

# Chapter 8: Selection bias

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## Important disclaimer

- The term “Selection bias” can refer to different things depending on the discipline
  - Economists’ selection bias (selection on observables) is actually epidemiologists’ confounding bias.
  - Survey statisticians use selection bias term to sample selection from population, which can lead to biased conclusions in descriptive research.
- Epidemiologists excelled in conceptualizing and making the distinction between confounding and selection.
- It’s a good habit to make sure that your collaborators are on the same page regarding terminology to avoid confusions.

## General guidelines

### General guidelines

- Causal effects are linked to specific **populations**
- In many epidemiologic studies, you end up analyzing a cohort that’s different from your original cohort
  - Survival analysis: we analyze uncensored individuals (we don’t see the outcome in censored individuals).
  - Case-control studies: we analyze individuals who got the outcome and a sample of patients who did not get the outcome.

## General guidelines

- The hope is that the estimate we get in this subset is the same as the one we would've had if we did the estimation on the original cohort.
- If this is not the case Selection bias
- In all the coming DAGs, selection is represented as a variable with a square around it.
- The square here does not mean statistical adjustment. It means that the analysis is done on one stratum of the selection variable.
  - DAGs are not just useful in helping identifying adjustment covariates. They can help in the design as well.

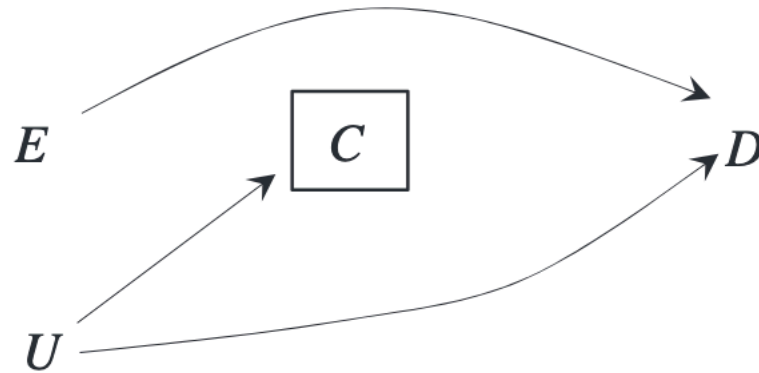
## Types of selection bias

### Selection bias under the null

- This type of selection bias can arise regardless of the true treatment effect being null or not.
- Conditioning on a collider (or a descendant of a collider) is necessary for this bias to happen.
- This is the topic of this book and Hernan's famous paper in 2004 (Hernán, Hernández-Díaz, and Robins 2004)

### Selection bias off the null

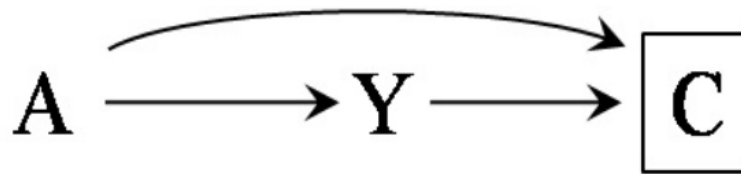
- This type of selection bias **cannot** happen under the null.
- This type of selection bias **can** happen even without conditioning on colliders.



- This bias is not covered in the chapter.

## Selection bias in cross-sectional studies

### Example 1



- $A$  : Treatment
- $Y$  : Fetal malformation
- $C$  : Live birth

We don't have  $Y$  for dead fetuses, so we essentially **restricting our analysis to** living fetuses.

### Example 1

- Our regression will give us this quantity

$$\frac{Pr[Y = 1|A = 1, C = 0]}{Pr[Y = 1|A = 0, C = 0]}$$

- Is this a valid estimate for our estimand?

$$\frac{Pr[Y^{a=1} = 1]}{Pr[Y^{a=0} = 1]}$$

The answer is no, because we have association transmitted through the path  $A \rightarrow C \leftarrow Y$

### Example 2



Figure 8.2

- $A$  : Treatment
- $Y$  : Fetal malformation
- $C$  : Live birth
- $S$ : Parental grief

A descendant of a collider is as dangerous as the collider itself.

