This print-out should have 16 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

## 001 10.0 points

Which of the following statements are true for all lines and planes in 3-space?

I. *two lines parallel to a third line are parallel*,

II. *two planes perpendicular to a third plane are parallel*,

III. *two lines perpendicular to a plane are parallel.*

1. I and III only
2. all of them
3. I only
4. I and II only
5. II and III only
6. none of them
7. II only
8. III only

## 002 10.0 points

Determine all unit vectors $\mathbf{v}$ orthogonal to

$$\mathbf{a} = 3\,\mathbf{i} + \mathbf{j} + 4\,\mathbf{k}, \quad \mathbf{b} = 3\,\mathbf{i} + 2\,\mathbf{j} + 6\,\mathbf{k}\,.$$

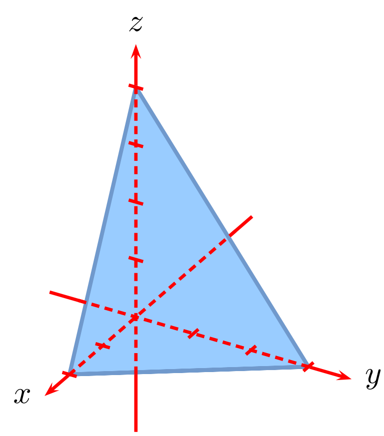
1. $\mathbf{v} = \pm\left(\frac{2}{7}\,\mathbf{i} + \frac{6}{7}\,\mathbf{j} - \frac{3}{7}\,\mathbf{k}\right)$
2. $\mathbf{v} = \frac{2}{7}\,\mathbf{i} - \frac{3}{7}\,\mathbf{j} + \frac{6}{7}\,\mathbf{k}$
3. $\mathbf{v} = 2\,\mathbf{i} - 3\,\mathbf{j} + 6\,\mathbf{k}$
4. $\mathbf{v} = \pm\left(\frac{2}{7}\,\mathbf{i} - \frac{3}{7}\,\mathbf{j} + \frac{6}{7}\,\mathbf{k}\right)$
5. $\mathbf{v} = -\frac{2}{7}\,\mathbf{i} - \frac{6}{7}\,\mathbf{j} + \frac{3}{7}\,\mathbf{k}$
6. $\mathbf{v} = -2\,\mathbf{i} - 6\,\mathbf{j} + 3\,\mathbf{k}$

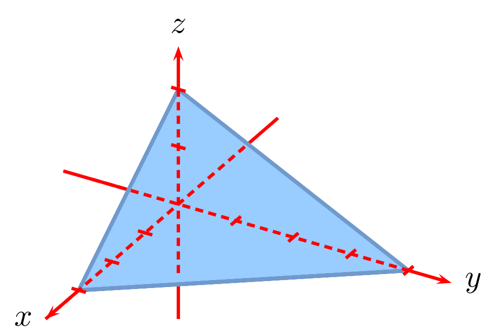
## 003 10.0 points

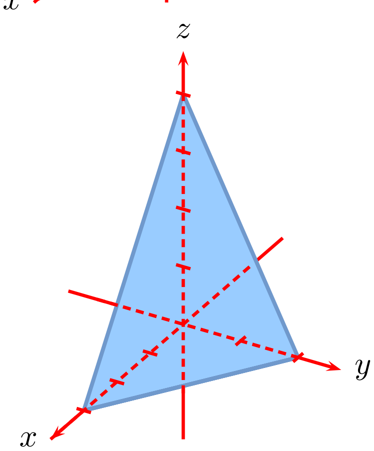
Which of the following surfaces is the graph of

$$6x + 4y + 3z = 12$$

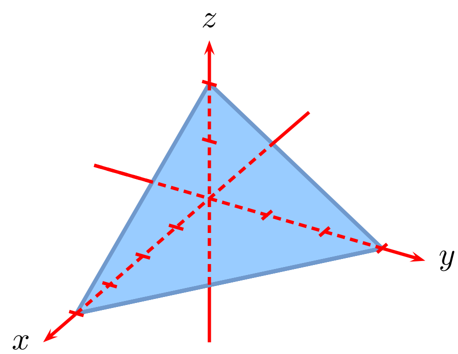
in the first octant?

1. 

