

$$\underbrace{x^5 + x^4 + x^3}_{x^3(x^2+x+1)} + \underbrace{x^2 + x + 1}_{} = 0$$

$$(x^3 + 1)(x^2 + x + 1) = 0$$

$$\rightarrow (x+1)(x^2 - x + 1)(x^2 + x + 1) = 0$$

$$\Rightarrow x+1=0 \text{ ou } x^2 - x + 1 = 0 \text{ ou } x^2 + x + 1 = 0$$

$$\Rightarrow x = -1 ; x = \frac{1 \pm i\sqrt{3}}{2} \text{ ou } x = \frac{-1 \pm i\sqrt{3}}{2}$$

2ème méthode:

$$x^5 + x^4 + x^3 + x^2 + x + 1 = 0$$

$$\begin{array}{r|l} x^5 + x^4 + x^3 + x^2 + x + 1 & x+1 \\ \hline -(x^5 + x^4) & x^4 + x^2 + 1 \\ \hline & + x^3 + x^2 + x + 1 \\ & -(x^3 + x^2) \\ \hline & x + 1 \\ & -(x+1) \\ \hline & 0 \end{array}$$

$$x^5 + x^4 + x^3 + x^2 + x + 1 = (x+1)(x^4 + x^2 + 1) = 0$$

$$\Rightarrow x+1=0 \text{ ou } x^4 + x^2 + 1 = 0$$

$$\frac{x^4}{x^2} + \frac{x^2}{x^2} + \frac{1}{x^2} = 0 \Rightarrow x^2 + 1 + \frac{1}{x^2} = 0$$

$$\Rightarrow \left(x^2 + \frac{1}{x^2}\right) + 1 = 0$$

Posons: $u = x + \frac{1}{x}$

$$u^2 = \left(x + \frac{1}{x}\right)^2 = x^2 + 2 + \frac{1}{x^2} = \left(x^2 + \frac{1}{x^2}\right) + 2$$

$$u^2 - 2 = x^2 + \frac{1}{x^2}$$

$$u^2 - 2 + 1 = 0 \Rightarrow u^2 - 1 = 0 \Rightarrow u^2 = 1 \Rightarrow u = 1 ; u = -1$$

$$* x + \frac{1}{x} = 1 \Rightarrow x^2 + 1 = x \Rightarrow x^2 - x + 1 = 0$$

$$\Rightarrow x = \frac{1 \pm i\sqrt{3}}{2}$$

$$* x + \frac{1}{x} = -1 \Rightarrow x^2 + 1 = -x \Rightarrow x^2 + x + 1 = 0$$

$$\Rightarrow x = \frac{-1 \pm i\sqrt{3}}{2}$$

3^e méthode:

$$\underbrace{x^5 + x^4}_{x^4(x+1)} + \underbrace{x^3 + x^2}_{x^2(x+1)} + \underbrace{x + 1}_{1} = 0$$

$$(x+1)(x^4 + x^2 + 1) = 0$$

$$\Rightarrow x+1=0 \quad \checkmark \quad \text{ou} \quad x^4 + \underbrace{x^2}_u + 1 = 0 \quad \checkmark$$

$$u = x^2; \quad u^2 + u + 1 = 0; \quad \Delta = 1^2 - 4(1) = -3$$

$$u = \frac{-1 \pm i\sqrt{3}}{2}; \quad x^2 = \frac{-1 \pm i\sqrt{3}}{2}$$

$$x^4 + x^2 + 1 = 0$$

$$x^4 + \underbrace{x^2}_u + 1 + \underbrace{x^2}_{-x^2} - x^2 = 0$$

$$\underbrace{x^4 + 2x^2 + 1}_{(x^2+1)^2} - x^2 = 0$$

$$(x^2+1)^2 - x^2 = 0$$

$$(\underbrace{x^2+1+x}) (\underbrace{x^2+1-x}) = 0$$

$$x = \frac{-1 \pm i\sqrt{3}}{2}; \quad x = \frac{1 \pm i\sqrt{3}}{2}$$