Lab Notes: Lab 3

Lab: Seeing is Believing (In Uncertainty Propagation)
Date: Oct 2, 2025
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Part 1
Methods
In this experiment we will be investigating a model to see how well it describes image formation by a convex lens. First we will be investigating if *P* is truly constant for various d_o and d_i, object distance and image distance. Once finding an accurate model, we will then want to measure *P* at the lowest possible uncertainty. P represents optical power and is found through the following equation: *P* = $\frac{1}{do} + \frac{1}{di}$, where *P* is measured in diopters. When designing our experiment, we prioritized minimizing uncertainty by finding a range of values where the image would be distinguishably sharp. We utilized the PASCO light source, the PASCO optics track, and a ruler. The optics track ensured precise alignment and reproducible placement of each component along the optical axis. The experimental systems that we used were the convex lens, and image screen. We hypothesized that *P* would stay constant for different combinations of d_o and d_i. We set up the image screen on one end of the PASCO optics track and the PASCO light source at different distances from the image screen. The object distances were as follows: 36.5 cm, 30.2 cm, 32.4 cm, 28.5 cm, and 45.8 cm. Each measurement was repeated to estimate uncertainty in distance readings. From the measured values of d_o and $d_{i,}$ we calculated *P* for each trial and compared the results to determine if the optical power remained consistent across all measurements. Everything was held constant other than the object distance, which was moving the light source different distances away from the convex lens.

- **Equipment**
    - Multicolor laser - helps make the light path straight through the center of the lens to minimize alignment error
    - Image Screen - used to get the focused image to determine distance
    - PASCO optics track - provides a fixed and aligned setup for measuring distances and images
    - PASCO light source - used a beam of light to form clean images through lens
    - PASCO convex lenses (+60 mm focal length)
    - Meterstick - used to record distances along the optics track
- **Variables**
    - Independent Variables: object distance
    - Dependent Variables: image distance
    - Constant Variables: lens type, light intensity, track alignment, and environment
- **Statistical and Analytical Tools**
    - T-score
        - $t = \frac{|P_{comb,meas} - P_{comb,theory}|}{\sqrt{\delta P_{comb,meas}^{2} + \delta P_{comb,theory}^{2}}}$
        - Used to compare each measured P initial with the average P
    - Propagation of Uncertainty

- $\delta P = \sqrt{(\frac{\delta d_o}{d_0^2})^2 + (\frac{\delta d_i}{d_i^2})^2}$
- Used to calculate uncertainties in measuring distances

Data

Trials	do (cm)	Sd0 (cm)	di (cm)	Sdi (cm)
1	36.5	36-37	45	44.4-45.9
2	30.2	91.6-92.7	62.2	61.7-63.3
3	32.4	87-85.8	54.1	53.4-55.1
4	28.5	99.5-98.5	70	69.5-71.6
5	45.8	82.6-81	36	37.8- 35.1

Trials	P (D) ($\frac{1}{cm}$)	SP(D) ($\frac{1}{cm}$)
1	0.0496194825	0.000527
2	0.0491897532	0.000638
3	0.04934848589	0.000641
4	0.04937343358	0.000652
5	0.04961183891	0.001109
Mean	0.04942859882	
Standard Deviation	0.0001847444353	
Uncertainty	0.00008262022317	

Trials	t-score	Decision
1	0.358	Indistinguishable
2	0.372	Indistinguishable
3	0.124	Indistinguishable
4	0.084	Indistinguishable
5	0.165	Indistinguishable

Sample Calculation

Given d_o and δd_o, and d_i and δd_i:

$$P = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{36.5} + \frac{1}{45.15} = 0.02740 + 0.02215 = 0.04955\, cm^{-1} = 4.955\, D$$

$$\delta P = \sqrt{(\frac{\delta d_o}{d_0^2})^2 + (\frac{\delta d_i}{d_i^2})^2} = \sqrt{(\frac{0.50}{36.5^2})^2 + (\frac{0.75}{45.15^2})^2} = 0.000526\, cm^{-1} = 0.0526\, D$$

Therefore,

$P = (0.04955 \pm 0.00053)\, cm^{-1}$ or $P = (4.955 \pm 0.053)\, D$

Conclusion

We concluded that using the thin-lens relation $P = \frac{1}{d_o} + \frac{1}{d_i}$, our five trials yielded mutually consistent optical-power estimates: every trial's t-score relative to the means was < 1, so no trial is statistically distinguishable from the mean within uncertainty. The weighted result for the lens is $P = 4.943 \pm 0.008\, D$ which corresponds to a focal length $f = \frac{1}{P} = 20.23 \pm 0.03\, cm$. Uncertainty was dominated by the finite "sharp image" windows in d_i, and to a lesser extent, d_o. Systemic effects likely include small misalignments of the optical axis, ruler, our subjective judgement of image sharpness. Overall, the agreement across trials and small scatter indicate our method is precise and the thin-lens model fits the data well. To reduce uncertainty in future iterations of this experiment, we recommend the use of finer distance steps, define the focus with a contrast/edge metric instead of by eye, and fit all data simultaneously with a linear model to extract P and its uncertainty from the dataset.

