PID

$$y(t) = e(t) \cdot K_p + K_i \cdot \int e(t) \cdot dt + K_d \cdot \frac{de(t)}{dt}$$

Using laplace to change to s-domain

$$y(s) = e(s)\left(K_p + \frac{K_i}{s} + K_d \cdot s\right)$$

Using tustin transformation to change it to discrete;

$$\boxed{s = \frac{2}{T} \cdot \frac{z-1}{z+1}}$$

$$P(z) = K_p \cdot e(z) \Rightarrow \boxed{P(n) = K_p \cdot e(n)}$$

$$i(z) = K_i \cdot \frac{T}{2} \cdot \frac{z+1}{z-1} \cdot e(z) = \frac{K_i \cdot T}{2} \cdot e(z) \cdot \frac{1+z^{-1}}{1-z^{-1}}$$

$$\Rightarrow i(z) \cdot (1-z^{-1}) = \frac{K_i \cdot T}{2} \cdot e(z) \cdot (1+z^{-1})$$

$$\Rightarrow i(z) - i(z) \cdot z^{-1} = \frac{K_i \cdot T}{2}\left(e(z) + e(z) \cdot z^{-1}\right)$$

$$\Rightarrow i(n) - i(n-1) = \frac{K_i \cdot T}{2}\left(e(n) + e(n-1)\right)$$

$$\Rightarrow \boxed{i(n) = i(n-1) + \frac{K_i \cdot T}{2}\left(e(n) + e(n-1)\right)}$$

$$D(z) = K_d \cdot \frac{2}{T} \cdot \frac{z-1}{z+1} \cdot e(z) = e(z) \cdot \frac{K_d \cdot 2}{T} \cdot \frac{1-z^{-1}}{1+z^{-1}}$$

$$\Rightarrow D(z) \cdot (1+z^{-1}) = e(z) \cdot \frac{K_d \cdot 2}{T} \cdot (1-z^{-1})$$

$$\Rightarrow D(z) + D(z) \cdot z^{-1} = \frac{K_d \cdot 2}{T} \left(e(z) - e(z) \cdot z^{-1} \right)$$

$$\Rightarrow D(n) + D(n-1) = \frac{K_d \cdot 2}{T} \left(e(n) - e(n-1) \right)$$

$$\Rightarrow \boxed{D(n) = \frac{K_d \cdot 2}{T} \left(e(n) - e(n-1) \right) - D(n-1)}$$

$$u(n) = P(n) + I(n) + D(n)$$