

LAB2**Student experiment 1:**

a) $y[n]=x[n]-x[n-1]-x[n-2]$

Linearity:

$$x_1[n] \rightarrow y_1[n] = x_1[n] - x_1[n-1] - x_1[n-2]$$

$$x_2[n] \rightarrow y_2[n] = x_2[n] - x_2[n-1] - x_2[n-2]$$

$$A.y_1[n] + B.y_2[n] = A.(x_1[n] - x_1[n-1] - x_1[n-2]) + B.(x_2[n] - x_2[n-1] - x_2[n-2]) \rightarrow (1)$$

$$x_3[n] = A.x_1[n] + B.x_2[n] \rightarrow y_3[n] = x_3[n] - x_3[n-1] - x_3[n-2]$$

$$y_3[n] = A.x_1[n] + B.x_2[n] - A.x_1[n-1] - B.x_2[n-1] - A.x_1[n-2] - B.x_2[n-2] \rightarrow (2)$$

since (1) = (2) then $y_3[n] = A.y_1[n] + B.y_2[n]$ then the system is **linear**.

Time Invariance:

$$x_1[n] \rightarrow y_1[n] = x_1[n] - x_1[n-1] - x_1[n-2] \rightarrow y_1[n-n_0] = x_1[n-n_0] - x_1[n-n_0-1] - x_1[n-n_0-2]$$

$$x_2[n] = x_1[n-n_0] \rightarrow y_2[n] = x_1[n-n_0] - x_1[n-n_0-1] - x_1[n-n_0-2]$$

since $y_2[n] = y_1[n]$ then the system is **time invariant**.

$$y[n]=\cos(x[n])$$

Linearity:

$$x_1[n] \rightarrow y_1[n] = \cos(x_1[n])$$

$$x_2[n] \rightarrow y_2[n] = \cos(x_2[n])$$

$$A.y_1[n] + B.y_2[n] = A.\cos(x_1[n]) + B.\cos(x_2[n]) \rightarrow (1)$$

$$x_3[n] = A.x_1[n] + B.x_2[n] \rightarrow y_3[n] = \cos(x_3[n])$$

$$y_3[n] = \cos(A.x_1[n] + B.x_2[n]) \rightarrow (2)$$

since (1) \neq (2) then $y_3[n] \neq A.y_1[n] + B.y_2[n]$ then the system is **nonlinear**.

Time Invariance:

$$x_1[n] \rightarrow y_1[n] = \cos(x_1[n]) \rightarrow y_1[n-n_0] = \cos(x_1[n-n_0])$$

$$x_2[n] = x_1[n-n_0] \rightarrow y_2[n] = \cos(x_1[n-n_0])$$

since $y_2[n] = y_1[n]$ then the system is **time variant**.

$$y[n] = nx[n]$$

Linearity:

$$x_1[n] \rightarrow y_1[n] = nx_1[n]$$

$$x_2[n] \rightarrow y_2[n] = nx_2[n]$$

$$A.y_1[n] + B.y_2[n] = A.nx_1[n] + B.nx_2[n] \rightarrow (1)$$

$$x_3[n] = A.x_1[n] + B.x_2[n] \rightarrow y_3[n] = nx_3[n]$$

$$y_3[n] = n(A.x_1[n] + B.x_2[n]) \rightarrow (2)$$

since $(1) = (2)$ then $y_3[n] = A.y_1[n] + B.y_2[n]$ then the system is **linear**.

