This print-out should have 16 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

001 10.0 points

Which of the following statements are true for all lines and planes in 3-space?

I. *two lines parallel to a third line are parallel*,

II. *two planes perpendicular to a third plane are parallel*,

III. *two lines perpendicular to a plane are parallel.*

1. I and III only **correct**
2. all of them
3. I only
4. I and II only
5. II and III only
6. none of them
7. II only
8. III only

Explanation:

I. TRUE: each of the two lines has a direction vector parallel to the direction vector of the third line, so must be scalar multiples of each other.

II. FALSE: the xy-plane and yz-plane are both perpendicular to the xz-pane, but are perpendicular to each other, not parallel .

III. TRUE: the two lines will have direction vectors parallel to the normal vector of the plane, and so be parallel, hence the two lines are parallel.

002 10.0 points

Determine all unit vectors $\mathbf{v}$ orthogonal to

$$\mathbf{a} = 3\,\mathbf{i} + \mathbf{j} + 4\,\mathbf{k}, \quad \mathbf{b} = 3\,\mathbf{i} + 2\,\mathbf{j} + 6\,\mathbf{k}\,.$$

1. $\mathbf{v} = \pm\Big(\frac{2}{7}\mathbf{i} + \frac{6}{7}\mathbf{j} - \frac{3}{7}\mathbf{k}\Big)$ **correct**
2. $\mathbf{v} = \frac{2}{7}\mathbf{i} - \frac{3}{7}\mathbf{j} + \frac{6}{7}\mathbf{k}$
3. $\mathbf{v} = 2\,\mathbf{i} - 3\,\mathbf{j} + 6\,\mathbf{k}$
4. $\mathbf{v} = \pm\Big(\frac{2}{7}\mathbf{i} - \frac{3}{7}\mathbf{j} + \frac{6}{7}\mathbf{k}\Big)$
5. $\mathbf{v} = -\frac{2}{7}\mathbf{i} - \frac{6}{7}\mathbf{j} + \frac{3}{7}\mathbf{k}$
6. $\mathbf{v} = -2\,\mathbf{i} - 6\,\mathbf{j} + 3\,\mathbf{k}$

Explanation:

The non-zero vectors orthogonal to $\mathbf{a}$ and $\mathbf{b}$ are all of the form

$$\mathbf{v} = \lambda(\mathbf{a}\times\mathbf{b})\,, \quad \lambda \neq 0\,,$$

with λ a scalar. The only unit vectors orthogonal to $\mathbf{a}$, $\mathbf{b}$ are thus

$$\mathbf{v} = \pm\frac{\mathbf{a}\times\mathbf{b}}{\|\mathbf{a}\times\mathbf{b}\|}\,.$$

But for the given vectors $\mathbf{a}$ and $\mathbf{b}$,

$$\mathbf{a}\times\mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 1 & 4 \\ 3 & 2 & 6 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 4 \\ 2 & 6 \end{vmatrix}\mathbf{i} - \begin{vmatrix} 3 & 4 \\ 3 & 6 \end{vmatrix}\mathbf{j} + \begin{vmatrix} 3 & 1 \\ 3 & 2 \end{vmatrix}\mathbf{k}$$

$$= -2\,\mathbf{i} - 6\,\mathbf{j} + 3\,\mathbf{k}\,.$$

In this case,

$$\|\mathbf{a}\times\mathbf{b}\|^2 = 49\,.$$

Consequently,

$$\boxed{\mathbf{v} = \pm\Big(\frac{2}{7}\mathbf{i} + \frac{6}{7}\mathbf{j} - \frac{3}{7}\mathbf{k}\Big)}\,.$$

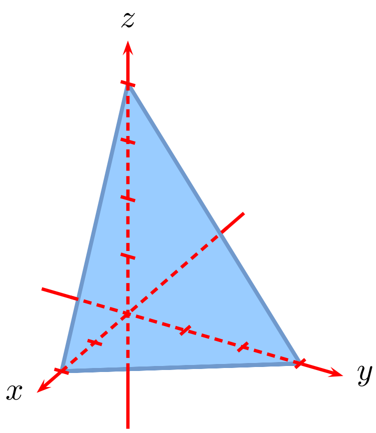
003 10.0 points

Which of the following surfaces is the graph of

$$6x + 4y + 3z \;=\; 12$$

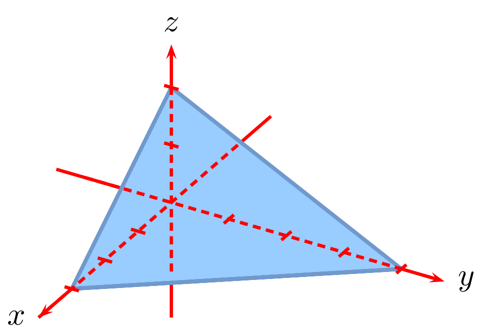
in the first octant?

1.



correct

2.



