Quaid-e-Awam University of Engineering, Science and Technology

بِسۡمِ ٱللَّهِ ٱلرَّحۡمَٰنِ ٱلرَّحِيمِ

In the name of Allah, the Most Merciful, the Most Kind

Date: 04-10-2021

BCS 103
Digital Logic & Computer Architecture

Lecture 17 and 18

Department of Computer Science

Boolean Algebra

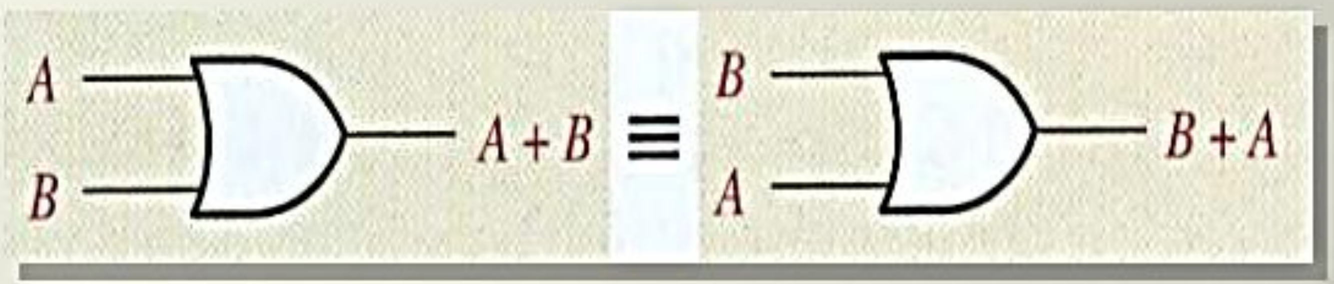
Boolean laws

Commutative law	Absorption law
$AB = BA$ $A + B = B + A$	$A + AB = A$ $A(A + B) = A$
Distributive law	**De Morgan's law**
$A(B + C) = AB + BC$ $A + BC = (A + B)(A + C)$	$\overline{A + B} = \overline{A} \bullet \overline{B}$ $\overline{A \bullet B} = \overline{A} + \overline{B}$
Associative law	**Note also**
$A(BC) = (AB)C$ $A + (B + C) = (A + B) + C$	$A + \overline{A}B = A + B$ $A(\overline{A} + B) = AB$

Laws of Boolean Algebra

- Commutative Law of Addition:

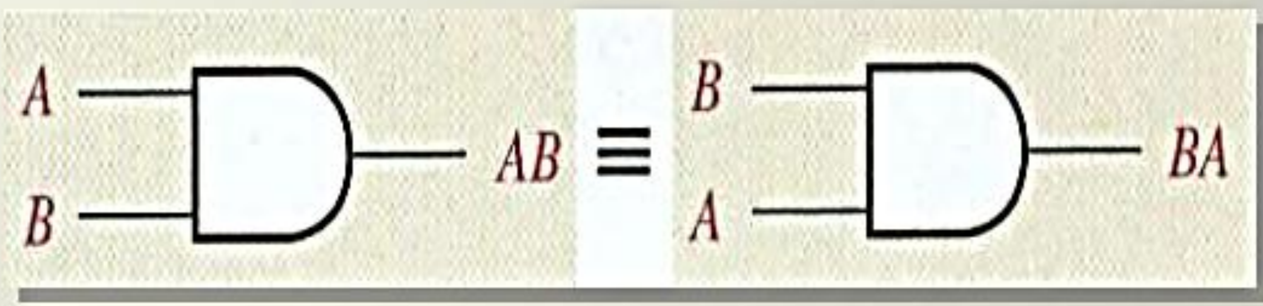
$$A + B = B + A$$



Laws of Boolean Algebra

- Commutative Law of Multiplication:

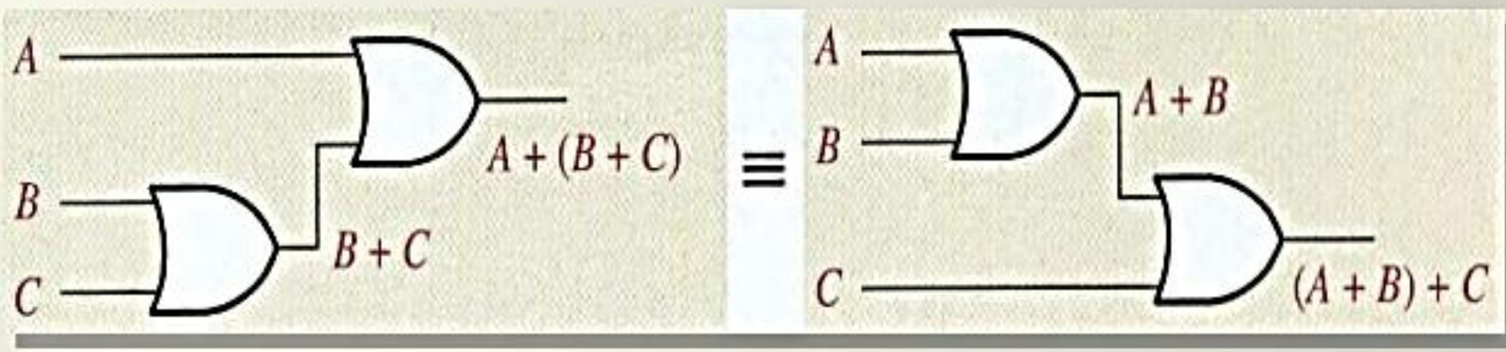
$$A * B = B * A$$



Laws of Boolean Algebra

- Associative Law of Addition:

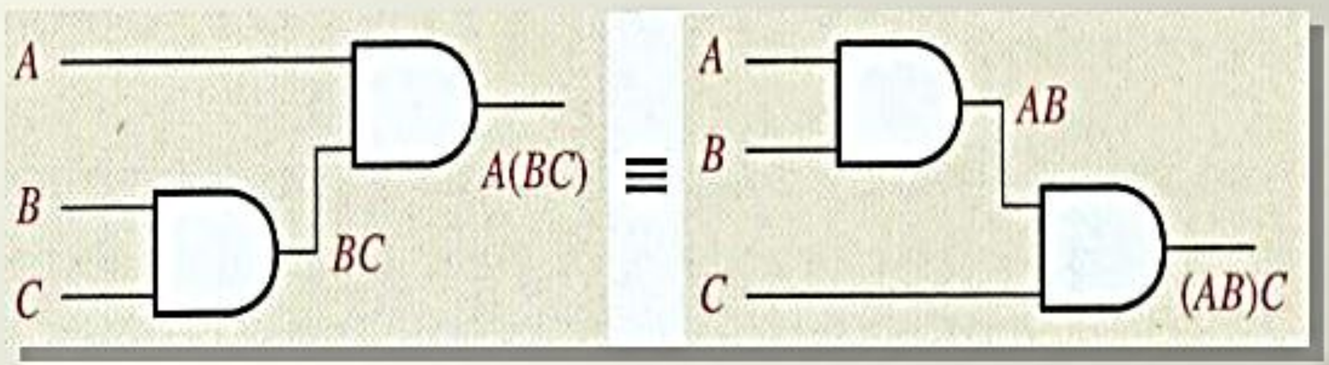
$$A + (B + C) = (A + B) + C$$



Laws of Boolean Algebra

- Associative Law of Multiplication:

