



بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

In the name of Allah, the Most Merciful, the Most Kind

Date: 25-10-2021

BCS 103

Digital Logic & Computer Architecture

Lecture 23 and 24

IN THE LAST LECTURE

We have discussed

- **Karnaugh Map (3 variables)**

TODAY

We will discuss about

- **Karnaugh Map (4 variables)**

Review

- 2 variables Karnaugh map

Q: Simplify the following expressions

$$A'B' + AB + A'B$$

		A	
		0	1
B	0		
	1		

Review

- 3 variables Karnaugh map

Q: Simplify the following expressions

$$AC + AB + B'C'$$

		BC			
		00	01	11	10
A	0				
	1				

Karnaugh Maps

- 4 variables Karnaugh map

AB		00	01	11	10
CD	00				
	01				
	11				
	10				

Four-Variable K-Maps

AB \ CD	00	01	11	10
	00	01	11	10
00	1	0	0	0
01	0	0	0	0
11	0	0	0	0
10	1	0	0	0

AB \ CD	00	01	11	10
	00	01	11	10
00	0	0	0	0
01	0	1	0	0
11	0	1	0	0
10	0	0	0	0

AB \ CD	00	01	11	10
	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	0	1	1	0
10	0	0	0	0

AB \ CD	00	01	11	10
	00	01	11	10
00	0	0	0	0
01	1	0	0	1
11	0	0	0	0
10	0	0	0	0

AB \ CD	00	01	11	10
	00	01	11	10
00	0	0	1	1
01	0	0	1	1
11	0	0	0	0
10	0	0	0	0

AB \ CD	00	01	11	10
	00	01	11	10
00	0	0	0	0
01	1	0	0	1
11	1	0	0	1
10	0	0	0	0

AB \ CD	00	01	11	10
	00	01	11	10
00	0	0	1	1
01	0	0	0	0
11	0	0	0	0
10	0	0	1	1

AB \ CD	00	01	11	10
	00	01	11	10
00	1	0	0	1
01	0	0	0	0
11	0	0	0	0
10	1	0	0	1

Four-Variable K-Maps

		CD			
		00	01	11	10
AB	00	1	0	0	0
	01	0	0	0	0
	11	0	0	0	0
	10	1	0	0	0

$$f = \sum(0,8) = \bar{B} \cdot \bar{C} \cdot \bar{D}$$

		CD			
		00	01	11	10
AB	00	0	0	0	0
	01	0	1	0	0
	11	0	1	0	0
	10	0	0	0	0

$$f = \sum(5,13) = B \cdot \bar{C} \cdot D$$

		CD			
		00	01	11	10
AB	00	0	0	0	0
	01	0	0	0	0
	11	0	1	1	0
	10	0	0	0	0

$$f = \sum(13,15) = A \cdot B \cdot D$$

		CD			
		00	01	11	10
AB	00	0	0	0	0
	01	1	0	0	1
	11	0	0	0	0
	10	0	0	0	0

$$f = \sum(4,6) = \bar{A} \cdot B \cdot \bar{D}$$

		CD			
		00	01	11	10
AB	00	0	0	1	1
	01	0	0	1	1
	11	0	0	0	0
	10	0	0	0	0

$$f = \sum(2,3,6,7) = \bar{A} \cdot C$$

		CD			
		00	01	11	10
AB	00	0	0	0	0
	01	1	0	0	1
	11	1	0	0	1
	10	0	0	0	0

$$f = \sum(4,6,12,14) = B \cdot \bar{D}$$

		CD			
		00	01	11	10
AB	00	0	0	1	1
	01	0	0	0	0
	11	0	0	0	0
	10	0	0	1	1

$$f = \sum(2,3,10,11) = \bar{B} \cdot C$$

		CD			
		00	01	11	10
AB	00	1	0	0	1
	01	0	0	0	0
	11	0	0	0	0
	10	1	0	0	1

$$f = \sum(0,2,8,10) = \bar{B} \cdot \bar{D}$$

Four-Variable K-Maps

		CD			
		00	01	11	10
AB	00	0	0	0	0
	01	1	1	1	1
	11	0	0	0	0
	10	0	0	0	0

		CD			
		00	01	11	10
AB	00	0	0	1	0
	01	0	0	1	0
	11	0	0	1	0
	10	0	0	1	0

		CD			
		00	01	11	10
AB	00	1	0	1	0
	01	0	1	0	1
	11	1	0	1	0
	10	0	1	0	1

		CD			
		00	01	11	10
AB	00	0	1	0	1
	01	1	0	1	0
	11	0	1	0	1
	10	1	0	1	0

		CD			
		00	01	11	10
AB	00	0	1	1	0
	01	0	1	1	0
	11	0	1	1	0
	10	0	1	1	0

		CD			
		00	01	11	10
AB	00	1	0	0	1
	01	1	0	0	1
	11	1	0	0	1
	10	1	0	0	1

		CD			
		00	01	11	10
AB	00	0	0	0	0
