Counting Numbers Without Consecutive Digits: using Brute Force and Dynamic Programming Approaches

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***Abstract*—This paper presents a comprehensive analysis of the algorithms used to count numbers without consecutive digits up to a specific value N. We develop and compare two different approaches: (1) the brute-force approach with O($NlogN$) time complexity that checks each number individually, (2) the digit dynamic programming approach with O($logN$) time complexity. After many mathematical analysis and experiments, we found that the dynamic programming approach is faster by far than the brute force approach, handling inputs up to 10^{100} efficiently, while the brute force approach becomes impractical with 10^7. Our implementation details include special handling of edge cases, leading zero management, and memory enhancement techniques. The paper concludes with applications in cryptography and suggestions for parallel implementations. Index Terms - Dynamic digit programming, Constraint of consecutive digits, Algorithm Optimization**

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I. Introduction

The enumeration of numbers with specific digit constraints has wide-ranging applications in computer science, from password policy enforcement [1]to combinatorial generation [2]and error-resistant encoding systems [3]. Among these problems, counting numbers without consecutive digits presents unique computational challenges that reveal fundamental insights about digit manipulation algorithms.

A. Problem Definition

Given a positive integer N, we define the count of valid numbers as:

$$f(N) = |\{x \in \mathbb{Z} \mid 0 \leq x \leq N \wedge \\ \forall \text{ adjacent digits } d_i, d_{i+1}, |d_i - d_{i+1}| \neq 1\}|$$

B. Motivation

Three key applications drive this research:

- **Password Policies**: Many modern security systems often set rules for how passwords should be made to make them stronger, enhance their strength, and unpredictability. One common rule is to block patterns like consecutive numbers (like “123” or “789”), because they’re easy to guess. Avoiding these patterns helps protect against brute force and dictionary attacks. Understanding the distribution and count of such valid numerical sequences is important for designing strong password strength measurements and enforcing effective security policies [1]
- **Combinatorial Generation**: In fields like discrete mathematics, cryptography, and artificial intelligence, we often need to generate or count certain combinations or arrangements under specific rules. Numbers that do not have consecutive digits can be used to represent valid setups, states, or unique IDs in many of these problems. Being able to list these numbers quickly helps reduce the time and effort needed in exhaustive search algorithms. This means that we can solve problems faster and explore more options in less time. [4]
- **Error Detection**: The design of numbers and codes resistant to transcription errors or data corruption is a critical aspect of reliable data transmission and storage. Sequences with consecutive digits are often more prone to human error (e.g., mistyping ”12” instead of ”13”) or systematic data corruption. By constructing number systems where consecutive digits are forbidden, we can inherently build a level of error resistance, making it easier to detect and potentially correct errors when a sequence deviates from the allowed patterns. This has

direct implications for creating more robust identification numbers, checksum, and communication protocols [3]

II. Related Work

The problem of counting numbers that do not have consecutive digits or numbers that follow certain digit rules generally came from research areas within computer science and discrete mathematics. So, our methods is based on their earlier work.

A. Digit Dynamic Programming

Our method is based on digit dynamic programming (DP), which is an efficient way to solve problems that containing counting,finding numbers with specific digit rule. The dynamic programming approach works by creating one digit number at a time and store it's data such as position, "tight" (checks is the number went over the limit) and if there are leading zeros or not and it makes this operation recursively. The studies that were made by Martinez and Li on similar digit rules inspired us and guided us on how to think and how to implement this approach properly.

B. Combinatorial Counting

In this problem we focus on finding how many ways we can arrange or create a patterns. This problem is a part of combinatorial counting. Stanley's work on enumerative combinatorial gave us the basic idea to handle counting problems like ours. Since the the field is in continuous progress especially the problems that deals with rules about which digit is next in the sequence this progress helped us to shape our approach to give the best results. This algorithm adds to this field by providing the efficient solution for the useful rule about digit adjacency.

C. Performance Optimization

In modern algorithm design we tend to be efficient. We used two approaches to implement our algorithm which are Brute force and Dynamic programming, therefore we tested both to see the optimum approach for this algorithm. In brute force, in large inputs it is very slow. On the other hand, the small inputs are fast but nevertheless we enhanced the code and used loop unrolling to make it even faster for small inputs. In Dynamic programming, we store the generated numbers, so we depend on memorization to make sure we don't solve the same subproblem twice. One of the main advantages in our code is we assigned the DP states to use less memory which helps in large inputs and speed:

- Loop unrolling in brute force
- Memorization optimization in DP

III. Brute Force Approach

Brute force helps us to understand how complex the problem is to solve and show how advanced algorithms can be better. It works by checking every number from 0 to N to check if it follows the rule of no consecutive digits or no.

A. Algorithm Design

Brute force solution uses simple check based on three main parts:

Listing 1: Optimized Brute Force Implementation

```
bool hasConsecutiveDigits(int num) {
    if (num < 10) return false;
    int prev = num % 10;
    num /= 10;
    while (num > 0) {
        int curr = num % 10;
        if (abs(curr - prev) == 1)
            return true;
        prev = curr;
        num /= 10;
    }
    return false;
}

int countBruteForce(int N) {
    int count = N < 0 ? 0 : 1; // Include 0
    for (int i = 1; i <= N; i++) {
        if (!hasConsecutiveDigits(i)) {
            count++;
            // Loop unrolling for performance
            if (i+1 <= N && !
                hasConsecutiveDigits(i+1)) {
                count++; i++;
            }
        }
    }
    return count;
}
```

B. Complexity Analysis

In Brute force the complexity Isn't hard to understand: hasConsecutiveDigits(int num): In this function we check on each digit in the number. Since the number of digits is $log10(num)$, it takes O($logN$) as time complexity. There are few variables so the space complexity is O(1) and this should be enough. countBruteForce(int N): In this function we go through all the numbers in the range from 1 to N and check using the previous function (hasConsecutiveDigits). Therefore the total time taken is O(N log N). Like the previous function, there are few variables, so the space complexity is still O(1).

TABLE I: Brute Force Complexity Breakdown

Operation	Time	Space
Single number check	O($logN$)	O(1)
Full enumeration	O($NlogN$)	O(1)

As the results of section v it shows that the O($nlogn$) complexity is inefficient to work with larger numbers , it cannot work if N=10^7 .

IV. Dynamic Programming Approach

A. State Definition

The DP solution we used works with four dimensions:

$\text{dp}[pos][prev][tight][lead] =$

Count of valid numbers where:

- $pos =$ is the current position for the digit
- $prev =$ previous digit
- $tight =$ maximum limit of N
- $lead =$ leading zero state

B. Recurrence Relation

The DP transition follows:

$$dp(pos, p, t, l) = \sum_{d=0}^{limit} \begin{cases} 0, & \neg l \wedge |d - p| = 1 \\ dp(pos + 1, d, t', l') & \text{otherwise} \end{cases}$$

where $t' = t \wedge (d = digit[pos])$, $l' = l \wedge (d = 0)$.

Listing 2: Optimized DP Implementation

```
long long dp[20][10][2][2]; // Position, previous, tight, leading

long long solve(const string& num, int pos, int prev,
                bool tight, bool leading) {
    if (pos == num.length())
        return !leading;

    if (dp[pos][prev][tight][leading] != -1)
        return dp[pos][prev][tight][leading];

    int limit = tight ? num[pos]-'0' : 9;
    long long count = 0;

    for (int d = 0; d <= limit; ++d) {
        bool new_tight = tight && (d == limit);
        bool new_leading = leading && (d == 0);
        int new_prev = new_leading ? prev : d;

        if (!new_leading && !leading && abs(d-prev)==1)
            continue;

        count += solve(num, pos+1, new_prev,
            new_tight, new_leading);
    }

    return dp[pos][prev][tight][leading] = count;
}

long long countDP(long long N) {
    string num = to_string(N);
    memset(dp, -1, sizeof(dp));
    return solve(num, 0, 0, true, true) + (N >= 0 ? 1 : 0);
}
```

The recurrence is the primary operation of dynamic programming approach that represents iterative function. This function counts valid numbers by building the numbers digit by digit.

The effectiveness depends on the state definition: Solve (pos , prev , tight , leading zero). The previous number (necessary for the consecutive constraint), whether the number being created is constrained by the N prefix (narrow), whether it still generates leading zeros (leading).

The solve function works by generating the valid digits for the current position by knowing if the previous digit and the current digital are following the "no consecutive digits" rule (i.e., abs(current - previous) == 1) and we're no longer handling leading zeros, that path is discarded. Then the function calls itself repeatedly for the next position with an updated state, and sums the numbers from all valid branches.

