

$$x^2 - y = 31)$$

$$y^2 - x = 31)$$

$$x^2 - y = y^2 - x \Rightarrow x^2 - y - y^2 + x = 0$$

$$\Rightarrow \underbrace{x^2 - y^2} + (\underbrace{x - y}) = 0$$

$$\Rightarrow (\underbrace{x - y})(\underbrace{x + y}) + (x - y) = 0$$

$$\Rightarrow (x - y)(x + y + 1) = 0$$

$$\Rightarrow x - y = 0 \text{ ou } x + y + 1 = 0$$

$$\textcircled{1} \quad x - y = 0 \Rightarrow x = y.$$

$$x^2 - y = 31 \Rightarrow x^2 - x = 31 \Rightarrow x^2 - x - 31 = 0$$

$$\Delta = (-1)^2 - 4(-31) = 1 + 124 = 125$$

$$x = \frac{1 \pm 5\sqrt{5}}{2} \quad \left| \begin{array}{l} x = y = \frac{1 + 5\sqrt{5}}{2} \\ x = y = \frac{1 - 5\sqrt{5}}{2} \end{array} \right.$$

$$\textcircled{2}: \quad x + y + 1 = 0 \checkmark \Rightarrow y = -(x + 1)$$

$$x^2 - y = 31 \Rightarrow x^2 - (-(x + 1)) = 31 \Rightarrow x^2 + x + 1 = 31$$

$$\Rightarrow x^2 + x - 30 = 0$$

$$\Delta = (-1)^2 - 4(-30) = 1 + 120 = 121 = 11^2$$

$$x = \frac{-1 \pm 11}{2} = -6; 5$$

$$\left| \begin{array}{l} x = 5; \quad y = -6 \\ x = -6; \quad y = 5 \end{array} \right.$$

$$S_{\mathbb{R}^2} = \left\{ \left( \frac{1+5\sqrt{5}}{2}, \frac{1+5\sqrt{5}}{2} \right); \left( \frac{1-5\sqrt{5}}{2}, \frac{1-5\sqrt{5}}{2} \right); (5; -6); (-6; 5) \right\}$$