Department of Systems and Biomedical Engineering

Cairo University

SBE 405 Medical Instrumentation IV: Ultrasound Imaging

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Wave Equation

The acoustic pressure p satisfies the three-dimensional wave Equation:

$$\frac{\partial^2 u}{\partial t^2} = c^2 \nabla^2 u$$

Or

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) p(x,y,z,t) = \frac{1}{c^2} \frac{\partial^2 p(x,y,z,t)}{\partial t^2}$$

It is a linear equation. When no shear stress is presented and only compression (longitudinal) wave in z direction is considered, this can be reduced to:

$$\frac{\partial^2 p(z,t)}{\partial z^2} = \frac{1}{c^2} \frac{\partial^2 p(z,t)}{\partial t^2}$$

Wave Equation

Solution form f(z±ct)

where the negative sign indicates a wave traveling in the +z direction and the positive sign indicates a wave traveling in the –z direction.

The sinusoidal solution for this equation is

$\overset{\mp}{p}(z,t) = \cos k(z \pm tc)$ (pressure) $W^{\mp}(z,t) = W_0 e^{j(\omega t \pm kz)}$ (displacement)

Note that *ω=2πf (angular frequency), and k= ω/c (wave number)*

Acoustic Impedance

The acoustic impedance of a medium is determined by its density (ρ) and stiffness (K).

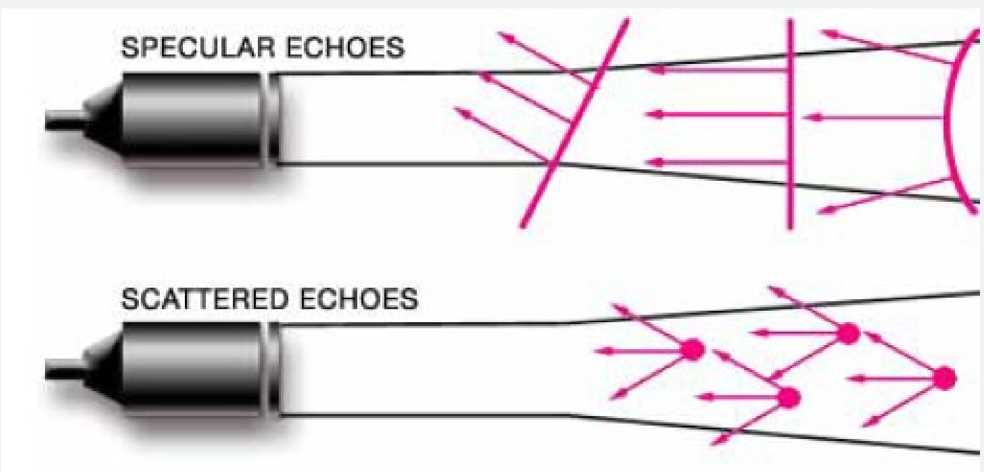
$$z = \sqrt{\rho k} \quad \text{or} \quad z = \rho c \quad \mathbf{kg\ m^{-2}\ s^{-1}},$$

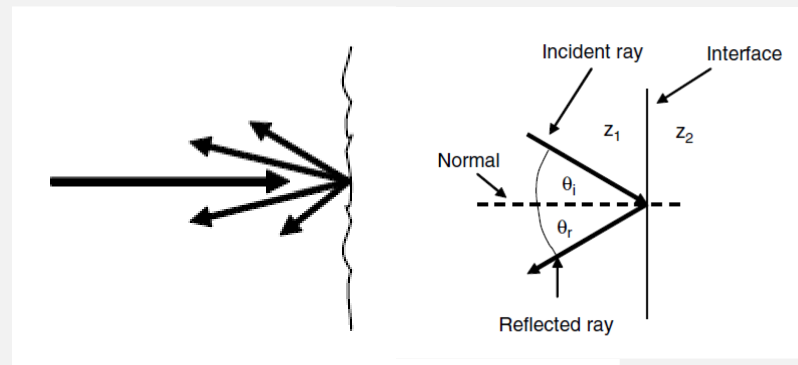
Table 2.3 Values of acoustic impedance.

