Investigating the Sensitivity of the Greeks to Underlying Asset Correlation for a Selection of Exotic Options

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Contents

1 Introduction

The main goal of this project is to estimate the Greeks for two types of exotic options, European basket put and call options, as well as European spread put and call options. This will be done for two companies in the technology industry. The payoff of a basket option is based on the price of a portfolio of underlying assets, and the price of a spread option has payoff based on the difference in price between two assets (Hull, 2022). The companies we chose are NVIDIA and AMD, as they are two companies that play similar roles in the semiconductor industry, suggesting that their stock returns are correlated.

Option Greeks measure the sensitivity of option prices to changes in model parameters, hence they play an important role in managing portfolio risk. The Greeks estimated in this project are Delta, Gamma, Theta and Vega, which measure sensitivity to changes in the underlying asset price, curvature of the aforementioned sensitivity, the effect of time decay, and sensitivity to changes in volatility respectively (Hull, 2022).

The estimation was conducted using a Monte Carlo simulation framework under a risk-neutral measure as described by Boyle et al. (1997). The underlying asset prices are modeled using correlated geometric Brownian motions, and option sensitivities are estimated using finite-difference approximations.

The analysis concludes that correlation plays an important role in both option prices and their Greeks. Basket option prices increase with higher correlation and spread option prices decrease with higher correlation. The estimated Greeks of basket and spread options behave in opposite ways, but for both options the impact of correlation is highest when they are near at-the-money, and weakens for deep in- or out-of-the-money, with some notable tail effects in Gamma.

The contribution of this paper is to provide a clear, simulation-based analysis of how correlation affects the risk sensitivities of multi-asset derivatives using empirically motivated data.

2 Methodology / Literature Review

To price the basket and spread options, we used the Monte Carlo method described by Boyle et al. (1997). The method involves simulating underlying asset prices under the risk-neutral measure and averaging the discounted payoffs of the options.

To simulate terminal stock prices, we used the Black-Scholes model (Black and Scholes, 1973):

$$X_t = X_0 \exp\left[\left(r_f - \frac{\sigma^2}{2}\right)t + \sigma W_t\right] \text{ where } W_t \text{ is a standard Brownian motion} \tag{1}$$

First, we simulated values of two independent Brownian motions using a standard normal distribution random number generator:

$$\begin{cases} W_t^{(1)} \sim \mathcal{N}(0,t) \\ W_t^{(2)} \sim \mathcal{N}(0,t) \end{cases}, \quad \mathrm{Cov}(W_t^{(1)}, W_t^{(2)}) = 0 \tag{2}$$

Next, we defined the Brownian motions governing the behaviour of the terminal stock prices, dependent on the correlation of their returns (ρ):

$$\begin{cases} W_t^{(NVDA)} = W_t^{(1)} \\ W_t^{(AMD)} = \rho W_t^{(1)} + \sqrt{1-\rho^2}\, W_t^{(2)} \end{cases} \tag{3}$$

Finally, we obtained our model for correlated terminal stock prices:

$$\begin{cases} NVDA_t = NVDA_0 \exp\left[\left(r_f - \frac{\sigma^2_{NVDA}}{2}\right) t + \sigma_{NVDA}\, W_t^{(NVDA)}\right] \\ AMD_t = AMD_0 \exp\left[\left(r_f - \frac{\sigma^2_{AMD}}{2}\right) t + \sigma_{AMD}\, W_t^{(AMD)}\right] \end{cases} \tag{4}$$

We defined the option payoff functions as the following:

$$\begin{cases} ((AMD_t + NVDA_t)/2 - K_B)^+, & \text{basket call} \\ (K_B - (AMD_t + NVDA_t)/2)^+, & \text{basket put} \\ ((AMD_t - NVDA_t) - K_S)^+, & \text{spread call} \\ (K_S - (AMD_t - NVDA_t))^+, & \text{spread put} \end{cases} \tag{5}$$

We defined the strike prices for the basket, K_B, and spread, K_S, using the stock prices at the last sample period (see table 1). $K_B = (AMD_0 + NVDA_0)/2 = 197.265$ and $K_S = AMD_0 - NVDA_0 = 40.53$.

To estimate the Greeks, we used a finite difference technique described by Glasserman (2003). We used the following estimators for each option sensitivity:

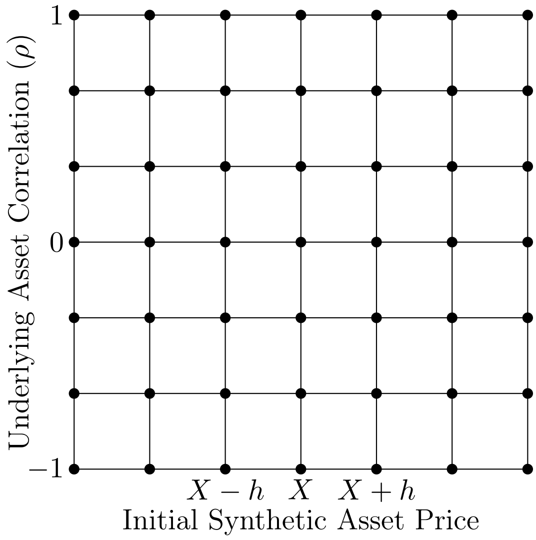
$$\widehat{\Delta} = \frac{\widehat{V}(X+h) - \widehat{V}(X-h)}{2h} \tag{6}$$

$$\widehat{\Gamma} = \frac{\widehat{V}(X+h) - 2\widehat{V}(X) + \widehat{V}(X-h)}{h^2} \tag{7}$$

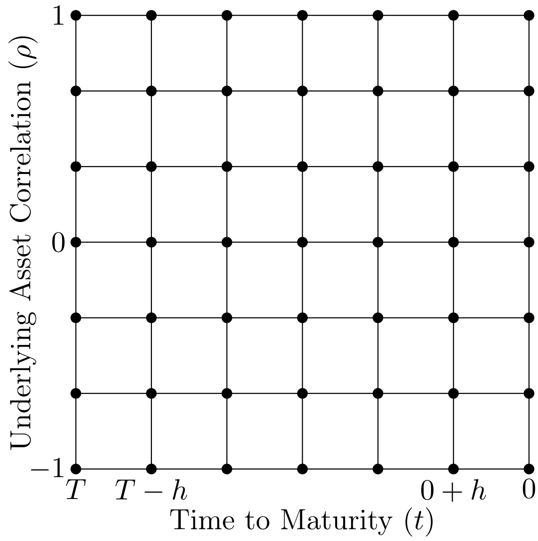
$$\widehat{\Theta} = \frac{\widehat{V}(t+h) - \widehat{V}(t)}{h} \tag{8}$$

$$\widehat{\mathcal{V}} = \frac{\widehat{V}(\sigma+h) - \widehat{V}(\sigma-h)}{2h} \tag{9}$$

where V represents an option value, X represents the price of the underlying asset, and h represents an arbitrary increment.



(a) Varying price of the underlying asset.



(b) Varying time to maturity.

Figure 1: Lattice structures for simulating option prices.

