Part A:

Database: A systemetic collection of data that supports data storage, data manipulation and make data management easier is called Database.

*** CT-1 1(a)

DBMS: A collection of programs, that enables users to –

- Access database.
- Manipulate data.
- Reporting or representation of data.

It also helps to controll the access of database.

1(a)

***

Application of DBMS:

1. Social media : Facebook, Twiter, LinkedIn.
2. Search engine : Google, Bing.
3. Play store : Google play.
4. Video archieve : Youtube, Netflix.
5. Online shopping : Amazon, Alibaba, Daraz.
6. ERP/MIS : Univercities, Buying houses, corporate officies.
7. Scientific Research : Artificial intelligence, genetics.
8. Scheduling : Airplane, bus, train.

'Purposes of DBMS' or 'File system Vs DBMS':

1. Data redundency and inconsistency.
2. Difficulty in accessing data.
3. Data isolation.
4. Integrity problems.
5. Atomicity problems.
6. Concurrent access anomalies.
7. Security problems.

Q*1 Describe data abstraction. Berifly describe the different levels ofdata abstraction?

Solve:

Data abstraction:

To ease the user interaction with database, the developers hide internal irrelevant details from users, this process is called data abstraction.

Different level of Data abstraction:

- Physical level: The lowest level of abstraction, describes how the data are actually stored.
- Logical level: The next highest level of abstraction, describes what data are stored and what relationship exist among those data.
- View level: The highest level of abstraction, describes only the intire part of the database.

Instance: A collection of information stored in a database at a particular moment is called an instance of that database.

Database Model:

Data Model: The database model is a logical structure of database

- It includes relationships & constraints.
- It also determine how the data can be stored, organized or accessed

Different types of data model:

Record based mode:

- The database is structured in a fixed format record of several types.
- Each table contains records of particular type.
- Each record type has a defined fixed number of fields or attributes.
- The columns of the table corresponding attributes of record type.

Relational model: ER model: OOP   semi stru...

Database Languages: ***.

CT-1   3. DDL vs DML

DDL (Data Definition Language):

...efines different structures in a database.
...orks on database object, such as tables, idexes & users.
...mmon DDL statements: CREATE, ALTER, ~~DELETE~~ DROP.

...ML (Data Manipulation Language):

...ables users to access or manipulate data as organized by appropriate data model.

...ctions:

- Retrival of information stored in the database
- Insertion of " in to the database.
- Deletion of " from ".
- Modification " " stored in the "

- Procedural DML requires users to specify what data are needed & how to get those data.
- Declarative DML requires users to specify what data are needed without specifying how to get those data.

DCL: (Data control language):

DCL includes commands such as Grant or Revoke. It mainly deals with rights, permissions and other controll in the database system.

GRANT: This command gives users access priviliges to the database.

Revoke: This command withdraws the users access priviliges given by using GRANT command.

Query: A query is a statement that request retrival of information.

Query Language: The portion of DML that involves information retrieval is called query language.

Intigrity Constraints:

Intigrity constraints provides a way of ensuring that changes made to the database by authorized users do not results in a loss of data consistency.

- Domain constraints
- Reffenential integrity constraints
- Assertion
- Authorization.

*** Database design process; MID 3.(a)

step 1: Define the purpose of database.
(requirement analysis)

step 2: Gather and organize data or information.
(by interviewing users, designer's own analysis).

step 3: The designer chooses a data model.
(by applying the concepts of choosen data model, translate the requirments into conceptual schema of the database.)

step 4: Refine and normalize the design.

- Client machine on which remote database user works.
- Server machine on which, database system runs.

Database Architecture:

Two-Tier Architecture:

In a two-tier architecture,

- The application is resides at the client machine, (where it invokes database system functionality at serve machine through query language statements)
- The application program interface standards like ODBC & JDBC are used for interaction betn client & server.

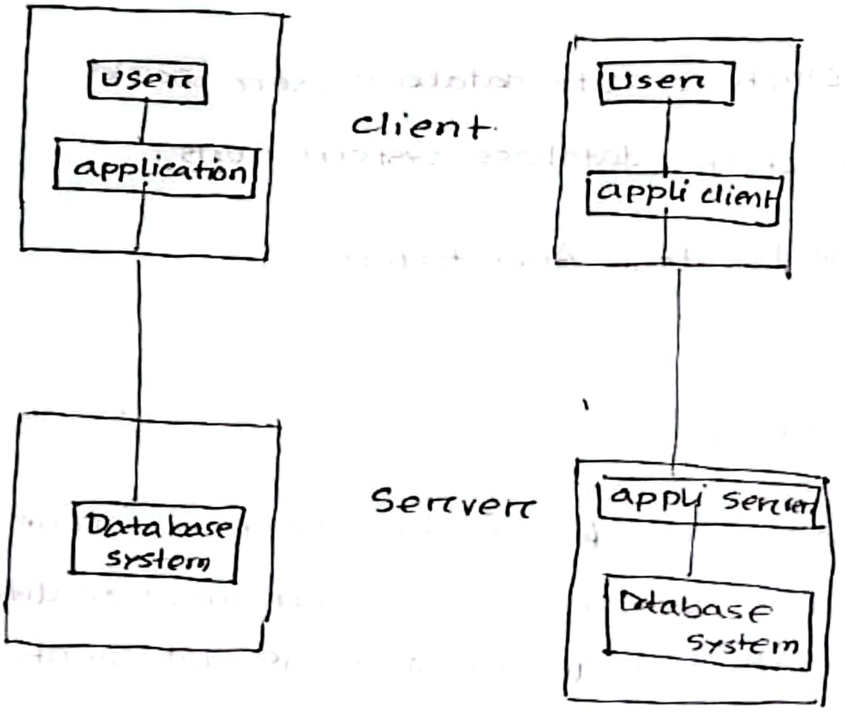
T-1 2.

Three-Tier Architecture:

In a three-tier architecture,

- The client machine acts as merely a front end. (doesn't accept any direct database call)
- Instead, the client ends communicates with an applicatio server. (usually throughs a forms interface)

- The application serverr in turn communicates with the database system.
  (to access data)
- The business logic of the application, which says what actions to carrry out underr what conditions.
  (is embedded that in the application serverr, insted of being distributed accross multiple alients).
- The three-tierr application is more apprcopriate forr large applications, and forr applications that run on the World wide web.



(a) Two-tierr arrchietecture (b) threee-tier architecturre.

Database Users:

Four types of database system users.

- Naive users.
- Application programmers.
- Sophisticated users.
- Specialized users.

Database Adminstrator (DBA) *** CT[1] (4)[no] + MID 1(D)

DBA: The main reasons of using DBMS is to have a central controll on both data and the programs that access those data.

A person who has such central controll over the system is called a Database Adminstrator (DBA).

Functions:

- Schema definition.
- Storage structure and access method definition.
- Schema and physical organization modification.
- Granting of authorization for data access.
- Routine management.

Structure of relational Database:

- For each attribute of a relation there is a set of permitted value called domain of that attribute.
- A domain is atomic if elements or column doesn't accept multiple value

Null: The null value is a special value that signifies the value is unknown or doesn't exist.

relation — Table.
record, tupple — Row
field, attribute — column

Database Schema: In generral a rrelation schema consist of set of attributes with their corresponding domain.

Example: department (dept_name Varchar, building Varchar, budget int)

Key:

Supper key: A supper key is a set of attributes within a table or relation whose value can uniquely identify a tupple.

Candidate key: A candidate key is a minimal set of attributes within a given table or relation whose value can uniquely identify a tupple.

Primary key:*** MTD 2.(b) Primary vs Foreign.

Primary key is being selected from the sets of candidate key by database designer.

A primary key is a special relational database table/relation, columns/attribute (combination of columns or attribute). is designated to uniquely identify all table/relation records.

The main features of primary key:

- It must have a unique value for each row of data.
- It can not contain any null values.
- One table contains only one primary key.

composite key: A composite key is a set of two or more attributes in one table/relation that is used to ~~identify~~ uniquely identify each row/tupple in a relation.

Foreign key

- A Foreign key is used to linked two table or relation together.
- A Foreign key is field/attributes (collection of field/attributes) used in one table/relation that can refers to the primary key of another table/relations.

Features of foreign key:

* linked betn two table.
* Refer another table primary key.
* One table contain more than one foreign key.
* Foreign key may not be unique.
* Every foreign key become composite key, if the refferred primary key is also a composite key.

ER model -- Database Modeling:

The ER data model was developed to facilitate database design by allowing specification of enterprise schema that represents the overall logical structure of a database.

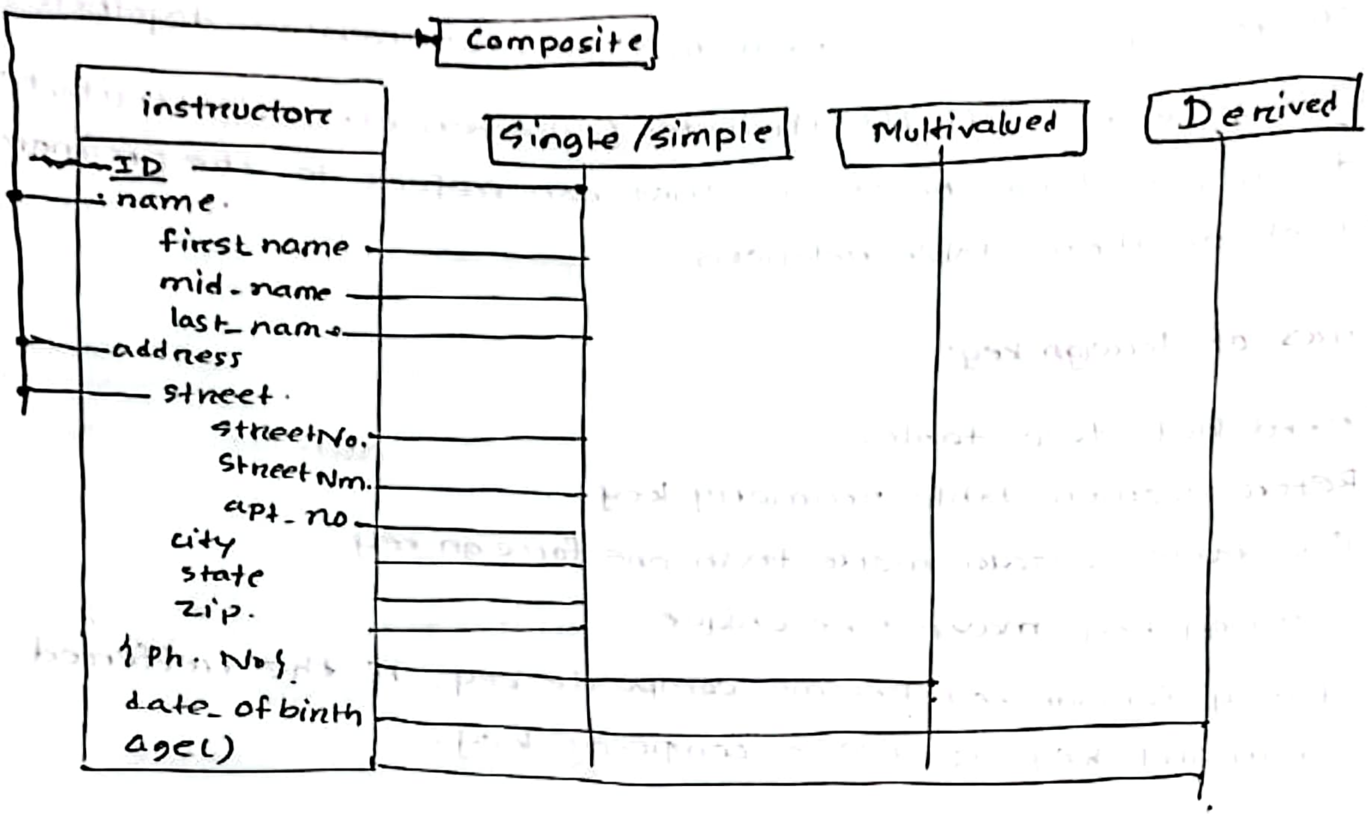
- Entity sets
- relationship sets
- attributes.

Complex Attributes

Attribute type:

- Simple & Composite attribute.
  - (attributes that divided into sub parts)
- Single valued and multivalued attribute.
  - Ex- Phone number.
- Derived attribute. can be computed from other attributes
  Ex- Age, date of birth.

Domain - the set of permitted values for each attribute



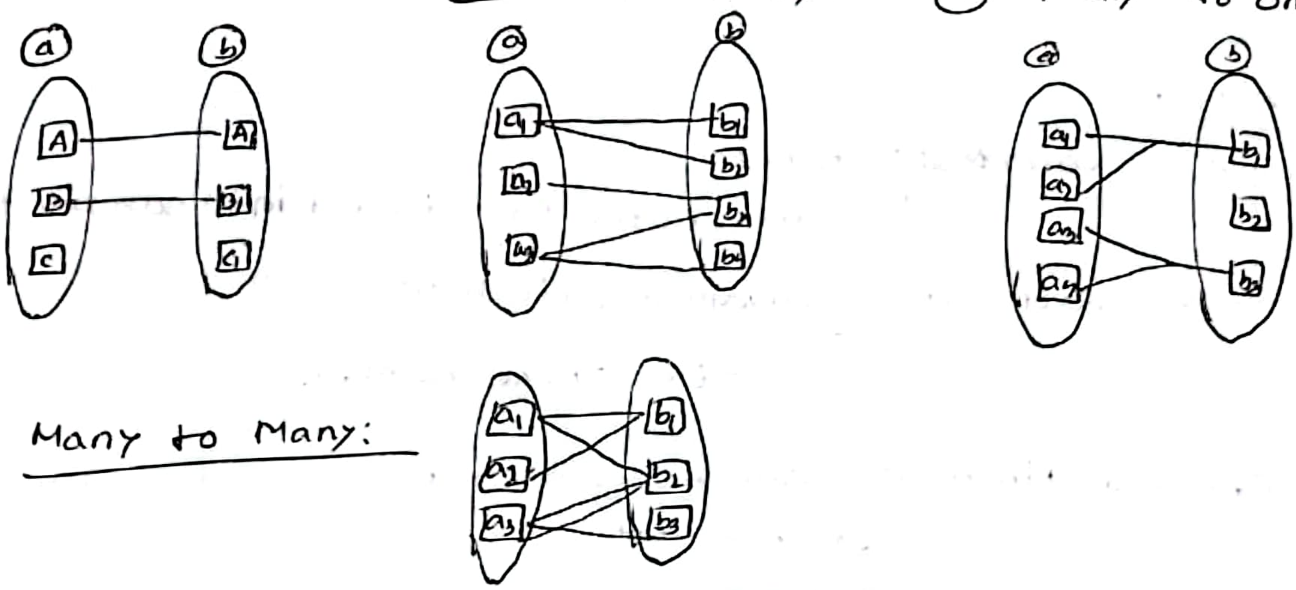
*** Mapping Cardinality constraints: MID 3 (B)

- Express the number of entities to which another entity can be associated via a relationship set.
- Most useful for describing binary relationship set.
- There are 4 types of mapping cardinality in a binary relationship set.

→ one
— Many.

[1.] one to one [2.] One to many (3) Many to One

(4) Many to Many:



Selection of primary key in Relationship set:

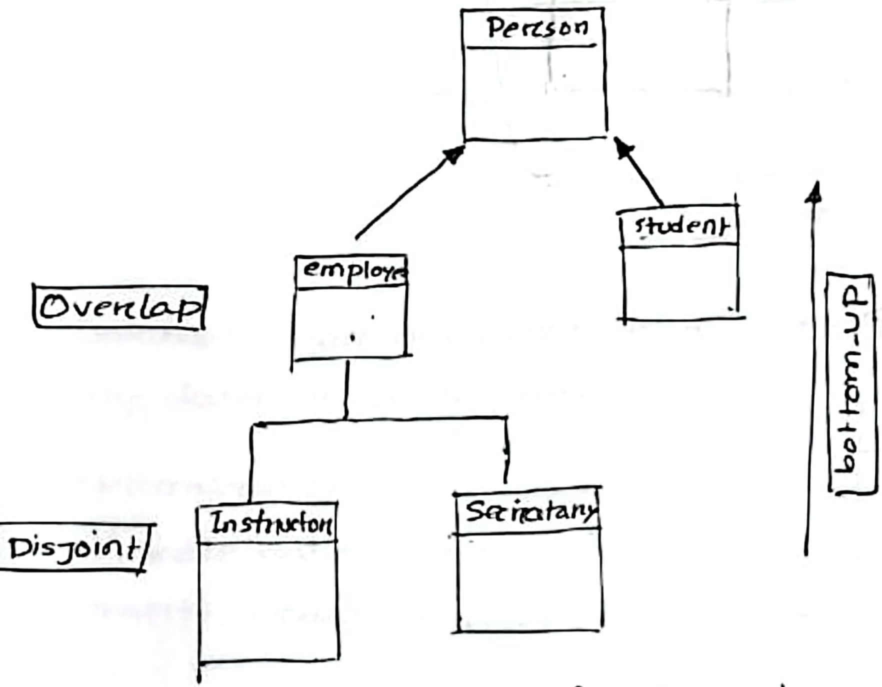
For: one to one / many to many:

advisor (stu-ID or Teacher-ID)
যে কোনো একটা কে Primary ধরলেই হবে।

For other maping cardinalities: সবসময় many side এর টাই primary key হবে।

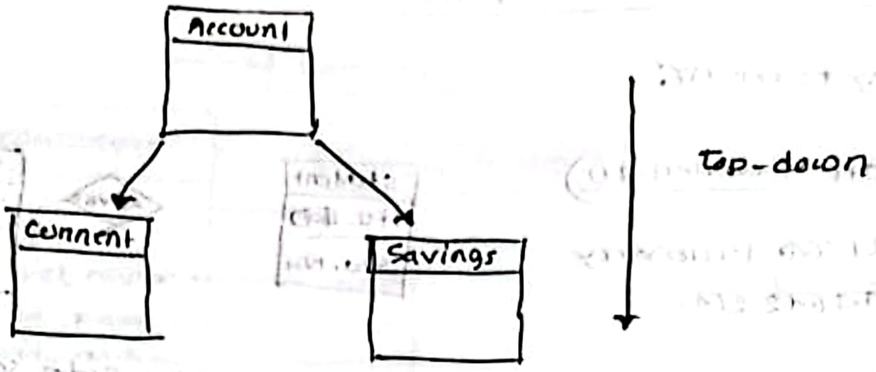
Generalization:

- It is a bottom up process.
- Lower level functions are combined to made higher entity
- Subclasses are combined to form super-class.