Time Invariance:

$$x_1[n] \rightarrow y_1[n] = nx_1[n] \rightarrow y_1[n-n_0] = (n-n_0).x_1[n-n_0]$$

$$x_2[n] = x_1[n-n_0] \rightarrow y_2[n] = n.x_1[n-n_0]$$

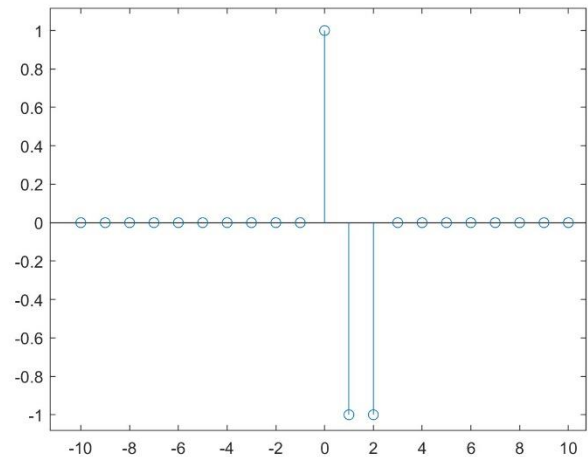
since $y_2[n] \neq y_1[n]$ then the system is **time variant**.

b) $y[n]=x[n]-x[n-1]-x[n-2]$

input = Delta(n)

```
>> nx1=[-10:10], x1=[zeros(1,10) 1 zeros(1,10)];  
>> y = zeros(1,length(x1));  
>> for c=3:length(x1)  
y(c) = x1(c)-x1(c-1)-x1(c-2)  
end  
>> stem(nx1, y)
```

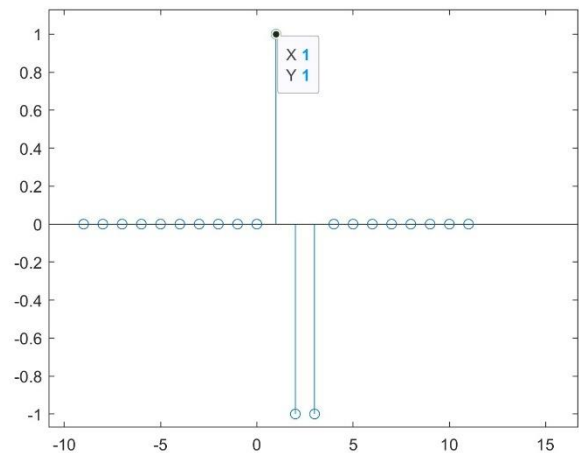
response



input = Delta(n-1)

```
>> nx1=[-10:10], x1=[zeros(1,10) 1 zeros(1,10)];  
>> y = zeros(1,length(x1));  
>> for c=3:length(x1)  
y(c) = x1(c)-x1(c-1)-x1(c-2)  
end  
>> stem(nx1+1, y)
```

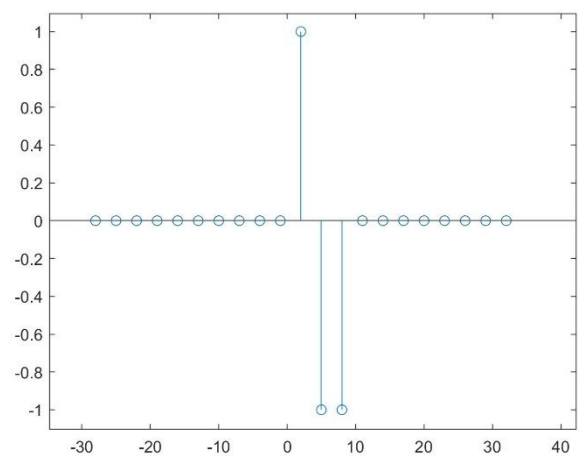
response



input = Delta(n)+2* Delta(n-1)

```
>> nx1=[-10:10], x1=[zeros(1,10) 1 zeros(1,10)];  
>> y = zeros(1,length(x1));  
>> for c=3:length(x1)  
y(c) = x1(c)-x1(c-1)-x1(c-2)  
end  
>> stem(nx1+2*(nx1+1), y)
```