The estimators for the Delta (6), Gamma (7), and Vega (9) use centered finite differences for the improved convergence rate of the estimator bias (Glasserman, 2003). The Theta (8) was estimated using a forward finite difference to obtain an estimate at time $t = 0$.

Prices of both exotic options were simulated at a discrete number of points for varying option parameters (price and volatility of the underlying asset, and time to maturity of the option) and varying correlation between the returns of the two stocks. This estimation approach is visualized in Figure 1. The Vega was estimated using two volatility levels of the underlying asset with varying initial asset prices and correlation.

2.1 Numerical simulation procedure

The simulation of terminal stock prices and exotic option prices under varying parameters was performed using R. A random seed of `1` was used for reproducibility.

The two independent Brownian motions were simulated using a standard normal distribution random number generator with 10,000 replications. The correlation values ranging from -1 to 1 were partitioned into 20 discrete increments. The prices of AMD and NVIDIA were incremented simultaneously — the prices were incremented in the same direction for the basket option, and were incremented in opposing directions for the spread option. To estimate the Delta, the prices were varied by $\pm 60\%$ in 1% increments from the initial prices reported in table 1. To estimate the Gamma, the prices were varied in 10% increments (a larger increment was used to achieve improved numerical stability). The option duration (1 year) was partitioned into 100 time steps. The volatility of the returns for AMD and NVIDIA were incremented simultaneously in 1% steps above and below the empirical volatility reported in table 1. We made the decision to increment the parameters simultaneously in order to emphasize the effects of the correlation on the Greeks through larger price and volatility variation.

In each case, the option sensitivities were evaluated with respect to the parameter for the synthetic asset (price and volatility). E.g., the Vega for the basket option was estimated with respect to the volatility of the logarithmic basket returns. The volatility of the spread returns were estimated using the standard deviation of the terminal spread value, as the spread can be less than or equal to zero.

A single value of the estimated Vega for both the spread call and put option was replaced using a linear interpolation of the surrounding values due to a numerical artifact.

3 Data

We used 61 months of daily closing stock price data for Advanced Micro Devices, Inc. and NVIDIA Corporation from Yahoo! Finance using a sample period of 2020-11-01 to 2025-11-01. We transformed the data to end of month prices and calculated the monthly logarithmic returns for both stocks to obtain a sample of 60 periods. Using the monthly returns, we calculated the monthly return volatility (standard deviation) of returns and annualized the results by multiplying by a factor of $\sqrt{12}$.

Using the same sample period as for the stock price data (minus the first sample month), we downloaded 60 months of 3-month treasury bill secondary market rate data from the Federal Reserve Bank of St. Louis. We calculated an average of the treasury bill rate data to estimate the risk-free rate.

3.1 Data source and access

Stock price data for AMD and NVIDIA was sourced from Yahoo! Finance:

- Advanced Micro Devices, Inc. (AMD)
- NVIDIA Corporation (NVDA)

Treasury bill rate data was sourced from the Federal Reserve Bank of St. Louis (FRED).

Both data sources were last accessed on Friday 12$^{\text{th}}$ December, 2025. We imported the data using the `tidyquant` package for R.

3.2 Descriptive statistics and patterns

Table 1 summarizes the relevant information of the 60 period sample.

Table 1: Data sample summary

Stock	Final Price ($)	Annualized Mean Log Return	Annualized Return Volatility (σ)
AMD	217.53	0.04927106	0.4959142
NVDA	177.00	0.14900158	0.5365392

The empirical correlation between the monthly returns for AMD and NVIDIA over the sample period was 0.715. Both stocks experienced high return volatility driven by developments in artificial intelligence technology.

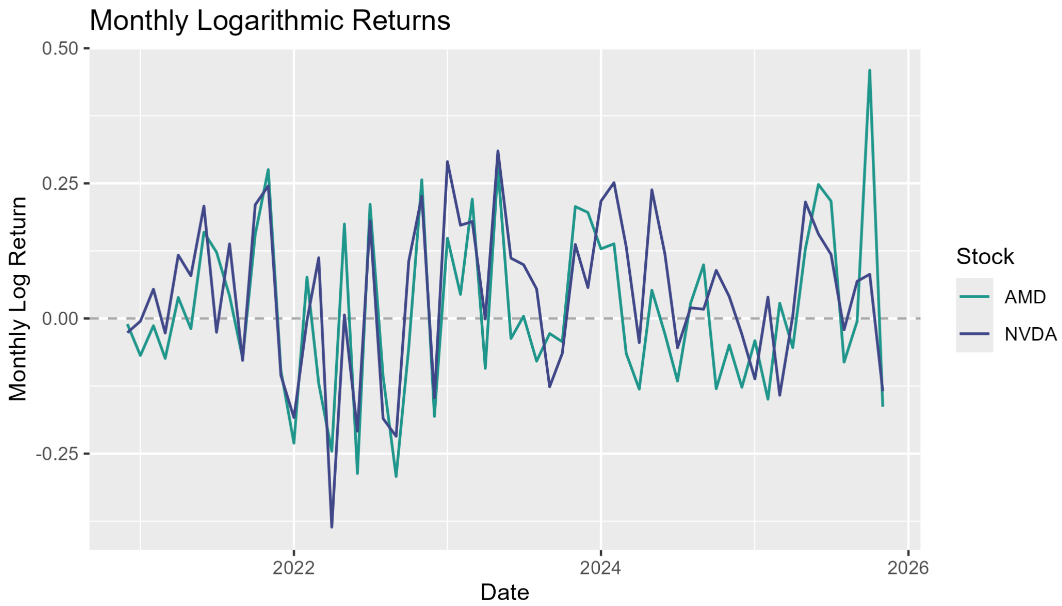


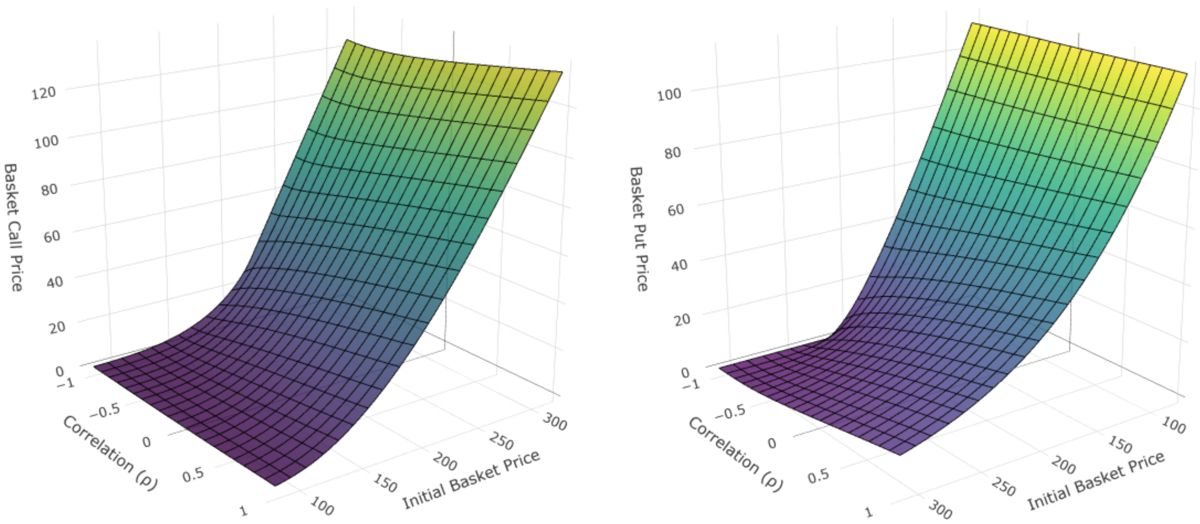
Figure 2: Monthly logarithmic returns for AMD and NVIDIA.