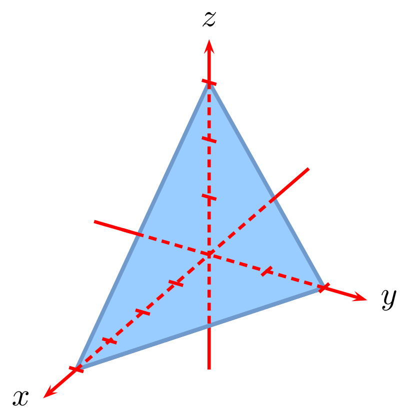

2. 


3. 

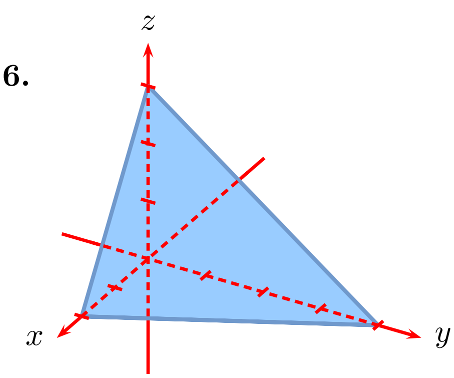
**4.**




**5.**




**6.**




**004 10.0 points**

Find parametric equations for the line passing through the point $P(3, -2, 3)$ and perpendicular to the plane

$$x + 3y - 2z = 6 .$$

**1.** $x = 3 - t,\ \ y = 2 - 3t,\ \ z = 3 - 2t$

**2.** $x = 3 + t,\ \ y = -2 + 3t,\ \ z = 3 - 2t$

**3.** $x = 1 - 3t,\ \ y = -3 + 2t,\ \ z = -2 + 3t$

**4.** $x = 1 + 3t,\ \ y = 3 + 2t,\ \ z = 2 - 3t$

**5.** $x = -3 + t,\ \ y = 2 + 3t,\ \ z = -3 - 2t$

**6.** $x = 1 + 3t,\ \ y = 3 - 2t,\ \ z = -2 + 3t$

**005 10.0 points**

Find parametric equations for the line through the point $P(5, 5, 4)$ that is parallel to the plane $x + y + z = 3$ and perpendicular to the line

$$x = 3 + t,\ \ y = 4 - t,\ \ z = 3t .$$

**1.** $x = 5 + 4t,\ \ y = 5 + 2t,\ \ z = 4 + t$

**2.** $x = 5 - 4t,\ \ y = 5 + 2t,\ \ z = 4 + t$

**3.** $x = 5 - 4t,\ \ y = 5 + 2t,\ \ z = 4 - 2t$

**4.** $x = 5 + 4t,\ \ y = 5 - 2t,\ \ z = 4 - 2t$

**5.** $x = 5 + t,\ \ y = 5 + t,\ \ z = 4 - 2t$

**006 10.0 points**

Find an equation for the plane passing through the points

$$Q(-2, -1, -1)\,, \qquad R(0, -2, -1)\,, \qquad S(-5, -1, -3)\,.$$

**1.** $2x - 4y + 3z + 5 = 0$

**2.** $2x + 4y - 3z + 5 = 0$

**3.** $2x + 3y - 4z - 5 = 0$

**4.** $2x + 4y - 3z - 5 = 0$

**5.** $2x - 3y - 4z - 5 = 0$

**6.** $2x - 3y - 4z + 5 = 0$

**007 10.0 points**

Find an equation for the plane passing through the point $P(-1, -1, -1)$ and parallel to the plane

$$3x + 2y + z = 4 .$$

**1.** $x + 3y + 2z = -6$

**2.** $2x + y + 3z = -10$

**3.** $x + 3y + 2z = -10$

**4.** $3x + 2y + z = -10$

**5.** $3x + 2y + z = -6$

**6.** $2x + y + 3z = -6$

### 008 10.0 points

Determine as a linear relation in $x$, $y$, $z$ the plane given in vector form by

$$\mathbf{x} = \mathbf{a} + u\,\mathbf{b} + v\,\mathbf{c}$$

when

$$\mathbf{a} = \mathbf{i} - \mathbf{j} + 2\mathbf{k}, \quad \mathbf{b} = 2\mathbf{i} - \mathbf{j} - 2\mathbf{k}$$

and

$$\mathbf{c} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}\,.$$

**1.** $7x + 4y + 5z - 13 = 0$

**2.** $3x + 4y - 5z - 13 = 0$

**3.** $7x - 4y - 5z - 13 = 0$

**4.** $3x - 4y + 5z + 13 = 0$

**5.** $3x - 4y - 5z + 13 = 0$

**6.** $7x + 4y + 5z + 13 = 0$

### 009 10.0 points

Describe the motion of a particle with position $P(x, y)$ when

$$x = 5\sin t\,, \quad y = 4\cos t$$

as $t$ varies in the interval $0 \le t \le 2\pi$.

