COMP 4900 Final

John Oommen

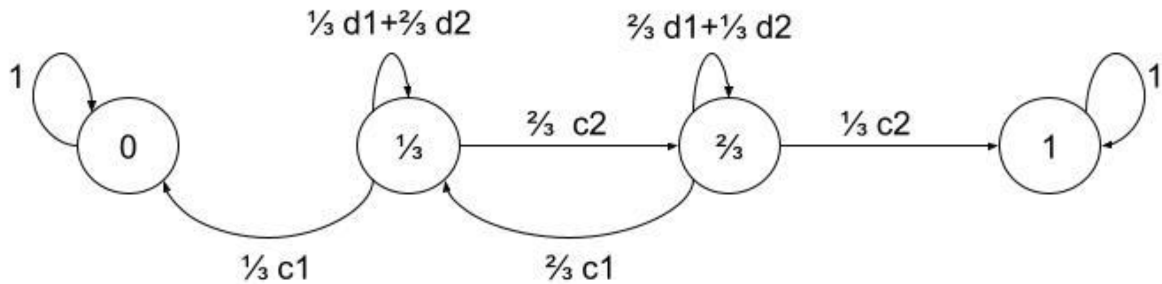
Abdulaah Emin

101130854

Carleton University

Q1

1.



$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{c1}{3} & \frac{d1}{3}+\frac{2}{3}d2 & \frac{2}{3}c2 & 0 \\ 0 & \frac{2}{3}c1 & \frac{2}{3}d1+\frac{d2}{3} & \frac{c2}{3} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} f13 \\ f23 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{c2}{3} \end{bmatrix} + \begin{bmatrix} \frac{d1}{3}+\frac{2}{3}d2 & \frac{2}{3}c2 \\ \frac{2}{3}c1 & \frac{2}{3}d1+\frac{d2}{3} \end{bmatrix} \begin{bmatrix} f13 \\ f23 \end{bmatrix}$$

$$f = [INV \begin{bmatrix} 1-\frac{d1}{3}+\frac{2}{3}d2 & -\frac{2}{3}c2 \\ -\frac{2}{3}c1 & 1-\frac{2}{3}d1+\frac{d2}{3} \end{bmatrix}] \begin{bmatrix} 0 \\ \frac{c2}{3} \end{bmatrix}$$

2.

$R_0=1$

$$R_1 = \frac{\frac{c1}{3}}{\frac{2}{3}c2}$$

$$R_2 = \frac{\frac{c1}{3}}{\frac{2}{3}c2} \frac{\frac{2}{3}c1}{\frac{1}{3}c2}$$

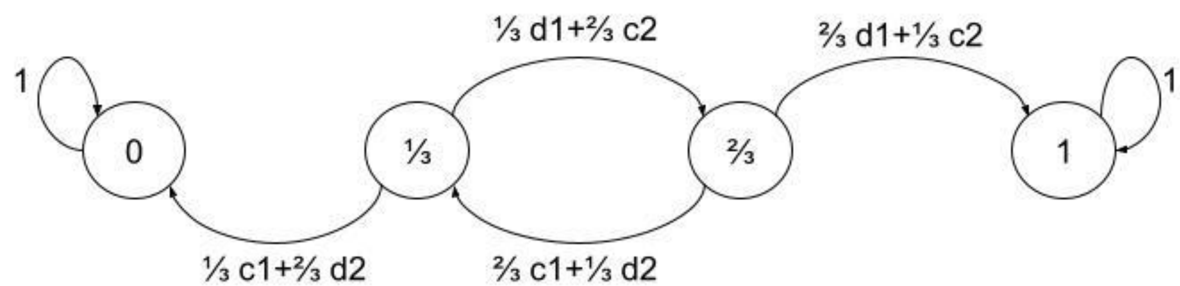
$$\Pr[\phi(\infty)=3|\phi(0)=1] = \frac{R0}{R0+R1+R2} = \frac{1}{1+\frac{\frac{c1}{3}}{\frac{2}{3}c2}+\frac{\frac{c1}{3}}{\frac{2}{3}c2}\frac{\frac{2}{3}c1}{\frac{1}{3}c2}}$$

3.

$$\Pr[\phi(\infty)=3|\phi(0)=2] = \frac{R0+R1}{R0+R1+R2} = \frac{1+\frac{\frac{c1}{3}}{\frac{2}{3}c2}}{1+\frac{\frac{c1}{3}}{\frac{2}{3}c2}+\frac{\frac{c1}{3}}{\frac{2}{3}c2}\frac{\frac{2}{3}c1}{\frac{1}{3}c2}}$$

Q2.