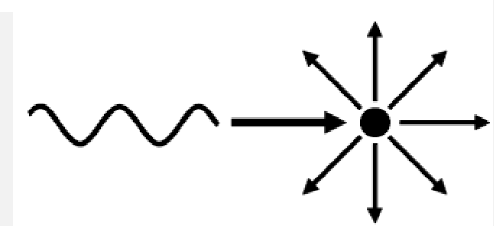
Material	z (kg m^{-2} s^{-1})
Liver	1.66×10^6
Kidney	1.64×10^6
Blood	1.67×10^6
Fat	1.33×10^6
Water	1.48×10^6
Air	430
Bone	6.47×10^6

Reflection and Scattering

- **Specular echoes** originate from relatively large, regularly shaped objects with smooth surfaces. These echoes are relatively intense and angle dependent. (i.e. valves)
- **Scattered echoes** originate from relatively small, weakly reflective, irregularly shaped objects are less angle dependant and less intense. (i.e.. blood cells)



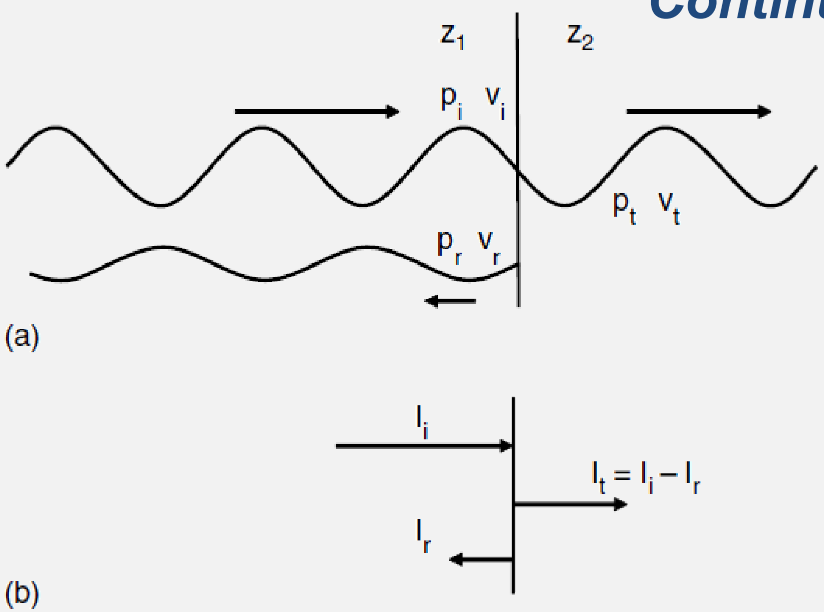




Reflection and Scattering

- When sound wave travels through the interface between two tissues of different z, part of the wave is transmitted and the other part is reflected.
- The amplitudes of the transmitted and reflected waves depend on the change in z.

Continuity of P and normal component of V



$$P_i + P_r = P_t$$

$$v_t + v_r = v_i$$

v here is particle velocity to be consistent with the book

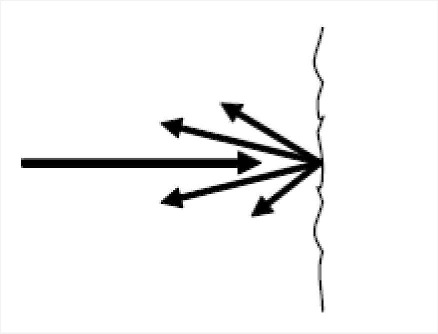
$$\frac{p_r}{p_i} = \frac{z_2 - z_1}{z_2 + z_1}$$

Scattering

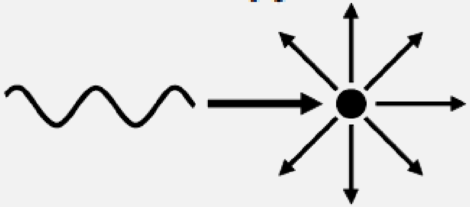
➢The total ultrasound power scattered by a very small target is much less than that for a large interface.

