This print-out should have 15 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

001 10.0 points

Find the directional derivative, $f_{\mathbf{v}}$, of

$$f(x, y) = \sqrt{3x - 2y}$$

at the point $(4, -3)$ in the direction

$$\mathbf{v} = \mathbf{i} + \mathbf{j}.$$

1. $f_{\mathbf{v}} = \dfrac{5}{12}$

2. $f_{\mathbf{v}} = \dfrac{7}{12}$

3. $f_{\mathbf{v}} = \dfrac{1}{4}$

4. $f_{\mathbf{v}} = \dfrac{1}{12}$ **correct**

5. $f_{\mathbf{v}} = \dfrac{3}{4}$

Explanation:

For an arbitrary vector $\mathbf{v}$,

$$f_{\mathbf{v}} = \nabla f \cdot \left(\frac{\mathbf{v}}{|\mathbf{v}|}\right),$$

where we have normalized the direction vector so that it has unit length.

Now the partial derivatives of

$$f(x, y) = \sqrt{3x - 2y}$$

are given by

$$\frac{\partial f}{\partial x} = \frac{3}{2\sqrt{3x - 2y}},$$

and

$$\frac{\partial f}{\partial y} = -\frac{1}{\sqrt{3x - 2y}}.$$

Thus

$$\nabla f(x, y) = \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j}$$

$$= \Big(\frac{3}{2\sqrt{3x - 2y}}\Big)\mathbf{i} - \Big(\frac{1}{\sqrt{3x - 2y}}\Big)\mathbf{j},$$

and so

$$\nabla f(4, -3) = \frac{1}{\sqrt{2}}\Big(\frac{1}{2}\mathbf{i} - \frac{1}{3}\mathbf{j}\Big).$$

On the other hand,

$$\mathbf{v} = \mathbf{i} + \mathbf{j} \quad \Longrightarrow \quad \frac{\mathbf{v}}{|\mathbf{v}|} = \frac{1}{\sqrt{2}}(\mathbf{i} + \mathbf{j}).$$

But then

$$\nabla f \cdot \left(\frac{\mathbf{v}}{|\mathbf{v}|}\right) = \frac{1}{2}\Big(\frac{1}{2}\mathbf{i} - \frac{1}{3}\mathbf{j}\Big) \cdot (\mathbf{i} + \mathbf{j}).$$

Consequently,

$$\boxed{f_{\mathbf{v}} = \frac{1}{2}\Big(\frac{1}{2} - \frac{1}{3}\Big) = \frac{1}{12}}.$$

keywords:

002 10.0 points

Find the directional derivative, $f_{\mathbf{v}}$, of the function

$$f(x, y) = 4 + 3x\sqrt{y}$$

at the point $P(3, 1)$ in the direction of the vector

$$\mathbf{v} = \langle 3, -4 \rangle.$$

1. $f_{\mathbf{v}} = -\dfrac{12}{5}$

2. $f_{\mathbf{v}} = -\dfrac{8}{5}$

3. $f_{\mathbf{v}} = -\dfrac{11}{5}$

4. $f_{\mathbf{v}} = -\dfrac{9}{5}$ **correct**

5. $f_{\mathbf{v}} = -2$

Explanation:

Now for an arbitrary vector $\mathbf{v}$,

$$f_{\mathbf{v}} = \nabla f \cdot \left(\frac{\mathbf{v}}{|\mathbf{v}|}\right),$$

where we have normalized so that the direction vector has unit length. But when

$$f(x, y) = 4 + 3x\sqrt{y},$$

then

$$\nabla f = (3\sqrt{y})\,\mathbf{i} + \frac{3}{2}\left(\frac{x}{\sqrt{y}}\right)\mathbf{j}.$$

At $P(3, 1)$, therefore,

$$\nabla f\Big|_P = 3\,\mathbf{i} + \frac{9}{2}\,\mathbf{j}.$$

Consequently, when $\mathbf{v} = \langle 3, -4 \rangle$,

$$\boxed{f_{\mathbf{v}}(3, 1) = \left\langle 3, \frac{9}{2} \right\rangle \cdot \left(\frac{\mathbf{v}}{|\mathbf{v}|}\right) = -\frac{9}{5}}.$$

keywords:

003 10.0 points

Find the directional derivative, $D_{\mathbf{v}}f$, of

$$f(x, y, z) = 4x\tan^{-1}\left(\frac{y}{z}\right)$$

at the point $P = (1, 1, 1)$ in the direction of the vector

$$\mathbf{v} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}.$$

1. $D_{\mathbf{v}}f|_P = \pi$

2. $D_{\mathbf{v}}f|_P = 1$

3. $D_{\mathbf{v}}f|_P = \frac{1}{3}$

4. $D_{\mathbf{v}}f|_P = \frac{4}{3}\pi$

5. $D_{\mathbf{v}}f|_P = \frac{1}{3}\pi$ **correct**

6. $D_{\mathbf{v}}f|_P = \frac{4}{3}$

Explanation:

For an arbitrary vector $\mathbf{v}$,

$$D_{\mathbf{v}}f = \nabla f \cdot \left(\frac{\mathbf{v}}{|\mathbf{v}|}\right),$$

where we have normalized so that the direction vector has unit length. But when

$$f(x, y, z) = 4x\tan^{-1}\left(\frac{y}{z}\right),$$

then

$$\begin{aligned}\nabla f &= \frac{\partial f}{\partial x}\,\mathbf{i} + \frac{\partial f}{\partial y}\,\mathbf{j} + \frac{\partial f}{\partial z}\,\mathbf{k} \\ &= 4\tan^{-1}\Big(\frac{y}{z}\Big)\,\mathbf{i} + \frac{4x}{z(1+(y/z)^2)}\,\mathbf{j} \\ &\qquad - \frac{4xy}{z^2(1+(y/z)^2)}\,\mathbf{k}.\end{aligned}$$

Thus

$$\nabla f = 4\tan^{-1}\Big(\frac{y}{z}\Big)\,\mathbf{i} + \frac{4xz}{z^2+y^2}\,\mathbf{j} - \frac{4xy}{z^2+y^2}\,\mathbf{k}.$$

At $P = (1, 1, 1)$, therefore,

$$\nabla f|_P = 4\Big(\frac{\pi}{4}\Big)\,\mathbf{i} + 2\,\mathbf{j} - 2\,\mathbf{k}.$$

Consequently, when

$$\mathbf{v} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k},$$

we see that

$$|\mathbf{v}| = \sqrt{1 + 2^2 + 2^2} = 3,$$

and

$$D_{\mathbf{v}}f|_P = \frac{1}{3}\Big(4\Big(\frac{\pi}{4}\Big)\,\mathbf{i} + 2\,\mathbf{j} - 2\,\mathbf{k}\Big) \cdot (\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}).$$

Consequently,

$$\boxed{D_{\mathbf{v}}f|_P = \frac{1}{3}(\pi + 4 - 4) = \frac{1}{3}\pi}.$$

keywords: directional derivative, gradient, dot product, unit vector,

004 10.0 points

Find the maximum slope on the graph of

$$f(x,\, y) \;=\; 4\sin(xy)$$

at the point $P(0,\, 3)$.

1. max slope $=$ 4

2. max slope $=$ 1

3. max slope $=$ 12π

4. max slope $=$ 12 **correct**

5. max slope $=$ π

6. max slope $=$ 4π

7. max slope $=$ 3π

8. max slope $=$ 3

Explanation:

At $P(0,\, 3,\, 0)$ the slope in the direction of $\mathbf{v}$ is given by

$$\nabla f\Big|_{(0,3)} \cdot \left(\frac{\mathbf{v}}{|\mathbf{v}|}\right) .$$

But when

$$f(x,\, y) \;=\; 4\sin(xy)\,,$$

the gradient of f is

$$\nabla f(x,\, y) \;=\; 4y\cos(xy)\,\mathbf{i} + 4x\cos(xy)\,\mathbf{j}\,,$$

so at $P(0,\, 3)$

$$\nabla f\Big|_{(0,3)} \;=\; 12\,\mathbf{i}\,.$$

Consequently, the slope at P will be maximized when $\mathbf{v} = \mathbf{i}$ in which case

$$\boxed{\text{max slope} \;=\; 12}\,.$$

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keywords: slope, gradient, trig function, maximum slope

