COMP 4900 Final

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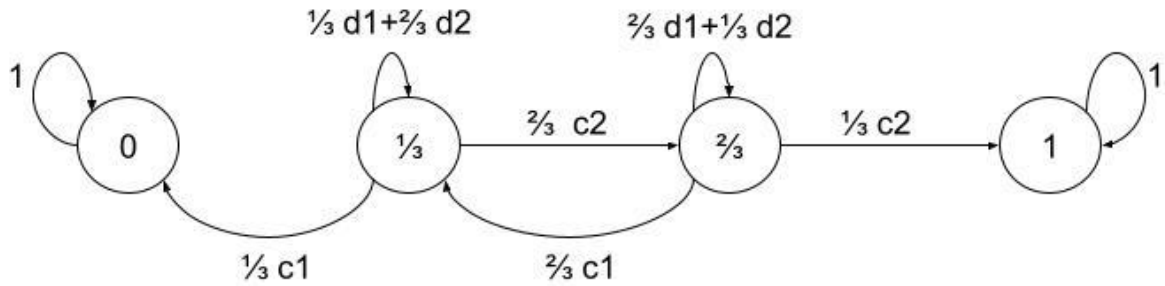
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Q1

1.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \dfrac{c1}{3} & \dfrac{d1}{3}+\dfrac{2}{3}d2 & \dfrac{2}{3}c2 & 0 \\ 0 & \dfrac{2}{3}c1 & \dfrac{2}{3}d1+\dfrac{d2}{3} & \dfrac{c2}{3} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} f13 \\ f23 \end{bmatrix} = \begin{bmatrix} 0 \\ \dfrac{c2}{3} \end{bmatrix} + \begin{bmatrix} \dfrac{d1}{3}+\dfrac{2}{3}d2 & \dfrac{2}{3}c2 \\ \dfrac{2}{3}c1 & \dfrac{2}{3}d1+\dfrac{d2}{3} \end{bmatrix} \begin{bmatrix} f13 \\ f23 \end{bmatrix}$$

$$f = [INV \begin{bmatrix} 1-\dfrac{d1}{3}+\dfrac{2}{3}d2 & -\dfrac{2}{3}c2 \\ -\dfrac{2}{3}c1 & 1-\dfrac{2}{3}d1+\dfrac{d2}{3} \end{bmatrix}] \begin{bmatrix} 0 \\ \dfrac{c2}{3} \end{bmatrix}$$

2.

$R_0=1$

$$R_1 = \dfrac{\dfrac{c1}{3}}{\dfrac{2}{3}c2}$$

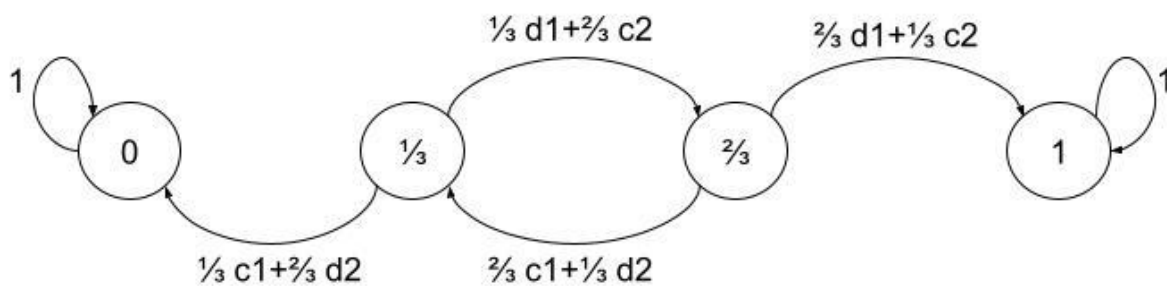$$R_2 = \dfrac{\dfrac{c1}{3} \quad \dfrac{2}{3}c1}{\dfrac{2}{3}c2 \quad \dfrac{1}{3}c2}$$

$$\Pr[\phi(\infty)=3 \mid \phi(0)=1] = \frac{R0}{R0+R1+R2} = \cfrac{1}{1+\cfrac{\frac{c1}{3}}{\frac{2}{3}c2}+\cfrac{\frac{c1}{3}\,\frac{2}{3}c1}{\frac{2}{3}c2\,\frac{1}{3}c2}}$$

3.

$$\Pr[\phi(\infty)=3 \mid \phi(0)=2] = \frac{R0+R1}{R0+R1+R2} = \cfrac{1+\cfrac{\frac{c1}{3}}{\frac{2}{3}c2}}{1+\cfrac{\frac{c1}{3}}{\frac{2}{3}c2}+\cfrac{\frac{c1}{3}\,\frac{2}{3}c1}{\frac{2}{3}c2\,\frac{1}{3}c2}}$$

Q2.

