This print-out should have 5 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

001 10.0 points

Evaluate the integral

$$I = \int x^2\sqrt{x^3+6}\,dx\,.$$

1. $I = \frac{2}{9}\left(x^3+6\right)^{3/2}+C$ **correct**

2. $I = \frac{1}{9}\left(x^3+6\right)^{3/2}+C$

3. $I = \frac{2}{9}\left(x^3+6\right)^{1/2}+C$

4. $I = \frac{1}{9}\left(x^3+6\right)^{1/2}+C$

5. $I = 3\left(x^3+6\right)^{1/2}+C$

6. $I = 3\left(x^3+6\right)^{3/2}+C$

Explanation:
Set $u = x^3+6$. Then

$$du = 3x^2\,dx\,,$$

in which case

$$I = \frac{1}{3}\int \sqrt{u}\,du = \frac{2}{9}u^{3/2}+C$$

with C an arbitrary constant. Consequently,

$$\boxed{I = \frac{2}{9}\left(x^3+6\right)^{3/2}+C}\,.$$

002 10.0 points

Evaluate the definite integral

$$I = \int_1^5 \frac{2x-7}{\sqrt{7x-x^2}}\,dx\,.$$

Correct answer: -1.42558.

Explanation:
Set $u = 7x-x^2$. Then

$$du = (7-2x)\,dx\,,$$

while

$$x = 1 \implies u = 6\,,$$

$$x = 5 \implies u = 10\,.$$

In this case,

$$I = -\int_6^{10}\frac{1}{\sqrt{u}}\,du = -\Big[2\sqrt{u}\Big]_6^{10}\,.$$

Consequently,

$$\boxed{I = -2(\sqrt{10}-\sqrt{6}) = -1.42558}\,.$$

003 10.0 points

Evaluate the integral

$$I = \int_0^1 3x\,\sqrt[3]{1-x^2}\,dx\,.$$

1. $I = \frac{9}{4}$

2. $I = \frac{3}{4}$

3. $I = -\frac{9}{8}$

4. $I = -\frac{9}{4}$

5. $I = \frac{9}{8}$ **correct**

6. $I = -\frac{3}{4}$

Explanation:
Set $u = 1-x^2$; then $du = -2x\,dx$, while

$$x = 0 \implies u = 1$$

$$x = 1 \implies u = 0\,.$$

Thus

$$I = 3\int_0^1 (1-x^2)^{1/3}\cdot x\,dx$$

$$= -\frac{3}{2}\int_1^0 u^{1/3}\,du = \frac{3}{2}\int_0^1 u^{1/3}\,du\,.$$

Consequently,

$$\boxed{I = \frac{3}{2}\Big[\frac{3}{4}u^{4/3}\Big]_0^1 = \frac{9}{8}}.$$

004 10.0 points

Evaluate the integral

$$I = \int_0^6 te^{-t}dt.$$

1. $I = 1 - \frac{6}{e^7}$
2. $I = 1 - \frac{7}{e^6}$ **correct**
3. $I = 1 + \frac{6}{e^7}$
4. $I = 1 + \frac{7}{e^7}$
5. $I = 1 + \frac{7}{e^6}$
6. $I = 1 - \frac{6}{e^6}$

Explanation:

After Integration by Parts,

$$I = \Big[-te^{-t}\Big]_0^6 + \int_0^6 e^{-t}dt$$

$$= \Big[-te^{-t} - e^{-t}\Big]_0^6.$$

Consequently,

$$\boxed{I = -6e^{-6} - e^{-6} + 1 = 1 - \frac{7}{e^6}}.$$

005 10.0 points

Find the area bounded by the graphs of

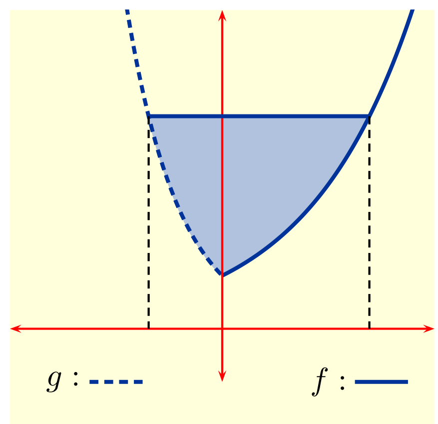
$$f(x) = e^{4x}, \quad g(x) = e^{-8x}$$

and the line $y = 4$.

1. Area $= \frac{3}{16}(4\ln 4 + 3)$ sq. units
2. Area $= \frac{3}{2}(\ln 4 + 1)$ sq. units
3. Area $= \frac{3}{2}(\ln 4 - 1)$ sq. units
4. Area $= \frac{3}{8}(4\ln 4 - 3)$ sq. units **correct**
5. Area $= \frac{3}{4}(\ln 4 - 1)$ sq. units
6. Area $= \frac{3}{16}(4\ln 4 - 3)$ sq. units

Explanation:

The graph of f is an exponentially increasing function, while the graph of g is an exponentially decreasing function. The y-intercept of both graphs is at $y = 1$. Thus the required area is the shaded area in the figure



To express the area in terms of definite integrals, we need to know the x-coordinates of the points of intersection of the horizontal line $y = 4$ with the graphs of f and g respectively, *i.e.*, the solutions of $f(x) = 4$ and $g(x) = 4$. Now

$$e^{4x} = 4 \quad \Longrightarrow \quad x = \frac{1}{4}\ln 4,$$

while

$$e^{-8x} = 4 \quad \Longrightarrow \quad x = -\tfrac{1}{8}\ln 4.$$

In terms of definite integrals, therefore, the required area is given by

$$\int_{-\frac{\ln 4}{8}}^{0} (4 - g(x))\,dx + \int_0^{\frac{1}{4}\ln 4} (4 - f(x))\,dx.$$

But

$$\int_{-\frac{1}{8}\ln 4}^{0} (4 - e^{-8x})\, dx = \left[4x + \frac{1}{8}e^{-8x}\right]_{-\frac{1}{8}\ln 4}^{0}$$
$$= \frac{1}{8}(4\ln 4 - 3),$$

while

$$\int_{0}^{\frac{1}{4}\ln 4} (4 - e^{4x})\, dx = \left[4x - \frac{1}{4}e^{4x}\right]_{0}^{\frac{1}{4}\ln 4}$$
$$= \frac{1}{4}(4\ln 4 - 3).$$

Consequently,

$$\boxed{\text{Area} = \frac{3}{8}(4\ln 4 - 3) \text{ sq.units}}.$$