### Selection bias in cohort studies

Including randomized trials

### Example

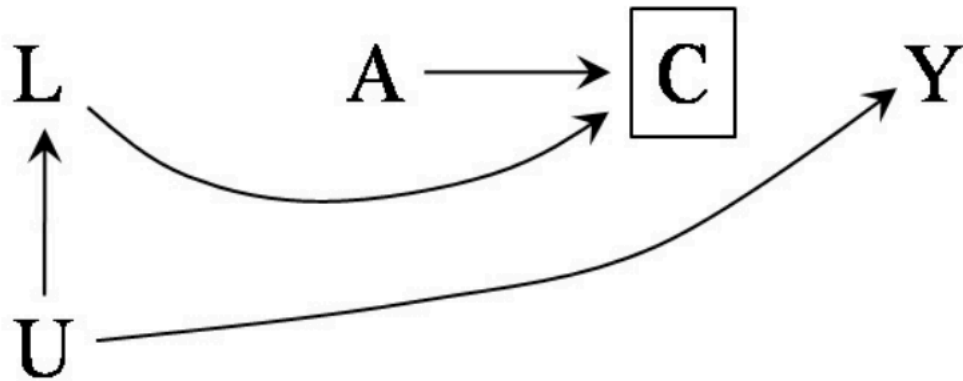


Figure 8.3

- $A$  : Antiretroviral treatment
- $Y$ : Death
- $L$ : Disease severity
- $U$ : High level of immunosuppression
- $C$ : Loss to follow-up

### Example

- Remember,  $C$  is not a variable we put in a regression model. It's a part of how your **analyzed data was formed**.
- In this example,  $A$  can show favorable result not because it's actually effective in reducing mortality, but because it caused sick people to leave the study. Although in reality,  $A$  and  $Y$  are not associated.
- The previous DAG is an example of selection bias due to **differential loss-to-follow-up** or **informative censoring**.