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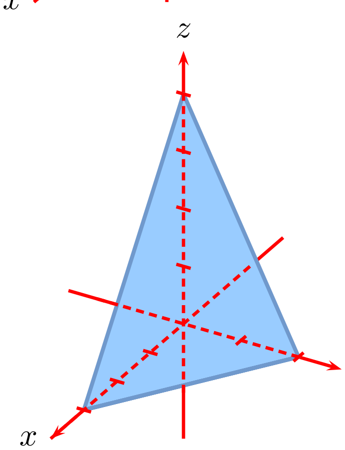

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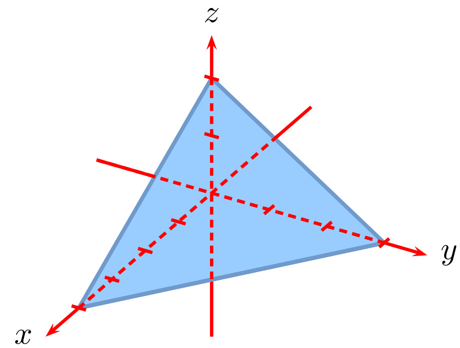

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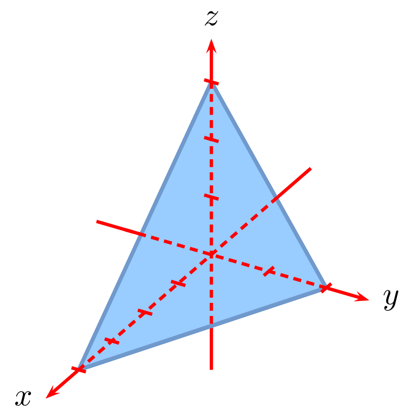

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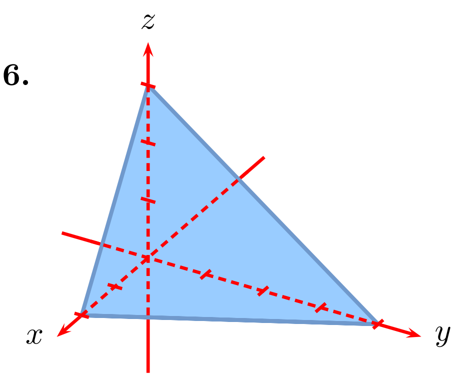
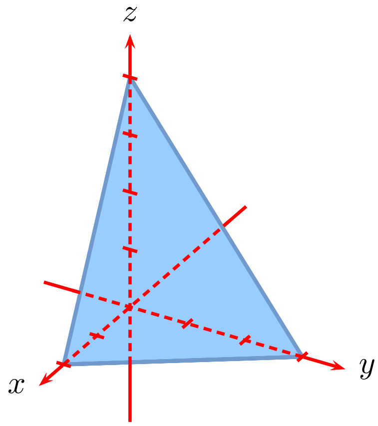


Explanation:

Since the equation is linear, it's graph will be a plane. To determine which plane, we have only to compute the intercepts of

$$6x + 4y + 3z \;=\; 12\,.$$

Now the x-intercept occurs at $y = z = 0$, *i.e.* at $x = 2$; similarly, the y-intercept is at $y = 3$, while the z-intercept is at $z = 4$. By inspection, therefore, the graph is



004 10.0 points

Find parametric equations for the line passing through the point $P(3, -2, 3)$ and perpendicular to the plane

$$x + 3y - 2z = 6.$$

1. $x = 3 - t, \quad y = 2 - 3t, \quad z = 3 - 2t$

2. $x = 3 + t, \quad y = -2 + 3t, \quad z = 3 - 2t$ **correct**

3. $x = 1 - 3t, \quad y = -3 + 2t, \quad z = -2 + 3t$

4. $x = 1 + 3t, \quad y = 3 + 2t, \quad z = 2 - 3t$

5. $x = -3 + t, \quad y = 2 + 3t, \quad z = -3 - 2t$

6. $x = 1 + 3t, \quad y = 3 - 2t, \quad z = -2 + 3t$

Explanation:

A line passing through a point $P(a, b, c)$ and having direction vector $\mathbf{v}$ is given parametrically by

$$\mathbf{r}(t) = \mathbf{a} + t\mathbf{v}, \quad \mathbf{a} = \langle a, b, c \rangle.$$

Now for the given line, its direction vector will be parallel to the normal to the plane

$$x + 3y - 2z = 6.$$

Thus

$$\mathbf{a} = \langle 3, 2, 3, \rangle, \qquad \mathbf{v} = \langle 1, 3, -2 \rangle,$$

and so

$$\mathbf{r}(t) = \langle 3 + t, 2 + 3t, 3 + 2t \rangle.$$

Consequently,

$$\boxed{x = 3 + t, \quad y = 2 + 3t, \quad z = 3 + 2t}$$

are parametric equations for the line.

