COMP 4900 Assignment 3

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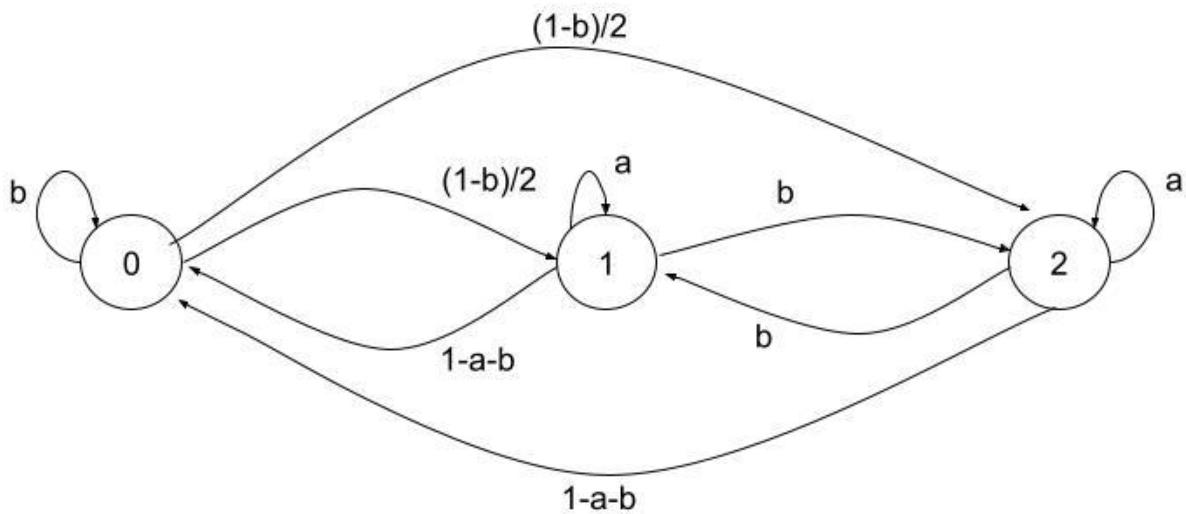
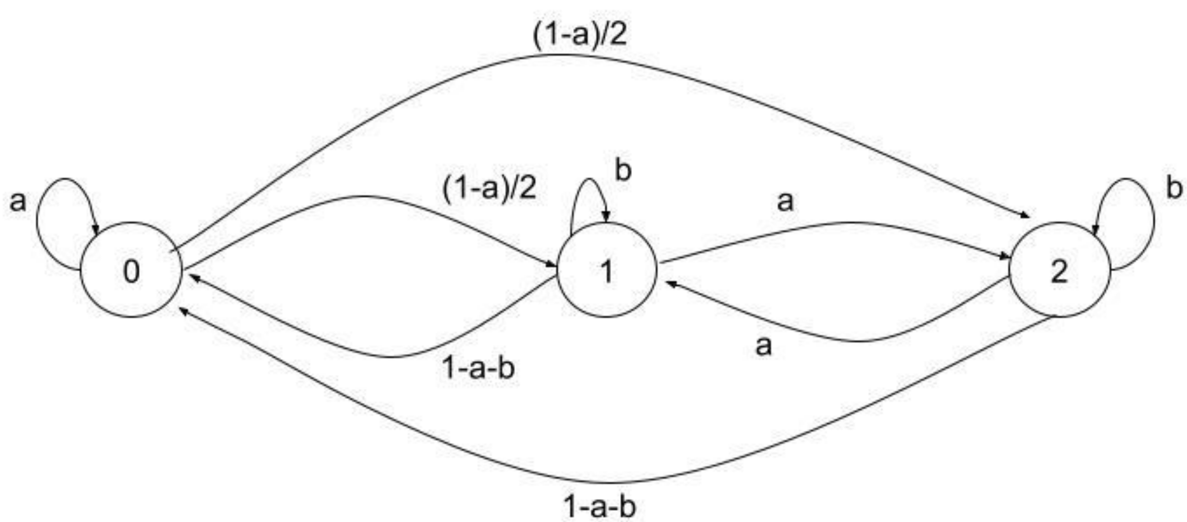
a.

