Solution of HW#4, (CE391F)

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Source code: https://goo.gl/TNeDXV

<u>I-Random numbers:</u> Generate 10,000 random numbers using the linear congruential generator x(n+1)=ax(n)+c mod m, where a= 22695477, c=1 and m= 2^{32}

The random numbers are defined as u(n)=$\frac{x(n)}{2^{32}}$, and are between 0 and 1. Initialize the sequence with your favorite number $0 \le x(0) \le 2^{32}$ (random seed).

1) *Compute the mean value of the random numbers that you generated, and compare with the mean value of a uniform distribution between 0 and 1.*

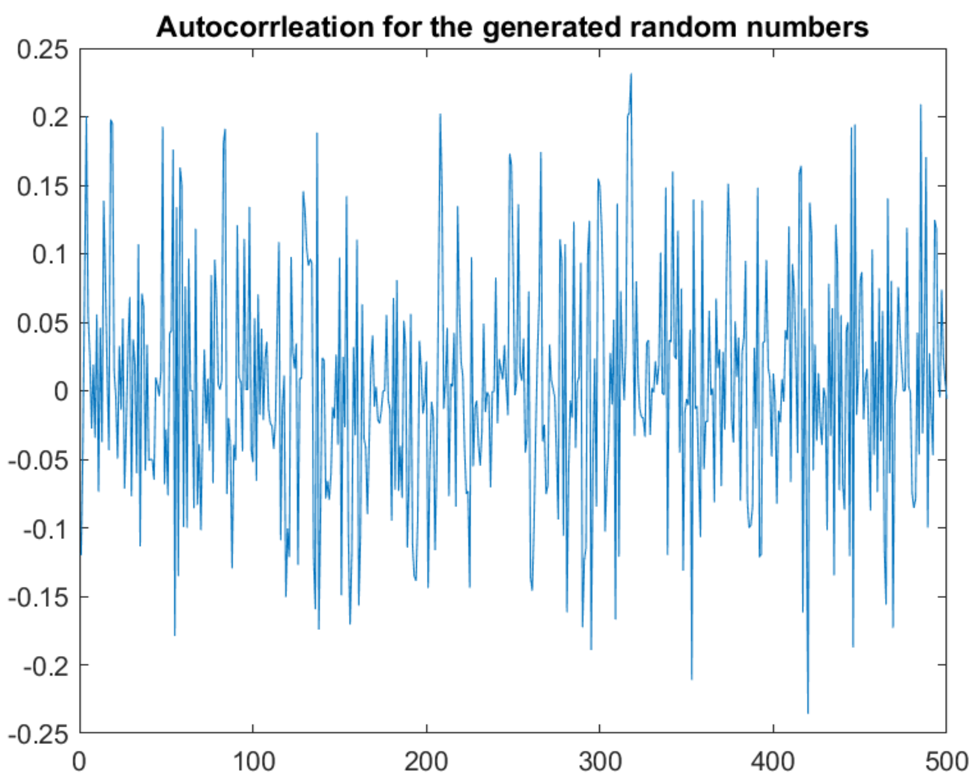
   The mean value of uniform distribution is: 0.506459637372816
   The mean value of the random number generator is: 0.503395717083760
   Which both are very similar.

2) *Compute the standard deviation of the random numbers that you generated, and compare with the standard deviation of a uniform distribution between 0 and 1.*

   The standard deviation of uniform distribution is: 0.289018075911333
   The standard deviation of the random number generator is: 0.290308764995758
   Which both are very similar.

3) *Compute the first 500 points of the autocorrelation function, and plot the results. Do the numbers appear to be uncorrelated?*

   The numbers are totally uncorrelated, due to the sudden fluctuations in the graph.