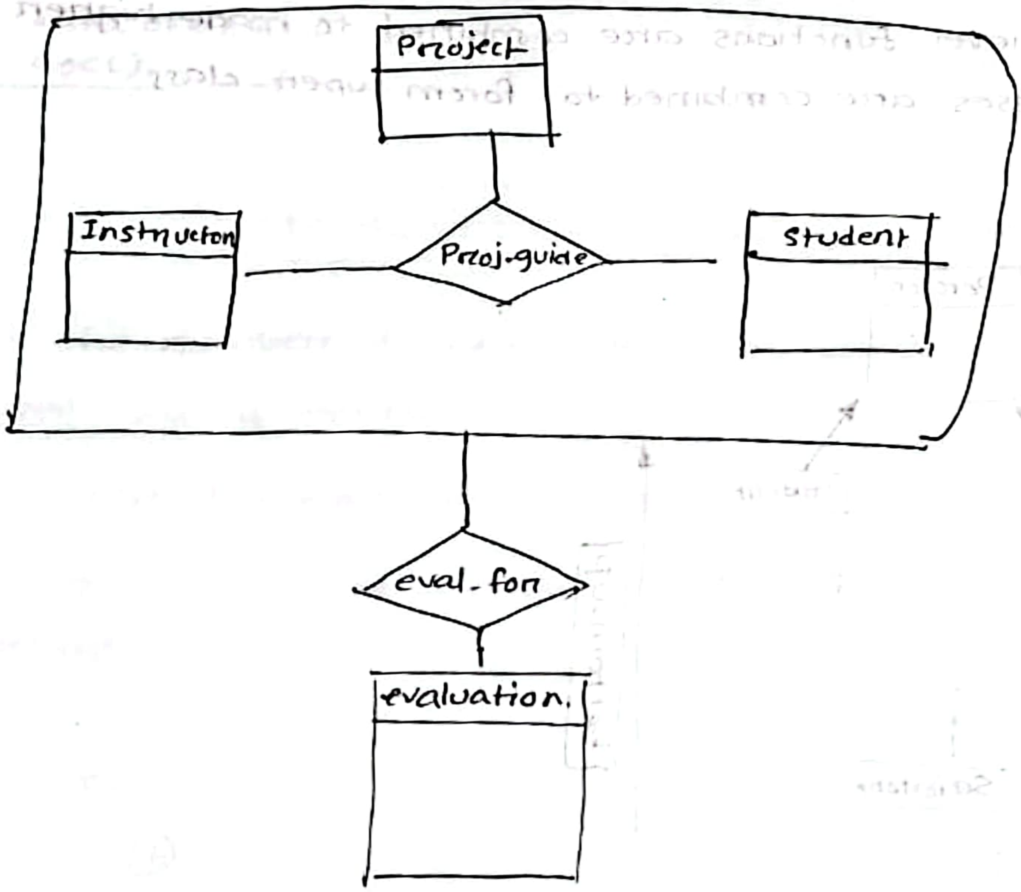
Specialization:

- It is a top down process
- By breaking upper level entity we made lower level entity.
- Devide super class to make sub class.



Aggregation:

- Treat relationship as abstract entity.
- Allows relationships between relationships.
- Eliminate relationship redundancy via 'aggregation'.



Functional Dependency:

A functional dependency is a relationship betn two attributes → generally primary key and non-key attributes

$\boxed{\text{determinent}}$

Typical Functional dependency : $x \longrightarrow y$

$\boxed{\text{dependent}}$

Trivial F.D:

If the dependent is a subset of determinent then it is called Trivial F.D.

$A \to B$

$\{A, B, C\} \longrightarrow B$

$A \to A, \quad B \to B.$

If A = Determinent, B = dependent.

$B \not\subseteq A$.

Then it is called Non-Trivial F.D..

Normalization:

Database normalization is a technique of ~~normalize~~ organizing the data in a database.

Normalization is a systemetic approach of decomposing tables to eliminate data redundency and undesirable characteristics like, insert, update & delet anomalies.

Normalization is used for mainly two purposes:

- Eliminating redundant (useless) data.
- Eliminating Insertion, Update & Delete anomalies.

Partial Dependency:

If an attributes in a table depends on only a part of primary key not the whole key then it is called partial dependency.

If the Schema: score table (student-ID, sub-name, teacher, Exam)

$\boxed{\text{Sub\_name} \rightarrow \text{Teacher}}$

Partial dependency.

Transitive dependency:

If a non-prime attributes depends on a non-prime attribute then it is called Transitive dependency.

$\boxed{\text{Teacher} \rightarrow \text{Exam}}$

Normalization:

▷ First Normal from:

- Only single valued attributes.
- Values stored in the column should be same domain.
- Unique column name.
- data stored order doesn't matter.

roll	subject
101	OS, CN
103	Java
102	C, C++

→

roll	subject.
101	OS
101	CN
103	Java.
102	C
102	C++.

$\boxed{\text{Not in First Normal from}}$.

▷ Second Normal from:

- Should be in first Normal from.
- No partial dependency.

Ex:

stud - ID	Sub_name	Teacher	Exm.name	Total.
10	OOP2	T_1	OOP2	150
10	DBMS	T_2	DBMS	210
11	OOP2	T_1	OOP2	150

sub_name ⟶ Teacher (Partial dependency occurs).

Now,

Sub-id	Sub_name	Teacher
1	OOP2	T_1
2	DBMS	T_2

Stud-id	sub-id	Exm-name.	total.
10	1	OOP2	150
10	2	DBMS	210
11	1	OOP2	150

Now Partial dependecy solve. Using this method we can solve any dependency.

Third normal: form:

- Should be in second normal form.
- Remove Transitive dependency.

3.5: BCNF (Boyce code Normal From):

- should be in third normal from.
- If any primitive attribute depends on non primitive attribute. A ⟶ B.

remove this dependency. | B → Prime. A - non prime.

Fourth Normal Form:

- Should be in the BCNF.
- Remove multivalued dependency

Relational Algebra

Slide link তারাও   Part B

Part 4: Storage Management: (Chapter: 12)

Physical storage systems

Classification of physical storage media:

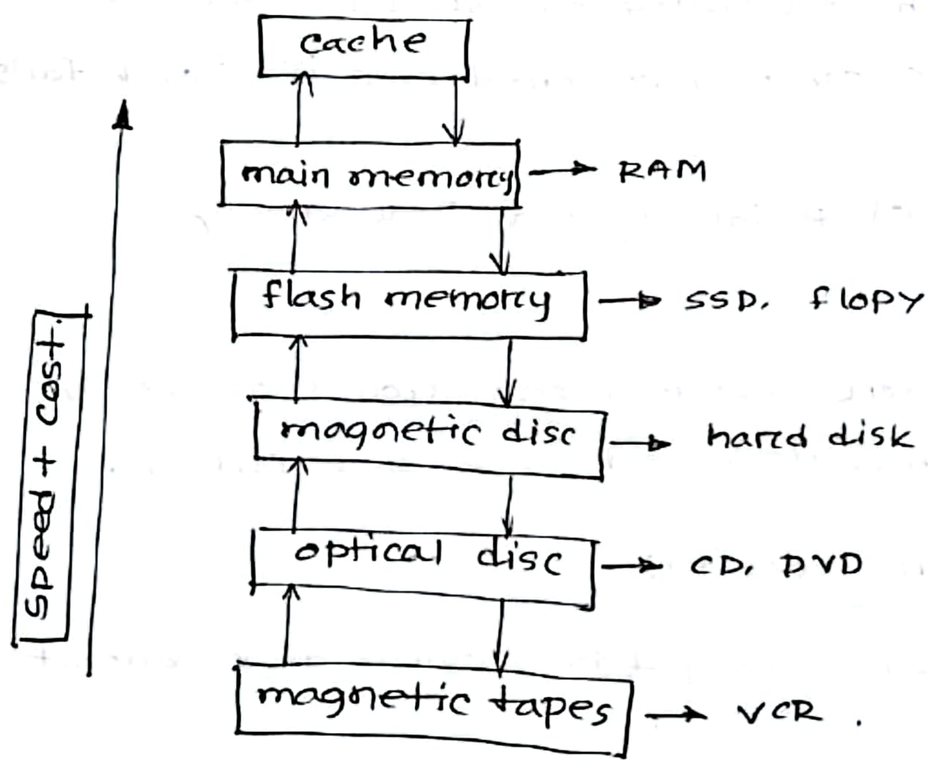
Can differentiate storage into—

- Volatile storage: losses contents when power is switched off
- Non volatile storage:
  - Contents persist even when power is switched off
  - Includes secondary and ternary storage, as well as batter backed-up main memory.

Factors affecting choice of storage media include—

- Speed with which data can be accessed.
- Cost per unit of data.
- Reliability.

Storage Hierarchy



- **Primary storage:** Fastest media but volatile (cache, main memory)
- **Secondary storage:** Non-volatile and moderately fast access time.
  - also called online storage
  - Ex - flash memory, magnetic disk.
- **Tertiary storage:** Lowest level in hierarchy, non-volatile, slow access time.
  - also called off-line storage, and used for archival storage
  - Ex — Optical disk, magnetic taper.

RAID:

Redundant Arrays of Independent Disks.:

Disk organization techniques that manage a Large number of disk providing a view of a single disk of.

- High capacity and high speed by using multiple disk in parallel.
- High reliability by storing data redundantly, so that data can be recovered even if a disk fails.

Improvement of Reliability via Redundancy:

- **Redundency:**

  Store extra information that can be used to rebuild information Lost in a disk failure.

- **Mirroring or Shadowing:**
  - Duplicate every disk. Logical disk consist of two physical disk.
  - Write on both disk.
  - Read from single disk.
  - If one disk become failed data still avaiable in the other.

• Data loss occurs when a disk with it's mirror disk also failed before repair.

[Mean time to data loss depends on = Mean time to failure + Mean time to repair]

RAID level: ***

Raid level 0: ~~non-redundant~~ Block striping, non-redundant.

- Used in high performance application where data loss is not critical.

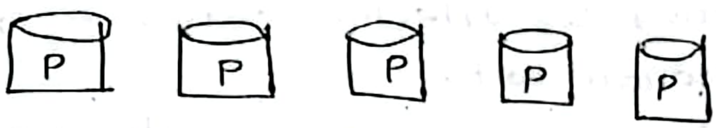
Raid level 1: Mirrored disks with block striping.

- Offers best write performance.
- Populars for applications such as log files in a database system

Raid 0

Raid 1.

Raid level 5: Block interleaved : Distributed parity. partitions data and parity among all N+1 disk rather than storing data in N disk and parity in 1 disk.



P0	0	1	2	3	4
5	P_1	6	7	8	9
10	11	P_2	12	13	14
15	16	17	P4	18	19

P_0	0	1	2	3
4	P_1	5	6	7
8	9	P_2	10	11
12	13	14	P_3	15
16	17	18	19	P_4

Raid level: 5.

Raid level 6: Similar to raid level 5. can recover two disc failure & store two parity.

- Two error correction blocks (P,Q)

- Better relibility than level 5 at a higher cost.

Raid level: 6 : (P+Q) redundency

Choice of RAID Level:

Factors in choosing RAID level.

- Monetary cost.
- Performance
- Performance during failure.
- Performance during rebuild.

RAID 0: RAID 0 is used when data safety is not important

- used in high performance application.
- Data can be recovered quickly from other device.

RAID 1:

- write performance best.
- ~~for~~ storing log files in a database system
- Higher storage cost.
- Preferred for applications with many small/random updates.
- Only requires two block writes.

Raid level:5

- sequential writes & large number of data storage.
- read frequently and write rarely.
- 1 parity.

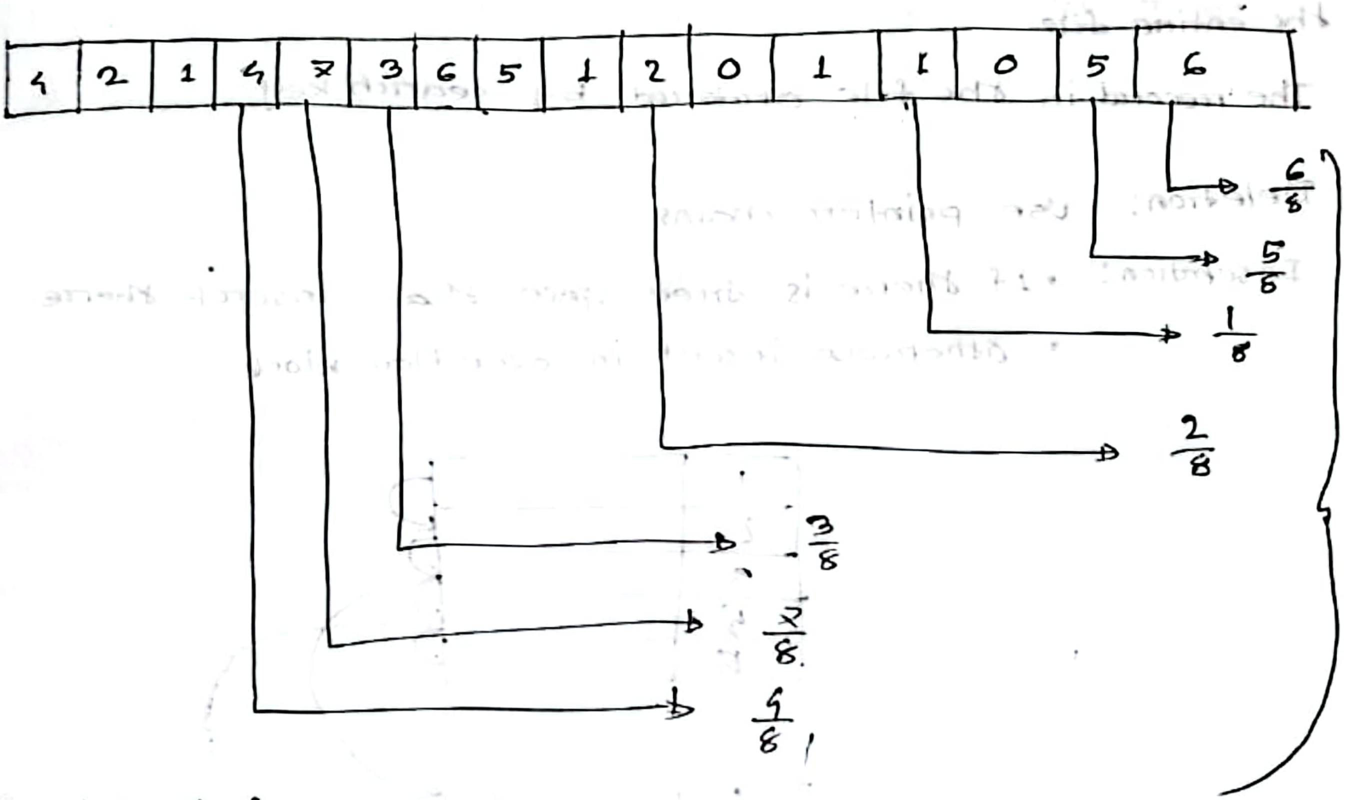
Raid level:6

- High data protection.
- it can tolarate two disk (failure)
- with High random input/output.

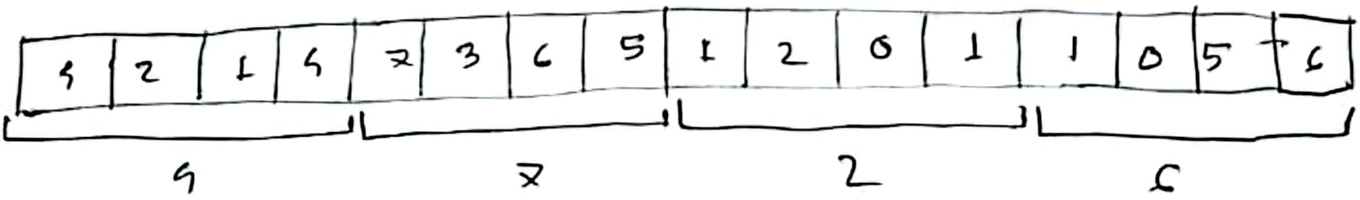
chapter 13: Data storages and structures: ***

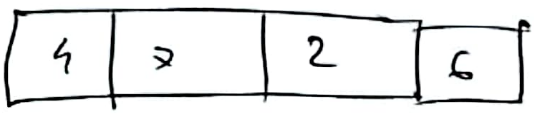
Two-level free space map:

3bit= $2^3=8$



Second level freespace:



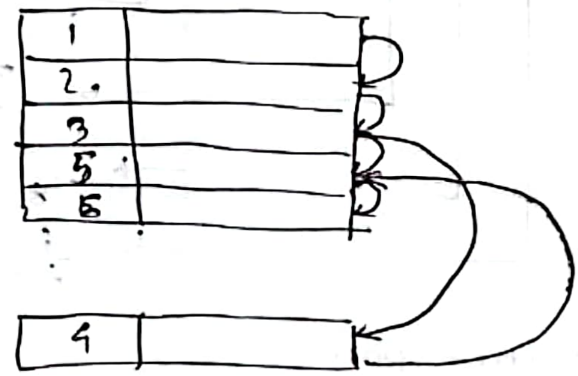


Organization of records in Files.

- Heap - record can be placed anywhere in the file where there is space.
- Sequential - store records in sequential order. based on the value of the search key of each record.
- Multitable clustering.
- B+ tree:
- Hashing:

Sequential file organization:

- Suitable for application that require sequential processing the entire file.
- The record in the file ordered by search key..
- Deletion: Use pointer chains.
- Insertion:
  - If there is free space than insert there
  - Otherwise insert in overflow block.



B+ Tree:

A B+ tree is a rooted tree satisfying the following properties.

- All paths from root to leaf are of the same length.
- Each node that is not a root are a leaf has between (n/2) and n ~~va~~ children.
- A leaf node has betewen ((n-1)/2) and (n-1) values. (selling values)

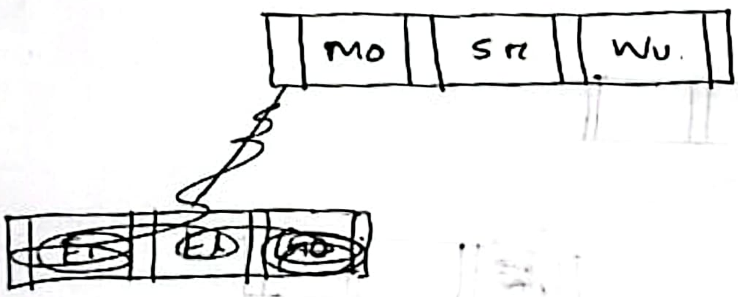
Special case:

- If the root is not a leaf then it has atleast two children.
- If the root is leaf it means it has no other nodes in the tree. it can have between 0 and (n-1) values.

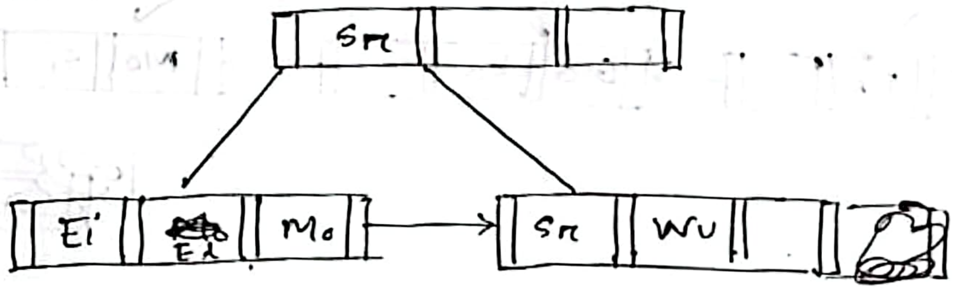
B+ Tree insertion:

Sr, Wu, Mo, Ei, El, Go, ka, Ca, Si, Cr, Br.

