Final

1. Let W be the region below the cone $z = \sqrt{x^2 + y^2}$, above the xy-plane and inside the sphere $x^2 + y^2 + z^2 = 4$. Find the flux of the vector field

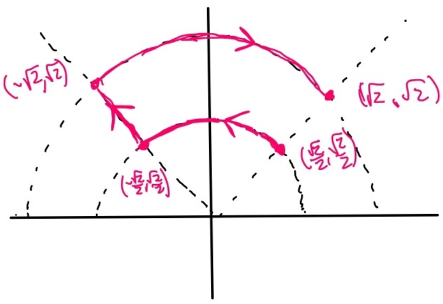
$$F = (3xz^2 + y,\, 3x^2y - \sin(z),\, 3y^2z - e^y)$$

through the boundary of W. In what direction do we have a positive flux: inward or outward?

2. An object starts at the point $(\sqrt{2}/2, \sqrt{2}/2)$, goes to the point $(-\sqrt{2}/2, \sqrt{2}/2)$ moving along the circle $x^2 + y^2 = 1$, then goes to the point $(-\sqrt{2}, \sqrt{2})$ moving along the line $y = -x$ and finally goes to the point $(\sqrt{2}, \sqrt{2})$ moving along the circle $x^2 + y^2 = 4$ as depicted. Find the work done by

$$F = (y^3 - \cos(x - y), \cos(x - y))$$

to move our object along the described path.



3. Compute the curl of the force field

$$F = (z - (1 + xy)e^{xy-z},\ 1 - x^2 e^{xy-z},\ x + xe^{xy-z})$$

showing that it is conservative. Use this knowledge to compute the work done by F to move an object along the path $c(t) = (t^2 + 1, t^3 - 1,\ t^4 - 1)$ with $t \in [-1, 0]$.

4. Let W be the region below the paraboloid $z = 1 + x^2 + y^2$, above the xy-plane and inside the cylinder $x^2 + y^2 = 4$. Find the flux of the vector field

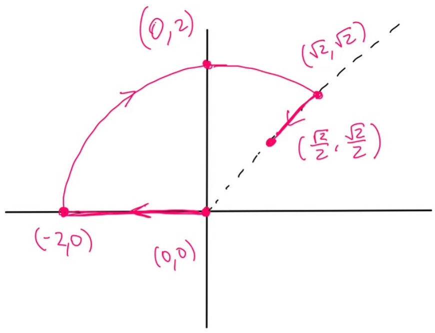
$$F = (x^2 - y,\, 3x^2y + \sin(z),\, e^y - 2xz)$$

through the boundary of W. In what direction do we have a positive flux: inward or outward?

5. An object starts at the origin $(0,0)$, goes towards the point $(-2,0)$ moving along the x-axis, then goes to the point $(\sqrt{2},\sqrt{2})$ moving along the circle $x^2+y^2=4$ and finally goes to the point $(\sqrt{2}/2,\sqrt{2}/2)$ moving along the line $y=x$. Find the work done by the force

$$F = (\cos(y-x),\, x^3 - \cos(y-x))$$

to move our object along the described path.



6. Compute the curl of the force field

$$F = (\cos(xy - z) - xy\sin(xy - z),\ z - x^2\sin(xy - z),\ y + x\sin(xy - z))$$

showing that it is conservative. Use this knowledge to compute the work done by F to move an object along the path $c(t) = (t^2 + 1, t^3 - 1,\ t^4 - 1)$ with $t \in [-1, 0]$.