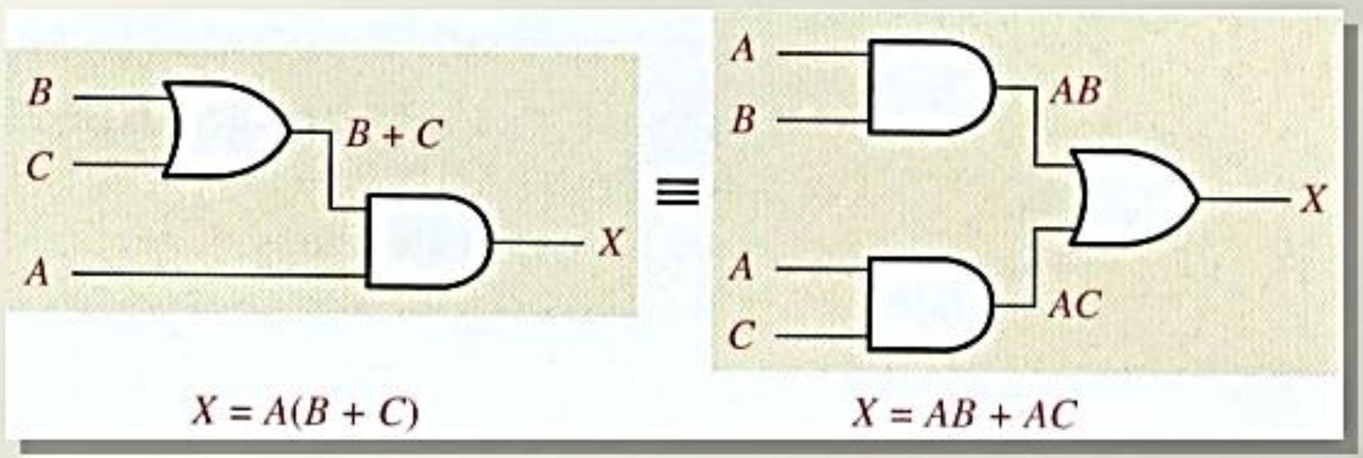
$$A * (B * C) = (A * B) * C$$



Laws of Boolean Algebra

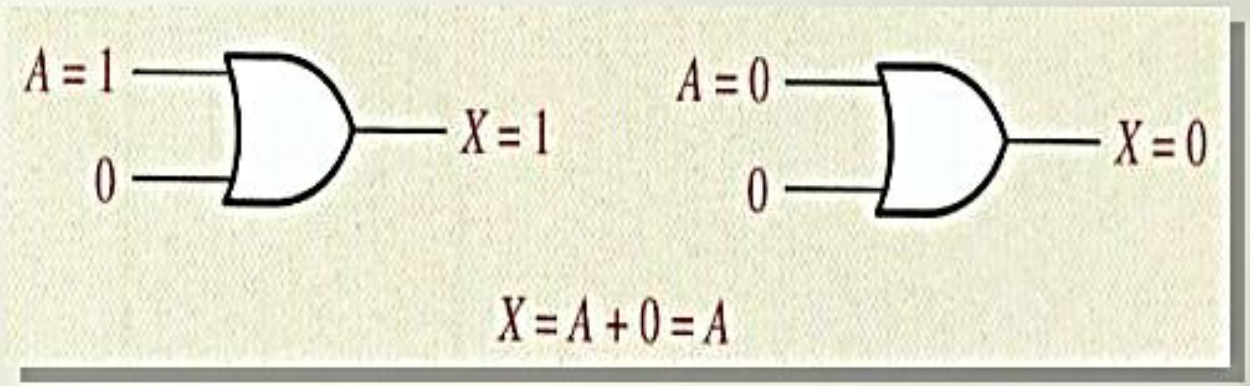
- Distributive Law:

