Quaid-e-Awam University of Engineering, Science and Technology

بِسۡمِ ٱللَّهِ ٱلرَّحۡمَٰنِ ٱلرَّحِيمِ

In the name of Allah, the Most Merciful, the Most Kind

Date: 29-10-2021

BCS 103
Digital Logic & Computer Architecture

Lecture 25 and 26

Department of Computer Science

IN THE LAST LECTURE

We have discussed

- **Karnaugh Map (4 variables)**

TODAY

Today we will learn about:

Combinational Logic Circuits

Adders

- **Half Adder**
- **Full Adder**

Combinational Logic Circuits:

Combinational logic circuits are those circuit which are composed of logic gates in other words we can say that the basic elements of combinational logic circuit are the logic gates.

In combinational logic circuits the output will be change whenever we apply the input or output depends upon present input.

Two types of Adder

Binary Adders

- Half Adder
- Full Adder

Half Adder

Half Adder

- Performs the addition of two binary bits.
- Four possible operations:
  - 0+0= 0
  - 0+1= 1
  - 1+0= 1
  - 1+1=10
- Circuit implementation requires 2 outputs; one to indicate the ***sum*** and another to indicate the ***carry***.

Truth Table

A_0	B_0	S_0	C_1
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

Half Adder

- It is a combinational logic circuit that performs addition, between two binary bits.
- It has two input x, y and two output sum and carry.
- The sum output will be high whenever their inputs have odd number of 1's.
- The carry output will be high whenever both inputs are logical 1's.

Truth Table

A_0	B_0	S_0	C_1
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

Half Adder

- Performs 1-bit addition.
- Inputs: A_0, B_0
- Outputs: S_0, C_1

Truth Table

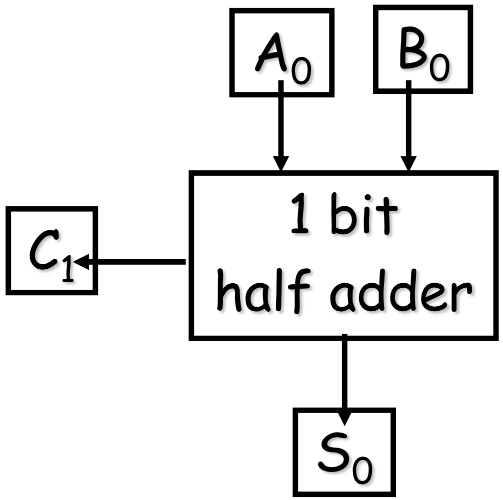
A_0	B_0	S_0	C_1
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

- Boolean equations:
  - $S_0 = A_0B_0' + A_0'B_0 = A_0 \oplus B_0$
  - $C_1 = A_0B_0$

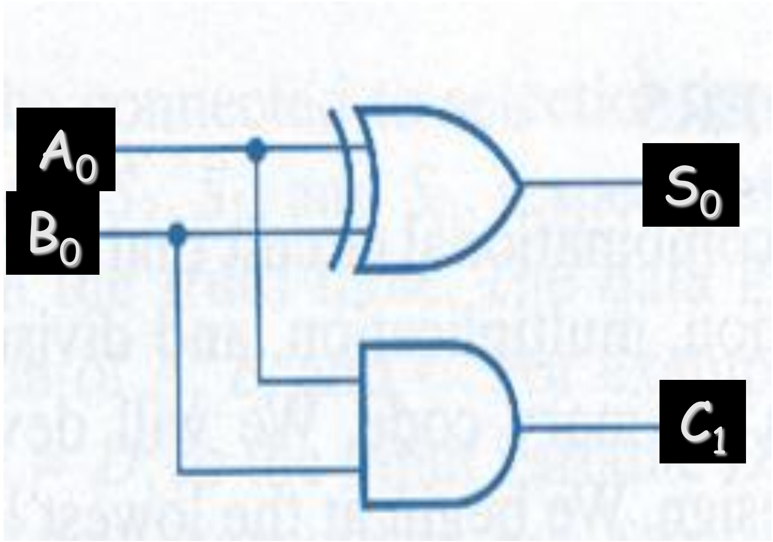
Half Adder (cont.)

