Quiz 08/13

martes, 12 de agosto de 2025 2:12 p. m.

Consider an object that moves along the curve C, which can be described as the upper half of the circle $x^2+y^2=1$ going from $(-1,0)$ to $(1,0)$. Our goal is to find the work done by the force:

$$F=\left(x^2+y^2,\ 2xy-3x+\sqrt[3]{y^5+1}\right)$$

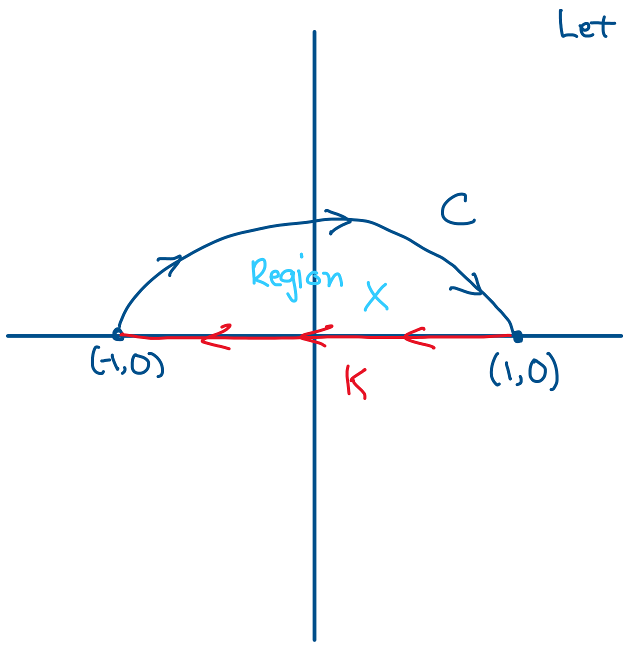
in this motion. Use stokes theorem.

a) Compute curl $(F)=F_{2x}-F_{1y}$ to show that F is not a conservative force.

$$\left.\begin{aligned} F_{2x} &= 2y-3 \\ F_{1y} &= 2y \end{aligned}\right\} \longmapsto F_{2x}-F_{1y}=\cancel{2y}-3-\cancel{2y}=-3$$

Therefore F is not a conservative force since curl$(F)\neq 0$.

b) Sketch the region X with the curves C and C' bounding it. Is the orientation on $C\cup C'$ compatible with the orientation on X?



Let $C'=K$

$X=\{(x,y): x^2+y^2\leq 1,\ y\geq 0\}$

$C(\theta): (\cos\theta, \sin\theta)$, $K(x)=(x,0)$

Furthermore, $C\cup K$ is not compatible with region X as region X is oriented in a CCW while $C\cup K$ is in a CW orient.

c) Explain why we can write

$$\int_C F\cdot dr = 3\,\text{Area}(U) - \int_{C'} F\cdot dr$$

Using Green's theorem for a CW boundary, we use the following:

$$(\text{cw})\int_{\partial u}(P\,dx+Q\,dy) = -\iint_u (Q_x-P_y)\,dA$$

From our F,

$$-\iint(-3)\,dA = 3\,\text{Area}(U)$$

Where

$$\iint_{C\cup K} F\cdot dr = \int_C F\cdot dr + \int_K F\cdot dr = 3\,\text{Area}(U) \Longrightarrow \int_C F\cdot dr = 3\,\text{Area}(U) - \int_K F\cdot dr$$

d) Use the formula above to compute the work done by F.

$$\int_C F\cdot dr = 3\,\text{Area}(U) - \int_K F\cdot dr$$

$$= \frac{3\pi}{2} - \int_1^{-1} F(K(x))\cdot K'(x)\,dx$$

$$= \frac{3\pi}{2} - \underbrace{\int_1^{-1} x^2\,dx}$$

$$\int_1^{-1} x^2\,dx \longrightarrow -\int_{-1}^{1} x^2\,dx = -\left[\frac{1}{3}x^3\right]_{-1}^{1}$$

$$= \left(\frac{1}{3}\right)-\left(-\frac{1}{3}\right)$$

$$= \frac{2}{3}$$

$$\boxed{\int_C F\cdot dr = \frac{3\pi}{2}+\frac{2}{3} = \frac{9\pi+4}{6}}$$