$$A(B + C) = AB + AC$$



Rules of Boolean Algebra

- Rule 1 IDENTITY w.r.t ADDITION

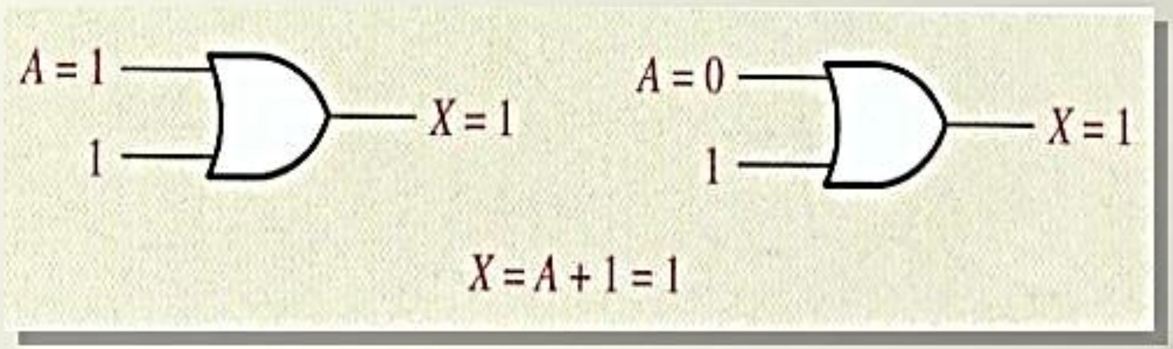


A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

OR Truth Table

Rules of Boolean Algebra

- Rule 2 NULL w.r.t ADDITION

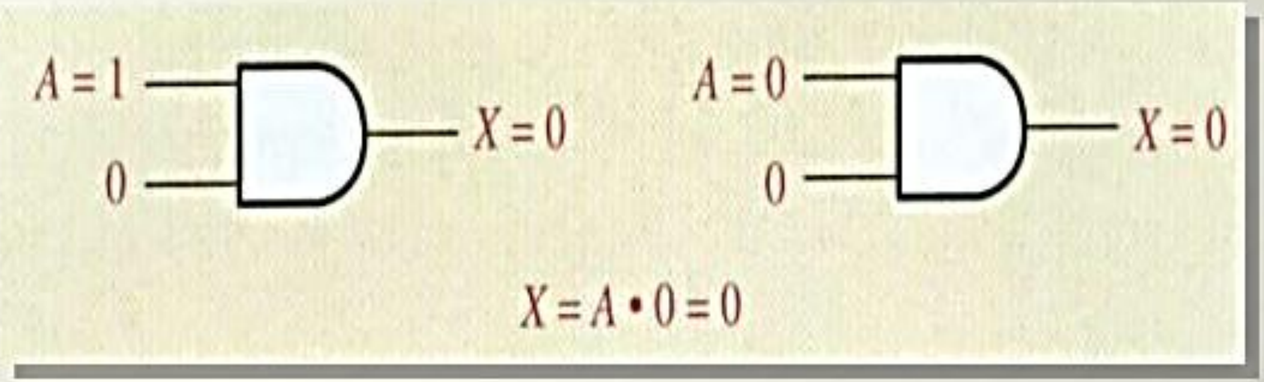


A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

OR Truth Table

Rules of Boolean Algebra

- Rule 3 NULL w.r.t MULTIPLICATION

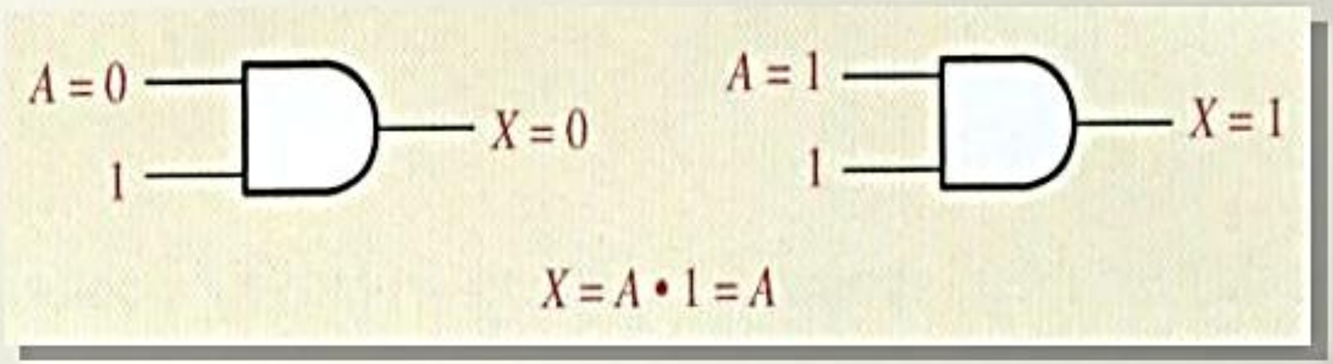


A	B	X
0	0	0
0	1	0
1	0	0
1	1	1

AND Truth Table

Rules of Boolean Algebra

- Rule 4 IDENTITY w.r.t MULTIPLICATION

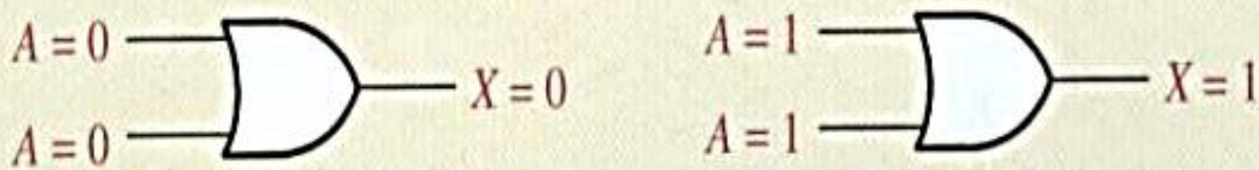


A	B	X
0	0	0
0	1	0
1	0	0
1	1	1

AND Truth Table

Rules of Boolean Algebra

- Rule 5 IDEMPOTENT w.r.t ADDITION



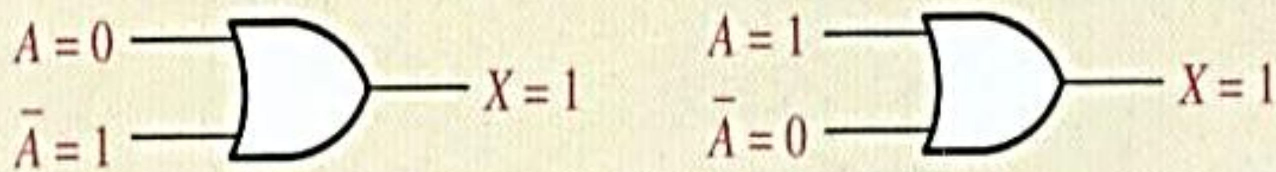
$$X = A + A = A$$

A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

OR Truth Table

Rules of Boolean Algebra

- Rule 6 COMPLEMENTARITY w.r.t ADDITION



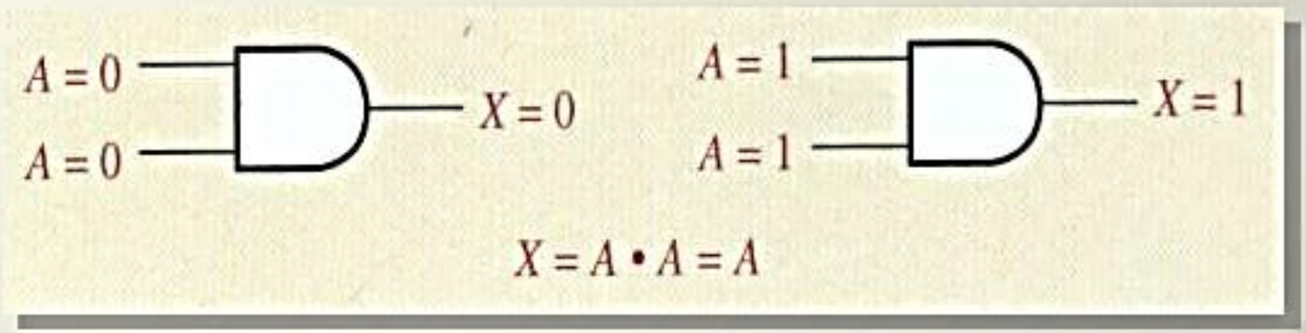
$$X = A + \bar{A} = 1$$

A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

OR Truth Table

Rules of Boolean Algebra

- Rule 7 IDEMPOTENT w.r.t MULTIPLICATION

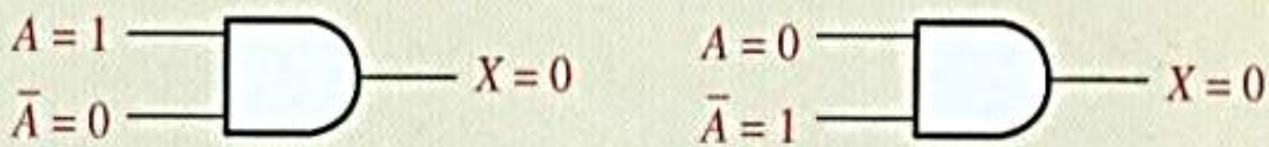


A	B	X
0	0	0
0	1	0
1	0	0
1	1	1

AND Truth Table

Rules of Boolean Algebra

- Rule 8 COMPLEMENTARITY w.r.t MULTIPLICATION



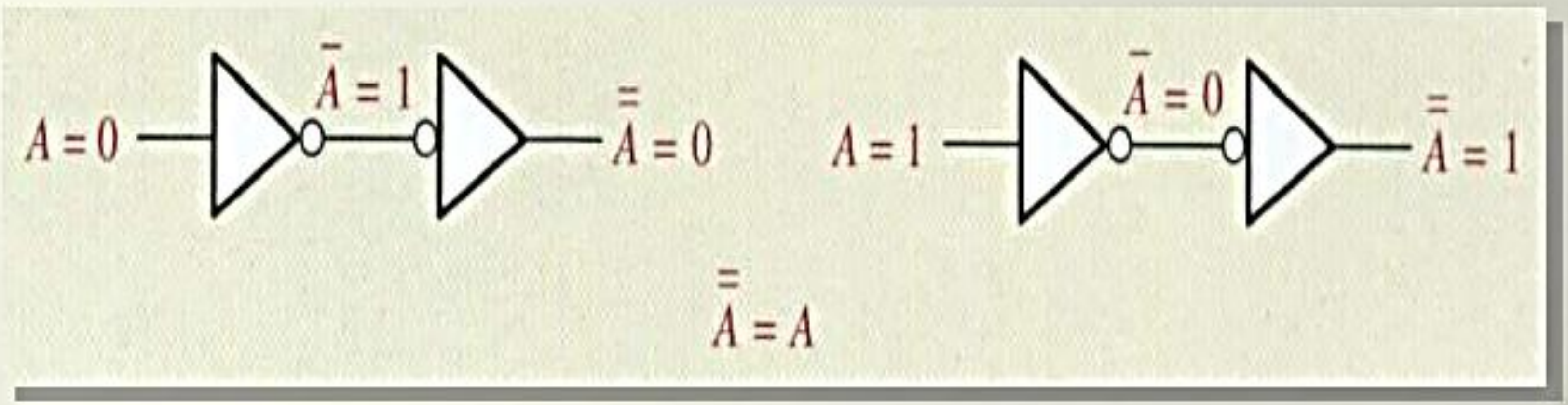
$$X = A \cdot \bar{A} = 0$$

A	B	X
0	0	0
0	1	0
1	0	0
1	1	1

AND Truth Table

Rules of Boolean Algebra

- Rule 9 INVOLUTION

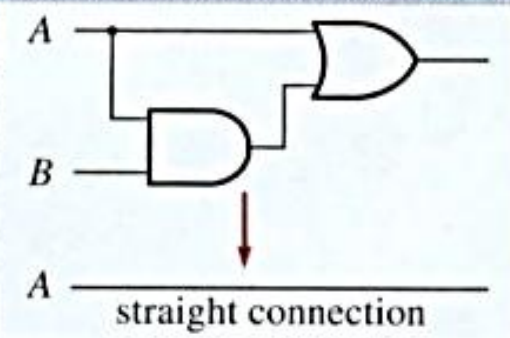


Rules of Boolean Algebra

- Rule 10: A + AB = A

A	B	AB	A + AB
0	0	0	0
0	1	0	0
1	0	0	1
1	1	1	1

equal



A	B	X
0	0	0
0	1	0
1	0	0
1	1	1

AND Truth Table

A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

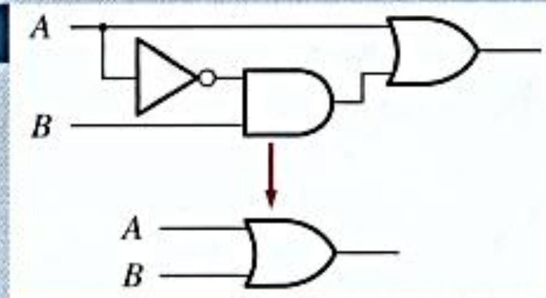
OR Truth Table

Rules of Boolean Algebra

- Rule 11: $A + \overline{A}B = A + B$

A	B	$\overline{A}B$	$A + \overline{A}B$	$A + B$
0	0	0	0	0
0	1	1	1	1
1	0	0	1	1
1	1	0	1	1

equal



A	B	X
0	0	0
0	1	0
1	0	0
1	1	1

AND Truth Table

A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

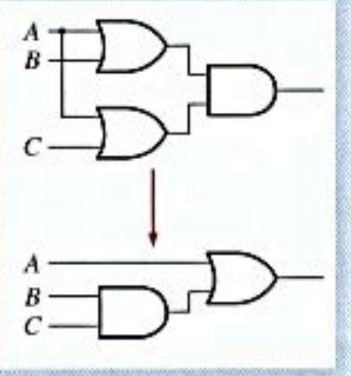