Part 2

Methods

For the second part of our experiment, we are designing a model that will further investigate a combination of lenses. We will be using the formula $P_{combined} = P_1 + P_2$ as a baseline model when designing a system. For Part 2 we will be utilizing the Eye Model rather than the PASCO convex lens and image screen. The Eye Model apparatus includes a movable retina screen to simulate focusing at different distances. Corrective lenses with different focal lengths will be used to correct the blurry image when the retina screen is either at far sighted or near sighted distances. We will be using the same technique as from Part 1 when finding a range of sharpness that was distinguishable from when the image began to get blurry, measuring with a ruler. We tested four corrective lenses: two positive lenses (+120mm and +400mm) for far-sighted trials, and two negative lenses (-128mm and -1000mm) for near-sighted trials. Each lens was tested in two trials to ensure consistency and account for observational variability. We did find that the -128mm lens was out of range at all distances for the near sided trials, meaning it could not produce a focused image within the limits of adjustment of the model. We hypothesize that the model for power addition will hold true when summing the optical power for the eye and the lens for both near sighted and far sighted. Using the measurements of focal positions and the uncertainty of the focal lengths, we will be able to determine how true our model holds to the thin-lens relationship. For analysis, we compared each lens's theoretical optical power $P = 1/f$ with the measured model and the calculated combined from objects and image distances

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  - Image Screen - used to get the focused image to determine distance
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  - PASCO light source - used a beam of light to form clean images through lens
  - PASCO convex lenses (+60 mm focal length)
  - Meterstick - used to record distances along the optics track
- **Variables**
  - Independent Variables: lens configuration (normal, nearsighted, farsighted)
  - Dependent Variables: image position and combined optical power
  - Constant Variables: object distance, eye model water volume, light intensity, and environment
- **Statistical and Analytical Tools**
  - T-score
    - $t = \frac{|P_{comb,meas} - P_{comb,theory}|}{\sqrt{\delta P_{comb,meas}^{2} + \delta P_{comb,theory}^{2}}}$
    - Used to compare each measured P initial with the average P
  - Propagation of Uncertainty
    - $\delta P = \sqrt{\left(\frac{\delta d_o}{d_0^{2}}\right)^2 + \left(\frac{\delta d_i}{d_i^{2}}\right)^2}$
    - Used to calculate uncertainties in measuring distances

Data

Eye-Only						
Conditions	d0	Sd0	di	Sdi	P(D)	SP(D)
Normal	31.5	31.4-31.6	13	12.9-13.1	0.108669	0.0006

Corrective Lens									
Conditions	f	P=1/f	SP(D)	d0	Sd0	di	Sdi	Pcomb/ measure (D)	Pcomb (D)
	120	0.008333 333333	1	12.5	11.3-13.3	8.5	0.68	19.765	11.7
	120	0.008333 333333	1.3	12.4	10.8-13.4	8.5	0.89	19.829	11.7
	400	0.0025	3.15	32.2	29.7-36	8.5	0.83	14.87	11.117

Far

	400	0.0025	3	31.8	28.5-34.5	8.5	0.80	14.909	11.117
	-1000	-0.001	3.65	33.4	28.3-35.6	11.4	1.25	11.766	10.767
	-1000	-0.001	4.25	33.3	28.5-37	11.4	1.25	11.764	10.767
Near	-128	out of range							

Sample Calculation

Eye-only Power

$$P_{eye} = \frac{1}{d_0} + \frac{1}{d_i} = \frac{1}{31.5} + \frac{1}{13} = 0.108669\ cm^{-1} = 10.867\ D$$

$$\delta P_{eye} = \sqrt{\left(\frac{\delta d_0}{d_0^2}\right)^2 + \left(\frac{\delta d_i}{d_i^2}\right)^2} = \sqrt{\left(\frac{0.10}{31.5^2}\right)^2 + \left(\frac{0.10}{13^2}\right)^2} = 0.000600\ cm^{-1} = 0.060\ D$$

$$P_{eye} = (0.108669 \pm 0.000600)\ cm^{-1} = (10.867 \pm 0.060)\ D$$

Lens Power

$$P_{lens} = \frac{1}{f},\ f = 120;\ P_{lens} = \frac{1}{120} = 0.008333\ cm^{-1} = 0.833\ D$$

$$P_{comb,theory} = P_{eye} + P_{lens} = 0.108669 + 0.008333 = 0.117002\ cm^{-1} = 11.7\ D$$

$$\delta P_{comb,theory} \approx \delta P_{eye} = 0.000600\ cm^{-1} = 0.060\ D$$

$$P_{comb,meas} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{12.5} + \frac{1}{8.5} = 0.08 + 0.117647 = 0.197647\ cm^{-1} = 19.765\ D$$

$$\delta P_{comb,meas} = \sqrt{\left(\frac{\delta d_o}{d_o^2}\right)^2 + \left(\frac{\delta d_i}{d_i^2}\right)^2}$$

$$t = \frac{|P_{comb,meas} - P_{comb,theory}|}{\sqrt{\delta P_{comb,meas}^2 + \delta P_{comb,theory}^2}}\ \text{(t-score)}$$

Conclusion

We concluded that the model $P_{combined} = P_1 + P_2$ represents how optical power is demonstrated in the eye-model system within limits of uncertainty. Using the results, we compared each lens's theoretical optical power $P = 1/f$ with the measured model and the calculated combined from objects and image distances. The positive lenses within the far-sighted trials increased the total optical power while the negative lenses in the near-sighted trials decreased the total optical power. Specifically, the +120mm lens was 8D higher than the calculated while the +400mm was 3-4D higher which is within the propagated uncertainty for error. The -1000m lens was about 1 D difference while -128D was out of range. The propagated uncertainty was about +/- 1D which means the measured data agrees within error. Some errors in this experiment included small misalignments between the light source, lens, and eye model,. In addition, slight measurement uncertainty (±0.8–1.0 cm) in object and image distances caused noticeable variation in the calculated optical power values. To reduce uncertainty in future iterations of this experiment, we recommend the use of finer distance steps, define the focus with a contrast/edge metric instead of by eye, and fit all data simultaneously with a linear model to extract P and its uncertainty from the dataset.