Quiz #5 - M 427L

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July 1st

a) Repeat this process to now find the linear approximation for the vector valued function

$F(x,y) = (e^{y-x}, 1 + x\cos(x-y), 2 - \sin(x+y))$

near the origin $P = (0,0)$. Note that it has 3 coordinate outputs, so there 3 gradients to compute! Also, in this particular case, because you are centered at the origin, its better to write $\Delta x = x - 0 = x$ and $\Delta y = y - 0 = y$, instead of keeping the deltas.

$f'g + g'f = \cos(x-y) + (-\sin(x-y) \cdot x)$
$= 0 + (+\sin(x-y) \cdot x)$

Let $P = (0,0)$, $f(x,y) = (e^{y-x}, 1 + x\cos(x-y), 2 - \sin(x+y))$

$f(P) = f(0,0) = (e^0, 1 + 0 \cdot \cos(0), 2 - \sin(0))$
$= (1,1,2)$

$f_1x = -e^{y-x}$ $f_1y = e^{y-x}$
$f_2x = \cos(x-y) - x\sin(x-y)$, $f_2y = x\sin(x-y)$
$f_3x = -\cos(x+y)$, $f_3y = -\cos(x+y)$

Let $f_1 = e^{y-x}$, $f_2 = 1 + x\cos(x-y)$, $f_3 = 2 - \sin(x+y)$

$\nabla f_1(x,y) = (-e^{y-x}, e^{y-x}) \longrightarrow \nabla f_1(P) = (-1,1)$

$\nabla f_2(x,y) = (\cos(x-y) - x\sin(x-y), x\sin(x-y)) \longrightarrow \nabla f_2(P) = (1,0)$

$\nabla f_3(x,y) = (-\cos(x+y), -\cos(x+y)) \longrightarrow \nabla f_3(P) = (-1,-1)$

$\Longrightarrow \therefore F(x,y) \approx (1,1,2) + (-x+y, x, -x-y) = (1-x+y, 1+x, 2-x-y)$
$L(x,y) = (1-x+y, 1+x, 2-x-y)$

b) "Cheat" method using the following approximations:

Let:

$e^t \approx 1 + t,$
$\cos(t) \approx 1 - \frac{t^2}{2},$
$\sin(t) \approx t - \frac{t^3}{3}$

Write down the 1st two non-zero terms of each Taylor series for the functions below and circle only the the terms that have degree at most 1.

$e^{y-x} = 1 + (y-x)$
$1 + x\cos(x-y) = 1 + x\left(1 - \frac{(x-y)^2}{2}\right)$
$2 - \sin(x+y) = 2 - \left((x+y) - \frac{(x+y)^3}{3}\right)$

c) Using $L(x,y)$, estimate $f(-0.01, 0.01)$

I'm using method a,

$L(-0.01, 0.01) = (1 - (-0.01) + (0.01), 1 + (-0.01), 2 - (-0.01) - (0.01))$
$= \boxed{(1.02, 0.99, 2.00)} \approx F(-0.01, 0.01)$