OR Truth Table

Rules of Boolean Algebra

- Rule 12: (A + B)(A + C) = A + BC

A	B	C	A + B	A + C	(A + B)(A + C)	BC	A + BC
0	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	1	1	1	1
1	0	0	1	1	1	0	1
1	0	1	1	1	1	0	1
1	1	0	1	1	1	0	1
1	1	1	1	1	1	1	1

equal



A	B	X
0	0	0
0	1	0
1	0	0
1	1	1

AND Truth Table

A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

OR Truth Table

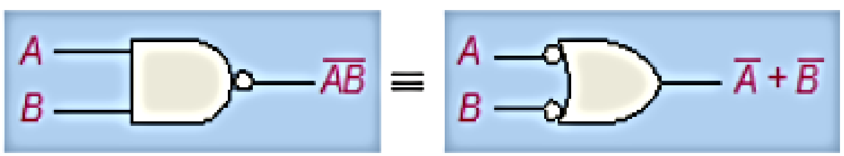
DeMorgan's Theorem

DeMorgan's 1st Theorem

The complement of a product of variables is equal to the sum of the complemented variables.

$$\overline{AB} = \overline{A} + \overline{B}$$

Applying DeMorgan's first theorem to gates:



NAND   Negative-OR

Inputs		Output	
A	B	$\overline{AB}$	$\overline{A} + \overline{B}$
0	0	1	1
0	1	1	1
1	0	1	1
1	1	0	0

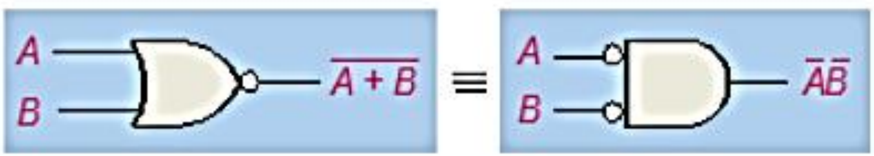
DeMorgan's Theorem

DeMorgan's 2nd Theorem

The complement of a sum of variables is equal to the product of the complemented variables.

$$\overline{A + B} = \overline{A} \cdot \overline{B}$$

Applying DeMorgan's second theorem to gates:



NOR Negative-AND

Inputs		Output	
A	B	$\overline{A + B}$	$\overline{A}\overline{B}$
0	0	1	1
0	1	0	0
1	0	0	0
1	1	0	0

Simplification

Simplification

- Simplify the following expression

$$A + AB$$

$\downarrow$ Factoring A out of both terms

$$A(1 + B)$$

$\downarrow$ Applying identity $A + 1 = 1$

$$A(1)$$

$\downarrow$ Applying identity $1A = A$

$$A$$

Simplification

- Simplify the following expression

$$A + \overline{A}B$$

$\downarrow$ Applying the previous rule to expand A term $A + AB = A$

$$A + AB + \overline{A}B$$

$\downarrow$ Factoring B out of 2nd and 3rd terms

$$A + B(A + \overline{A})$$

$\downarrow$ Applying identity $A + \overline{A} = 1$

$$A + B(1)$$

$\downarrow$ Applying identity $1A = A$

$$A + B$$

Simplification

- Simplify the following expression

$$(A + B)(A + C)$$

$$\downarrow \quad \text{Distributing terms}$$

$$AA + AC + AB + BC$$

$$\downarrow \quad \text{Applying identity } AA = A$$

$$A + AC + AB + BC$$

$$\downarrow \quad \text{Applying identity } A + AB = A \text{ to the } A + AC \text{ term}$$

