Quaid-e-Awam University of Engineering, Science and Technology

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In the name of Allah, the Most Merciful, the Most Kind

Date: 30-09-2021

BCS 103
Digital Logic & Computer Architecture

Lecture 15 and 16

Department of Computer Science

Boolean Algebra

Boolean Algebra

Boolean Algebra is the mathematics we use to analyse digital gates and circuits. We can use these “Laws of Boolean” to both reduce and simplify a complex Boolean expression in an attempt to reduce the number of logic gates required. Boolean Algebra is therefore a system of mathematics based on logic that has its own set of rules or laws which are used to define and reduce Boolean expressions.

Boolean Algebra

The variables used in Boolean Algebra only have one of two possible values, a logic “0” and a logic “1” but an expression can have an infinite number of variables all labelled individually to represent inputs to the expression, For example, variables A, B, C etc, giving us a logical expression of A + B = C, but each variable can ONLY be a 0 or a 1.

Boolean Algebra

- **Boolean Constants**
  - these are ‘0’ (false) and ‘1’ (true)
- **Boolean Variables**
  - variables that can only take the vales ‘0’ or ‘1’
- **Boolean Functions**
  - each of the logic functions (such as AND, OR and NOT) are represented by symbols
- **Boolean Theorems**
  - a set of **identities** and **laws**

Boolean Algebra Functions

Using the information above, simple 2-input AND, OR and NOT Gates can be represented by 16 possible functions as shown in the following table.

Function	Description	Expression
1.	NULL	0
2.	IDENTITY	1
3.	Input A	A
4.	Input B	B

Boolean Algebra Functions

Function	Description	Expression
5.	NOT A	$\overline{A}$
6.	NOT B	$\overline{B}$
7.	A AND B (AND)	$A . B$
8.	A AND NOT B	$A . \overline{B}$
9.	NOT A AND B	$\overline{A} . B$
10.	NOT AND (NAND)	$\overline{A . B}$

Boolean Algebra Functions

Function	Description	Expression
11.	A OR B (OR)	$A + B$
12.	A OR NOT B	$A + \overline{B}$
13.	NOT A OR B	$\overline{A} + B$
14.	NOT OR (NOR)	$\overline{A + B}$
15.	Exclusive-OR	$A . \overline{B} + \overline{A} . B$
16.	Exclusive-NOR	$A . B + \overline{A} . \overline{B}$

Boolean identities

AND Function	OR Function	NOT function
$0 \bullet 0 = 0$	$0 + 0 = 0$	$\overline{0} = 1$
$0 \bullet 1 = 0$	$0 + 1 = 1$	$\overline{1} = 0$
$1 \bullet 0 = 0$	$1 + 0 = 1$	$\overline{\overline{A}} = A$
$1 \bullet 1 = 1$	$1 + 1 = 1$	
$A \bullet 0 = 0$	$A + 0 = A$	
$0 \bullet A = 0$	$0 + A = A$	
$A \bullet 1 = A$	$A + 1 = 1$	
$1 \bullet A = A$	$1 + A = 1$	
$A \bullet A = A$	$A + A = A$	
$A \bullet \overline{A} = 0$	$A + \overline{A} = 1$	

Boolean laws

Commutative law $AB = BA$ $A + B = B + A$	**Absorption law** $A + AB = A$ $A(A + B) = A$
Distributive law $A(B + C) = AB + BC$ $A + BC = (A + B)(A + C)$	**De Morgan's law** $\overline{A + B} = \overline{A} \bullet \overline{B}$ $\overline{A \bullet B} = \overline{A} + \overline{B}$
Associative law $A(BC) = (AB)C$ $A + (B + C) = (A + B) + C$	**Note also** $A + \overline{A}B = A + B$ $A(\overline{A} + B) = AB$

Thanks