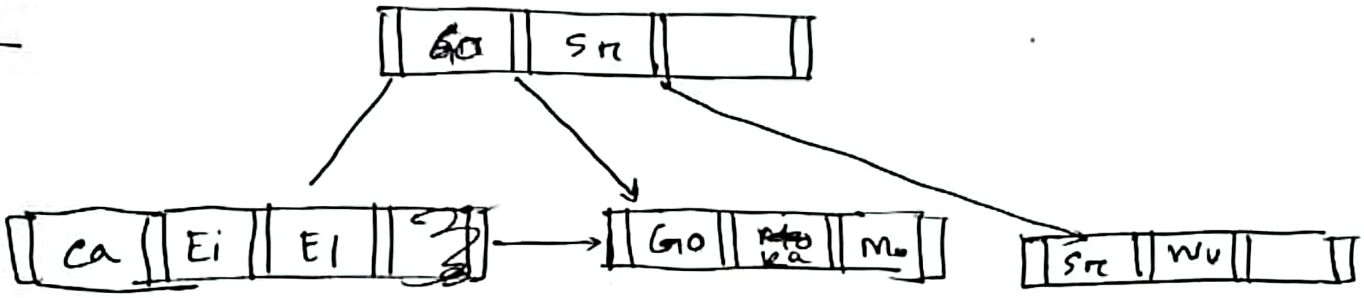
Empty node, n = 4

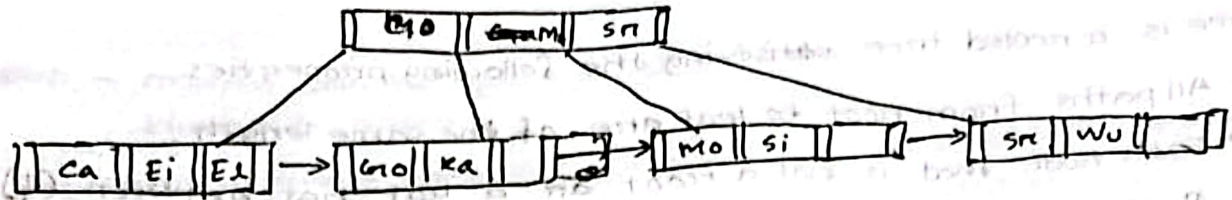


Ei Insert: El:

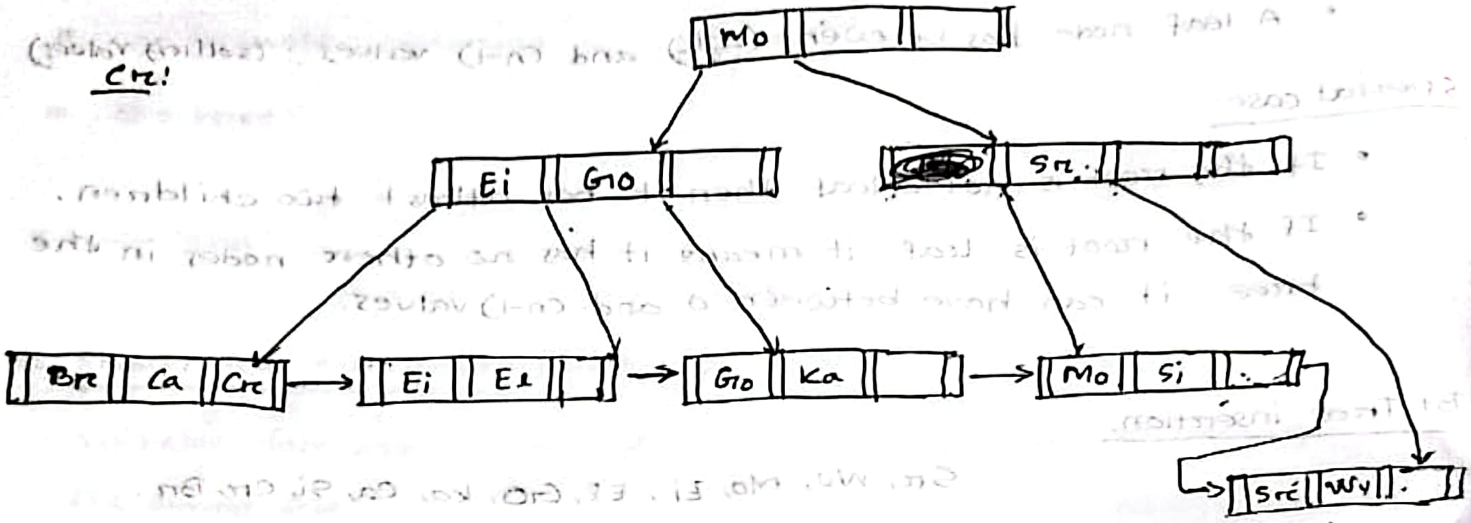


GO:



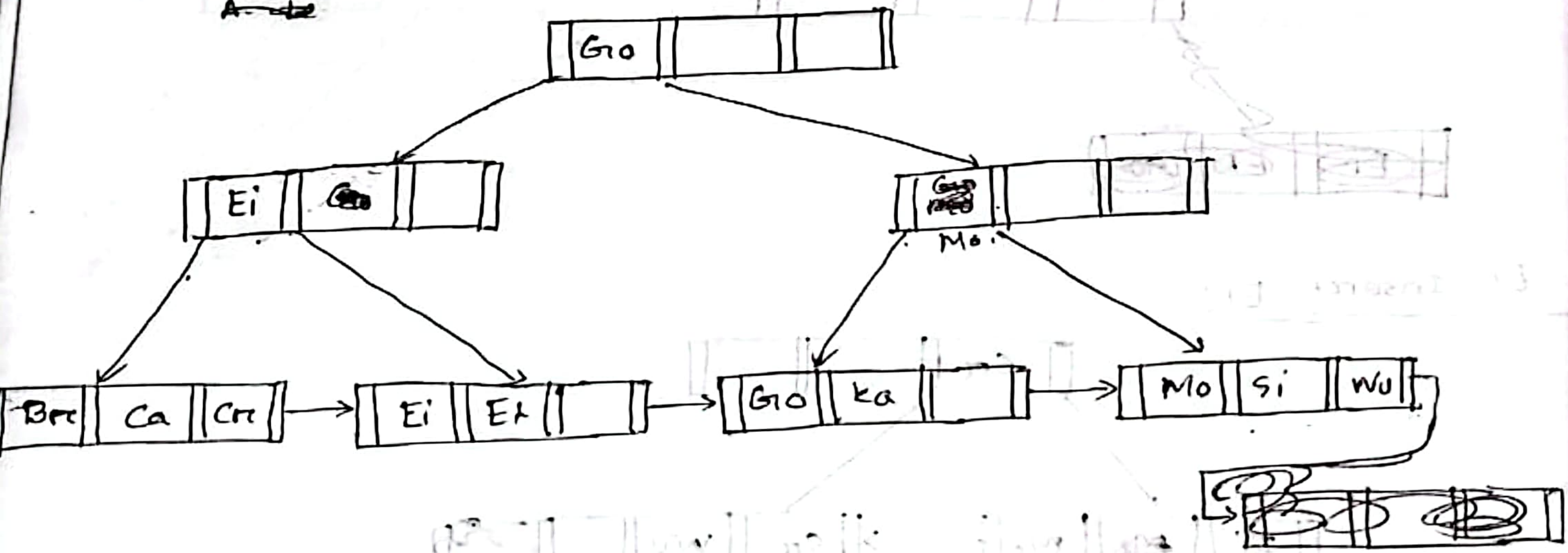


Cr:

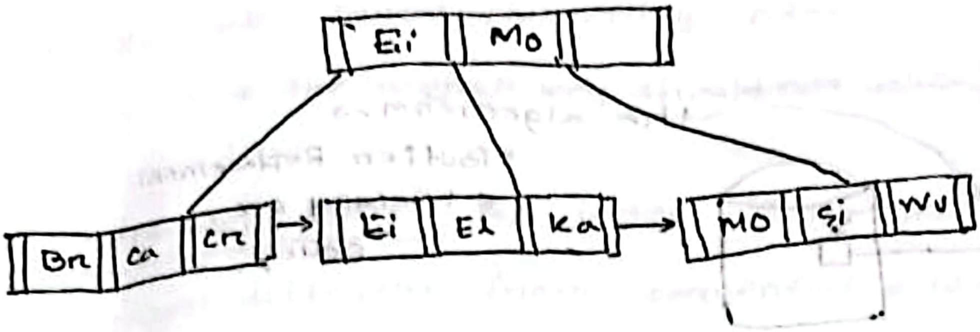


Deletion:

Delete Sr:

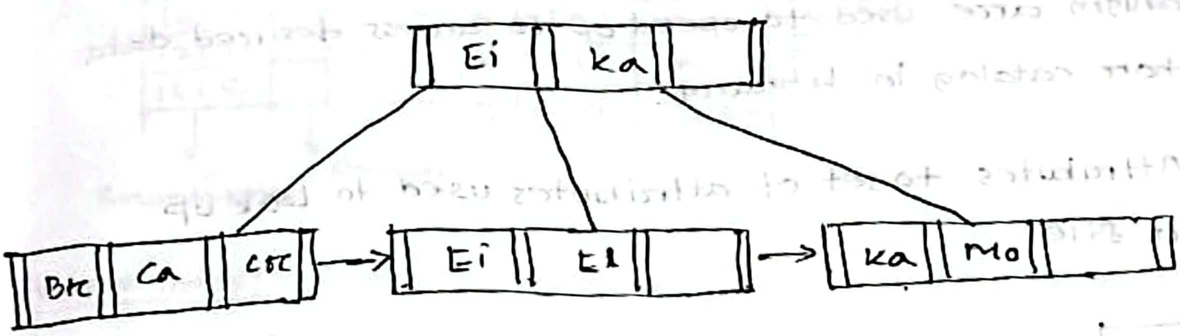


Deletion(no)!



Delete Wu, Si

Wu, Si



Multitable clustering:

Usually use for join.

Dept:

dpt-name	building
CSE	X
EEE	Y
ME	Z

Teacher:

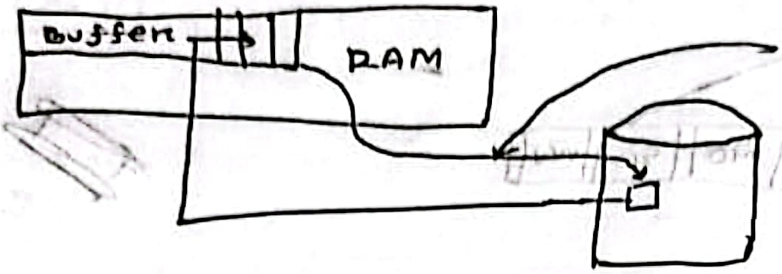
ID	Name	dept-name
1	A	CSE
2	B	EEE
3	C	CSE.

multitable clustering:

CSE	X	
1	A	CSE
3	C	CSE
EEE	Y	
2	B	EEE
ME	Z	

Storage access:

Buffer -> A portion of a main memory.



-> the algorithm -> "Buffer Replacement strategy or policy.

Doing the whole system by Buffer manger.

Indexing:

Indexing mechanism are used to speed up to access desired data.

Ex- Author catalog in Library.

Search key: Attributes to set of attributes used to look up records of a file.

Search key	Pointer

- Index files are much smaller than original file.
- Ordered indicies: Search keys are stored in a sorted order.
- Hashing idicies: Search keys are distributed uniformly accross buckets using hash functions.

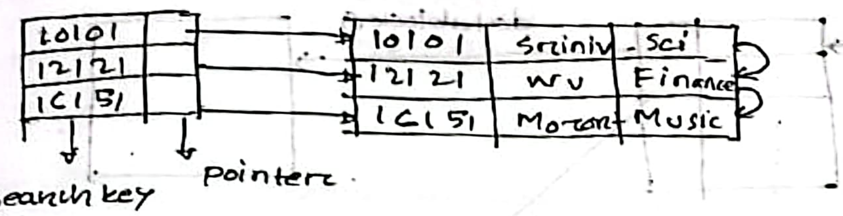
Ordered indicies: In an ordered index the index entries are stored sorted on the search key values.

Primary index: In a sequentially ordered file. ~~whose search key~~ the index whose search key specifies an sequential order in a file, also called clustering index.

- The search key should not always be primary key.

Secondary index: An index whose search key specifies an order different from sequential order of a file is called secondary or non-clustering index. (secondary index have to be dense)

Dense index: Index records appears for each search key value.



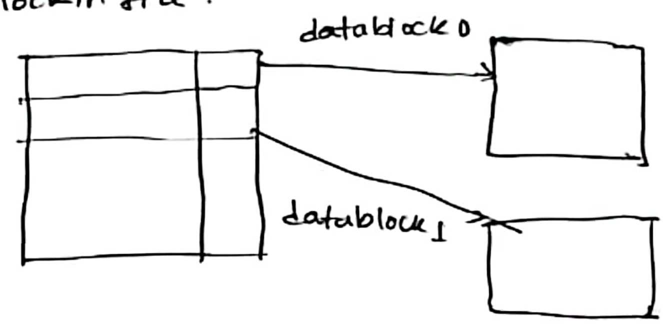
Sparse index: Contains index records for only some search key values. in a file.

- sparse index is applicable when the search keys are sequentially ordered.

Compared to Dense index:

- Less space and less maintenace overhead for insertion and deletion
- Generally slower than dense index for locating records.

Good trade off: Sparse index with ~~each search key value~~ index entry for ~~each search value~~ every blockin file. corresponding the least search key value ~~for a~~ in the block.



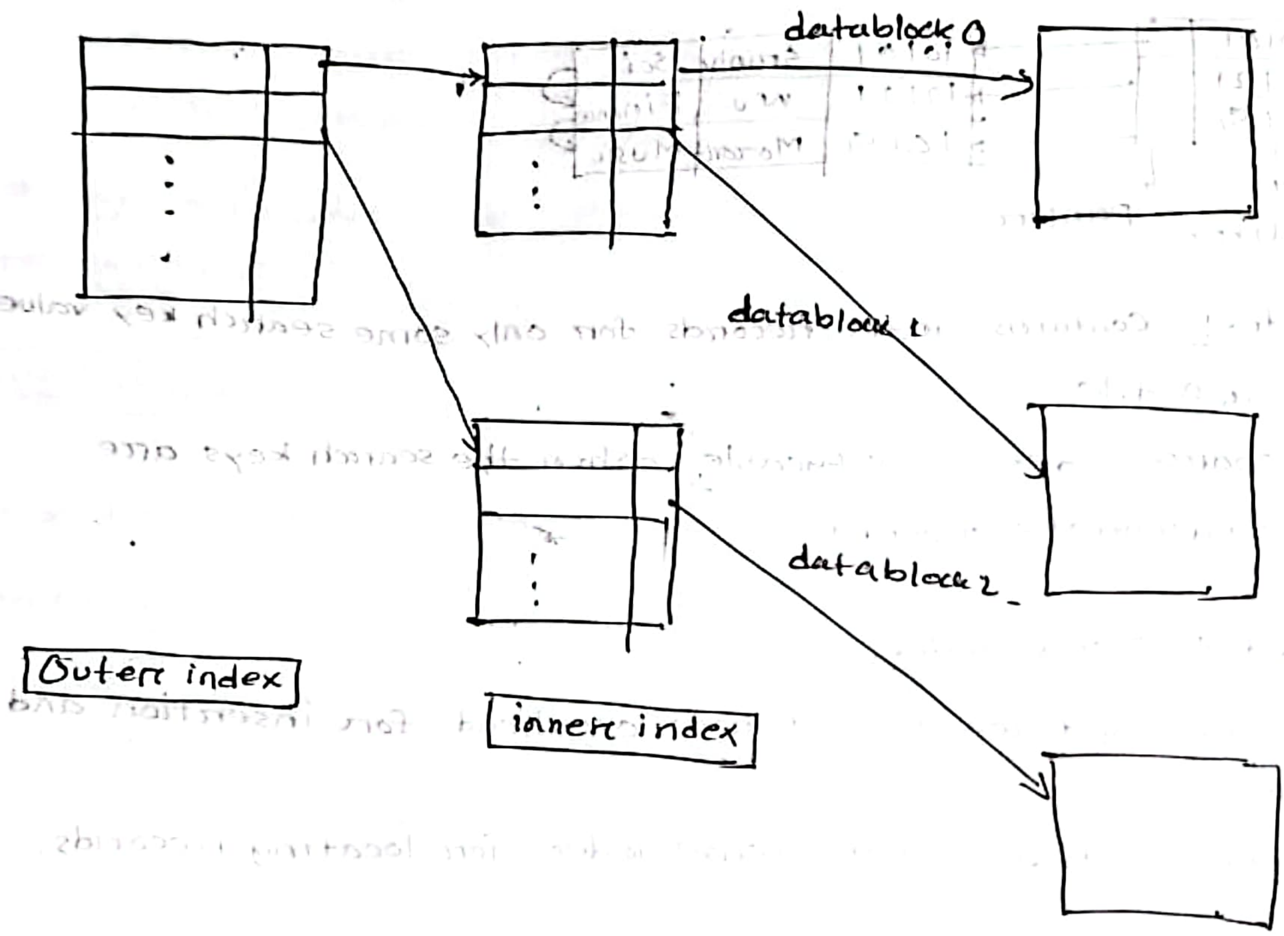
Multilevel Index: CT②

If primary index doesn't fit in memory access become too expensive.

Solution: Treat primary index kept on a sequential file and construct sparse on it.

Outer index: sparse of primary index.

inner index: the primary index file.



Static Hashing:

A bucket is unit of storage containing one or more ~~storage~~ entries.

Bitmap Indices:

ID	Course
1	CSE 2205
2	CSE 2203
3	CSE 2205
4	CSE 2205
5	CSE 2204
6	CSE 2203
7	CSE 2205
8	CSE 2201
9	CSE 2201
10	CSE 2203

Bitmap for:

CSE 2201 = $\boxed{0000000110}$

CSE 2203 = $\boxed{0100010001}$

CSE 2204 = $\boxed{0000100000}$

CSE 2205 = $\boxed{1011001000}$

Summer, 2022

~~Given~~.

Stating Hashing using (hashfunction)

~~Given~~.

Hash : In a hash index bucket store entires with pointer records.

- Worst hash function means all search key values in ~~at~~ same bucket.
- An ideal hash function is uniform i.e ~~the~~ same number of ~~hash fun~~ search key values in each bucket.
- Ideal hash function is random.

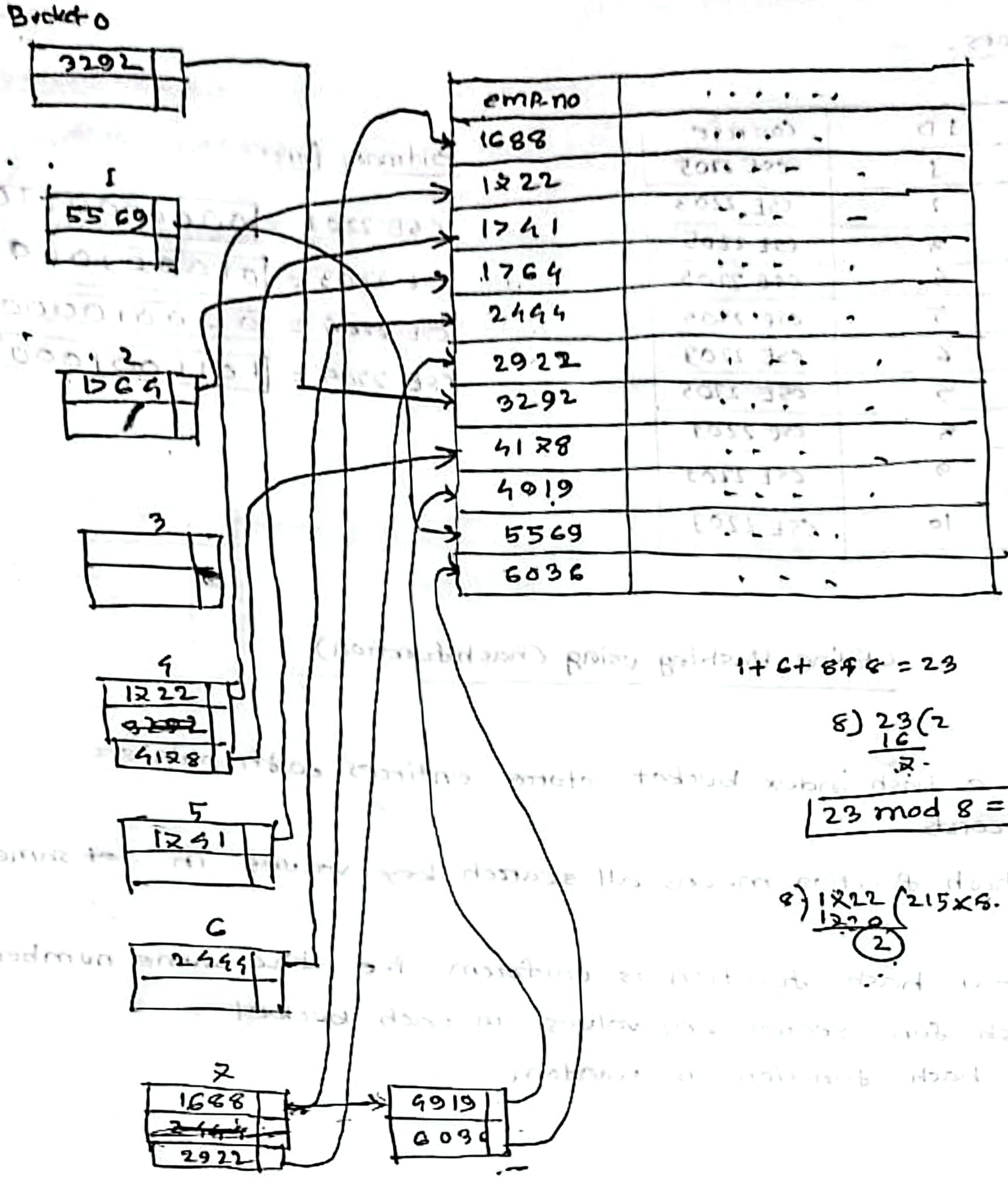
Summer: 2022

4(a)

Hash function:

$H(M) = M \bmod 2^k \quad [k=3)$

$= M \bmod 8.$



$1+6+8+8 = 23$

$8)\ 23\ (2$
$\underline{16}$
7

$\boxed{23 \bmod 8 = 7}$

$8)\ 1222\ (215 \times 8.$
$\underline{1220}$
2

Bucket Overflow: ***

Reason:

- Insufficient storage in buckets.
- Skew in distribution of records:
  - ~~has~~ multiple records have same search key values.
  - chosen non-uniform distribution of key values.