response

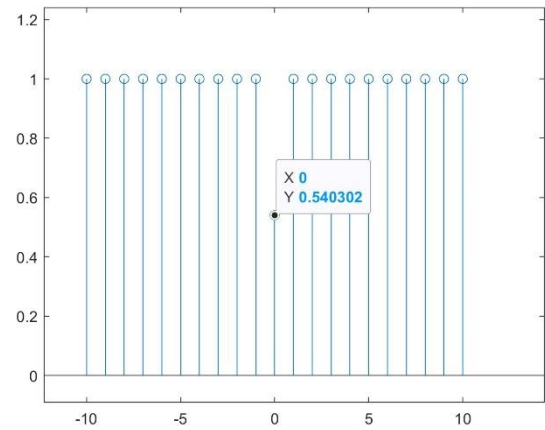


$$y[n] = \cos(x[n])$$

input = Delta(n)

```
>> nx1=[-10:10], x1=[zeros(1,10) 1 zeros(1,10)];
>> y = zeros(1,length(x1));
>> for c=1:length(x1)
y(c) = cos(x1(c));
end
>> stem(nx1,y)
```

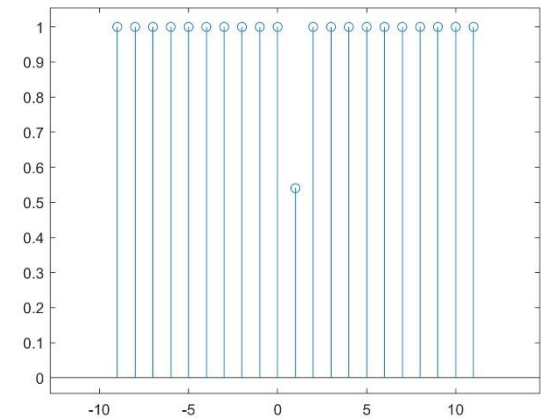
response



input = Delta(n-1)

```
>> nx1=[-10:10], x1=[zeros(1,10) 1 zeros(1,10)];
>> y = zeros(1,length(x1));
>> for c=1:length(x1)
y(c) = cos(x1(c));
end
>> stem(nx1+1,y)
```

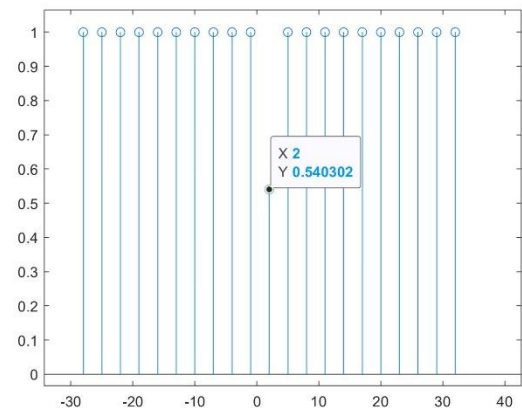
response



input = Delta(n)+2* Delta(n-1)

```
>> nx1=[-10:10], x1=[zeros(1,10) 1 zeros(1,10)];
>> y = zeros(1,length(x1));
>> for c=1:length(x1)
y(c) = cos(x1(c));
end
>> stem(nx1+2*(nx1+1), y)
```

response

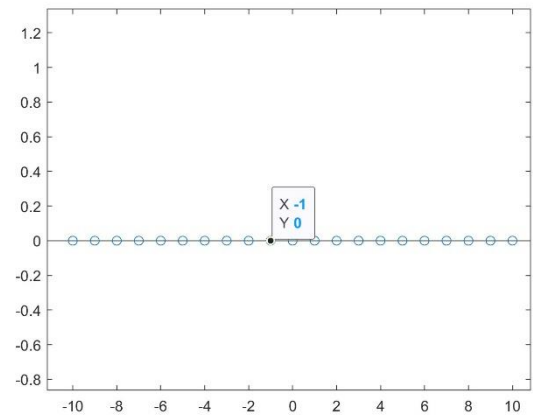


$$y[n] = nx[n]$$

input = Delta(n)

```
>> nx1=[-10:10], x1=[zeros(1,10) 1 zeros(1,10)];
>> y = zeros(1,length(x1));
>> for c=1:length(x1)
y(c) = nx1(c) * x1(c)
end
>> stem(nx1,y)
```