Similar DAGs for differential loss-to-follow-up

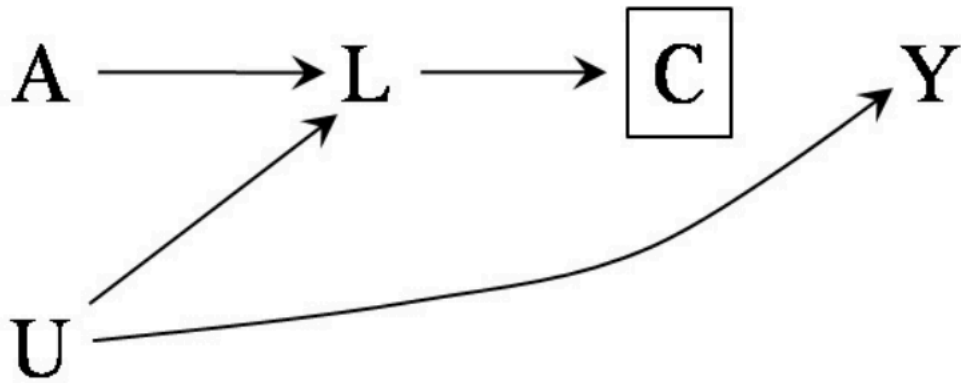


Figure 8.4

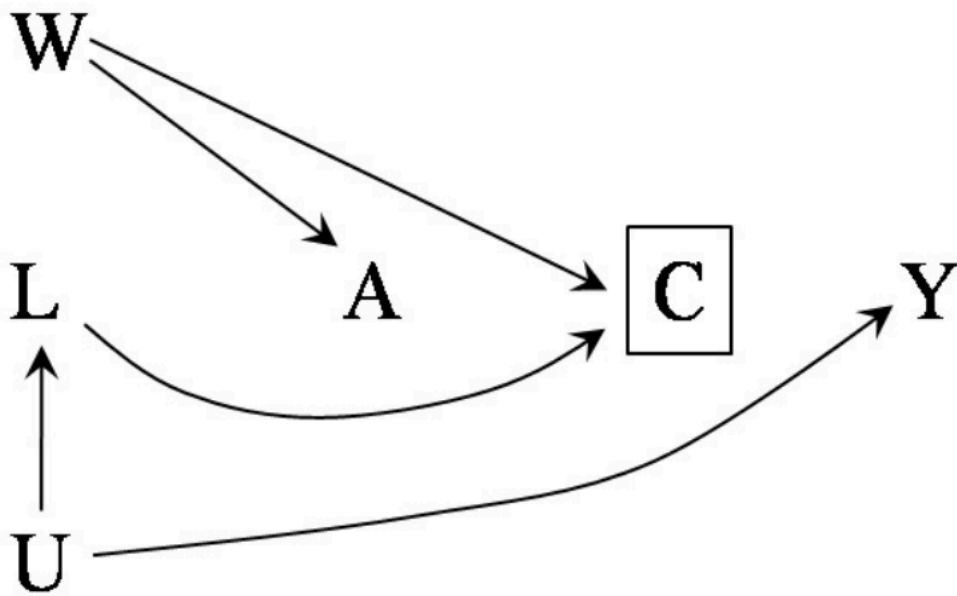


Figure 8.5

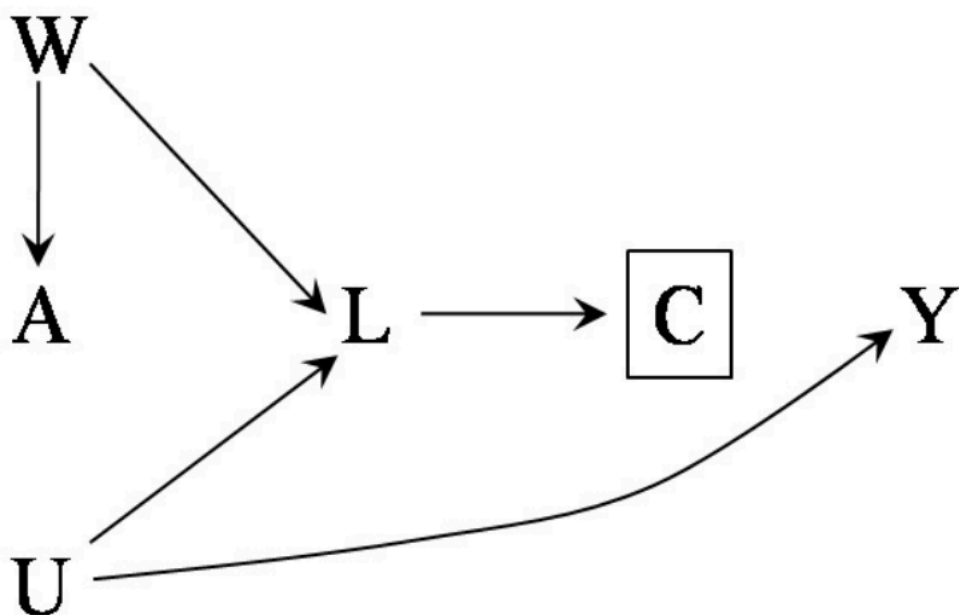
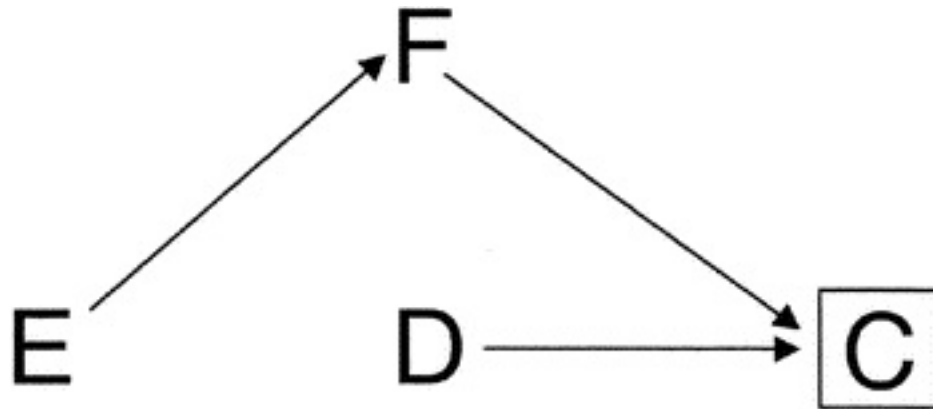