The high correlation and volatility of returns can be observed in figure 2.

The sample average of the treasury bill data used in the simulation as the risk-free rate was $r_f = 0.03175167$.

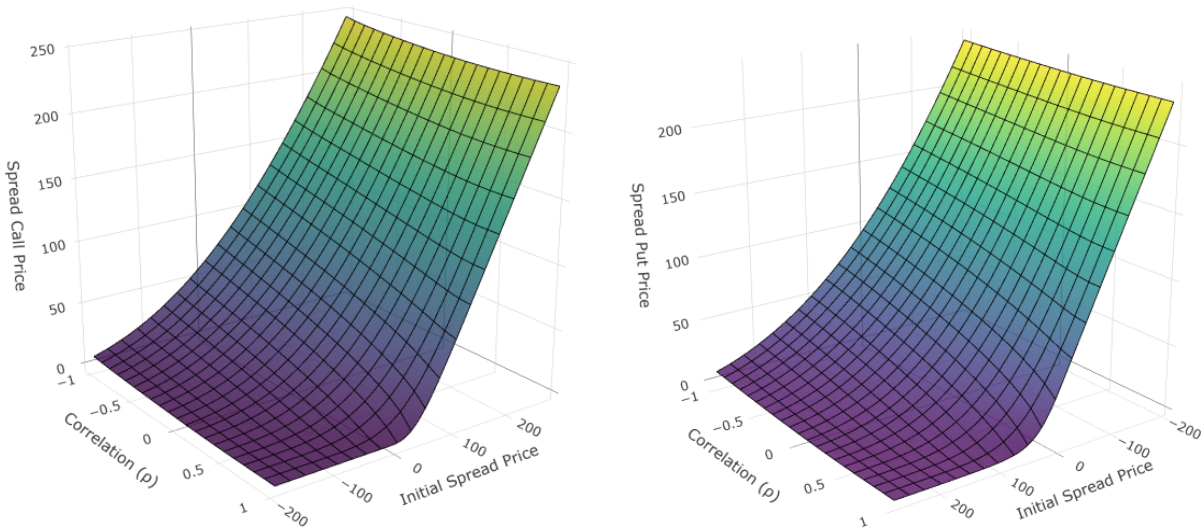
4 Results: Figures and Tables

4.1 Option price sensitivity to correlation



(a) The price of a basket call. (b) The price of a basket put.

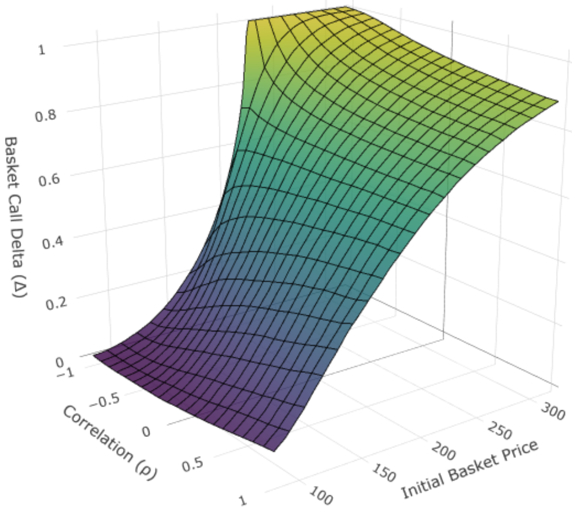
Figure 3: Basket option prices.



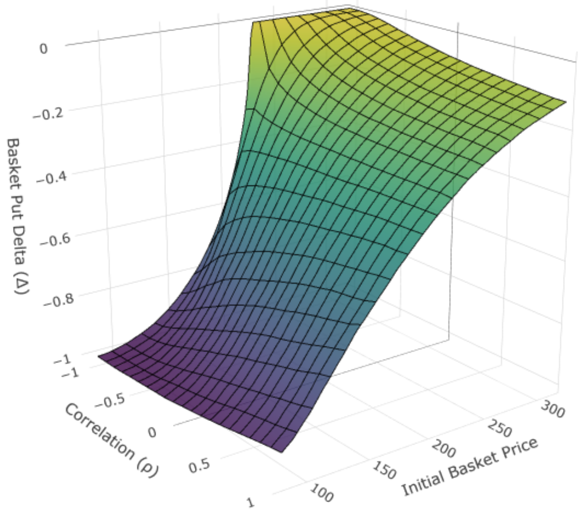
(a) The price of a spread call. (b) The price of a spread put.

Figure 4: Spread option prices.

4.2 Delta estimation

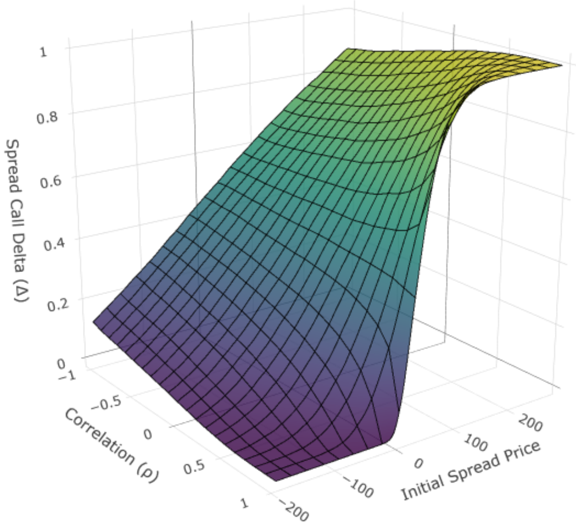


(a) The Delta of a basket call.

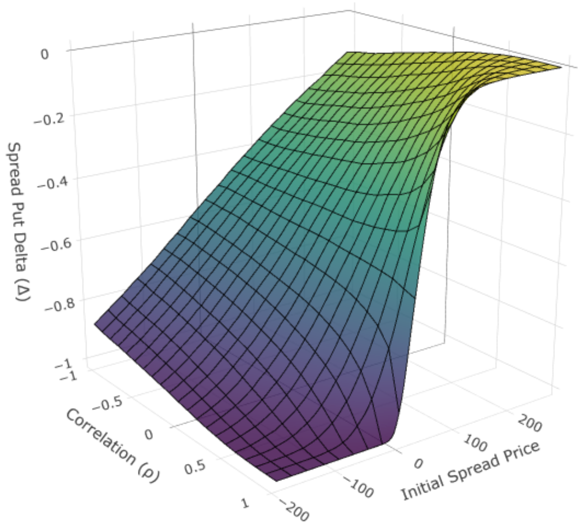


(b) The Delta of a basket put.

Figure 5: Basket option Deltas.