1.



$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{c1}{3}+\frac{2}{3}d2 & 0 & \frac{d1}{3}+\frac{2}{3}c2 & 0 \\ 0 & \frac{2}{3}c1+\frac{d2}{3} & 0 & \frac{2}{3}d1+\frac{c2}{3} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$R_0=1$

$$R_1=\frac{\frac{c1}{3}+\frac{2}{3}d2}{\frac{d1}{3}+\frac{2}{3}c2}$$

$$R_2=\frac{\frac{c1}{3}+\frac{2}{3}d2}{\frac{d1}{3}+\frac{2}{3}c2}\frac{\frac{2c1}{3}+\frac{1}{3}d2}{\frac{2d1}{3}+\frac{1}{3}c2}$$

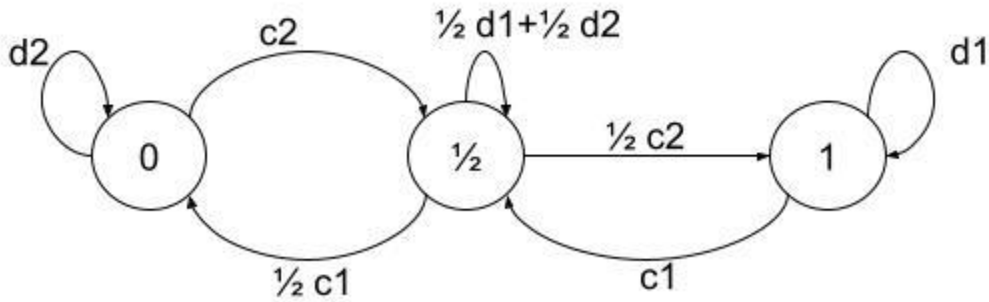
$$\Pr[\phi(\infty)=3|\phi(0)=1]=\frac{R0}{R0+R1+R2}=\frac{1}{1+\frac{\frac{c1}{3}+\frac{2}{3}d2}{\frac{d1}{3}+\frac{2}{3}c2}+\frac{\frac{c1}{3}+\frac{2}{3}d2\frac{2c1}{3}+\frac{1}{3}d2}{\frac{d1}{3}+\frac{2}{3}c2\frac{2d1}{3}+\frac{1}{3}c2}}$$

2. $E[P_1(\infty)]=0.5*\Pr[\phi(\infty)=3|\phi(0)=1]+0.5\Pr[\phi(\infty)=3|\phi(0)=2]$

$$\Pr[\phi(\infty)=3|\phi(0)=2]=\frac{R0+R1}{R0+R1+R2}=\frac{1+\frac{\frac{c1}{3}+\frac{2}{3}d2}{\frac{d1}{3}+\frac{2}{3}c2}}{1+\frac{\frac{c1}{3}+\frac{2}{3}d2}{\frac{d1}{3}+\frac{2}{3}c2}+\frac{\frac{c1}{3}+\frac{2}{3}d2\frac{2c1}{3}+\frac{1}{3}d2}{\frac{d1}{3}+\frac{2}{3}c2\frac{2d1}{3}+\frac{1}{3}c2}}$$

Q3.

1.



$$\begin{bmatrix} d2 & c2 & 0 \\ \frac{c1}{2} & \frac{d1}{2}+\frac{d2}{2} & \frac{c2}{2} \\ 0 & c1 & d1 \end{bmatrix}$$

Since this is an ergodic Markov chain, we can find the eigenvector for eigenvalue=1, which will give us π *, then we can find P_1* by this equation.