1.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \dfrac{c1}{3}+\dfrac{2}{3}d2 & 0 & \dfrac{d1}{3}+\dfrac{2}{3}c2 & 0 \\ 0 & \dfrac{2}{3}c1+\dfrac{d2}{3} & 0 & \dfrac{2}{3}d1+\dfrac{c2}{3} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$R_0=1$

$$R_1=\frac{\frac{c1}{3}+\frac{2}{3}d2}{\frac{d1}{3}+\frac{2}{3}c2}$$

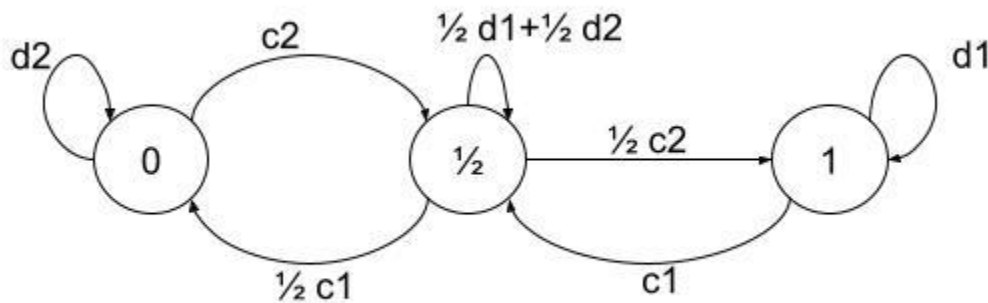$$R_2=\frac{\frac{c1}{3}+\frac{2}{3}d2}{\frac{d1}{3}+\frac{2}{3}c2}\frac{\frac{2c1}{3}+\frac{1}{3}d2}{\frac{2d1}{3}+\frac{1}{3}c2}$$

$$\Pr[\phi(\infty)=3|\phi(0)=1]=\frac{R0}{R0+R1+R2}=\frac{1}{1+\frac{\frac{c1}{3}+\frac{2}{3}d2}{\frac{d1}{3}+\frac{2}{3}c2}+\frac{\frac{c1}{3}+\frac{2}{3}d2}{\frac{d1}{3}+\frac{2}{3}c2}\frac{\frac{2c1}{3}+\frac{1}{3}d2}{\frac{2d1}{3}+\frac{1}{3}c2}}$$

2. $E[P_1(\infty)]=0.5* \Pr[\phi(\infty)=3|\phi(0)=1]+0.5\,\Pr[\phi(\infty)=3|\phi(0)=2]$

$$\Pr[\phi(\infty)=3|\phi(0)=2]=\frac{R0+R1}{R0+R1+R2}=\frac{1+\frac{\frac{c1}{3}+\frac{2}{3}d2}{\frac{d1}{3}+\frac{2}{3}c2}}{1+\frac{\frac{c1}{3}+\frac{2}{3}d2}{\frac{d1}{3}+\frac{2}{3}c2}+\frac{\frac{c1}{3}+\frac{2}{3}d2}{\frac{d1}{3}+\frac{2}{3}c2}\frac{\frac{2c1}{3}+\frac{1}{3}d2}{\frac{2d1}{3}+\frac{1}{3}c2}}$$

Q3.

1.

$$\begin{bmatrix} d2 & c2 & 0 \\ \dfrac{c1}{2} & \dfrac{d1}{2}+\dfrac{d2}{2} & \dfrac{c2}{2} \\ 0 & c1 & d1 \end{bmatrix}$$

Since this is an ergodic Markov chain, we can find the eigenvector for eigenvalue=1, which will give us π *, then we can find P_1* by this equation.