β=0



β=1



$$F^0=\begin{bmatrix} b & \frac{1-b}{2} & \frac{1-b}{2} \\ 1-a-b & a & b \\ 1-a-b & b & a \end{bmatrix} F^1=\begin{bmatrix} a & \frac{1-a}{2} & \frac{1-a}{2} \\ 1-a-b & b & a \\ 1-a-b & a & b \end{bmatrix} G=\begin{bmatrix} 0.5 & 0.5 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

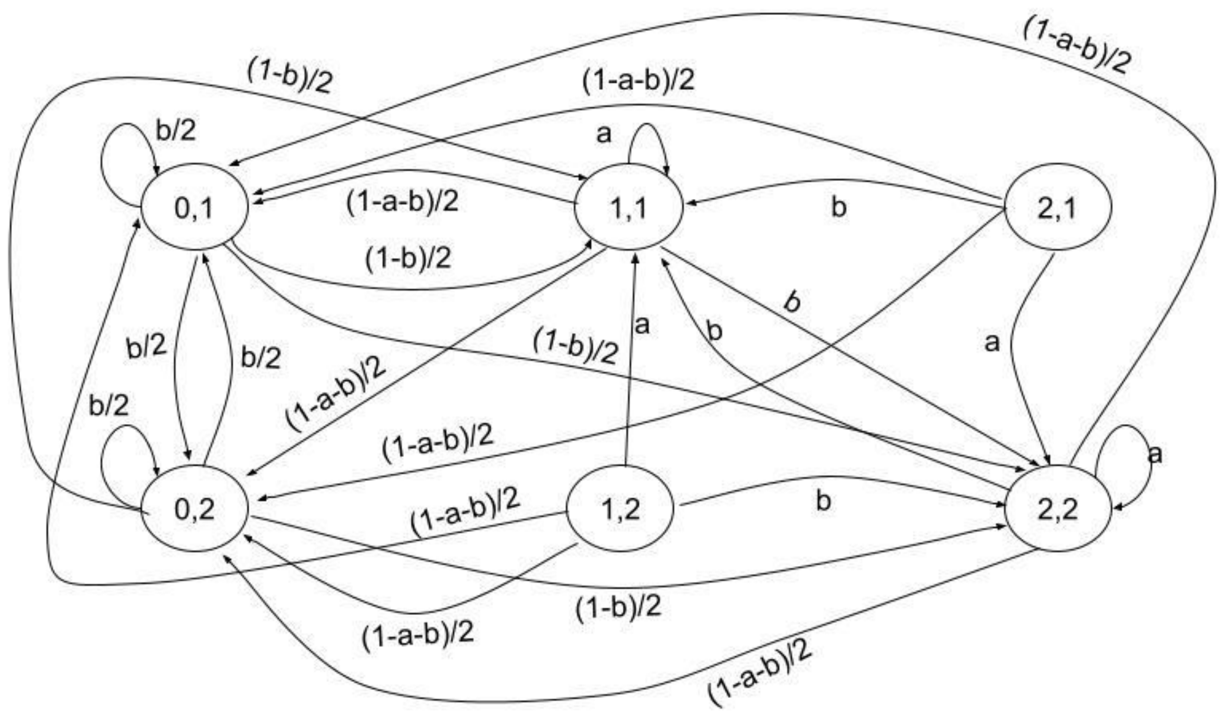
b. New states are:

