AE301 Tutorials

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1 Tutorial 1

Exercise 1.1
Verify the dimensions in both the FLT and MLT systems, of the following quantities which appear in Table 1.1:

a) volume,

b) acceleration,

c) mass,

d) moment of inertia (area),

e) work

Solution 1.1

a) Since volume is the result of cubing the dimension of length, its dimension will be $[L^3]$

b) Acceleration is defined as the time rate of change of velocity. Thus its dimension will be equal to the dimension of velocity divided by the dimension of time.

$$\Rightarrow \frac{[L\,T^{-1}]}{[T]} \Rightarrow [L\,T^{-2}]$$

c) Mass is one of the fundamental dimensions (MLT) and hence its dimension is simply M. We can convert this to the FLT system using Newton's second law: $F = ma$ where F is force and m is mass and a is acceleration. Using dimensions, we get:

$$[F] = [M][L\,T^{-2}]$$
$$\Rightarrow [M] = [F\,L^{-1}\,T^{2}]$$

d) Area moment of inertia, also called the second moment of area is a measure of an object resistance to change. For example the second moment of area about the $x-$axis is given by:

$$I_{xx} = \int y^2\, dy\, dx$$

Where y is the distance to the axis. It's clear from the equation that the dimension is $[L^4]$

e) Work is a product of force and distance. Hence, its dimension is $[F\, L]$. In the MLT system, the dimension of work is $[M\, L^2\, T^{-2}]$

Exercise 1.2

If P is a force and x a length, what are the dimensions (in the FLT system) of:

a) $\dfrac{dP}{dx}$

b) $\dfrac{d^3P}{dx^3}$

c) $\displaystyle\int P\ dx$

Solution 1.2

a) This a rate of change of P with respect to x. Finding the rate of a quantity *does not change* its dimension (e.g. the dimension of P is $[L]$ and the dimension of dp is also $[L]$). The dimension of $\dfrac{dP}{dx}$ is:

$$\frac{[F]}{[L]} \Rightarrow [F\, L^{-1}]$$

b) This is the *third derivative* of P with respect to x. It is easy to make a mistake in determining the dimension of this quantity but by simplifying the expression, the dimension will be clear:

$$\frac{d^3P}{dx^3} = \frac{d}{dx}\left(\frac{d}{dx}\left(\frac{dP}{dx}\right)\right)$$

It can be seen that the dimension of d^3P does not change because it is the rate of the rate of dP. For the denominator, it's different as we are multiplying rates hence the dimension of dx^3 is $[L^3]$. Finally, we get the dimension to be $[F\, L^{-3}]$.

c) The integral evaluates to $P\,x$ (plus a constant but we can ignore it here). The dimension of the integral is $[F\,L]$.

Exercise 1.3
Determine the dimension of the coefficients A and B which appear in the dimensionally homogeneous equation

$$\frac{d^2x}{dt^2} + A\,\frac{dx}{dt} + Bx = 0$$

Where x is a length and t is time.

Solution 1.3
The equation is dimensionally homogeneous which means all terms in the equation must have the same dimension. The dimension of $\dfrac{d^2x}{dt^2}$ is $[L\,T^{-2}]$ and all other terms should have this dimension:

Dimension of $A\,\dfrac{dx}{dt}$ is $[L\,T^{-2}]$, knowing the dimension of $\dfrac{dx}{dt}$ to be $[L\,T^{-1}]$. We get the dimension of A: $[T^{-1}]$

Dimension of Bx is also $[L\,T^{-2}]$. x is a length so its dimension is $[L]$ and hence the dimension of B is $[T^{-2}]$

Exercise 1.4
According to information found in an old hydraulics book, the energy loss per unit weight of fluid flowing through a nozzle connected to a hose can be estimated by the formula

$$h = (0.04 \text{ to } 0.09)(D/d)^4 V^2/2g$$

where h is the energy loss per unit weight, D the hose diameter, d the nozzle tip diameter, V the fluid velocity in the hose, and g the acceleration of gravity. Do you think this equation is valid in any system of units? Explain.

Solution 1.4
Energy has the dimension $[F\,L]$ and thus energy per unit weight has a dimension $[\dfrac{F\,L}{F}]$. The dimension of other terms in the equation has already been found previously. By substituting each dimension in the equation we get:

$$\begin{aligned} \left[\frac{F\,L}{F}\right] &= (0.04 \text{ to } 0.09)\left[\frac{L^4}{L^4}\right](\frac{1}{2})\left[\frac{L^2}{T^2}\right]\left[\frac{T^2}{L}\right] \\ [L] &= [L] \end{aligned}$$

The dimensions are equal and hence this equation is dimensionally homogeneous and is valid in any system of units.