005 10.0 points

Find parametric equations for the line through the point $P(5, 5, 4)$ that is parallel to the plane $x + y + z = 3$ and perpendicular to the line

$$x = 3 + t, \quad y = 4 - t, \quad z = 3t.$$

1. $x = 5 + 4t, \quad y = 5 + 2t, \quad z = 4 + t$

2. $x = 5 - 4t, \quad y = 5 + 2t, \quad z = 4 + t$

3. $x = 5 - 4t, \quad y = 5 + 2t, \quad z = 4 - 2t$

4. $x = 5 + 4t, \quad y = 5 - 2t, \quad z = 4 - 2t$ **correct**

5. $x = 5 + t, \quad y = 5 + t, \quad z = 4 - 2t$

Explanation:

Two vectors which are perpendicular to the required line are the normal, $\langle 1, 1, 1 \rangle$, of the given plane and a direction vector, $\langle 1, -1, 3 \rangle$, for the given line. So a direction vector for the required line is

$$\langle 1, 1, 1 \rangle \times \langle 1, -1, 3 \rangle = \langle 4, -2, -2 \rangle.$$

Thus L is given by

$$\langle x, y, z \rangle = \langle 5, 5, 4 \rangle + t \langle 4, -2, -2 \rangle,$$

which can be written in parametric form as

$$\boxed{x = 5 + 4t, \quad y = 5 - 2t, \quad z = 4 - 2t}.$$

006 10.0 points

Find an equation for the plane passing through the points

$$Q(-2, -1, -1), \qquad R(0, -2, -1),$$
$$S(-5, -1, -3).$$

1. $2x - 4y + 3z + 5 = 0$

2. $2x + 4y - 3z + 5 = 0$ **correct**

3. $2x + 3y - 4z - 5 = 0$

4. $2x + 4y - 3z - 5 = 0$

5. $2x - 3y - 4z - 5 = 0$

6. $2x - 3y - 4z + 5 = 0$

Explanation:

Since the points Q, R, and S lie in the plane, the displacement vectors

$$\overrightarrow{QR} = \langle 2, -1, 0 \rangle,$$

$$\overrightarrow{QS} = \langle -3, 0, -2 \rangle,$$

lie in the plane. Thus the cross product

$$\mathbf{n} = \overrightarrow{QR} \times \overrightarrow{QS} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -1 & 0 \\ -3 & 0 & -2 \end{vmatrix}$$

is normal to the plane.

On the other hand, if $P(x, y, z)$ is an arbitrary point on the plane, then the displacement vector

$$\mathbf{v} = \overrightarrow{PQ} = \langle x+2, y+1, z+1 \rangle$$

lies in the plane, so

$$\mathbf{v} \cdot \mathbf{n} = 0.$$

Now

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -1 & 0 \\ -3 & 0 & -2 \end{vmatrix} = \langle 2, 4, -3 \rangle.$$

But then

$$\mathbf{v} \cdot \mathbf{n} = 2(x+2) + 4(y+1) - 3(z+1)$$

$$= 2x + 4y - 3z + 5 = 0.$$

Consequently, the plane

$$\boxed{2x + 4y - 3z + 5 = 0}$$

passes through Q, R and S.

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keywords: plane, cross product, plane determined by three points, dot product

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007 10.0 points

Find an equation for the plane passing through the point $P(-1, -1, -1)$ and parallel to the plane

$$3x + 2y + z = 4.$$

1. $x + 3y + 2z = -6$

2. $2x + y + 3z = -10$

3. $x + 3y + 2z = -10$

4. $3x + 2y + z = -10$

5. $3x + 2y + z = -6$ **correct**

6. $2x + y + 3z = -6$

Explanation:

The scalar equation for the plane through $P(a, b, c)$ with normal vector

$$\mathbf{n} = A\mathbf{i} + B\mathbf{j} + C\mathbf{k}$$

is

$$A(x-a) + B(y-b) + C(z-c) = 0.$$

In this question

$$P(a, b, c) = (-1, -1, -1),$$

while

$$\mathbf{n} = 3\mathbf{i} + 2\mathbf{j} + \mathbf{k}$$

if the plane is parallel to

$$3x + 2y + z = 4$$

since parallel planes have parallel normal vectors.

Consequently, the plane has equation

$$\boxed{3x + 2y + z = -6}.$$

keywords: plane, normal vector, point on plane, scalar equation

008 10.0 points

Determine as a linear relation in x, y, z the plane given in vector form by

$$\mathbf{x} = \mathbf{a} + u\,\mathbf{b} + v\,\mathbf{c}$$

when

$$\mathbf{a} = \mathbf{i} - \mathbf{j} + 2\mathbf{k}, \quad \mathbf{b} = 2\mathbf{i} - \mathbf{j} - 2\mathbf{k}$$

and

$$\mathbf{c} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}.$$

1. $7x + 4y + 5z - 13 = 0$ **correct**

2. $3x + 4y - 5z - 13 = 0$

3. $7x - 4y - 5z - 13 = 0$

4. $3x - 4y + 5z + 13 = 0$

5. $3x - 4y - 5z + 13 = 0$

6. $7x + 4y + 5z + 13 = 0$

Explanation:

The plane

$$\mathbf{x} = \mathbf{a} + u\,\mathbf{b} + v\,\mathbf{c}$$

is the unique plane passing through the points P, Q, and R where

$$\overrightarrow{OP} = \mathbf{a}, \quad \overrightarrow{OQ} = \mathbf{a} + \mathbf{b}, \quad \overrightarrow{OR} = \mathbf{a} + \mathbf{c}.$$

