$$\int_0^{2\pi} \sqrt{\frac{1+\cos x}{2}}\,dx$$

$\cos^2 x + \sin^2 x = 1$

$\cos 2x = \cos(x+x)$

$\cos(a+b) = \cos a \cos b - \sin a \sin b$

$\cos(a+a) = \cos a \cos a - \sin a \sin a$

$\cos 2a = \cos^2 a - \underbrace{\sin^2 a}_{1-\cos^2 a}$

$= \cos^2 a - 1 + \cos^2 a = 2\cos^2 a - 1$

$2\cos^2 a = 1 + \cos 2a$

$\cos^2 a = \frac{1+\cos 2a}{2}$

$2a = x \Rightarrow a = \frac{x}{2}$

$\sqrt{\cos^2 \frac{x}{2}} = \sqrt{\frac{1+\cos x}{2}}$

$\sqrt{A^2} = |A|$

$\sqrt{(-2)^2} = \sqrt{4} = 2$

$$\int_0^{2\pi} \sqrt{\frac{1+\cos x}{2}}\,dx = \int_0^{2\pi} \sqrt{\cos^2 \frac{x}{2}}\,dx$$

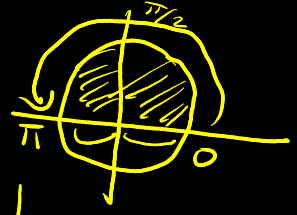
$$= \int_0^{2\pi} \left|\cos \frac{x}{2}\right| dx$$

Posons: $u = \frac{x}{2}$; $du = \frac{dx}{2} \Rightarrow dx = 2du$

si $x = 0$, $u = 0$

si $x = 2\pi$, $u = \pi$

$$\int_0^{2\pi} \left|\cos\frac{x}{2}\right| dx = \int_0^{\pi} 2\left|\cos u\right| du$$



$$= 2\int_0^{\pi/2} \cos u\, du + 2\int_{\pi/2}^{\pi} -\cos u\, du$$

$$= 2\int_0^{\pi/2} \cos u\, du - 2\int_{\pi/2}^{\pi} \cos u\, du$$

$$= 2\left[\sin u\right]_0^{\pi/2} - 2\left[\sin u\right]_{\pi/2}^{\pi}$$

$$= 2(1-0) - 2(0-1)$$

$$= 2+2 = 4$$

$$\boxed{\int_0^{2\pi} \sqrt{\frac{1+\cos x}{2}}\, dx = 4}$$