**1.** Moves once counterclockwise along the ellipse

$$\frac{x^2}{25} + \frac{y^2}{16} = 1\,,$$

starting and ending at (0, 4).

**2.** Moves along the line

$$\frac{x}{5} + \frac{y}{4} = 1\,,$$

starting at (0, 4) and ending at (5, 0).

**3.** Moves along the line

$$\frac{x}{5} + \frac{y}{4} = 1\,,$$

starting at (5, 0) and ending at (0, 4).

**4.** Moves once clockwise along the ellipse

$$(5x)^2 + (4y)^2 = 1\,,$$

starting and ending at (0, 4).

**5.** Moves once counterclockwise along the ellipse

$$(5x)^2 + (4y)^2 = 1\,,$$

starting and ending at (0, 4).

**6.** Moves once clockwise along the ellipse

$$\frac{x^2}{25} + \frac{y^2}{16} = 1\,,$$

starting and ending at (0, 4).

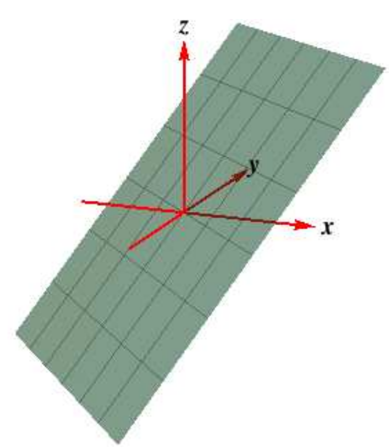
### 010 10.0 points

For which one of the following surfaces is

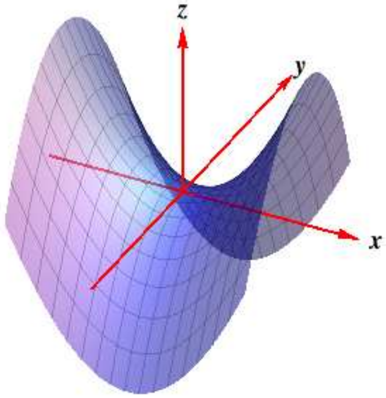
$$\boldsymbol{\Phi}(u, v) = u\cos v\,\mathbf{i} + u^2\,\mathbf{j} + u\sin v\,\mathbf{k}$$

a parametrization?