P_1*= $0 * \pi_1 + \frac{1}{2} *\pi_2 + \pi_3$

2.Assuming c1=0.3 & c2=0.4

π *= {0.183673, 0.489796, 0.326531}

P_1*= $\frac{1}{2} *\pi_2 + \pi_3$ = 0.57

$L_{4,2}$ accuracy is 0.77 given this formula

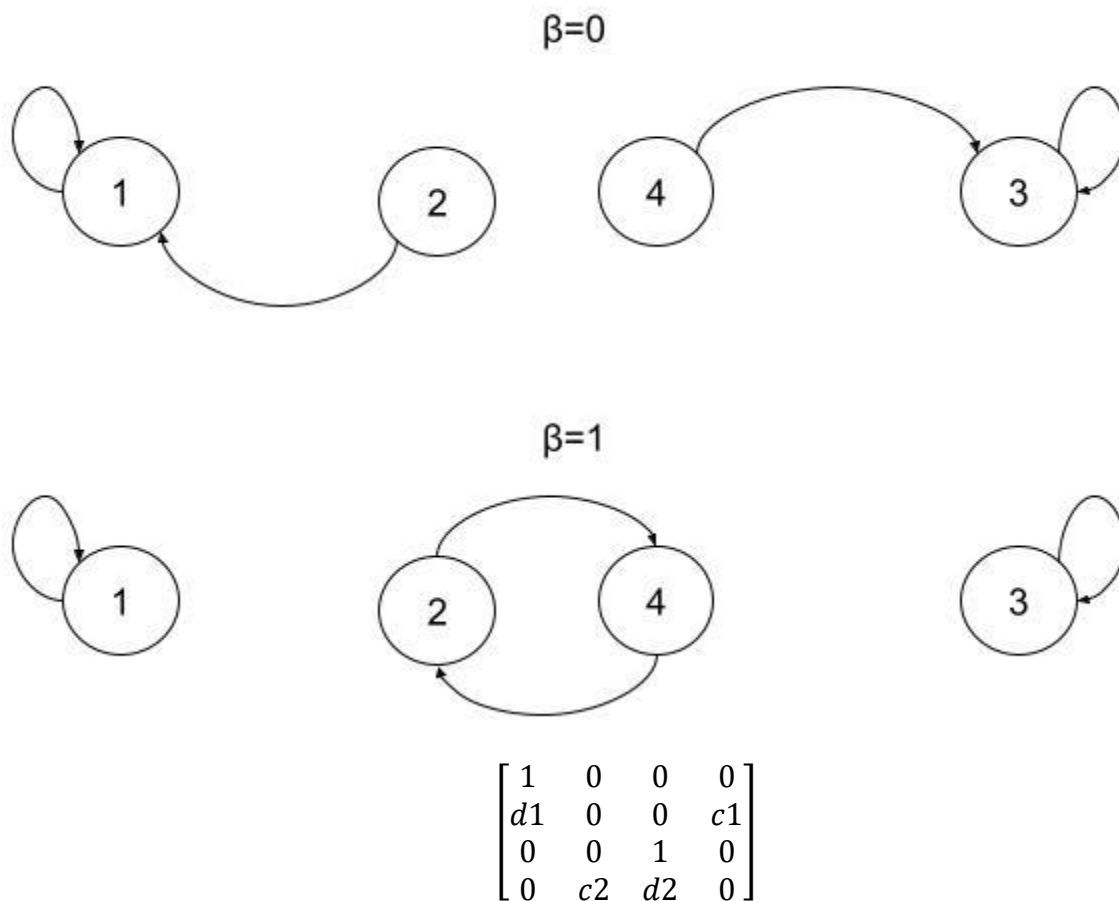$$\frac{1}{1+\left(\dfrac{c1}{c2}\right)^{4}\dfrac{c1-d1}{c2-d2}\dfrac{c2^4-d2^4}{c1^4-d1^4}}$$

Q4.

1.

$\frac{1}{R-1}$ is there because we need to distribute the difference evenly to all other actions (R-1 actions)

2.

The following probability vector (n+1) would no longer be a valid probability distribution, since the sum of all entries would be greater than 1

Q5.

$\beta=0$

$\beta=1$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ d1 & 0 & 0 & c1 \\ 0 & 0 & 1 & 0 \\ 0 & c2 & d2 & 0 \end{bmatrix}$$

We can determine its accuracy by either simulation or by using the Chapman-Kolmogorov formula on the corresponding matrix.

$$\begin{bmatrix} f21 \\ f41 \end{bmatrix} = \begin{bmatrix} d1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ c2 & d2 \end{bmatrix} \begin{bmatrix} f21 \\ f41 \end{bmatrix}$$

$$f = [INV \begin{bmatrix} 1 & 0 \\ -c2 & 1 - d2 \end{bmatrix}] \begin{bmatrix} f21 \\ f41 \end{bmatrix}$$

$$f = [INV \begin{bmatrix} 1 & 0 \\ -c2 & 1 - d2 \end{bmatrix}] \begin{bmatrix} f21 \\ f41 \end{bmatrix}$$