005 10.0 points

Suppose that over a certain region of space the electrical potential V is given by

$$V(x, y, z) = 6x^2 - 6xy + xyz\,.$$

Find the rate of change of the potential at $P(2, 1, 7)$ in the direction of the vector

$$\mathbf{v} = \mathbf{i} + \mathbf{j} - \mathbf{k}\,.$$

1. 25

2. $\dfrac{25}{\sqrt{3}}$ **correct**

3. -25

4. $-\dfrac{25}{\sqrt{3}}$

5. $-\dfrac{25}{3}$

Explanation:

The rate of change at $P(2, 1, 7)$ is given by

$$D_u V \;=\; \nabla V(2, 1, 7) \cdot \frac{\mathbf{v}}{|\mathbf{v}|}\,.$$

Now, when

$$V(x, y, z) = 6x^2 - 6xy + xyz$$

it follows that

$$\nabla V \;=\; \langle 12x - 6y + yz,\; -6x + xz,\; xy\rangle$$

and

$$\nabla V(2, 1, 7) \;=\; \langle 25,\; 2,\; 2\rangle\,.$$

Consequently,

$$D_u V \;=\; \langle 25,\; 2,\; 2\rangle \cdot \frac{\langle 1,\; 1,\; -1\rangle}{\sqrt{3}} \;=\; \boxed{\frac{25}{\sqrt{3}}}\,.$$

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keywords:

006 10.0 points

Find the linearization of $z = f(x, y)$ at $P(2, -1)$ when

$$f(2, -1) = 1$$

and

$$f_x(2, -1) = -2, \quad f_y(2, -1) = 3.$$

1. $L(x, y) = 1 - 2x + 3y$

2. $L(x, y) = 8z + 2x - 3y$

3. $L(x, y) = 1 - 2x - 3y$

4. $L(x, y) = 1 + 2x - 3y$

5. $L(x, y) = 8 - 2x + 3y$ **correct**

6. $L(x, y) = 8z - 2x + 3y$

Explanation:

The linearization of $z = f(x, y)$ at $P(a, b)$ is given by

$$L(x, y) = f(a, b) + f_x(a, b)(x - a) + f_y(a, b)(y - b),$$

and so at $P(2, -1)$,

$$L(x, y) = f(2, -1) + f_x(2, -1)(x - 2) + f_y(2, -1)(y + 1).$$

Consequently, the linearization of f at P is

$$L(x, y) = 1 - 2(x - 2) + 3(y + 1),$$

which after rearrangement becomes

$$\boxed{L(x, y) = 8 - 2x + 3y}.$$

keywords: linearization, partial derivative, radical function, square root function,

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007 10.0 points

Find the quadratic approximation to

$$f(x, y) = \cos(x + y) + 2\sin(x - y)$$

at $P(0, 0)$.

1. $Q(x, y) = 2 + 2x - 2y + \frac{1}{2}x^2 + xy - \frac{1}{2}y^2$

2. $Q(x, y) = 2 + 2x - 2y - \frac{1}{2}x^2 + xy - \frac{1}{2}y^2$

3. $Q(x, y) = 2 + 2x - 2y + \frac{1}{2}x^2 - xy + y^2$

4. $Q(x, y) = 1 - 2x + 2y - \frac{1}{2}x^2 + xy + y^2$

5. $Q(x, y) = 1 - 2x + 2y + \frac{1}{2}x^2 - xy + y^2$

6. $Q(x, y) = 1 + 2x - 2y - \frac{1}{2}x^2 - xy - \frac{1}{2}y^2$ **correct**

Explanation:

The Quadratic Approximation to $f(x, y)$ at $P(0, 0)$ is given by

$$Q(x, y) = f(0, 0) + f_x(0, 0)x + f_y(0, 0)y + \frac{1}{2}f_{xx}(0, 0)x^2 + f_{xy}(0, 0)xy + \frac{1}{2}f_{yy}(0, 0)y^2.$$

But when

$$f(x, y) = \cos(x + y) + 2\sin(x - y)$$

we see that

$$f_x = -\sin(x + y) + 2\cos(x - y),$$
$$f_y = -\sin(x + y) - 2\cos(x - y),$$

so that $f(0, 0) = 1$ and

$$f_x(0, 0) = 2, \quad f_y(0, 0) = -2,$$

while

$$f_{xx} = -\cos(x + y) - 2\sin(x - y),$$
$$f_{xy} = \cos(x + y) + 2\sin(x - y),$$
$$f_{yy} = \cos(x + y) - 2\sin(x - y),$$

so that $f_{xx}(0, 0) = 1$ and

$$f_{xy}(0, 0) = -1, \quad f_{yy}(0, 0) = -1,$$

Consequently, the Quadratic Approximation to f at $P(0, 0)$ is

$$\boxed{Q(x, y) \;=\; 1 + 2x - 2y - \frac{1}{2}x^2 - xy - \frac{1}{2}y^2}\,.$$

keywords: quadratic approximation, partial derivative, second order partial derivative, trig function,

008 10.0 points

Find the quadratic approximation to

$$f(x, y) \;=\; \sqrt{1 - x + 2y}$$

at $P(0, 0)$.

1. $Q(x, y) = 1 - \frac{1}{2}x + y - \frac{1}{8}x^2 + \frac{1}{2}xy - \frac{1}{2}y^2$ **correct**

2. $Q(x, y) = 1 - \frac{1}{2}x + y - \frac{1}{8}x^2 + \frac{1}{2}xy + \frac{1}{2}y^2$

3. $Q(x, y) = 1 - \frac{1}{2}x + y + \frac{1}{8}x^2 - \frac{1}{2}xy - y^2$

4. $Q(x, y) = 1 - \frac{1}{2}x - y - \frac{1}{8}x^2 - \frac{1}{2}xy - \frac{1}{2}y^2$

5. $Q(x, y) = 1 - \frac{1}{2}x - y + \frac{1}{8}x^2 + \frac{1}{2}xy + y^2$

6. $Q(x, y) = 1 - \frac{1}{2}x - y + \frac{1}{8}x^2 - \frac{1}{2}xy + y^2$

Explanation:

The Quadratic Approximation to $f(x, y)$ at $P(0, 0)$ is given by

$$Q(x, y) \;=\; f(0, 0) + f_x(0, 0)x + f_y(0, 0)y$$

$$+\, \frac{1}{2}f_{xx}(0, 0)x^2 + f_{xy}(0, 0)xy + \frac{1}{2}f_{yy}(0, 0)y^2.$$

But when

$$f(x, y) \;=\; \sqrt{1 - x + 2y}$$

we see that

$$f_x \;=\; -\frac{1}{2}\frac{1}{\sqrt{1 - x + 2y}}\,,$$

$$f_y \;=\; \frac{1}{\sqrt{1 - x + 2y}}\,,$$

so that $f(0, 0) = 1$ and

$$f_x(0, 0) = -\frac{1}{2}, \quad f_y(0, 0) = 1\,,$$

while

$$f_{xx} \;=\; -\frac{1}{4}\frac{1}{(1 - x + 2y)^{3/2}}\,,$$

$$f_{xy} \;=\; \frac{1}{2}\frac{1}{(1 - x + 2y)^{3/2}}\,,$$

$$f_{yy} \;=\; -\frac{1}{(1 - x + 2y)^{3/2}}\,,$$

so that

$$f_{xx}(0, 0) = -\frac{1}{4}, \quad f_{xy}(0, 0) = \frac{1}{2}\,,$$

and $f_{yy}(0, 0) = -1$.