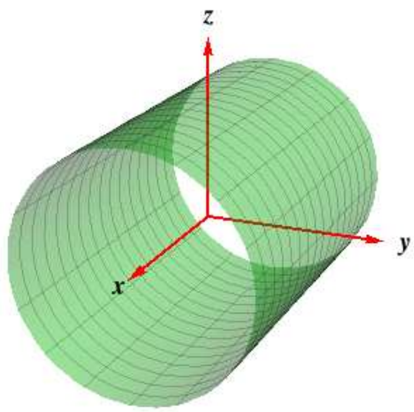
**1.**



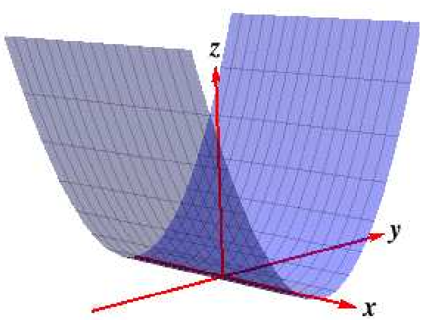
**2.**



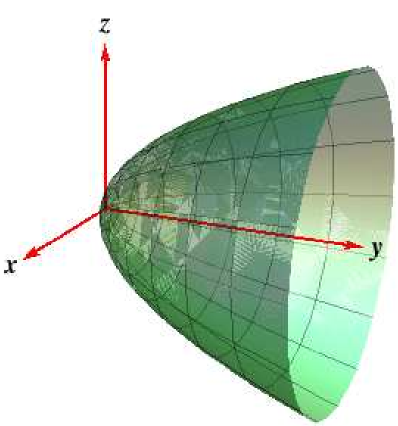

**3.**



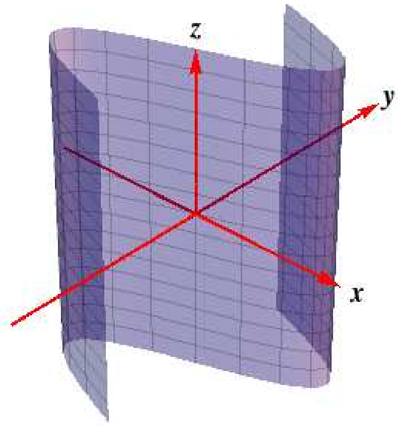

**4.**




**5.**




**6.**

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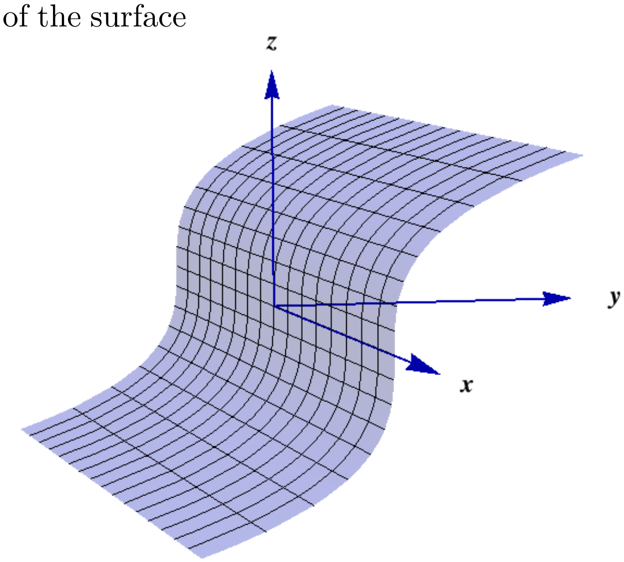
**011 10.0 points**

Which of the following is a parametrization of the surface




1. $\mathbf{\Phi} = (u,\, u+v,\, v)$

2. $\mathbf{\Phi} = (\cos u \sin v,\, 3\cos u \sin v,\, \cos v)$

3. $\mathbf{\Phi} = (u,\, u\cos v,\, u \sin v)$

4. $\mathbf{\Phi} = (u,\, \cos v,\, \sin v)$

5. $\mathbf{\Phi} = (u,\, v^3,\, v)$

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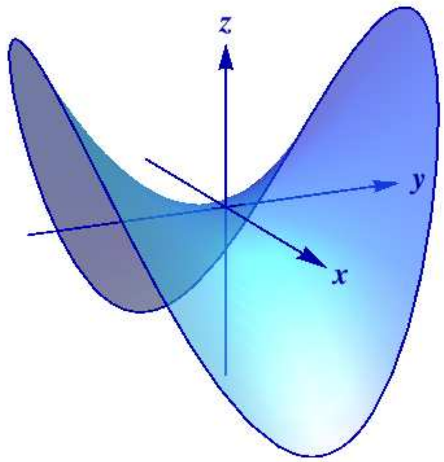
**012 10.0 points**

Express the graph of

$$z = y^2 - x^2\,, \quad x^2 + y^2 \le 9\,,$$

shown in




as a surface parametrized in terms of cylindrical polar coordinates.

**1.** For $0 \le r \le 3, \;\; 0 \le \theta \le 2\pi$,
$\mathbf{\Phi}(r,\,\theta) = (r\cos\theta,\, r\sin\theta,\, -r^2\sin 2\theta)$

**2.** For $0 \le r \le 3, \;\; 0 \le \theta \le 2\pi$,
$\mathbf{\Phi}(r,\,\theta) = (r\cos\theta,\, r\sin\theta,\, -r^2\cos 2\theta)$

**3.** For $0 \le r \le 0, \;\; 0 \le \theta \le 2\pi$,
$\mathbf{\Phi}(r,\,\theta) = (r\cos\theta,\, r\sin\theta,\, r^2\cos 2\theta)$

**4.** For $0 \le r \le 3, \;\; 0 \le \theta \le 2\pi$,
$\mathbf{\Phi}(r,\,\theta) = (r\cos\theta,\, r\sin\theta,\, r^2\cos 2\theta)$

**5.** For $0 \le r \le 0, \;\; 0 \le \theta \le 2\pi$,
$\mathbf{\Phi}(r,\,\theta) = (r\cos\theta,\, r\sin\theta,\, r^2\sin 2\theta)$

