

**LAB2****Student experiment 1:**

a)  $y[n] = x[n] - x[n-1] - x[n-2]$

**Linearity:**

$$x_1[n] \rightarrow y_1[n] = x_1[n] - x_1[n-1] - x_1[n-2]$$

$$x_2[n] \rightarrow y_2[n] = x_2[n] - x_2[n-1] - x_2[n-2]$$

$$A.y_1[n] + B.y_2[n] = A.(x_1[n] - x_1[n-1] - x_1[n-2]) + B(x_2[n] - x_2[n-1] - x_2[n-2]) \rightarrow (1)$$

$$x_3[n] = A.x_1[n] + B.x_2[n] \rightarrow y_3[n] = x_3[n] - x_3[n-1] - x_3[n-2]$$

$$y_3[n] = A.x_1[n] + B.x_2[n] - A.x_1[n-1] - B.x_2[n-1] - A.x_1[n-2] - B.x_2[n-2] \rightarrow (2)$$

since (1) = (2) then  $y_3[n] = A.y_1[n] + B.y_2[n]$  then the system is **linear**.

**Time Invariance:**

$$x_1[n] \rightarrow y_1[n] = x_1[n] - x_1[n-1] - x_1[n-2] \rightarrow y_1[n-n_0] = x_1[n-n_0] - x_1[n-n_0-1] - x_1[n-n_0-2]$$

$$x_2[n] = x_1[n-n_0] \rightarrow y_2[n] = x_1[n-n_0] - x_1[n-n_0-1] - x_1[n-n_0-2]$$

since  $y_2[n] = y_1[n]$  then the system is **time invariant**.

**$y[n] = \cos(x[n])$**

**Linearity:**

$$x_1[n] \rightarrow y_1[n] = \cos(x_1[n])$$

$$x_2[n] \rightarrow y_2[n] = \cos(x_2[n])$$

$$A.y_1[n] + B.y_2[n] = A.\cos(x_1[n]) + B.\cos(x_2[n]) \rightarrow (1)$$

$$x_3[n] = A.x_1[n] + B.x_2[n] \rightarrow y_3[n] = \cos(x_3[n])$$

$$y_3[n] = \cos(A.x_1[n] + B.x_2[n]) \rightarrow (2)$$

since (1) = (2) then  $y_3[n] \neq A.y_1[n] + B.y_2[n]$  then the system is **nonlinear**.

**Time Invariance:**

$$x_1[n] \rightarrow y_1[n] = \cos(x_1[n]) \rightarrow y_1[n-n_0] = \cos(x_1[n-n_0])$$

$$x_2[n] = x_1[n-n_0] \rightarrow y_2[n] = \cos(x_1[n-n_0])$$

since  $y_2[n] = y_1[n]$  then the system is **time variant**.

## **y[n]=nx[n]**

### Linearity:

$$x_1[n] \rightarrow y_1[n] = nx_1[n]$$

$$x_2[n] \rightarrow y_2[n] = nx_2[n]$$

$$A.y_1[n] + B.y_2[n] = A.nx_1[n] + B.nx_2[n] \rightarrow (1)$$

$$x_3[n] = A.x_1[n] + B.x_2[n] \rightarrow y_3[n] = nx_3[n]$$

$$y_3[n] = n(A.x_1[n] + B.x_2[n]) \rightarrow (2)$$

since (1) = (2) then  $y_3[n] = A.y_1[n] + B.y_2[n]$  then the system is **linear**.