$$A + AB + BC$$

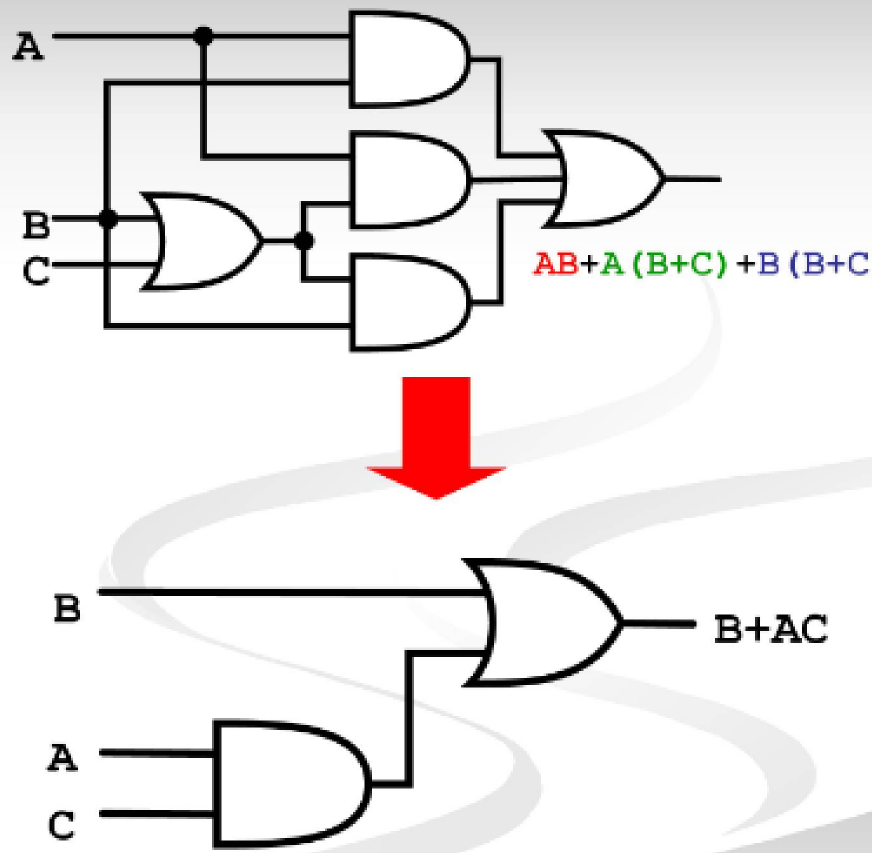
$$\downarrow \quad \text{Applying identity } A + AB = A \text{ to the } A + AB \text{ term}$$

$$A + BC$$

Simplification

$AB+A(B+C)+B(B+C)$

- (distributive law)
  - $AB+AB+AC+BB+BC$
- (rule 7; $BB=B$)
  - $AB+AB+AC+\mathbf{B}+BC$
- (rule 5; $AB+AB=AB$)
  - $\mathbf{AB}+AC+B+BC$
- (rule 10; $B+BC=B$)
  - $AB+AC+\mathbf{B}$
- (rule 10; $AB+B=B$)
  - $\mathbf{B}+AC$



Simplification

- Simplify the following expression

Expression	Rule(s) Used
$\overline{AB}(\overline{A}+B)(\overline{B}+B)$	Original Expression
$\overline{AB}(\overline{A}+B)$	Complement law, Identity law.
$(\overline{A}+\overline{B})(\overline{A}+B)$	DeMorgan's Law
$\overline{A}+\overline{B}B$	Distributive law. This step uses
$\overline{A}$	Complement, Identity.

Simplification

- Simplify the following expression

Expression	Rule(s) Used
$(A + C)(AD + A\overline{D}) + AC + C$	Original Expression
$(A + C)A(D + \overline{D}) + AC + C$	Distributive.
$(A + C)A + AC + C$	Complement, Identity.
$A((A + C) + C) + C$	Commutative, Distributive.
$A(A + C) + C$	Associative, Idempotent.
$AA + AC + C$	Distributive.
$A + (A + T)C$	Idempotent, Identity, Distributive.
$A + C$	Identity, twice.

Simplification

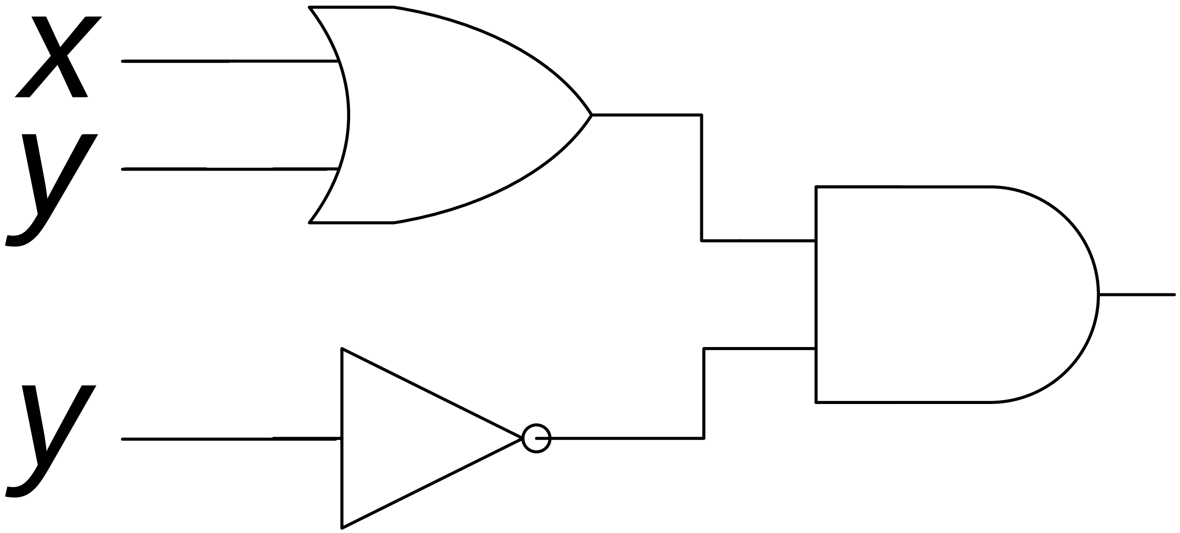
- Simplify the following expression

Simplify: $\overline{A}(A+B)+(B+AA)(A+\overline{B})$:

Expression	Rule(s) Used
$\overline{A}(A+B)+(B+AA)(A+\overline{B})$	Original Expression
$\overline{A}A+\overline{A}B+(B+A)A+(B+A)\overline{B}$	Idempotent (AA to A), then Distributive, used twice.
$\overline{A}B+(B+A)A+(B+A)\overline{B}$	Complement, then Identity. (Strictly speaking, we also used the Commu
$\overline{A}B+BA+AA+B\overline{B}+A\overline{B}$	Distributive, two places.
$\overline{A}B+BA+A+A\overline{B}$	Idempotent (for the A's), then Complement and Identity to remove $B\overline{B}$.
$\overline{A}B+AB+A\mathit{T}+A\overline{B}$	Commutative, Identity; setting up for the next step.
$\overline{A}B+A(B+\mathit{T}+\overline{B})$	Distributive.
$\overline{A}B+A$	Identity, twice (depending how you count it).
$A+\overline{A}B$	Commutative.
$(A+\overline{A})(A+B)$	Distributive.
$A+B$	Complement, Identity.

