Quaid-e-Awam University of Engineering, Science and Technology

بِسْمِ اللّٰهِ الرَّحْمٰنِ الرَّحِيْمِ

In the name of Allah, the Most Merciful, the Most Kind

Date: 05-10-2021

BCS 103
Digital Logic & Computer Architecture

Lecture 19 and 20

Department of Computer Science

IN THE LAST LECTURE

We have discussed

- **Simplification of Expressions**
- **From Boolean Expression to Circuit**
- **From Circuit to Boolean Expression**
- **Expression validity**

TODAY

We will discuss about

- **Sum of Product (minterm)**
- **Product of Sum (Maxterm)**
- **Karnaugh Map**

Review: Minterm

- A product term in which all the variables appear exactly once, either complemented or uncomplemented, is called a minterm
- A minterm represents exacly one combination of the binary variables in a truth table. It has the value of 1 for that combination and 0 for the others

X	Y	Z	Product Term	Symbol	m_0	m_1	m_2	m_3	m_4	m_5	m_6	m_7
0	0	0	$\overline{X}\,\overline{Y}\,\overline{Z}$	m_0	1	0	0	0	0	0	0	0
0	0	1	$\overline{X}\,\overline{Y}Z$	m_1	0	1	0	0	0	0	0	0
0	1	0	$\overline{X}Y\overline{Z}$	m_2	0	0	1	0	0	0	0	0
0	1	1	$\overline{X}YZ$	m_3	0	0	0	1	0	0	0	0
1	0	0	$X\overline{Y}\,\overline{Z}$	m_4	0	0	0	0	1	0	0	0
1	0	1	$X\overline{Y}Z$	m_5	0	0	0	0	0	1	0	0
1	1	0	$XY\overline{Z}$	m_6	0	0	0	0	0	0	1	0
1	1	1	XYZ	m_7	0	0	0	0	0	0	0	1

Table 2-6 Minterms for Three Variables

Review: Maxterm

- A sum term in which all the variables appear exactly once, either complemented or uncomplemented, is called a maxterm
- A maxterm represents exacly one combination of the binary variables in a truth table. It has the value of 0 for that combination and 1 for the others

X	Y	Z	Sum Term	Symbol	M_0	M_1	M_2	M_3	M_4	M_5	M_6	M_7
0	0	0	$X+Y+Z$	M_0	0	1	1	1	1	1	1	1
0	0	1	$X+Y+\overline{Z}$	M_1	1	0	1	1	1	1	1	1
0	1	0	$X+\overline{Y}+Z$	M_2	1	1	0	1	1	1	1	1
0	1	1	$X+\overline{Y}+\overline{Z}$	M_3	1	1	1	0	1	1	1	1
1	0	0	$\overline{X}+Y+Z$	M_4	1	1	1	1	0	1	1	1
1	0	1	$\overline{X}+Y+\overline{Z}$	M_5	1	1	1	1	1	0	1	1
1	1	0	$\overline{X}+\overline{Y}+Z$	M_6	1	1	1	1	1	1	0	1
1	1	1	$\overline{X}+\overline{Y}+\overline{Z}$	M_7	1	1	1	1	1	1	1	0

Table 2-7 Maxterms for Three Variables

- A minterm and maxterm with the same subscript are the complements of each other, i.e., $M_j = m'_j$

Review: Sum of Minterms

- A Boolean function can be represented algebraically from a given truth table by forming the logical sum of all the minterms that produce a 1 in the function. This expression is called a sum of minterms

(a) X	Y	Z	F	$\overline{F}$
0	0	0	1	0
0	0	1	0	1
0	1	0	1	0
0	1	1	0	1
1	0	0	0	1
1	0	1	1	0
1	1	0	0	1
1	1	1	1	0

$F = X'Y'Z' + X'YZ' + XY'Z + XYZ$

$= m_0 + m_2 + m_5 + m_7$

$F(X,Y,Z) = \Sigma m(0,2,5,7)$

Review: Product of Maxterms

- A Boolean function can be represented algebraically from a given truth table by forming the logical product of all the maxterms that produce a 0 in the function. This expression is called a product of maxterms

(a) X	Y	Z	F	$\overline{F}$
0	0	0	1	0
0	0	1	0	1
0	1	0	1	0
0	1	1	0	1
1	0	0	0	1
1	0	1	1	0
1	1	0	0	1
1	1	1	1	0

$$F = (X+Y+Z')(X+Y'+Z')(X'+Y+Z)(X'+Y'+Z)$$

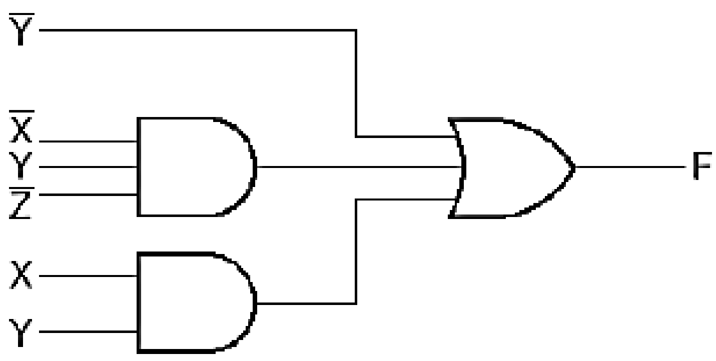
$$= M_1 . M_3 . M_4 . M_6$$

$$F(X,Y,Z) = \prod M(1,3,4,6)$$

- To convert a Boolean function F from SoM to PoM:
  - Find F' in SoM form
  - Find F= (F')' in PoM form

Review: Sum-of-Products

- The sum-of-minterms form is a standard algebraic expression that is obtained from a truth table
- When we simplify a function in SoM form by reducing the number of product terms or by reducing the number of literals in the terms, the simplified expression is said to be in Sum-of-Products form
- Sum-of-Products expression can be implemented using a two-level circuit



$$F = \Sigma m(0,1,2,3,4,5,7) \qquad \text{(SoM)}$$
$$= Y' + X'YZ' + XY \qquad \text{(SoP)}$$

Fig. 2-5 Sum-of-Products Implementation

Review: Product-of-Sums

- The product-of-maxterms form is a standard algebraic expression that is obtained from a truth table
- When we simplify a function in PoM form by reducing the number of sum terms or by reducing the number of literals in the terms, the simplified expression is said to be in Product-of-Sums form
- Product-of-Sums expression can be implemented using a two-level circuit

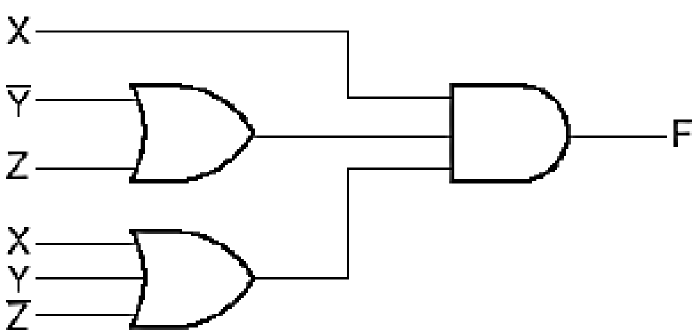


Fig. 2-7 Product-of-Sums Implementation

$$F = \prod M(0,2,3,4,5,6) \qquad \text{(PoM)}$$

$$= X(Y' + Z)(X + Y + Z') \qquad \text{(PoS)}$$

The Karnaugh Map

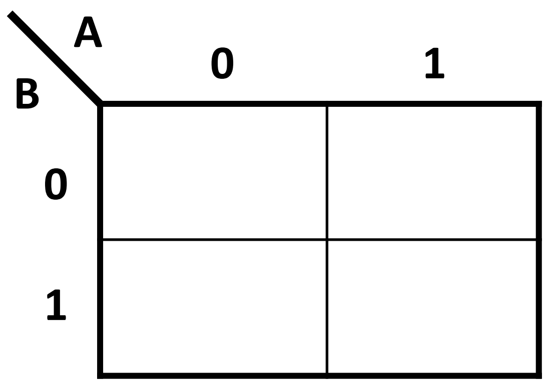
- Feel a little difficult using Boolean algebra laws, rules, and theorems to simplify logic?
- A K-map provides a systematic method for simplifying Boolean expressions and, if properly used, will produce the simplest SOP or POS expression possible, known as the <u>minimum expression</u>.

Karnaugh Maps

- Karnaugh maps provide an alternative way of simplifying logic circuits.
- Instead of using Boolean algebra simplification techniques, you can transfer logic values from a Boolean statement or a truth table into a Karnaugh map.
- The arrangement of 0's and 1's within the map helps you to visualise the logic relationships between the variables and leads directly to a simplified Boolean statement.

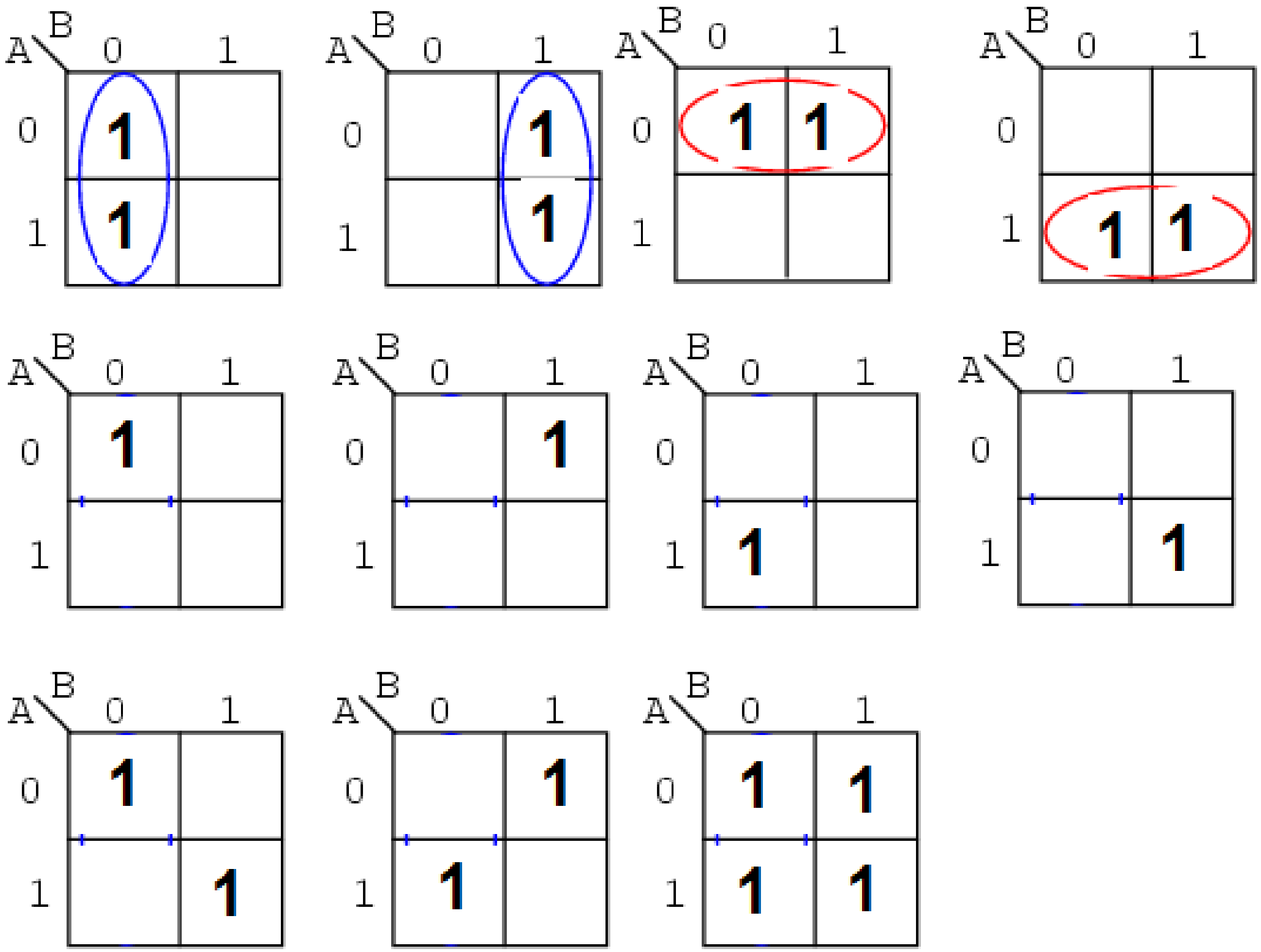
Karnaugh Maps

- 2 variables Karnaugh map



Karnaugh Maps

- 2 variables Karnaugh map



Thanks