### Time Invariance:

$$x_1[n] \rightarrow y_1[n] = nx_1[n] \rightarrow y_1[n-n_0] = (n-n_0).x_1[n-n_0]$$

$$x_2[n] = x_1[n-n_0] \rightarrow y_2[n] = n.x_1[n-n_0]$$

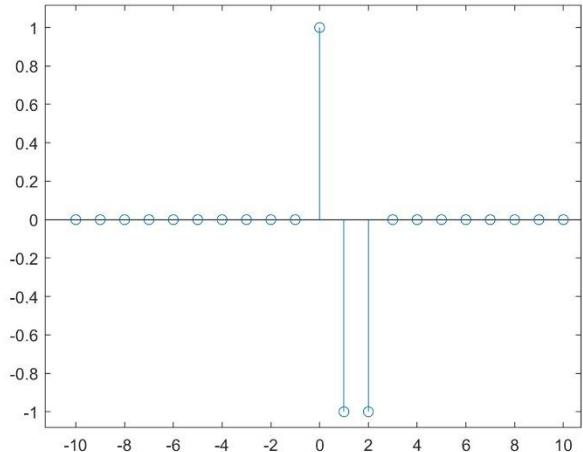
since  $y_2[n] = y_1[n]$  then the system is **time variant**.

$$b) y[n] = x[n] - x[n-1] - x[n-2]$$

input = Delta(n)

```
>> nx1=[-10:10], x1=[zeros(1,10) 1 zeros(1,10)];
>> y = zeros(1,length(x1));
>> for c=3:length(x1)
y(c) = x1(c)-x1(c-1)-x1(c-2)
end
>> stem(nx1, y)
```

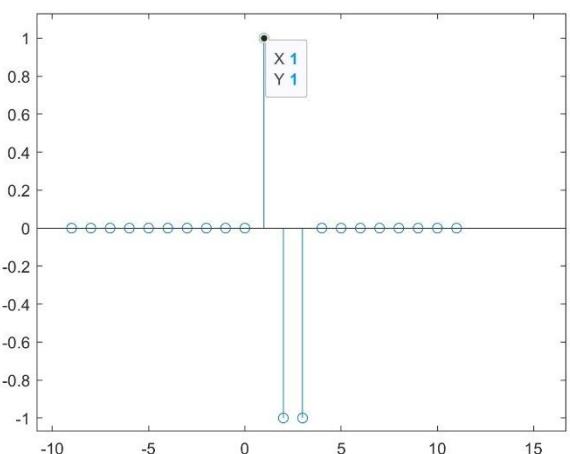
response



input = Delta(n-1)

```
>> nx1=[-10:10], x1=[zeros(1,10) 1 zeros(1,10)];
>> y = zeros(1,length(x1));
>> for c=3:length(x1)
y(c) = x1(c)-x1(c-1)-x1(c-2)
end
>> stem(nx1+1, y)
```

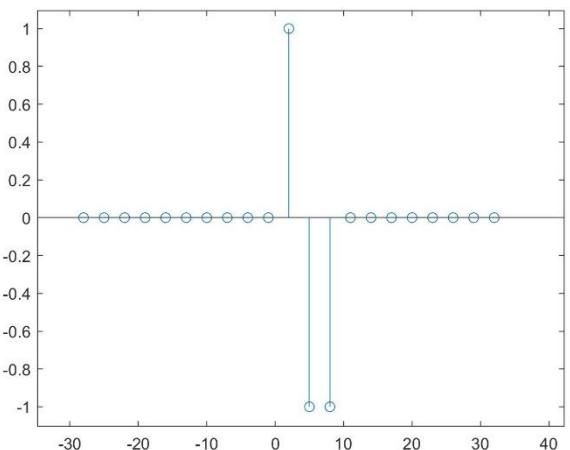
response



input = Delta(n)+2\* Delta(n-1)

```
>> nx1=[-10:10], x1=[zeros(1,10) 1 zeros(1,10)];
>> y = zeros(1,length(x1));
>> for c=3:length(x1)
y(c) = x1(c)-x1(c-1)-x1(c-2)
end
>> stem(nx1+2*(nx1+1), y)
```

