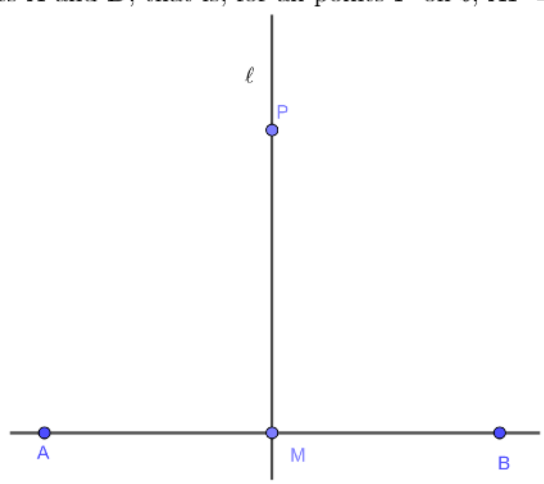
MTH 335 SAMPLE MIDTERM EXAMS

0. Introduction

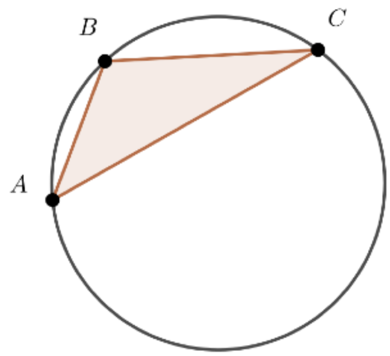
This is a collection of problems given on previous midterm exams. Not every subcollection was an actual exam, but I have tried to separate them into single exam size subcollections.

1. Exam 1

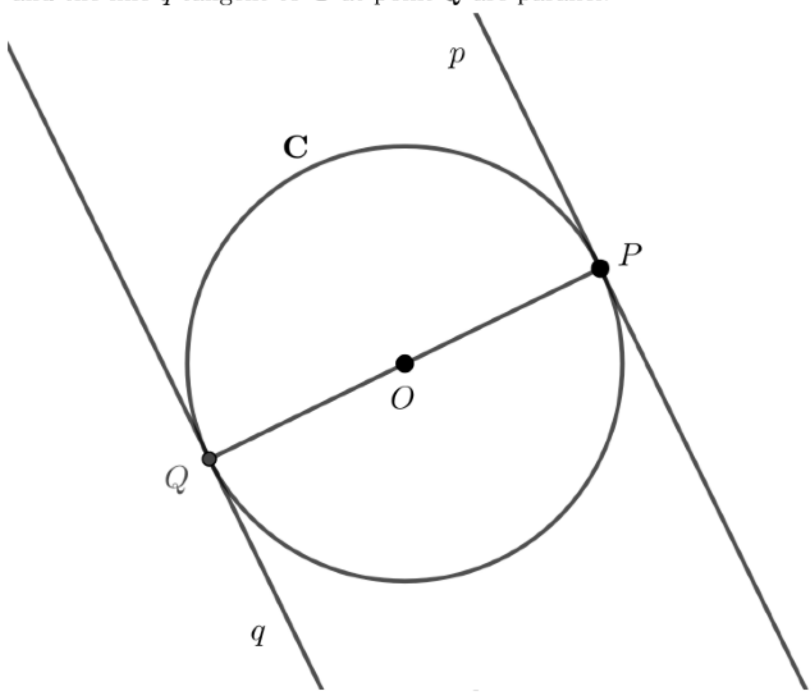
Problem 1. Let AB be a line segment and let ℓ be a line perpendicular to $\overleftrightarrow{AB}$ meeting AB at the midpoint M of AB. Show that every point of ℓ is equidistant from the endpoints A and B, that is, for all points P on ℓ, $AP \cong BP$.