**6.** For $0 \le r \le 0, \;\; 0 \le \theta \le 2\pi$,
$\mathbf{\Phi}(r,\,\theta) = (r\cos\theta,\, r\sin\theta,\, -r^2\sin 2\theta)$

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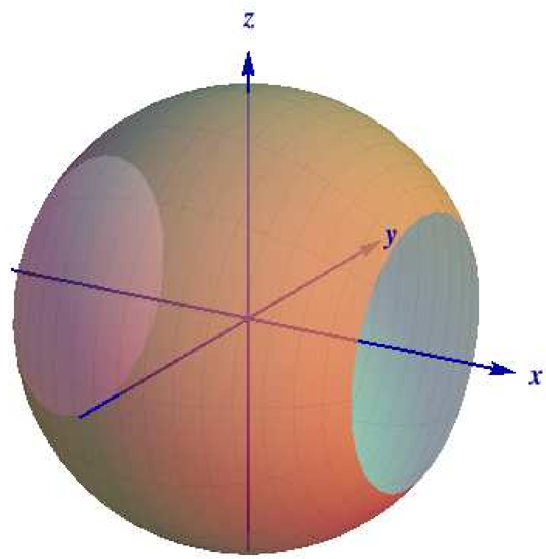
**013 10.0 points**

The surface $S$ shown in




consists of the portion of the sphere

$$x^2 + y^2 + z^2 \;=\; 25$$

where

$$y^2 + z^2 \;\ge\; 9$$

Use spherical polar coordinates $(\rho,\,\theta,\,\phi)$ to describe $S$.

**1.** $S =$ all points $P(3,\,\theta,\,\phi)\}$ with

$$0 \le \theta \le 2\pi, \;\; 0 \le \phi \le \pi, \;\; \sin^2\phi\cos^2\theta \le \frac{2}{5}\,.$$

**2.** $S =$ all points $P(3,\,\theta,\,\phi)\}$ with

$$0 \le \theta \le 2\pi, \;\; 0 \le \phi \le \pi, \;\; \sin^2\phi\sin^2\theta \le \frac{16}{25}\,.$$

**3.** $S =$ all points $P(3,\,\theta,\,\phi)\}$ with

$$0 \le \theta \le 2\pi, \;\; 0 \le \phi \le \pi, \;\; \cos^2\phi\cos^2\theta \le \frac{2}{5}\,.$$

**4.** $S =$ all points $P(5,\,\theta,\,\phi)$ with

$$0 \le \theta \le 2\pi, \;\; 0 \le \phi \le \pi, \;\; \sin^2\phi\cos^2\theta \le \frac{16}{25}\,.$$

**5.** $S =$ all points $P(5,\, \theta,\, \phi)\}$ with

$$0 \le \theta \le 2\pi, \;\; 0 \le \phi \le \pi, \;\; \cos^2\phi \sin^2\theta \le \frac{2}{5}\,.$$

**6.** $S =$ all points $P(5,\, \theta,\, \phi)\}$ with

$$0 \le \theta \le 2\pi, \;\; 0 \le \phi \le \pi, \;\; \cos^2\phi \sin^2\theta \le \frac{16}{25}\,.$$

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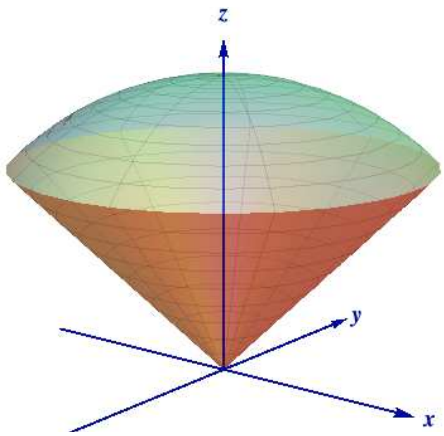
**014 10.0 points**

The solid $W$ shown in




consists of all points enclosed by the sphere

$$x^2 + y^2 + z^2 \;=\; 1$$

and the cone

$$z^2 \;=\; 3(x^2 + y^2), \quad z \ge 0\,.$$

Describe $W$ as a set of points $\{(\rho,\, \theta,\, \phi)\}$ in spherical polar coordinates.