Figure 8.6

- These DAGs are modified versions of Figure 8.3
  - For instance, in figure 8.4, the association between  $A$  and  $L$  is represented by mediation while in figure 8.5 presented by a backdoor path  $A \leftarrow W \rightarrow C$  and presented by both in figure 8.6

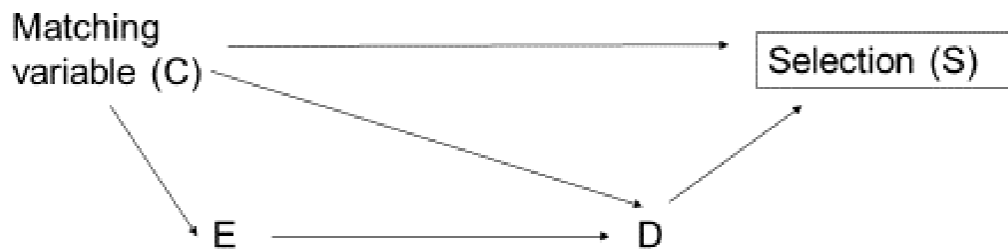
## Selection bias in case-control studies

DAG



- *E*: Estrogen use
- *D*: CHD
- *F*: Hip fracture
- *C*: Selection into the study

## Matched case-control designs are inherently biased





## The structural definition of selection bias

**Selection bias** to refer to all biases that arise from conditioning on a common effect of two variables, one of which is either the treatment or a cause of treatment, and the other is either the outcome or a cause of the outcome.

**Selection bias**, similar to confounding bias, is a violation of the exchangeability assumption.

## Common examples of selection bias

- Differential loss to follow-up or informative censoring.
- Missing data bias, or non-response bias.
- Healthy worker bias.
- Self-selection bias or volunteer bias.
- Selection affected by treatment received before study entry aka **prevalent-user bias**
- Immortal-time bias is a mix of selection and misclassification bias.

## Which designs are prone to selection bias?

- All of them, even randomized experiments.
- Randomization fixes confounding but not selection.
- Selection bias is more likely to occur with designs that are built on selection by default i.e. case-control design
  - Friendly advice, whenever you have the full cohort, please don't conduct a case-control study.

## Which analyses are prone to selection bias?

- Conventional covariate adjustment in treatment-confounder feedback setting.
- Cox regression.

## Selection without bias

- RCTs are conducted among volunteers willing to enter the experiment. So those volunteers select into the trial.
- However, this is not what we mean here by selection bias.
- Based on our definition, the selection variable should be a **common effect** of the treatment or a cause of the treatment and the outcome or cause of the outcome.
- Since volunteering participation happened **before** treatment assignment, there is no bias.
- The self-selection bias we mentioned earlier is about agreeing to continue in the trial after being treated.

## The distinction between confounding and selection bias

### Example

- $A$ : Physical activity.
- $Y$ : Heart disease
- $C$  : Being a firefighter
- $L$ : Parental socioeconomic status
- $U$ : Attraction towards physical activity