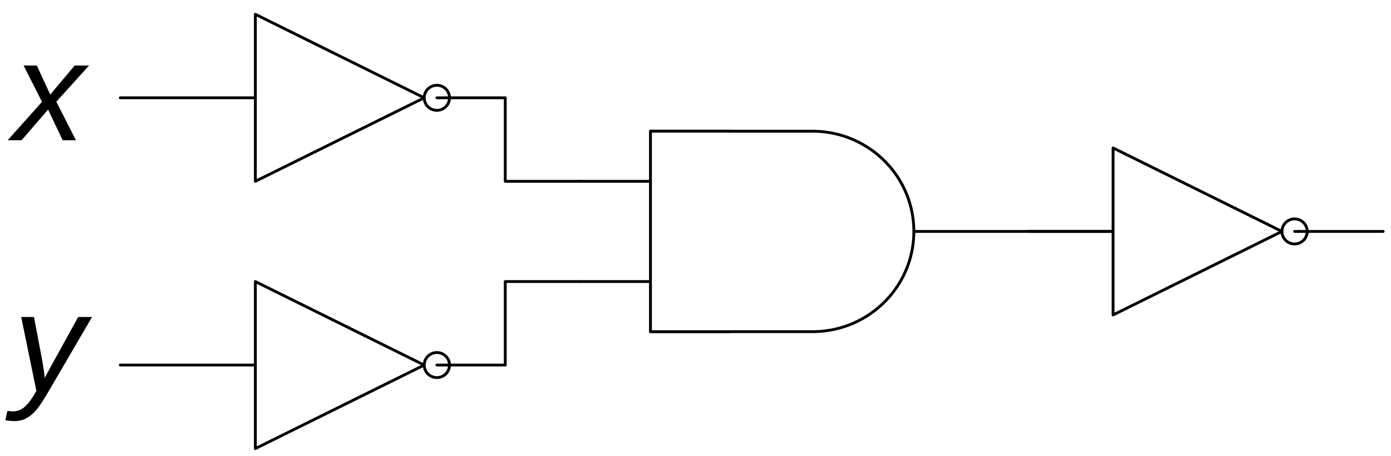
Generate Equation

- Find the output of the following circuit



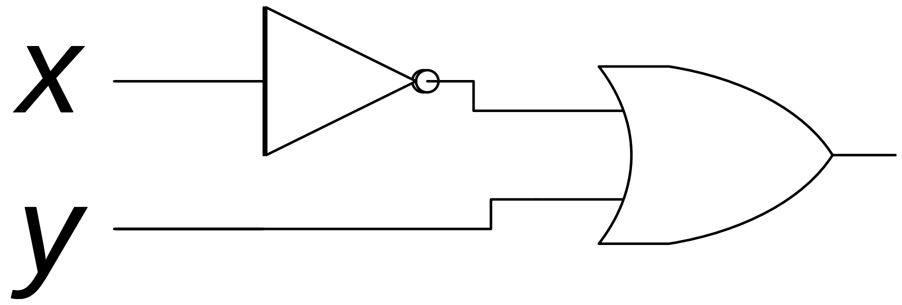
Generate Equation

- Find the output of the following circuit



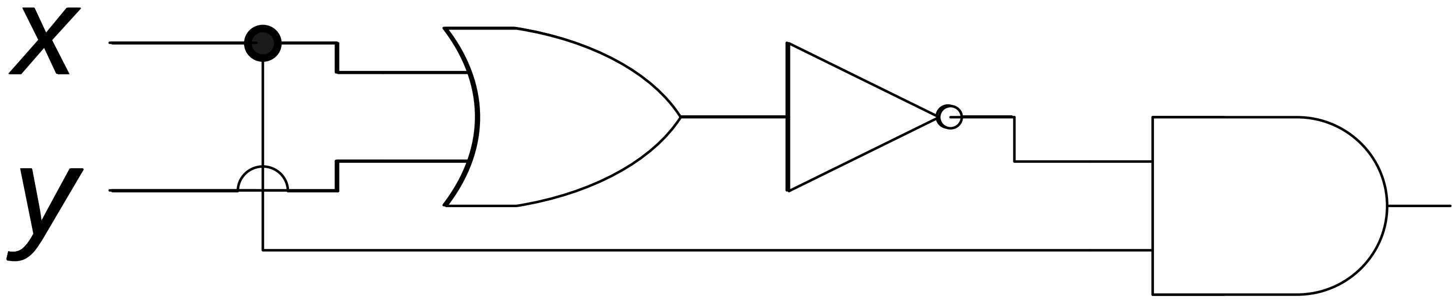
Generate Equation

- Find the output of the following circuit



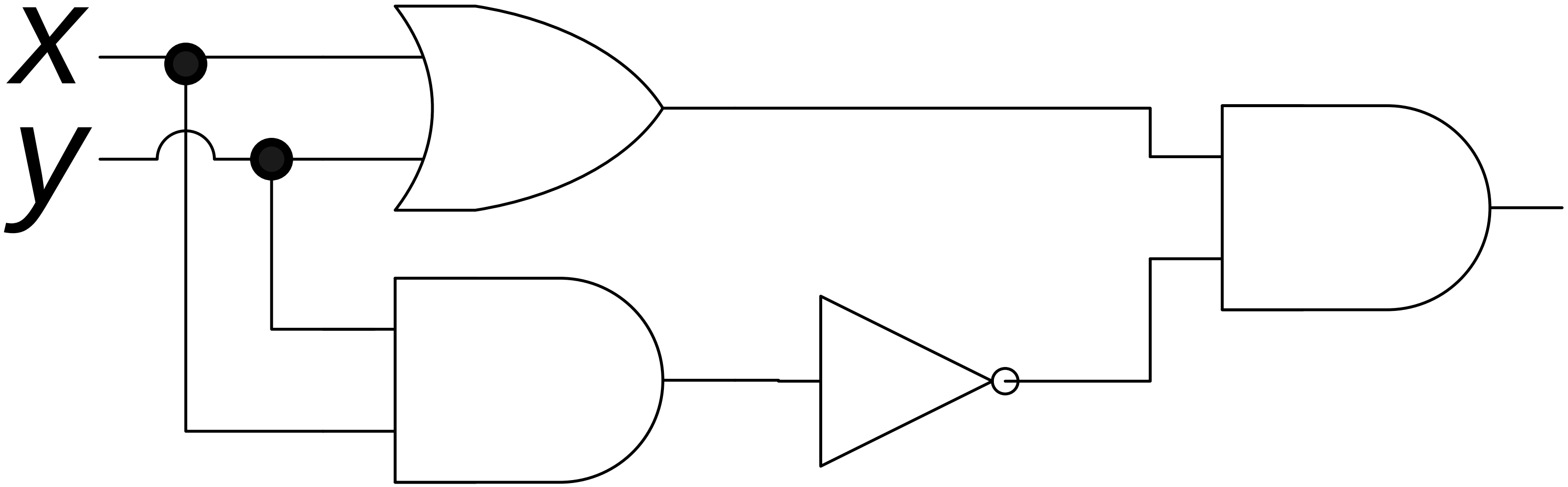
Generate Equation

- Find the output of the following circuit



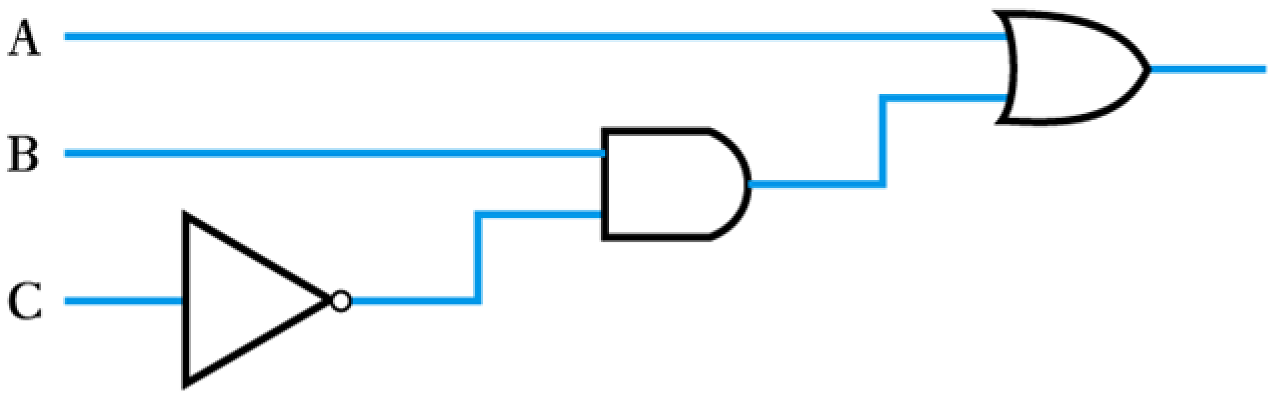
Generate Equation

- Find the output of the following circuit



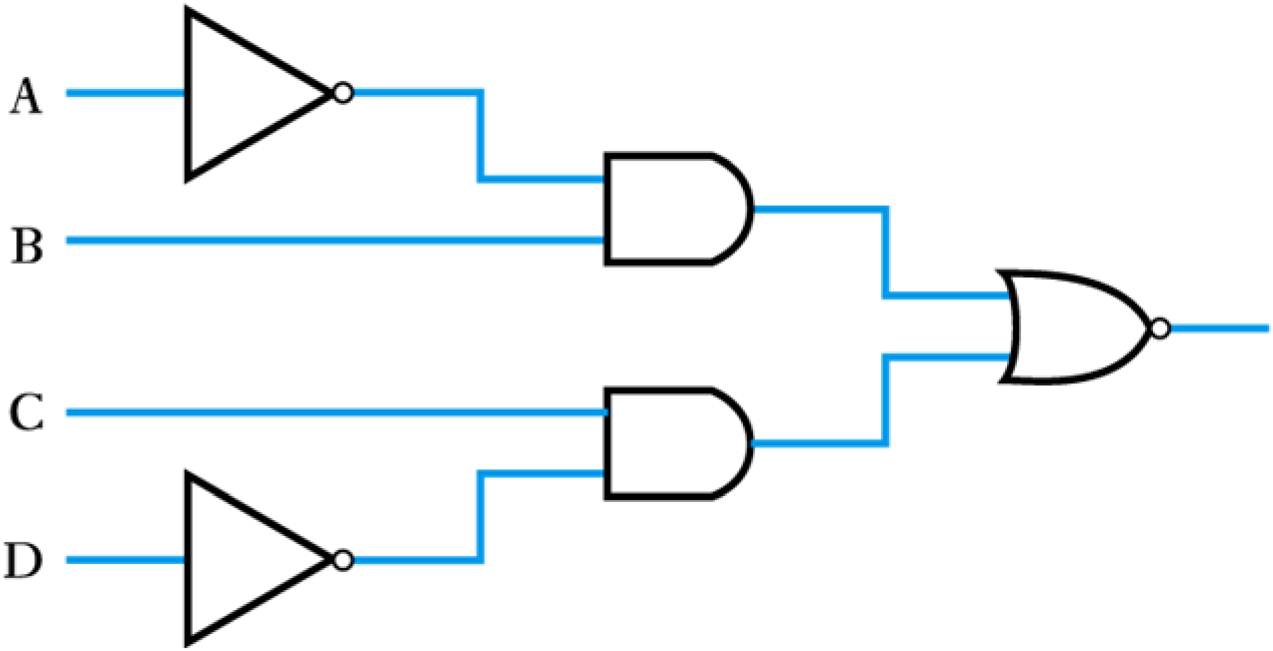
Generate Equation