	01	1	1	1	1
	11	1	1	1	1
	10	0	0	0	0

		CD			
		00	01	11	10
AB	00	1	1	1	1
	01	0	0	0	0
	11	0	0	0	0
	10	1	1	1	1

Four-Variable K-Maps

AB \ CD				
	00	01	11	10
00	0	0	0	0
01	1	1	1	1
11	0	0	0	0
10	0	0	0	0

$$f = \sum(4,5,6,7) = \bar{A} \bullet B$$

AB \ CD				
	00	01	11	10
00	0	0	1	0
01	0	0	1	0
11	0	0	1	0
10	0	0	1	0

$$f = \sum(3,7,11,15) = C \bullet D$$

AB \ CD				
	00	01	11	10
00	1	0	1	0
01	0	1	0	1
11	1	0	1	0
10	0	1	0	1

$$f = \sum(0,3,5,6,9,10,12,15)$$

$$f = A \otimes B \otimes C \otimes D$$

AB \ CD				
	00	01	11	10
00	0	1	0	1
01	1	0	1	0
11	0	1	0	1
10	1	0	1	0

$$f = \sum(1,2,4,7,8,11,13,14)$$

$$f = A \oplus B \oplus C \oplus D$$

AB \ CD				
	00	01	11	10
00	0	1	1	0
01	0	1	1	0
11	0	1	1	0
10	0	1	1	0

$$f = \sum(1,3,5,7,9,11,13,15)$$

$$f = D$$

AB \ CD				
	00	01	11	10
00	1	0	0	1
01	1	0	0	1
11	1	0	0	1
10	1	0	0	1

$$f = \sum(0,2,4,6,8,10,12,14)$$

$$f = \bar{D}$$

AB \ CD				
	00	01	11	10
00	0	0	0	0
01	1	1	1	1
11	1	1	1	1
10	0	0	0	0

$$f = \sum(4,5,6,7,12,13,14,15)$$

$$f = B$$

AB \ CD				
	00	01	11	10
00	1	1	1	1
01	0	0	0	0
11	0	0	0	0
10	1	1	1	1

$$f = \sum(0,1,2,3,8,9,10,11)$$

$$f = \bar{B}$$

Four-Variable K-Maps

		CD			
		00	01	11	10
AB	00	1	1		1
	01	1	1		1
	11	1	1		1
	10	1	1		

		CD			
		00	01	11	10
AB	00	1	1		1
	01				1
	11				
	10	1	1		1

		CD			
		00	01	11	10
AB	00				
	01	1	1	1	
	11	1	1		1
	10	1			

		CD			
		00	01	11	10
AB	00		1	1	
	01	1	1	1	1
	11	1		1	1
	10			1	

		CD			
		00	01	11	10
AB	00				
	01				
	11				
	10				

		CD			
		00	01	11	10
AB	00				
	01				
	11				
	10				

Four-Variable K-Maps

AB \ CD				
	00	01	11	10
00				
01				
11				
10				

AB \ CD				
	00	01	11	10
00				
01				
11				
10				

AB \ CD				
	00	01	11	10
00				
01				
11				
10				

AB \ CD				
	00	01	11	10
00				
01				
11				
10				

AB \ CD				
	00	01	11	10
00				
01				
11				
10				

AB \ CD				
	00	01	11	10
00				
01				
11				
10				

Exercise

4.7 Using a Karnaugh map, simplify the following functions and implement them with basic gates.

(a) $F(A, B, C, D) = \Sigma(0, 2, 3, 6, 7, 8, 10, 11, 12, 15)$

(b) $F(A, B, C, D) = \Sigma(0, 2, 3, 5, 7, 8, 13) + d(1, 6, 12)$ *

(c) $F(A, B, C, D) = \Sigma(1, 7, 9, 10, 12, 13, 14, 15) + d(4, 5, 8)$ *

(d) $F(A, B, C, D) = \pi(0, 8, 10, 11, 14) + d(6)$ *

(e) $F(A, B, C, D) = \pi(2, 8, 11, 15) + d(3, 12, 14)$ *

(f) $F(W, X, Y, Z) = \pi(0, 2, 6, 11, 13, 15) + d(1, 9, 10, 14)$ *

4.8 Prepare a Karnaugh map for the following functions.

(a) $F = ABC + A'BC + B'C'$

(b) $F = A + B + C'$

(c) $Y = AB + B'CD$

4.9 Using the Karnaugh map method, simplify the following functions, obtain their sum of the products form, and product of the sums form. Realize them with basic gates.

(a) $F(W, X, Y, Z) = \Sigma(1, 3, 4, 5, 6, 7, 9, 12, 13)$

(b) $F(W, X, Y, Z) = \Sigma(1, 5, 6, 7, 11, 12, 13, 15)$

* Questions with don't care conditions

Thanks