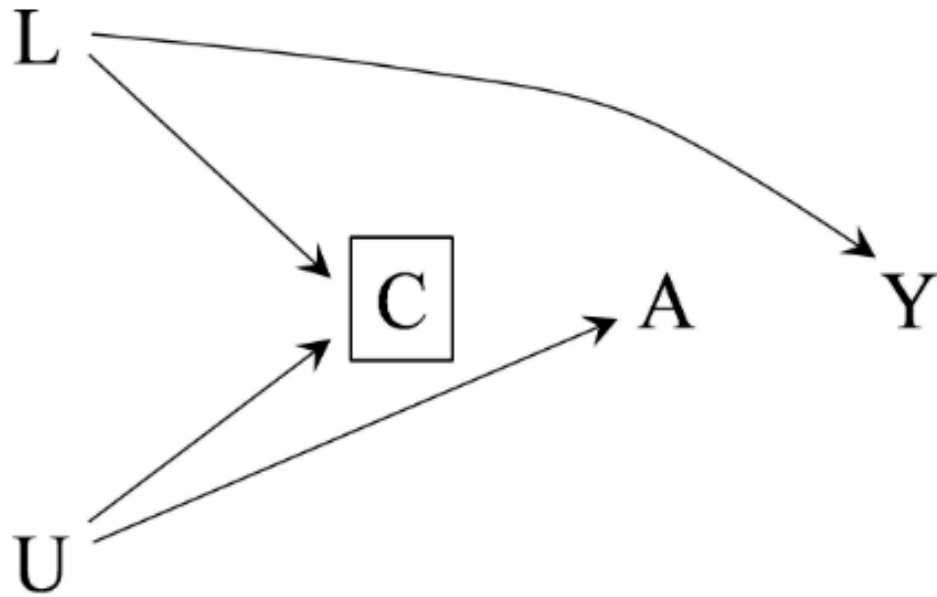
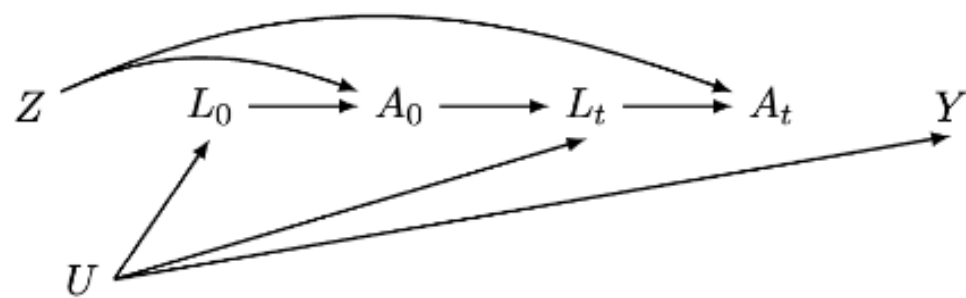


Figure 8.7

**Advantages of using the structural approach**

**1**

It can guide the choice of the analytic method



2

It can help is study design and data collection.

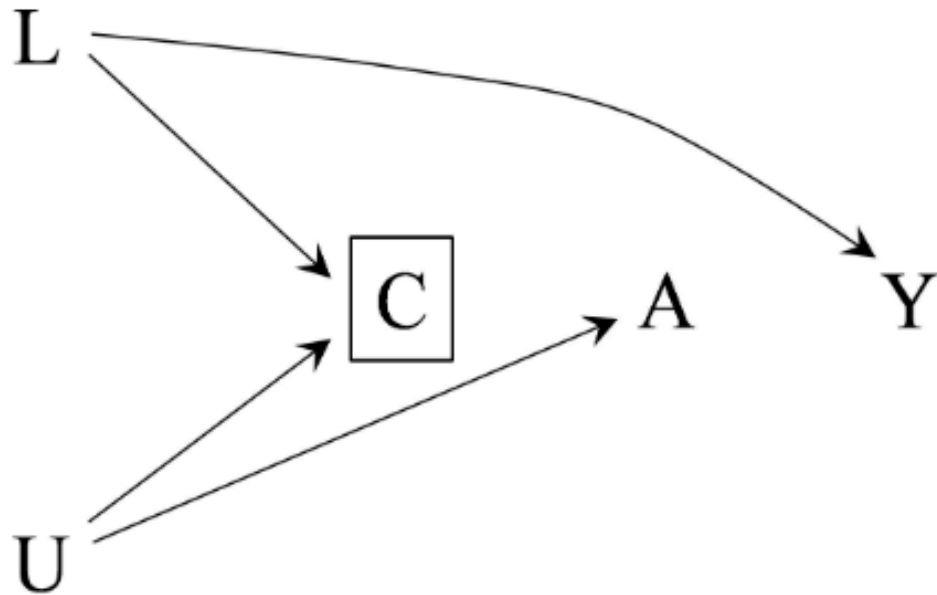


Figure 8.7

3

Selection bias resulting from conditioning on pre-treatment variables (e.g., being a firefighter) could explain why certain variables behave as “confounders” in some studies but not others.

4

Causal diagrams enhance communication among investigators and may decrease the occurrence of misunderstandings.

**Important reminder**

- DAGs ignore the magnitude or direction of selection bias and confounding.

- It is possible that some noncausal paths opened by conditioning on a collider are weak and thus induce little bias.
- It is not an “all or nothing” issue, in practice, it is important to consider the expected direction and magnitude of the bias

### Selection bias in hazard ratios

- $A$ : Treatment (protective)
- $Y_1$  and  $Y_2$ : Death at time 1 and time 2.
- $U$ : Protective Haplotype

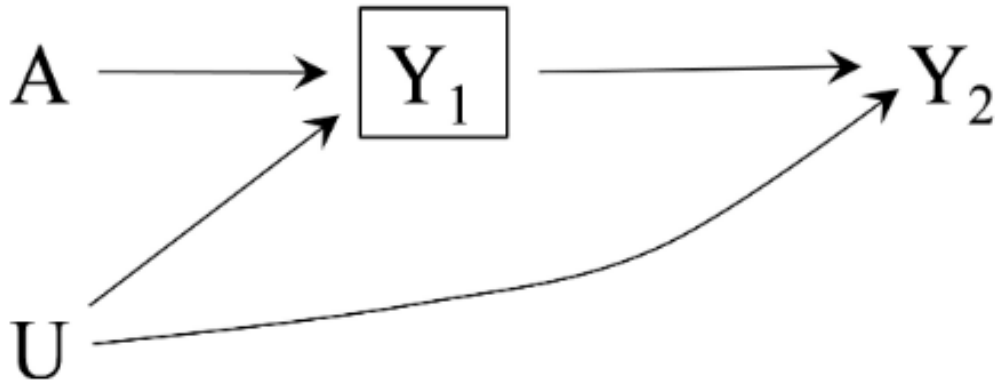


Figure 8.8

### Measures of effects

#### Risk ratio

$$aRR_{AY_1} = \frac{Pr[Y_1 = 1|A = 1]}{Pr[Y_1 = 1|A = 0]}$$

$$aRR_{AY_2} = \frac{Pr[Y_2 = 1|A = 1]}{Pr[Y_2 = 1|A = 0]}$$

## Hazard ratio

$$HR_{AY_1} = aRR_{AY_1} = \frac{Pr[Y_1 = 1|A = 1]}{Pr[Y_1 = 1|A = 0]}$$

$$HR_{AY_2} = aRR_{AY_2|Y_1=0} = \frac{Pr[Y_2 = 1|A = 1, Y_1 = 0]}{Pr[Y_2 = 1|A = 0, Y_1 = 0]}$$

In conclusion, we have two issues:

- The estimand changed.
- Selection bias

## Avoiding selection bias

### New estimand

- Similar to the interaction chapter, we will view selection or censoring as an intervention. If we are able to satisfy the causal identification assumption with  $c$ , then this estimand can be estimated using observed data

$$\frac{Pr[Y^{a=1,c=0} = 1]}{Pr[Y^{a=0,c=0} = 1]}$$

- This reads as the effect of  $A$  on  $Y$  had everyone got  $A$  and remained uncensored vs everyone not getting  $A$  and remained uncensored.
- Weighting can be a good approach to achieve this (See example).

## References

Hernán, Miguel A., Sonia Hernández-Díaz, and James M. Robins. 2004. “A Structural Approach to Selection Bias.” *Epidemiology* 15 (5): 615–25. <http://www.jstor.org/stable/20485961>.