It is handled by using overflow buckets.

Handling Bucket Overflows

- **Overflow chaining:** The overflow buckets of a given bucket are chain together in a linked list. Above scheme is called closed addressing or closed hashing.

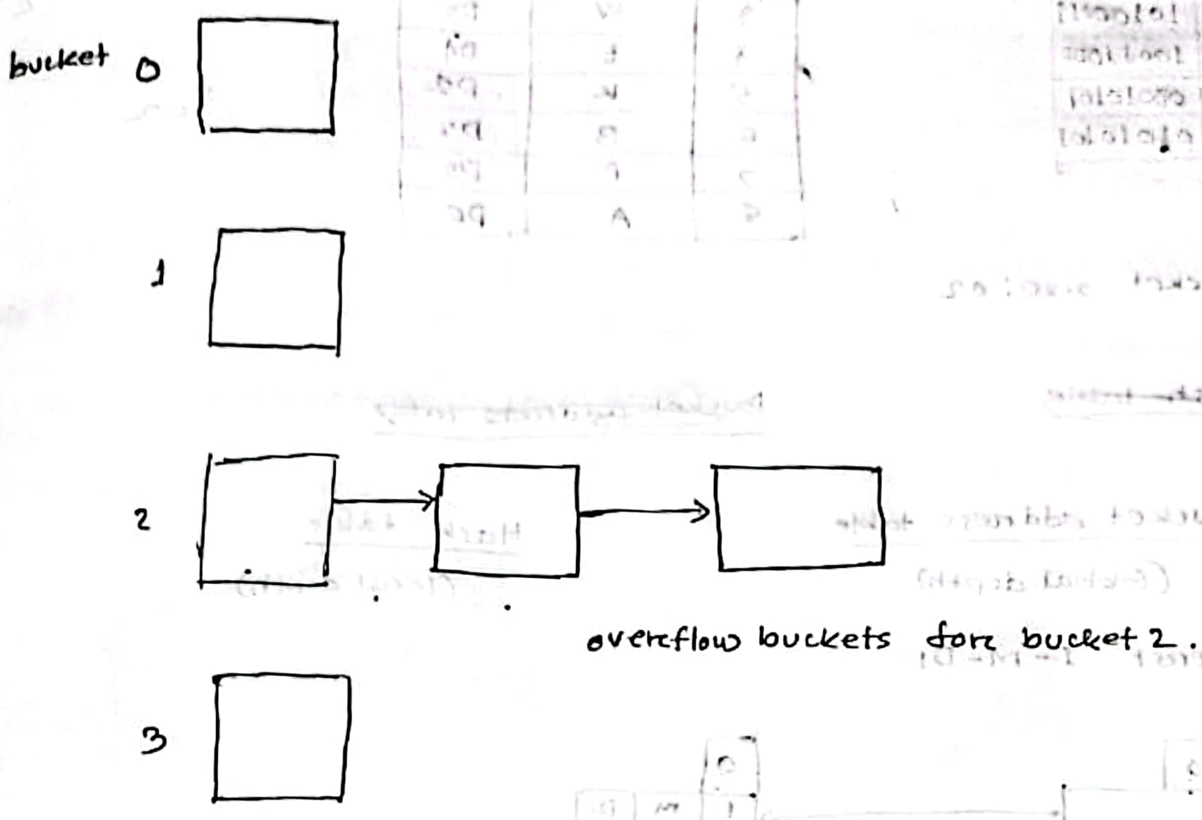
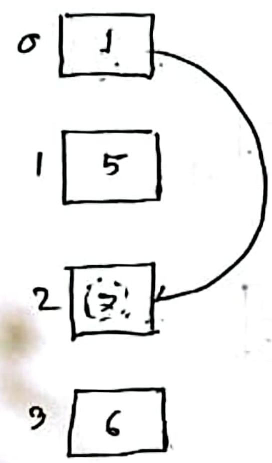


fig 1. Closed addressing / Hasing.

- **Open addressing:**

fun: $h(2) = 0$.

{ As, 0 bucket is full so it will point the next empty bucket. 2 }.



** Deficiencies of static Hashing: Chapter 14 : slid no: 58

2^n

Dynamic Hashing:

Extendable hashing

Dept	h(dept)
D1	00110101
D2	11110001
D3	10100011
D4	10011000
D5	00010101
D6	01010101
⋮	⋮

ID	Name	Dept
1	M	D1
2	S	D2
3	W	D3
4	E	D4
5	K	D2
6	B	D2
7	F	D5
8	A	D6

2^2

$2^1 = 2$

Bucket size: 02

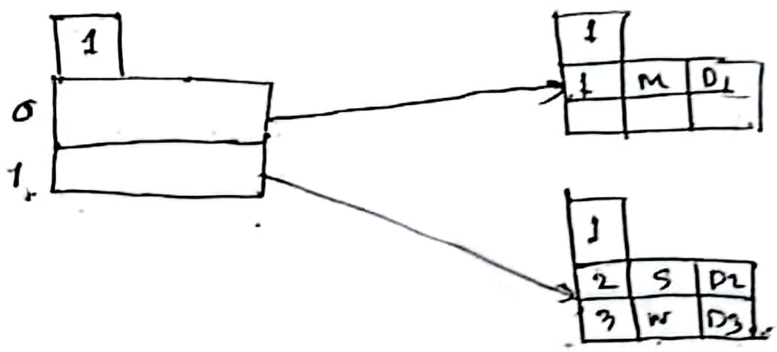
~~Hash table~~ ~~bucket address table~~

Bucket address table (Global depth)

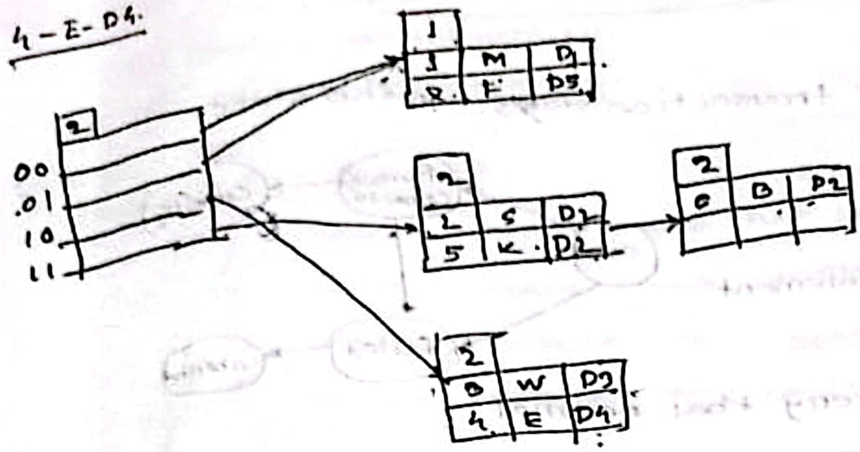
Hash table (local depth)

Insert 1 - M - D1:

Insert 3 - W - D3

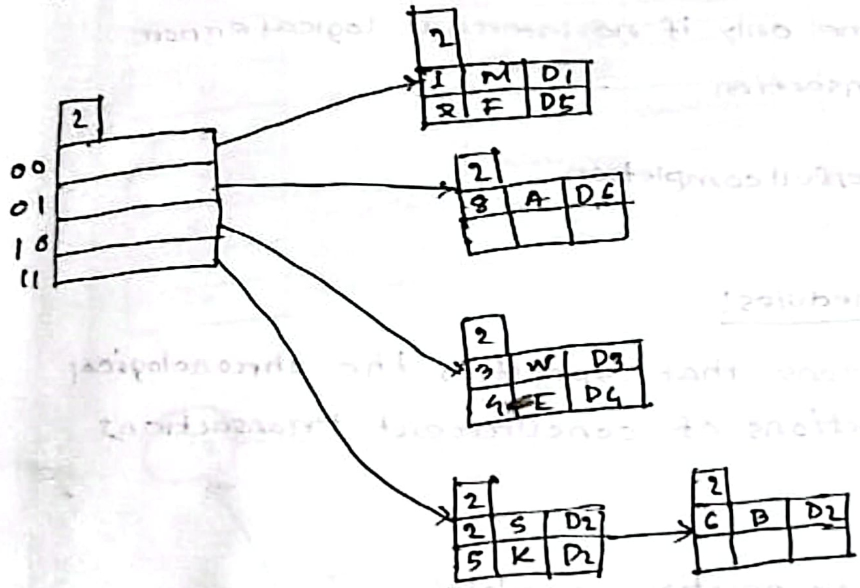


4 - E - D4.



~~C-B-D2~~

8 - A - D6 (Global depth > local depth)



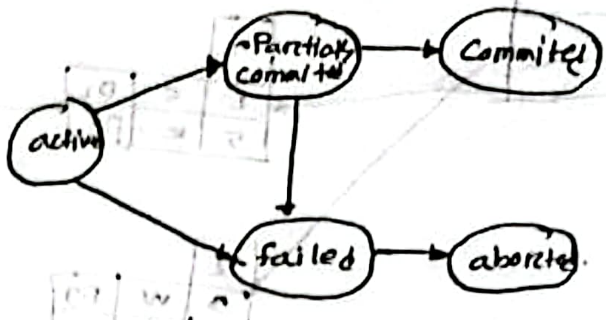
Chapter 12: "Transaction".

A transaction is a unit of program execution that acceses and possibly updates various data items.

*** ACID properties. পড়তে হবে chapter 12: slide: 7.

Life cycle of a Transaction state:

Active: The initial state; the transaction stays in this state while it is executing.



Partially commited:
after the final statement has been executed.

Failed: after the discovery that normal execution can no longer proceed.

Aborted: after the transaction has been rolled back and the database restored to it's state prior to the start of the transaction. Two options after it has been aborted.

- Restart the transaction.
  - can be done only if no internal logical error.
- kill the transaction.

Committed: after succesfull completion.

Schedules:

A sequences of instructions that specifies the chronological order in which instructions of concurrent transactions are executed.

- **Serial:** first execute a transaction completely. then execute another transaction.
- **No serial:** executing two transaction parallely.
- **Serializability:** test a schedule is serial or not.
- **serializable:** If a not serial -schedule have a equivalent seriali schedule.

* Two types of serializability.
  1. Conflict serializability.
  2. View serializability.

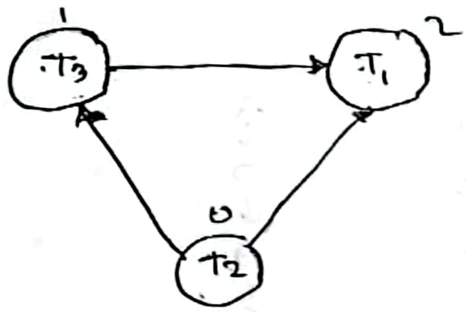
1. conflict:

Condition for conflict:

1. read read → not conflict.
2. write read → conflict.
3. read write → conflict.
4. write write → conflict.

Presedence Graph:

Instruction	T_1	T_2	T_3
1	~~Read(x)~~		
2			~~Read(Y)~~
3			~~Read(x)~~
4		~~Read(Y)~~	
5		~~Read(Z)~~	
6			~~Write(Y)~~
7		~~Write(Z)~~	
8	Read(Z)		
9	Write(x)		
10	Write(Z)		



- As there is no cycle so it is conflict serializable.
- where indegree is low there parority high.

$T_2 \rightarrow T_3 \rightarrow T_1$

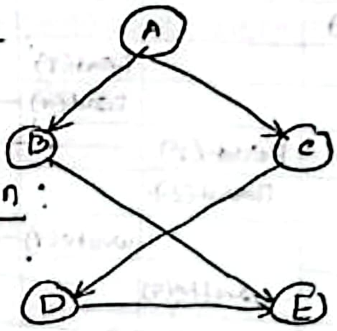
Ins	T_1	T_2	T_3
1.		R(Y)	
2		R(Z)	
3		W(Z)	
4			R(Y)
5			R(X)
6			W(Y)
7	R(X)		
8	R(Z)		
9	W(X)		
10	W(Z)		

$2^3 = 8$

3!

Ques. Prev

No cycle exist so it is conflict serializable.



- in degree list of the transaction:

A = 0
B = 1
C = 1
D = 1
E = 2

So possible combination:

A → B → C → D → E

A → B → D → C → E

A → C → B → D → E

A → C → D → B → E

A → D → C → B → E

A → D → B → C → E

View serializibility:

$3! = 3 \times 2 = 6$

condition:

i) conflict serializable &

ii) blind write check.

If one condition is satisfy then we checking view serializible or not.

To check view serializability:

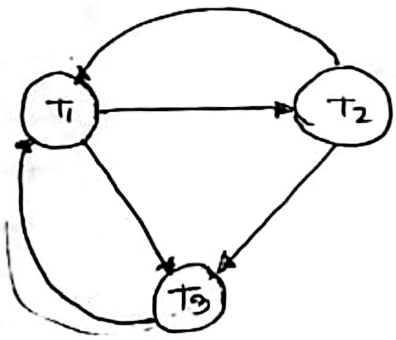
- Initial read: $S_i : T_x = S_j : T_x$.
- Updated read: $S_i : T_x \rightarrow T_y = S_j : T_x \rightarrow T_y$.
- Final write: $S_i : T_x = S_j : T_x$.

Blind write: write without read.

S_1:

T_1	T_2	T_3
Read (Q)		
	$\boxed{\text{write(Q)}}$	
		Read (Q)
write(Q)		
		write (Q)

Blind write



Here cycle → so not conflict serializable so we check blind write,

Combination:

1. $S_2 = \langle T_1, T_2, T_3 \rangle$
2. $S_3 = \langle T_2, T_1, T_3 \rangle$
3. $S_4 = \langle T_3, T_1, T_2 \rangle$
4. $S_5 = \langle T_2, T_3, T_1 \rangle$
5. $S_6 = \langle T_3, T_2, T_1 \rangle$
6. $S_7 = \langle T_1, T_3, T_2 \rangle$

$\langle T_1, T_2, T_3 \rangle$

$\langle T_1, T_3, T_2 \rangle$

$\langle T_2, T_1, T_3 \rangle$

$\langle T_2, T_3, T_1 \rangle$

$\langle T_3, T_1, T_2 \rangle$

$\langle T_3, T_2, T_1 \rangle$

S_2

T_1	T_2	T_3
Read (Q)		
write(Q)		
	write(Q)	
		Read (Q)
		write(Q)

Initial read:

S1 : T_1 = S2: T_1

Updated read:

S1: $T_2 \rightarrow T_3$ = S2 : $T_2 \rightarrow T_3$.

Final write:

S1: T_3 = S2: T_3.

So this is view serializable.

Chapter 18

Lock Based Protocols

A lock is a mechanism to controll concurrent access to a data item.

Exclusive — Read + write → x-lock.

shared — Read → s-lock.

***** Lock compatibility matrix**

T_1 (rows) / T_2 (columns)

	S	X
S	T	F
X	F	F

~~A set of tr~~

Deadlock prevention: (Strategies)

Wait die: scheme – non-preemptive

- older transaction may wait for younger one to release data item.
- younger transaction never wait for older one, they are rolled back instead.

Wound wait: scheme-preemptive

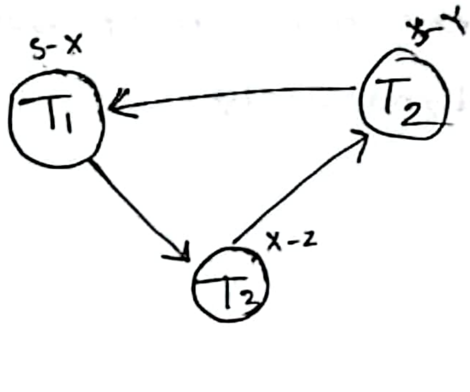
- younger transaction may wait for older one to release data item.
- older transaction never wait for younger one, they are rolled back instead.

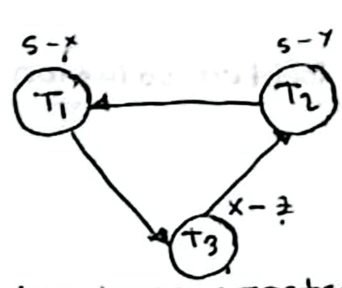
Timeout-Based Schemes:

A transaction waits for a lock only for a specified amount of time. After that the wait times out and the transition is rolled back.

- PrevQ:

Ins. No	T_1	T_2	T_3
1	Lock-S(X)		
2	Read(X)		
3			Lock-X(Z)
4			Read(Z)
5	Lock-X(Z)		
6		Lock-S(Y)	
7		Read(Y)	
8		Lock X(X)	
9			Lock-X(Y)
10	--	---	---





A cycle has been created so a dead lock present here.

Solution:

i) Let T_3 is the transaction with minimum cost factor, we choose T_3 as victim.

ii) We must force T_3 for rollback. T_3 must aborted.

iii) After breaking deadlock T_3 must be restarted.

iv) we have to include number of rollback in the cost factor of T_3 to avoid starvation.

GAP:

*** Deficiencies of static Hashing:

- In static hashing. function h maps search key values to a fixed set of B bucket addresses. Databases grow or shrink with time.
  - If the initial number of bucket is too small and file grows, performance will degrade due to too much overflows.
  - If space is alocated for anticipated growth a significant amount of space will be wasted initially. (and buckets will be underfull).
  - If database shrink space will be wasted again.
- One solution: periodic re-organization of file with new hash function.
  - Expensive, disrupt normal operation.
- Better solution: Allow the number of buckets to modify dynamicaly.

ACID Properties ***

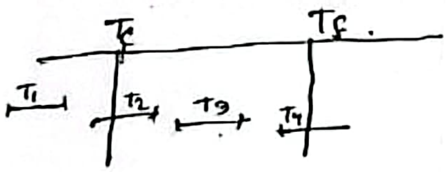
A Transaction is a unit of program execution that accesses or possibly updates various data items. To preserve the data integrity database must ensure—

- **Atomicity:** Either all operations of the transaction are properly reflected in the database or none are.

- **Consistency:** Execution of transaction in isolation, preserves the consistency of a database.

- **Isolation:** Although multiple transactions are executed concurrently, each transaction must unware of other concurrently executed transaction. Intermediate transaction results hidden from other cocurrently executed transaction.

- **Durability:** After complete a transaction succesfuly, the changes it has made to the database persist, even if there are system failures.



Here, T_2 and T_3 = redo
T_4 = Undo.

