Westminster International University in Tashkent

Week 2: Probability Topics

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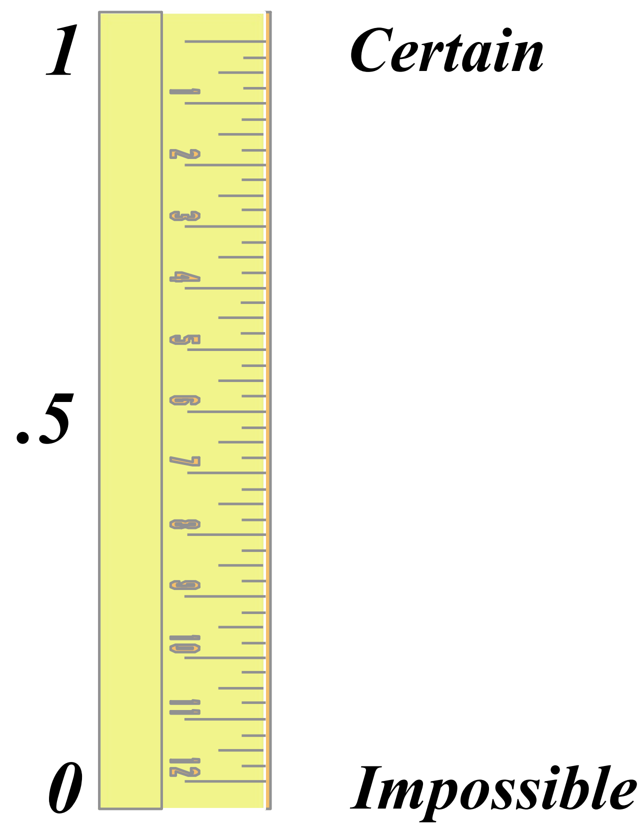
Agenda

- Events, Sample Spaces, Probability
- Unions and Intersections
- Complementary Events
- Additive Rule, Mutually Exclusivity
- Multiplicative Rule
- Combination, Permutation

What is Probability?

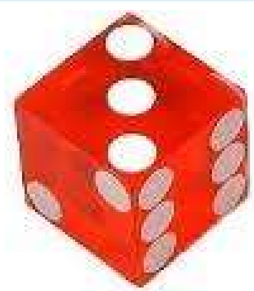
1. Numerical measure of the likelihood that event will occur
   - *P*(Event)
   - *P*(A)
   - *Prob*(A)
2. Lies between 0 and 1
3. Sum is 1



Concept of probability

An **experiment** is an act or process of observation that leads to a single outcome that cannot be predicted with certainty.

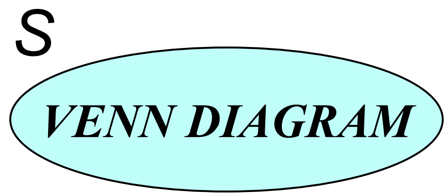
A **sample point** is the most basic outcome of an experiment.



Sample space and events

Consider an experiment

Sample space S:

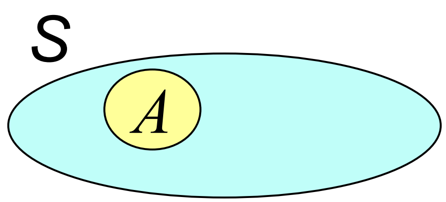


Example:

S={1,2,…,6} rolling a dice

S={head, tail} flipping a coin

Event A:

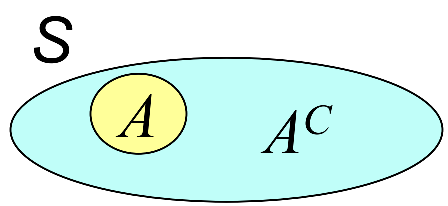


Example:

A={1,6} when rolling a dice

Complementary event A^C (or A'):

$P(A) + P(A^C) = 1$



Example:

A´={2,3,4,5} rolling a dice

Unions and intersections

The **union** of two events A and B is the event that occurs if either A or B (or both) occurs on a single performance of the experiment. We denote the union of events A and B by the symbol $A \cup B$. $A \cup B$ consists of all the sample points that belong to *A or B or both*. (See Figure 3.7a.)