Exercise 1.5
Make use of Table 1.3 to express the following quantities in SI units:

a) $10.2\,\text{in/min}$

b) $4.81\,\text{slug}$

c) $3.02\,\text{lb}$

d) $73.1\,\text{ft/s}^2$

e) $0.0234\,\text{lb}\cdot\text{s/ft}^2$

Solution 1.5

a)

$$\begin{aligned}10.2\,\text{in/min} &\Rightarrow (10.2\,\text{in/min})(2.540\times10^{-2}\,\text{m/in})(\frac{1}{60}\,\text{min/s})\\ &= 4.32\times10^{-3}\,\text{m/s} = 4.32\,\text{mm/s}\end{aligned}$$

b)

$$\begin{aligned}4.81\,\text{slug} &\Rightarrow (4.81\,\text{slug})(14.59\,\text{kg/slug})\\ &= 70.2\,\text{kg}\end{aligned}$$

c)

$$\begin{aligned}3.02\,\text{lb} &\Rightarrow (3.02\,\text{lb})(4.448\,\text{N/lb})\\ &= 13.4\,\text{N}\end{aligned}$$

d)

$$\begin{aligned}73.1\,\text{ft/s}^2 &\Rightarrow (73.1\,\text{ft/s}^2)(3.0481\times10^{-1}\,\frac{\text{m/s}^2}{\text{ft/s}^2})\\ &= 22.3\,\text{m/s}^2\end{aligned}$$

e)

$$0.0234\,\text{lb.s/ft}^2 \Rightarrow (0.0234\,\text{lb.s/ft}^2)(47.88\,\frac{\text{N.s/m}^2}{\text{lb.s/ft}^2})$$
$$= 1.12\,\text{N.s/m}^2$$

Exercise 1.6
An important dimensionless parameter in certain types of fluid flow problems is the Froude number defined as $V/\sqrt{gl}$, where V is a velocity, g the acceleration of gravity, and l a length. Determine the value of the Froude number for $V = 10\,\text{ft/s}$, and $l = 2\text{ft}$. Recalculate the Froude number using SI units for V, g and l. Explain the significance of the results of these calculations.

Exercise 1.7
The information on a can of pop indicates that the can contains 355 mL. The mass of a full can of pop is 0.369 kg while an empty can weighs 0.153 N. Determine the specific weight, density, and specific gravity of the pop and compare your results with the corresponding values for water at 20 °C. Express your results in SI units.

Solution 1.7
We first find the specific weight using the equation:

$$\gamma = \frac{\text{weight of fluid}}{\text{volume of fluid}}$$

Weight of the fluid can be obtained by subtracting the weight of the can from the total weight (weight of can + weight of pop). We are given the weight of the can (0.153 N) but not the total weight. A total mass is rather given (0.369 kg). We can find the weight using:

$$W_{total} = m_{total} * g$$
Where g is the acceleration due to gravity.
$$W_{total} = 3.62\,\text{N}$$
$$\therefore \text{weight of fluid} = 3.62 - 0.153\text{N} = 3.467\,\text{N}$$

We now need the volume of fluid, it is given in units of mL. This needs to be converted to m^3.

$$1\,\text{m}^3 = 10^6\,\text{mL}^3 \therefore 355\,\text{mL} \qquad = 355 \times 10^{-6}\,\text{m}^3$$

Finally the specific weight is equal to: $\gamma = 9770\,\text{N/m}^3$.

To obtain the density ρ we use γ. These are related by the gravitational constant g (Specific weight is a measure of the force exerted on a unit volume of fluid by gravity).

$$\gamma = \rho\, g$$
$$\therefore \rho = \frac{\gamma}{g} = \frac{9770\,\mathrm{N/m^3}}{9.81\,\mathrm{m/s^2}} = 996\,\mathrm{N \cdot s^2/m^4} = 996\,\mathrm{kg/m^3}$$

We now calculate the specific gravity using the density of water at 4 °C as a reference density:

$$SG = \frac{\rho}{\rho_{H_2O@4\,^\circ\mathrm{C}}} = 0.996$$

Exercise 1.8
The temperature and pressure at the surface of Mars during a Martian spring day were determined to be −50 °C and 900 Pa, respectively.

a) Determine the density of the Martian atmosphere for these conditions if the gas constant for the Martian atmosphere is assumed top be equivalent to that of carbon dioxide.

b) Compare the answer from part (*a*) with the density of the earth's atmosphere during a spring day when the temperature is 18 °C and the pressure 101.6 kPa (abs).

Exercise 1.9
Calculate the Reynolds numbers for the flow of water and for air through a 4 mm diameter tube, if the mean velocity is 3 m/s and the temperature is 30 °C in both cases. Assume the air is at standard atmospheric pressure.

Exercise 1.10
Estimate the increase in pressure (in psi) required to decrease a unit volume of mercury by 0.1 %

Exercise 1.11
Often the assumption is made that the flow of a certain fluid can be considered as incompressible flow if the density of the fluid changes by less than 2 %. If air is flowing through a tube such that the air pressure at one section is 9.0 psi (gauge) and at a downstream section it is 8.5 psi (gauge) at the same temperature, do you think that this flow could be considered an incompressible flow? Support your answer with the necessary calculations. Assume standard atmospheric pressure.

Exercise 1.12

Natural gas at 70 °F and standard atmospheric pressure of 14.7 psi (abs) is compressed isentropically to a new absolute pressure of 70 psi. Determine the final density and temperature of the gas.

Exercise 1.13

Compare the isentropic bulk modulus of air at 101 kPa (abs) with that of water at the same pressure.