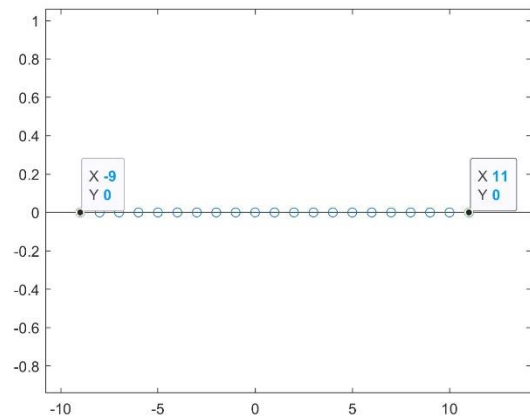
response



input = Delta(n-1)

```
>> nx1=[-10:10], x1=[zeros(1,10) 1 zeros(1,10)];
>> y = zeros(1,length(x1));
>> for c=1:length(x1)
y(c) = nx1(c) * x1(c)
end
>> stem(nx1+1,y)
```

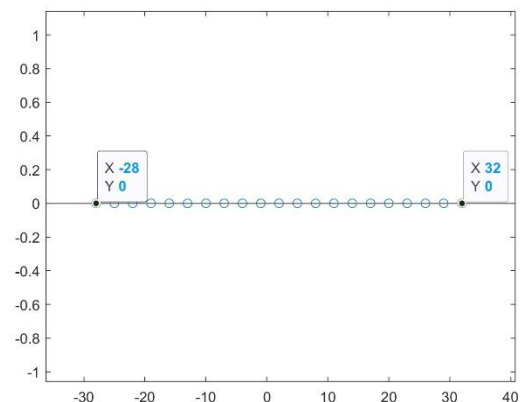
response



input = Delta(n)+2* Delta(n-1)

```
>> nx1=[-10:10], x1=[zeros(1,10) 1 zeros(1,10)];
>> y = zeros(1,length(x1));
>> for c=1:length(x1)
y(c) = nx1(c) * x1(c)
end
>> stem(nx1+2*(nx1+1), y)
```

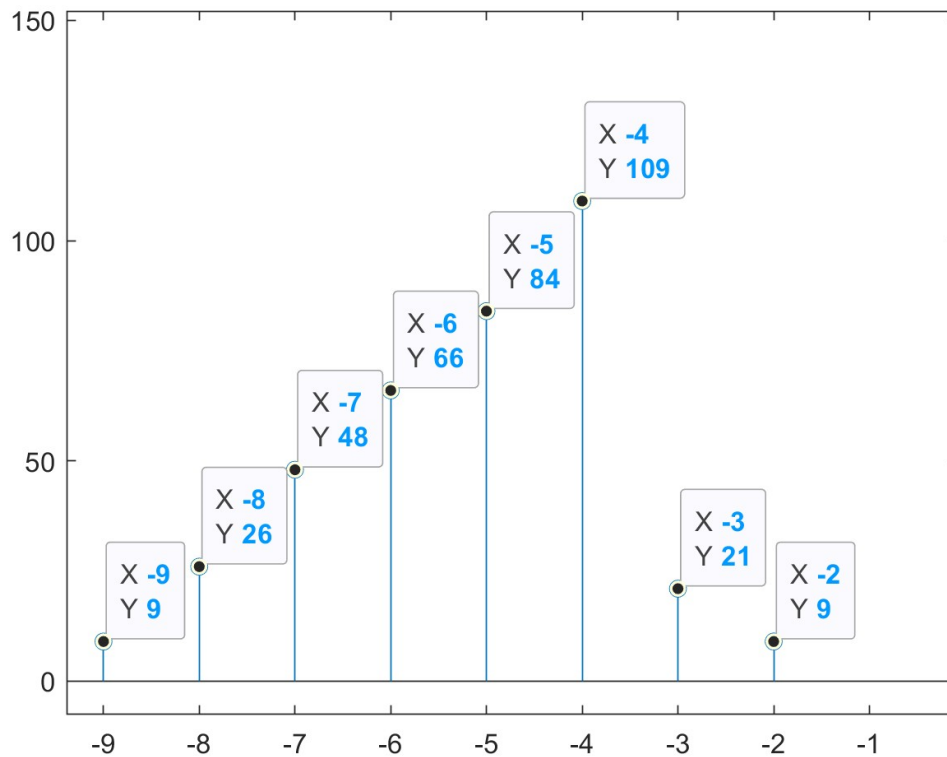
response



Student experiment 2:

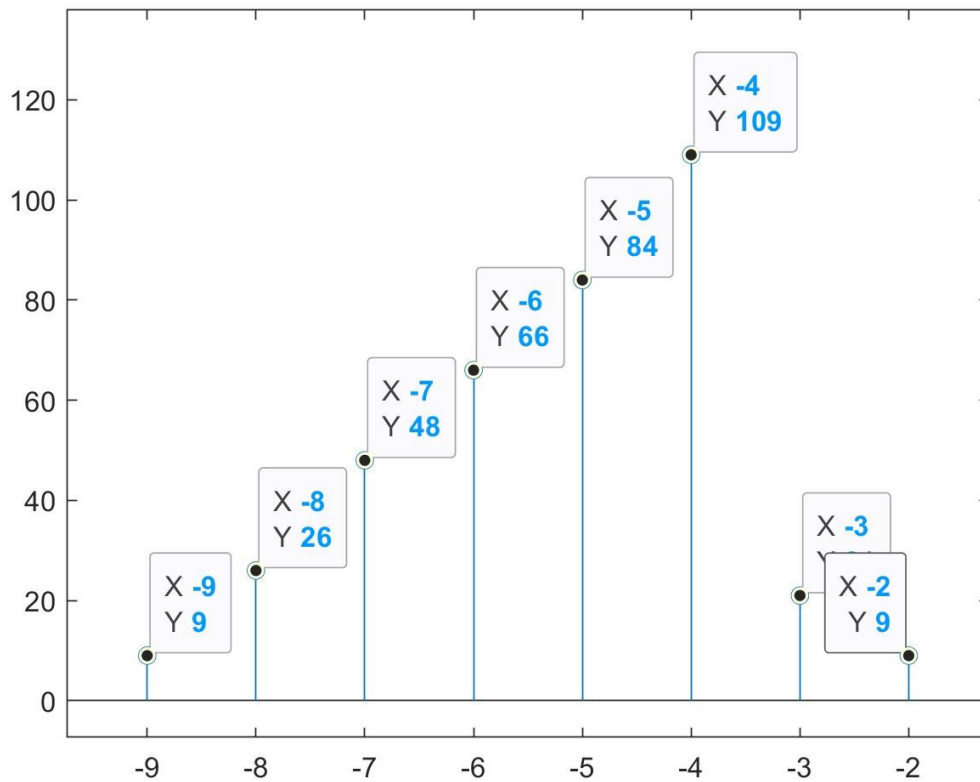
a)

```
>> nx=[-3 -2 -1];  
>> x=[1 2 3];  
>> nh=[-6 -5 -4 -3 -2 -1];  
>> h=[9 8 5 32 5 3];  
>> M=length(x);  
>> N=length(h);  
>> ny=[-9:-2]  
>> y=zeros(1,M+N-1);  
>> for u=1:(N)  
x1=h(u)*[zeros(1,u-1) x zeros(1,length(y)-u-2)];  
y=y+x1;  
end
```



b)

```
>> nx=[-3 -2 -1];  
>> x=[1 2 3];  
>> nh=[-6 -5 -4 -3 -2 -1];  
>> h=[9 8 5 32 5 3];  
>> M=length(x);  
>> N=length(h);  
>> ny=[-9:-2]  
>> y=conv(x,h);
```