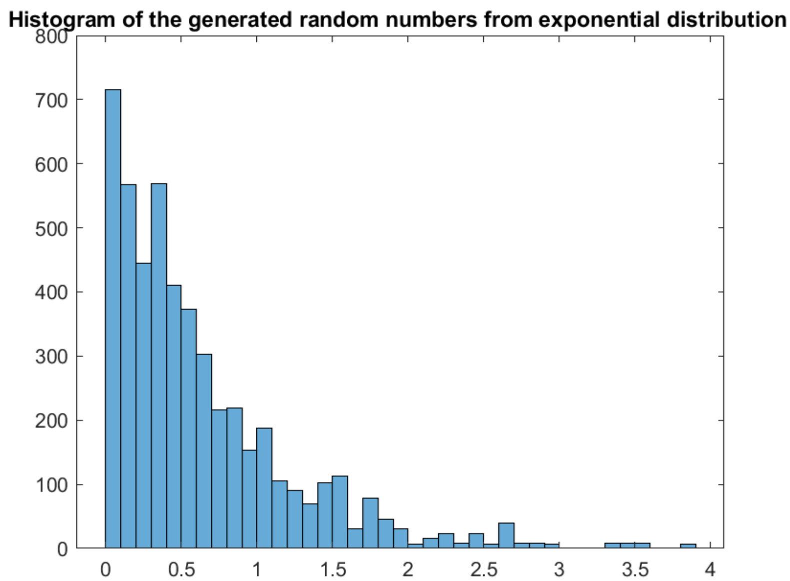


4) *Using the rejection sampling method seen in class, and the above random number generator defined above, generate 5,000 samples following an exponential distribution $\lambda e^{-\lambda x}$ with $\lambda = 1.5$. Plot the histogram of the samples.*

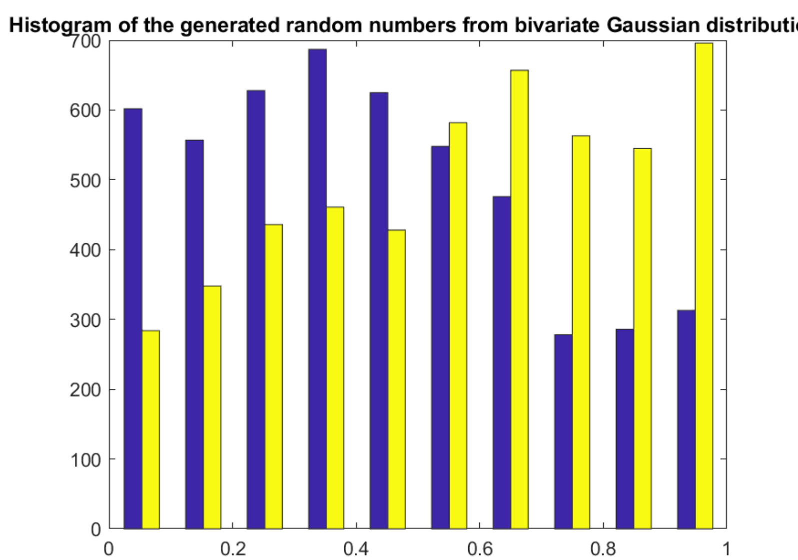
Rejection sampling in a nutshell,

- Define the maximum range (max of p.d.f) around your distribution
- Define the domain
- Choose a point on this domain
- If the point is less the the p.d.f value of this point, accept it
  In the case of exponential distribution, we need to find the maximum value, which is an optimization problem

$$f(x) = \lambda e^{-\lambda x}, which\ the\ maximu\ value\ is\ \lambda$$



5) *Same question for generating a bivariate Gaussian distribution, defined as in: http://www.statisticshowto.com/bivariate-normal-distribution/, with $\mu_1 = 0.5$, $\mu_2 = 1$, $\sigma_1 = 0.75$, $\sigma_2 = 1$, and $\rho = 0.7$*