- Find the output of the following circuit



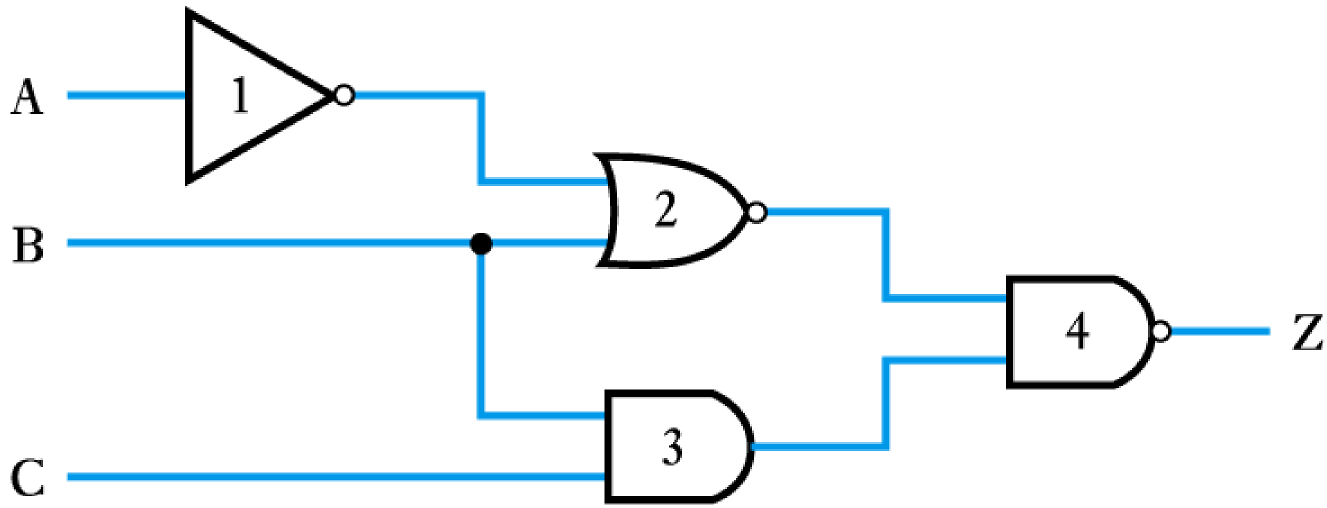
Generate Equation

- Find the output of the following circuit



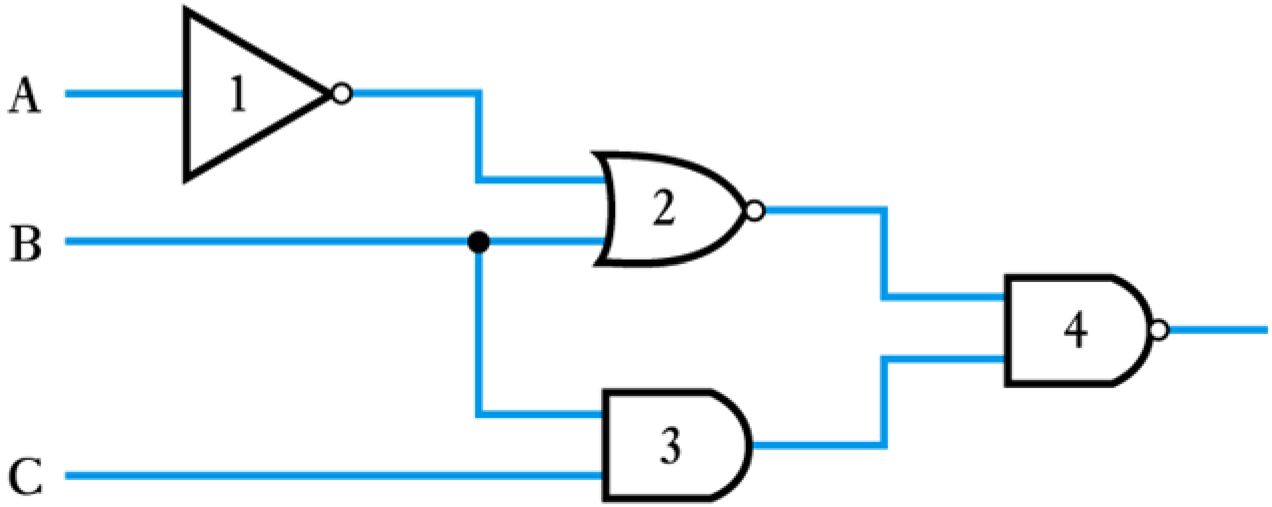
Generate Equation

- Find the output of the following circuit



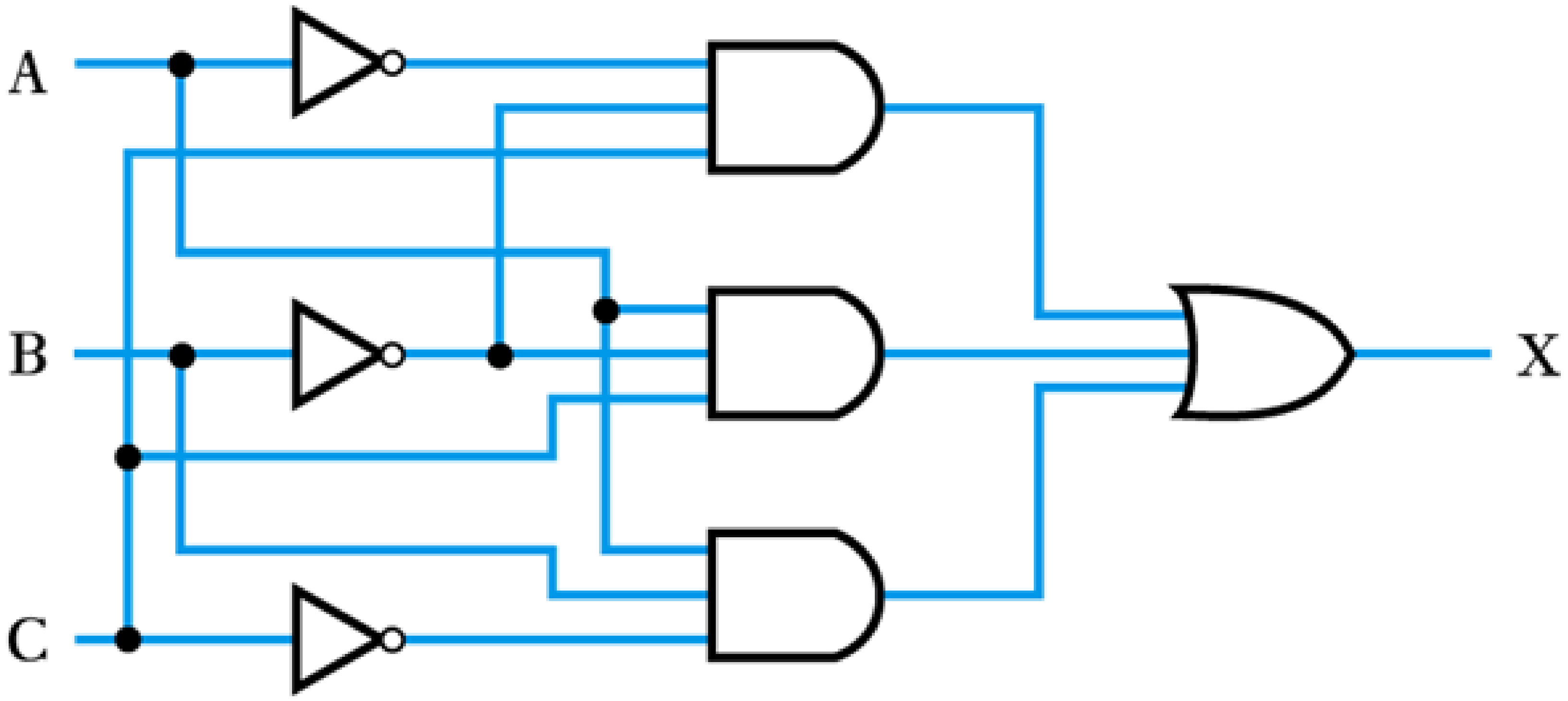
Generate Equation

- Find the output of the following circuit



Generate Equation

- Find the output of the following circuit



Example 1

Determine if the following equation is valid

$$\bar{x}_1\bar{x}_3 + x_2x_3 + x_1\bar{x}_2 = \bar{x}_1x_2 + x_1x_3 + \bar{x}_2\bar{x}_3$$

Left-Hand Side (LHS)