The **intersection** of two events A and B is the event that occurs if both A and B occur on a single performance of the experiment. We write $A \cap B$ for the intersection of A and B. $A \cap B$ consists of all the sample points belonging to *both A and B*. (See Figure 3.7b.)

Example

Suppose my air conditioner broke and heat in my apartment raised high. I went to a "person" to ask him to take a look at it, he came to my apartment, used a bunch of spare parts and then fixed it. I paid him for the repairs.

Now what is more likely,

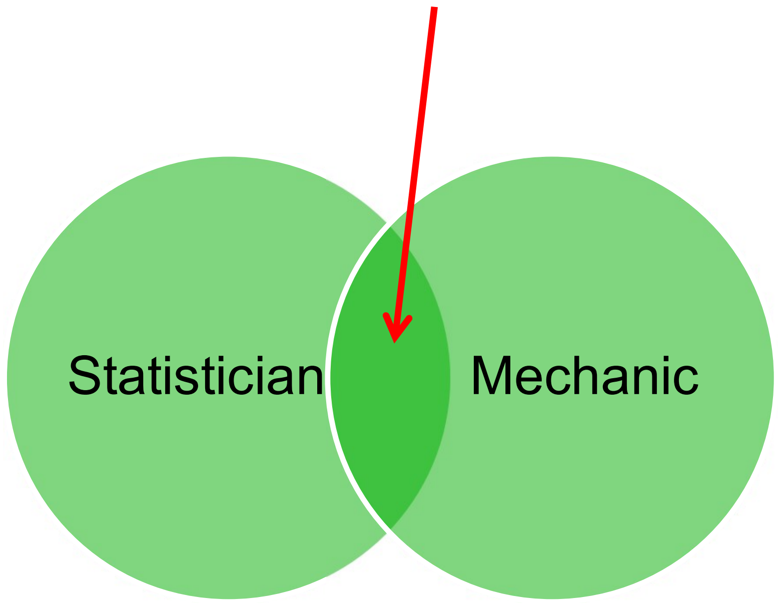
1) He is a statistician,

2) He is a statistician and AC mechanic.