Student experiment 3:

a) Inverse Fourier Series Code

```
function b=If_series(x)
N=length(x);
n=1:N;
b = zeros(1,N);
for k=1:N
b(k)=sum(x(k)*exp(-2*pi*i*k*n/N));
end
```

b) Fourier series of the three signals:

➤ $a_k = \frac{1}{T} \int_T x(t) e^{-jk\omega t} dt$

➤ $X(t) = \sum_{-\infty}^{\infty} a(k) e^{-jk\omega t}$

X = [1 2 3 4]

Fourier Coefficient:

ak =

Columns 1 through 3

2.5000 + 0.0000i -0.5000 + 0.5000i -0.5000 - 0.0000i

Column 4

-0.5000 - 0.5000i

X signal:

Xt =

Columns 1 through 3

-0.0000 + 0.0000i 0.0000 - 0.0000i -0.0000 - 0.0000i

Column 4

-2.0000 - 2.0000i

$$X = [1 \ 2 \ 2 \ 1]$$

Fourier Coefficient:

$a_k =$

Columns 1 through 3

$1.5000 + 0.0000i \quad -0.2500 - 0.2500i \quad 0.0000 - 0.0000i$

Column 4

$-0.2500 + 0.2500i$

X signal:

$X_t =$

Columns 1 through 3

$-0.0000 + 0.0000i \quad 0.0000 + 0.0000i \quad 0.0000 - 0.0000i$

Column 4

$-1.0000 + 1.0000i$

$$X = [0 \ 1 \ 2 \ -2 \ -1]$$

Fourier Coefficient:

$a_k =$

Columns 1 through 3

$0.0000 + 0.0000i \quad 0.0000 - 0.8507i \quad -0.0000 + 0.5257i$

Columns 4 through 5

$0.0000 - 0.5257i \quad -0.0000 + 0.8507i$

X signal:

$X_t =$

Columns 1 through 3

$0.0000 + 0.0000i \quad 0.0000 + 0.0000i \quad -0.0000 + 0.0000i$

Columns 4 through 5

$0.0000 - 0.0000i \quad -0.0000 + 4.2533i$

c) Fourier series algorithm:

$$x[n] = \cos(2\pi n * 3/7)$$

take point $n = 0$ as the start point then: $\omega = 2\pi * \frac{3}{7}$, and since $T = \frac{2\pi}{\omega}$ then $T = \frac{2\pi}{2\pi * \frac{3}{7}} = \frac{7}{3}$

then take $n = [0 : 7/3]$

Analytical solution:

$$\cos(2\pi n * \frac{3}{7}) \rightarrow \frac{e^{\frac{6}{7}\pi nj} + e^{-\frac{6}{7}\pi nj}}{2}$$

Then $a_0 = 0$

$a_6 = 0.5$

$a_{-6} = 0.5$

Fourier series algorithm on MATLAB:

```
>> n = [0:0.1:7/3]
```

```
>> x = cos(2*pi*n*3/7)
```

```
>> ak = f_series(x);
```

```
>> ak
```

```
ak =
```

Columns 1 through 3

```
0.0278 + 0.0000i    0.5045 + 0.0441i   -0.0098 - 0.0017i
```

Columns 4 through 6

```
-0.0035 - 0.0009i   -0.0018 - 0.0006i   -0.0011 - 0.0005i
```

Columns 7 through 9

```
-0.0007 - 0.0003i   -0.0005 - 0.0003i   -0.0003 - 0.0002i
```

Columns 10 through 12

```
-0.0003 - 0.0001i   -0.0002 - 0.0001i   -0.0002 - 0.0000i
```

Columns 13 through 15

```
-0.0002 - 0.0000i   -0.0002 + 0.0000i   -0.0002 + 0.0001i
```

Columns 16 through 18

```
-0.0003 + 0.0001i   -0.0003 + 0.0002i   -0.0005 + 0.0003i
```

Columns 19 through 21

```
-0.0007 + 0.0003i   -0.0011 + 0.0005i   -0.0018 + 0.0006i
```