হলেই abor
rollback = পুরাত
val
commit = updat

<To start>
<To, B, 2000, 2050>.
<T1 start>
<checkpoint {To, T1}>
<T1, C, 700, 600>
<T1, commit>
<T2, start>
<T2, A, 500, 400>
<To, B, 2000>
<To, abort>

crash.

Redo phase:

undo = { ~~To~~, ~~T1~~ } T2 (commit)

B = 2000, 20~~5~~0
C = ~~700~~, 600
A = 500, ~~400~~

∴ Undo list = T2

Undo phase;

<T2 A, 500>
<T2 abort>

Recovery Algorithm : X=500, Y=100

T_1	T_2
R(X)	
X=X+5	
	R(Y)
	Y=Y+10
W(X)	
checkpoint	
commit	
	W(Y)

Log report:

<T1 start>
<T1 start>
<T1, X, 500, 505>
<checkpoint {T1,T2}>
<T1, commit>
<T2, Y, 100, 110>

Winter 2022

1 (a). case 1:

<To start>
<To, A, 1000, 950>
<To, B, 2000, 2050>

Undo phase: ↑ bottom-up.

<To, B, 2000>
<To, A, 1000>
<To ~~commit~~ abort>.

(b) case2:

<To start>
<To, A, 1000, 950>
<To, B, 2000, 2050>
<To. commit>
<T1 start>
<T1, C, 700, 600>
<T2 start>
<T2, D, 1000, 600>
<T1 commit>

redo phase: ↓ Top-bottom

Undo = {~~T0~~, ~~T1~~, T2}

A = ~~1000~~, 950
B = ~~2000~~, 2050
C = ~~700~~, 600
D = 1000, ~~600~~

Undo list: T2

Undo phase:

<T2, D, 1000>
<T2 abort>

case 3:

<T0 start>
<T0, A, 1000, 950>
<T0, B, 2000, 2050>
<T0, commit>
<T1 start>
<T1, C, 700, 600>
<T2 start>
<T2, D, 1000, 600>
<T1 commit>
<T2 commit>

redo phase:

Undo = { ~~T0~~, ~~T1~~, ~~T2~~ }

A = ~~1000~~, 950
B = ~~2000~~, 2050
C = ~~700~~, 600
D = ~~1000~~, 600

Top- bottom.

Part- A (বাকি অংশ)

- The relational algebra is a procedural query language.
- It consist of a set operations that take one or two relation as input and produce a new relation as ther

Basic Relational Algebra operation:

i) Unary Relational operations:

- SELECT (σ)
- PROJECT (π)
- RENAME (ρ)

ii) Set theory Relational Algebra Operation:

- UNION (∪)
- INTERSECTION (∩)
- DIFFERENCE (-)
- CARTESIAN PRODUJCT (×)

iii) Binary Relational Operation:

- JOIN
- DIVISION

- **SELECT (σ):**

$\boxed{\sigma_p (R)}$

σ = SELECT symbol
P = predicate or condition.
R = Relation or table.

Ex- select tupples from tutorials where topic = 'Database'

$$\sigma_{topic = \text{"Database"}} (Tutorials).$$

- **Projection (Π):** (all)

$\boxed{\Pi_{attribute\_name} (Relation)}$

Ex- Find the name of all instructors in the physics Department.

$$\Pi_{name}\ \sigma_{dept = \text{"physics"}} (instructors).$$

- **UNION:** $\cup$

Ex- Consider a query to Find the set of all courses taught In the Fall 2009 semester, the spring 2010 semester, or both the information is containing by 'section' Relation.

$$\Pi_{courses} \left(\sigma_{sem = Fall \wedge year = 2009} (section) \right) \cup$$

$$\Pi_{courses} \left(\sigma_{sem = spring \wedge year = 2010} (section) \right)$$

for set diff: (but not)

same as ↑ just (–) in middle.

Intersection: (both)

'$\cap$'

Cartesian Product (A×B):

Example:
Find all the instructor name from physics department with their taught course-id from "instructor" and "teaches" relation respectiv

$$\Pi_{name,\ course\text{-}id}\left(\sigma_{ins\text{-}id = teaches\text{-}id}\left(\sigma_{dept = phy}(instructor \times teaches)\right)\right)$$

Rename:

$\rho_x(E)$

| ρ = rename notation
| x = new-name
| E = old name.

- Find Highest salary?

ID	Salary
1	45
2	50
3	40

- instructor × d. instructor

i.ID	i.Salary	d.ID	d.Salary
1	45	1	45
1	45	2	50
1	45	3	40
2	50	1	45
2	50	2	50
2	50	3	40
3	40	1	45
3	40	2	50
3	40	3	40

$\pi_{i.salary<d.salary}(\rho_i(instructor) \times \rho_d(d.instructor))$

$\pi_{salary}(instructor)$

$\pi_{salary}(instructor)$ $(\rho_i(instructor) \times \rho_d(d.instructor))$

- set-diff = $\pi_{i.salary<d.salary}$

2 (a)

(i) $\rho_{ship}(Boat)$

(ii) $\pi_{name, rating}(Sailor)$ $(\sigma_{sid=id}(Sailor \bowtie Reservation))$

(iii) dar.name G AVG(rating)

~~(iv) σ $\sigma_{color=red}$ (Sailor ⋈ (Sailor $\sigma_{sid=id}$~~

~~(iv) π_{name} ($\sigma_{color=red}$~~

(iv) $\pi_{name}(\sigma_{color=red}.(Sailor \bowtie_{sid=id} Reservation) \bowtie_{bid=id} Boat)$

(iv) $\pi_{name}(\sigma_{color=red}(Sailor \bowtie_{sid=id} Reservation) \bowtie_{bid=id} Boat)$

Our Mid

2 (a) $\pi_{productname}$ (Products) $\frown$ ~~(Products ⋈ sale)~~) A.

~~(a) $\pi_{productname}$ ($\sigma_{saledate}$ >= '2023-08-1'.~~

~~π pro~~

(b) $\pi_{productname}$ ($\sigma_{saledate >= '2023-08-01' \wedge saledate <= '2023-08-31'}$ (product ⋈ sale))

(c) category_name G sum (price*qty) (categories ⋈ sale).

4. (a) SELECT
product code, product.name, description
FROM
products.

(b) SELECT.
product code, product name
FROM
products
WHERE productnam LIKE '19._ _%r%'

(c) SELECT. ordernumber, ~~status~~. SUM (qty * priceeach),
FROM
orders Left JOIN orderdetails USING (ordernumber).
GROUP BY status.

$\pi_{i.salary < d.salary} (\rho_i (instructor) \times \rho_d (d.instructor))$

$\pi_{salary} (instructor)$

$\pi_{salary} (instructor)$

• set-diff - $\pi_{i.salary < d.salary} (\rho_i (instructor) \times \rho_d (d.instructor))$

2 (a)

(i)

$\rho_{ship} (Boat)$

(ii) $\pi_{name, rating} (Sailor)$

(iii) $_{dar.name} G_{AVG(rating)} (\sigma_{sid=id} (Sailor \bowtie Reservation))$

~~(iv) $\sigma_{color=red}$ (Sailor ⋈~~

~~(iv) π_{name} ($\sigma_{color=red}$ (Sailor $\sigma_{sid=id}$~~

(iv) $\pi_{name} (\sigma_{color=red} (Sailor \bowtie_{sid=id} Reservation) \bowtie_{bid=id} Boat)$

(iv) $\pi_{name} (\sigma_{color=red} (Sailor \bowtie_{sid=id} Reservation) \bowtie_{bid=id} Boat)$

Dure Mid

2.(a) $\pi_{productname}$ (Products)

~~(a) $\pi_{productname}$ ($\sigma_{saledate}$ > '2023-08-1'.~~ (Products ⋈ sale)) A

~~π pro~~

(b) $\pi_{productname}$ ($\sigma_{saledate}$ >= '2023-08-01' ∧ saledate <= '2023-08-31' (Product ⋈ sale))

(c) categorey_nm G sum (price*qty) (categorries ⋈ sale).

4.(a) SELECT

productcode, product.name, description

FROM

products.

(b) SELECT

product code, product name

FROM

products

WHERE productnam LIKE '19._ _%r%'

(c) SELECT. ordernumber, ~~status~~, SUM (qty * priceeach),

FROM

orders Left JOIN orderdetails USING (ordernum

GROUP BY status.

Winter 2022

5 (b) (i.) SELECT name, rating, age

FROM. sailor

WHERE

rating >5 AND age ~~= 20 AND~~ between

20 and 30.

(ii) SELECT name.

FROM sailor

WHERE.

name LIKE 'a--%d'.

(iii) SELECT

b.name, rating, day

From sailor Left JOIN Reservation on R.SID = S.SID.

Left JOIN Boat ON R.bid = b.id.

WHERE rating >10 OR day <'2021-06-26'.

(iv)

Stored Procedure;

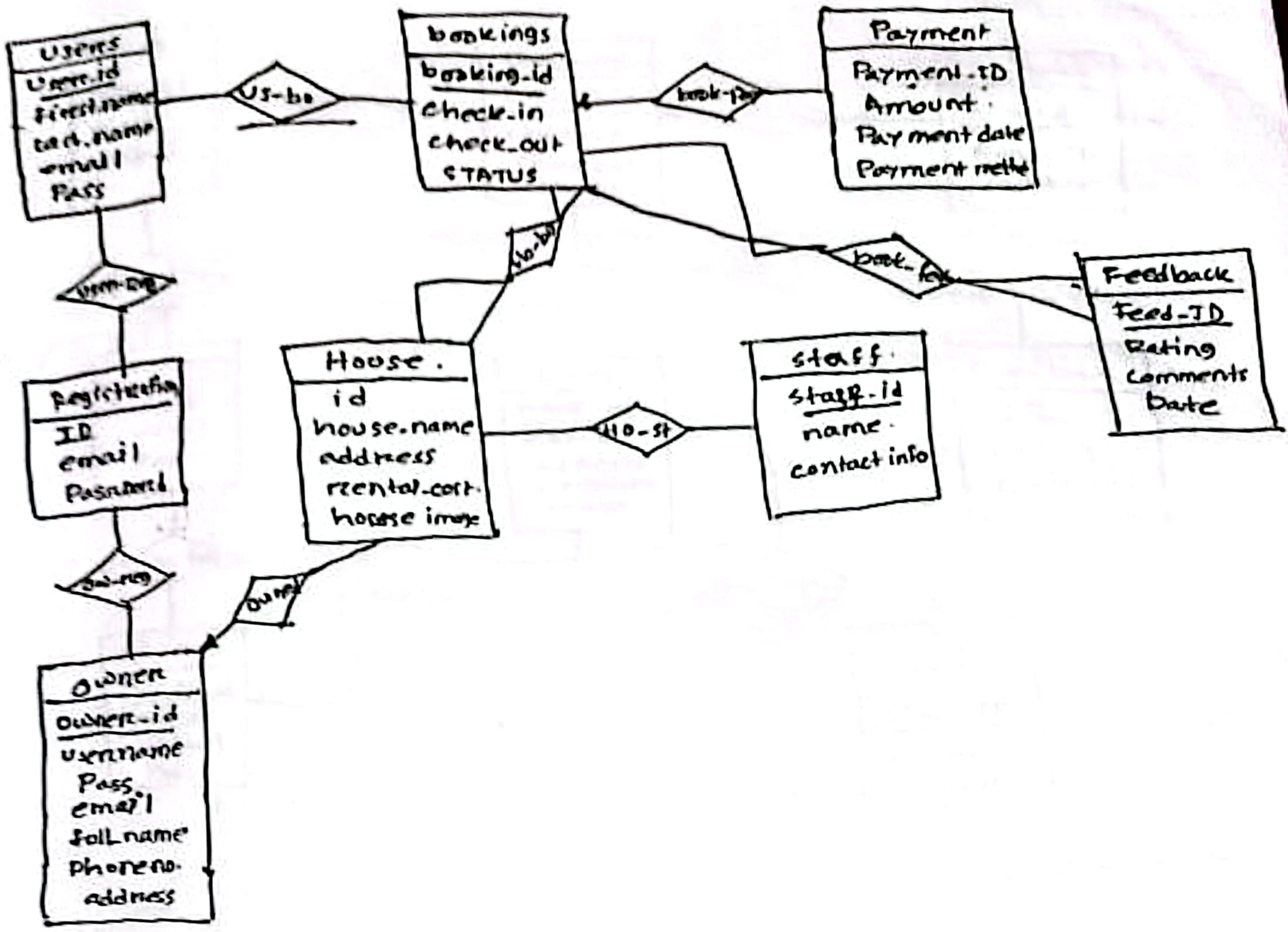
```
DELIMITER $$
CREATE PROCEDURE StatusDetails (
    IN  P_cust_name  VARCHAR(50);
    IN  P_sname  VARCHAR(50))

BEGIN
    DECLARE C.ID INT;
    DECLARE S.ID INT.

    SELECT sid INTO S.ID WHERE s.name = P_s name.
    SELECT cid INTO C-ID WHERE cust_name = P_cust_name.

    SELECT
    SUM(CASE WHEN status = 'shipped' THEN qty ELSE 0 END)
                                                        shipped
    SUM(CASE WHEN status = 'canceled' THEN qty ELSE 0 END)
                                                        canceled.
    SUM (CASE WHEN status = 'disputed' THEN qty ELSE 0 END)
                                                        disputed.
    FROM Orders
    WHERE cust_no = C-ID AND sid = S_ID
END $$
DELIMITER .
```

```
DELIMITER $$.
CREATE FUNCTION CalculateGrade (fmark INT)
 RETURNS VARCHAR(5)
 DETERMINISTIC
 BEGIN
 DECLARE grade VARCHAR(5)
 IF fmark > 89 THEN SET grade = 'A+';
 ELSE IF fmark ≥ 80 AND fmark ≤ 89 THEN SET grade='A';
 ELSEIF fmark ≥ 70 AND fmark ≤ 79 THEN SET grade= 'B+';
 ELSEIF fmark ≥ 60 AND fmark ≤ 69 THEN SET grade = 'B'
 ELSE SET grade = 'F'
 RETURN (grade);
 END
$$ DELIMETER.
```

(b) (i)
```
SELECT name, address
    FROM customers.
```

(ii)
```
SELECT order no. , orderdate
    FROM Orders
   WHERE
       orderdate BETWEEN '2020-12-01' AND
                         '2020-12-31'
```

(iii)
```
SELECT customer name, order no, ship_date
 FROM. CUSTOMERS LEFT JOIN Orders UsinG
                     Customer. custid = Order.cid
 LEFT JOIN Shipments using Orders. orderno =
 GROUP BY customer no                Shipment oid
```



users (user_id, firstname . . .)

us-bo (User_id, booking - id)

bookings (booking-id, - - -)

book-Pay (Payment-ID)

Payment (Payment_ID, - - -)

Registration (ID, email, pass)

User-reg (user-id, ID)

House (ID - - -) owner-reg (ID, owner-id)

Hou-book (booking_id, id)

staff (staff-id . - - -)

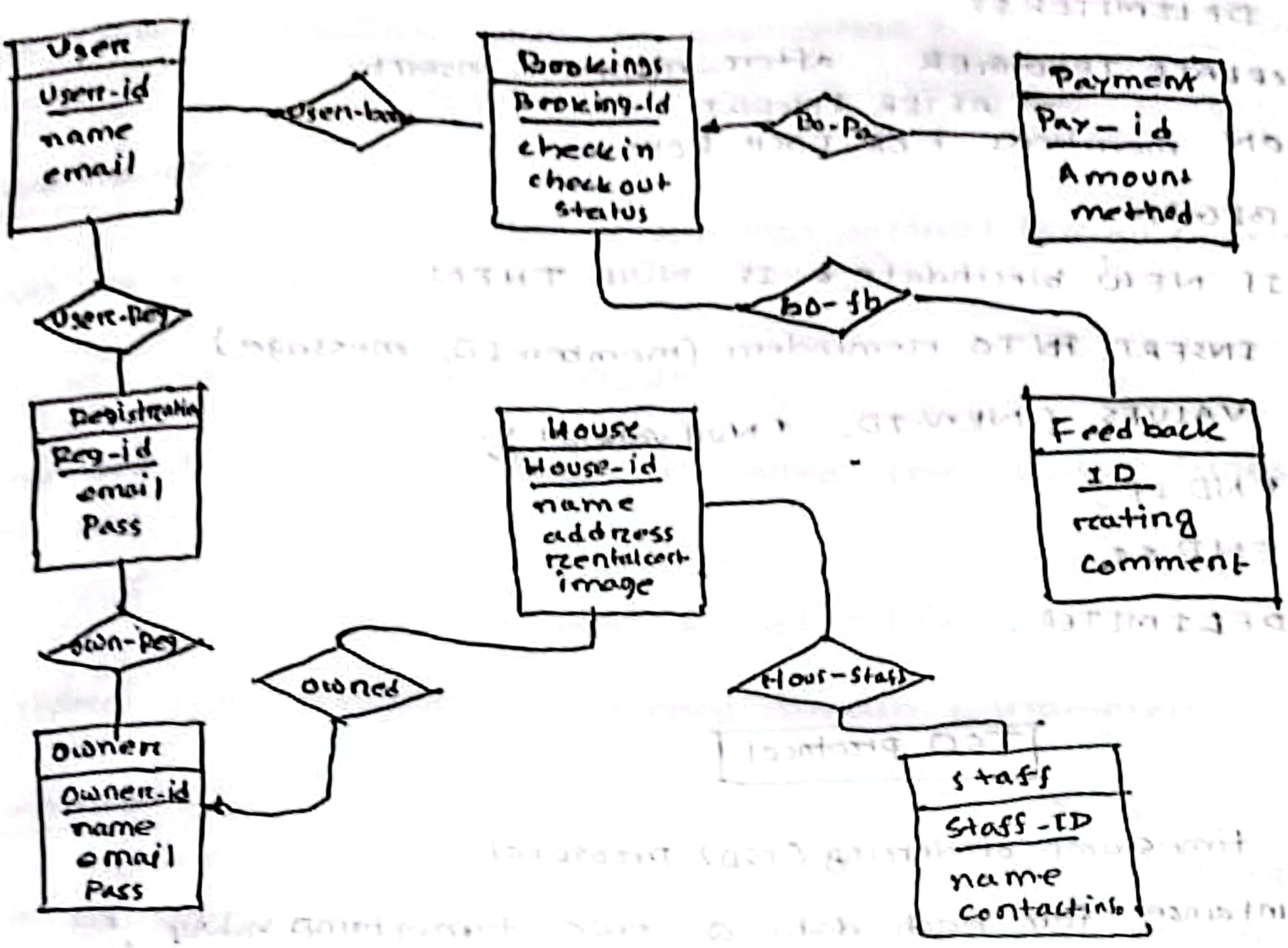
Ho-st (id, staff-id)

Feedback (Fee-ID, . . .

book-feed (booking_id, feed-id)

Owner (owner_id, . . .

Owned (id)


User
User-id
name
email
User-bo
Bookings
Booking-id
checkin
checkout
status
Bo-Pa
Payment
Pay-id
Amount
method
bo-fb
Feedback
ID
rating
comment
User-Reg
Registration
Reg-id
email
Pass
own-Reg
Owner
Owner-id
name
email
Pass
owned
House
House-id
name
address
rentalcost
image
Hous-Staff
Staff
Staff-ID
name
contactinfo

Summer 2021:

4(b)

```
DELIMITER $$
CREATE TRIGGER after_member_insert
AFTER INSERT
ON members FOR EACH ROW
BEGIN
IF NEW.birthdate IS NULL THEN
INSERT INTO reminders (memberID, message)
VALUES (NEW.ID, 'Null value');
END IF;
END $$
DELIMITER ;
```

TSO protocol

The timestamp ordering (TSO) protocol maintains for each data Q two timestamp values.