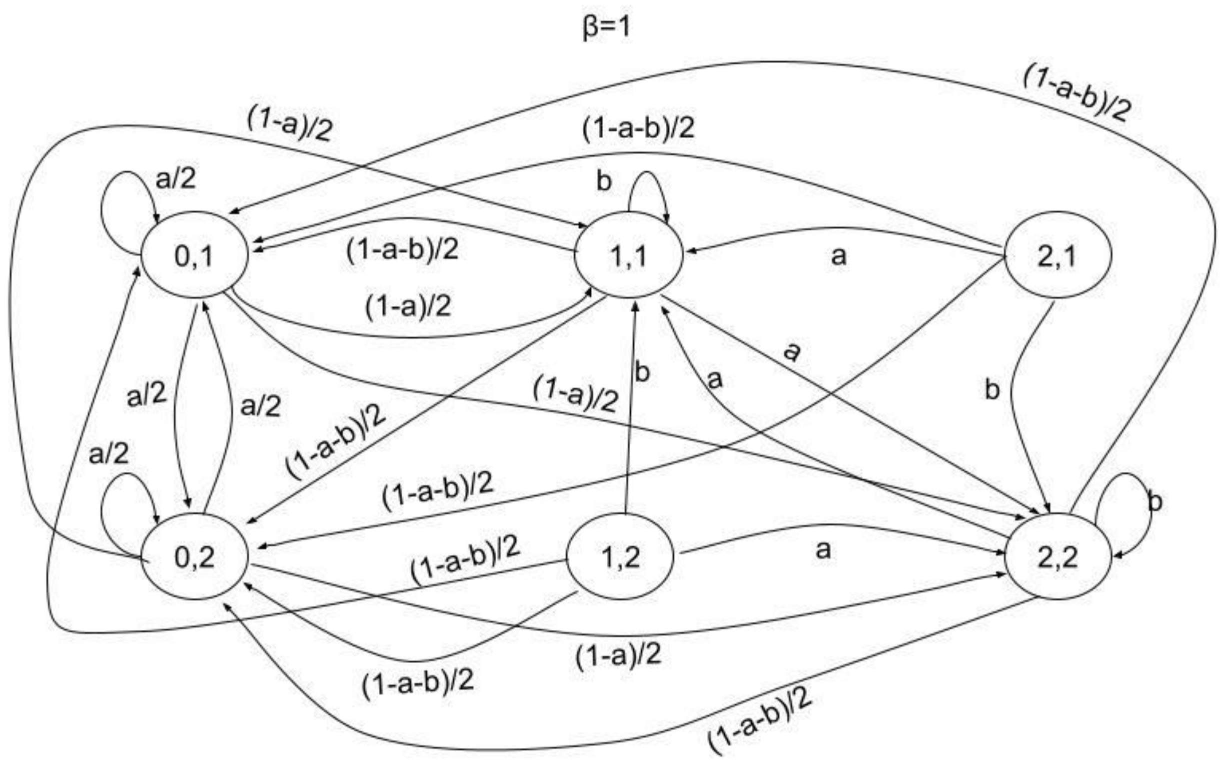
0 → (0,1) & (0,2)

1 → (1,1) & (1,2)

2 → (2,1) & (2,2)

β=0





Using this formula,

$$F^{0,1}_{<i,j><k,l>} = F^{0,1\ old}_{i,k} * G_{k,l}$$

F^0 and F^1 can be generated as follow:

$$F^0 = \begin{bmatrix} \frac{b}{2} & \frac{1-b}{2} & 0 & \frac{b}{2} & 0 & \frac{1-b}{2} \\ \frac{1-a-b}{2} & a & 0 & \frac{1-a-b}{2} & 0 & b \\ \frac{1-a-b}{2} & b & 0 & \frac{1-a-b}{2} & 0 & a \\ \frac{b}{2} & \frac{1-b}{2} & 0 & \frac{b}{2} & 0 & \frac{1-b}{2} \\ \frac{1-a-b}{2} & a & 0 & \frac{1-a-b}{2} & 0 & b \\ \frac{1-a-b}{2} & b & 0 & \frac{1-a-b}{2} & 0 & a \end{bmatrix} \quad F^1 = \begin{bmatrix} \frac{a}{2} & \frac{1-a}{2} & 0 & \frac{a}{2} & 0 & \frac{1-a}{2} \\ \frac{1-a-b}{2} & b & 0 & \frac{1-a-b}{2} & 0 & a \\ \frac{1-a-b}{2} & a & 0 & \frac{1-a-b}{2} & 0 & b \\ \frac{a}{2} & \frac{1-a}{2} & 0 & \frac{a}{2} & 0 & \frac{1-a}{2} \\ \frac{1-a-b}{2} & b & 0 & \frac{1-a-b}{2} & 0 & a \\ \frac{1-a-b}{2} & a & 0 & \frac{1-a-b}{2} & 0 & b \end{bmatrix}$$