Thus the vector $\mathbf{n} = \mathbf{b} \times \mathbf{c}$ is normal to the plane, and in point-normal form the plane is given by

$$(\mathbf{x} - \mathbf{a}) \cdot \mathbf{n} = 0.$$

Now

$$\mathbf{n} = \mathbf{b} \times \mathbf{c} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -1 & -2 \\ 1 & 2 & -3 \end{vmatrix}$$

$$= \begin{vmatrix} -1 & -2 \\ 2 & -3 \end{vmatrix} \mathbf{i} - \begin{vmatrix} 2 & -2 \\ 1 & -3 \end{vmatrix} \mathbf{j} + \begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix} \mathbf{k}.$$

Thus

$$\mathbf{n} = 7\,\mathbf{i} + 4\,\mathbf{j} + 5\,\mathbf{k}.$$

On the other hand,

$$\mathbf{x} - \mathbf{a} = (x\mathbf{i} + y\mathbf{j} + z\mathbf{k}) - (\mathbf{i} - \mathbf{j} + 2\mathbf{k})$$
$$= (x-1)\,\mathbf{i} + (y+1)\,\mathbf{j} + (z-2)\,\mathbf{k}.$$

Consequently,

$$(\mathbf{x} - \mathbf{a}) \cdot \mathbf{n} = 7x + 4y + 5z - (13),$$

so as a linear relation in x, y, z the plane is given by

$$\boxed{7x + 4y + 5z - 13 = 0}.$$

009 10.0 points

Describe the motion of a particle with position $P(x, y)$ when

$$x = 5\sin t, \quad y = 4\cos t$$

as t varies in the interval $0 \le t \le 2\pi$.

1. Moves once counterclockwise along the ellipse

$$\frac{x^2}{25} + \frac{y^2}{16} = 1,$$

starting and ending at (0, 4).

2. Moves along the line

$$\frac{x}{5} + \frac{y}{4} = 1,$$

starting at (0, 4) and ending at (5, 0).

3. Moves along the line

$$\frac{x}{5} + \frac{y}{4} = 1,$$

starting at (5, 0) and ending at (0, 4).

4. Moves once clockwise along the ellipse

$$(5x)^2 + (4y)^2 = 1,$$

starting and ending at (0, 4).

5. Moves once counterclockwise along the ellipse

$$(5x)^2 + (4y)^2 = 1,$$

starting and ending at (0, 4).

6. Moves once clockwise along the ellipse

$$\frac{x^2}{25} + \frac{y^2}{16} = 1,$$

starting and ending at (0, 4). **correct**

Explanation:

Since

$$\cos^2 t + \sin^2 t = 1$$

for all t, the particle travels along the curve given in Cartesian form by

$$\frac{x^2}{25} + \frac{y^2}{16} = 1;$$

this is an ellipse centered at the origin. At $t = 0$, the particle is at $(5\sin 0, 4\cos 0)$, *i.e.*, at the point (0, 4) on the ellipse. Now as t increases from $t = 0$ to $t = \pi/2$, $x(t)$ increases from $x = 0$ to $x = 5$, while $y(t)$ decreases from $y = 4$ to $y = 0$; in particular, the particle moves from a point on the positive y-axis to a point on the positive x-axis, so it is moving *clockwise.*

In the same way, we see that as t increases from $\pi/2$ to π, the particle moves to a point on the negative y-axis, then to a point on the negative x-axis as t increases from π to $3\pi/2$, until finally it returns to its starting point on the positive y-axis as t increases from $3\pi/2$ to 2π.

Consequently, the particle moves clockwise once around the ellipse

$$\boxed{\frac{x^2}{25} + \frac{y^2}{16} = 1},$$

starting and ending at (0, 4).

keywords: motion on curve, ellipse

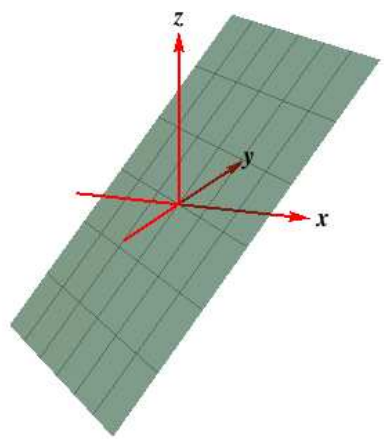
010 10.0 points

For which one of the following surfaces is

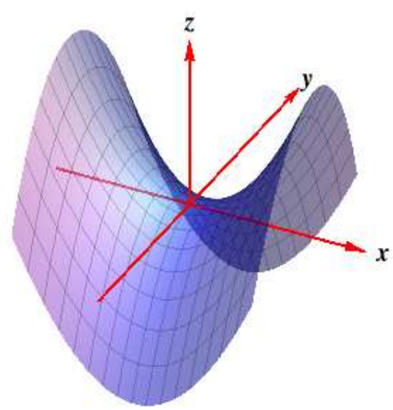
$$\mathbf{\Phi}(u, v) = u\cos v\,\mathbf{i} + u^2\,\mathbf{j} + u\sin v\,\mathbf{k}$$

a parametrization?