response

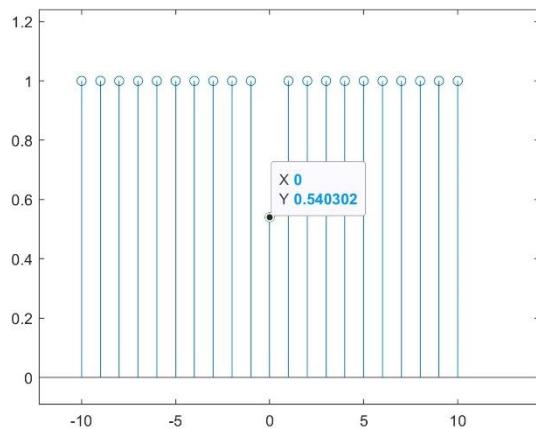


## $y[n]=\cos(x[n])$

### input = Delta(n)

```
>> nx1=[-10:10], x1=[zeros(1,10) 1 zeros(1,10)];
>> y = zeros(1,length(x1));
>> for c=1:length(x1)
y(c) = cos(x1(c));
end
>> stem(nx1 ,y)
```

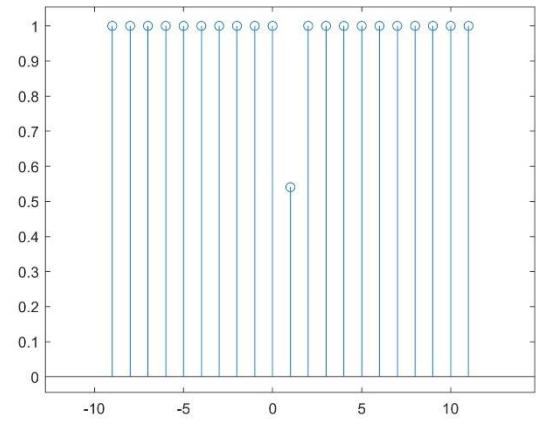
### response



### input = Delta(n-1)

```
>> nx1=[-10:10], x1=[zeros(1,10) 1 zeros(1,10)];
>> y = zeros(1,length(x1));
>> for c=1:length(x1)
y(c) = cos(x1(c));
end
>> stem(nx1+1 ,y)
```

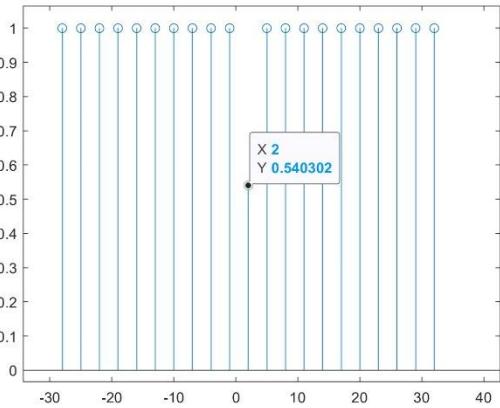
### response



### input = Delta(n)+2\* Delta(n-1)

```
>> nx1=[-10:10], x1=[zeros(1,10) 1 zeros(1,10)];
>> y = zeros(1,length(x1));
>> for c=1:length(x1)
y(c) = cos(x1(c));
end
>> stem(nx1+2*(nx1+1), y)
```

### response

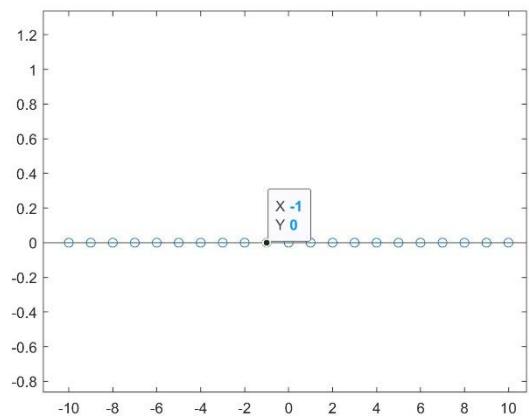


## $y[n] = nx[n]$

input = Delta(n)

```
>> nx1=[-10:10], x1=[zeros(1,10) 1 zeros(1,10)];
>> y = zeros(1,length(x1));
>> for c=1:length(x1)
y(c) = nx1(c) * x1(c)
end
>> stem(nx1,y)
```

