This print-out should have 5 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

001 10.0 points

Evaluate the integral

$$I = \int_0^{\pi/2} \cos^3 x \, dx \,.$$

1. $I = 1$

2. $I = \dfrac{5}{6}$

3. $I = \dfrac{1}{3}$

4. $I = \dfrac{2}{3}$ **correct**

5. $I = \dfrac{1}{6}$

Explanation:
Since

$$\cos^2 x = 1 - \sin^2 x \,,$$

we see that

$$I = \int_0^{\pi/2} (1 - \sin^2 x) \cos x \, dx \,.$$

This suggests using the substitution $u = \sin x$. For then $du = \cos x \, dx$, while

$$x = 0 \implies u = 0 \,,$$

$$x = \frac{\pi}{2} \implies u = 1 \,.$$

In this case,

$$I = \int_0^1 (1 - u^2) \, du \,.$$

Thus

$$\boxed{I = \Big[u - \frac{1}{3} u^3 \Big]_0^1 = \frac{2}{3}}.$$

002 10.0 points

Determine the indefinite integral

$$I = \int 3 \sin^2 x \cos^3 x \, dx \,.$$

1. $I = \sin^3 x - \dfrac{3}{5} \sin^5 x + C$ **correct**

2. $I = -\dfrac{3}{5} \sin^3 x - \cos^5 x + C$

3. $I = \dfrac{3}{5} \cos^3 x - \sin^5 x + C$

4. $I = -\cos^3 x + \dfrac{3}{5} \cos^5 x + C$

5. $I = \sin^3 x + \dfrac{3}{5} \sin^5 x + C$

6. $I = \dfrac{3}{5} \cos^3 x + \sin^5 x + C$

Explanation:
Since

$$\sin^2 x \cos^3 x = \sin^2 x \cos^2 x \cos x$$
$$= \sin^2 x (1 - \sin^2 x) \cos x \,,$$

we see that I can be written as the sum

$$I = \int 3 \sin^2 x (1 - \sin^2 x) \cos x \, dx$$
$$= \int 3 \sin^2 x \cos x \, dx$$
$$- 3 \int \sin^4 x \cos x \, dx \,,$$

of two integrals, both of which can be evaluated using the substitution $u = \sin x$. For then

$$du = \cos x \, dx \,,$$

in which case

$$I = \int 3u^2 \, du - \int 3u^4 \, du$$
$$= u^3 - \frac{3}{5} u^5 + C \,.$$

Consequently,

$$\boxed{I = \sin^3 x - \frac{3}{5}\sin^5 x + C}.$$

003 10.0 points

Evaluate the integral

$$I = \int_0^{\pi/4} (1 + 2\sin^2(\theta))\, d\theta\,.$$

1. $I = \frac{1}{2}\pi - \frac{1}{2}$ **correct**

2. $I = 1 - \frac{1}{4}\pi$

3. $I = -\frac{1}{2}\pi$

4. $I = \frac{1}{4}\pi - 1$

5. $I = -\pi$

6. $I = \pi$

Explanation:
Since

$$\sin^2(\theta) = \frac{1}{2}\Big(1 - \cos(2\theta)\Big),$$

the integral can be rewritten as

$$I = \int_0^{\pi/4} (2 - \cos(2\theta))\ d\theta$$

$$= \Big[2\theta - \frac{1}{2}\sin(2\theta)\Big]_0^{\pi/4}.$$

Consequently

$$\boxed{I = \frac{1}{2}\pi - \frac{1}{2}}.$$

004 10.0 points

Evaluate the integral

$$I = \int_0^{\pi/2} (2\cos^2(x) + \sin^2(x))\, dx$$

1. $I = \frac{3}{2}\pi$

2. $I = 3$

3. $I = \frac{3}{4}\pi$ **correct**

4. $I = \frac{3}{2}$

5. $I = 3\pi$

6. $I = \frac{3}{4}$

Explanation:
Since

$$\cos^2(x) = \frac{1}{2}(1+\cos(2x)),\ \ \sin^2(x) = \frac{1}{2}(1-\cos(2x)),$$

we see that

$$2\cos^2(x) + \sin^2(x) = \frac{1}{2}\Big(3 + \cos(2x)\Big).$$

Thus

$$I = \frac{1}{2}\int_0^{\pi} \Big(3 + \cos(2x)\Big)\, dx$$

$$= \frac{1}{2}\Big[3x + \frac{1}{2}\sin(2x)\Big]_0^{\pi/2}.$$

Consequently,

$$\boxed{I = \frac{3}{4}\pi}.$$

005 10.0 points

Evaluate the integral

$$I = \int_0^{\pi/2} 3\ \sin^2(x)\cos^3(x)\, dx\,.$$

1. $I = \frac{4}{5}$

2. $I = \frac{2}{5}$ **correct**

3. $I = \frac{6}{5}$

4. $I = \frac{1}{5}$

5. $I = \frac{8}{5}$

Explanation:

Since

$$\sin^2(x)\cos^3(x) = (\sin^2(x)\cos^2(x))\cos(x)$$

$$= \sin^2(x)(1-\sin^2(x))\cos(x)$$

$$= (\sin^2(x) - \sin^4(x))\cos(x),$$

the integrand is of the form $\cos(x) f(\sin(x))$, suggesting use of the substitution $u = \sin(x)$. For then

$$du = \cos(x)\, dx,$$

while

$$x = 0 \implies u = 0$$

$$x = \frac{\pi}{2} \implies u = 1.$$

In this case

$$I = \int_0^1 3(u^2 - u^4)\, du.$$

Consequently,

$$\boxed{I = \left[u^3 - \frac{3}{5}u^5\right]_0^1 = \frac{2}{5}}.$$

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keywords: Stewart5e, indefinite integral, powers of sin, powers of cos, trig substitution,