- W-timestamp(Q) is the largest timestamp of any transaction that write(Q) successfully.
- R-timestamp(Q) is the largest - - that read(Q) successfully

Imposes rules on read and write operation to ensure that-

- Any conflicting operations are executed in timestamp order.
- Out of order operations cause transaction rollback.

Part-A: (MATH)

Q. Define Laplace transform with its application?

Solve:

Laplace transform:

Let $f(t)$ be a function defined for all positive values of t. then,

$$F(s) = \int_0^{\alpha} e^{-st} f(t)\,dt$$

provided the integral exist, is called the Laplace transform of $f(t)$ is denoted by,

$$\mathcal{L}\{f(t)\} = \int_0^{\alpha} e^{-st}.\ f(t)dt = F(s)$$

where s is complex frequency domain parameter.

Application:

Laplace transform helps in solving differential equation with boundary values, without finding the general solutions and values of arbitrary constant.

Formula:

1. $\mathcal{L}\{1\} = \frac{1}{s}$
2. $\mathcal{L}\{t\} = \frac{1}{s^2}$
3. $\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}$
4. $\mathcal{L}\{e^{at}\} = \frac{1}{s-a}$
5. $\mathcal{L}\{\sin at\} = \frac{a}{s^2+a^2}$
6. $\mathcal{L}\{\cos at\} = \frac{s}{s^2+a^2}$
7. $\mathcal{L}\{\sinh at\} = \frac{a}{s^2-a^2}$
8. $\mathcal{L}\{\cosh at\} = \frac{s}{s^2-a^2}$

Q. If $F(t) = 7e^{2t} + 9e^{-2t} + 5\cos t + 7t^3 + 5\sin 3t + 2$. Find $\mathcal{L}\{F(t)\} = ?$

Solution:

$$\mathcal{L}\{F(t)\} = 7\mathcal{L}\{e^{2t}\} + 9\mathcal{L}\{e^{-2t}\} + 5\mathcal{L}\{\cos t\} + 7\mathcal{L}\{t^3\} + 5\mathcal{L}\{\sin 3t\} + 2\mathcal{L}\{1\}.$$

$$= 7\frac{1}{s-2} + 9\cdot\frac{1}{s+2} + 5\frac{s}{s^2+1} + 7\cdot\frac{6}{s^4} + 5\frac{3}{s^2+9} + \frac{2}{s}$$

$$= \frac{7}{s-2} + \frac{9}{s+2} + \frac{5s}{s^2+1} + \frac{42}{s^4} + \frac{15}{s^2+9} + \frac{2}{s}$$

Ans

Q. If $F(t) = 3t^4 - 2t^3 + 4e^{-3t} + 3\cos t - 2\sin 5t$. $\mathcal{L}\{F(t)\} = ?$

Solve:

$$\mathcal{L}\{F(t)\} = 3\mathcal{L}\{t^4\} - 2\mathcal{L}\{t^3\} + 4\mathcal{L}\{e^{-3t}\} + 3\mathcal{L}\{\cos t\} - 2\mathcal{L}\{\sin 5t\}$$

$$= \frac{72}{s^5} - \frac{12}{s^4} + \frac{4}{s+3} + \frac{3s}{s^2+1} - \frac{10}{s^2+25}$$

Ans

Q. Prove that $\mathcal{L}\{1\} = \frac{1}{s}$.

proof:

we know,

$$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\, dt$$

$$\therefore \mathcal{L}\{1\} = \int_0^\infty e^{-st}\cdot 1\cdot dt$$

$$= \left[\frac{e^{-st}}{-s}\right]_0^\infty$$

$$= -\frac{1}{s}\left[e^{-\infty} - 1\right]$$

$$= -\frac{1}{s}\left[0 - 1\right]$$

$$= \frac{1}{s} \text{ (proved)}$$

Prove that $\mathcal{L}\{t\} = \frac{1}{s^2}$.

Proof:

We know,

$$\mathcal{L}\{f(t)\} = \int_0^{\alpha} e^{-st} f(t)\, dt.$$

$$\Rightarrow \mathcal{L}\{t\} = \int_0^{\alpha} e^{-st} \cdot t\, dt.$$

$$= \left[t \int e^{-st} \cdot dt - \left\{ \int \frac{d}{dt} \cdot (t) \int e^{-st} dt \right]_0^{\alpha} dt$$

$$= t \left[\frac{e^{-st}}{-s} \right]_0^{\alpha} + \frac{1}{s} \int_0^{\alpha} e^{-st} \cdot dt$$

$$= \frac{t}{-s} \left[e^{-\alpha} - 1 \right] + \frac{1}{s} \left[\frac{e^{-st}}{-s} \right]_0^{\alpha}$$

$$= \frac{t}{s} + \frac{1}{s^2}$$

$$= \left[t \cdot \frac{e^{-st}}{-s} \right]_0^{\alpha} + \frac{1}{s} \int_0^{\alpha} e^{-st} \cdot dt$$

$$= 0 + \frac{1}{s} \left[\frac{e^{-st}}{-s} \right]_0^{\alpha}$$

$$= -\frac{1}{s^2} \cdot \left[e^{\alpha} - 1 \right]$$

$$= \frac{1}{s^2} \text{ (proved)}.$$

$\boxed{Q}$ Prove that $\mathcal{L}\{e^{at}\} = \frac{1}{s-a}$

Proof:

We know,

$$\mathcal{L}\{f(t)\} = \int_0^{\alpha} e^{-st} . f(t)\, dt .$$

$$\Rightarrow \mathcal{L}\{e^{at}\} = \int_0^{\alpha} e^{-st} . e^{at} . dt .$$

$$= \int_0^{\alpha} e^{-st+at} . dt .$$

$$= \int_0^{\alpha} e^{-t(s-a)} . dt .$$

$$= \left[\frac{e^{-(s-a)t}}{-(s-a)} \right]_0^{\alpha}$$

$$= -\frac{1}{(s-a)} \left[e^{-\alpha} - 1 \right]$$

$$= \frac{1}{(s-a)} \quad \text{(proved)}$$

$\boxed{Q.}$ prove that $\mathcal{L}\{e^{-2t}\} = \frac{1}{s+2}$

Proof:

we know,

$$\mathcal{L}\{f(t)\} = \int_0^{\alpha} e^{-st} f(t)\, dt$$

$$\mathcal{L}\{e^{-2t}\} = \int_0^{\alpha} e^{-st} . e^{-2t}\, dt .$$

$$= \int_0^{\alpha} e^{-st-2t} . dt$$

$$= \int_0^{\alpha} e^{-t(s+2)} . dt$$

$$= \left[\frac{e^{-t(s+2)}}{-(s+2)} \right]_0^{\alpha}$$

$$= -\frac{1}{(s+2)} \left[e^{-\alpha} - 1 \right]$$

$$= \frac{1}{s+2} \quad \text{(proved)}$$

5) prove that, $\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}$, Here $n = 0, 1, 2, 3$.

Proof:
we know,

$$\mathcal{L}\{f(t)\} = \int_0^\alpha e^{-st} \cdot f(t)\, dt$$

$$\mathcal{L}\{t^n\} = \int_0^\alpha e^{-st} \cdot t^n \cdot dt.$$

let,
$st = u$
$\Rightarrow t = \frac{u}{s}$

$s \cdot \frac{d}{dt}(t) = \frac{du}{dt}$
$\Rightarrow s = \frac{du}{dt}$
$\Rightarrow dt \cdot s = du.$
$\Rightarrow dt = \frac{du}{s}$

Limit.
when, $t = 0$, $u = 0$.
when, $t = \alpha$, $u = \alpha$.

Now,

$$\mathcal{L}\{t^n\} = \int_0^\alpha e^{-u} \cdot \left(\frac{u}{s}\right)^n \cdot \frac{du}{s}.$$

$$= \int_0^\alpha e^{-u} \frac{u^n}{s^n} \cdot \frac{du}{s}$$

$$= \frac{1}{s^{n+1}} \int_0^\alpha e^{-u} u^{(n+1)-1} \cdot du.$$

$$= \frac{1}{s^{n+1}} \Gamma n+1 \qquad \left[\int_0^\alpha e^{-u} u^{n-1} \cdot d = \Gamma n\right]$$

$$= \frac{n!}{s^{n+1}} \text{ (proved)}$$

(b) Prove that, $\mathcal{L}\{\sin at\} = \frac{a}{s^2+a^2}$

Proof:

We know,

$$\mathcal{L}\{f(t)\} = \int_0^\alpha e^{-st} . f(t)\, dt$$

$$\mathcal{L}\{\sin at\} = \int_0^\alpha e^{-st} \sin at \cdot dt$$

$$\sin at = \frac{e^{iat} + e^{-iat}}{2i}$$

Now,

$$\mathcal{L}\{\sin at\} = \int_0^\alpha e^{-st} . \left(\frac{e^{iat} - e^{-iat}}{2i}\right) dt$$

$$= \frac{1}{2i}\int_0^\alpha e^{-st} . e^{iat} . dt - \frac{1}{2i}\int_0^\alpha e^{-st} . e^{-iat} . dt$$

$$= \frac{1}{2i}\int_0^\alpha e^{-st+iat} . dt - \frac{1}{2i}\int_0^\alpha e^{-st-iat} . dt$$

$$= \frac{1}{2i}\int_0^\alpha e^{-t(s-ia)} . dt - \frac{1}{2i}\int_0^\alpha e^{-t(s+ia)} . dt$$

$$= \frac{1}{2i}\left[\frac{e^{-t(s-ia)}}{-(s-ia)}\right]_0^\alpha - \frac{1}{2i}\left[\frac{e^{-t(s+ia)}}{-(s+ia)}\right]_0^\alpha$$

$$= \frac{1}{2i(s-ia)} + \frac{1}{2i(s+ia)}$$

$$= \frac{\cancel{s} + ia - \cancel{s} + ia}{2i(s-ia)(s+ia)}$$

$$= \frac{a}{s^2 - (ia)^2} = \frac{a}{s^2+a^2} \quad [i^2 = -1]$$

(proved)

6 Prooved that $\mathcal{L}\{\cos at\} = \frac{s}{s^2+a^2}$

Proof-

we know,

$$\mathcal{L}\{f(t)\} = \int_0^\infty \cancel{f(t)\,dt}\cdot e^{-st}\cdot f(t)\,dt$$

$$\mathcal{L}\{\cos at\} = \int_0^\infty \cancel{\cos at}\; e^{-st}\cdot \cos at\,dt$$

$$\cos at = \frac{e^{iat} + e^{-iat}}{2}$$

Now,

$$\mathcal{L}\{\cos at\} = \int_0^\infty e^{-st}\cdot \frac{e^{iat} + e^{-iat}}{2}\cdot dt$$

$$= \frac{1}{2}\int_0^\infty e^{-t(s-ia)}\cdot dt + \frac{1}{2}\int_0^\infty e^{-t(s+ia)}\cdot dt.$$

$$= \frac{1}{2}\left[\frac{e^{-t(s-ia)}}{-(s-ia)}\right]_0^\infty + \frac{1}{2}\left[\frac{e^{-t(s+ia)}}{-(s+ia)}\right]_0^\infty$$

$$= \frac{1}{2(s-ia)} + \frac{1}{2(s+ia)}$$

$$= \frac{s+\cancel{ia}+s-\cancel{ia}}{2(s+ia)(s-ia)}$$

$$= \frac{s}{s^2+a^2} \quad \text{(proved)}$$

Q. Prove that $\mathcal{L}\{\sinh at\} = \frac{a}{s^2-a^2}$.

Proof:

We know,

$$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\, dt$$

$$\Rightarrow \mathcal{L}\{\sinh at\} = \int_0^\infty e^{-st} \sinh at \cdot dt$$

$$\sinh at = \frac{e^{at} - e^{-at}}{2}$$

Now,

$$\mathcal{L}\{\sinh at\} = \int_0^\infty e^{-st} \cdot \frac{e^{at} - e^{-at}}{2} \cdot dt$$

$$= \frac{1}{2}\int_0^\infty e^{-t(s-a)} \cdot dt - \frac{1}{2}\int_0^\infty e^{-t(s+a)} \cdot dt$$

$$= \frac{1}{2}\left[\frac{e^{-t(s-a)}}{-(s-a)}\right]_0^\infty - \frac{1}{2}\left[\frac{e^{-t(s+a)}}{-(s+a)}\right]_0^\infty$$

$$= \frac{1}{2(s-a)} - \frac{1}{2(s+a)}$$

$$= \frac{s+a - s+a}{2(s-a)(s+a)}$$

$$= \frac{a}{s^2-a^2} \quad \text{(proved)}$$

(8) prove that $\mathcal{L}\{\cosh at\} = \dfrac{s}{s^2-a^2}$

Proof:

we know,

$$\mathcal{L}\{f(t)\} = \int_0^\infty \cancel{f(t)}\; e^{-st}\cdot f(t)\,dt$$

$$\mathcal{L}\{\cosh at\} = \int_0^\infty e^{-st} \cosh at \; dt.$$

$$\cosh at = \frac{e^{at}+e^{-at}}{2}$$

Now,

$$\mathcal{L}\{\cosh at\} = \int_0^\infty e^{-st}\cdot \frac{e^{at}+e^{-at}}{2}\cdot dt$$

$$= \frac{1}{2}\int_0^\infty e^{-t(s-a)}\,dt + \frac{1}{2}\int_0^\infty e^{-t(s+a)}\cdot dt$$

$$= \frac{1}{2(s-a)} + \frac{1}{2(s+a)}$$

$$= \frac{s+a+s-a}{2(s-a)(s+a)}$$

$$= \frac{s}{s^2-a^2} \quad \text{(proved)}$$

② Derive first translation theorem or shifting property of Laplace transform

Proof:

We know,

$$\mathcal{L}\{f(t)\} = F(s)$$

then $\mathcal{L}\{e^{at}. f(t)\} = F(s-a)$

$$\therefore \mathcal{L}\{e^{at}. f(t)\} = \int_0^\infty e^{-st}. e^{at}. f(t)\, dt.$$

$$= \int_0^\infty e^{-st+at} f(t)\, dt.$$

$$= \int_0^\infty e^{-t(s-a)}. f(t)\, dt$$

$$= \int_0^\infty e^{-ut}. f(t)\, dt \qquad [(s-a) = u]$$

$$= F(u) = F(s-a)$$

(proved)

Problem:

Find the laplace transform of

(i) $e^{-2t}. \sin 4t$ (ii) $e^{-t}. \cos 2t$ (iii) $e^{2t} \sin at$

(i) $e^{-2t}. \sin 4t$.

$$\mathcal{L}\{\sin 4t\} = \frac{4}{s^2+16}.$$

$$\therefore \mathcal{L}\{e^{-2t}. \sin 4t\} = \frac{4}{(s+2)^2+16}$$

Ans

(ii) $e^{-t}. \cos 2t$.

$$\mathcal{L}\{\cos 2t\} = \frac{s}{s^2+4}$$

$$\mathcal{L}\{e^{-2t}. \cos 2t\} = \frac{s+2}{(s+2)^2+4}$$

Ans

(iii) $e^{2t} \cdot \sin at$.

$$\mathcal{L}\{\sin at\} = \frac{a}{s^2+a^2}$$

$$\therefore \mathcal{L}\{e^{2t} \cdot \sin at\} = \frac{a}{(s-2)^2+a^2}$$

Q) Derive the change of scale property theorem?

Proof:

we know,

$$\mathcal{L}\{f(t)\} = F(s)$$

$$\Rightarrow \mathcal{L}\{f(at)\} = \frac{1}{a} F\left(\frac{s}{a}\right)$$

we know,

$$\mathcal{L}\{f(t)\} = F(s) = \int_0^\infty e^{-st} \cdot f(t)\, dt$$

$$\mathcal{L}\{f(at)\} = \int_0^\infty e^{-st} f(at)\, dt$$

let,

$at = u$

$\Rightarrow t = \frac{u}{a}$

$\Rightarrow dt = \frac{du}{a}$

Now,

$$\mathcal{L}\{f(at)\} = \frac{1}{a}\int_0^\infty e^{-\frac{s}{a} \cdot u} \cdot f(u)\, du$$

$$= \frac{1}{a} F\left(\frac{s}{a}\right)$$

proved.

Q Using shifting property find the L.T. of

(i) sin 3t.

$$\mathcal{L}\{\sin t\} = \frac{1}{s^2+1}$$

$$\mathcal{L}\{\sin 3t\} = \frac{1}{3}\frac{1}{\left(\frac{s}{3}\right)^2+1}$$

$$= \frac{1}{3}\frac{1}{\frac{s^2}{9}+1}$$

$$= \frac{1}{3}\frac{1}{\frac{s^2+9}{9}}$$

$$= \frac{1}{\cancel{3}} \times \frac{\cancel{9}\,3}{s^2+9} = \frac{3}{s^2+9}$$

(ii) cos 3t.

$$\mathcal{L}\{\cos t\} = \frac{s}{s^2+1}$$

$$\mathcal{L}\{\cos 3t\} = \frac{1}{3} \cdot \frac{\frac{s}{3}}{\frac{s^2}{9}+1}$$

$$= \frac{1}{\cancel{3}} \cdot \frac{s}{\cancel{3}} \times \frac{\cancel{9}\,3}{s^2+9}$$

$$= \frac{s}{s^2+9}$$

Linearity Property:

Q Find $\mathcal{L}\{3t^2e^{2t} - 6t^3 - \sin 2t + \cosh 3t\}$.

Solve:

$$3\mathcal{L}\{t^2 \cdot e^{2t}\} - 6\mathcal{L}\{t^3\} - \mathcal{L}\{\sin 2t\} + \mathcal{L}\{\cosh 3t\}$$

$$= 3\frac{2}{(s-2)^3} - 6\cdot\frac{6}{s^4} - \frac{2}{s^2+4} + \frac{s}{s^2-9}$$

$$= \frac{6}{(s-2)^3} - \frac{36}{s^4} - \frac{2}{s^2+4} + \frac{s}{s^2-9}$$ Ans

Q Find $\mathcal{L}\{3e^{2t}\cdot\sin 3t - 4\cos 4t\cdot e^t - t^3\cdot e^t + t^2 + \cosh 2t - 2\}$

Solve:

$$3\mathcal{L}\{e^{2t}\cdot\sin 3t\} - 4\mathcal{L}\{\cos 4t\cdot e^t\} - \mathcal{L}\{t^3\cdot e^t\} + \mathcal{L}\{t^2\} + \mathcal{L}\{\cosh 2t\} - 2\mathcal{L}\{1\}$$

$$= 3\cdot\frac{3}{(s-2)^2+9} - 4\cdot\frac{(s-1)}{(s-1)^2+16} - \frac{6}{(s-1)^4} + \frac{2}{s^3} + \frac{s}{s^2-4} - \frac{2}{s}$$

Multiplication by power of t.

$$\mathcal{L}\{F(t)\cdot t^n\} = (-1)^n\frac{d^n}{ds^n}\cdot[F(s)] \quad \text{where, } n = 0, 1, 2, 3, -$$

Q. Find the laplace transform of $\mathcal{L}\{t\sin at\}$

Solve:

Given, $\mathcal{L}\{t\sin at\}$

$$\mathcal{L}\{\sin at\} = \frac{a}{s^2+a^2}$$

$$\mathcal{L}\{t\cdot\sin at\} = (-1)^1\cdot\frac{d}{ds}\left(\frac{a}{s^2+a^2}\right)$$

$$= -\frac{(s^2+a^2)\cdot 0 - a\,2s}{(s^2+a^2)^2}$$

$$= \frac{2as}{(s^2+a^2)^2}$$ ans

Q Find the Laplace transform of $\mathcal{L}\{t^2 \sin at\}$.