$P_1^* = 0 * \pi_1 + ½ * \pi_2 + \pi_3$

2.Assuming c1=0.3 & c2=0.4

π *= {0.183673, 0.489796, 0.326531}

$P_1^* = ½ * \pi_2 + \pi_3 = 0.57$

$L_{4,2}$ accuracy is 0.77 given this formula

$$p_1(\infty) = \frac{1}{1 + \left(\frac{c_1}{c_2}\right)^N \frac{c_1 - d_1}{c_2 - d_2} \frac{(c_2^N - d_2^N)}{(c_1^N - d_1^N)}}$$

$$\frac{1}{1 + \left(\frac{c1}{c2}\right)^4 \frac{c1 - d1}{c2 - d2} \frac{c2^4 - d2^4}{c1^4 - d1^4}}$$

Q4.

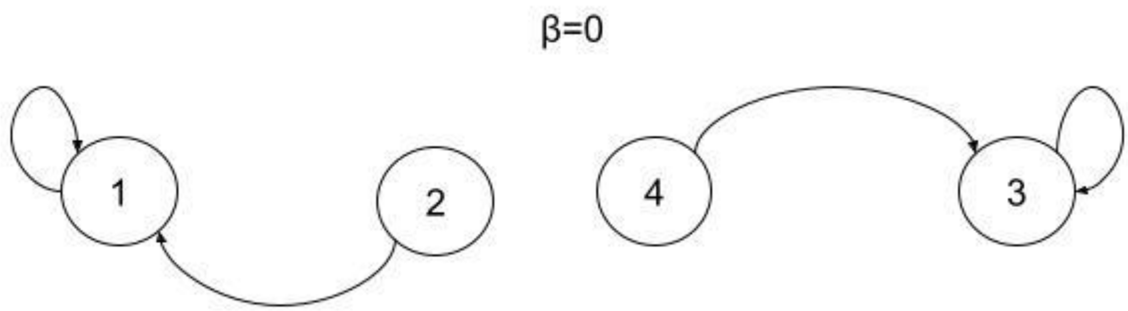
1.

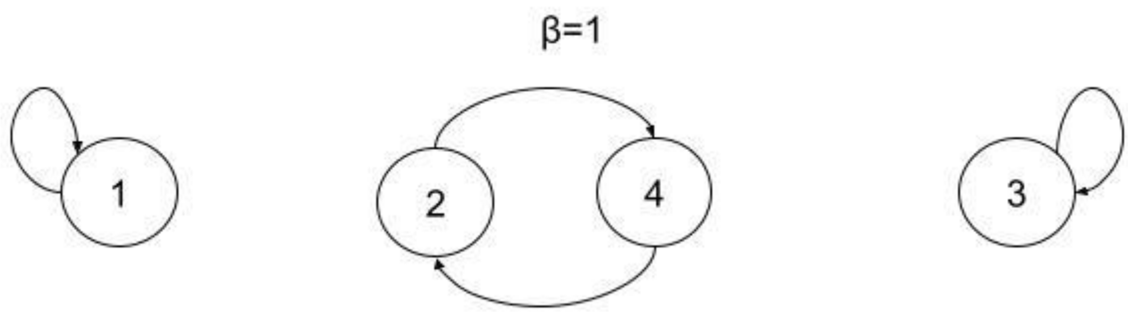
$\frac{1}{R-1}$ is there because we need to distribute the difference evenly to all other actions (R-1 actions)

2.

The following probability vector (n+1) would no longer be a valid probability distribution, since the sum of all entries would be greater than 1

Q5.





$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ d1 & 0 & 0 & c1 \\ 0 & 0 & 1 & 0 \\ 0 & c2 & d2 & 0 \end{bmatrix}$$

We can determine its accuracy by either simulation or by using the Chapman-Kolmogorov formula on the corresponding matrix.

$$\begin{bmatrix} f21 \\ f41 \end{bmatrix} = \begin{bmatrix} d1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ c2 & d2 \end{bmatrix} \begin{bmatrix} f21 \\ f41 \end{bmatrix}$$

$$f = [INV \begin{bmatrix} 1 & 0 \\ -c2 & 1 - d2 \end{bmatrix}] \begin{bmatrix} d1 \\ 0 \end{bmatrix}$$