This print-out should have 8 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

001 10.0 points

Find $\lim_{(x,y)\to(6,-3)} \left(x^5 + 2x^3y - 3xy^2\right)$.

1. 6642

2. 9234

3. 8910

4. 6318 **correct**

5. 6156

Explanation:

002 10.0 points

Find $\lim_{(x,y)\to(0,0)} \dfrac{6xy^2}{x^2+y^2}$, if it exists.

1. 3

2. The limit does not exist.

3. 0 **correct**

4. 12

5. 6

Explanation:

003 10.0 points

Find $\lim_{(x,y)\to(0,0)} \dfrac{4xy}{\sqrt{x^2+y^2}}$, if it exists.

1. The limit does not exist.

2. 2

3. 0 **correct**

4. 8

5. 4

Explanation:

004 10.0 points

Find $\lim_{(x,y)\to(0,0)} \dfrac{5\left(x^2+y^2\right)}{\sqrt{x^2+y^2+16}-4}$, if it exists.

1. 20

2. 0

3. The limit does not exist.

4. 5

5. 40 **correct**

Explanation:
Rationalize the denominator:

$$\frac{5\left(x^2+y^2\right)}{\sqrt{x^2+y^2+16}-4}\left(\frac{\sqrt{(x^2+y^2+4^2)}+4}{\sqrt{(x^2+y^2+4^2)}+4}\right)$$

which now simplifies to x^2+y^2:

$$\frac{5\left(x^2+y^2\right)[\sqrt{(x^2+y^2+4^2)}+4]}{(x^2+y^2)}$$

That factor also appears in the numerator, so they cancel on the domain of the original function.

Now the resulting function has a continuous numerator (for (x,y) near (0,0) but not equal to it) and continuous denominator WHICH WILL NOT EQUAL 0.

Since the ratio of two continuous functions is continuous (if the denominator does not approach 0), we conclude that the limit as (x,y) approaches the origin is the value of the rewritten function at (0,0):

$$\begin{aligned} 5(\sqrt{(0^2+0^2+4^2)}+4) &= 5(\sqrt{4^2}+4) \\ &= 40 \end{aligned}$$

005 10.0 points

Find $\lim_{(x,y)\to(0,0)} \dfrac{2xy^4}{x^2+y^8}$, if it exists.

1. 1

2. The limit does not exist. **correct**

3. 0

4. 2

5. 4

Explanation:

006 10.0 points

Determine $f_x - f_y$ when

$$f(x, y) = 4x^2 + xy - 4y^2 + 2x + y\,.$$

1. $f_x - f_y = 9x - 7y + 3$

2. $f_x - f_y = 9x - 7y + 1$

3. $f_x - f_y = 7x + 9y + 3$

4. $f_x - f_y = 7x - 7y + 1$

5. $f_x - f_y = 7x + 9y + 1$ **correct**

6. $f_x - f_y = 9x + 9y + 3$

Explanation:

After differentiation we see that

$$f_x = 8x + y + 2\,, \quad f_y = x - 8y + 1\,.$$

Consequently,

$$\boxed{f_x - f_y = 7x + 9y + 1}\,.$$

007 10.0 points

Determine f_x when

$$f(x,\, y) = \frac{2x - y}{2x + y}\,.$$

1. $f_x = -\dfrac{5x}{(2x+y)^2}$

2. $f_x = -\dfrac{3y}{(2x+y)^2}$

3. $f_x = \dfrac{5y}{(2x+y)^2}$

4. $f_x = -\dfrac{4x}{(2x+y)^2}$

5. $f_x = \dfrac{3x}{(2x+y)^2}$

6. $f_x = \dfrac{4y}{(2x+y)^2}$ **correct**

Explanation:

From the Quotient Rule we see that

$$f_x = \frac{2(2x+y) - 2(2x-y)}{(2x+y)^2}\,.$$

Consequently,

$$\boxed{f_x = \frac{4y}{(2x+y)^2}}\,.$$

008 10.0 points

Determine f_x when

$$f(x, y) = (2x - y)\, e^{x/y}\,.$$

1. $f_x = \left(\dfrac{x}{y} - 1\right) e^{x/y}$

2. $f_x = \left(\dfrac{2x}{y} + 1\right) e^{x/y}$ **correct**

3. $f_x = \left(\dfrac{x}{y} - 3\right) e^{x/y}$

4. $f_x = \left(\dfrac{x}{y} + 3\right) e^{x/y}$

5. $f_x = \left(\dfrac{2x}{y} - 1\right) e^{x/y}$

6. $f_x = \left(\dfrac{2x}{y} + 3\right) e^{x/y}$

Explanation:

Differentiating with respect to x keeping y fixed, we see that

$$f_x = 2e^{x/y} + \left(\frac{2x-y}{y}\right)e^{x/y}\,.$$

Consequently,

$$\boxed{f_x = \left(\frac{2x}{y}+1\right)e^{x/y}}\,.$$