➢ For small targets (*d* << *λ*), *scattered* power is proportional to the sixth power of the size *d* and inversely proportional to the fourth power of the wavelength λ:

$$W_s \propto \frac{d^6}{\lambda^4} \propto d^6 f^4$$

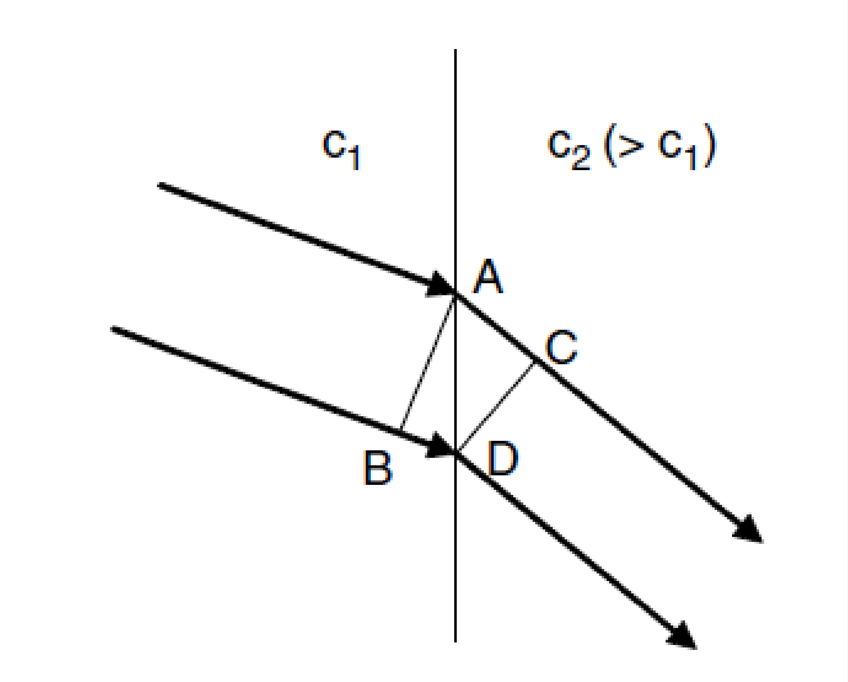


Diffuse Reflection



This frequency dependence is often referred to as Rayleigh scattering .

Refraction



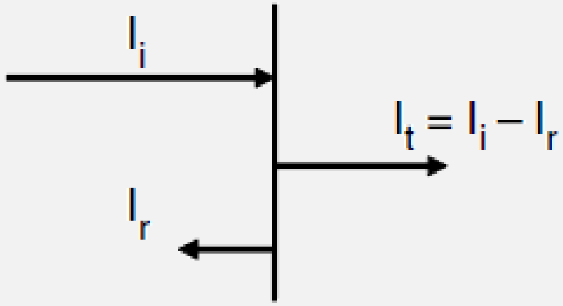
Snell's law

$$\frac{\sin\theta_i}{\sin\theta_t} = \frac{c_1}{c_2}$$

Acoustic Intensity

- The energy transported in an ultrasound wave is usually characterized by an acoustic intensity *I.*
- The acoustic intensity can be calculated in terms of the sound pressure as follows:

$$I = \frac{P^2}{Z} \quad (\text{W·cm−2})$$



$$\frac{I_r}{I_i} = R_i = R_A^2 = \left(\frac{z_2 - z_1}{z_2 + z_1}\right)^2$$ Intensity reflection coefficient

$T_i = 1 - R_i$ Intensity transmission coefficient

Normal Incidence

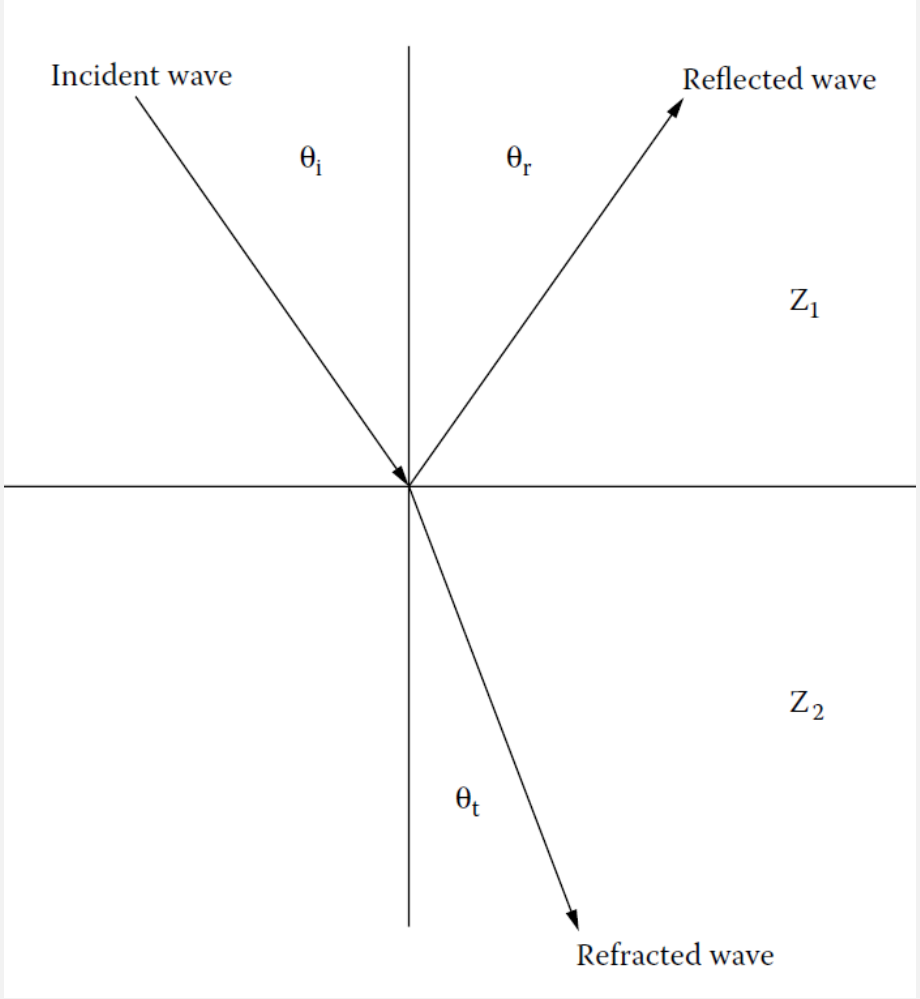
For Incidence Angle ≠ 90º

$$R = \frac{p_r}{p_i} = \frac{Z_2 \cos\theta_i - Z_1 \cos\theta_t}{Z_2 \cos\theta_i + Z_1 \cos\theta_t}$$

$$T = \frac{p_t}{p_i} = \frac{2Z_2 \cos\theta_i}{Z_2 \cos\theta_i + Z_1 \cos\theta_t}$$

$$\frac{I_r}{I_i} = \left(\frac{Z_2 \cos\theta_i - Z_1 \cos\theta_t}{Z_2 \cos\theta_i + Z_1 \cos\theta_t} \right)^2$$

$$\frac{I_t}{I_i} = \frac{4Z_2 Z_1 \cos\theta_i}{(Z_2 \cos\theta_i + Z_1 \cos\theta_t)^2}$$



http://books.google.com.eg/books?id=Hwb5D60vb5IC&pg=PA54&dq=reflection+coefficient+ultrasound+cos&hl=en&sa=X&ei=TIeDUrClBMS80QWNuoH4DA&ved=0CCsQ6AEwAA#v=onepage&q=reflection%20coefficient%20ultrasound%20cos&f=true

Attenuation

- Attenuation means the loss of the signal's amplitude with increasing propagation distance.
- There are two major causes of attenuation

  1- Absorption: the conversion of acoustic energy into heat via viscosity, relaxation, heat conduction, etc.

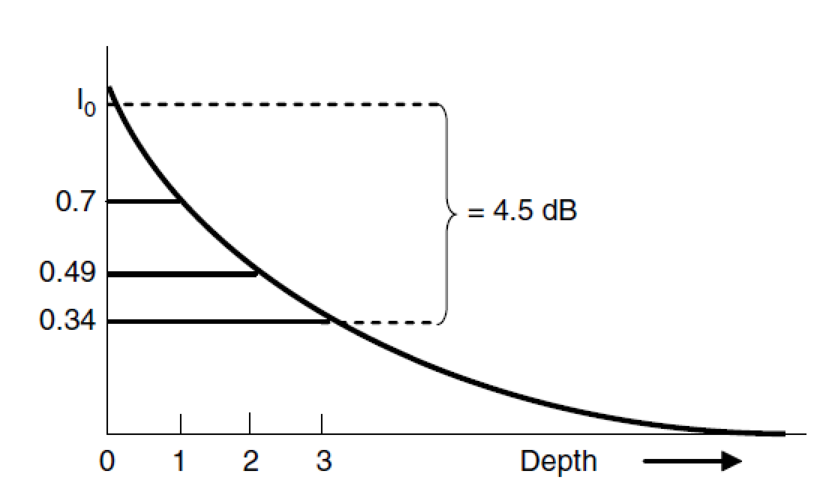
  2- Scattering: the conversion of the energy of the coherent, collimated beam into incoherent, divergent waves as a result of wave interaction with inhomogeneities in the material.

Absorption is the dominant mechanism for ultrasonic attenuation in biological tissues.

Attenuation

$$p(z) = p(z=0)e^{-\alpha z}$$

➢ Where P(z=0) is the pressure at z=0 and α is the pressure attenuation coefficient.



$$\alpha = \frac{1}{z}\ln\left[\frac{p(z=0)}{p(z)}\right]$$

➢The attenuation coefficient has a unit of nepers per centimeter and is expressed in units of decibels per centimeter

$$\alpha(\mathrm{dB/cm}) = 20(\log_{10}e)\alpha(\mathrm{np/cm}) = 8.686\alpha(\mathrm{np/cm})$$

dB ~ $20\log P_2/P_1$ in terms of pressure or $10\log I_2/I_1$ in terms of intensity

Attenuation

The attenuation coefficient of tissues, when expressed in dB cm^{-1} , increases approximately linearly with frequency.

Common unit for α is
dB cm^{-1} MHz^{-1}

Table 2.5 Values of attenuation for some human tissues.

Tissue	Attenuation (dB cm^{-1} MHz^{-1})
Liver	0.399
Brain	0.435
Muscle	0.57
Blood	0.15
Water	0.02
Bone	22

Ultrasound Wave Interference

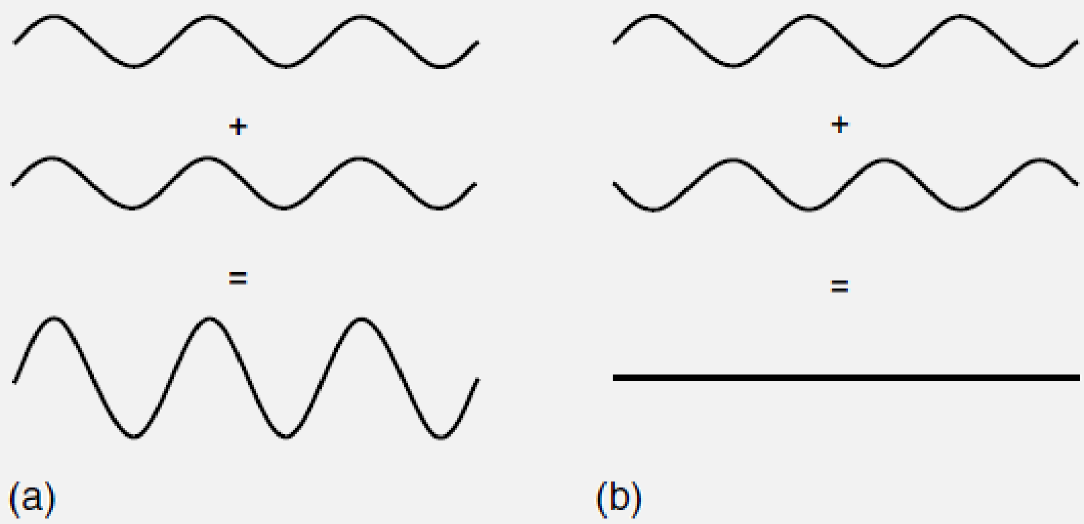


Fig. 2.12 The effects of two waves travelling through the same medium are added, i.e. the waves interfere with each other: (a) waves with the same phase interfere constructively (add); (b) waves with opposite phase interfere destructively (cancel).