<u>II-Uncertainty propagation:</u> We consider the classical Godunov scheme of HW 3, with a trapezoidal diagram (with parameters identical to those of HW 3). Assume the parameters to be v=30 m/s, w=5m/s and k_{max}=0.2/m. 10 cells of 100m each, and a time step of 2 seconds. We consider an initial condition in which the initial density k is discontinuous: k(0,x)=a if 0<x<500 and k(0,x)=b for x>500, but now a and b are random variables, with probability density function (the variables a and b are independent)

$P(a,b) = \frac{1}{\Delta a \Delta b}$ *if* $a_0 - \frac{\Delta a}{2} \leq a \leq a_0 + \Delta a/2$ *and* $b_0 - \frac{\Delta b}{2} \leq b \leq b_0 + \Delta b/2$

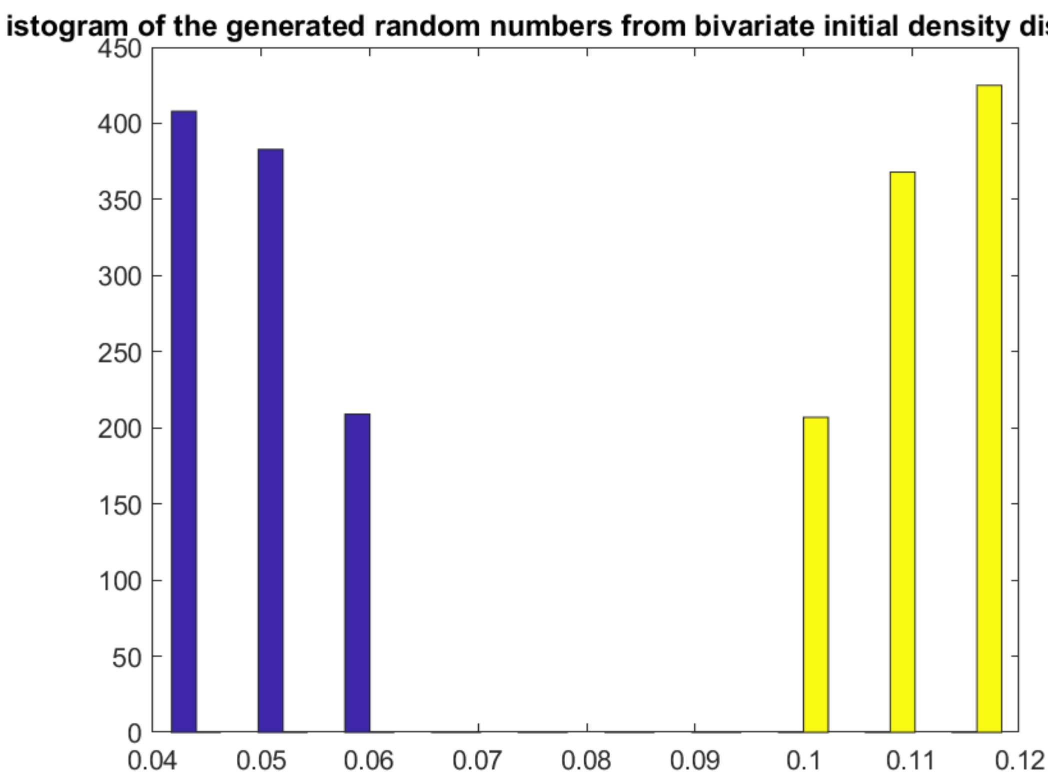
$P(a,b) = 0$ *Otherwise*

Use $a_0 = 0.03/m$ *and* $b_0 = 0.09/m$, *with* $\Delta a = \Delta b = 0.02$

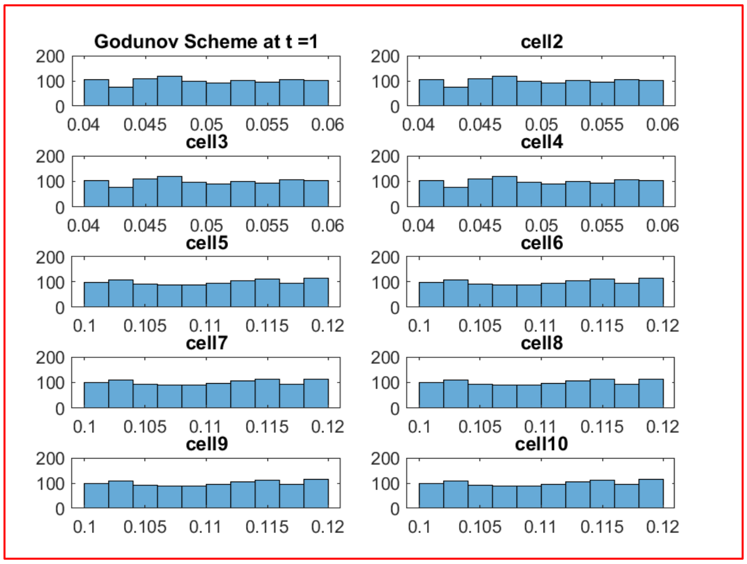
The upstream boundary demand is d(t)=0.1/s if 0<t<10s, and the downstream boundary supply is s(t)=0.2/s if 0<t<10s.

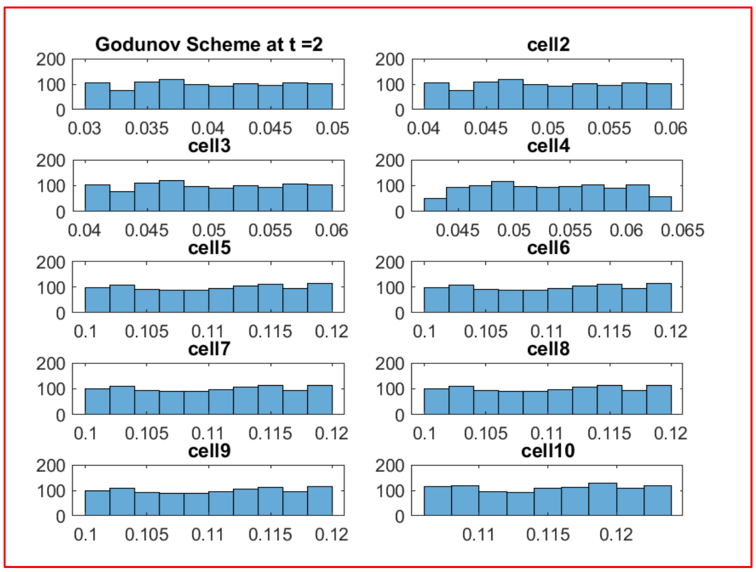
1) *Compute 1,000 samples of a and b according to the above distribution*

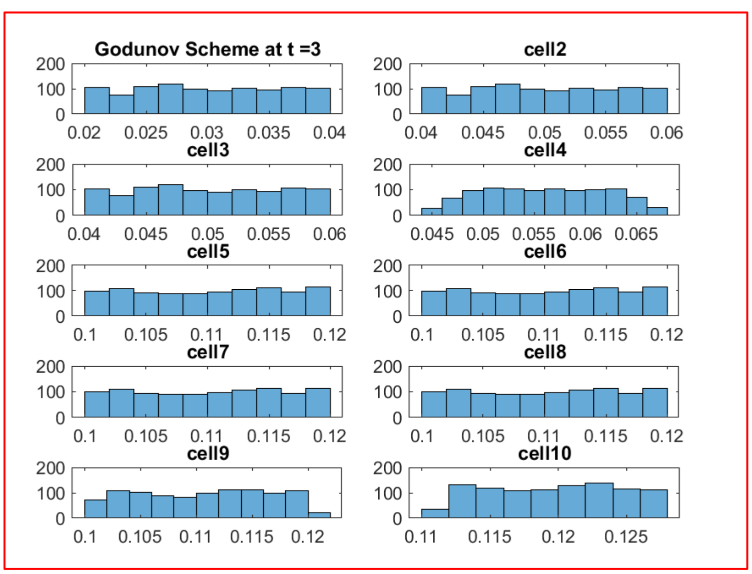
   I defined the domain boundaries within the domain of the p.d.f where it's value not zero.

