M 427L - Quiz 8

Optimization

July 16th, 2025

Suppose the temperature (measured in °F) in the elliptic ball

$$W=\{(x,y,z): x^2+4y^2+z^2\le 4\}$$

is given by

$$T(x,y,z)=x^2-2x+y^2-z^2+70$$

a) Find the points located in the interior of W that are candidates for max/min of our temperature function. (Critical points)

$\nabla T=(2x-2,\ 2y,\ -2z)$

$$\nabla T=0 \Longrightarrow \begin{cases} 2x-2=0 \\ 2y=0 \\ -2z=0 \end{cases} \longrightarrow \begin{cases} x=1 \\ y=0 \\ z=0 \end{cases}$$

$W=(1^2+0^2+0^2<4)$

$W=(1\le 4)\longrightarrow T$

$\therefore$ $(1,0,0)$ lies in the int W by T

b) Now, use the method of Lagrange multipliers to find the points in the boundary of W that are candidates for max and min of our temperature function.

$\nabla T=\lambda\nabla G$, where $G(x,y,z)=x^2+4y^2+z^2-4=0$

$$\begin{bmatrix}2x-2\\2y\\-2z\end{bmatrix}=\lambda\begin{bmatrix}2x\\8y\\2z\end{bmatrix}$$

$$\begin{bmatrix}2x-2\\2y\\-2z\end{bmatrix}=\begin{bmatrix}2\lambda x\\8\lambda y\\2\lambda z\end{bmatrix}=\begin{cases}2x-2=2\lambda x\\2y=8\lambda y\\-2z=2\lambda z\end{cases}\longrightarrow$$

$y(4-3x)=0$

$z(2x-1)=0$

$x^2+4y^2+z^2=0$

$2(x-1)=2\lambda x \longrightarrow x-1=\lambda x$

$x(2y)=(8\lambda y)x$

$2xy=8\lambda xy$

$2xy=8(x-1)y$

$2xy-8(x-1)y=0$ $\rightarrow y(4-3x)=0$

$y(2x-(8(x-1)))=0$

$(2x-8x+8)=0$

$y(8-6x)=0$

$x(-2z)=(2\lambda z)x$

$-2xz=2\lambda xz$

$-2xz=2z(x-1)$

$-2xz-2z(x-1)=0$

$z(-2x-2(x-1))=0$

$z(-2x-2x+2)$

$z(-4x+2)=0$

$-2z(2x-1)=0$

$z(2x-1)=0$

$y=0$

- $z=0$: $x^2=4$, $x=\pm 2$, $\therefore (\pm 2,0,0)$
- $2x-1=0$: $x=1/2$, $(1/2)^2+z^2=4$, $z=\pm\sqrt{\frac{15}{2}}$, $\therefore (1/2,\ 0,\ \pm\sqrt{15/2})$

$4-3x=0$

- $x=\frac{4}{3}$, $z=0$, $\therefore (4/3,\ \pm\sqrt{\frac{5}{3}},\ 0)$

c) Compile the data from a and b to determine the max and min temperatures alongside the points where those temperatures are attained?

(x,y,z)	$T(x,y,z)$
$(1,0,0)$	69
$(-2,0,0)$	78
$(2,0,0)$	70
$(1/2,0,+\sqrt{\frac{15}{2}})$	65.5
$(1/2,0,-\sqrt{\frac{15}{2}})$	65.5
$(4/3,+\sqrt{\frac{5}{3}},0)$	$69.\overline{667}$
$(4/3,-\sqrt{\frac{5}{3}},0)$	$69.\overline{667}$

MAX = 78

MIN = 65.5