Consequently, the Quadratic Approximation to f at $P(0, 0)$ is

$$\boxed{Q(x, y) = 1 - \frac{1}{2}x + y - \frac{1}{8}x^2 + \frac{1}{2}xy - \frac{1}{2}y^2}\,.$$

keywords: quadratic approximation, partial derivative, second order partial derivative, trig function,

009 10.0 points

Find the quadratic approximation to

$$f(x, y) \;=\; e^{-x+2y^2}$$

at $P(0, 0)$.

1. $Q(x, y) \;=\; 1 + x + \frac{1}{2}xy + 2y^2$

2. $Q(x, y) \;=\; 1 + 2x + \frac{1}{2}x^2 + 2y^2$

3. $Q(x, y) \;=\; 1 + 2y + 2xy + \frac{1}{2}y^2$

4. $Q(x, y) \;=\; 1 - 2x + \frac{1}{2}x^2 - 2y^2$

5. $Q(x, y) = 1 - x + \frac{1}{2}x^2 - 2y^2$

6. $Q(x, y) = 1 - x + \frac{1}{2}x^2 + 2y^2$ **correct**

Explanation:

The Quadratic Approximation to $f(x, y)$ at $P(0, 0)$ is given by

$$Q(x, y) = f(0, 0) + f_x(0, 0)x + f_y(0, 0)y$$

$$+ \frac{1}{2}f_{xx}(0, 0)x^2 + f_{xy}(0, 0)xy + \frac{1}{2}f_{yy}(0, 0)y^2.$$

But when

$$f(x, y) = e^{-x+2y^2}$$

we see that

$$f_x = -e^{-x+2y^2}, \quad f_y = 4ye^{-x+2y^2},$$

so that $f(0, 0) = 1$ and

$$f_x(0, 0) = -1, \quad f_y(0, 0) = 0,$$

while

$$f_{xx} = e^{-x+2y^2}, \quad f_{xy} = -4ye^{-x+2y^2},$$

and

$$f_{yy} = 4e^{-x+2y^2} + 16y^2e^{-x+2y^2},$$

so that

$$f_{xx}(0, 0) = 1, \quad f_{xy}(0, 0) = 0,$$

and $f_{yy}(0, 0) = 4$.

Consequently, the Quadratic Approximation to f at $P(0, 0)$ is

$$\boxed{Q(x, y) = 1 - x + \frac{1}{2}x^2 + 2y^2}.$$

keywords: quadratic approximation, partial derivative, second order partial derivative, trig function,

010 10.0 points

Find the quadratic approximation to

$$f(x, y) = \ln(1 + 4x^2 - 2y)$$

at $P(0, 0)$.

1. $Q(x, y) = 1 - 2x + 2x^2 - 4y^2$

2. $Q(x, y) = 1 - 2y + 2x^2 + 4y^2$

3. $Q(x, y) = -2y + 4x^2 + 2y^2$

4. $Q(x, y) = -2x + 2x^2 + 4y^2$

5. $Q(x, y) = 1 - 2y + 4x^2 - 2y^2$

6. $Q(x, y) = -2y + 4x^2 - 2y^2$ **correct**

Explanation:

The Quadratic Approximation to $f(x, y)$ at $P(0, 0)$ is given by

$$Q(x, y) = f(0, 0) + f_x(0, 0)x + f_y(0, 0)y$$

$$+ \frac{1}{2}f_{xx}(0, 0)x^2 + f_{xy}(0, 0)xy + \frac{1}{2}f_{yy}(0, 0)y^2.$$

But when

$$f(x, y) = \ln(1 + 4x^2 - 2y)$$

we see that

$$f_x = \frac{8x}{1 + 4x^2 - 2y}, \quad f_y = -\frac{2}{1 + 4x^2 - 2y}.$$

Thus $f(0, 0) = 0$ and

$$f_x(0, 0) = 0, \qquad f_y(0, 0) = -2.$$

Differentiating once more we get

$$f_{xx} = \frac{8}{1 + 4x^2 - 2y} - \frac{64x^2}{(1 + 4x^2 - 2y)^2}$$

while

$$f_{xy} = \frac{16x}{(1 + 4x - 2y)^2},$$

and

$$f_{yy} = -\frac{4}{(1 + 4x - 2y)^2}.$$

This gives

$$f_{xx}(0,\, 0) = 8, \quad f_{xy}(0,\, 0) = 0\,,$$

and $f_{yy}(0,\, 0) = -4$.