Row number	x_1	x_2	x_3	$\bar{x}_1 \bar{x}_3$	$x_2 x_3$	$x_1 \bar{x}_2$	f
0	0	0	0	1	0	0	1
1	0	0	1	0	0	0	0
2	0	1	0	1	0	0	1
3	0	1	1	0	1	0	1
4	1	0	0	0	0	1	1
5	1	0	1	0	0	1	1
6	1	1	0	0	0	0	0
7	1	1	1	0	1	0	1

Right-Hand Side (RHS)

Row number	x_1	x_2	x_3	$\overline{x_1} x_2$	$x_1 x_3$	$\overline{x_2}\,\overline{x_3}$	f
0	0	0	0	0	0	1	1
1	0	0	1	0	0	0	0
2	0	1	0	1	0	0	1
3	0	1	1	1	0	0	1
4	1	0	0	0	0	1	1
5	1	0	1	0	1	0	1
6	1	1	0	0	0	0	0
7	1	1	1	0	1	0	1

$$\underbrace{\bar{x}_1\bar{x}_3 + x_2x_3 + x_1\bar{x}_2}_{\text{LHS}} \stackrel{?}{=} \underbrace{\bar{x}_1x_2 + x_1x_3 + \bar{x}_2\bar{x}_3}_{\text{RHS}}$$

f
1
0
1
1
1
1
0
1

f
1
0
1
1
1
1
0
1

Thanks