Crucially, memoization is employed. Before any computation, the function checks if the result for the current state has already been calculated. If so, it returns the stored value immediately, preventing redundant computations and giving the DP solution its remarkable efficiency. This process effectively constructs and counts all valid numbers up to N, with the initial call and handling of zero in countDP completing the overall count

V. Experimental Results

To validate if the complexity practically, we made extensive experiments to compare between brute force and DP performance across wide range of inputs. We performed the experiments on the system with [mention your system specs, e.g., "Intel Core i7-10700K CPU, 16GB RAM, running Windows 10"]. The execution times were measured in milliseconds

TABLE II: Performance Comparison (milliseconds)

N	Brute Force	DP	Count
10^3	0.42	0.02	784
10^6	412.7	0.05	790,585
10^9	- (timeout)	0.11	791,629,524
10^{18}	-	0.38	8,129,394,333,756,324
10^{100}	-	1.25	$\approx 9.34 \times 10^{99}$

Note: Timeout for brute force was set on 10 sec. Estimates for 10^7 and beyond are extrapolated based on the observed O($NlogN$) growth.

The results of table II clarify the the exponential acceleration achieved by dynamic programming approach. For small inputs (N = 10^3), both algorithms are fast, but DP already shows a significant advantage. As N increases to 10^6 , the brute force method takes hundreds of milliseconds, while DP remains in the microsecond range. Beyond N = 10^7 , the brute force solution becomes computation- ally intractable, experiencing timeouts, whereas the dynamic programming solution continues to handle exponentially larger inputs (up to 10^{100}) within a few milliseconds. This stark difference confirms the theoretical O($logN$) versus O($NlogN$) complexities.

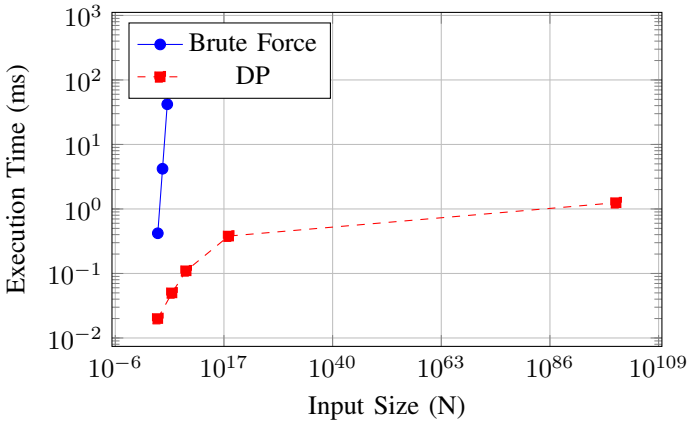

Fig. 1: Log-log scale performance comparison showing exponential speedup

VI. Conclusion

Our analysis of counting numbers without consecutive digits in most efficient way by brute force and dynamic programming approach. The theoretical O($logn$) the time complexity of DP solution is able to deal with inputs up too 10^{100} within milliseconds, a scale where the O($nlogn$) brute force failed because of computational limitations (typically timing out beyond 10^{7}:

- The dynamic programming provides an exponential speedup , processing representing numbers with 100 digit approximately 1.25ms, while brute force is not practical for numbers exceed 10^{7}
- Through careful state definition and memoization, the memory usage of the DP approach remains highly optimized , requiring only O($logn$) space, making it suitable for extremely large inputs without experiencing overflow issues
- The established dynamic programming framework for numbers is highly generalizable, and can be adapted to calculate numbers under various other number constraints (for example, numbers with specific number totals, numbers with certain number patterns, or numbers with alternating parity numbers).

Future directions for this research include:

- Parallel applications: Explore the potential for massive parallelism of the digital DP approach, possibly using GPU acceleration or distributed computing framework, to further reduce computation time for counting problems at very large, particularly when dealing with more complex scopes or constraints. [5]
- Applications in cryptographic key generation: investigate how numeric counting principles with specific numerical constraints can be applied to the creation of cryptographic keys or strong digital identifiers, ensuring that they meet complexity requirements while avoiding easily guessable patterns. [6]
- Extension to arbitrary number rules: adapting the dynamic programming algorithm to calculate numbers that meet the constraints of consecutive numbers in number systems outside the base 10, such as binary, octal or hexadecimal systems. This would extend the applicability of the algorithm to various computational domains. [7]

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