} 4 & 3 \\ 6 & 3 \end{vmatrix} \mathbf{i} - \begin{vmatrix} 1 & 3 \\ 2 & 3 \end{vmatrix} \mathbf{j} + \begin{vmatrix} 1 & 4 \\ 2 & 6 \end{vmatrix} \mathbf{k}$$

$$= -6\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}.$$

In this case,

$$\|\mathbf{a} \times \mathbf{b}\|^2 = 49.$$

Consequently,

$$\boxed{\mathbf{v} = \pm\left(\frac{6}{7}\mathbf{i} - \frac{3}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}\right)}.$$

022 10.0 points

Find the area of the triangle having vertices

$$P(-3, -1), \quad Q(-2, -2), \quad R(3, 3).$$

1. area $= 5$ **correct**

2. area $= \frac{9}{2}$

3. area $= 4$

4. area $= 6$

5. area $= \frac{11}{2}$

Explanation:

To use vectors we shall identify a line segment with the corresponding directed line segment.

Since the area of the parallelogram having adjacent edges $\overline{PQ}$ and $\overline{PR}$ is given by

$$|\overrightarrow{PQ} \times \overrightarrow{PR}|,$$

ΔPQR has

$$\text{area} = \frac{1}{2}|\overrightarrow{PQ} \times \overrightarrow{PR}|\,.$$

Now

$$\overrightarrow{PQ} = \langle 1, -1, 0\rangle\,, \quad \overrightarrow{PR} = \langle 6, 4, 0\rangle\,.$$

But then

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -1 & 0 \\ 6 & 4 & 0 \end{vmatrix} = \begin{vmatrix} 1 & -1 \\ 6 & 4 \end{vmatrix} \mathbf{k}\,.$$

Consequently, ΔPQR has

$$\boxed{\text{area} = 5}\,.$$

---

keywords: vectors, cross product area, triangle, parallelogram

---

023 10.0 points

Find a vector $\mathbf{v}$ orthogonal to the plane through the points

$$P(5, 0, 0), \quad Q(0, 4, 0), \quad R(0, 0, 2)\,.$$

1. $\mathbf{v} = \langle 8, 5, 20\rangle$

2. $\mathbf{v} = \langle 8, 2, 20\rangle$

3. $\mathbf{v} = \langle 4, 10, 20\rangle$

4. $\mathbf{v} = \langle 8, 10, 20\rangle$ **correct**

5. $\mathbf{v} = \langle 2, 10, 20\rangle$

Explanation:

Because the plane through P, Q, R contains the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$, any vector $\mathbf{v}$ orthogonal to both of these vectors (such as their cross product) must therefore be orthogonal to the plane.

Here

$$\overrightarrow{PQ} = \langle -5, 4, 0\rangle\,, \quad \overrightarrow{PR} = \langle -5, 0, 2\rangle\,.$$

Consequently,

$$\boxed{\mathbf{v} = \overrightarrow{PQ} \times \overrightarrow{PR} = \langle 8, 10, 20\rangle}$$

is othogonal to the plane through P, Q and R.