G will become deterministic, therefore when in state $S_{i,j}$ choose action j with probability 1

$$G=\begin{array}{c}(0,1)\\(1,1)\\(2,1)\\(0,2)\\(1,2)\\(2,2)\end{array}\begin{bmatrix}1 & 0\\1 & 0\\1 & 0\\0 & 1\\0 & 1\\0 & 1\end{bmatrix}$$

This new machine does not need to have 6 states, it can have only 4, since from old G matrix, state 1 action 2 (1,2) never occurs and state 2 action 1 (2,1) never occurs. As can be seen in our new matrix F^0 and F^1, columns for (2,1) and (1,2) are all zero entries.

c.(See code for calculations)

Using these formulas,

$$F^{\sim}_{i,j} = F^0_{i,j}\,[G_{i,1} * (1-C_1) + G_{i,2} * (1-C_2)] + F^1_{i,j}\,[G_{i,1} * C_1 + G_{i,2} * C_2]$$

$$\pi(n) = (F^{\sim})^T * \pi(n-1)$$

$$P(n) = G^T * \pi(n)$$

Given a=0.3, b=0.7 and c=[0.4 , 0.6], $F^{\sim}$ can be generated as follow:

$$F^{\sim}=\begin{bmatrix}0.5 & 0.25 & 0.25\\0 & 0.46 & 0.54\\0 & 0.46 & 0.54\end{bmatrix}$$

Given $\pi(0) = [0.2 , 0.4 , 0.4]$

We can find P(0) , $\pi(1)$ and P(1)

$P(0) = G^T * \pi(0) = [0.5 , 0.5]$

$\pi(1) = (F^{\sim})^T * \pi(0) = [0.1 , 0.418, 0.482]$

$P(1) = G^T * \pi(1) = [0.468 , 0.532]$

d. (See code for calculations)

Using same formulas and same $\pi(0)$, a,b and c values from question c,

$F^{\sim}$ can be generated as follow:

$$F^{\sim}=\begin{bmatrix}0.27 & 0.23 & 0 & 0.27 & 0 & 0.23\\0 & 0.46 & 0 & 0 & 0 & 0.54\\0 & 0.54 & 0 & 0 & 0 & 0.46\\0.23 & 0.27 & 0 & 0.23 & 0 & 0.27\\0 & 0.54 & 0 & 0 & 0 & 0.46\\0 & 0.46 & 0 & 0 & 0 & 0.54\end{bmatrix}$$

Given $\pi(0) = [0.2 , 0.4 , 0.4]$

We can convert to our new $\pi(0)$ using the old G, following this formula:

$$\pi(0)_{s=(i,j)}^{new} = \pi(0)_{s=i}^{old} * G_{i,j}^{old}$$

States order: (0,1),(1,1),(2,1),(0,2),(1,2),(2,2)

$\pi(0) = [0.1, 0.4, 0.0, 0.1, 0.0, 0.4]$

We can find P(0) , $\pi(1)$ and P(1)

$P(0) = (G)^T * \pi(0) = [0.5 , 0.5]$

$\pi(1) = (F^{\sim})^T * \pi(0) = [0.05 , 0.418 , 0 , 0.05 , 0 , 0.482]$

$P(1) = G^T * \pi(1) = [0.468 , 0.532]$