This print-out should have 15 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

001 10.0 points

Find the directional derivative, $f_{\mathbf{v}}$, of

$$f(x, y) = \sqrt{3x - 2y}$$

at the point $(4, -3)$ in the direction

$$\mathbf{v} = \mathbf{i} + \mathbf{j}.$$

1. $f_{\mathbf{v}} = \dfrac{5}{12}$
2. $f_{\mathbf{v}} = \dfrac{7}{12}$
3. $f_{\mathbf{v}} = \dfrac{1}{4}$
4. $f_{\mathbf{v}} = \dfrac{1}{12}$
5. $f_{\mathbf{v}} = \dfrac{3}{4}$

002 10.0 points

Find the directional derivative, $f_{\mathbf{v}}$, of the function

$$f(x, y) = 4 + 3x\sqrt{y}$$

at the point $P(3, 1)$ in the direction of the vector

$$\mathbf{v} = \langle 3, -4 \rangle.$$

1. $f_{\mathbf{v}} = -\dfrac{12}{5}$
2. $f_{\mathbf{v}} = -\dfrac{8}{5}$
3. $f_{\mathbf{v}} = -\dfrac{11}{5}$
4. $f_{\mathbf{v}} = -\dfrac{9}{5}$
5. $f_{\mathbf{v}} = -2$

003 10.0 points

Find the directional derivative, $D_{\mathbf{v}}f$, of

$$f(x, y, z) = 4x \tan^{-1}\left(\frac{y}{z}\right)$$

at the point $P = (1, 1, 1)$ in the direction of the vector

$$\mathbf{v} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}.$$

1. $D_{\mathbf{v}}f|_P = \pi$
2. $D_{\mathbf{v}}f|_P = 1$
3. $D_{\mathbf{v}}f|_P = \dfrac{1}{3}$
4. $D_{\mathbf{v}}f|_P = \dfrac{4}{3}\pi$
5. $D_{\mathbf{v}}f|_P = \dfrac{1}{3}\pi$
6. $D_{\mathbf{v}}f|_P = \dfrac{4}{3}$

004 10.0 points

Find the maximum slope on the graph of

$$f(x, y) = 4\sin(xy)$$

at the point $P(0, 3)$.

1. max slope $= 4$
2. max slope $= 1$
3. max slope $= 12\pi$
4. max slope $= 12$
5. max slope $= \pi$
6. max slope $= 4\pi$
7. max slope $= 3\pi$

8. max slope $= 3$

---

005 10.0 points

Suppose that over a certain region of space the electrical potential V is given by

$$V(x, y, z) = 6x^2 - 6xy + xyz\,.$$

Find the rate of change of the potential at $P(2, 1, 7)$ in the direction of the vector

$$\mathbf{v} = \mathbf{i} + \mathbf{j} - \mathbf{k}\,.$$

1. 25

2. $\dfrac{25}{\sqrt{3}}$

3. -25

4. $-\dfrac{25}{\sqrt{3}}$

5. $-\dfrac{25}{3}$

---

006 10.0 points

Find the linearization of $z = f(x, y)$ at $P(2, -1)$ when

$$f(2, -1) = 1$$

and

$$f_x(2, -1) = -2\,, \quad f_y(2, -1) = 3\,.$$

1. $L(x, y) = 1 - 2x + 3y$

2. $L(x, y) = 8z + 2x - 3y$

3. $L(x, y) = 1 - 2x - 3y$

4. $L(x, y) = 1 + 2x - 3y$

5. $L(x, y) = 8 - 2x + 3y$

6. $L(x, y) = 8z - 2x + 3y$

---

007 10.0 points

Find the quadratic approximation to

$$f(x, y) = \cos(x + y) + 2\sin(x - y)$$

at $P(0, 0)$.

1. $Q(x, y) = 2 + 2x - 2y + \dfrac{1}{2}x^2 + xy - \dfrac{1}{2}y^2$

2. $Q(x, y) = 2 + 2x - 2y - \dfrac{1}{2}x^2 + xy - \dfrac{1}{2}y^2$

3. $Q(x, y) = 2 + 2x - 2y + \dfrac{1}{2}x^2 - xy + y^2$

4. $Q(x, y) = 1 - 2x + 2y - \dfrac{1}{2}x^2 + xy + y^2$

5. $Q(x, y) = 1 - 2x + 2y + \dfrac{1}{2}x^2 - xy + y^2$

6. $Q(x, y) = 1 + 2x - 2y - \dfrac{1}{2}x^2 - xy - \dfrac{1}{2}y^2$

---

008 10.0 points

Find the quadratic approximation to

$$f(x, y) = \sqrt{1 - x + 2y}$$

at $P(0, 0)$.

