This print-out should have 5 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

001 10.0 points

Find $f'(x)$ when

$$f(x) = \frac{1}{\sqrt{4x - x^2}}.$$

1. $f'(x) = \dfrac{x-2}{(4x-x^2)^{3/2}}$ **correct**

2. $f'(x) = \dfrac{x-2}{(4x-x^2)^{1/2}}$

3. $f'(x) = \dfrac{x-2}{(x^2-4x)^{3/2}}$

4. $f'(x) = \dfrac{2-x}{(x^2-4x)^{1/2}}$

5. $f'(x) = \dfrac{2-x}{(x^2-4x)^{3/2}}$

6. $f'(x) = \dfrac{2-x}{(4x-x^2)^{3/2}}$

Explanation:
By the Chain Rule,

$$f'(x) = -\frac{1}{2(4x-x^2)^{3/2}}(4-2x).$$

Consequently,

$$\boxed{f'(x) = \frac{x-2}{(4x-x^2)^{3/2}}}.$$

002 10.0 points

Find all the critical points of f when

$$f(x) = \frac{x}{x^2+4}.$$

1. $x = -4, 4$

2. $x = -2, 0$

3. $x = -4, 2$

4. $x = -2, 4$

5. $x = -2, 2$ **correct**

6. $x = 0, 2$

Explanation:
By the Quotient Rule,

$$f'(x) = \frac{(x^2+4) - 2x^2}{(x^2+4)^2}$$

$$= \frac{4-x^2}{(x^2+4)^2}.$$

Since f is differentiable everywhere, the only critical points occur at the solutions of $f'(x) = 0$, *i.e.*, at the solutions of

$$4 - x^2 = 0.$$

Consequently, the only critical points are

$$\boxed{x = -2, 2}.$$

003 10.0 points

Determine the absolute maximum value of

$$f(x) = \sin(x) - \cos^2(x)$$

on $[0, 2\pi]$.

1. abs. max. value $= -\dfrac{3}{4}$

2. abs. max. value $= \dfrac{5}{4}$

3. abs. max. value $= 1$ **correct**

4. abs. max. value $= \dfrac{3}{4}$

5. abs. max. value $= -1$

6. abs. max. value $= -\dfrac{5}{4}$

Explanation:
By the Chain Rule,

$$f'(x) = \cos(x) + 2\cos(x)\sin(x).$$

Thus

$$f'(x) = \cos(x)(2\sin(x) + 1).$$

Since f is differentiable everywhere, the critical points are the solutions of

$$f'(x) = \cos(x)(2\sin(x) + 1) = 0,$$

so in $(0, 2\pi)$ the critical points of f are

$$x = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}.$$

Now the absolute maximum value of f on $[0, 2\pi]$ occurs either at an endpoint or at a critical point. But

$$f(0) = -1, \ f\left(\frac{\pi}{2}\right) = 1, \ f\left(\frac{7\pi}{6}\right) = -\frac{5}{4},$$

while

$$f\left(\frac{3\pi}{2}\right) = -1, \qquad f\left(\frac{11\pi}{6}\right) = -\frac{5}{4},$$

and $f(2\pi) = -1$. Consequently,

$$\boxed{\text{abs. max. value} = 1}.$$

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004 10.0 points

Find the absolute minimum value of

$$f(x) = \frac{1}{3}x^3 - 4x^2 + 7x + 9$$

on the interval $[0, 3]$.

1. none of the other answers

2. abs. min. value $= 1$

3. abs. min. value $= 2$

4. abs. min. value $= 4$

5. abs. min. value $= 3$ **correct**

Explanation:
The absolute minimum value of f on $[0, 3]$ occurs either at an endpoint of $[0, 3]$ or at a critical point of f in $(0, 3)$. Now

$$f'(x) = x^2 - 8x + 7 = (x - 1)(x - 7),$$

so the critical points of f occur at $x = 1, 7$. But only $x = 1$ lies in $(0, 3)$. On the other hand,

$$f(0) = 9, \quad f(1) = \frac{37}{3}, \quad f(3) = 3.$$

Consequently,

$$\boxed{\text{abs. min. value} = 3}.$$

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005 10.0 points

Determine the absolute maximum value of

$$f(x) = \frac{3 + 2x}{x^2 + 4}$$

on the interval $[-1, 2]$.

1. none of the other answers

2. abs max $= \dfrac{1}{5}$

3. abs max $= \dfrac{7}{8}$

4. abs max $= \dfrac{3}{2}$

5. abs max $= 1$ **correct**

Explanation:
By the Quotient Rule

$$f'(x) = \frac{2x^2 + 8 - 6x - 4x^2}{(x^2 + 4)^2} = \frac{8 - 6x - 2x^2}{(x^2 + 4)^2}$$

for all x. Hence the critical points x_1, x_2 of f are the solutions of the equation

$$2x^2 + 6x - 8 \;=\; 0\,.$$

But

$$2x^2 + 6x - 8 \;=\; 2(x-1)(x+4),$$

so $x_1 = 1$ which lies inside $[-1, 2]$, while $x_2 = -4$ which lies outside the interval $[-1, 2]$. Thus the absolute maximum of f on this interval is attained at the point $x = 1$, $x = -1$ or $x = 2$. Computing the values of f at these points and comparing values we see that the absolute maximum of f on $[-1, 2]$ is

$$\boxed{\text{abs max} \;=\; 1}\,.$$