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M 427L Quiz #3, June 18, 2025

Consider the planar curve given by the equation:

$$x^4(x^2+y^2)=y^4$$

1. Use polar coordinates to rewrite the equation into the form $r=f(\theta)$.

Let $x=r\cos\theta$ $\quad (x,y)\mapsto(r,\theta)$
$y=r\sin\theta$

$$r^4\cos^4\theta\,(r^2\cos^2\theta+r^2\sin^2\theta)=r^4\sin^4\theta$$

$$r^4\cdot r^2\cos^4\theta\,(\cancel{\cos^2\theta+\sin^2\theta})^{1}=r^4\sin^4\theta$$

$$\frac{r^6\cos^4\theta}{\cos^4\theta}=\frac{r^4\sin^4\theta}{r^4}$$

$$\frac{r^6}{r^4}=\frac{\sin^4\theta}{\cos^4\theta}$$

$$\sqrt{r^2}=\sqrt{\frac{\sin^4\theta}{\cos^4\theta}}$$

$$r=\sqrt{\tan^4\theta}$$

$$\boxed{r=\tan^2\theta}$$

2. Does your equation in (1) impose any restrictions for θ?

There is a restriction on θ for where $\tan\theta$ is undefined when $\cos\theta=0$:

$$\theta\neq\frac{\pi}{2}+k\pi \quad k\in\mathbb{Z} \quad \vee \quad \cos\theta\neq 0$$

In addition to $r=\tan^2\theta$, we are already guaranteed

$$r=\tan^2\theta\geq 0$$

$$\therefore \rightarrow r\geq 0$$

3. Using the answer in (1), find a parameterization $c(\theta)=(x(\theta),y(\theta))$ describing the portion of the curve

$x^4(x^2+y^2)=y^4$ when $x\geq 0$ $\qquad -\frac{\pi}{2}<\theta<\frac{3\pi}{2}$

$\rightarrow \cos\theta\geq 0 \longrightarrow \cos\theta\geq 0\rightarrow\theta\in[-\frac{\pi}{2},\frac{\pi}{2}]$

$$c(\theta)=(\tan^2\theta\cos\theta,\ \tan^2\theta\sin\theta):\ \theta\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$$