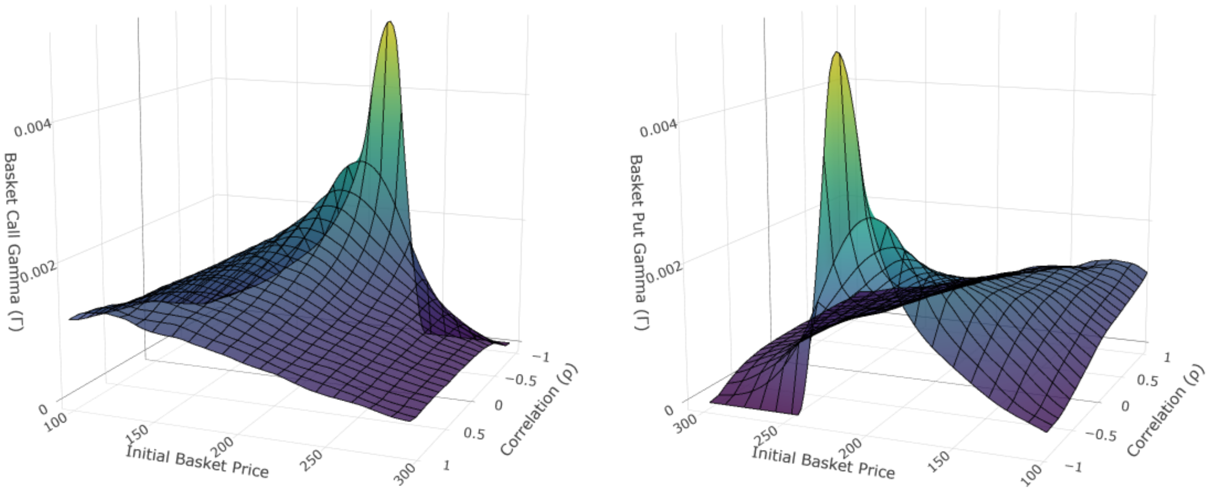
(a) The Delta of a spread call.



(b) The Delta of a spread put.

Figure 6: Spread option Deltas.

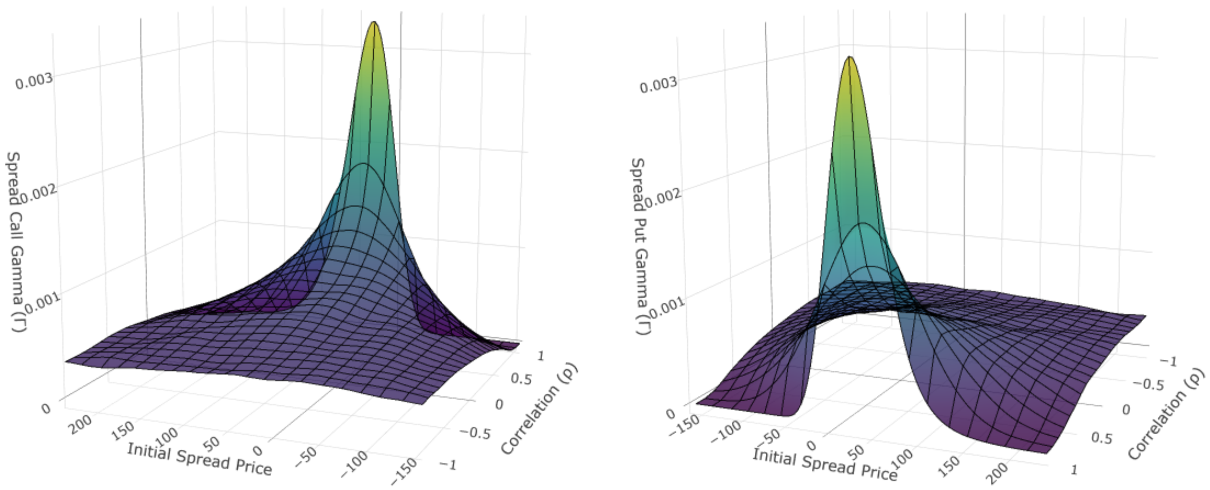
4.3 Gamma estimation



(a) The Gamma of a basket call.

(b) The Gamma of a basket put.

Figure 7: Basket option Gammas.

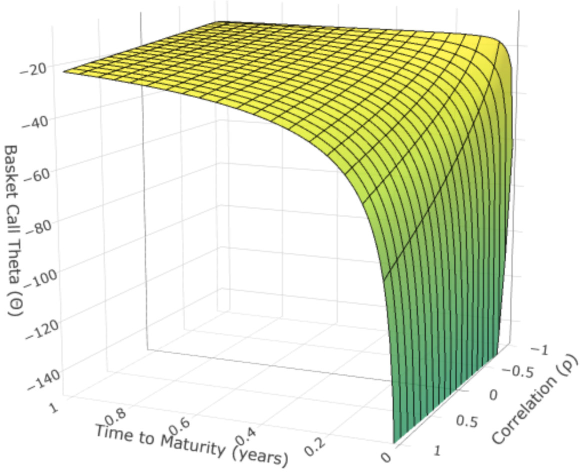


(a) The Gamma of a spread call.

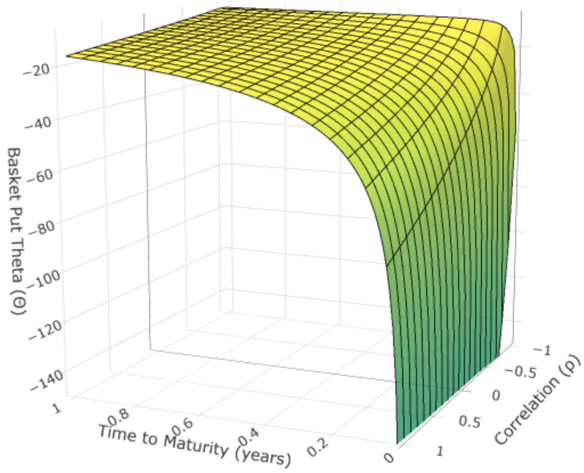
(b) The Gamma of a spread put.

Figure 8: Spread option Gammas.

4.4 Theta estimation

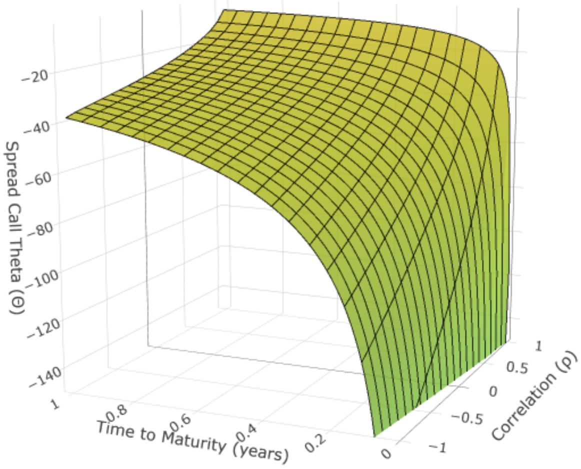


(a) The Theta of a basket call.