Solve:

Given,

$\mathcal{L}\{t^2 \sin at\}$

$$\mathcal{L}\{\sin at\} = \frac{a}{s^2+a^2}$$

$$\therefore \mathcal{L}\{t^2 \sin at\} = (-1)^2 \cdot \frac{d^2}{ds^2}\left[\frac{a}{s^2+a^2}\right]$$

$$= \frac{d}{ds}\left[\frac{d}{ds}\left(\frac{a}{s^2+a^2}\right)\right]$$

$$= -\frac{d}{ds} \frac{2as}{(s^2+a^2)^2}$$

$$= -\frac{(s^2+a^2)^2 \cdot 2a - 2as \cdot 2(s^2+a^2) \cdot 2s}{(s^2+a^2)^4}$$

$$= -\frac{(s^2+a^2)\{(s^2+a^2) \cdot 2a - 8as^2\}}{(s^2+a^2)^4}$$

$$= -\frac{2as^2 + 2a^3 - 8as^2}{(s^2+a^2)^3}$$

$$= \frac{6as^2 - 2a^3}{(s^2+a^2)^3}$$ ans

2. Find the laplace transform of $\mathcal{L}\{t^2 \sin 2t\}$.

Solve:

$$\mathcal{L}\{\sin 2t\} = \frac{2}{s^2+4}$$

$$\therefore \mathcal{L}\{t^2 \sin 2t\} = (-1)^2 \cdot \frac{d^2}{ds^2}\left(\frac{2}{s^2+4}\right)$$

$$= \frac{d}{ds}\left[\frac{d}{ds}\left(\frac{2}{s^2+4}\right)\right]$$

$$= \frac{d}{ds} \frac{(s^2+4)\cdot 0 - 2\cdot 2s}{(s^2+4)^2}$$

$$= -\frac{d}{ds}\left(\frac{4s}{(s^2+4)^2}\right)$$

$$= -\frac{(s^2+4)^2 \cdot 4 - 4s\, 2(s^2+4)\cdot 2s}{(s^2+4)^4}$$

$$= -\frac{(s^2+4)\{(s^2+4)\cdot 4 - 16s^2\}}{(s^2+4)^4}$$

$$= -\frac{4s^2+16-16s^2}{(s^2+4)^3}$$

$$= \frac{12s^2-16}{(s^2+4)^3}$$ Ans

Find the laplace transform of $\mathcal{L}\{t^2 \sin 2t\}$

Solve

$\mathcal{L}\{\sin 2t\} = \frac{2}{s^2+4}$

$\therefore \mathcal{L}\{t^2 \sin 2t\} = (-1)^2 \frac{d^2}{ds^2}\left(\frac{2}{s^2+4}\right)$

$$= \frac{d}{ds}\left[\frac{d}{ds}\left(\frac{2}{s^2+4}\right)\right]$$

$$= \frac{d}{ds} \frac{(s^2+4) \cdot 0 - 2 \cdot 2s}{(s^2+4)^2}$$

$$= -\frac{d}{ds}\left(\frac{4s}{(s^2+4)^2}\right)$$

$$= -\frac{(s^2+4)^2 \cdot 4 - 4s \; 2(s^2+4) \cdot 2s}{(s^2+4)^4}$$

$$= -\frac{(s^2+4)\{(s^2+4) \cdot 4 - 16s^2\}}{(s^2+4)^4}$$

$$= -\frac{4s^2+16-16s^2}{(s^2+4)^3}$$

$$= \frac{12s^2-16}{(s^2+4)^3}$$

Ans

] Find the laplace transform of $\mathcal{L}\{t^2 \cdot \sin 2t\}$

Solve

$\mathcal{L}\{\sin 2t\} = \frac{2}{s^2+4}$

$\therefore \mathcal{L}\{t^2 \cdot \sin 2t\} = (-1)^2 \cdot \frac{d^2}{ds^2}\left(\frac{2}{s^2+4}\right)$

$= \frac{d}{ds}\left[\frac{d}{ds}\left(\frac{2}{s^2+4}\right)\right]$

$= \frac{d}{ds} \frac{(s^2+4)\cdot 0 - 2\cdot 2s}{(s^2+4)^2}$

$= -\frac{d}{ds}\left(\frac{4s}{(s^2+4)^2}\right)$

$= -\frac{(s^2+4)^2 \cdot 4 - 4s\ 2(s^2+4)\cdot 2s}{(s^2+4)^4}$

$= -\frac{(s^2+4)\{(s^2+4)\cdot 4 - 16s^2\}}{(s^2+4)^4}$

$= -\frac{4s^2+16-16s^2}{(s^2+4)^3}$

$= \frac{12s^2-16}{(s^2+4)^3}$ Ans

Q) Find LT of $\mathcal{L}\{t\cdot\cos at\}$.

Solve:

$$\mathcal{L}\{\cos at\} = \frac{s}{s^2+a^2}$$

$$\mathcal{L}\{t\cdot\cos at\} = (-1)^1 \frac{d}{ds}\left(\frac{s}{s^2+a^2}\right)$$

$$= -\frac{(s^2+a^2)1 - s\,2s}{(s^2+a^2)^2}$$

$$= -\frac{s^2+a^2-2s^2}{(s^2+a^2)^2}$$

$$= \frac{s^2-a^2}{(s^2+a^2)^2}$$

Ans

Q) Find the LT of $\mathcal{L}\{t\cdot\cos 3t\}$.

Solve:

$$\mathcal{L}\{\cos 3t\} = \frac{s}{s^2+9}$$

$$\mathcal{L}\{t\cdot\cos 3t\} = (-1)^1\cdot\frac{d}{ds}\left(\frac{s}{s^2+9}\right)$$

$$= -\frac{(s^2+9)\cdot 1 - s\cdot 2s}{(s^2+9)^2}$$

$$= -\frac{s^2+9-2s^2}{(s^2+9)^2}$$

$$= \frac{s^2-9}{(s^2+9)^2}$$

Ans

Q) show that $\mathcal{L}\{t^2 \cdot e^{3t}\} = \frac{2}{(s-3)^3}$

Solve:

$$\mathcal{L}\{e^{3t}\} = \frac{1}{s-3}$$

$$\mathcal{L}\{t^2 \cdot e^{3t}\} = (-1)^2 \cdot \frac{d^2}{ds^2}\left(\frac{1}{(s-3)}\right)$$

$$= \frac{d}{ds}\left[\frac{d}{ds}(s-3)^{-1}\right]$$

$$= -\frac{d}{ds}(s-3)^{-2}.$$

$$= 2(s-3)^{-3}.$$

$$= \frac{2}{(s-3)^3}$$

Ans

Q) show that $\mathcal{L}\{t^3 \cdot e^{-3t}\} = \frac{6}{(s+3)^4}$.

Solve:

$$\mathcal{L}\{e^{-3t}\} = \frac{1}{s+3}$$

$$\mathcal{L}\{t^3 \cdot e^{-3t}\} = (-1)^3 \cdot \frac{d^3}{ds^3}(s+3)^{-1}$$

$$= -\frac{d^2}{ds^2}\left[\frac{d}{ds}(s+3)^{-1}\right]$$

$$= \frac{d^2}{ds^2} \cdot (s+3)^{-2}.$$

$$= \frac{d}{ds}\left[\frac{d}{ds}(s+3)^{-2}\right]$$

$$= -2\frac{d}{ds}(s+3)^{-3}.$$

$$= \frac{6}{(s+3)^4}$$ (proved).

[5] Find the laplace transform of $t^2(\sin at + e^{at})$

Solve:

$\mathcal{L}\{\sin at\} = \frac{a}{s^2+a^2}$

$\mathcal{L}\{e^{at}\} = \frac{1}{s-a}$.

$$\therefore \mathcal{L}\{t^2(\sin at + e^{at})\} = (-1)^2 \frac{d^2}{ds^2}\left[\frac{a}{s^2+a^2} + \frac{1}{s-a}\right]$$

$$= \frac{d}{ds} \cdot \frac{d}{ds}\left[\frac{a}{s^2+a^2} + \frac{1}{s-a}\right].$$

$$= \frac{d}{ds}\left[\frac{(s^2+a^2)\cdot 0 - a\cdot 2s}{(s^2+a^2)^2} + \frac{1}{(s-a)^2}\right]$$

$$= -\frac{d}{ds}\left[\frac{2as}{(s^2+a^2)^2} - (s-a)^{-2}\right]$$

$$= -\frac{(s^2+a^2)^2 \cdot 2a - 2as \cdot 2(s^2+a^2)\cdot 2s}{(s^2+a^2)^4} + \frac{2}{(s-a)^3}$$

$$= -\frac{(s^2+a^2)\{(s^2+a^2)\cdot 2a - 8as^2\}}{(s^2+a^2)^4} + \frac{2}{(s-a)^3}.$$

$$= -\frac{2as^2 + 2a^3 - 8as^2}{(s^2+a^2)^3} + \frac{2}{(s-a)^3}.$$

$$= \frac{6as^2 - 2a^3}{(s^2+a^2)^3} + \frac{2}{(s-a)^3}$$

[6] Find the laplace transform of $\{e^{4t}.\cos 5t + t.\sin 2t\}$.

Solve:

$\mathcal{L}\{\cos 5t\} = \frac{s}{s^2+25}$

$\mathcal{L}\{e^{4t}.\cos 5t\} = \frac{(s-4)}{(s-4)^2+25}$

Again.

$\mathcal{L}\{\sin 2t\} = \frac{2}{s^2+4}$.

$\mathcal{L}\{t.\sin 2t\} = (-1)^1 \frac{d}{ds}\left(\frac{2}{s^2+4}\right)$.

$= - \frac{(s^2+4)\cdot 0 - 2\cdot 2s}{(s^2+4)^2}$.

$= \frac{4s}{(s^2+4)^2}$.

$\therefore \mathcal{L}\{e^{4t}.\cos 5t + t.\sin 2t\}$

$= \left\{\frac{(s-4)}{(s-4)^2+25} + \frac{4s}{(s^2+4)^2}\right\}$ Ans

[Q] Find the Laplace transform of 1st, 2nd and 3rd derivatives.

Solve:

We know

$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st}.f(t)\,dt$.

$\mathcal{L}\{f'(t)\} = \int_0^\infty e^{-st} f'(t)\,dt$.

$= \left[e^{-st}\int f'(t)\,dt - \int\left\{\frac{d}{dt}e^{-st}\int f'(t)\,dt\right\}dt\right]_0^\infty$

$= \left[e^{-st}.f(t)\right]_0^\infty + s\int_0^\infty e^{-st}f(t)\,dt$.

$= -f(0) + s\,\mathcal{L}\{f(t)\}$.

$\mathcal{L}\{f'(t)\} = s\mathcal{L}\{f(t)\} - f(0)$.

Now, replacing ~~putting~~ $f(t)$ by $f'(t)$ and $f'(t)$ by $f''(t)$

$$\mathcal{L}\{f''(t)\} = s\mathcal{L}\{f'(t)\} - f'(0)$$

$$= s\{s\mathcal{L}\{f(t)\} - f(0)\} - f'(0)$$

$$= s^2\mathcal{L}\{f(t)\} - sf(0) - f'(0).$$

Again,

$$\mathcal{L}\{f'''(t)\} = s^2\mathcal{L}\{f'(t)\} - sf'(0) - f''(0)$$

$$= s^2\{s\mathcal{L}\{f(t)\} - f(0)\} - sf'(0) - f''(0)$$

$$= s^3\mathcal{L}\{f(t)\} - s^2f(0) - sf'(0) - f''(0)$$

[Q] Prove that, $\mathcal{L}\left\{\int_0^t f(t)\,dt\right\} = \frac{F(s)}{s}$ where, $\mathcal{L}\{f(t)\} = F(s)$

Or, Find the L.T of $\int_0^t f(t)\,dt$.

Solve!

Let,

$$\phi(t) = \int_0^t f(t)\,dt$$

$$\phi(0) = 0$$

$$\phi'(t) = f(t)$$

We know, the Laplace transform of $\phi(t)$ then,

$$\mathcal{L}\{\phi'(t)\} = s\mathcal{L}\{\phi(t)\} - \phi(0)$$

$$\Rightarrow \mathcal{L}\{\phi'(t)\} = s\mathcal{L}\{\phi(t)\} - 0$$

$$\Rightarrow \mathcal{L}\{\phi(t)\} = \frac{\mathcal{L}\{\phi'(t)\}}{s}$$

$$\Rightarrow \mathcal{L}\left\{\int_0^t f(t)\,dt\right\} = \frac{\mathcal{L}\{f(t)\}}{s} = \frac{F(s)}{s}$$

(proved)

$\boxed{Q}$ If $\mathcal{L}\{f(t)\} = F(s)$, then show that $\mathcal{L}\left\{\frac{F(t)}{t}\right\} = \int_s^{\alpha} f(u)\,du$.

Solve:

Let,

$$G(t) = \frac{F(t)}{t}$$

$$\Rightarrow \mathcal{L}\{tG(t)\} = \mathcal{L}\{F(t)\}$$

$$\Rightarrow (-1)^1 \cdot \frac{d}{ds} g(s) = f(s)$$

$$\Rightarrow -d\,g(s) = -f(s)\,ds$$

Now integrating both side from α to s,

$$\int_{\alpha}^{s} d[g(s)] = -\int_{\alpha}^{s} f(s)\,ds$$

$$\Rightarrow [g(s)]_{\alpha}^{s} = \int_s^{\alpha} f(s)\,ds$$

$$\Rightarrow g(s) = \int_s^{\alpha} f(s)\,ds$$

$$\Rightarrow \mathcal{L}\{G(t)\} = \int_s^{\alpha} f(u)\,du$$

$$\Rightarrow \mathcal{L}\left\{\frac{F(t)}{t}\right\} = \int_s^{\alpha} f(u)\,du$$

(proved)

(Q) Prove that $\int_0^\infty \frac{F(t)}{t} \cdot dt = \int_0^\infty f(u)\, du$

Solve!

let,

$$G(t) = \frac{F(t)}{t}$$

$$\Rightarrow \mathcal{L}\{t \cdot G(t)\} = \mathcal{L}\{F(t)\}$$

$$\Rightarrow d \cdot [g(s)] = -f(s) \cdot ds$$

Now integrating on both side from α to s

$$\int_\alpha^s d\,[g(s)] = -\int_\alpha^s f(s)\, ds.$$

$$\Rightarrow g(s) = \int_s^\alpha f(s)\, ds$$

$$\Rightarrow \mathcal{L}\{G(t)\} = \int_s^\alpha f(u)\, du.$$

$$\Rightarrow \mathcal{L}\left\{\frac{F(t)}{t}\right\} = \int_s^\alpha f(u)\, du.$$

taking Laplace transform of both side.

$$\int_0^\alpha \frac{F(t)}{t} \cdot dt = \int_s^\alpha f(u)\, du.$$

If $s = 0$, we can write,

$$\int_0^\alpha \frac{F(t)}{t} \cdot dt = \int_0^\alpha f(u)\, du.$$

(Q) Show that, $\int_0^{\alpha} \frac{\sin at}{t} \cdot dt = \frac{\pi}{2}$.

Solve:

We know,

$$\int_0^{\alpha} \frac{F(t)}{\cancel{dt}\, t} \cdot dt = \int_0^{\alpha} f(u)\,du.$$

Here,

$$F(t) = \sin at$$

$$\Rightarrow \mathcal{L}\{F(t)\} = \mathcal{L}\{\sin at\}.$$

$$\Rightarrow f(s) = \frac{a}{s^2+a^2}.$$

$$\therefore f(u) = \frac{a}{u^2+a^2}.$$

Now,

$$\int_0^{\alpha} \frac{\sin at}{t} \cdot dt = \int_0^{\alpha} \frac{a}{u^2+a^2} \cdot du.$$

$$= a \int_0^{\alpha} \frac{1}{u^2+a^2}\, du.$$

$$= \cancel{a} \cdot \frac{1}{\cancel{a}} \left[\tan^{-1}\frac{u}{a}\right]_0^{\alpha}.$$

$$= \tan^{-1}\alpha - \tan^{-1} 0.$$

$$= \tan^{-1}\tan\frac{\pi}{2} - \tan^{-1}\tan 0.$$

$$= \frac{\pi}{2} \quad \text{(proved)}.$$

[Q.] Using Laplace transform prove that,

$$\int_0^\alpha \frac{\sin t}{t}\, dt = \frac{\pi}{2s}$$

Solve:

We know,

$$\int_0^\alpha \frac{\sin at}{t} \cdot dt = \frac{\pi}{2}.$$

Now,

$$\int_0^\alpha \frac{F(t)}{t} \cdot dt = \int_0^\alpha f(u)\, du$$

Here,

$F(t) = \sin t$.

$\Rightarrow \mathcal{L}\{F(t)\} = \mathcal{L}\{\sin t\}$.

$$f(s) = \frac{1}{s^2+1}$$

$$f(u) = \frac{1}{u^2+1}$$

$$\therefore \int_0^\alpha \frac{\sin t}{t} \cdot dt = \int_0^\alpha \frac{1}{u^2+1} \cdot du$$

$$= [\tan^{-1} u]_0^\alpha$$

$$= \tan^{-1}\alpha - \tan^{-1} 0$$

$$= \tan^{-1} \tan \frac{\pi}{2} - \tan^{-1} \tan 0$$

$$= \frac{\pi}{2}.$$

$$\therefore \int_0^\alpha \frac{\sin t}{t} \cdot dt = \frac{\pi}{2}$$

Taking L.T on both side,

$$\mathcal{L}\left\{\int_0^\alpha \frac{\sin t}{t} \cdot dt\right\} = \frac{\pi}{2}\mathcal{L}\{1\}.$$

$$\Rightarrow \int_0^\alpha \frac{\sin t}{t} \cdot dt = \frac{\pi}{2s} \quad \text{(proved)}$$

Find $\mathcal{L}\left\{\frac{e^{-at}-e^{-bt}}{t}\right\}$ and hence prove that $\int_0^{\infty}\frac{e^{-at}-e^{-bt}}{t}$

$= \ln\frac{b}{a}$.

Ans:

We know,

$$\mathcal{L}\left\{\frac{F(t)}{t}\right\} = \int_s^{\infty} f(u)\,du$$

Here,

$$F(t) = e^{-at}.$$

$$\Rightarrow \mathcal{L}\{F(t)\} = \mathcal{L}\{e^{-at}\}.$$

$$\Rightarrow f(s) = \frac{1}{s+a}.$$

$$f(u) = \frac{1}{u+a}.$$

$$\mathcal{L}\left\{\frac{e^{-at}}{t}\right\} = \int_s^{\infty}\frac{1}{u+a}\,du.$$

Now,

$$\mathcal{L}\left\{\frac{e^{-at}-e^{-bt}}{t}\right\}$$

$$= \mathcal{L}\left\{\frac{e^{-at}}{t}\right\} - \mathcal{L}\left\{\frac{e^{-bt}}{t}\right\}.$$

$$= \int_s^{\infty}\frac{1}{u+a}\,du - \int_s^{\infty}\frac{1}{u+b}\,du.$$

$$= \left[\ln(u+a)\right]_s^{\infty} - \left[\ln(u+b)\right]_s^{\infty}.$$

$$= \left[\frac{\ln \cancel{u}\left(1+\frac{a}{u}\right)}{\ln \cancel{u}\left(1+\frac{b}{u}\right)}\right]_s^{\infty}$$

$$= \frac{\ln\left(1+\frac{a}{\infty}\right)}{\ln\left(1+\frac{b}{\infty}\right)} - \frac{\ln\left(1+\frac{a}{s}\right)}{\ln\left(1+\frac{b}{s}\right)}$$

$$= \ln 1 - \frac{\ln\left(\frac{s+a}{s}\right)}{\ln\left(\frac{s+b}{s}\right)}$$

$$= 0 - \ln\left(\frac{s+a}{s+b}\right)$$

$$= -\left[\ln(s+a) - \ln(s+b)\right]$$

$$= \ln(s+b) - \ln(s+a)$$

$$= \ln\frac{(s+b)}{(s+a)}$$

Ans

2nd part:

L.H.S.

$$\int_0^{\alpha} \frac{e^{-at}}{t} \cdot dt - \int_0^{\alpha} \frac{e^{-bt}}{t} \cdot dt$$

$$= \int_0^{\alpha} \frac{1}{u+a} \cdot du - \int_0^{\alpha} \frac{1}{u+b} \cdot du$$

$$= \left[\frac{\ln u\left(1+\frac{a}{u}\right)}{\ln u\left(1+\frac{b}{u}\right)}\right]_0^{\alpha}$$

$$= \frac{\ln\left(1+\frac{a}{\alpha}\right)}{\ln\left(1+\frac{b}{\alpha}\right)} - \lim_{s \to 0} \frac{\ln\left(1+\frac{a}{s}\right)}{\ln\left(1+\frac{b}{s}\right)}$$

$$= \ln 1 - \lim_{s \to 0} \frac{\ln \frac{s+a}{s}}{\ln \frac{s+b}{s}}$$

$$= 0 - \ln\frac{a}{b}$$

$$= -\left[\ln a - \ln b\right] = \ln b - \ln a$$

$$= \ln\frac{b}{a} \text{ (proved)}$$

Q Let $F(t)$ have period $T>0$, so that $F(t+T) = F(t)$ then show that $\mathcal{L}\{F(t)\} = \dfrac{\int_0^T e^{-st} F(t)\,dt}{1-e^{-sT}}$.