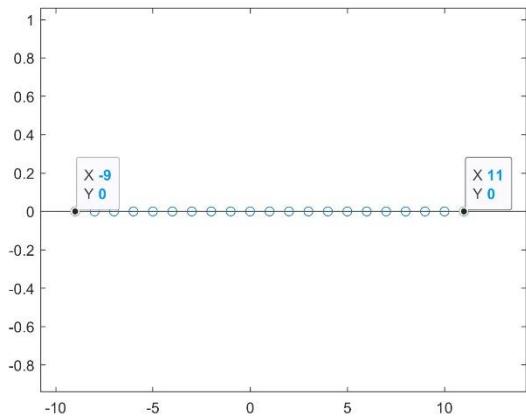
response



input = Delta(n-1)

```
>> nx1=[-10:10], x1=[zeros(1,10) 1 zeros(1,10)];
>> y = zeros(1,length(x1));
>> for c=1:length(x1)
y(c) = nx1(c) * x1(c)
end
>> stem(nx1+1,y)
```

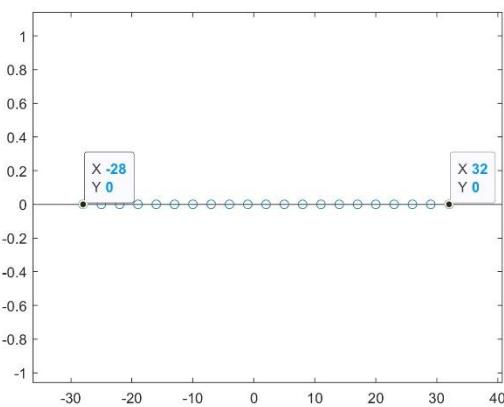
response



input = Delta(n)+2\* Delta(n-1)

```
>> nx1=[-10:10], x1=[zeros(1,10) 1 zeros(1,10)];
>> y = zeros(1,length(x1));
>> for c=1:length(x1)
y(c) = nx1(c) * x1(c)
end
>> stem(nx1+2*(nx1+1), y)
```