1. $Q(x, y) = 1 - \dfrac{1}{2}x + y - \dfrac{1}{8}x^2 + \dfrac{1}{2}xy - \dfrac{1}{2}y^2$

2. $Q(x, y) = 1 - \dfrac{1}{2}x + y - \dfrac{1}{8}x^2 + \dfrac{1}{2}xy + \dfrac{1}{2}y^2$

3. $Q(x, y) = 1 - \dfrac{1}{2}x + y + \dfrac{1}{8}x^2 - \dfrac{1}{2}xy - y^2$

4. $Q(x, y) = 1 - \dfrac{1}{2}x - y - \dfrac{1}{8}x^2 - \dfrac{1}{2}xy - \dfrac{1}{2}y^2$

5. $Q(x, y) = 1 - \dfrac{1}{2}x - y + \dfrac{1}{8}x^2 + \dfrac{1}{2}xy + y^2$

6. $Q(x, y) = 1 - \dfrac{1}{2}x - y + \dfrac{1}{8}x^2 - \dfrac{1}{2}xy + y^2$

---

009 10.0 points

Find the quadratic approximation to

$$f(x, y) = e^{-x+2y^2}$$

at $P(0, 0)$.

1. $Q(x, y) = 1 + x + \frac{1}{2}xy + 2y^2$

2. $Q(x, y) = 1 + 2x + \frac{1}{2}x^2 + 2y^2$

3. $Q(x, y) = 1 + 2y + 2xy + \frac{1}{2}y^2$

4. $Q(x, y) = 1 - 2x + \frac{1}{2}x^2 - 2y^2$

5. $Q(x, y) = 1 - x + \frac{1}{2}x^2 - 2y^2$

6. $Q(x, y) = 1 - x + \frac{1}{2}x^2 + 2y^2$

---

010 10.0 points

Find the quadratic approximation to

$$f(x, y) = \ln(1 + 4x^2 - 2y)$$

at $P(0, 0)$.

1. $Q(x, y) = 1 - 2x + 2x^2 - 4y^2$

2. $Q(x, y) = 1 - 2y + 2x^2 + 4y^2$

3. $Q(x, y) = -2y + 4x^2 + 2y^2$

4. $Q(x, y) = -2x + 2x^2 + 4y^2$

5. $Q(x, y) = 1 - 2y + 4x^2 - 2y^2$

6. $Q(x, y) = -2y + 4x^2 - 2y^2$

---

011 10.0 points

Find an equation for the plane passing through the origin that is parallel to the tangent plane to the graph of

$$z = f(x, y) = x^2 - 2y^2 + 2x + y$$

at the point $(1, -1, f(1, -1))$.

1. $z + 4x - 5y - 9 = 0$

2. $z - 4x - 5y = 0$

3. $z - 4x + 5y + 9 = 0$

4. $z + 4x + 5y + 1 = 0$

5. $z - 4x + 5y = 0$

6. $z + 4x - 5y = 0$

---

012 10.0 points

Find the equation of the tangent plane to the surface

$$4x^2 + 2y^2 + 5z^2 = 79$$

at the point $(2, -3, 3)$.

1. $8x - 6y + 15z = 79$

2. $4x - 2y + 5z = 79$

3. $8x - 6y + 15z = 43$

4. $8x + 6y + 15z = 79$

5. $8x + 6y + 15z = 43$

---

013 10.0 points

Find an equation for the tangent plane to the graph of

$$z = xe^y \cos z - 7$$

at the point $(7, 0, 0)$.

1. $x + 7y - z = 7$

2. $x - 7y - z = 7$

3. $x + 7y + z = -7$

4. $x + 7y + z = 7$

5. $x + 7y - z = -7$

014 10.0 points

If $\mathbf{r}(x)$ is the vector function whose graph is trace of the surface

$$z = f(x, y) = 3x^2 - y^2 - x - 2y$$

on the plane $y+2x = 0$, determine the tangent vector to $\mathbf{r}(x)$ at $x = 1$.

1. tangent vector $= \langle 2, 0, 3 \rangle$

2. tangent vector $= \langle 1, 0, 1 \rangle$

3. tangent vector $= \langle 1, -2, 1 \rangle$

4. tangent vector $= \langle 1, -2, 3 \rangle$

5. tangent vector $= \langle 2, 1, 3 \rangle$

6. tangent vector $= \langle 2, 0, 1 \rangle$

015 10.0 points

If $\mathbf{r}(x)$ is the vector function whose graph is trace of the surface

$$z = f(x, y) = 3x^2 - 2y^2 - 2x + 3y$$

on the plane $y = 2x$, determine the tangent vector to $\mathbf{r}(x)$ at $x = 1$.

1. tangent vector $= \langle 1, 2, -6 \rangle$

2. tangent vector $= \langle 1, 2, 4 \rangle$

3. tangent vector $= \langle 1, 0, -6 \rangle$

4. tangent vector $= \langle 2, 1, 4 \rangle$

5. tangent vector $= \langle 2, 2, -6 \rangle$

6. tangent vector $= \langle 2, 0, 4 \rangle$