Quiz #1 - MA27L Vector Calculus

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Abdon Morales (am226923)

1. Consider the following vectors

$$v_1 = (2, -1, 1, 0, 1) \text{ and } v_2 = (1, 1, 0, -1, 2)$$

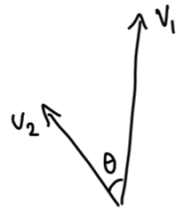
in $\mathbb{R}^5$.

A. Dot Product and Planar Sketch

$$v_1 \cdot v_2 = \begin{bmatrix} 2 \\ -1 \\ 1 \\ 0 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \\ 2 \end{bmatrix} = 2(1) + (-1)(1) + (1)0 + 0(-1) + 1(2)$$

$$= 2 - 1 + 2$$

$$= \boxed{3}$$



Which implies that the two vectors form an acute angle due to result of $v_1 \cdot w_1 > 0$.

B. Scalar Projection v_1 onto v_2:

$$\text{proj } v_1 = v_1 \cdot \hat{v}_2 = |v_1| \cos\theta$$

$$\text{proj}(v_1 \mapsto v_2) = \sqrt{7} \cdot \frac{3}{7}$$

$$= \boxed{\frac{3}{\sqrt{7}}}$$

NOTE! $\text{proj } v_1 \equiv \frac{v_1 \cdot v_2}{|v_1||v_2|}$

$$= \frac{v_1 \cdot v_2}{|v_2|}$$

$$\cos\theta = \frac{v_1 \cdot v_2}{|v_1||v_2|}$$

$$= \frac{3}{\sqrt{7} \cdot \sqrt{7}}$$

$$\cos\theta = \frac{3}{7}$$

$$\text{norm}(v_1) = \sqrt{(2)^2 + (-1)^2 + (1)^2 + (0)^2 + (1)^2}$$

$$= \sqrt{4 + 1 + 1 + 1}$$

$$= \sqrt{7}$$

$$\text{norm}(v_2) = \sqrt{(1)^2 + (1)^2 + (0)^2 + (-1)^2 + (2)^2}$$

$$= \sqrt{1 + 1 + 1 + 4}$$

$$= \sqrt{7}$$

$\text{proj}(v_1 \mapsto v_2)$ represents the length of the shadow that v_1 casts onto the direction of v_2