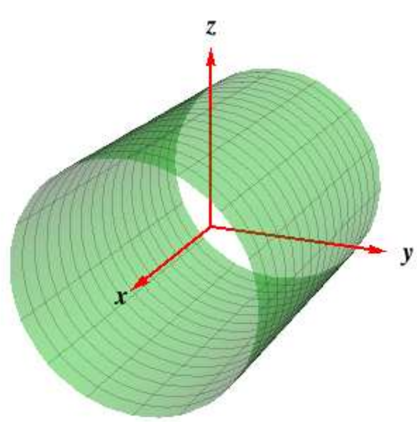
1.



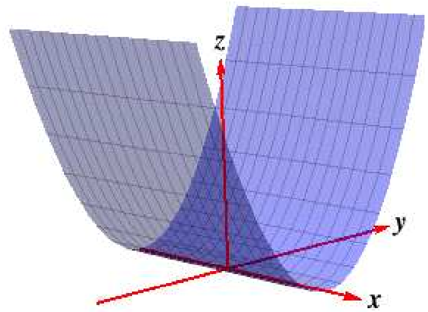
2.



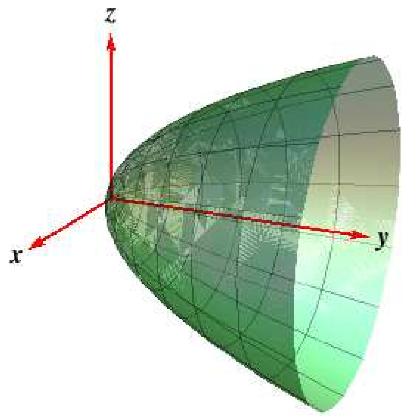
3.



4.



5. **correct**



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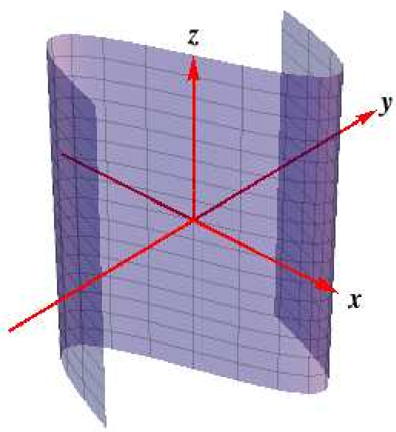


Explanation:

To determine the surface parametrized by

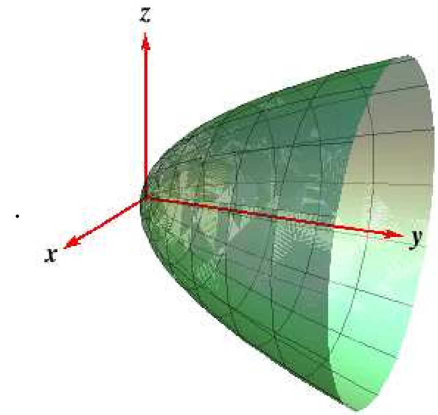
$$\mathbf{\Phi}(u,\, v) \;=\; u \cos v\, \mathbf{i} + u^2\, \mathbf{j} + u \sin v\, \mathbf{k}$$

we take slices parallel to a coordinate plane.

Now for fixed u the vertical plane $y = u$ parallel to the xz-plane intersects the surface in the curve parametrized by

$$\mathbf{\Phi}(u,\, v) \;=\; u \cos v\, \mathbf{i} + u^2\, \mathbf{j} + u \sin v\, \mathbf{k}$$

as v varies, *i.e.*, the circle $x^2 + z^2 = u^2$ whose radius increases as u increases. The only surface having this property is



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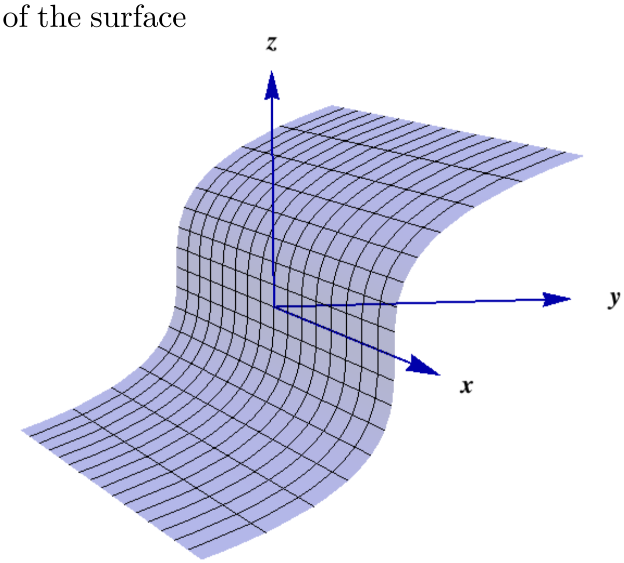
011 10.0 points

Which of the following is a parametrization of the surface



1. $\mathbf{\Phi} = (u,\, u + v,\, v)$

2. $\mathbf{\Phi} = (\cos u \sin v,\, 3 \cos u \sin v,\, \cos v)$

3. $\mathbf{\Phi} = (u,\, u \cos v,\, u \sin v)$

4. $\mathbf{\Phi} = (u,\, \cos v,\, \sin v)$

5. $\mathbf{\Phi} = (u,\, v^3,\, v)$ **correct**

Explanation:

Cross-sections of the surface perpendicular to the x-axis are the graph of the same cubic relation in y, z. The only parametrization with this property is

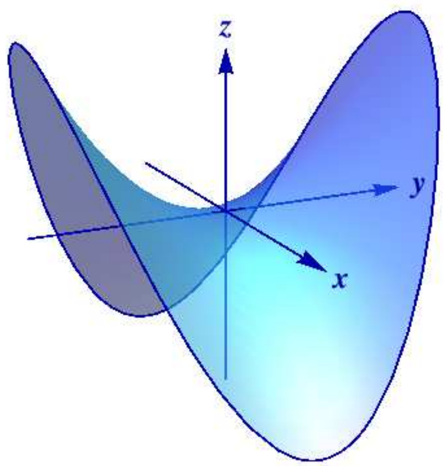
$$\boxed{\mathbf{\Phi}(u,\, v) \;=\; (u,\, v^3,\, v)}\,.$$

012 10.0 points

Express the graph of

$$z = y^2 - x^2, \quad x^2 + y^2 \le 9,$$

shown in



as a surface parametrized in terms of cylindrical polar coordinates.

1. For $0 \le r \le 3, \ 0 \le \theta \le 2\pi$,
$\mathbf{\Phi}(r, \theta) = (r\cos\theta, r\sin\theta, -r^2 \sin 2\theta)$

2. For $0 \le r \le 3, \ 0 \le \theta \le 2\pi$,
$\mathbf{\Phi}(r, \theta) = (r\cos\theta, r\sin\theta, -r^2 \cos 2\theta)$
correct

3. For $0 \le r \le 0, \ 0 \le \theta \le 2\pi$,
$\mathbf{\Phi}(r, \theta) = (r\cos\theta, r\sin\theta, r^2 \cos 2\theta)$

4. For $0 \le r \le 3, \ 0 \le \theta \le 2\pi$,
$\mathbf{\Phi}(r, \theta) = (r\cos\theta, r\sin\theta, r^2 \cos 2\theta)$

5. For $0 \le r \le 0, \ 0 \le \theta \le 2\pi$,
$\mathbf{\Phi}(r, \theta) = (r\cos\theta, r\sin\theta, r^2 \sin 2\theta)$

6. For $0 \le r \le 0, \ 0 \le \theta \le 2\pi$,
$\mathbf{\Phi}(r, \theta) = (r\cos\theta, r\sin\theta, -r^2 \sin 2\theta)$

Explanation:

In cylindrical polars the coordinates of a point $P(x, y, z)$ are given by

$$x = r\cos\theta, \quad y = r\sin\theta, \quad z = z.$$

But when

$$z = y^2 - x^2, \quad x^2 + y^2 \le 9,$$

then

$$z = r^2(\sin^2\theta - \cos^2\theta) = -r^2\cos 2\theta,$$

while

$$r^2 = x^2 + y^2 \le 9.$$

Consequently,

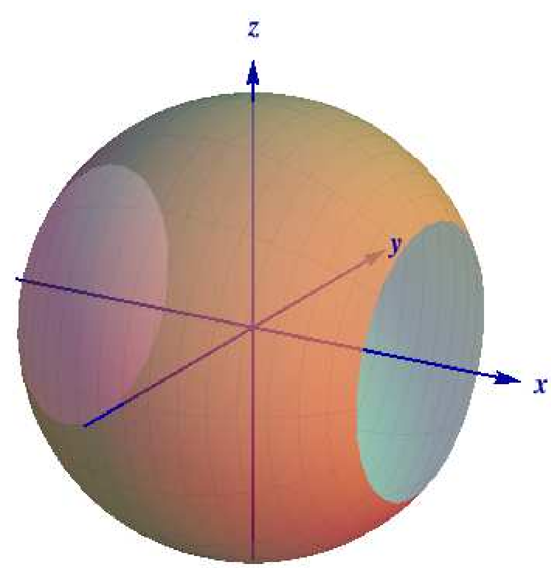
$$\boxed{\mathbf{\Phi}(r, \theta) = (r\cos\theta, r\sin\theta, -r^2\cos 2\theta)}$$

with

$$0 \le r \le 3, \quad 0 \le \theta \le 2\pi.$$

013 10.0 points

The surface S shown in



consists of the portion of the sphere

$$x^2 + y^2 + z^2 = 25$$

where

$$y^2 + z^2 \ge 9$$

Use spherical polar coordinates (ρ, θ, ϕ) to describe S.