**1.** $0 \le \rho \le 4,\; 0 \le \theta \le 2\pi,\; 0 \le \phi \le \dfrac{\pi}{3}$

**2.** $0 \le \rho \le 1,\; 0 \le \theta \le 2\pi,\; 0 \le \phi \le \dfrac{\pi}{3}$

**3.** $0 \le \rho \le 4,\; 0 \le \theta \le 2\pi,\; 0 \le \phi \le \dfrac{\pi}{6}$

**4.** $0 \le \rho \le 4,\; 0 \le \theta \le 2\pi,\; 0 \le \phi \le \dfrac{\pi}{4}$

**5.** $0 \le \rho \le 1,\; 0 \le \theta \le 2\pi,\; 0 \le \phi \le \dfrac{\pi}{6}$

**6.** $0 \le \rho \le 1,\; 0 \le \theta \le 2\pi,\; 0 \le \phi \le \dfrac{\pi}{4}$

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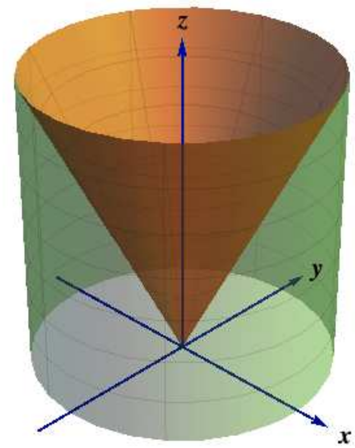
**015 10.0 points**

The solid $W$ shown in




that lies above the $xy$-plane, below the cone

$$z^2 \;=\; 9x^2 + 9y^2\,,$$

and within the cylinder

$$x^2 + y^2 \;=\; 1\,.$$

Describe $W$ as a set of points $\{(r,\, \theta,\, z)\}$ in cylindrical coordinates.

**1.** $0 \le r \le 1, \quad 0 \le \theta \le \pi, \quad 0 \le z \le 3r$

**2.** $0 \le r \le 3, \quad 0 \le \theta \le 2\pi, \quad 0 \le z \le 9r$

**3.** $0 \le r \le 1, \quad 0 \le \theta \le 2\pi, \quad 0 \le z \le 3r$

**4.** $0 \le r \le 1, \quad 0 \le \theta \le 2\pi, \quad 0 \le z \le 9r$

**5.** $0 \le r \le 3, \quad 0 \le \theta \le \pi, \quad 0 \le z \le 9r$

**6.** $0 \le r \le 3, \quad 0 \le \theta \le \pi, \quad 0 \le z \le 3r$

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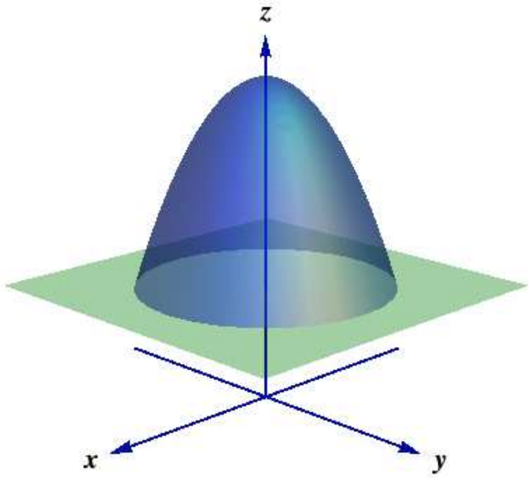
**016 10.0 points**

The solid $W$ shown in




is bounded by the paraboloid

$$z = 11 - x^2 - y^2$$

and the plane $z = 2$. Describe $W$ as a set of points $\{(r, \theta, z)\}$ in cylindrical coordinates.

**1.** $0 \le r \le 9,\ \ 0 \le \theta \le \pi,\ \ 2 \le z \le 11 - r^2$

**2.** $0 \le r \le 3,\ \ 0 \le \theta \le \pi,\ \ 2 \le z \le 11 - r^2$

**3.** $0 \le r \le 3,\ \ 0 \le \theta \le 2\pi,\ \ 2 \le z \le 11 - r^2$

**4.** $0 \le r \le 3,\ \ 0 \le \theta \le 2\pi,\ \ 2 \le z \le 11 - r$

**5.** $0 \le r \le 9,\ \ 0 \le \theta \le 2\pi,\ \ 2 \le z \le 11 - r$

**6.** $0 \le r \le 9,\ \ 0 \le \theta \le \pi,\ \ 2 \le z \le 11 - r$