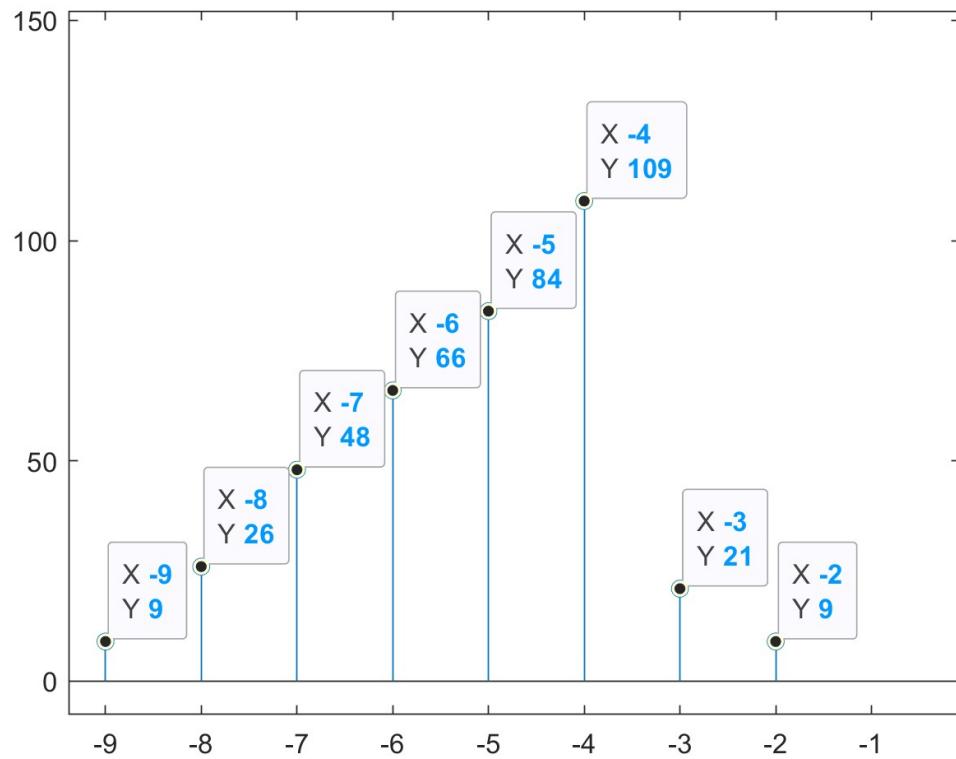
response



## Student experiment 2:

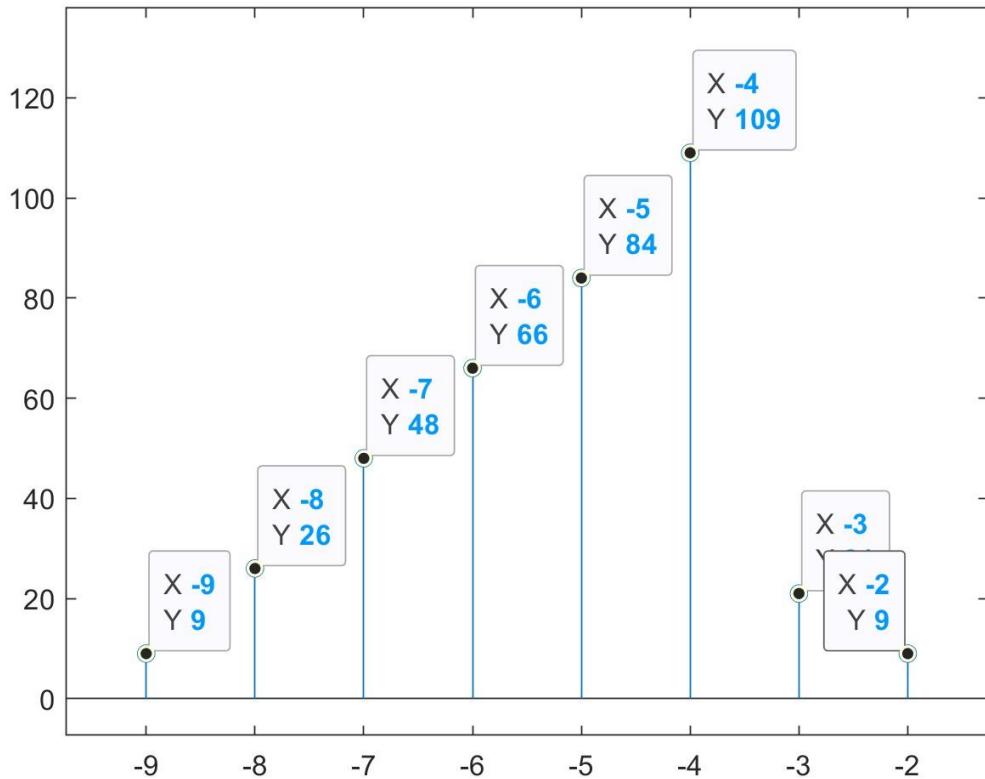
a)

```
>> nx=[-3 -2 -1];  
>> x =[1 2 3];  
>> nh=[-6 -5 -4 -3 -2 -1];  
>> h =[9 8 5 32 5 3];  
>> M=length(x);  
>> N=length(h);  
>> ny=[-9:-2]  
>> y=zeros(1,M+N-1);  
>> for u=1:(N)  
x1=h(u)*[zeros(1,u-1) x zeros(1,length(y)-u-2)];  
y=y+x1;  
end
```



b)

```
>> nx=[-3 -2 -1];  
>> x =[1 2 3];  
>> nh=[-6 -5 -4 -3 -2 -1];  
>> h =[9 8 5 32 5 3];  
>> M=length(x);  
>> N=length(h);  
>> ny=[-9:-2]  
>> y=conv(x,h);
```