Consequently, the Quadratic Approximation to f at $P(0,\, 0)$ is

$$\boxed{Q(x,\, y) = -2y + 4x^2 - 2y^2}\,.$$

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keywords: quadratic approximation, partial derivative, second order partial derivative, trig function,

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011 10.0 points

Find an equation for the plane passing through the origin that is parallel to the tangent plane to the graph of

$$z \;=\; f(x,\, y) \;=\; x^2 - 2y^2 + 2x + y$$

at the point $(1,\, -1,\, f(1,\, -1))$.

1. $z + 4x - 5y - 9 \;=\; 0$

2. $z - 4x - 5y \;=\; 0$ **correct**

3. $z - 4x + 5y + 9 \;=\; 0$

4. $z + 4x + 5y + 1 \;=\; 0$

5. $z - 4x + 5y \;=\; 0$

6. $z + 4x - 5y \;=\; 0$

Explanation:

Parallel planes have parallel normals. On the other hand, the tangent plane to the graph of $z = f(x,\, y)$ at the point $(a,\, b,\, f(a,\, b))$ has normal

$$\mathbf{n} \;=\; \langle -f_x(a,\, b),\, -f_y(a,\, b), 1 \rangle\,.$$

But when

$$f(x,\, y) \;=\; x^2 - 2y^2 + 2x + y$$

we see that

$$f_x \;=\; 2x + 2\,, \qquad f_y \;=\; -4y + 1\,,$$

and so when $a = 1$, $b = -1$,

$$\mathbf{n} \;=\; \langle -4,\, -5, 1 \rangle\,.$$

Thus an equation for the plane through the origin with normal parallel to $\mathbf{n}$ is

$$\langle x,\, y,\, z \rangle \cdot \mathbf{n} \;=\; \langle x,\, y,\, z \rangle \cdot \langle -4,\, -5,\, 1 \rangle \;=\; 0\,,$$

which after evaluation becomes

$$\boxed{\; z - 4x - 5y \;=\; 0 \;}\,.$$

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keywords:

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012 10.0 points

Find the equation of the tangent plane to the surface

$$4x^2 + 2y^2 + 5z^2 = 79$$

at the point $(2, -3, 3)$.

1. $8x - 6y + 15z = 79$ **correct**

2. $4x - 2y + 5z = 79$

3. $8x - 6y + 15z = 43$

4. $8x + 6y + 15z = 79$

5. $8x + 6y + 15z = 43$

Explanation:

Let

$$F(x) \;=\; 4x^2 + 2y^2 + 5z^2\,.$$

The equation to the tangent plane to the surface at the point $P(2, -3, 3)$ is given by

$$F_x\Big|_P (x-2) + F_y\Big|_P (y+3) + F_z\Big|_P (z-3) = 0\,.$$

Since

$$F_x \;=\; 8x\,, \qquad F_x\Big|_P \;=\; 16,$$

$$F_y = 4y\,, \qquad F_y\Big|_P = -12,$$

and

$$F_z = 10z\,, \qquad F_z\Big|_P = 30$$

it follows that the equation of the tangent plane is

$$\boxed{8x - 6y + 15z = 79}\,.$$

keywords:

013 10.0 points

Find an equation for the tangent plane to the graph of

$$z = xe^y \cos z - 7$$

at the point $(7, 0, 0)$.