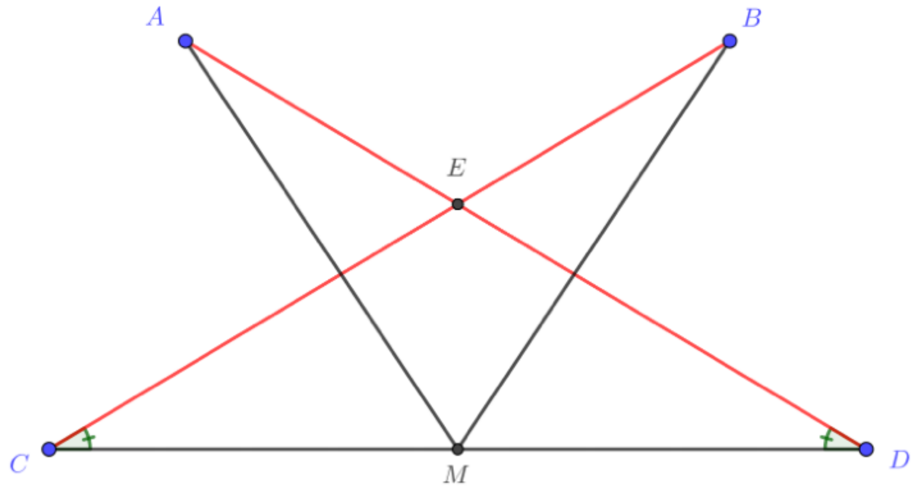
Problem 2. Let $\triangle ABC$ be a triangle. Construct a circle passing through all three vertices. (Remember: A correct answer is a list of steps of the construction followed by a description of why those steps accomplish the desired construction.) You may use the result in Problem 1 even if you cannot solve Problem 1.



Problem 3. Let PQ be a diameter of circle $\mathbf{C}$. Show that the line p tangent to $\mathbf{C}$ at P and the line q tangent to $\mathbf{C}$ at point Q are parallel.



Problem 4. Given four points, A, B, C, D with A and D on opposite sides of $\overleftrightarrow{BC}$ and B and C on opposite sides of $\overleftrightarrow{AD}$, let E be the point of intersection of AD and BC. Let M be the midpoint of CD. Assuming $AD \cong BC$ and $\angle ADC \cong \angle BCD$, show that $\angle AMC \cong \angle BMD$.



2. Exam 2

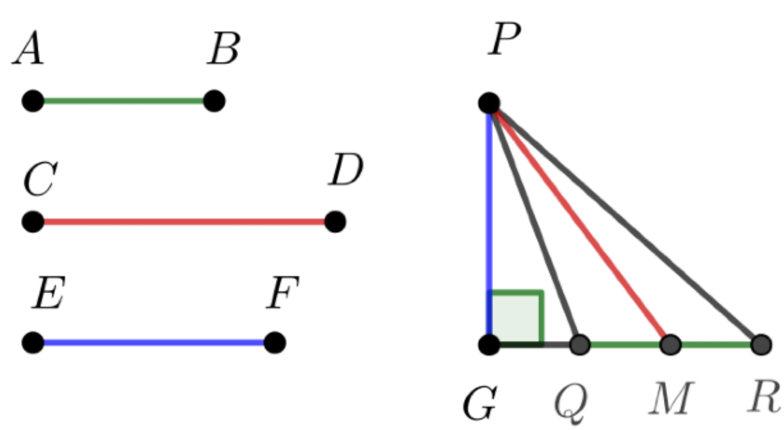
Problem 1. First, two definitions.

Definition. Given a triangle, a **median** of that triangle is a line segment from a vertex to the midpoint of the opposite side.

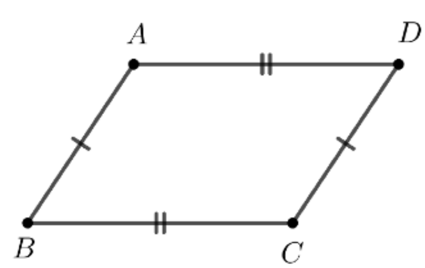
Definition. Given a triangle, an **altitude** of that triangle is a line segment from a vertex to the line containing the opposite side that is perpendicular to that line.

Given line segments AB, CD and EF, with $\mathcal{L}(CD) \geq \mathcal{L}(EF)$, construct a triangle $\triangle PQR$ so that $QR \cong AB$, the median from P is congruent to CD and the altitude from P is congruent to EF.

Hint: It's easier if you don't start with QR.

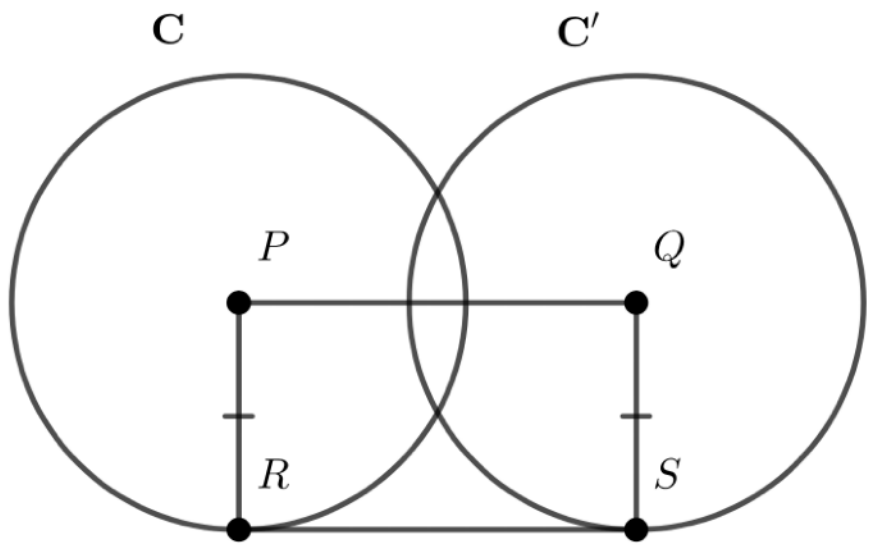


Problem 2. Given four points, A, B, C and D such that A and C are on the opposite sides of $\overleftrightarrow{BD}$, $AB \cong CD$ and $BC \cong AD$, show that $\overleftrightarrow{BC}$ is parallel to $\overleftrightarrow{AD}$ and $\overleftrightarrow{AB}$ is parallel to $\overleftrightarrow{CD}$.



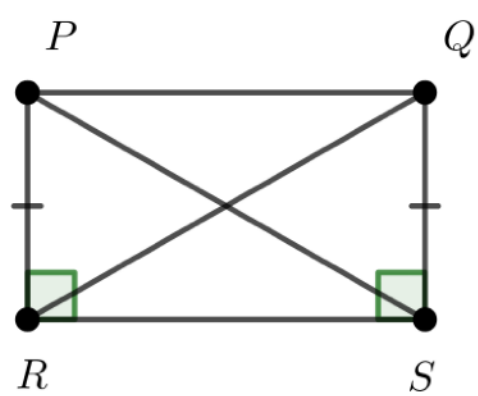
Problem 3. This is a five-part problem; you may use the result of a part in a later part even if you were not successful in the earlier part.

Let circle $\mathbf{C}$ have center P and radius r, circle $\mathbf{C}'$ center Q and the same radius r. Let R be a point on $\mathbf{C}$ and S a point on $\mathbf{C}'$ so that R and S are on the same side of $\overleftrightarrow{PQ}$ and $\overleftrightarrow{RS}$ is tangent to both $\mathbf{C}$ and $\mathbf{C}'$.

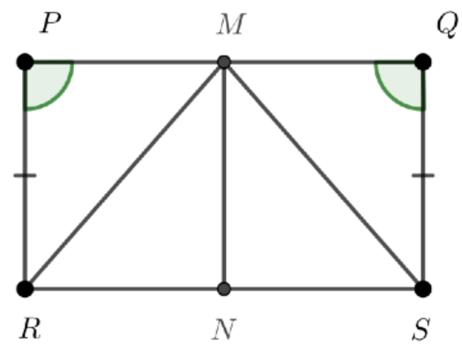


I. Show that $\overleftrightarrow{PR}$ is perpendicular to $\overleftrightarrow{RS}$ and that $\overleftrightarrow{QS}$ is perpendicular to $\overleftrightarrow{RS}$.

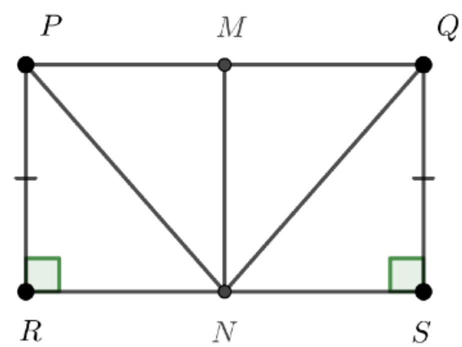
II. Show that $\angle RPQ \cong \angle PQS$. (Note: This is to be done *without* the Euclidean parallel axiom, so Theorem 64 does not apply.)



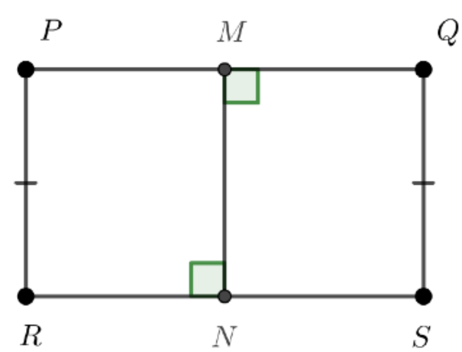
III. Let M be the midpoint of PQ and N the midpoint of RS. Show that $\overleftrightarrow{MN}$ is perpendicular to $\overleftrightarrow{RS}$.



IV. Show that $\overleftrightarrow{MN}$ is perpendicular to $\overleftrightarrow{PQ}$.

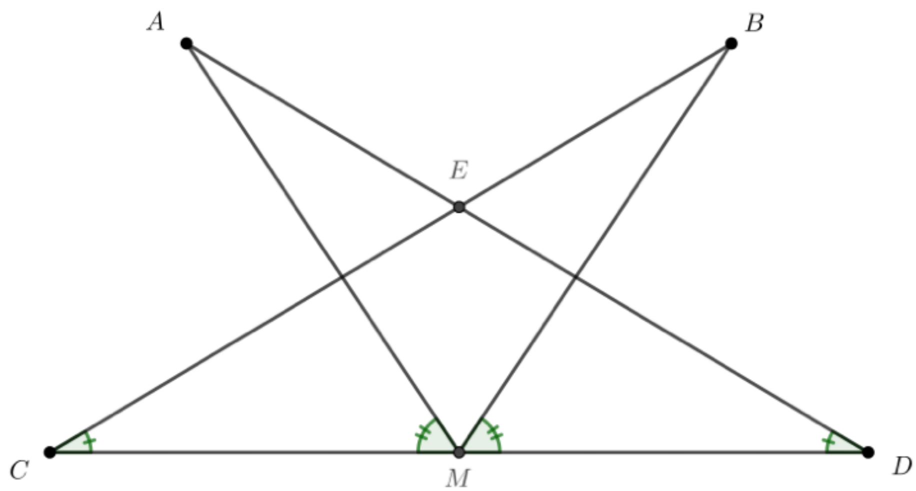


V. Show that $\overleftrightarrow{RS}$ is parallel to $\overleftrightarrow{PQ}$.

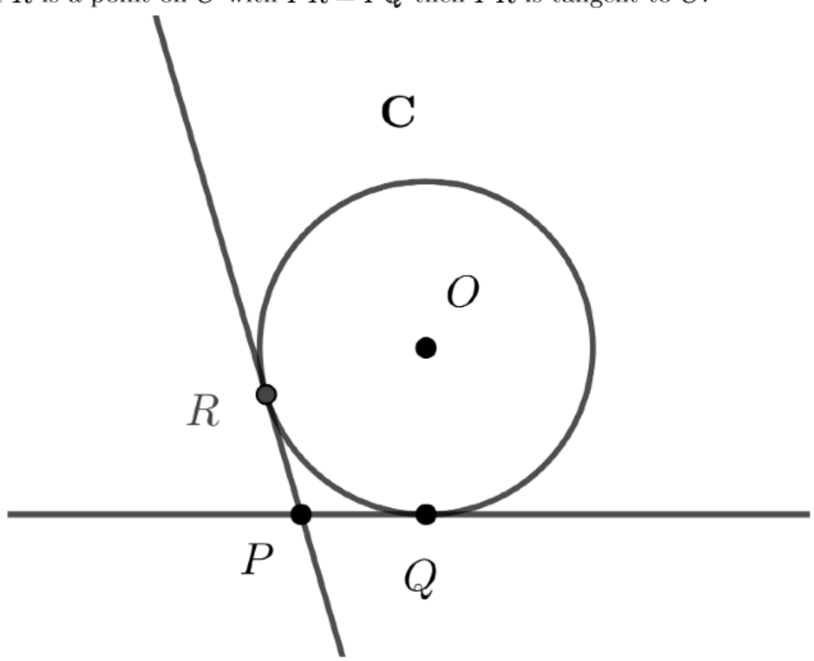


3. Exam 3

Problem 1. Let point E be between points A and D and assume E is also between points C and B. Let M be the midpoint of CD. Assume $\angle ADC \cong \angle BCD$ and $\angle AMC \cong \angle BMD$. Show that $AD \cong BC$.



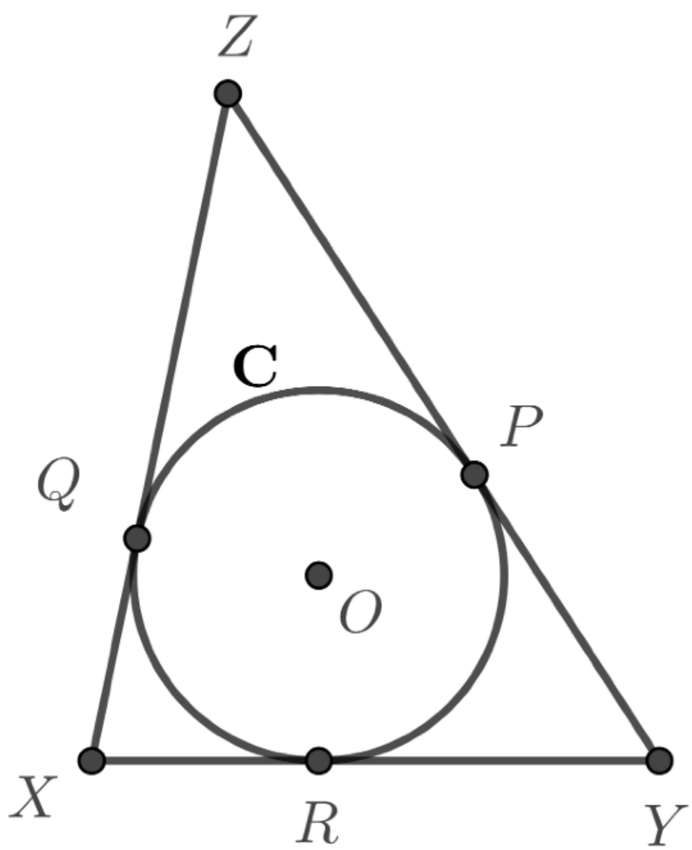
Problem 2. Let C be a circle, Q a point on C, $\overleftrightarrow{PQ}$ a line tangent to C. Show that if R is a point on C with $PR \cong PQ$ then $\overleftrightarrow{PR}$ is tangent to C.



Problem 3. Let $\triangle XYZ$ be a triangle and let C be a circle with

- $\overleftrightarrow{XY}$ tangent to C at point R on XY,
- $\overleftrightarrow{YZ}$ tangent to C at point P on YZ,
- $\overleftrightarrow{XZ}$ tangent to C at point Q on XZ,

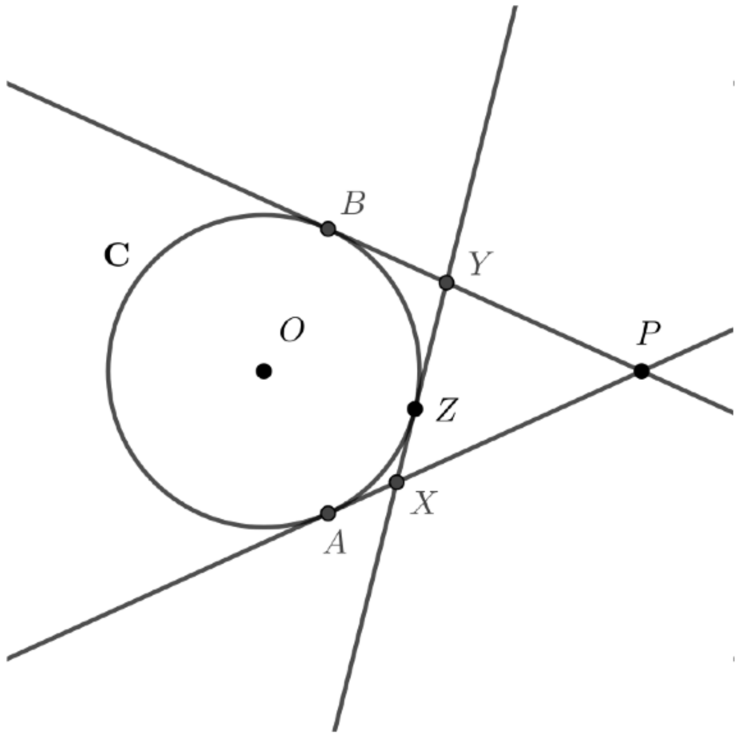
and suppose $\mathcal{L}(XY) = 15$, $\mathcal{L}(YZ) = 21$, and $\mathcal{L}(XR) = 6$. Find $\mathcal{L}(QZ)$.



Problem 4. Construct a triangle given two sides and a length along one side to where the incircle intersects the side. More exactly, given segment AB, segment FD and point E between A and B, with $\mathcal{L}(AB) < \mathcal{L}(AE) + \mathcal{L}(FD)$, construct a triangle $\triangle XYZ$ with $XY \cong AB$, $YZ \cong FD$ and so that there is a circle C tangent to each of the three sides of $\triangle XYZ$ and tangent to $\overleftrightarrow{XY}$ at point R on XY with $XR \cong AE$. Remember, a construction is a list of steps followed by an explanation of why those steps accomplish the desired task.

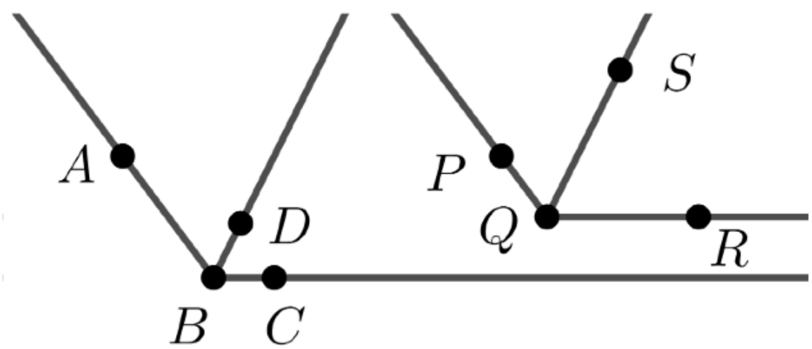
4. Exam 4

Problem 1. Let C be a circle with center O. Let P be a point outside of C and let A and B be points on C with $\overleftrightarrow{AP}$ and $\overleftrightarrow{BP}$ tangent to C. Let X be a point on AP, Y a point on BP so that $\overleftrightarrow{XY}$ is tangent to C at point Z. Show that $\mathscr{L}(PX) + \mathscr{L}(XZ) = \mathscr{L}(PY) + \mathscr{L}(YZ)$.

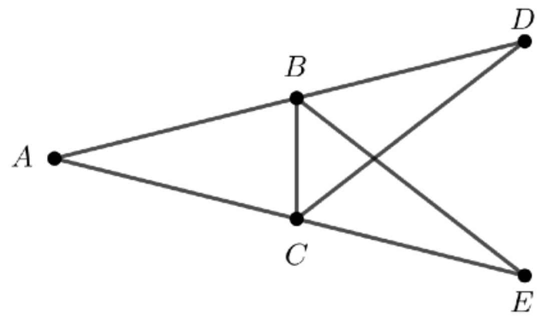


Problem 2. Let $\angle ABC \cong \angle PQR$, and let $\overleftrightarrow{BD}$ be the bisector of $\angle ABC$ and $\overleftrightarrow{QS}$ be the bisector of $\angle PQR$. Show that $\angle ABD \cong \angle PQS$C.

Remember: You are *not* allowed to use angle measure.



Problem 3. Let ΔABC be a triangle with $AB \cong AC$, let D be a point on $\overleftrightarrow{AB}$ with B be between points A and D, and let E be a point on $\overleftrightarrow{AC}$ with C be between points A and E. Assume that $\overrightarrow{BE}$ bisects $\angle CBD$ and $\overrightarrow{CD}$ bisects $\angle BCE$. Show that $\Delta BCD \cong \Delta CBE$. You may use the result stated in the previous problem even if you have not been able to solve the previous problem.

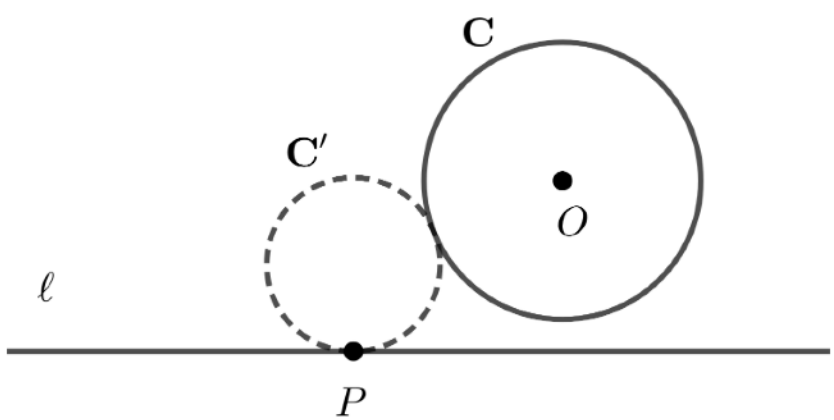


Problem 4. We first make the following definition.

Definition. Two circles are tangent if they intersect at a point where they have a common tangent line.

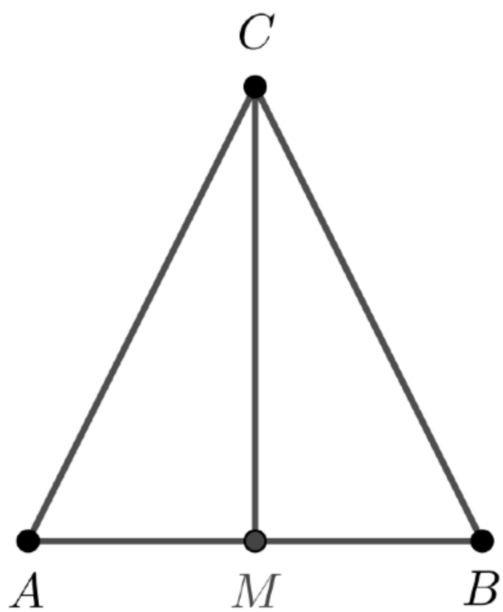
Given a circle C with center O, a line ℓ that does not intersect C, and a point P on line ℓ with $\overleftrightarrow{OP}$ not perpendicular to ℓ, construct a circle C' which is tangent to ℓ at P, tangent to circle C and, except for the one point of intersection, has circle C on the outside.

Remember: Your answer should consist of a numbered list of steps followed by an explanation of why those steps accomplish the desired result. In your explanation you may use, without proof, the fact the perpendicular bisector of a line segment consists exactly of the points equidistant from the two endpoints. You may use any construction we have done as a single step in your construction. *Hint:* What must be the relationship between the radius of C, the radius of C', and the distance from the center of C' to the center of C?

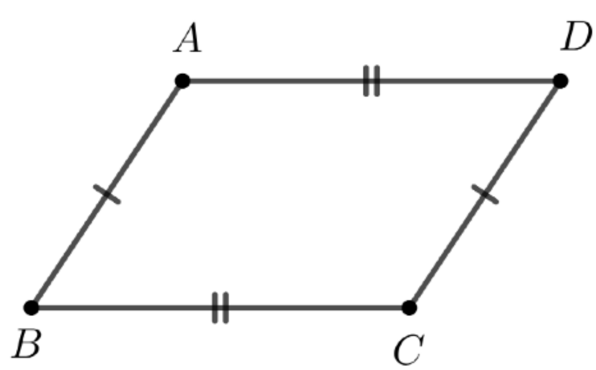


5. Exam 5

Problem 1. Let ΔABC be a triangle and let M be the midpoint of AB. Show that if $\overleftrightarrow{CM} \perp \overleftrightarrow{AB}$ then ΔABC is isosceles.



Problem 2. [Yes, this is the same as problem 2 of Exam 2. It's just here to make this exam the proper length] Let A, B, C, D be four points with B and D on opposite sides of $\overleftrightarrow{AC}$, A and C on opposite sides of $\overleftrightarrow{BD}$, $AB \cong CD$, and $AD \cong BC$. Show that $\overleftrightarrow{AB}$ is parallel to $\overleftrightarrow{CD}$.

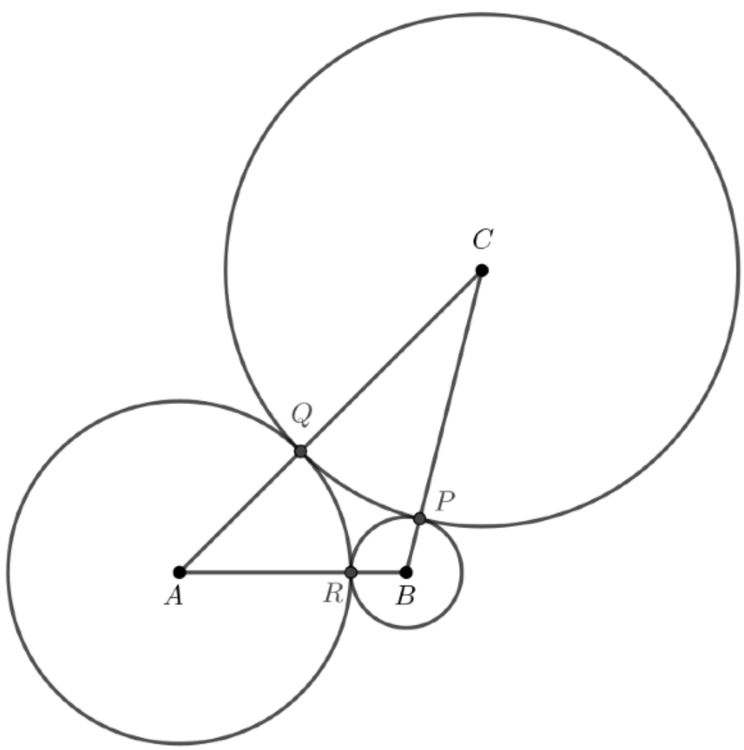


Problem 3. Given triangle ΔABC, construct points P, Q, R on BC, AC, AB, respectively so that $AQ \cong AR$, $BP \cong BR$, and $CP \cong CQ$. (*Hint:* First show, for example, that if we have such points,

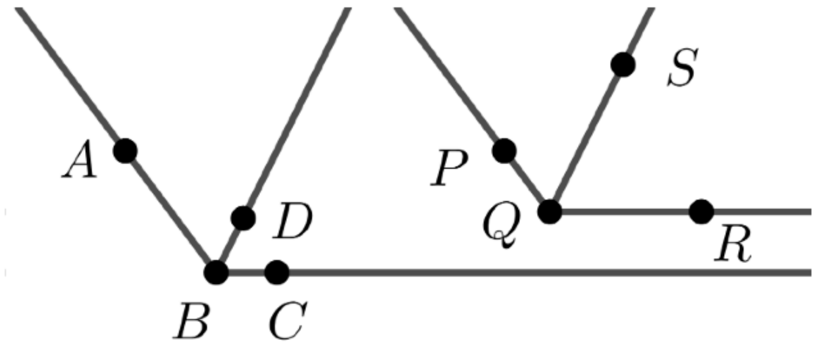
$$\mathscr{L}(AR) = \frac{\mathscr{L}(AB) + \mathscr{L}(AC) - \mathscr{L}(BC)}{2},$$

then show how to construct a segment of such a length. (You may assume without proof the triangle inequality: That the sum of lengths of two sides of a triangle is greater than the length of the third side.) If you've done it right, once you have constructed one of the three points you can say for the remaining steps "The other two points are constructed similarly." This still leaves the explanation do be done properly.

Remember, a correct answer consists of a numbered list of steps and an explanation of why those steps succeed in construct the desired objects.



Problem 4. Assume D is in the interior of $\angle ABC$, that S is in the interior of $\angle PQR$, that$\angle ABD \cong \angle PQS$ and $\angle CBD \cong \angle RQS$. Show that $\angle ABC \cong \angle PQR$.
Remember: You may *not* use angle measure.



Hint: Without loss of generality we may assume $AB \cong CB \cong PQ \cong QR$, that D lies on AC and that S lies on PR. Let S' be a point on ray $\overrightarrow{QS}$ so that $QS' \cong BD$. Using some triangle congruences, show that if $S \neq S'$ we get a contradiction to Axiom 4, so $S = S'$. You should then be able to complete the proof using some triangle congruences and Axiom 1.