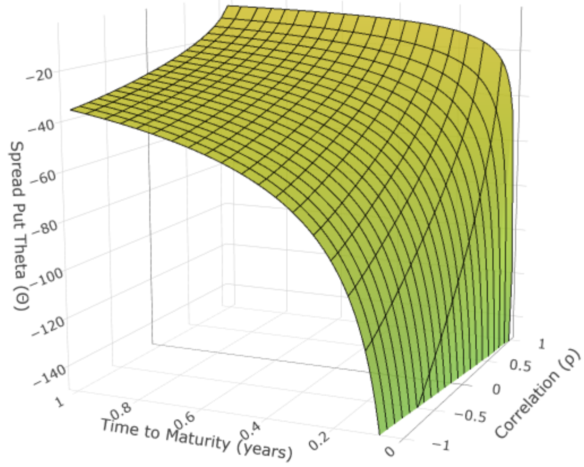


(b) The Theta of a basket put.

Figure 9: Basket option Thetas.



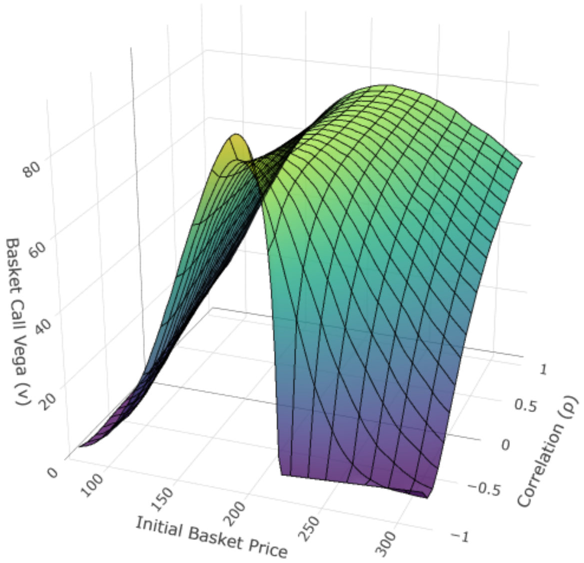
(a) The Theta of a spread call.



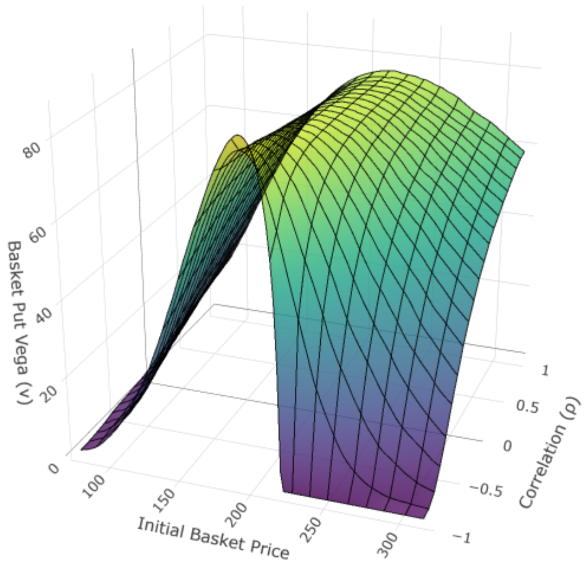
(b) The Theta of a spread put.

Figure 10: Spread option Thetas.

4.5 Vega estimation

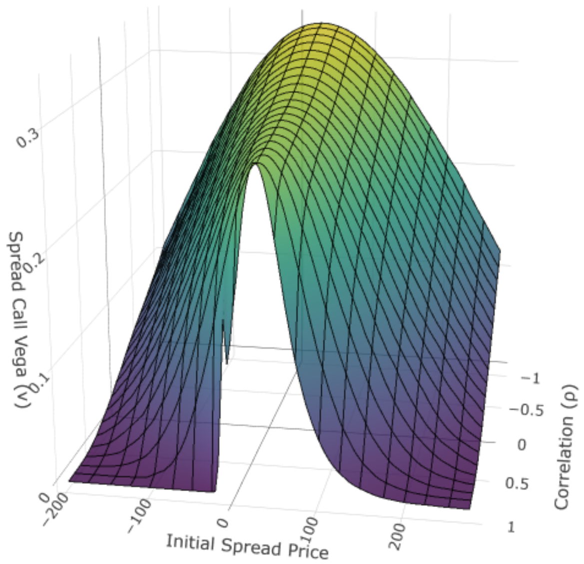


(a) The Vega of a basket call.

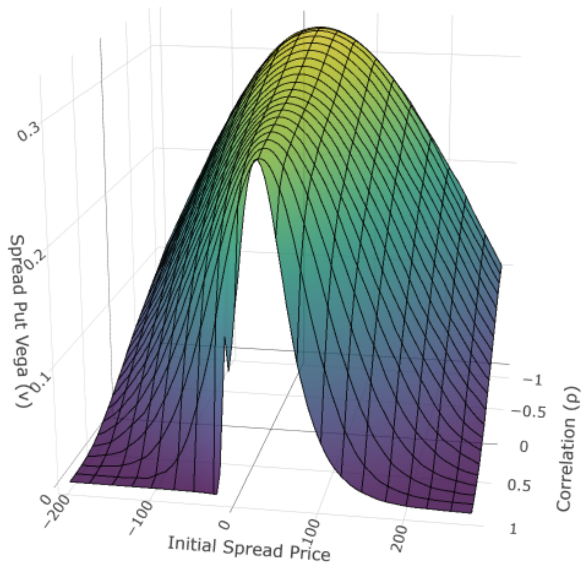


(b) The Vega of a basket put.

Figure 11: Basket option Vegas.



(a) The Vega of a spread call.



(b) The Vega of a spread put.

Figure 12: Spread option Vegas.

5 Results: Numerical Analysis

In this section we present the Monte Carlo estimates of option prices and sensitivities for both the basket and spread options. For each quantity (price, Delta, Gamma, Theta, Vega), we examine how the surface changes with the correlation ρ between AMD and NVIDIA and with the relevant underlying parameter (initial synthetic asset price or time to maturity).

5.1 Option price sensitivity to correlation

In this section, we investigate how the values of the basket and spread options react to changes in the correlation ρ between AMD and NVIDIA, as well as changes in the initial synthetic price (the basket level or the spread). All option prices are computed using Monte Carlo simulation under the Black–Scholes framework introduced in Section 2. To ensure consistency between simulations, we use the same set of random normal draws for every value of ρ.

5.1.1 Basket option prices

Figure 3 reports the simulated prices of the basket call and basket put as functions of the initial basket price and the correlation $\rho \in [-1, 1]$. For the basket call, the price surface increases in both dimensions: the option becomes more valuable when the basket starts at a higher level and when the correlation between the two assets is more positive. The monotonicity in the initial basket level follows directly from the call payoff (see equation 5)

$$(B_T - K_B)^+,$$

which is increasing in B_0. The monotonicity with respect to correlation is explained by the variance of the basket:

$$\mathrm{Var}(B_T) = \frac{1}{4}\sigma_{AMD}^2 + \frac{1}{4}\sigma_{NVDA}^2 + \frac{1}{2}\sigma_{AMD}\,\sigma_{NVDA}\,\rho, \tag{10}$$

which increases with ρ for positive weights. Because the call payoff is convex, higher variance (i.e., lower diversification) leads to a higher expected payoff. This explains why basket call prices rise as the correlation becomes more positive.

The basket put surface shows the opposite monotonicity in the initial basket price: for fixed correlation, put prices decrease as the initial basket price grows, since the probability of finishing below the strike diminishes. However, for fixed initial basket level, basket put prices also *increase* with correlation. Although a positive correlation makes the joint downward movement of both assets more probable, the dominant effect is that the correlation increases the total basket variance. Since the payoff $(K_B - B_T)^+$ is convex, increases in variance increase the expected value of the put option. Thus, both basket calls and basket puts exhibit positive sensitivity to correlation, despite reacting differently to the initial basket level.

5.1.2 Spread option prices

Figure 4 displays the simulated prices of the spread call and spread put as functions of the initial spread and the correlation ρ. The spread call price increases monotonically with the initial spread: a larger initial gap between AMD and NVIDIA makes it more likely that the spread remains above the strike at maturity.

However, the dependence on correlation is fundamentally opposite to that of basket options. The variance of the spread is

$$\mathrm{Var}(S_T) = \sigma_{AMD}^2 + \sigma_{NVDA}^2 - 2\sigma_{AMD}\,\sigma_{NVDA}\,\rho, \tag{11}$$

which is decreasing in ρ. Higher correlation therefore reduces the volatility of the spread and lowers the value of the call option. In the limit $\rho \to 1$, the two assets move almost in parallel, making the difference nearly deterministic and significantly reducing the value of the option.

The spread put behaves as expected with respect to the initial spread: prices decrease when the spread begins far above the strike. As with spread calls, spread put prices decrease with correlation because reduced volatility lowers the probability of ending deep in the money. Thus, both spread calls and puts exhibit a negative sensitivity to correlation, which is exactly the reverse of the pattern observed for basket options.

5.1.3 Comparison and implication

Figures 3 and 4 together highlight how the effect of correlation is critically dependent on the payoff structure.

- For basket options, correlation increases the volatility of the sum of assets, increasing both call and put prices.
- For spread options, correlation decreases the volatility of the difference between the assets, lowering both call and put prices.

This fundamental contrast driven by the opposing effects of correlation on basket variance versus spread variance will reappear in the analysis of the Greeks (Delta, Gamma, Theta, and Vega) in the following subsections.

5.2 Delta sensitivity to correlation and initial price

Delta measures the sensitivity of an option's price to small changes in the underlying asset. Figures 5 and 6 display how Delta varies with the initial synthetic price and the correlation ρ for basket and spread options.

5.2.1 Basket option Delta

For basket options, both call and put Deltas exhibit the expected monotonic relationship with respect to the initial basket price: call Delta increases from 0 to 1 as the basket becomes deep in-the-money, while put Delta decreases from 0 to -1 as the basket moves further below the strike.

A central pattern in Figure 5 is that **Delta steepens as correlation increases**. Since basket volatility satisfies equation 10, higher correlation increases the volatility of the basket, making the option value more sensitive to changes in the underlying. As a result, both call and put Delta become more responsive to price movements when ρ is high.

From a hedging perspective, this means that delta-hedging basket options becomes more reactive when correlation increases, as even small changes in the basket value have amplified effects on the option price.

5.2.2 Spread option Delta

For spread options, the behavior is structurally opposite. Spread volatility (see equation 11), **decreases** with correlation. Consequently, as seen in Figure 6, both call and put Deltas flatten as ρ increases: the spread option becomes less sensitive to changes in the underlying spread.

Practically, this implies that high correlation simplifies hedging of spread positions. Because Delta varies more gradually with the underlying, rebalancing needs are reduced. However, this also reduces the potential for directional gains from spread divergence.

5.2.3 Comparison and implication

These opposite behaviors arise from correlation's inverse effects on basket versus spread volatility:

- **Basket options:** Correlation increases volatility, causing Delta to steepen.
- **Spread options:** Correlation decreases volatility, causing Delta to flatten.

A subtle but important nuance appears at **extreme initial price levels**. For both basket and spread options, the dependence of Delta on correlation weakens—and can even appear to reverse when the synthetic price is very high or very low.

This phenomenon is explained by **payoff saturation**. When the option is almost surely deep ITM or deep OTM:

- The payoff becomes nearly linear in the underlying,
- Delta approaches its limiting values (0, 1, or -1),
- Sensitivity to volatility and therefore to correlation—vanishes.

Thus, while Delta behaves monotonically with correlation in the economically meaningful region near the strike, its dependence naturally fades at extreme prices. Apparent reversals in the Delta–correlation trend in those regions are simply a consequence of the option already being effectively locked into its eventual payoff state.

5.3 Gamma sensitivity to correlation and initial price

Gamma measures the rate of change of Delta with respect to the underlying price, capturing the curvature of the option value. High Gamma indicates that Delta reacts sharply to small price moves, requiring frequent hedging adjustments. Figures 7–8 display the Gamma surfaces for basket and spread options across moneyness and correlation ρ.

5.3.1 Basket option Gamma

For basket options, Gamma exhibits a **non-monotonic** dependence on correlation that varies with moneyness.

ATM region: decreasing Gamma with higher correlation. When the basket is near the strike price, where Gamma naturally peaks, we observe a clear pattern: **ATM basket Gamma decreases as correlation increases.** At $\rho \approx -1$, the terminal basket distribution is tightly concentrated around the strike, producing a sharp Gamma peak. As correlation increases, the basket variance (see equation 10) rises, spreading probability mass away from the strike. This dispersion reduces the curvature of the option value at the strike, flattening the Gamma peak. Thus the **maximum ATM Gamma typically occurs at $\rho = -1$ for basket calls**.

Deep ITM/OTM region: Gamma increases with correlation. Away from the strike, where Gamma is usually small, increasing correlation tends to **increase** Gamma. Higher volatility introduces curvature in regions where the payoff would otherwise behave nearly linearly, making Gamma larger in magnitude.

Basket put tail effect. Basket puts display an additional tail-driven phenomenon: **Far OTM put Gamma is maximized at $\rho = 1$.** High positive correlation increases the probability of simultaneous large downside movements, making Delta extremely sensitive as the basket approaches the strike from above. In this case Gamma reflects tail-induced curvature rather than local strike curvature.

5.3.2 Spread option Gamma

For spread options, the behavior is more monotonic and structurally opposite to the basket case.

Spread volatility satisfies equation 11 which **decreases** as correlation increases. The resulting distribution becomes more concentrated around its mean.

ATM region: Gamma increases with correlation. Near the strike, this concentration steepens the curvature of the spread payoff, causing **ATM Gamma to increase with ρ.** The Gamma peak becomes sharper as correlation rises.

Away from ATM: Gamma decreases with correlation. When the spread is far ITM or OTM, the payoff becomes nearly linear, and Gamma rapidly drops toward zero. Since higher correlation further reduces spread volatility, **Gamma decays even faster away from the strike**.

5.3.3 Comparison and implications

The structural contrast between basket and spread options produces opposite Gamma behaviors:

- **Basket options:** ATM Gamma decreases with correlation, while ITM/OTM Gamma increases; puts exhibit a tail-risk spike at $\rho = 1$.
- **Spread options:** ATM Gamma increases with correlation, while off-ATM Gamma monotonically decreases with correlation.

From a hedging perspective, this means:

- Basket options require the most active hedging when correlation is low (sharp ATM Gamma).
- Spread options require the most active hedging when correlation is high.

5.4 Theta sensitivity to correlation and time to maturity

Theta (Θ) measures the sensitivity of an option's value to the passage of time. For European options, Theta is typically negative, reflecting the erosion of time value as maturity approaches. Figures 9–10 display the Theta surfaces for basket and spread options as functions of correlation and time to maturity. The vertical axis of each plot was restricted as the Theta decreases rapidly as the option approaches maturity.

5.4.1 Basket option Theta

Figures 9a and 9b show that Theta for basket calls and puts is negative across all maturities and correlations, and becomes sharply more negative as maturity approaches. A key observation is that **Theta becomes more negative as correlation increases**. Theta begins to decrease more rapidly for $\rho \approx 1$ than for $\rho \approx -1$.

This behavior is consistent with the effect of correlation on basket volatility. The variance of the basket (see equation 10) increases with ρ. Higher variance increases the option's time value, which means more time value is lost per unit of time as expiration approaches, producing a more negative Theta. As expected from put–call parity, the call and put Theta surfaces are nearly identical.

5.4.2 Spread option Theta

Figures 10a and 10b reveal that spread Theta behaves in a **fundamentally opposite** way from basket Theta. For spread options, **Theta becomes less negative as correlation decreases**. Theta begins to decrease more rapidly for $\rho \approx -1$ than for $\rho \approx 1$.

This behavior arises because correlation *reduces* spread volatility (see equation 11) becomes smaller as ρ increases. A less volatile spread has *less* extrinsic time value. Therefore, as ρ rises, the option has less time value to lose, and its Theta becomes **less negative**. In other words, high correlation causes the spread distribution to collapse toward its mean, reducing the speed at which uncertainty decays over time. Call and put Theta surfaces again coincide almost perfectly.

5.4.3 Comparison and implications

The Theta results highlight an important structural contrast:

- **Basket options:** Higher correlation ⇒ higher variance ⇒ more time value ⇒ *more negative* Theta.
- **Spread options:** Higher correlation ⇒ lower variance ⇒ less time value ⇒ *less negative* Theta.

For practitioners, this means:
- Basket options lose time value more rapidly as underlying assets become more correlated.
- Spread options lose time value more slowly in high-correlation environments.

These findings confirm that correlation alters not only price levels and curvature sensitivities (Gamma, Vega), but also the *speed* of time decay in multi-asset derivatives.

5.5 Vega sensitivity to correlation and initial price

Vega measures the sensitivity of an option's value to changes in the volatility of the underlying assets. For long European options, Vega is positive, since higher volatility increases the probability of favorable extreme outcomes and therefore raises the option's time value. Figures 11–12 show the Vega surfaces for basket and spread options across initial prices and correlation levels.

Across all four surfaces, Vega displays the classical bell-shaped form: it reaches its maximum when the option is at-the-money (ATM) and decays rapidly as the option becomes deep in- or out-of-the-money, where payoff uncertainty is reduced.

5.5.1 Basket option Vega

Figures 11a and 11b illustrate the Vega surfaces for the basket call and basket put. A central finding is that **basket Vega generally increases with correlation**. This follows directly from the basket variance (see equation 10) which increases as ρ increases. A higher baseline volatility makes the option more sensitive to further changes in volatility, resulting in a larger Vega.

However, in the region near the strike, Vega exhibits a clear **non-monotonic dependence** on correlation. In particular, the Vega of both the basket call and put can attain a *local maximum at* $\rho = -1$. This behavior occurs because strongly negative correlation concentrates probability mass near the strike, sharpening the sensitivity of the option's value to volatility precisely where Vega is naturally largest. Away from the ATM region, Vega increases smoothly with correlation. The call and put Vega surfaces are nearly identical, as expected from put–call parity in volatility sensitivity.

5.5.2 Spread option Vega

Figures 12a and 12b reveal a different structure for the Vega of spread options. In contrast to the basket case, **spread Vega decreases with correlation**, although this effect is mild near the strike. The variance of the spread satisfies equation 11 and thus *decreases* as ρ increases. Because the spread becomes more stable when the underlying assets move together, its value becomes less sensitive to additional changes in volatility.

Near the ATM region, Vega remains relatively stable with respect to correlation, exhibiting only a slight downward slope. Away from the strike, Vega decreases *monotonically* as correlation rises, since the payoff becomes more linear and less sensitive to changes in volatility. As with basket options, the call and put Vega surfaces coincide almost perfectly.

5.5.3 Comparison and implication

The Vega results highlight how correlation shapes volatility exposure differently across payoff structures:

- **Basket options:** Higher correlation $\Rightarrow$ higher baseline variance $\Rightarrow$ *higher* Vega, with possible non-monotonic peaks near the ATM region.
- **Spread options:** Higher correlation $\Rightarrow$ lower baseline variance $\Rightarrow$ *lower* Vega, especially away from the strike.

Despite the opposite effects on underlying variance, the sensitivity patterns reflect how correlation reshapes the distribution of terminal outcomes. In high-correlation environments, basket options exhibit amplified volatility exposure, while spread options experience diminished exposure.

6 Discussion and Future Work

6.1 Discussion

In summary, the primary objective of this project was achieved: we estimated the Greeks of put and call options for two types of exotic options, basket and spread options, written on two firms from the same industry, NVIDIA and AMD. We then examined how correlation affected option prices and Greeks using a Monte Carlo framework under Black–Scholes assumptions.

From the results, we could clearly see that correlation is a key driver for option prices and that the effects of said correlation were heavily dependent on the structure of the option payoff. For basket options, we saw that increases in correlation led to higher option prices for both the calls and puts consistently. This result is within expectations, because higher correlation increases the variance of the basket, which implies that since both the call and put payoffs are convex functions of the basket terminal value, higher variance increases expected payoffs as implied by Jensen's inequality. However, when it comes to spread options we saw that their prices are affected in a fundamentally opposite way from basket options. In fact, we can see from our results that, as correlation increases, the variance of the price difference between the two assets decreases, which in turn reduces the likelihood of the realization of high or extreme spread realizations and causes both spread put and call prices to drop.

Moving beyond prices, the analysis of Greeks reveals that correlation effects are highly non-linear and strongly dependent on moneyness, and this is especially true for higher-order Greeks. In fact, while Delta varies smoothly with correlation in a simple monotonic relationship, Gamma, Vega and Theta exhibit more complicated patterns.

For basket options, Gamma and Vega generally increase with correlation, which reflects the expected amplification of curvature and volatility sensitivity as the basket variance rises. However, when the initial basket price is near the ATM region, both Greeks display non-monotonic dependence on correlation, as changes in correlation near ATM alter local curvature and sensitivity to volatility as ρ varies. This causes the location of the maximum sensitivity when near ATM to shift across correlation values and away from high positive correlation, sometimes occurring at strongly negative correlation. Indeed, we can see that both the Vega of the basket put and call and the Gamma of the basket call can locally attain their maximum at $\rho = -1$. This is not the case for the Gamma of the basket put, since its maximum is instead reached when $\rho = 1$ and when the initial basket price is way higher than the strike price, which means that the put is deeply out-of-the-money. Therefore, this large Gamma reflects tail-driven curvature, where small changes in the initial basket price significantly increase the probability of crash/extreme downside, rather than local curvature around the strike price. Since, this probability of extreme downside increases with volatility, the local maximum is reached at $\rho = 1$.

Regarding the Theta of the basket option, increasing correlation raises basket variance and time value, causing Theta to become more negative, which is consistent with what we observe in the graphs.

For spread options, the surfaces of the prices and the Greeks display a different structure from those of the basket options. This is because increasing correlation reduces the variance of the spread, thus leading the price of the spread options to decrease monotonically with correlation. Near the strike, we saw that Gamma and Vega responded differently to changes. In fact, near the ATM region, Gamma shows a noticeable dependence on correlation in a similar way to what we saw for the basket options, but this time an increase in correlation leads to a lower variance and less dispersion in the distribution of the spread, which means a sharper payoff curvature around this region. In contrast, the Vega of the spread options remains relatively stable across different correlation values ATM, and we can only notice a mild downward trend as correlation increases. Away from the strike, both Gamma and Vega decrease monotonically with correlation, however this effect is more relevant for Vega, since Gamma rapidly falls toward zero, as the payoff becomes close to linear.

Regarding the Theta of spread options, it appears less sensitive to moneyness than Gamma or Vega, reflecting the fact that Theta is driven primarily by the amount of remaining uncertainty about the payoff rather than by local payoff curvature. As correlation increases, the variance and time value of the spread decrease, causing Theta to become less negative, an effect that is more pronounced for longer maturities.

Finally, when examining the Deltas for both our basket and spread options, we can notice that while Deltas generally vary monotonically with correlation, at extreme initial basket or spread levels, the dependence weakens and seems to flatten out or change sign locally. This can be explained by the fact, that at these initial levels, options are almost surely ITM or OTM, so payoff saturation limits the impact of variance on directional exposure, causing the dependence of Delta on correlation to weaken or appear to reverse.

6.2 Future work

The following are several ways that this project could be extended in the future:

- The estimation of the Greeks with respect to the parameters (price and volatility) of one underlying asset.
- The estimation of more Greeks in addition to the ones in the project such as Rho and higher order Greeks. This would allow us to further investigate sources of risk and how options react to changes in interest rates and non-linear effects.
- The use of a stochastic volatility model instead of a constant volatility one to more accurately simulate the behavior of real markets.
- The use of a dynamic correlation model instead of a constant correlation one in order to simulate changing market conditions.
- The inclusion of basket option with more than two underlying assets to see how these more complex correlation structures affect the Greeks.
- The inclusion of types of exotic options, or the selection of underlying assets that are not in the same industry to see if the Greeks patterns are similar.
- The performance of stress tests by simulating extreme market conditions in order to see how stable the models are.

7 Bibliography

Black, F. and Scholes, M. (1973). The pricing of options and corporate liabilities. *Journal of political economy*, 81(3):637–654.

Boyle, P., Broadie, M., and Glasserman, P. (1997). Monte carlo methods for security pricing. *Journal of Economic Dynamics and Control*, 21(8):1267–1321. Computational financial modelling.

Glasserman, P. (2003). *Monte Carlo Methods in Financial Engineering*, chapter 7, pages 377–385. Springer New York, NY, New York, USA.

Hull, J. C. (2022). *Options, Futures, and Other Derivatives, Global Edition*, chapter 19 and 26, pages 420–436, 627–628. Pearson.

A Appendix: Code and Data Processing

The R code for the simulation and plots is stored on GitHub. The below code is a sample of the code used to simulate option prices for varying correlation.

```
# Iterating over correlation values
for (rho in rho_partition){
  # Defining asset price paths based on correlation
  W_NVDA_T <- W_1_T
  W_AMD_T <- rho * W_1_T + sqrt(1 - rho^2) * W_2_T

  NVDA_path <- exp((rf - NVDA_vol^2 / 2) * T + NVDA_vol * W_NVDA_T)
  AMD_path <- exp((rf - AMD_vol^2 / 2) * T + AMD_vol * W_AMD_T)

  # Iterating over initial asset prices
  for (percentage_change in price_incremements){
    # Calculating initial asset prices
    basket_0 <- (NVDA_0 * (1 + percentage_change) + AMD_0 * (1 +
       percentage_change)) / 2

    # Simulating terminal asset prices
    NVDA_t <- NVDA_0 * (1 + percentage_change) * NVDA_path
    AMD_t <- AMD_0 * (1 + percentage_change) * AMD_path
    basket_t <- (NVDA_t + AMD_t) / 2

    # Discounting average payoffs at maturity
    basket_call <- exp(-rf * T) * mean(vapply(basket_t - basket_strike,
        payoff, FUN.VALUE = numeric(1)))
    basket_put <- exp(-rf * T) * mean(vapply(basket_strike - basket_t,
       payoff, FUN.VALUE = numeric(1)))

    # Capturing simulation results
    b_call_vec <- c(b_call_vec, basket_call)
    b_put_vec <- c(b_put_vec, basket_put)
    b_0_vec <- c(b_0_vec, basket_0)
    b_rho_vec <- c(b_rho_vec, rho)
  }
}
```

Project Stage	**Description of AI Use**	**% AI-Generated Content**	**AI Tools Used**
(The table below should be filled by all team members; leave blank if no AI was used for that step.)			
Conceptualization of Research Goals			
Literature Review			
Data Gathering			
Data Cleaning			
Coding / Model Implementation	AI was used to write a portion of the code used to add wire frames to the surface plots.	<2%	Microsoft Copilot
Mathematical Derivations			
Results Analysis	AI was used to only assist with minor language editing, rephrasing and reorganization of the written result analysis.	5-10%	Overleaf AI, Microsoft Copilot
Writing – Introduction	Minimal vocabulary suggestions and language clarity.	5%	Microsoft Copilot
Writing – Methods	AI was used to create a template for the lattice diagrams.	5%	Microsoft Copilot
Writing – Results and Discussion			
Editing / Grammar			
Bibliography			

Table 2: AI Use Disclosure Table

Human Contribution Statement

For each stage of the project where you disclosed using AI, clearly describe the oversight and validation made by human team members. Focus on intellectual input, decision-making, interpretation, and validation of results.

- Conceptualization of Research Goals:

- Literature Review:

- Data Gathering:

- Data Cleaning and Preparation:

- Model Coding / Implementation:
  The code generated by AI was validated by testing the plots to ensure that they accurately represented the simulation results.

- Mathematical Derivations:

- Numerical Results and Analysis:

- Writing – Introduction and Methods:
  The code generated by AI for the lattice diagrams was modified to achieve the desired presentation. For the introduction the main ideas and text was originally all human, AI just made minor suggestions to make sentences clearer. Important ideas and wording are intact.

- Writing – Results and Discussion:

- Editing / Proofreading:

- Bibliography and Citation Management: