The given default parameters are listed as,

$$m_1 := 1 \cdot kg \qquad m_2 := 0.1 \cdot kg \qquad c_1 := 0.01 \cdot \frac{N \cdot s}{m} \qquad k_1 := 1 \cdot \frac{N}{m} \qquad F_1 := 1 \cdot N$$

The mass, stiffness, damping and force matrixes are,

$$M := \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0.1 \end{bmatrix} kg \qquad F := \begin{bmatrix} F_1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} N$$

$$K(k_2) := \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \qquad C(c_2) := \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix}$$

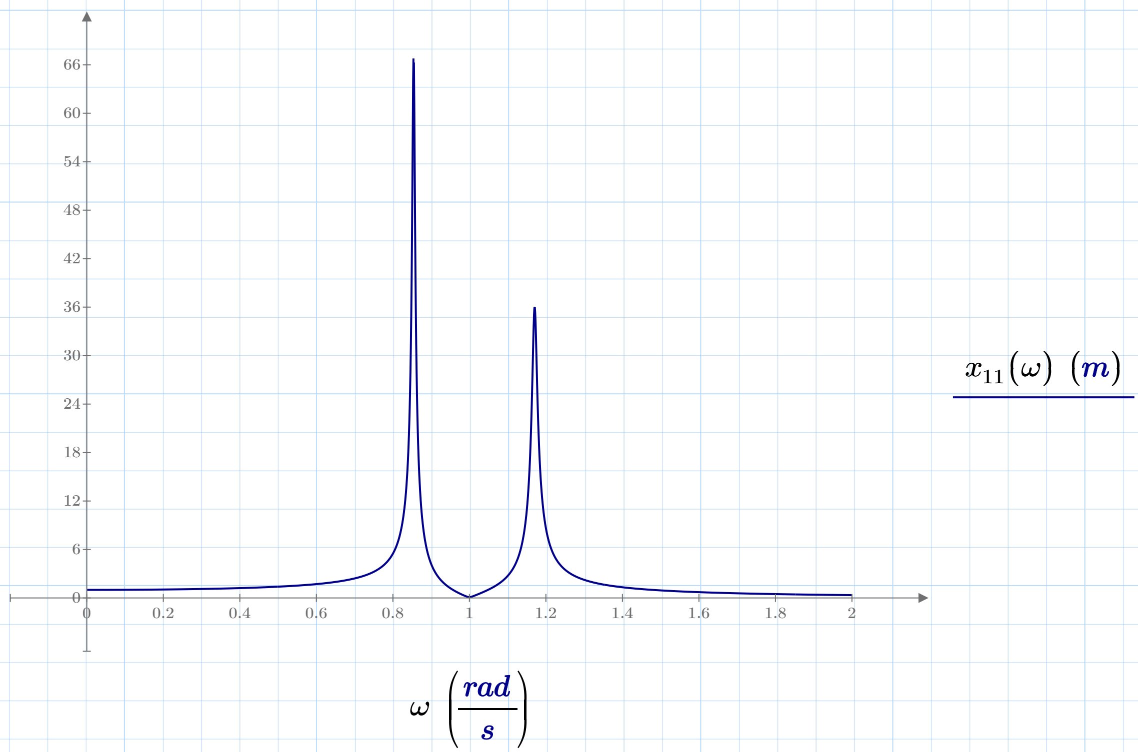
The response of the main mass might be calculated as,

$$A(w, k_2, c_2) := -w^2 \cdot M + K(k_2) + 1\mathrm{i} \cdot w \cdot C(c_2) \qquad X_1(w, k_2, c_2) := \left| \left(A(w, k_2, c_2)^{-1} \cdot F \right)_0 \right|$$

By using the second damping and stiffness parameters, the response might be determined and plotted as,

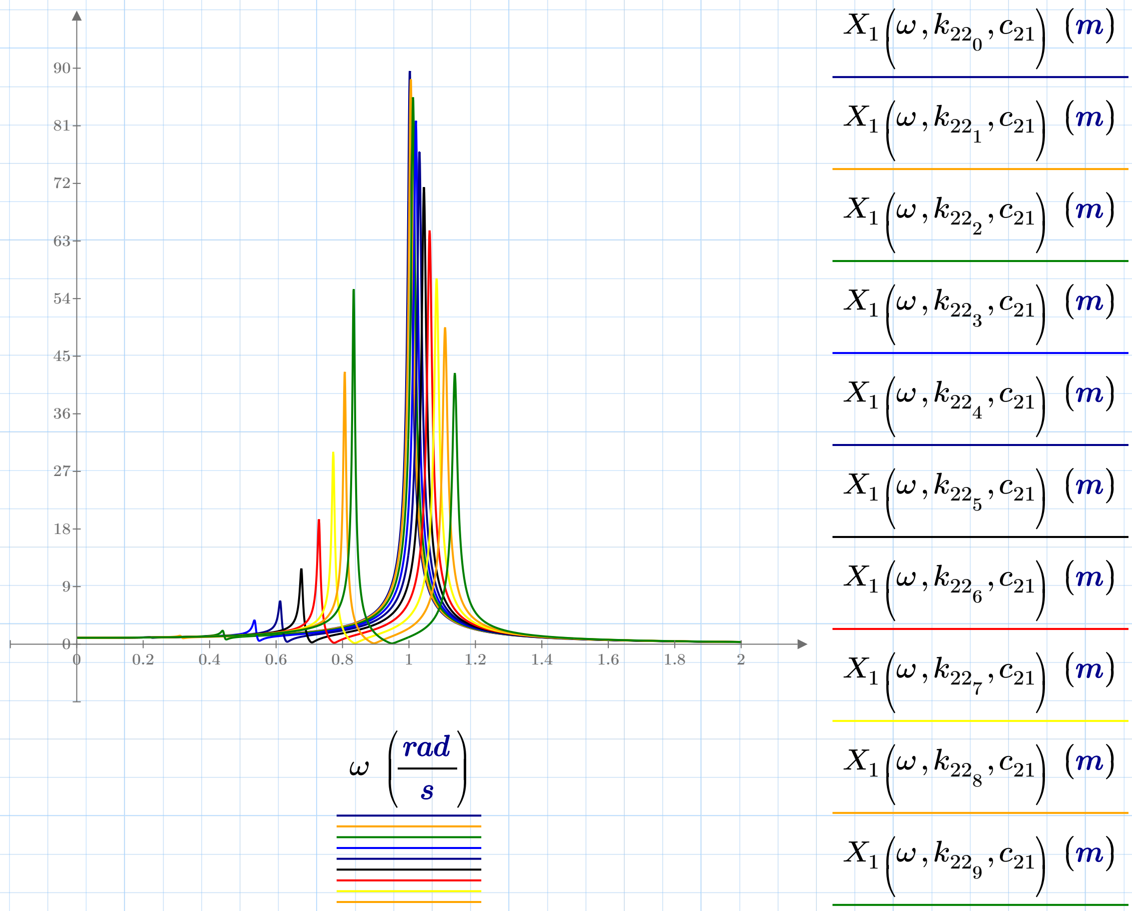
$$k_{21} := 0.1 \cdot \frac{N}{m} \qquad c_{21} := 0.001 \cdot \frac{N \cdot s}{m}$$

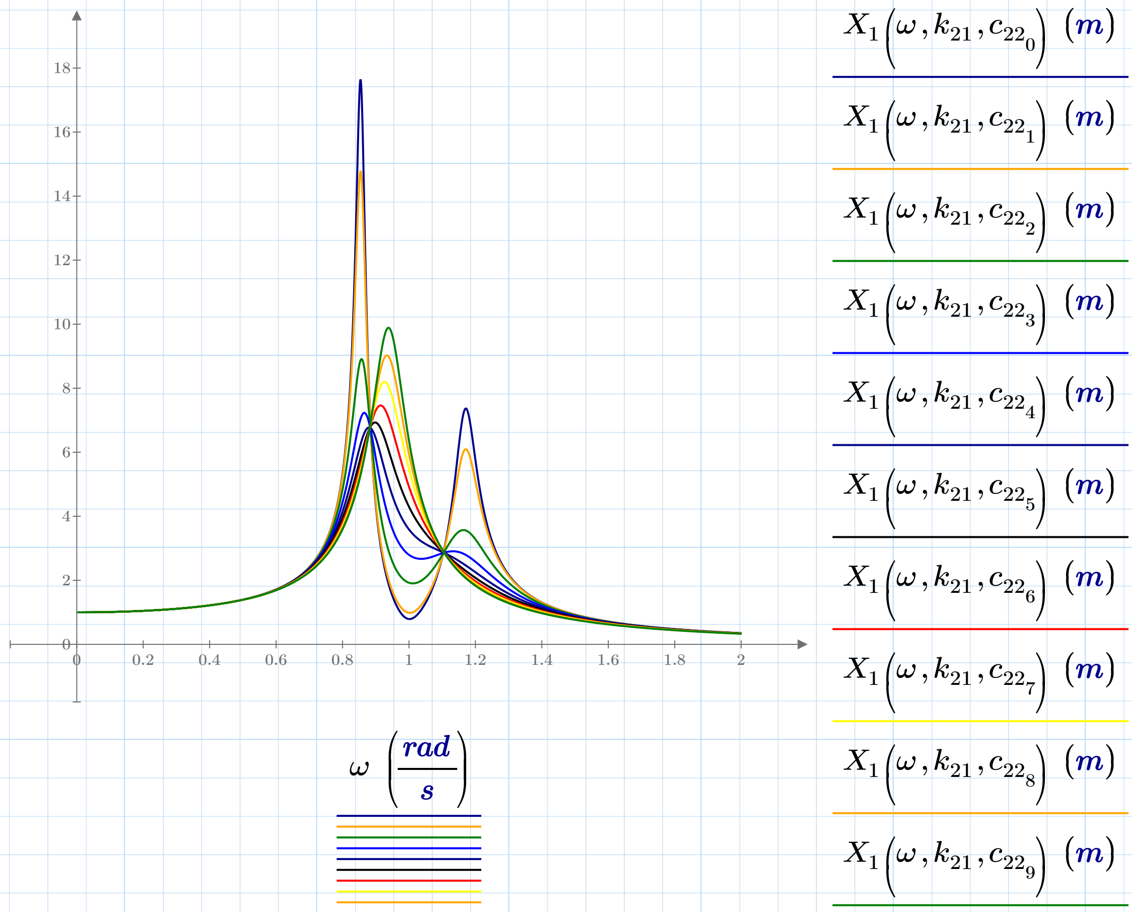
$$x_{11}(w) := X_1(w, k_{21}, c_{21}) \qquad \omega := 0, 0.001 \cdot \frac{rad}{s} .. \pi \cdot \frac{rad}{s}$$



For variable damping and stiffness values,

$$k_{22} := \begin{bmatrix} 0.005 \\ 0.01 \\ 0.02 \\ 0.03 \\ 0.04 \\ 0.05 \\ 0.06 \\ 0.07 \\ 0.08 \\ 0.09 \\ 0.1 \end{bmatrix} \cdot \frac{N}{m} \qquad c_{22} := \begin{bmatrix} 0.008 \\ 0.01 \\ 0.02 \\ 0.03 \\ 0.04 \\ 0.05 \\ 0.06 \\ 0.07 \\ 0.08 \\ 0.09 \\ 0.1 \end{bmatrix} \cdot \frac{N \cdot s}{m}$$





The integral of the curves,

$$I\left(w_1, w_2, k, c\right) := \int_{w_1}^{w_2} X_1\left(w, k, c\right) \mathrm{d}w$$

For the given values, the integral might be calcuated as,

$$\omega_1 := 0.7 \cdot \frac{rad}{s} \qquad \omega_2 := 1.2 \cdot \frac{rad}{s} \qquad k_{23} := 0.1 \cdot \frac{N}{m} \qquad c_{23} := 0.001 \cdot \frac{N \cdot s}{m}$$

$$I_d := I\left(\omega_1, \omega_2, k_{23}, c_{23}\right) = 3.497 \ \frac{m}{s}$$

The maximum peak of the curves,

$$\omega_i := 0.9 \cdot \frac{rad}{s} \qquad f(X) := \mathbf{maximize}\left(X, \omega_i\right)$$

For default configuration,

$$f_{11} := f\left(x_{11}\right) = 0.854 \ \frac{1}{s} \qquad x_{11}\left(f_{11}\right) = 67.7 \ m$$

Minimum integral configuration,

$$I_v(k_2, c_2) := I(\omega_1, \omega_2, k_2, c_2)$$

Guess Values

$$k_i := 0.1 \cdot \frac{N}{m} \qquad c_i := 0.02 \cdot \frac{N \cdot s}{m}$$

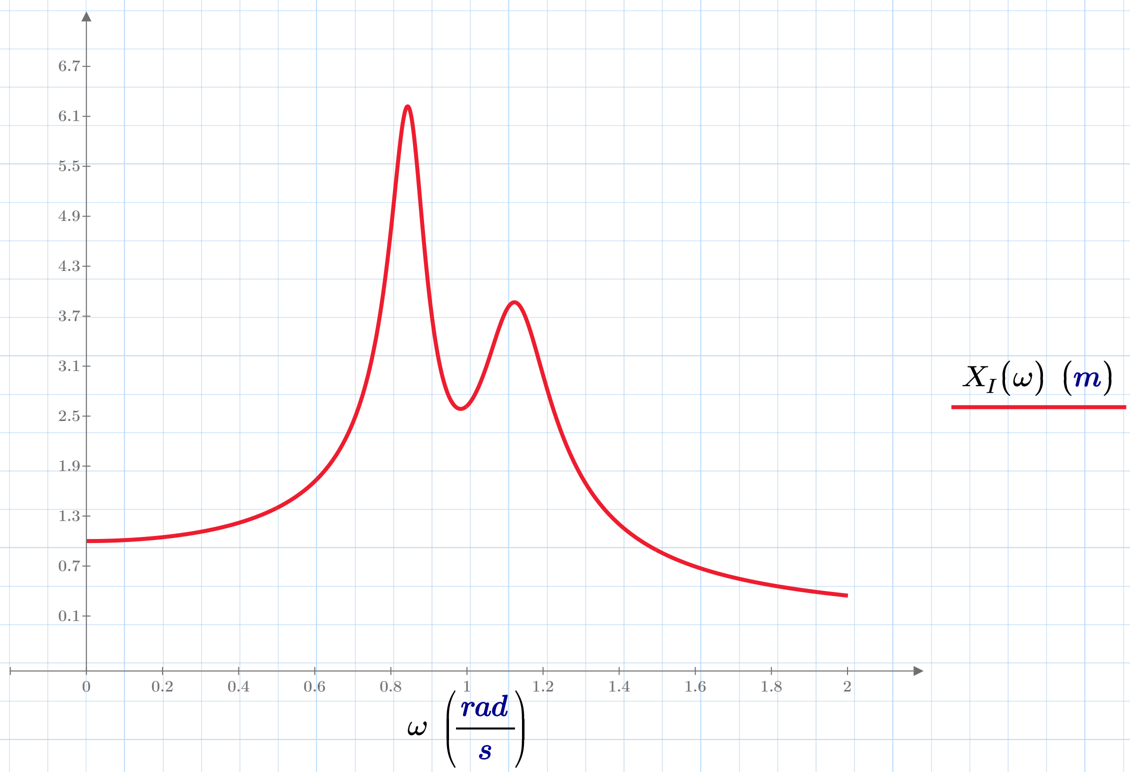
Constraints

$$k_i > 0 \cdot \frac{N}{m} \qquad c_i > 0 \cdot \frac{N \cdot s}{m}$$

Solver

$$\begin{bmatrix} k_{2i} \\ c_{2i} \end{bmatrix} := \mathbf{minimize}\left(I_v, k_i, c_i\right) = \begin{bmatrix} 0.09185 \ \frac{kg}{s^2} \\ 0.02425 \ \frac{kg}{s} \end{bmatrix}$$

$$X_I(w) := X_1(w, k_{2i}, c_{2i})$$



Minimum peak configuration,

$$X_{max}(\omega_i, k_i, c_i) := \textbf{maximize}\,(X_1, \omega_i, k_i, c_i)$$

$$\omega_n(k_2) := \sqrt{\text{eigenvals}\left(M^{-1} \cdot K(k_2)\right)} \qquad \omega_n(k_{21}) = \begin{bmatrix} 1.171 \\ 0.854 \end{bmatrix} \frac{rad}{s}$$

$$P_{max}(k_2, c_2) := \left\| \begin{array}{l} X_t(w) \leftarrow X_1(w, k_2, c_2) \\ \omega_1 \leftarrow \omega_n(k_2)_0 \\ \omega_1 \leftarrow \textbf{maximize}\,(X_t, \omega_1) \\ \omega_2 \leftarrow \omega_n(k_2)_1 \\ \omega_2 \leftarrow \textbf{maximize}\,(X_t, \omega_2) \\ X_{max} \leftarrow \max\left(X_t(\omega_1), X_t(\omega_2)\right) \\ \text{return } X_{max} \end{array} \right.$$

$$\begin{bmatrix} k_{2p} \\ c_{2p} \end{bmatrix} := \textbf{minimize}\,(P_{max}, k_{21}, c_{21}) = \begin{bmatrix} 0.08235 \dfrac{kg}{s^2} \\ 0.03314 \dfrac{kg}{s} \end{bmatrix} \qquad X_P(w) := X_1(w, k_{2p}, c_{2p})$$