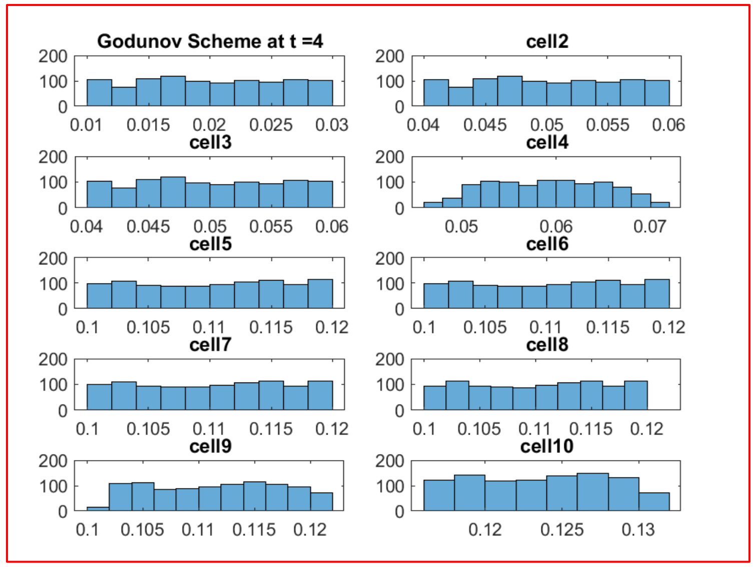


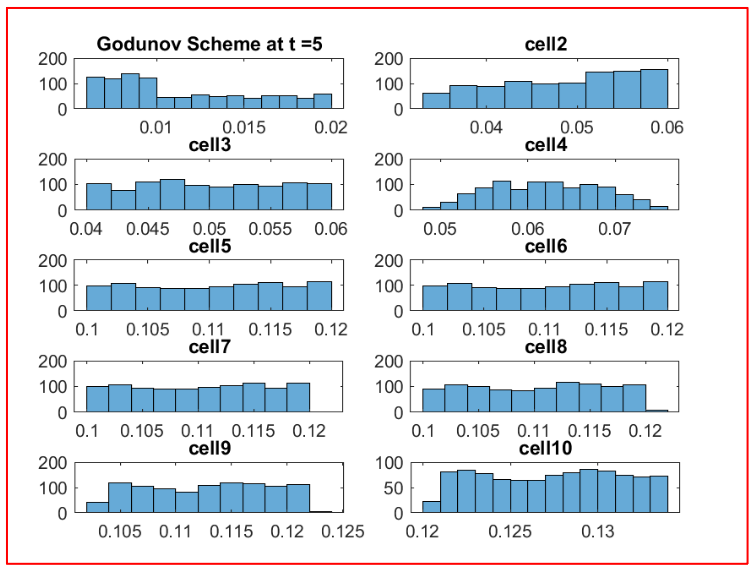
2) *Using these initial conditions, compute the state of the traffic at time t=10s, using the Godunov scheme, for each sample. Plot the distribution (histogram) of the densities in each cell.*











References:

[1] Exponential distribution, https://en.wikipedia.org/wiki/Exponential_distribution

[2] Bivariate Normal distribution, www.statisticshowto.com/bivariate-normal-distribution/

[3] LCR examples using Matlab,http://www.eeng.dcu.ie/~ee317/Matlab_Examples/random/tutinfo[1].htm

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[6] Solution of Godunov-scheme-For-a-Trapezoidal-fundamental-diagram, Abduallah Mohamed**,** https://goo.gl/iss8go