1. $S =$ all points $P(3,\, \theta,\, \phi)\}$ with

$$0 \le \theta \le 2\pi, \;\; 0 \le \phi \le \pi, \;\; \sin^2\phi\cos^2\theta \le \frac{2}{5}\,.$$

2. $S =$ all points $P(3,\, \theta,\, \phi)\}$ with

$$0 \le \theta \le 2\pi, \;\; 0 \le \phi \le \pi, \;\; \sin^2\phi\sin^2\theta \le \frac{16}{25}\,.$$

3. $S =$ all points $P(3,\, \theta,\, \phi)\}$ with

$$0 \le \theta \le 2\pi, \;\; 0 \le \phi \le \pi, \;\; \cos^2\phi\cos^2\theta \le \frac{2}{5}\,.$$

4. $S =$ all points $P(5,\, \theta,\, \phi)$ with

$$0 \le \theta \le 2\pi, \;\; 0 \le \phi \le \pi, \;\; \sin^2\phi\cos^2\theta \le \frac{16}{25}\,.$$

correct

5. $S =$ all points $P(5,\, \theta,\, \phi)\}$ with

$$0 \le \theta \le 2\pi, \;\; 0 \le \phi \le \pi, \;\; \cos^2\phi\sin^2\theta \le \frac{2}{5}\,.$$

6. $S =$ all points $P(5,\, \theta,\, \phi)\}$ with

$$0 \le \theta \le 2\pi, \;\; 0 \le \phi \le \pi, \;\; \cos^2\phi\sin^2\theta \le \frac{16}{25}\,.$$

Explanation:

In spherical polar coordinates $(\rho,\, \theta,\, \phi)$,

$$x \;=\; \rho\sin\phi\cos\theta, \quad y \;=\; \rho\sin\phi\sin\theta\,,$$

and

$$z \;=\; \rho\cos\phi\,,$$

with $0 \le \theta \le 2\pi$ and $0 \le \psi \le \pi$. We need to find further restrictions on ρ, θ, and ϕ so that

$$x^2 + y^2 + z^2 \;=\; 25, \quad y^2 + z^2 \;\ge\; 9\,.$$

Now

$$\rho^2 \;=\; x^2 + y^2 + z^2 \;=\; 25\,,$$

i.e., $\rho = 5$. But then,

$$x^2 \;=\; 25\sin^2\phi\cos^2\theta \;=\; 25 - y^2 - z^2 \;\le\; 16\,.$$

Consequently, S consists of all points P with $\rho = 5$ and

$$\boxed{0 \le \theta \le 2\pi, \;\; 0 \le \phi \le \pi, \;\; \sin^2\phi\cos^2\theta \le \frac{16}{25}}\,.$$

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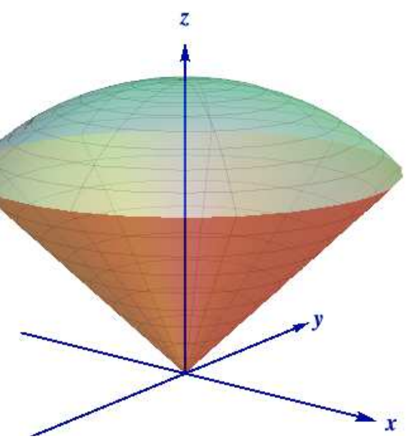
014 10.0 points

The solid W shown in



consists of all points enclosed by the sphere

$$x^2 + y^2 + z^2 \;=\; 1$$

and the cone

$$z^2 \;=\; 3(x^2 + y^2), \quad z \ge 0\,.$$

Describe W as a set of points $\{(\rho,\, \theta,\, \phi)\}$ in spherical polar coordinates.

1. $0 \le \rho \le 4,\; 0 \le \theta \le 2\pi,\; 0 \le \phi \le \dfrac{\pi}{3}$

2. $0 \le \rho \le 1,\; 0 \le \theta \le 2\pi,\; 0 \le \phi \le \dfrac{\pi}{3}$

3. $0 \le \rho \le 4,\; 0 \le \theta \le 2\pi,\; 0 \le \phi \le \dfrac{\pi}{6}$

4. $0 \le \rho \le 4,\; 0 \le \theta \le 2\pi,\; 0 \le \phi \le \dfrac{\pi}{4}$

5. $0 \le \rho \le 1,\; 0 \le \theta \le 2\pi,\; 0 \le \phi \le \dfrac{\pi}{6}$

correct

6. $0 \le \rho \le 1,\ 0 \le \theta \le 2\pi,\ 0 \le \phi \le \dfrac{\pi}{4}$

Explanation:

In spherical coordinates $(\rho,\, \theta, \phi)$,

$$x \;=\; \rho \cos\theta \sin\phi, \quad y \;=\; \rho \sin\theta \sin\phi,$$

and

$$z \;=\; \rho \cos\phi\,.$$

So the cone

$$z^2 \;=\; 3(x^2 + y^2), \quad z \ge 0\,,$$

can be written as

$$\rho\cos\phi \;=\; \sqrt{3}\,\rho \sin\phi(\cos^2\theta + \sin^2\theta)^{1/2}\,;$$

in other words,

$$\tan\phi \;=\; \frac{1}{\sqrt{3}}\,, \qquad \phi \;=\; \frac{\pi}{6}\,.$$

Since $\phi = 0$ at the North Pole, W thus consists of all points $(\rho,\, \theta, \phi)$ such that

$$0 \le \rho \le 1, \quad 0 \le \theta \le 2\pi, \quad 0 \le \phi \le \frac{\pi}{6}\,.$$

$$\boxed{0 \le \rho \le 1,\ 0 \le \theta \le 2\pi,\ 0 \le \phi \le \frac{\pi}{6}}\,.$$