- $S_0 = A_0B_0' + A_0'B_0 = A_0 \oplus B_0$
- $C_1 = A_0B_0$

Block Diagram



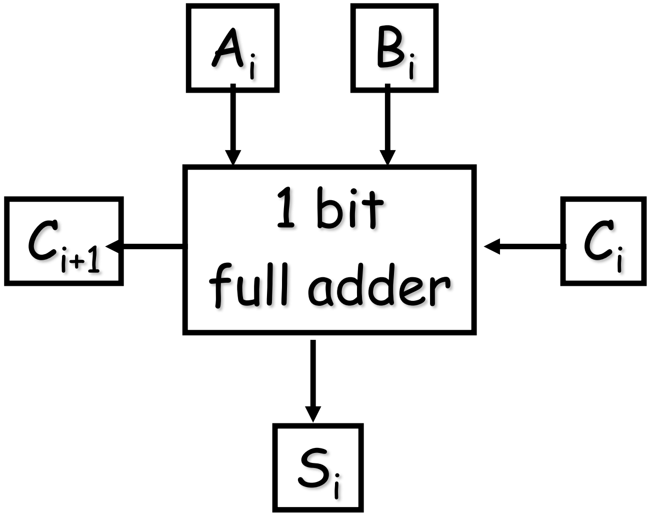
Logic Diagram



Full Adder

Full Adder

- Full adder (for higher-order bit addition)
- Combinational circuit that performs the additions of 3 bits (two bits and a carry-in bit)



Full Adder

- It is a combinational logic circuit that performs addition, between three binary bits.
- It has three inputs x , y , z and two output sum and carry.
- The sum output will be high whenever their inputs have odd number of 1’s.
- The carry output will be high whenever two or more inputs are logical 1’s.

Truth Table

A_i	B_i	C_i		S_i	C_{i+1}
0	0	0		0	0
0	0	1		1	0
0	1	0		1	0
0	1	1		0	1
1	0	0		1	0
1	0	1		0	1
1	1	0		0	1
1	1	1		1	1

Full Adder (cont.)

- The K-maps for
  - C_{i+1}:

B_iC_i / A_i

0	0	1	0
0	1	1	1

  - S_i:

B_iC_i / A_i

0	1	0	1
1	0	1	0

Truth Table

A_i	B_i	C_i		S_i	C_{i+1}
0	0	0		0	0
0	0	1		1	0
0	1	0		1	0
0	1	1		0	1
1	0	0		1	0
1	0	1		0	1
1	1	0		0	1
1	1	1		1	1

Full Adder (cont.)

A_i \ B_iC_i				
	0	0	1	0
	0	1	1	1

$$C_{i+1} = A_iB_i + A_iC_i + B_iC_i$$

A_i \ B_iC_i				
	0	1	0	1
	1	0	1	0

$$S_i = A_iB_i'\,C_i' + A_i'B_i'C_i + A_i'B_iC_i' + A_iB_iC_i$$

Full Adder (cont.)

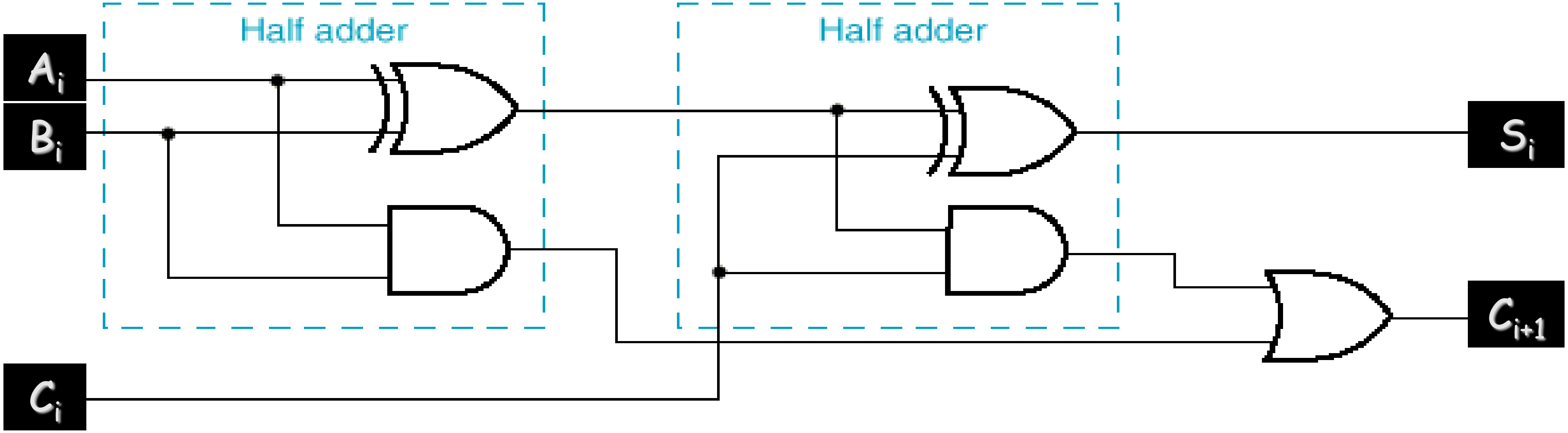
- Boolean equations:
  - $C_{i+1} = A_iB_i + A_iC_i + B_iC_i$
  - $S_i = A_iB_i'C_i' + A_i'B_i'C_i + A_i'B_iC_i' + A_iB_iC_i$
    $= A_i \oplus B_i \oplus C_i$
- You can design full adder circuit *directly* from the above equations (requires 3 ANDs and 1 OR for C_{i+1} and 2 XORs for S_i)

Full Adder using 2 Half Adders

- A full adder can also be realized with two half adders and an OR gate, since C_{i+1} can also be expressed as:
- $C_{i+1} = A_iB_i + A_iB_i'C_i + A_i'B_iC_i$

  $= A_iB_i + (A_iB_i' + A_i'B_i)C_i$

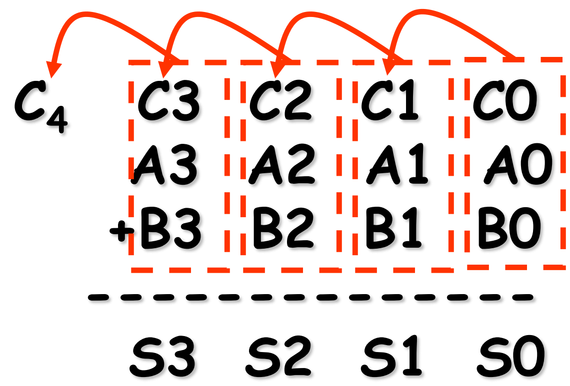
  $= A_iB_i + (A_i \oplus B_i)C_i$
- and $S_i = A_i \oplus B_i \oplus C_i$

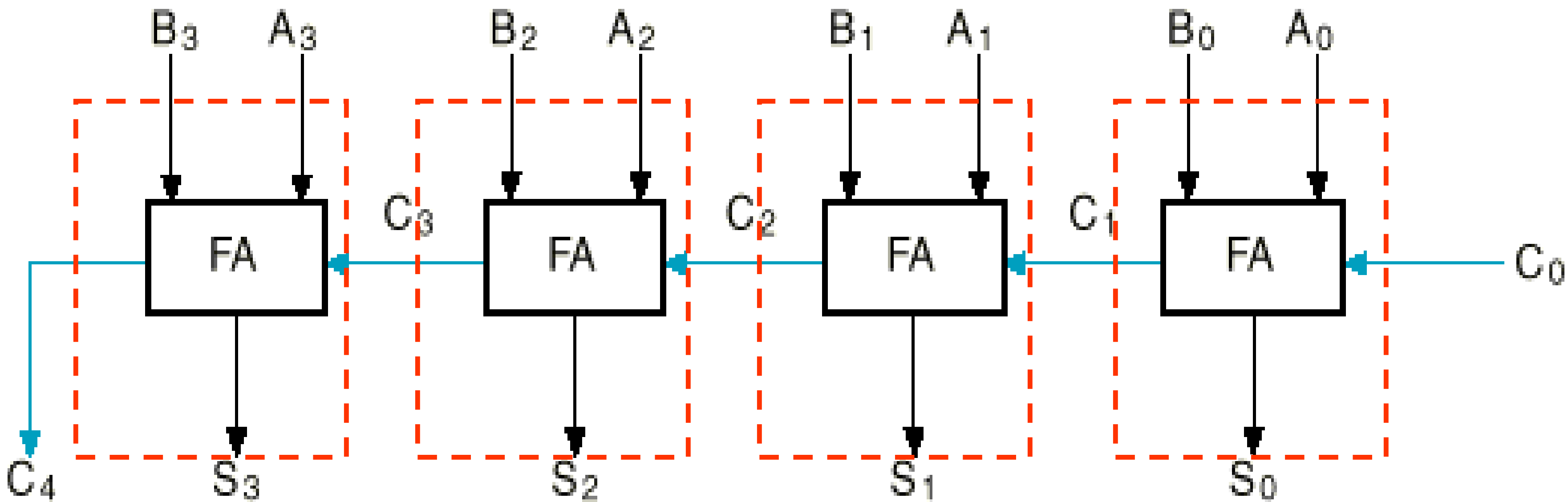


n-bit Ripple Carry Adder

- Constructed using ***n*** 1-bit full adder blocks in parallel.
- Cascade the full adders so that the carry out from one becomes the carry in to the next higher bit position.

Example: 4-bit Ripple Carry Adder





Thanks