### Student experiment 3:

#### a) Inverse Fourier Series Code

```
function b=If_series(x)
N=length(x);
n=1:N;
b = zeros(1,N);
for k=1:N
b(k)=sum(x(k)*exp(-2*pi*i*k*n/N));
end
```

#### b) Fourier series of the three signals:

- $a_k = \frac{1}{T} \int_T x(t) e^{-jk\omega t} dt$
- $X(t) = \sum_{-\infty}^{\infty} a(k) e^{-jk\omega t}$

$$X = [1 \ 2 \ 3 \ 4]$$

Fourier Coefficient:

ak =

Columns 1 through 3

$$2.5000 + 0.0000i \quad -0.5000 + 0.5000i \quad -0.5000 - 0.0000i$$

Column 4

$$-0.5000 - 0.5000i$$

X signal:

Xt =

Columns 1 through 3

$$-0.0000 + 0.0000i \quad 0.0000 - 0.0000i \quad -0.0000 - 0.0000i$$

Column 4

$$-2.0000 - 2.0000i$$

**X = [1 2 2 1]**

Fourier Coefficient:

ak =

Columns 1 through 3

1.5000 + 0.0000i -0.2500 - 0.2500i 0.0000 - 0.0000i

Column 4

-0.2500 + 0.2500i

X signal:

Xt =

Columns 1 through 3

-0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 - 0.0000i

Column 4

-1.0000 + 1.0000i

**X = [0 1 2 -2 -1]**

Fourier Coefficient:

ak =

Columns 1 through 3

0.0000 + 0.0000i 0.0000 - 0.8507i -0.0000 + 0.5257i

Columns 4 through 5

0.0000 - 0.5257i -0.0000 + 0.8507i

X signal:

Xt =

Columns 1 through 3

0.0000 + 0.0000i 0.0000 + 0.0000i -0.0000 + 0.0000i

Columns 4 through 5

0.0000 - 0.0000i -0.0000 + 4.2533i

### c) Fourier series algorithm:

$$x[n] = \cos(2\pi n * 3/7)$$

take point  $n = 0$  as the start point then:  $\omega = 2\pi * \frac{3}{7}$ , and since  $T = \frac{2\pi}{\omega}$  then  $T = \frac{2\pi}{2\pi * \frac{3}{7}} = \frac{7}{3}$   
 then take  $n = [0 : 7/3]$

Analytical solution:

$$\cos(2\pi n * \frac{3}{7}) \rightarrow \frac{e^{\frac{6\pi}{7}jn} + e^{-\frac{6\pi}{7}jn}}{2}$$

Then  $a_0 = 0$   
 $a_6 = 0.5$   
 $a_{-6} = 0.5$

Fourier series algorithm on MATLAB:

```
>> n = [0:0.1:7/3]
>> x = cos(2*pi*n*3/7)
>> ak = f_series(x);
>> ak
ak =
Columns 1 through 3
0.0278 + 0.0000i  0.5045 + 0.0441i  -0.0098 - 0.0017i
Columns 4 through 6
-0.0035 - 0.0009i  -0.0018 - 0.0006i  -0.0011 - 0.0005i
Columns 7 through 9
-0.0007 - 0.0003i  -0.0005 - 0.0003i  -0.0003 - 0.0002i
Columns 10 through 12
-0.0003 - 0.0001i  -0.0002 - 0.0001i  -0.0002 - 0.0000i
Columns 13 through 15
-0.0002 - 0.0000i  -0.0002 + 0.0000i  -0.0002 + 0.0001i
Columns 16 through 18
-0.0003 + 0.0001i  -0.0003 + 0.0002i  -0.0005 + 0.0003i
Columns 19 through 21
-0.0007 + 0.0003i  -0.0011 + 0.0005i  -0.0018 + 0.0006i
Columns 22 through 24
-0.0035 + 0.0009i  -0.0098 + 0.0017i  0.5045 - 0.0441i
```

$$x[n] = \sin(2\pi n * 3/7)$$

take point  $n = 0$  as the start point then:  $\omega = 2\pi * \frac{3}{7}$ , and since  $T = \frac{2\pi}{\omega}$  then  $T = \frac{2\pi}{2\pi * \frac{3}{7}} = \frac{7}{3}$

then take  $n = [0 : 7/3]$

Analytical solution:

$$\cos(2\pi n * \frac{3}{7}) \rightarrow \frac{e^{\frac{6\pi n j}{7}} - e^{-\frac{6\pi n j}{7}}}{2j}$$

Then  $a_0 = 0$

$$a_6 = 1/(2j)$$

$$a_{-6} = -1/(2j)$$

Fourier series algorithm on MATLAB:

```
>> n = [0:0.1:7/3]
>> x = sin(2*pi*n*3/7)
>> ak = f_series(x);
>> ak
ak =
Columns 1 through 3
-0.0012 + 0.0000i  0.0417 - 0.4905i  -0.0046 + 0.0190i
Columns 4 through 6
-0.0041 + 0.0102i  -0.0039 + 0.0069i  -0.0038 + 0.0051i
Columns 7 through 9
-0.0038 + 0.0039i  -0.0038 + 0.0029i  -0.0038 + 0.0022i
Columns 10 through 12
-0.0038 + 0.0016i  -0.0038 + 0.0010i  -0.0038 + 0.0005i
Columns 13 through 15
-0.0038 + 0.0000i  -0.0038 - 0.0005i  -0.0038 - 0.0010i
Columns 16 through 18
-0.0038 - 0.0016i  -0.0038 - 0.0022i  -0.0038 - 0.0029i
Columns 19 through 21
-0.0038 - 0.0039i  -0.0038 - 0.0051i  -0.0038 - 0.0069i
Columns 22 through 24
-0.0041 - 0.0102i  -0.0046 - 0.0190i  0.0417 + 0.4905i
```

$$x[n] = \exp(j \cdot 2 \cdot \pi \cdot n \cdot 3/7)$$

take point  $n = 0$  as the start point then:  $\omega = 2\pi * \frac{3}{7}$ , and since  $T = \frac{2\pi}{\omega}$  then  $T = \frac{2\pi}{2\pi * \frac{3}{7}} = \frac{7}{3}$

then take  $n = [0 : 7/3]$

Analytical solution:

$$e^{j \frac{6}{7} \pi n}$$

Then  $a_0 = 0$   
 $a_6 = 1$

Fourier series algorithm on MATLAB:

```
>> n = [0:0.1:7/3]
>> x = exp(j*2*pi*n*3/7)
>> ak = f_series(x);
ak =

Columns 1 through 3

0.0278 - 0.0012i  0.9950 + 0.0858i  -0.0288 - 0.0063i

Columns 4 through 6

-0.0138 - 0.0050i  -0.0087 - 0.0045i  -0.0062 - 0.0043i

Columns 7 through 9

-0.0046 - 0.0042i  -0.0034 - 0.0041i  -0.0025 - 0.0040i

Columns 10 through 12

-0.0018 - 0.0039i  -0.0012 - 0.0039i  -0.0007 - 0.0038i

Columns 13 through 15

-0.0002 - 0.0038i  0.0003 - 0.0037i  0.0008 - 0.0037i

Columns 16 through 18

0.0013 - 0.0036i  0.0019 - 0.0036i  0.0025 - 0.0035i

Columns 19 through 21

0.0032 - 0.0035i  0.0040 - 0.0034i  0.0051 - 0.0033i

Columns 22 through 24

0.0067 - 0.0031i  0.0092 - 0.0029i  0.0140 - 0.0025i
```