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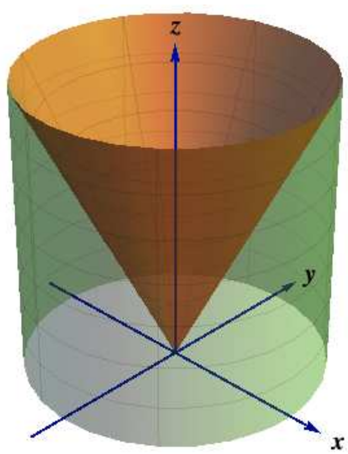
015 10.0 points

The solid W shown in



that lies above the xy-plane, below the cone

$$z^2 \;=\; 9x^2 + 9y^2\,,$$

and within the cylinder

$$x^2 + y^2 \;=\; 1\,.$$

Describe W as a set of points $\{(r,\, \theta,\, z)\}$ in cylindrical coordinates.

1. $0 \le r \le 1, \quad 0 \le \theta \le \pi, \quad 0 \le z \le 3r$

2. $0 \le r \le 3, \quad 0 \le \theta \le 2\pi, \quad 0 \le z \le 9r$

3. $0 \le r \le 1, \quad 0 \le \theta \le 2\pi, \quad 0 \le z \le 3r$ **correct**

4. $0 \le r \le 1, \quad 0 \le \theta \le 2\pi, \quad 0 \le z \le 9r$

5. $0 \le r \le 3, \quad 0 \le \theta \le \pi, \quad 0 \le z \le 9r$

6. $0 \le r \le 3, \quad 0 \le \theta \le \pi, \quad 0 \le z \le 3r$

Explanation:

In rectangular coordinates, W consists of all $(x,\, y,\, z)$ such that

$$x^2 + y^2 \le 1, \quad 0 \le z \le 3(x^2 + y^2)^{1/2}\,.$$

But in cylindrical coordinates $(r,\, \theta,\, z)$,

$$x \;=\; r\cos\theta, \quad y \;=\; r\sin\theta, \quad z \;=\; z\,,$$

so the cylinder becomes $r = 1$ while the cone becomes $z = 3r$.

Consequently, in cylindrical coordinates W consists of all points $(r,\, \theta,\, z)$ with

$$\boxed{0 \le r \le 1, \quad 0 \le \theta \le 2\pi, \quad 0 \le z \le 3r}\,.$$

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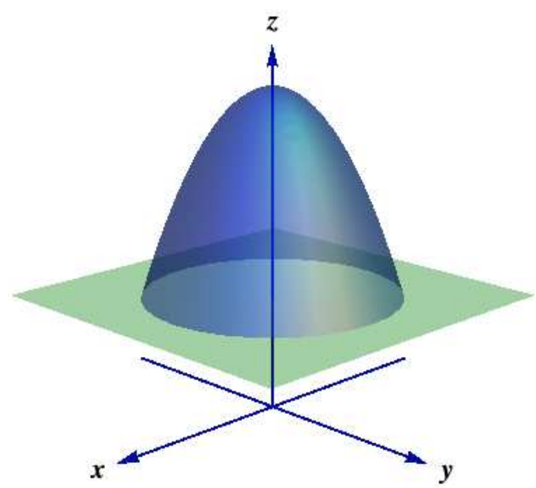
016 10.0 points

The solid W shown in



is bounded by the paraboloid

$$z = 11 - x^2 - y^2$$

and the plane $z = 2$. Describe W as a set of points $\{(r, \theta, z)\}$ in cylindrical coordinates.

1. $0 \le r \le 9, \ \ 0 \le \theta \le \pi, \ \ 2 \le z \le 11 - r^2$

2. $0 \le r \le 3, \ \ 0 \le \theta \le \pi, \ \ 2 \le z \le 11 - r^2$

3. $0 \le r \le 3, \ \ 0 \le \theta \le 2\pi, \ \ 2 \le z \le 11 - r^2$
correct

4. $0 \le r \le 3, \ \ 0 \le \theta \le 2\pi, \ \ 2 \le z \le 11 - r$

5. $0 \le r \le 9, \ \ 0 \le \theta \le 2\pi, \ \ 2 \le z \le 11 - r$

6. $0 \le r \le 9, \ \ 0 \le \theta \le \pi, \ \ 2 \le z \le 11 - r$

Explanation:

Since the plane $z = 2$ intersects the paraboloid

$$z = 11 - x^2 - y^2$$

when

$$2 = 11 - x^2 - y^2,$$

i.e., when $x^2 + y^2 = 9$. Thus in rectangular coordinates, W consists of all (x, y, z) such that

$$x^2 + y^2 \le 9, \quad 2 \le z \le 11 - x^2 - y^2 .$$

But in cylindrical coordinates (r, θ, z),

$$x = r\cos\theta, \quad y = r\sin\theta, \quad z = z .$$

Consequently, in cylindrical coordinates W consists of all points (r, θ, z) with

$$\boxed{0 \le r \le 3, \quad 0 \le \theta \le 2\pi, \quad 2 \le z \le 11 - r^2}.$$