

# Synthesizing Optimized Programs

## 1 Introduction

Our dependence on technology has increased exponentially in recent decades. We use technology in every aspect of our life, from daily mundane tasks like checking the news to life critical and expensive tasks like flying airplanes. The complexity of hardware and software also increased as a result. As we depend more on technology, there is an increasing need to ensure that those ever more complex technologies work as intended, especially in critical applications. To address this concern, engineers utilize *Program Verification* tools to formally (offering mathematical proofs) check if a program satisfies some constraints specified as first-order logic formulas. Those constraints are encodings of the relationships between the inputs and outputs of a program. Such tools have uncovered many bugs and raised the standard of reliability to a new level that is now required in certain industries. Program Verification techniques in some fields have advanced to the point where instead of verifying that a program satisfies some given correctness specifications, they can synthesize a program which satisfies those specifications by construction.

Program Synthesis has the potential to impact the quality of software even more than program verification. Program Synthesis, however, is a much harder problem to solve with a high potential for a tool to get stuck on a wrong part of the infinite search space (TODO: give a simple example). To guide the tool and limit the search space the tools are often also provided with a set of syntactic operators (i.e., a context-free grammar) it can compose together to generate programs in the search space (set of allowed programs). The original set of semantic constraints combined with the new set of syntactic constraints constitute a Syntax-Guided Synthesis (SyGuS) Problem.

## 2 Motivating example: matrix multiplication

Assume that we want to write a program that multiplies two matrices efficiently. Not only do we care about its functional correctness, but also about its running time. The problem can be described as: given  $2 \times 2$  matrices  $A$  and  $B$ , then find a program that efficiently computes the product defined as

$$AB = \begin{bmatrix} a_{11} * b_{11} + a_{12} * b_{21} & a_{11} * b_{12} + a_{12} * b_{22} \\ a_{21} * b_{11} + a_{22} * b_{21} & a_{21} * b_{12} + a_{22} * b_{22} \end{bmatrix} \quad (1)$$

Using SyGuS solver to synthesize a solution, we can use the following simple grammar:

$$T ::= a_{ij} \mid b_{ij}, \quad I ::= T \mid I + I \mid I - I \mid I * I, \quad S ::= \begin{bmatrix} I & I \\ I & I \end{bmatrix}$$

where  $S$  is the start symbol. SyGuS solver returns a naive implementation that performs the exact operation in the formulation using 8 multiplications and 4 additions. Multiplication, however, is known to be more computationally expensive than addition. So, we would prefer implementations that reduce the number of multiplications like Strassen's algorithm, which performs only 7 multiplications, over the naive version.

Similar problems occur in other synthesis applications: minimizing the number of logic gates in circuits, aggregate function applications in SQL queries, etc.

### 3 Proposal

In this project, our main goal is to equip SyGuS solvers with extra functionality that can generate optimal programs according to some given cost constraints. First, we need to communicate to the solver the cost of each term in the search space. We can do so by adding *weight annotations* to the grammar rules of the original formulation of the SyGuS problem. For the matrix multiplication example, the extension would look like so

$$T ::= a_{ij}^0 \mid b_{ij}^0, \quad I ::= T \mid I +^1 I \mid I -^1 I \mid I *^{16} I, \quad S ::= \begin{bmatrix} I & I \\ I & I \end{bmatrix}^0$$

We then define the *total weight* of the program to be the sum of the individual weights of each rule applied to construct the program. For example,  $weight(a_{11} * b_{11} + a_{12} * b_{21}) = 4 * 0 + 1 * 1 + 2 * 16 = 33$ .

Adding weight annotations to grammar rules changes the original SyGuS problem into an optimization problem: synthesize a program that satisfies the (1) semantic (2) syntactic constraints while minimizing the total (3) weight. Our proposal is to solve this new problem by extending current strategies to account for weights. We explore two strategies

**modify enumerators to generate terms with increasing weights:** current enumerators are designed to enumerate all terms in the language specified by the grammar. They start by enumerating the simplest terms (e.g., terminal symbols) and then iteratively apply production rules on them. Our aim here is for the enumerators to generate terms in order of increasing weights. This modification will force SyGuS solvers to generate minimal weight solutions that satisfy the provided semantic constraints. This change can be thought of as an extension of the current behavior of enumerators, which can be recovered by simply assigning terminal symbols a weight of 0 and other rules a weight of 1.

**improve unification to generate terms with decreasing weights:** unification is a process of pattern matching the structure of the desired solution against the grammar rules. If a match succeeds, the SyGuS problem is then divided into smaller sub-problems that are easier to solve. For matrix multiplication problem, we know that the solution is a  $2 \times 2$  matrix. So, instead of enumerating all  $2 \times 2$  matrices

$$\begin{bmatrix} a_{11} & a_{11} \\ a_{11} & \textcolor{blue}{a_{11}} \end{bmatrix}, \begin{bmatrix} a_{11} & a_{11} \\ a_{11} & \textcolor{blue}{a_{12}} \end{bmatrix}, \begin{bmatrix} a_{11} & a_{11} \\ a_{11} & \textcolor{blue}{a_{21}} \end{bmatrix}, \dots, \quad (2)$$

we can “pattern match” the rule  $\begin{bmatrix} I & I \\ I & I \end{bmatrix}$  against  $AB$  and try to solve the sub-problems

$$\begin{aligned} mul(A, B)_{11} &= a_{11} * b_{11} + a_{12} * b_{21} & mul(A, B)_{12} &= a_{11} * b_{12} + a_{12} * b_{22} \\ mul(A, B)_{21} &= a_{21} * b_{11} + a_{22} * b_{21} & mul(A, B)_{22} &= a_{21} * b_{12} + a_{22} * b_{22}. \end{aligned}$$

Solving these sub-problems using enumeration is orders-of-magnitude faster than enumerating (2). State of the art solvers utilize this technique to trim the size of large search spaces. However, it not clear to us yet how to support weights with unification. For example, following greedy approach by choosing minimum rules for pattern matching may not lead us to a

minimal solution. One approach to resolve that might be to ignore weights and find an initial solution to the problem using unification. Then, iteratively use enumeration over the initial solution to reduce the weights of the sub-terms. It might be clear how such iterative process may terminate for simple problems, but it gets more difficult with more complex problems. For example, such solution does not scale for problems like matrix multiplication. instead, we can use some heuristics to recover termination for these type of problems (e.g., rules with 0 weight like  $\begin{bmatrix} I & I \\ I & I \end{bmatrix}^0$ ).