Statistician and Mechanic

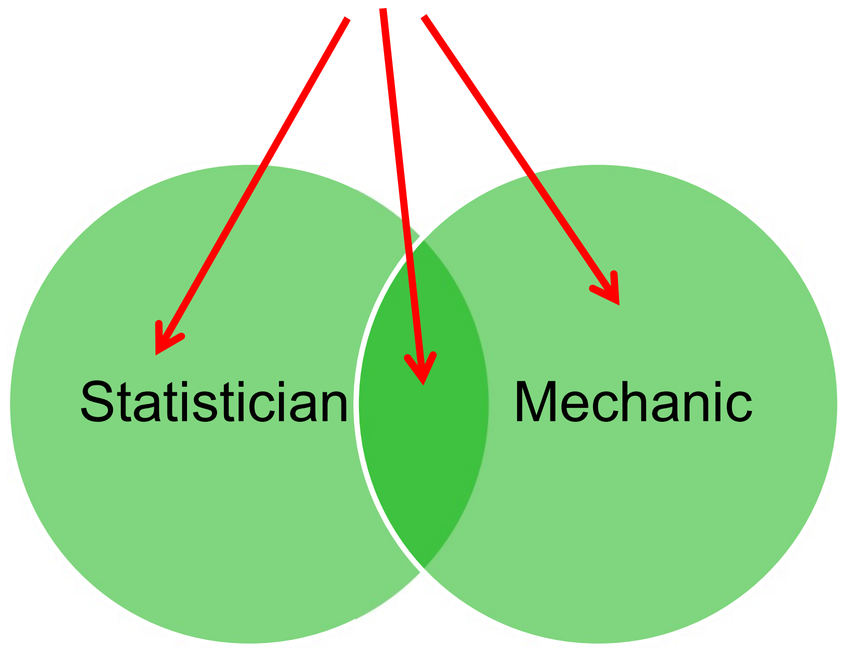
$A \cap B$

Statistician **AND** *Mechanic*



$A \cup B$

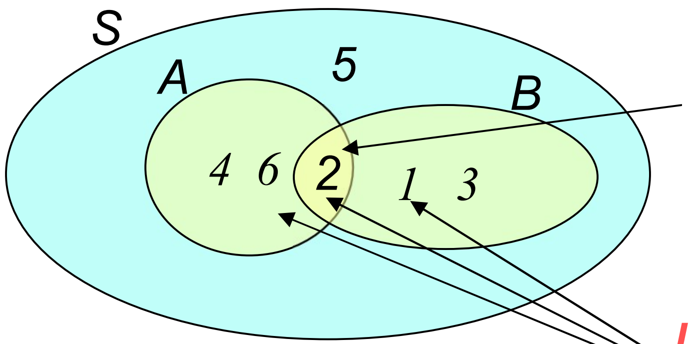
Statistician ***OR*** *Mechanic*



Probability theory: Events

Example:
Rolling a dice

$S=\{1,2,3,4,5,6\}$
$A=\{2,4,6\}$
$B=\{1,2,3\}$

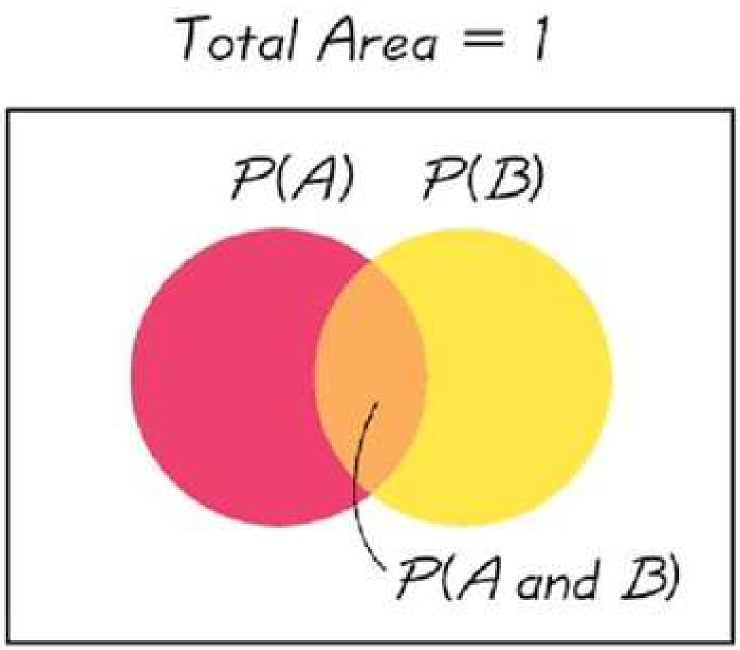


Intersection:
$A \cap B=\{2\}$

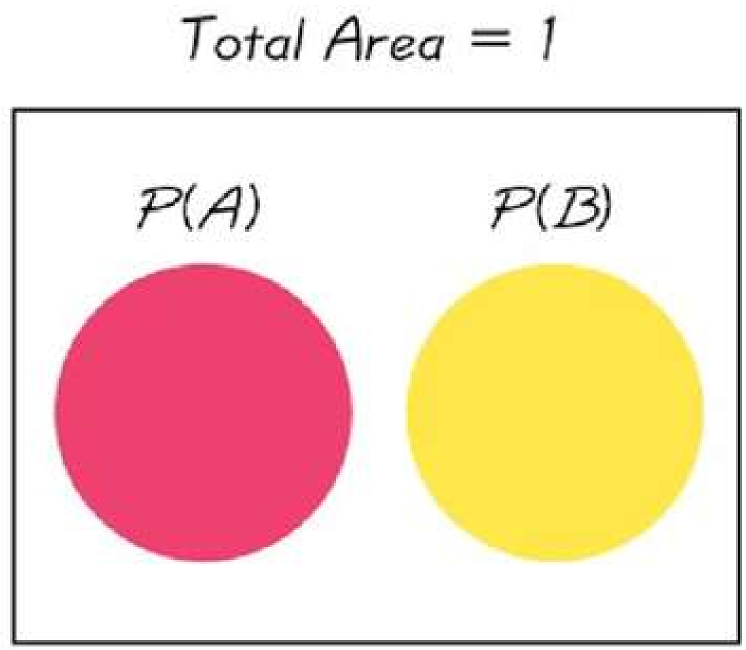
Union:
$A \cup B=\{1,2,3,4,6\}$

Disjoint or Mutually Exclusive

Events *A* and *B* are disjoint (or mutually exclusive) if they cannot occur at the same time. (That is, disjoint events do not overlap.)