1. $x + 7y - z = 7$ **correct**

2. $x - 7y - z = 7$

3. $x + 7y + z = -7$

4. $x + 7y + z = 7$

5. $x + 7y - z = -7$

Explanation:

Note that

$$xe^y \cos z - z = 7$$

Let

$$F(x) = xe^y \cos z - z\,.$$

The equation to the tangent plane to the surface at the point $P(7, 0, 0)$ is given by

$$F_x\Big|_P(x-7) + F_y\Big|_P(y-0) + F_z\Big|_P(z-0)\,.$$

Since

$$F_x = e^y \cos z\,, \qquad F_x\Big|_P = 1,$$

$$F_y = xe^y \cos z\,, \qquad F_y\Big|_P = 7,$$

and

$$F_z = -xe^y \sin z - 1\,, \qquad F_z\Big|_P = -1$$

it follows that the equation of the tangent plane is

$$\boxed{x + 7y - z = 7}\,.$$

keywords:

014 10.0 points

If $\mathbf{r}(x)$ is the vector function whose graph is trace of the surface

$$z = f(x,\, y) = 3x^2 - y^2 - x - 2y$$

on the plane $y+2x = 0$, determine the tangent vector to $\mathbf{r}(x)$ at $x = 1$.

1. tangent vector $= \langle\, 2,\, 0,\, 3\,\rangle$

2. tangent vector $= \langle\, 1,\, 0,\, 1\,\rangle$

3. tangent vector $= \langle\, 1,\, -2,\, 1\,\rangle$ **correct**

4. tangent vector $= \langle\, 1,\, -2,\, 3\,\rangle$

5. tangent vector $= \langle\, 2,\, 1,\, 3\,\rangle$

6. tangent vector $= \langle\, 2,\, 0,\, 1\,\rangle$

Explanation:

The graph of

$$z = f(x,\, y) = 3x^2 - y^2 - x - 2y$$

is the set of all points

$$(x,\, y,\, f(x,\, y))$$

as x, y vary in 3-space. So the intersection of the surface with the plane $y + 2x = 0$ is the set of all points

$$(x,\, -2x,\, f(x,\, -2x))\,, \quad -\infty < x < \infty\,.$$

But

$$f(x,\, -2x) = -x^2 + 3x\,.$$

Thus the surface and the plane $y = 2x$ intersect in the graph of

$$\mathbf{r}(x) = \langle\, x,\, -2x,\, -x^2 + 3x\,\rangle\,.$$

Now the tangent vector to the graph of $\mathbf{r}(x)$ is the derivative

$$\mathbf{r}'(x) = \langle 1, -2, -2x+3 \rangle .$$

Consequently, at $x = 1$ the graph of $\mathbf{r}(x)$ has

$$\boxed{\text{tangent vector} = \langle 1, -2, 1 \rangle} .$$

keywords:

015 10.0 points

If $\mathbf{r}(x)$ is the vector function whose graph is trace of the surface

$$z = f(x, y) = 3x^2 - 2y^2 - 2x + 3y$$

on the plane $y = 2x$, determine the tangent vector to $\mathbf{r}(x)$ at $x = 1$.

1. tangent vector $= \langle 1, 2, -6 \rangle$ **correct**

2. tangent vector $= \langle 1, 2, 4 \rangle$

3. tangent vector $= \langle 1, 0, -6 \rangle$

4. tangent vector $= \langle 2, 1, 4 \rangle$

5. tangent vector $= \langle 2, 2, -6 \rangle$

6. tangent vector $= \langle 2, 0, 4 \rangle$

Explanation:

The graph of

$$z = f(x, y) = 3x^2 - 2y^2 - 2x + 3y$$

is the set of all points

$$(x, y, f(x, y))$$

as x, y vary in 3-space. So the intersection of the surface with the plane $y = 2x$ is the set of all points

$$(x, 2x, f(x, 2x)), \quad -\infty < x < \infty .$$

But

$$f(x, 2x) = -5x^2 + 4x .$$

Thus the surface and the plane $y = 2x$ intersect in the graph of

$$\mathbf{r}(x) = \langle x, 2x, -5x^2 + 4x \rangle .$$

Now the tangent vector to the graph of $\mathbf{r}(x)$ is the derivative

$$\mathbf{r}'(x) = \langle 1, 2, -10x + 4 \rangle .$$

Consequently, at $x = 1$ the graph of $\mathbf{r}(x)$ has

$$\boxed{\text{tangent vector} = \langle 1, 2, -6 \rangle} .$$

keywords: