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Mathematics

Problem 1.1

$$\frac{y^{58}}{y^{4} \cdot y^{12}} = \frac{y^{58}}{y^{16}} = y^{42}$$

Problem 1.2

$$8^{2} \cdot 2^{x} = 2^{9}$$

$$(2^{3})^{2} \cdot 2^{x} = 2^{9}$$

$$2^{6} \cdot 2^{x} = 2^{9}$$

$$2^{x} = \frac{2^{9}}{2^{6}}$$

$$2^{x} = 2^{3}$$

$$x = 3$$

Problem 1.3

$$\frac{x}{y} = 3 \qquad x^{-2} \cdot y^{2} = \frac{y^{2}}{x^{2}}$$

$$\frac{y}{x} = 3^{-1}$$

$$\frac{y}{x} = \frac{1}{3}$$

$$\frac{y^{2}}{x^{2}} = \left(\frac{1}{3}\right)^{2} = \frac{1}{9}$$

Problem 1.4

$$\frac{\sqrt{2^{13}}}{\sqrt{8^3}} = \frac{\sqrt{2^{13}}}{\sqrt{(2^3)^3}} = \frac{\sqrt{2^{13}}}{\sqrt{2^9}} = 2^{\frac{13}{2}-\frac{9}{2}} = 2^{\frac{4}{2}} = 2^2 = 4$$

Problem 1.5

a) $x+y = y+x$ $\rightarrow$ True

b) $x(y+z) = xy + xz$ $\rightarrow$ True

~~z~~ c) $x^{y+z} = x^y x^z$ $\rightarrow$ True

d) $\frac{x^y}{x^z} = x^{y-z}$ $\Rightarrow$ True

Problem 1.6

$$\frac{x^2-25}{x-5} = 3$$

$$\frac{\cancel{(x-5)}(x+5)}{\cancel{x-5}} = 3$$

$$x+5 = 3$$

$$x = -2$$

Problem 2.1.

$f(x) = ax + b$ (linear)

$$\begin{cases} -460 = b \text{ (interept)} \\ 1340 = 1000a + b \end{cases}$$

$1340 + 460 = 1000a$

$\frac{1800}{1000} = a$

$a = 1.8$

$F = 1.8 \cdot K - 460$

$x = 1.8x - 460$

$x - 1.8x = -460$

$-0.8x = -460$

$0.8x = 460$

$x = \frac{460}{0.8} = 575$

Answer: $575\,°F = 575\,°K$

Problem 2.2

$y = f(x)$

$2x + 3 = 17$

$2x = 14$

$x = 7$

Problem 2.3

$3^{2x^2 - 4x + 3} = 3^3$

$2x^2 - 4x + 3 = 3$

$2x^2 - 4x = 0$

$2x(x - 2) = 0$

$x_1 = 0 \qquad x_2 = 2$

Problem 2.4

$$X \cdot (1.01)^t = 2X$$

$$1.01^t = 2$$

$$\ln 1.01^t = \ln 2$$

$$t \cdot \ln 1.01 = \ln 2$$

$$t = \frac{\ln 2}{\ln 1.01} \approx 69.6607$$

Answer: It takes the economy approximately 70 years to double its GDP

Problem 2.5

$$\ln\left(\frac{e^2}{e^3}\right) = \ln e^{-1} = -1 \ln e = -1 \cdot 1 = -1$$

Problem 3.1

$$\sum_{i=0}^{\infty}\left(\frac{1}{6^i}+\frac{1}{4^i}\right)=\frac{1}{6^1}+\frac{1}{6^2}+\frac{1}{6^3}\ \dots\ +\frac{1}{4^1}+\frac{1}{4^2}+\frac{1}{4^3}=S_1+S_2$$

$\Downarrow$ leading to 2 separate decreasing geometric progressions

$b_1=\frac{1}{6}$ $q=\frac{1}{6}$

$$S_1=\frac{\frac{1}{6}}{1-\frac{1}{6}}=\frac{1}{5}$$

$b_2=\frac{1}{4}$ $b_3=\frac{1}{4}$

$$S_2=\frac{\frac{1}{4}}{1-\frac{1}{4}}=\frac{1}{3}$$

$$S_1+S_2=\frac{1}{5}+\frac{1}{3}=\frac{8}{15}$$

Problem 3.2

$$\lim_{x\to 3}\frac{x^2-9}{x-3}=\lim_{x\to 3}\frac{\cancel{(x-3)}(x+3)}{\cancel{x-3}}=\lim_{x\to 3}(x+3)=3+3=6$$

Problem 3.3

$f(x)=x^3-4$

$(-1;\ -5)$

$f'(x)=3x^2$

Slope $=3\cdot(-1)^2=3$

Problem 3.4

$$\left(\frac{x^2+3}{x+2}\right)' = \frac{(x^2+3)'(x+2)-(x^2+3)(x+2)'}{(x+2)^2} =$$

$$= \frac{2x(x+2)-(x^2+3)}{(x+2)^2} = \frac{2x^2+4x-x^2-3}{(x+2)^2} = \frac{x^2+4x-3}{(x+2)^2}$$

Problem 3.5

$f(x) = x^7 + 4x^2$

$f'(x) = 7x^6 + 8x \qquad f''(x) = 42x^5 + 8$

Problem 3.6

$$f(x) = \frac{x^4+4^x}{\ln x} = \frac{(x^4+4^x)'\ln x - (\ln x)'(x^4+4^x)}{\ln x^2} =$$

$$= \frac{(4x^3+4^x)\ln x - \frac{1}{x}(x^4+4^x)}{\ln x^2}$$

Problem 3.7

$f(x) = 3x^3 - 9x$

$f'(x) = 9x^2 - 9 = 0 \Rightarrow \quad x^2 - 1 = 0$

$x = \pm 1$

$f''(x) = 18x > 0$

minima point at $x = 1$, $x = -1 \Rightarrow$ maxima

Problem 3.8

$f(x, y) = x^2 + 2y^3 \qquad f(2, 3) = 2^2 + 2 \cdot 3^3 = 58$

Problem 3.9

$$f(x, y) = \ln(2x - y)$$

$$2x - y > 0$$

Problem 3.10

$$f(x, y) = x^5 \cdot e^y + x^2 \cdot y^3$$

$$f_x' = 5x^4 \cdot e^y + 2x \cdot y^3$$

$$f_y' = x^5 \cdot e^y + 3y^2 \cdot x^2$$

Problem 3.11

$$f(x, y) = \underbrace{\sqrt{xy}}_{0} - 0.7x - 0.7y$$

$$x \cdot y = 0 \Rightarrow \text{min}$$

$$f_x' = \frac{\sqrt{y}}{2\sqrt{x}} = 0.7$$

$$f''(x) = -\frac{\sqrt{y}}{4\sqrt{x^3}} < 0 \text{ ; so } f'(x) = 0 \text{ maxima}$$

$$f''(y) = \frac{1}{2}\sqrt{\frac{x}{y}} - 0 = 0$$

$$f''(y) = -\frac{1}{4}\frac{\sqrt{x}}{\sqrt{y^3}} < 0 \Rightarrow \text{maxima}$$

$$\cancel{\frac{\sqrt{y}}{2\sqrt{x}}} \neq \frac{\sqrt{y}}{2\sqrt{x}} - 0.7 = \frac{\sqrt{x}}{\sqrt{y}} - 0.7$$

$$x = y \Rightarrow \text{maxima}$$

Problem 3.12

$$\max\ x^2y^2 \quad \text{s.t. } x+y=10$$

$$\mathbb{L} = x^2y^2 + \lambda(10-x-y)$$

$$\mathbb{L}_x = 2xy^2 - \lambda = 0$$

$$\mathbb{L}_y = 2yx^2 - \lambda = 0$$

$$\mathbb{L}_\lambda = 10-x-y = 0$$

$$2xy^2 - \lambda = 2yx^2 - \lambda$$

$$y = x$$

$$10-x-y=0$$

$$10-2y=0$$

$$\underline{y = 5 = x}$$

$$\lambda = 2xy^2 = 2\cdot 5^3$$

$$\max\ x^2y^2 = 5^4$$

Problem 4.1

$$\begin{bmatrix} 2 & 3 \\ 4 & 1 \\ 1 & 2 \end{bmatrix} \times \begin{bmatrix} 1 & 4 & 1 \\ 2 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 2\cdot 1 + 3\cdot 2 & 2\cdot 4 + 3\cdot 1 & 2\cdot 1 + 3\cdot 2 \\ 4\cdot 1 + 1\cdot 2 & 4\cdot 4 + 1\cdot 1 & 4\cdot 1 + 1\cdot 2 \\ 1\cdot 1 + 2\cdot 2 & 1\cdot 4 + 2\cdot 1 & 1\cdot 1 + 2\cdot 2 \end{bmatrix} =$$

$$= \begin{bmatrix} 8 & 11 & 8 \\ 6 & 17 & 6 \\ 5 & 6 & 5 \end{bmatrix}$$

Problem 4.2

$$\mathbb{B}\cdot A = \begin{bmatrix} 1\cdot 2+4\cdot 4+1\cdot 1 & 1\cdot 3+4\cdot 1+1\cdot 2 \\ 2\cdot 2+1\cdot 4+1\cdot 2 & 2\cdot 3+1\cdot 1+2\cdot 2 \end{bmatrix} =$$

$$= \begin{bmatrix} 2+16+1 & 3+4+2 \\ 4+4+2 & 6+1+4 \end{bmatrix} = \begin{bmatrix} 19 & 9 \\ 10 & 11 \end{bmatrix}$$

Problem 4.3

$$\mathbb{A}^T = \begin{bmatrix} 3.3 & 6.1 & 45.75 \\ 5.1 & 1.23 & 0 \end{bmatrix}$$

Problem 4.4

$$\det(A) = \begin{matrix} 2 & 3 & 0 & 2 & 3 \\ 4 & 5 & 2 & 4 & 5 \\ 2 & 5 & 3 & 2 & 5 \end{matrix}$$

$$\begin{matrix} 2 & 3 & 0 & 2 & 3 \\ 4 & 5 & 2 & 4 & 5 \\ 2 & 5 & 3 & 2 & 5 \end{matrix}$$

$$\det(A) = 2\cdot 5\cdot 3 + 3\cdot 2\cdot 2 + 0\cdot 4\cdot 5 -$$
$$- 3\cdot 4\cdot 3 - 2\cdot 2\cdot 5 - 0\cdot 5\cdot 2 =$$
$$= 30 + 12 + 0 - 36 - 20 - 0 =$$
$$= -14$$

Problem 5.1

We toss a coin twice $\rightarrow 2^2 = 4$

$$\Omega = \{(H,H), (H,T), (T,T), (T,H)\}$$

Problem 5.2

$n = 30$
$k = 3$

$$\# \mathbb{P}_k^n = \frac{30!}{(30-3)!} = \frac{30!}{27!} = 28 \cdot 29 \cdot 30 =$$

$$= 24360$$

Answer: 24360 possible outcomes exist for the 1st, 2nd and 3rd place

Problem 5.3

$$\Omega = \{ (1,1), (1,2) \dots (6,6) \}$$

$$\# \Omega = 36$$

$$A = \{ (1,1), (1,2), (1,3), (1,4), (1,5), (1,6),$$
$$(2,1), (2,3), (2,5),$$
$$(3,1), (3,2), (3,3), (3,4), (3,5), (3,6),$$
$$(4,1), (4,3), (4,5),$$
$$(5,1), (5,2), (5,3), (5,4), (5,5), (5,6),$$
$$(6,1), (6,3), (6,5) \} \qquad \# A = 27$$

$$\mathbb{P}(A) = \frac{27}{36} = \frac{3}{4}$$

Answer: With 75% probability that at least one of the tosses end up being odd