Solⁿ:

We know,

$$\mathcal{L}\{F(t)\} = \int_0^\infty e^{-st} F(t)\,dt.$$

$$\mathcal{L}\{F(t)\} = \int_0^T e^{-st} F(t)\,dt + \int_T^{2T} e^{-st} F(t)\,dt + \int_{2T}^{3T} e^{-st} F(t)\,dt + \cdots$$

$$\therefore \mathcal{L}\{F(t)\} = T_1 + T_2 + T_3 + \cdots \quad \text{——— (1)}$$

Now

$$T_1 = \int_0^T e^{-st} F(t)\,dt.$$

$$T_2 = \int_T^{2T} e^{-st} F(t)\,dt.$$

Let,

$$t = U + T$$

$$\Rightarrow \frac{d}{dt}(t) = \frac{dU}{dt}$$

$$\Rightarrow dt = du$$

Limit:

when $t = T$, then $U = 0$

when $t = 2T$ then $U = T$.

$$\therefore T_2 = \int_0^T e^{-s(U+T)} \cdot F(U+T) \cdot dt.$$

$$= \int_0^T e^{-sU} \cdot e^{-sT} \cdot F(U+T)\,dt$$

$$\therefore T_2 = e^{-sT} \int_0^T e^{-st} F(t)\,dt.$$

$$T_3 = \int_{2T}^{3T} e^{-st} \cdot F(t)\, dt$$

Let,

$t = u + 2T$

Limit:

when $t = 2T$, $u = 0$

" $t = 3T$ $u = T$

$$\Rightarrow \frac{d}{dt}(t) = \frac{d}{dt}(u+2T)$$

$$\Rightarrow dt = du.$$

$$T_3 = \int_0^T e^{-s(u+2T)} \cdot F(u+2T) \cdot dt$$

$$= \int_0^T e^{-su} \cdot e^{-2sT} \cdot F(u+2T)\, dt.$$

$$= e^{-2sT} \int_0^T e^{-st} \cdot F(t)\, dt.$$

Now from ① $\Rightarrow$

$$\mathcal{L}\{F(t)\} = \int_0^T e^{-st} \cdot F(t)\, dt + e^{-sT} \int_0^T e^{-st} \cdot F(t)\, dt$$

$$+ e^{-2sT} \int_0^T e^{-st} \cdot F(t)\, dt.$$

$$= \int_0^T e^{-st} F(t)\, dt \left(1 + e^{-sT} + e^{-2sT}\right).$$

$$= \frac{\int_0^T e^{-st} F(t)\, dt}{(1 - e^{-sT})}. \quad \left[(1 - e^{-sT})^{-1} = 1 + e^{-sT} + e^{-2sT}\right.$$

(proved)

Inverse Laplace Transform:

Definition: If the laplace transform of a function, $f(t)$ is $F(s)$. Then $f(t)$ is called the inverse laplace transform of $F(s)$. we write symbolically.

$$f(t) = \mathcal{L}^{-1} F(s)$$

where, $\mathcal{L}^{-1}$ is inverse Laplace transform operator.

Formula:

1. $\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} = 1$.
2. $\mathcal{L}^{-1}\left\{\frac{1}{s^n}\right\} = \frac{t^{n-1}}{(n-1)!}$
3. $\mathcal{L}^{-1}\left\{\frac{1}{s-a}\right\} = e^{at}$.
4. $\mathcal{L}^{-1}\left\{\frac{1}{s^2+a^2}\right\} = \frac{\sin at}{a}$.
5. $\mathcal{L}^{-1}\left\{\frac{s}{s^2+a^2}\right\} = \cos at$.
6. $\mathcal{L}^{-1}\left\{\frac{1}{s^2-a^2}\right\} = \frac{\sinh at}{a}$.
7. $\mathcal{L}^{-1}\left\{\frac{s}{s^2-a^2}\right\} = \cosh at$.
8. $\mathcal{L}^{-1}\left\{\frac{1}{(s-b)^2+a^2}\right\} = e^{bt} \cdot \frac{\sin at}{a}$.
9. $\mathcal{L}^{-1}\left\{\frac{(s-b)}{(s-b)^2+a^2}\right\} = e^{bt} \cos at$.
10. $\mathcal{L}^{-1}\left\{\tan^{-1}\frac{1}{s}\right\} = \frac{\sin t}{t}$.

Q. Find $\mathcal{L}^{-1}\left\{\frac{5}{(s-2)^2} + 2\tan^{-1}\frac{1}{s} + \frac{s+2}{s^2+4s+13}\right\} = ?$

Given,

$$5\,\mathcal{L}^{-1}\left\{\frac{1}{(s-2)^2}\right\} + 2\,\mathcal{L}^{-1}\left\{\tan^{-1}\frac{1}{s}\right\} + \mathcal{L}^{-1}\left\{\frac{s+2}{s^2+4s+13}\right\}.$$

$$= 5t\,e^{2t} + 2\frac{\sin t}{t} + \mathcal{L}^{-1}\left\{\frac{s+2}{(s+2)^2+9}\right\}$$

$$= 5t\cdot e^{2t} + 2\frac{\sin t}{t} + e^{-2t}\cdot\cos 3t$$

Q Find $\mathcal{L}^{-1}\left\{\frac{5s-6}{s^2+9} - \frac{s-15}{s^2-25}\right\} = ?$

$$5\,\mathcal{L}^{-1}\left\{\frac{s}{s^2+9}\right\} - 6\mathcal{L}\left\{\frac{1}{s^2+9}\right\} - \mathcal{L}^{-1}\left\{\frac{s}{s^2-25}\right\} + 15\,\mathcal{L}^{-1}\left\{\frac{1}{s^2-25}\right\}$$

$$= 5\cos 3t - 6\,\frac{\sin 3t}{3} - \cosh 5t + 15\cdot\frac{\sinh 5t}{5}$$

$$= 5\cos 3t - 2\sin 3t - \cosh 5t + 3\sinh 5t$$

*** Find the inverse LT of $\frac{s^2-3s+4}{s^3}$

Soln:

$$\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} - 3\,\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\} + 4\,\mathcal{L}^{-1}\left\{\frac{1}{s^3}\right\}$$

$$= 1 - 3t + 4^2\,\frac{t^2}{2}$$

$$= 1 - 3t + 2t^2$$ Ans

** Find the inverse LT of $\frac{s-2}{s^2-4s+13}$.

Soln:

$$L^{-1}\left\{\frac{s-2}{s^2-4s+13}\right\}$$

$$= L^{-1}\left\{\frac{s-2}{(s-2)^2+9}\right\}$$

$$= \cos 3t \cdot e^{2t}$$

** Find the inverse LT of $\frac{2s-5}{s^2-4}$

Soln:

$$L^{-1}\left\{\frac{2s-5}{s^2-4}\right\}$$

$$= 2\,L^{-1}\left\{\frac{s}{s^2-4}\right\} - 5\,L^{-1}\left\{\frac{1}{s^2-4}\right\}$$

$$= 2\cosh 2t - 5\,\frac{\sinh 2t}{2}$$

Convolution Theorem:

Let $F(t)$ and $G(t)$ be two function then convolution of $F(t)$ and $G(t)$ is denoted by, $F(t) * G(t)$ and defined by,

$$F(t) * G(t) = \int_0^t F(u)\, G(t-u) \cdot du.$$

Q. Evaluate $\mathcal{L}^{-1}\left\{\frac{1}{s(s+a)}\right\}$ using convolution theorem.

Soln:

Given,

$$\mathcal{L}^{-1}\left\{\frac{1}{s(s+a)}\right\}.$$

We can write,

$$\frac{1}{s(s+a)} = \frac{1}{s} \cdot \frac{1}{(s+a)}.$$

Let,

$$f(s) = \frac{1}{s+a}.$$

$$\Rightarrow \mathcal{L}^{-1}\{f(s)\} = \mathcal{L}^{-1}\left\{\frac{1}{s+a}\right\}.$$

$$\Rightarrow F(t) = e^{-at}.$$

$$F(u) = e^{-au}.$$

Again.

$$g(s) = \frac{1}{s}.$$

$$\Rightarrow \mathcal{L}^{-1}\{g(s)\} = \mathcal{L}^{-1}\left\{\frac{1}{s}\right\}$$

$$\Rightarrow G(t) = 1$$

$$G(t-u) = 1.$$

According to convolution theorem,

$$\mathcal{L}^{-1}\{f(s)\, g(s)\} \int_0^t \cancel{f(s) * g(s)} = F(t) * G(t)$$

$$= \int_0^t F(u) \cdot G(t-u)\, du.$$

$$= \int_0^t e^{-au} \cdot 1 \cdot du$$

$$= \left[\frac{e^{-au}}{-a}\right]_0^t$$

$$= -\frac{1}{a}\left[e^{-au}\right]$$

$$= -\frac{1}{a}\left[e^{-at} - e^0\right]$$

$$= -\frac{1}{a}\left[e^{-at} - 1\right]$$

$$= -\frac{1}{a}\cdot e^{-at} + \frac{1}{a}$$

$$= \frac{1}{a} - \frac{1}{a}\cdot e^{-at}$$ Ans

$\boxed{10}$ Evaluate $\mathcal{L}^{-1}\left\{\frac{1}{s^2(s+a)}\right\}$ convolution theorem?

Soln:

We can write,

$$\frac{1}{s^2(s+a)} = \frac{1}{s^2}\cdot\frac{1}{(s+a)}.$$

Now,

let,

$$f(s) = \frac{1}{(s+a)}.$$

$$\Rightarrow \mathcal{L}^{-1}\{f(s)\} = \mathcal{L}^{-1}\left\{\frac{1}{(s+a)}\right\}.$$

$$\Rightarrow F(t) = e^{-at}.$$

$$F(u) = e^{-au}.$$

Again,

$$g(s) = \frac{1}{s^2}$$

$$\Rightarrow \mathcal{L}^{-1}\{g(s)\} = \mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\}.$$

$$\Rightarrow G(t) = t.$$

$$\Rightarrow G(t-u) = (t-u)$$

ording to convolution theorem,

$$L^{-1}\{f(s)\,g(s)\} = F(t) * G(t)$$

$$= \int_0^t F(u)\, G(t-u)\cdot du$$

$$= \int_0^t e^{-au}\cdot (t-u)\cdot du$$

$$= \int_0^t e^{-au}\cdot t\cdot du - \int_0^t e^{-au}\cdot u\, du.$$

$$= t\left[\frac{e^{-au}}{-a}\right]_0^t - \left[u\int e^{-au}\cdot du - \int \frac{d}{du}(u)\int e^{-au}\cdot du\, du\right]_0^t$$

$$= \frac{t}{-a}\left[e^{-at}-1\right] - \left[u\cdot\frac{e^{-au}}{-a}\right]_0^t + \frac{1}{a}\int_0^t e^{-au}\cdot du.$$

$$= -\frac{t}{a}\cdot\left[e^{-at}-1\right] + \frac{1}{a}\left[t\, e^{-at} - 0\right] + \frac{1}{a^2}\left[e^{-au}\right]_0^t$$

$$= -\cancel{\frac{t\,e^{-at}}{a}} + \frac{t}{a} + \cancel{\frac{t\,e^{-at}}{a}} - \frac{1}{a^2}\left[e^{-at}-1\right]$$

$$= \frac{t}{a} - \frac{e^{-at}}{a^2} + \frac{1}{a^2}.$$

$$= \frac{t}{a} - \frac{e^{-at}}{a^2} + \frac{1}{a^2}$$ Ans

[10] Using convolution theorem find $\mathcal{L}^{-1}\left\{\frac{1}{s^2(s^2+4)}\right\}$?

Soln:

Given,

$$\mathcal{L}^{-1}\left\{\frac{1}{s^2(s^2+4)}\right\}$$

we can write,

$$\frac{1}{s^2(s^2+4)} = \frac{1}{s^2} \cdot \frac{1}{s^2+4}.$$

let,

$$f(s) = \frac{1}{s^2+4}.$$

$$\Rightarrow \mathcal{L}^{-1}\{f(s)\} = \mathcal{L}^{-1}\left\{\frac{1}{s^2+4}\right\}$$

$$\Rightarrow F(t) = \frac{\sin 2t}{2}.$$

$$F(u) = \frac{\sin 2u}{2}.$$

Again,

$$g(s) = \frac{1}{s^2}$$

$$\Rightarrow \mathcal{L}^{-1}\{g(s)\} = \mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\}$$

$$\Rightarrow G(t) = t.$$

$$\therefore G(t-u) = (t-u)$$

According to convolution theorem we can write,

$$\mathcal{L}^{-1}\{f(s)\, g(s)\} = F(t) * G(t)$$

$$= \int_0^t F(u)\, G(t-u)\cdot du,$$

$$= \int_0^t \frac{\sin 2u}{2} \cdot (t-u)\, du$$

$$= \frac{1}{2}\int_0^t \sin 2u\, (t-u)\, du$$

$$= \frac{1}{2}\int_0^t t\cdot \sin 2u \, du - \frac{1}{2}\int_0^t u \sin 2u \cdot du.$$

$$= -\frac{t}{2}\left[\frac{\cos 2u}{2}\right]_0^t - \frac{1}{2}\left[u\int \sin 2u\,du - \left\{\int \frac{d}{du}(u)\int \sin 2u\,du\right\}du\right]_0^t$$

$$= -\frac{t}{4}\cdot\left[\cos 2t - 1\right] + \frac{1}{4}\cdot\left[u\cos 2u\right]_0^t - \frac{1}{4}\int_0^t \cancel{\sin 2}\cos 2u\,du$$

$$= -\frac{t\cos 2t}{4} + \frac{t}{4} + \frac{1}{4}\left[t\cos 2t - 0\right] - \frac{1}{4}\left[\frac{\sin 2u}{2}\right]_0^t$$

$$= -\cancel{\frac{t\cos 2t}{4}} + \frac{t}{4} + \cancel{\frac{t\cos 2t}{4}} - \frac{1}{8}\left[\sin 2t - 0\right]$$

$$= \frac{t}{4} - \frac{\sin 2t}{8}.$$

$$= \frac{1}{4}\left(t - \frac{1}{2}\sin 2t\right)$$

Ans

Q. Using convolution theorem find $\mathcal{L}^{-1}\left\{\frac{1}{s^2(s^2+a^2)}\right\}$.

We can write,

$$\frac{1}{s^2(s^2+a^2)} = \frac{1}{s^2}\cdot\frac{1}{(s^2+a^2)}.$$

Let,

$$f(s) = \frac{1}{s^2+a^2}.$$

$$\Rightarrow \mathcal{L}^{-1}\{f(s)\} = \mathcal{L}^{-1}\left\{\frac{1}{s^2+a^2}\right\}.$$

$$\Rightarrow F(t) = \frac{\sin at}{a}.$$

$$F(u) = \frac{\sin au}{a}.$$

Again,

$$g(s) = \frac{1}{s^2}$$

$$\Rightarrow \mathcal{L}^{-1}\{g(s)\} = \mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\}.$$

$$\Rightarrow G(t) = t.$$

$$\therefore G(t-u) = (t-u).$$

According to convolution theorem we can write,

$$L^{-1}\{f(s)\, g(s)\} = F(t) * G(t)$$

$$= \int_0^t F(u)\, G(t-u)\, du$$

$$= \int_0^t \frac{\sin au}{a} \cdot (t-u) \cdot du.$$

$$= \frac{1}{a}\int_0^t \sin au \cdot t \cdot du - \frac{1}{a}\int_0^t \sin au \cdot u \cdot du.$$

$$= -\frac{t}{a^2}\left[\cos au\right]_0^t - \frac{1}{a}\left[u\int \sin au\, du - \left\{\int \frac{d}{du}(u) \int \sin au\, du\right\} du\right]_0^t$$

$$= -\frac{t}{a^2}\left[\cos at - 1\right] + \frac{1}{a^2}\left[u \cos au\right]_0^t - \frac{1}{a^2}\int_0^t \cos au\, du.$$

$$= -\frac{t\cancel{\cos at}}{\cancel{a^2}} + \frac{t}{a^2} + \frac{t\cancel{\cos at}}{\cancel{a^2}} - \frac{1}{a^3}\left[\sin au\right]_0^t$$

$$= \frac{t}{a^2} - \frac{\sin at}{a^3}.$$

$$= \frac{1}{a^2}\left(t - \frac{1}{a} \cdot \sin at\right)$$

Q. Using convolution theorem find $L^{-1}\left\{\frac{3}{s^2(s+2)}\right\}$

we can write,

$$\frac{3}{s^2(s+2)} = \frac{1}{(s+2)} \quad \frac{3}{s^2}.$$

Let,

$$f(s) = \frac{1}{(s+2)}$$

$$\Rightarrow L^{-1}\{f(s)\} = L^{-1}\left\{\frac{1}{(s+2)}\right\}.$$

$$\Rightarrow F(t) = e^{-2t}.$$

$$F(u) = e^{-2u}.$$

Again,

$$g(s) = \frac{3}{s^2}.$$

$$\Rightarrow L^{-1}\{g(s)\} = 3\, L^{-1}\left\{\frac{1}{s^2}\right\}.$$

$$\Rightarrow G(t) = 3t.$$

$$G(t-u) = 3(t-u)$$

According to convolution theorem we can write,

$$\mathcal{L}^{-1}\{F(s)\,G(s)\} = F(t) * G(t)$$

$$= \int_0^t F(u) \cdot G(t-u) \cdot du$$

$$= \int_0^t e^{-2u} \cdot 3\,(t-u)\, du$$

$$= 3 \int_0^t e^{-2u} \cdot t \cdot du - 3 \int_0^t e^{-2u} \cdot u\, du$$

$$= 3t \left[\frac{e^{-2u}}{-2}\right]_0^t - 3\left[u \int e^{-2u}\,du - \int \left\{\frac{d}{du}(u) \int e^{-2u}\,du\right\} du\right]$$

$$= -\frac{3t}{2}\left[e^{-2t} - 1\right] + \frac{3}{2}\left[u \cdot e^{-2u}\right]_0^t + \frac{3}{2}\int_0^t e^{-2u}\,du$$

$$= -\frac{3t \cdot e^{-2t}}{2} + \frac{3t}{2} + \frac{3}{2}\left[t\,e^{-2t} - 0\right] + \frac{3}{4}\left[e^{-2u}\right]_0^t$$

$$= -\cancel{\frac{3t \cdot e^{-2t}}{2}} + \frac{3t}{2} + \cancel{\frac{3t \cdot e^{-2t}}{2}} + \frac{3}{4}\left[e^{-2t} - 1\right]$$

$$= \frac{3t}{2} + \frac{3e^{-2t}}{4} + \frac{3}{4}.$$