Columns 22 through 24

```
-0.0035 + 0.0009i   -0.0098 + 0.0017i    0.5045 - 0.0441i
```

$$x[n] = \sin(2\pi n \cdot 3/7)$$

take point $n = 0$ as the start point then: $\omega = 2\pi \cdot \frac{3}{7}$, and since $T = \frac{2\pi}{\omega}$ then $T = \frac{2\pi}{2\pi \cdot \frac{3}{7}} = \frac{7}{3}$

then take $n = [0 : 7/3]$

Analytical solution:

$$\cos(2\pi n \cdot \frac{3}{7}) \rightarrow \frac{e^{\frac{6\pi nj}{7}} - e^{-\frac{6\pi nj}{7}}}{2j}$$

Then $a_0 = 0$

$$a_6 = 1/(2j)$$

$$a_{-6} = -1/(2j)$$

Fourier series algorithm on MATLAB:

```
>> n = [0:0.1:7/3]
```

```
>> x = sin(2*pi*n*3/7)
```

```
>> ak = f_series(x);
```

```
>> ak
```

```
ak =
```

Columns 1 through 3

```
-0.0012 + 0.0000i    0.0417 - 0.4905i   -0.0046 + 0.0190i
```

Columns 4 through 6

```
-0.0041 + 0.0102i   -0.0039 + 0.0069i   -0.0038 + 0.0051i
```

Columns 7 through 9

```
-0.0038 + 0.0039i   -0.0038 + 0.0029i   -0.0038 + 0.0022i
```

Columns 10 through 12

```
-0.0038 + 0.0016i   -0.0038 + 0.0010i   -0.0038 + 0.0005i
```

Columns 13 through 15

```
-0.0038 + 0.0000i   -0.0038 - 0.0005i   -0.0038 - 0.0010i
```

Columns 16 through 18

```
-0.0038 - 0.0016i   -0.0038 - 0.0022i   -0.0038 - 0.0029i
```

Columns 19 through 21

```
-0.0038 - 0.0039i   -0.0038 - 0.0051i   -0.0039 - 0.0069i
```

Columns 22 through 24

```
-0.0041 - 0.0102i   -0.0046 - 0.0190i    0.0417 + 0.4905i
```

$$x[n] = \exp(j*2*\pi*n*3/7)$$

take point $n = 0$ as the start point then: $\omega = 2\pi * \frac{3}{7}$, and since $T = \frac{2\pi}{\omega}$ then $T = \frac{2\pi}{2\pi * \frac{3}{7}} = \frac{7}{3}$

then take $n = [0 : 7/3]$

Analytical solution:

$$e^{j\frac{6}{7}\pi n}$$

Then $a_0 = 0$

$a_6 = 1$

Fourier series algorithm on MATLAB:

```
>> n = [0:0.1:7/3]
```

```
>> x = exp(j*2*pi*n*3/7)
```

```
>> ak = f_series(x);
```

```
ak =
```

Columns 1 through 3

```
0.0278 - 0.0012i    0.9950 + 0.0858i   -0.0288 - 0.0063i
```

Columns 4 through 6

```
-0.0138 - 0.0050i   -0.0087 - 0.0045i   -0.0062 - 0.0043i
```

Columns 7 through 9

```
-0.0046 - 0.0042i   -0.0034 - 0.0041i   -0.0025 - 0.0040i
```

Columns 10 through 12

```
-0.0018 - 0.0039i   -0.0012 - 0.0039i   -0.0007 - 0.0038i
```

Columns 13 through 15

```
-0.0002 - 0.0038i    0.0003 - 0.0037i    0.0008 - 0.0037i
```

Columns 16 through 18

```
0.0013 - 0.0036i    0.0019 - 0.0036i    0.0025 - 0.0035i
```

Columns 19 through 21

```
0.0032 - 0.0035i    0.0040 - 0.0034i    0.0051 - 0.0033i
```

Columns 22 through 24

```
0.0067 - 0.0031i    0.0092 - 0.0029i    0.0140 - 0.0025i
```