$$\underbrace{(xy-7)}^2 = x^2+y^2$$

1ère méthode :

$$(xy)^2 - 14xy + 49 + 2xy = \underbrace{x^2+y^2+2xy}$$

$$\underbrace{(xy)^2 - \overset{2.6.xy}{12}xy} + 49 = (x+y)^2$$

$$\underbrace{(xy)^2 - 2(6)(xy) + 36} + 13 = (x+y)^2$$

$$(xy-6)^2 + 13 = (x+y)^2$$

$$13 = \underbrace{(x+y)^2 - (xy-6)}$$

$$13 = (x+y+xy-6)(x+y-xy+6)$$

$$\begin{cases} x+y+xy-6 = 13;\ 1;\ -13;\ -1 \\ x+y-xy+6 = 1;\ 13;\ -1;\ -13 \end{cases}$$

$+$

$$2(x+y) = 14,\ 14;\ -14;\ -14$$

$$x+y = 7;\ 7;\ -7;\ -7$$

$\ominus$ : $2xy - 12 = 12;\ -12;\ -12,\ 12$

$$xy = 12;\ 0;\ 0;\ 12$$

$$x+y = 7,\ 7;\ -7;\ -7$$

$$(x,y) \in \begin{cases} (3;4);(0;7);(0;-7);(-3;-4) \\ (4;3);(7;0);(-7;0);(-4;-3) \end{cases}$$

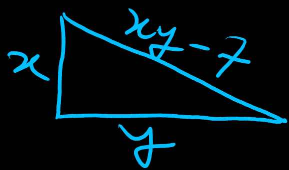
2nd méthode :

$$\boxed{(xy-7)^2 = x^2+y^2}.$$

* $xy=0$ ; $49 = x^2+y^2$

$\Rightarrow (7,0)\,;\,(7,0)$

* $xy>0$




$(x, y, xy-7)$ triplet pythagorien

$\exists\, a, b \in \mathbb{N}^*$ ; $0<b<a$

$x = a^2-b^2$ -

$y = 2ab$ -

$xy-7 = a^2+b^2$

$a \geq 2$ ✓

$b \geq 1$ ✓

$(a^2-b^2)(2ab) - 7 = a^2+b^2$

$a-b \geq 1$

$2ab(a+b)(a-b) - 7 = a^2+b^2$

$ab \geq a$

$ab \geq b+1$

$2ab(a+b)(a-b) = a^2+b^2+7$

$$\boxed{2ab(a+b)(a-b)} = (\underbrace{ab}_{\geq a}+\underbrace{ab}_{\geq b+1})(a+b)\underbrace{(a-b)}_{\geq 1}$$

$$a^2+b^2+7 \geq (a+b+1)(a+b)$$

$$\cancel{a^2}+\cancel{b^2}+7 \geq \cancel{a^2}+2ab+\cancel{b^2}+a+b$$

$$7 \geq 2ab+\underbrace{a}_{\geq 2}+\underbrace{b}_{\geq 1} \geq 2ab+3$$

$$7 \geq 2ab+3$$

2ab

$ab \leq 2$ , $a \geq 2$ , $b \geq 1$

$(3,4)$
$(4,3)$
$(0,7)$
$(7,0)$

$\Rightarrow a=2\,;\,b=1$ $\longrightarrow a^2-b^2$

$x = a^2-b^2 = 4-1 = 3\,;\; y = 2ab = 2(2) = 4$