Venn Diagram for Events That Are Not Disjoint



Venn Diagram for Disjoint Events

Union

Additive Rule of Probability

The probability of the union of events A and B is the sum of the probability of event A and the probability of event B, minus the probability of the intersection of events A and B; that is

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

For disjoint events: $P(A \cup B) = P(A) + P(B)$

Example: A: driving to the university

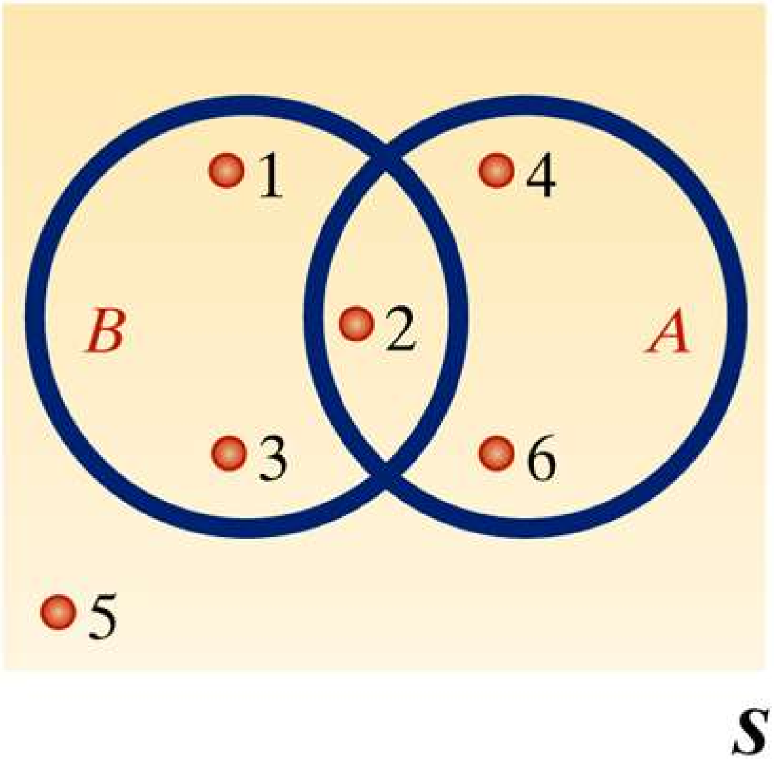
B: taking public transit to the university

Venn diagrams for die toss

B – observe a score no higher than 3

$P(B)=3/6$

$P(A \cup B)=5/6$



A – observe an even score

$P(A)=3/6$

$P(A \cap B)=1/6$

$$P(A \cup B)=P(A)+P(B)-P(A \cap B)=3/6+3/6-1/6=5/6$$

Conditional Probability

Conditional Probability Formula

To find the *conditional probability that event A occurs given that event B occurs*, divide the probability that *both* A and B occur by the probability that B occurs; that is,

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} \text{ [We assume that } P(B) \neq 0.\text{]}$$

Example. 70% of your friends like Chocolate, and 35% like Chocolate AND like Strawberry. What percent of those who like Chocolate also like Strawberry?

Solution.

P(Strawberry | Chocolate) = $\frac{0.35}{0.70} = 0.5 \; or \; 50\%$

Independent events

Multiplicative Rule of Probability

$P(A \cap B) = P(A)P(B|A)$ or, equivalently, $P(A \cap B) = P(B)P(A|B)$

Events A and B are **independent events** if the occurrence of B does not alter the probability that A has occurred; that is, events A and B are independent if

$$P(A|B) = P(A)$$

When events A and B are independent, it is also true that

$$P(B|A) = P(B)$$

Events that are not independent are said to be **dependent.**

Examples for independent events

Independent events:

Event A: It rains in Tashkent;

Event B: Donald Trump plays golf in Mar-a-Lago.

Dependent events:

Event A: It rains in Tashkent;

Event B: Football game in Tashkent is cancelled.

Table 3.6 Distribution of Product Complaints

	Reason for Complaint			
	Electrical	Mechanical	Appearance	Totals
During Guarantee Period	18%	13%	32%	63%
After Guarantee Period	12%	22%	3%	37%
Totals	30%	35%	35%	100%

Event A: {Cause of the complaint is electrical problem.}
Event B: {Complaint happened during guarantee period.}

$P(B) = 0.63 \quad P(A) = 0.30 \quad P(A \cap B) = 0.18$

If A and B are independent events, then $P(A \cap B) = P(A)*P(B)$

But,

$$P(A \cap B) = 0.18 \quad \neq \quad P(A)*P(B) = 0.30*0.63 = 0.189$$

A and B events are not independent.

Combination vs Permutation

A combination is an arrangement of all or part of a set of objects, *without* regard to the order of the arrangement.

A permutation is an arrangement of all or part of a set of objects, *with* regard to the order of the arrangement.

Number of permutations of n things taken r at a time: $${}_nP_r = \frac{n!}{(n-r)!}$$

Number of combinations of n things taken r at a time: $${}_nC_r = \frac{n!}{(n-r)!r!}$$

Examples

1. An English test contains five different essay questions labeled A, B, C, D, and E. You are supposed to choose 2 essays to answer. How many different ways are there to do this?

$$_n\mathbf{C}_r = \binom{5}{2} = \frac{5!}{2!(5-2)!} = 10$$

2. A family of 3 plans to sit in the same row at a movie theater. How many ways can the family be seated in 3 seats?

$$_n\mathbf{P}_r = {}_3P_3 = \frac{3!}{(3-3)!} = 3 * 2 * 1 = 6$$

3. A group of 8 swimmers are swimming in a race. Prizes are given for first, second, and third place. How many different outcomes can there be?

$$_n\mathbf{P}_r = {}_8P_3 = \frac{8!}{(8-3)!} = 6 * 7 * 8 = 336$$

THANK YOU!

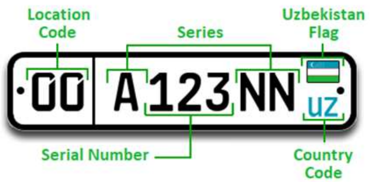
QUESTIONS?

Brain Teaser Problems

- If there are 20 students in your classroom and everyone greets each other with handshakes, then how many handshakes take place in you class? (190)
- If there are 50 students in the room, what is the probability that at least two students share the same birthday? (0.97)
- Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, chickens. You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a chicken. He then says to you, "Do you want to pick door No. 2?" Is it to your advantage to switch your choice? (Yes)
- Ahmed is in a dark room selecting socks from his drawer. He has only six socks in his drawer, a mixture of black and white. If he chooses two socks, the chances that he picks a white pair is 2/3. Then what are the chances that he selects a black pair? (0)

Brain Teaser Problems

- Hasan and Zukhra are arguing over who gets the last cookie in the jar, so their mother decides to create a game to settle. First, Hasan flips a coin twice, and each time Zukhra calls heads or tails in the air. If Zukhra gets both calls right, she gets the last cookie. If not, Hasan picks a number between one and six and then rolls a die. If he gets the number right, he gets the last cookie. If not, Zukhra picks two numbers between one and five, then spins a spinner with numbers one through five on it. If the spinner lands on one of Zukhra's two numbers, she gets the last cookie. If not, Hasan does.
  Who is more likely to win the last cookie, Hasan or Zukhra? And what is the probability that person wins it? (Each has equal chance of winning, that is ½)
- Suppose a car plate number with "777" costs the buyer $10,000 on average. How much money in total can the agency generate by selling all these plate numbers in Uzbekistan? There are 14 regions in the country and the plate number looks like as the following:
  ($2,460, 640, 000)



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